Floquet topological phases protected by dynamical symmetry

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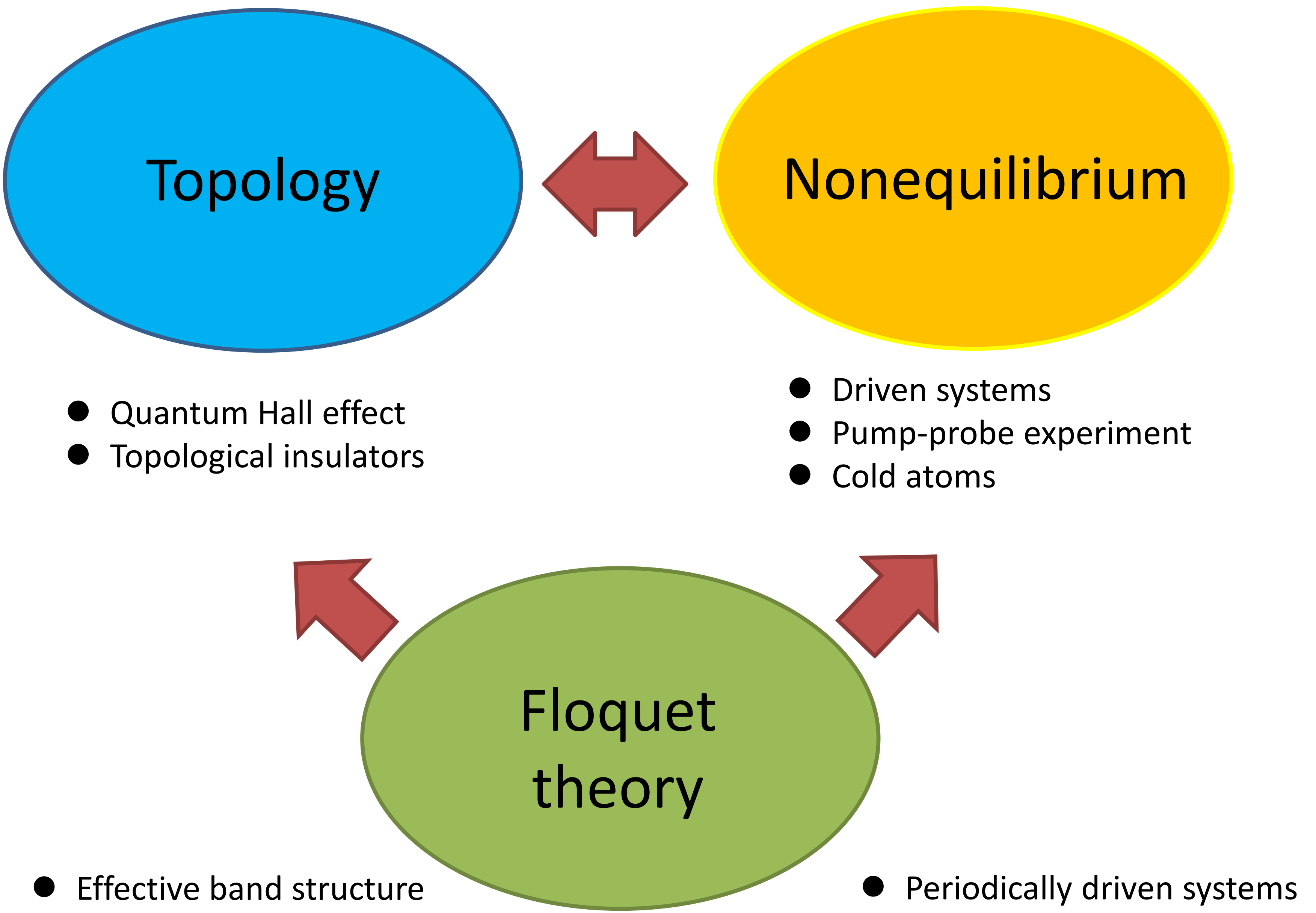

Topology
Nonequilibrium
Quantum Hall effect
Topological insulators
Driven systems
Pump-probe experiment
Cold atoms
Floquet theory
Effective band structure
Periodically driven systems

Plan of this talk

- Introduction
  - Floquet topological phases
  - Anomalous Hall state without Chern number

- Floquet topological phases in noninteracting systems
  - Time glide symmetry
  - Ten-fold way classification

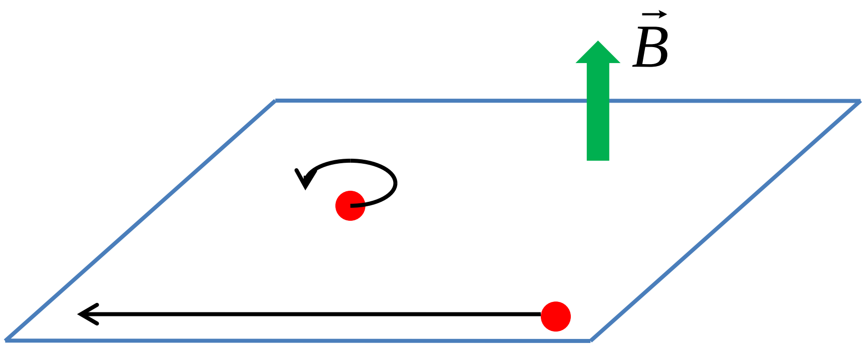
Morimoto, Po, Vishwanath, PRB (2017)

- Floquet topological phases in interacting systems
  - Group cohomology classification
  - 1D and 2D models

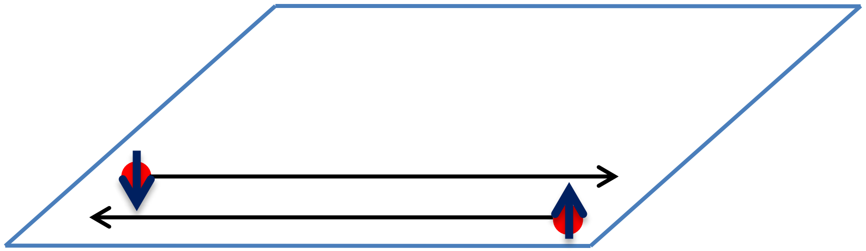
Potter, Morimoto, Vishwanath, PRX (2016)

Topological phases

Quantum Hall effect



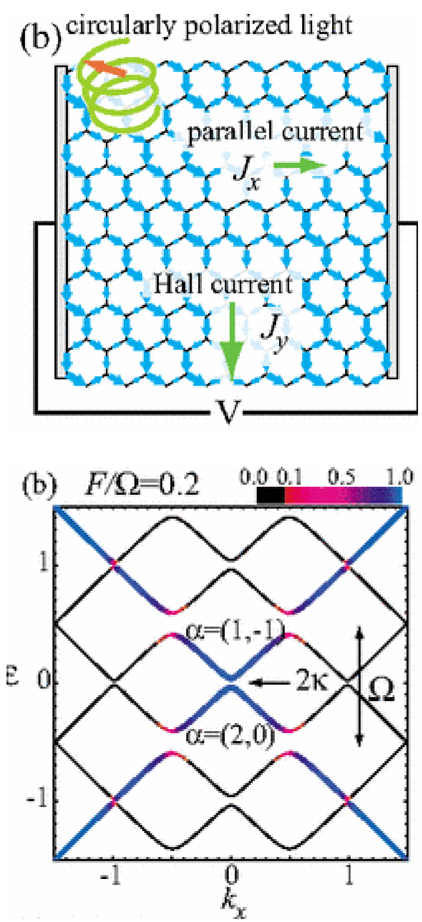
2D topological insulator



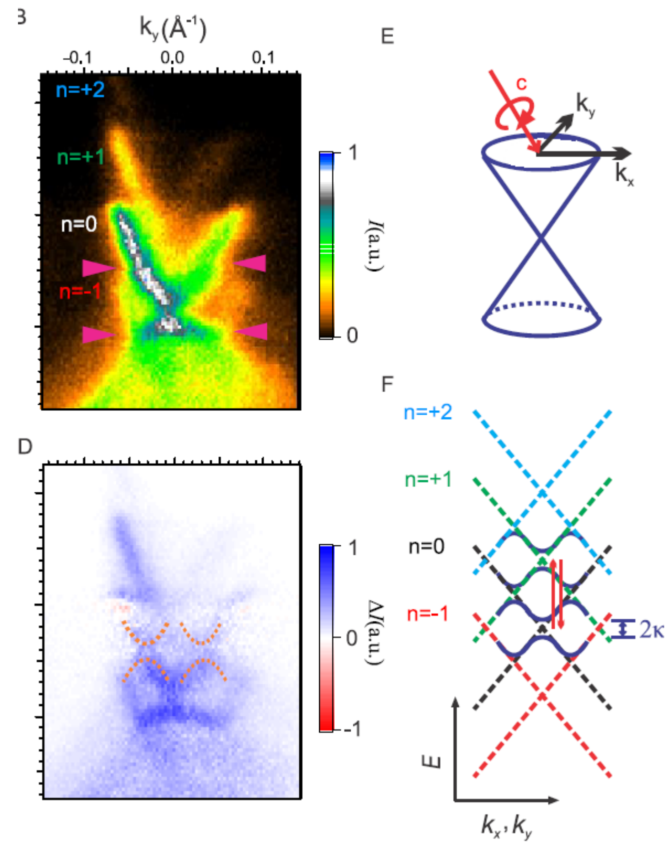
- Ground states with
  - Bulk excitation gap
  - Nontrivial gapless surface state

Topological phases in nonequilibrium states?

Dynamical Chern insulator with circularly polarized light



Oka, Aoki, PRB (2009)



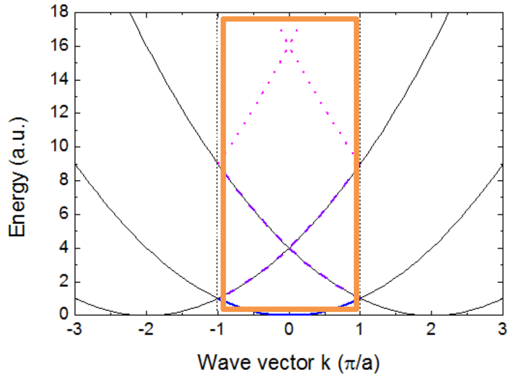
Wang et al, Science (2013)

Floquet theory

- Time direction analog of Bloch theorem

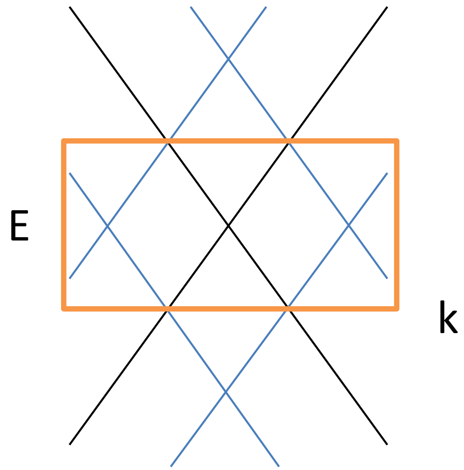
- Bloch: H(x+L) = H(x)

$$\psi(x) = e^{ikx}u(x), \qquad u(x+a) = u(x),$$



- Floquet: H(t+T) = H(t)

$$\psi(t) = e^{-i\epsilon t/\hbar}\phi(t), \qquad \phi(t+T) = \phi(t),$$



Floquet theory

- Time dependent Schroedinger equation

$$i\hbar\partial_t\psi(t) = H_0(t)\psi(t)$$

$$\psi(t) = e^{-i\epsilon t/\hbar}\phi(t), \quad \Rightarrow \quad (i\hbar\partial_t + \epsilon)\phi(t) = H_0(t)\phi(t).$$

$$\phi(t) = \sum_m e^{-im\Omega t}\phi_m, \quad \Rightarrow \quad (m\hbar\Omega + \epsilon)\phi_m = \widetilde{H}_{0mn}\phi_n,$$

$$\widetilde{H}_{0mn} = \frac{1}{T}\int_0^T dt e^{i(m-n)\Omega t} H_0(t).$$

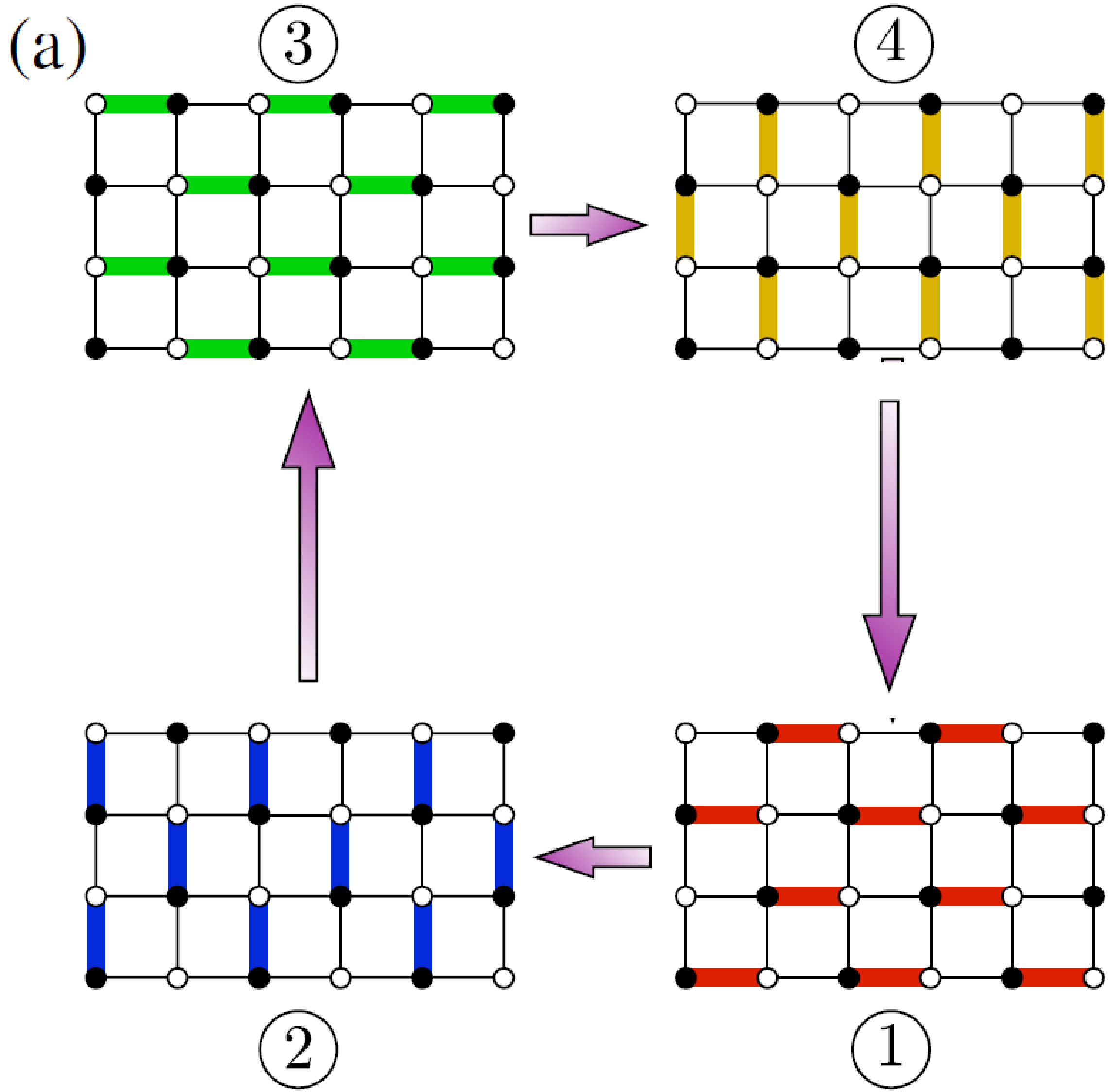
- Floquet Hamiltonian: $H_F\phi = \epsilon\phi,$

$$(H_F)_{mn} = \frac{1}{T}\int_0^T dt e^{i(m-n)\Omega t} H_0(t) - \delta_{mn} m\hbar\Omega.$$

Floquet anomalous Hall insulator

- 4 step drive

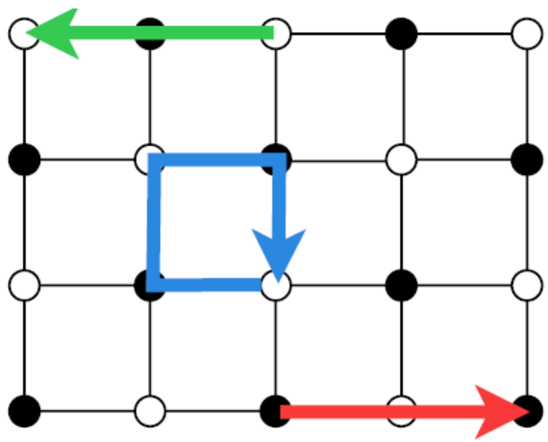
$$H(t) = \sum_{ij} t_{ij}(t) c_i^\dagger c_j$$



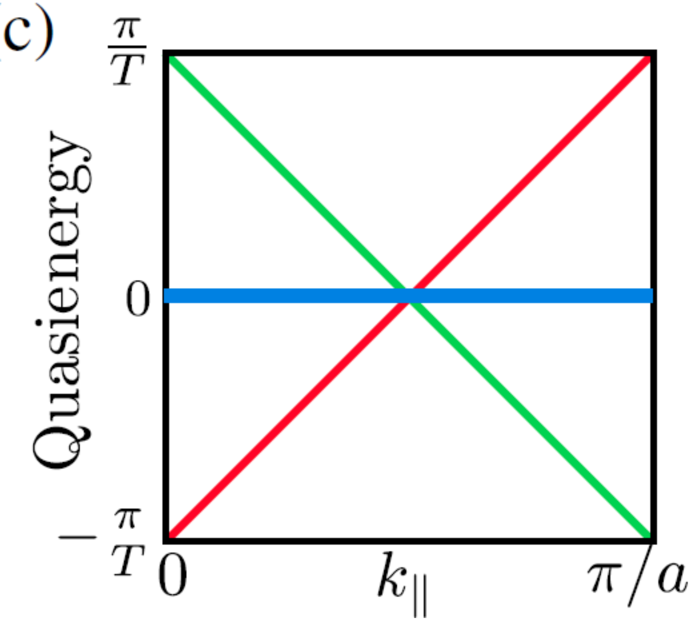
Rudner, Lindner, Berg, Levin, PRX (2013)

Floquet anomalous Hall insulator

(b)



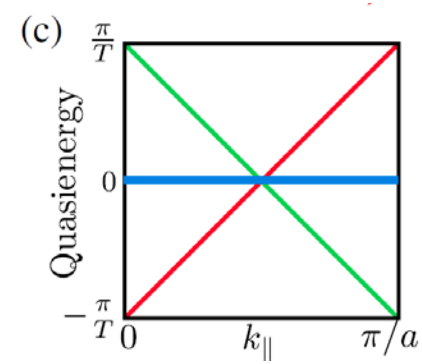
(c)



Rudner, Lindner, Berg, Levin, PRX (2013)

Topological number?

- No Chern number
  - An alternative way to obtain Floquet Hamiltonian:
    $H_F = i \log U(0 \rightarrow T)$
  - U=1 in the bulk → $H_F = 0$
  - Trivial bulk band



- Protection of edge states does not come from H_F. Instead, it originates from t dependence of U(t).

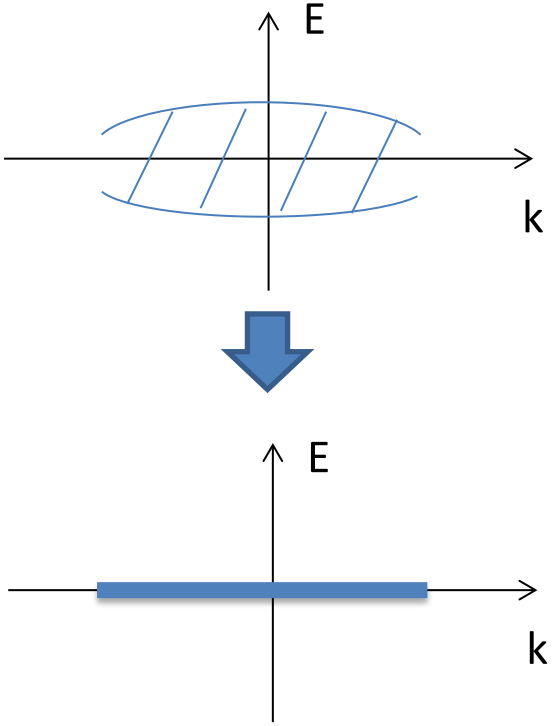
Winding number of U

- Spectral flattening

$$\tilde{H}(t) = \begin{cases} H(t), & 0 < t < T \\ -H_F, & T < t < 2T \end{cases}$$

$$\tilde{U}(t) = \mathcal{T} \exp[-i \int_0^t H(t')dt']$$

– U(t=T)=U(t=0)=1, periodic for (t,k)



- Winding number π_3(U(N))=Z

$$W = \frac{\epsilon_{ab}}{8\pi^2} \int dt dk d\lambda \mathrm{tr}[(\tilde{U}^\dagger \partial_t \tilde{U})(\tilde{U}^\dagger \partial_a \tilde{U})(\tilde{U}^\dagger \partial_b \tilde{U})],$$

a,b=kx,ky

Meaning of W

$$W = \frac{\epsilon_{ab}}{8\pi^2} \int dt dk d\lambda \mathrm{tr}[(\tilde{U}^\dagger \partial_t \tilde{U})(\tilde{U}^\dagger \partial_a \tilde{U})(\tilde{U}^\dagger \partial_b \tilde{U})],$$

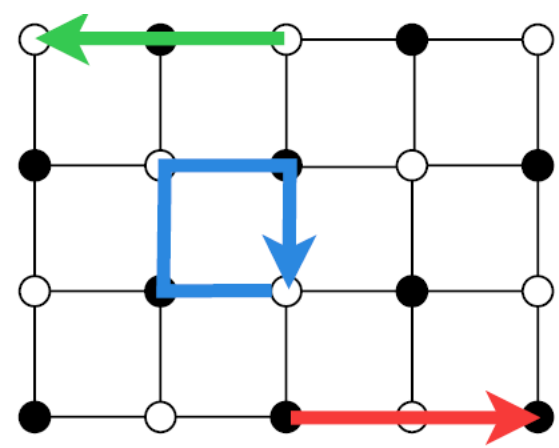
a,b=kx,ky

Integrand ~ i[H,x] y – i[H,y] x ~ px y – py x

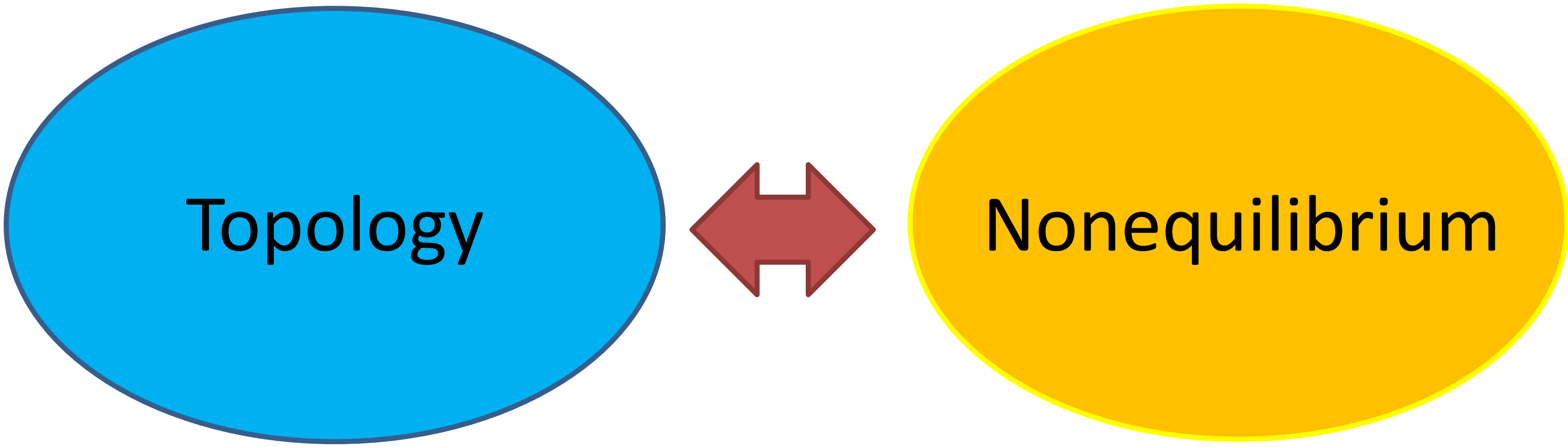
~ orbital magnetization

- Quantized magnetization in the bulk

(b)



Nathan et al., PRL (2017)



- Floquet topological phases
  protected by time glide symmetry

Adrian Po

Ashvin Vishwanath

Morimoto, Po, Vishwanath, PRB (2017)

Symmetries that protect TIs

- Ten fold way
  - Time reversal symmetry
  - Particle-hole symmetry
  - Chiral symmetry

$$gH(t)g^\dagger = H(t)$$

- Topological crystalline symmetry
  - Reflection symmetry
  - Rotation symmetry

$$gH(t,k)g^\dagger = H(t,g(k))$$

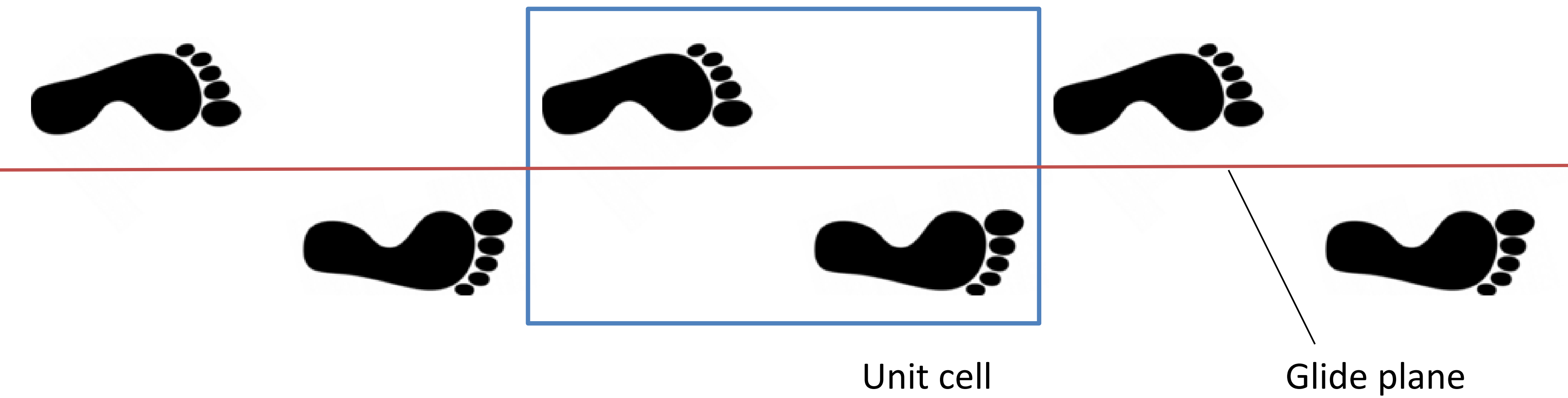
- Static symmetries: g: t→t

Symmetry that only appears in dynamical systems?

$$gH(t,k)g^\dagger = H(g(t), g(k))$$

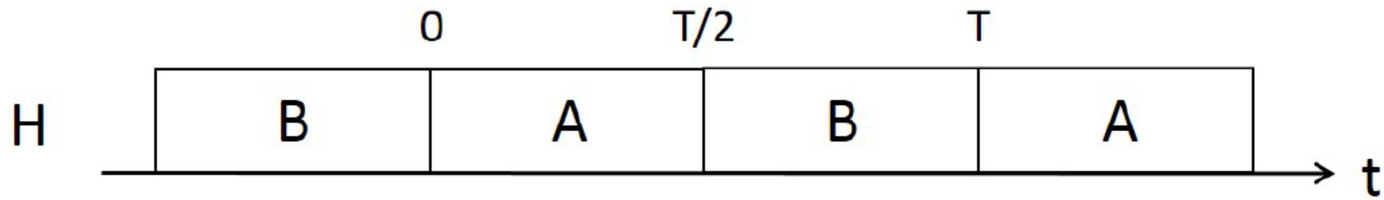
- Partial time translation: g(t)=t+t0
  - Time nonsymmorphic symmetry

Nonsymmorphic symmetry: glide symmetry

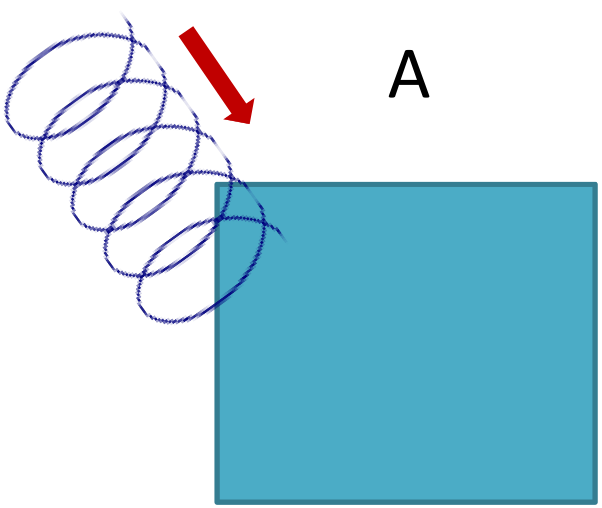


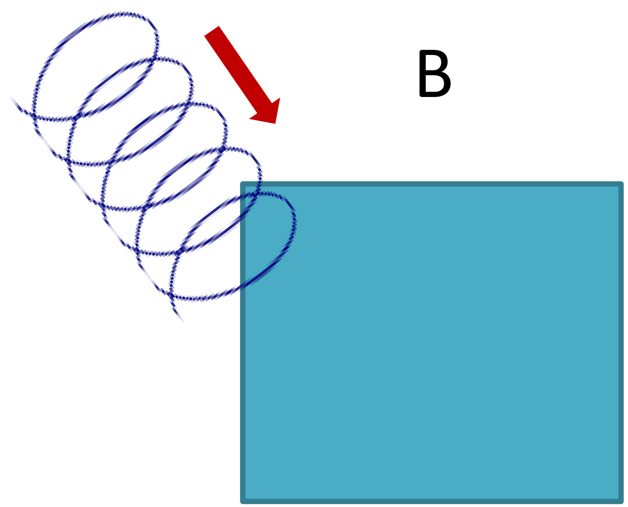
$$gH(x,y)g^{\dagger} = H(x+\frac{L}{2},-y)$$

Time glide symmetry

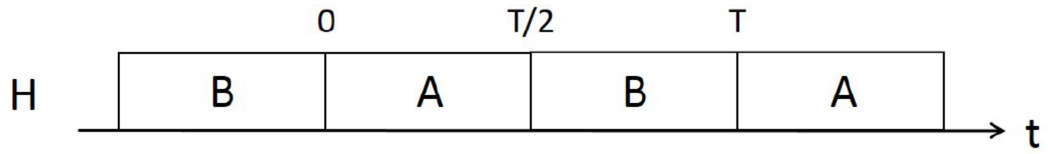


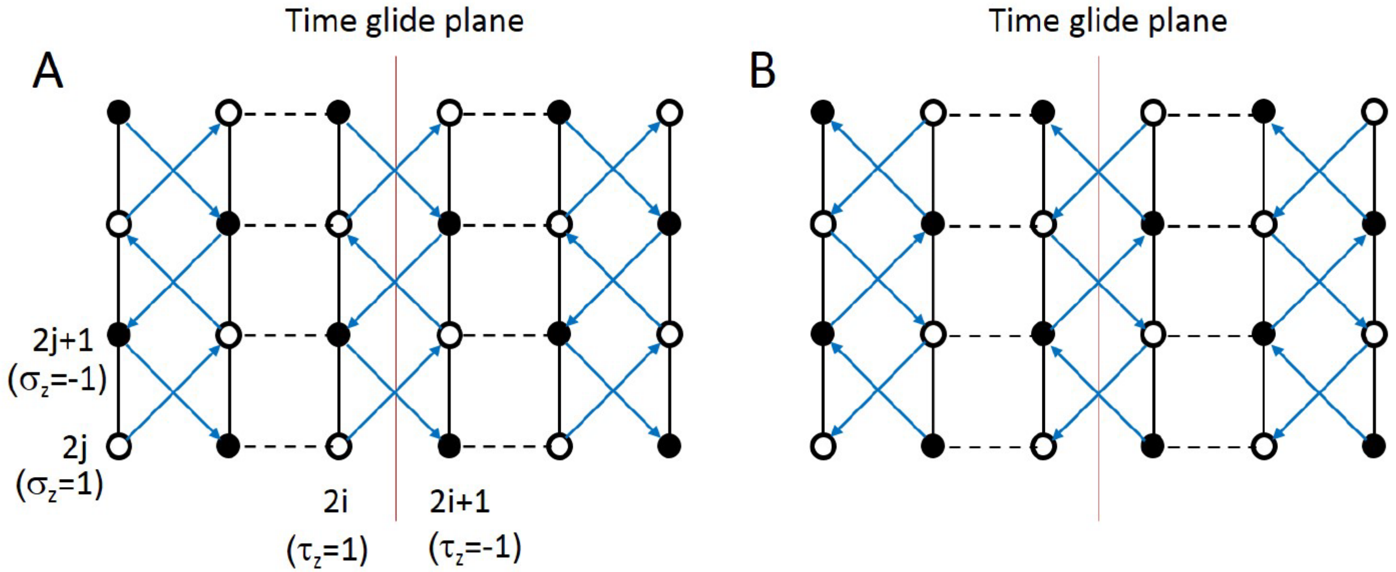
$$gH(t,k_x,k_y)g^\dagger = H(t+\frac{T}{2},-k_x,k_y)$$





2D toy model: time glide + sublattice symmetry

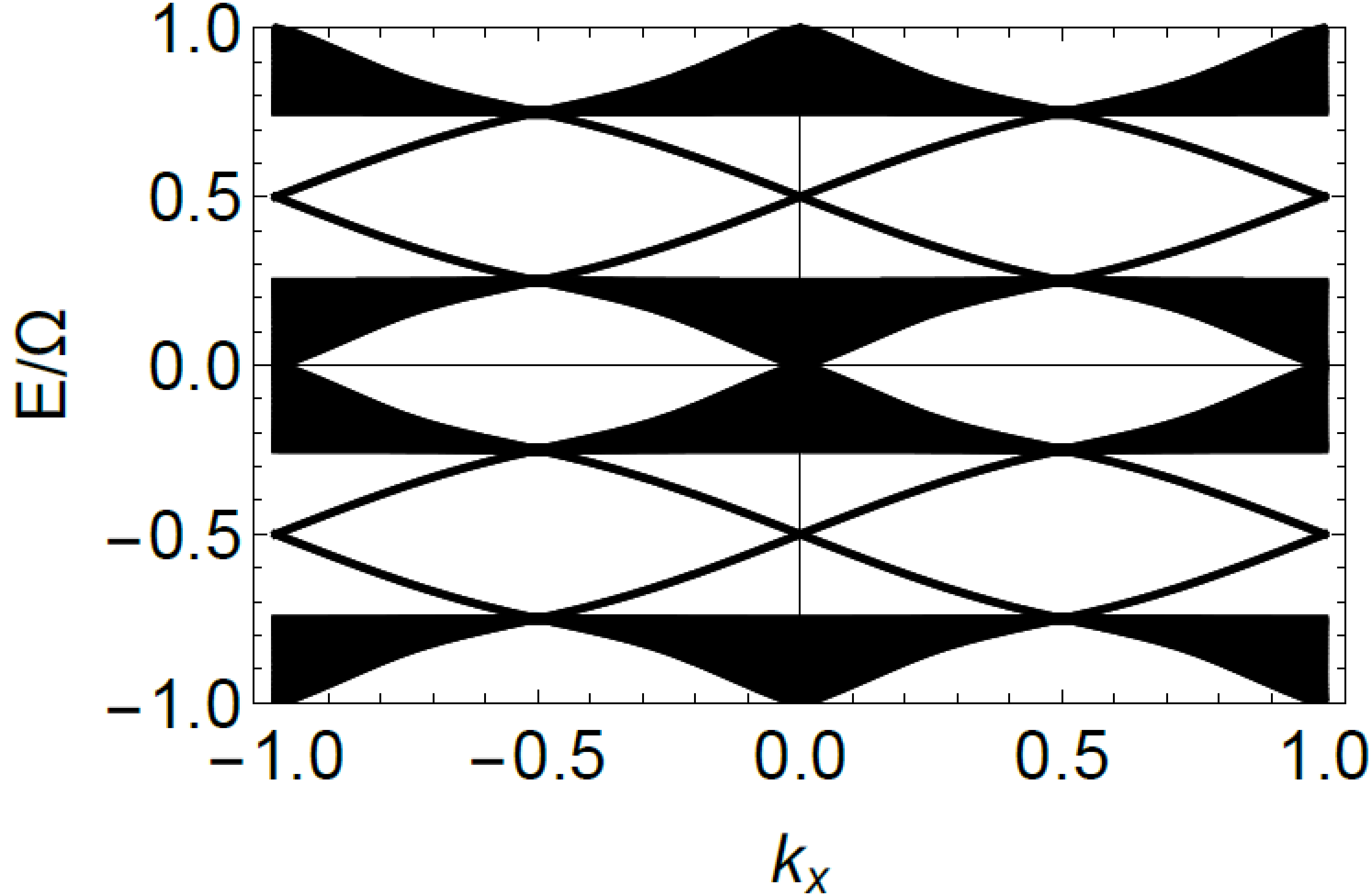




$$H(\boldsymbol{k},t)=2t\sigma_x\cos\frac{k_y}{2}+2t\eta(t)\sigma_y\tau_y\sin\frac{k_y}{2}+t'\tau_x\cos k_x+t'\tau_y\sin k_x,$$

$$\eta(t)=\begin{cases}+1, & (0\le t<\frac{T}{2})\\ -1. & (\frac{T}{2}\le t<T)\end{cases}$$

Quasienergy spectrum



Topological number for 1D class AIII

- Action of chiral symmetry: $\Gamma H(t)\Gamma = -H(-t),$

$$\Gamma U(k, 0 \to T/2)\Gamma = U^\dagger(k, T/2 \to T).$$

- When U(0→T)=1 (with spectral flattening),

$U(k, 0 \to T/2) = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$ and $\Gamma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$ commute

$\Rightarrow$ $b = c = 0.$

- We can define winding number as

$$\nu_\pi = \nu[d]. \qquad \nu[g(k)] = \frac{1}{2\pi i}\int dk\, \mathrm{tr}\left(g^\dagger \frac{dg}{dk}\right),$$

Topological number for time glide symmetric 2D model

- Focus on glide invariant plane (at kx=0)

$$M_T H(0,k_y,t) M_T^{-1} = H\left(0,k_y,t+\frac{T}{2}\right).$$

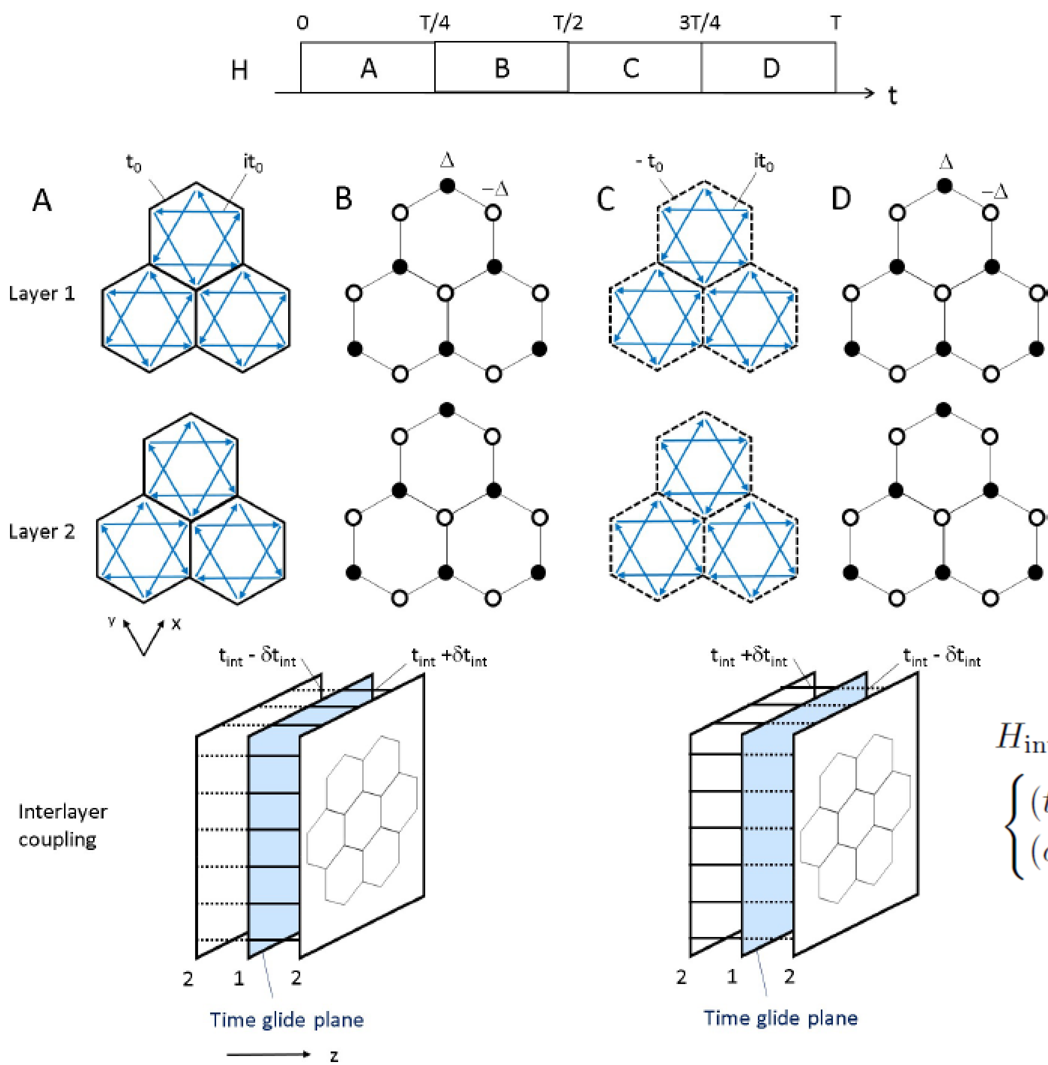
$$\Gamma M_T H(0,k_y,t)(\Gamma M_T)^{-1} = -H(0,k_y,T/2-t),$$

$$\Gamma M_T U\left(0,-\frac{T}{4}\to\frac{T}{4}\right)(\Gamma M_T)^{-1} = U^\dagger\left(0,-\frac{T}{4}\to\frac{T}{4}\right)$$

$$U\left(0,-\frac{T}{4}\to\frac{T}{4}\right) = \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}, \qquad \Gamma M_T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

- Topological number: ν[d']

3D model with time glide

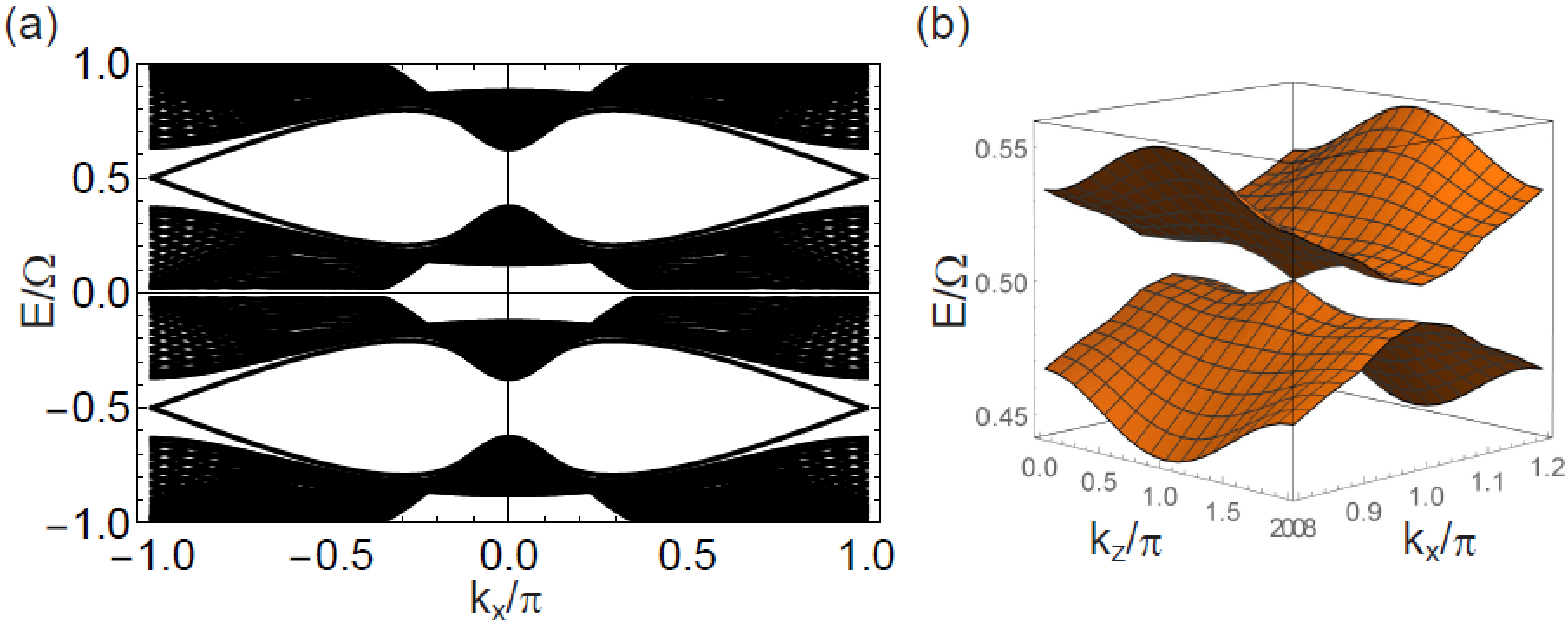


$$H_{\text{intra}} = \begin{cases} t_0 \boldsymbol{d}_{\text{H}} \cdot \boldsymbol{\sigma}, & (0 \le t < \frac{T}{4}) \\ \Delta \sigma_z, & (\frac{T}{4} \le t < \frac{T}{2}) \\ t_0 \sigma_z (\boldsymbol{d}_{\text{H}} \cdot \boldsymbol{\sigma}) \sigma_z, & (\frac{T}{2} \le t < \frac{3T}{4}) \\ \Delta \sigma_z, & (\frac{3T}{4} \le t < T) \end{cases}$$

$$\boldsymbol{d}_{\text{H}} = (1 + \cos k_x + \cos k_y, \sin k_x + \sin k_y, [-2\sin k_x + 2\sin k_y + 2\sin(k_x - k_y)]\tau_z)$$

$$H_{\text{inter}} = \begin{cases} (t_{\text{int}} + \delta t_{\text{int}} \cos k_z)\tau_x + \delta t_{\text{int}} \sin k_z \tau_y, & (0 \le t < \frac{T}{2}) \\ (\delta t_{\text{int}} + t_{\text{int}} \cos k_z)\tau_x - t_{\text{int}} \sin k_z \tau_y, & (\frac{T}{2} \le t < T) \end{cases}$$

Band structure



Energy spectrum at kz=pi

Topological number

- With time glide symmetry,

$$M^T H(k_x, k_y, k_z^0, t)(M^T)^{-1} = H(k_x, k_y, k_z^0, t + T/2).$$

we can write U(0→T) with half period evolution operator U_h as

$$U(k_x, k_y, 0 \to T) = g_T U_h g_T U_h, \qquad U_h(k_x, k_y) = U\left(k_x, k_y, 0 \to \frac{T}{2}\right).$$

- When U(0→T)=1, $g_T U_h$ becomes hermitian:

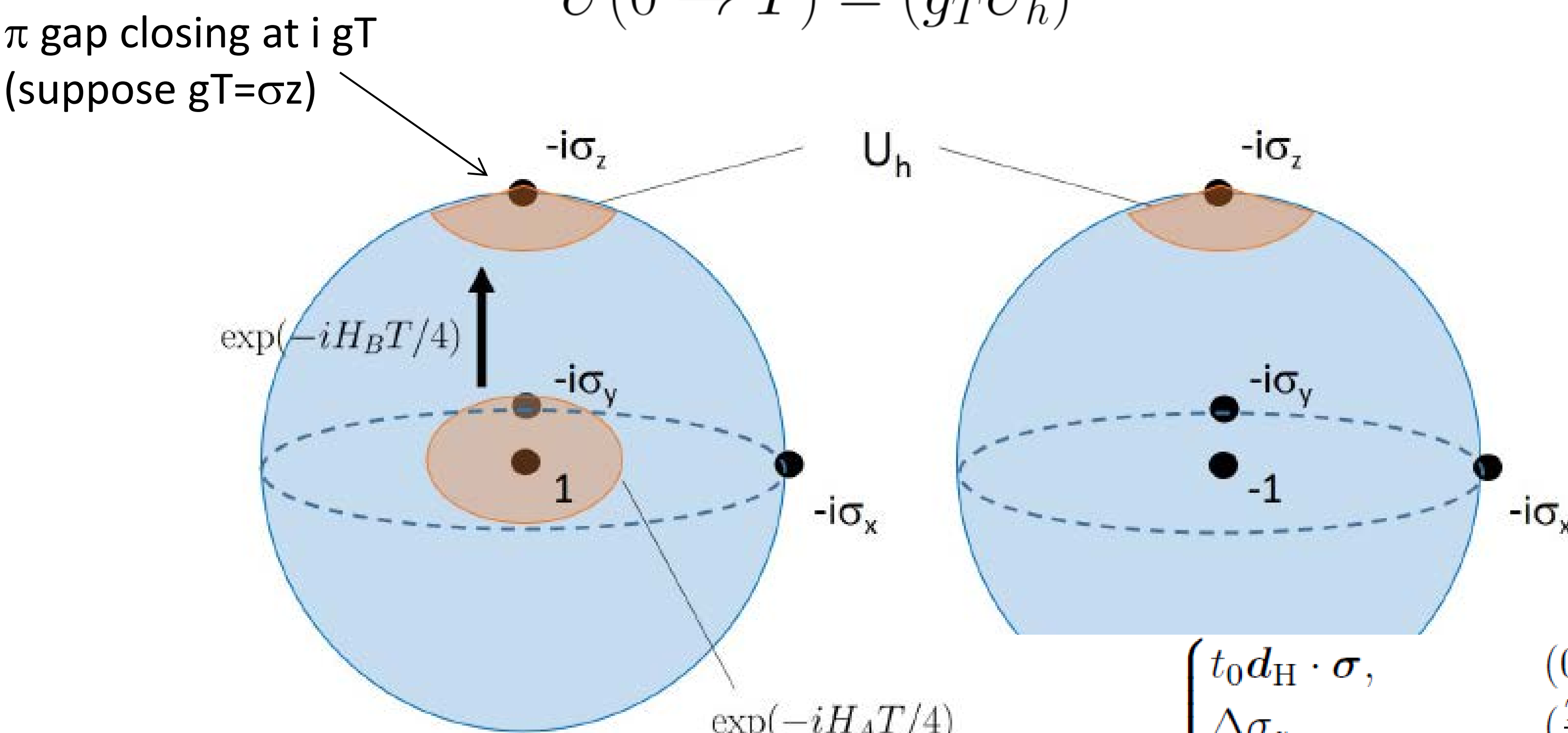
$$g_T U_h = (g_T U_h)^\dagger,$$

- We can define a Chern number for $g_T U_h$ (kx,ky)

Topological characterization

- U_h belongs to SU(2) and defines a point in S^3

$$U(0 \to T) = (g_T U_h)^2$$



$$H_{\text{intra}} = \begin{cases} t_0 \boldsymbol{d}_{\text{H}} \cdot \boldsymbol{\sigma}, & (0 \le t < \frac{T}{4}) \\ \Delta\sigma_z, & (\frac{T}{4} \le t < \frac{T}{2}) \\ t_0 \sigma_z (\boldsymbol{d}_{\text{H}} \cdot \boldsymbol{\sigma}) \sigma_z, & (\frac{T}{2} \le t < \frac{3T}{4}) \\ \Delta\sigma_z, & (\frac{3T}{4} \le t < T) \end{cases}$$

General classification of Floquet TIs?

Ten fold way in the equilibrium

	d													
Cartan	0	1	2	3	4	5	6	7	8	9	10	11	…	
Complex case:														
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0		period
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$		d = 2
Real case:														
AI	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	…	
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	…	
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0		
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$		period
AII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$		d = 8
CII	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	…	
C	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	…	
CI	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	…	

- Tenfold way for Floquet topological phases?

Classification scheme with U

- Define an effective Hamiltonian H from U and classify H

$$U_S(\boldsymbol{k},t) = \mathcal{T}\exp\left[-i\int_{\frac{T-t}{2}}^{\frac{T+t}{2}} dt' H(\boldsymbol{k},t')\right]$$

$$H_S(\boldsymbol{k},t) = \begin{pmatrix} 0 & U_S(\boldsymbol{k},t) \\ U_S^\dagger(\boldsymbol{k},t) & 0 \end{pmatrix},$$

- Gapped Hamiltonian E=$\pm$1
- Periodic in k and t $\in$ T^{d+1}

Roy, Harper, PRB (2017)
Morimoto, Po, Vishwanath, PRB (2017)

Symmetry actions

$$H_S(\boldsymbol{k},t) = \begin{pmatrix} 0 & U_S(\boldsymbol{k},t) \\ U_S^\dagger(\boldsymbol{k},t) & 0 \end{pmatrix},$$

- Symmetries of H leads to

$$\begin{aligned} T' H_S(\boldsymbol{k},t) T'^{-1} &= H_S(-\boldsymbol{k},t), \quad & T' &= T \otimes \sigma_x, \\ C' H_S(\boldsymbol{k},t) C'^{-1} &= H_S(-\boldsymbol{k},t), \quad & C' &= C \otimes \sigma_0, \\ \Gamma' H_S(\boldsymbol{k},t) \Gamma'^{-1} &= H_S(\boldsymbol{k},t), \quad & \Gamma' &= \Gamma \otimes \sigma_x, \end{aligned}$$

- Inherent sublattice symmetry

$$\tilde{\Gamma} H_S(\boldsymbol{k},t) \tilde{\Gamma}^{-1} = -H_S(\boldsymbol{k},t), \qquad \tilde{\Gamma} = 1 \otimes \sigma$$

Apply classification method for TIs in the equilibrium

Case of Class A and AIII

- dD class A systems ➔ d+1D class AIII systems

$$H_S(\boldsymbol{k},t) = \begin{pmatrix} 0 & U_S(\boldsymbol{k},t) \\ U_S^\dagger(\boldsymbol{k},t) & 0 \end{pmatrix}, \qquad \tilde{\Gamma} = 1 \otimes \sigma_z.$$

- dD class AIII systems ➔ d+1D class A systems

$$\Gamma' H_S(\boldsymbol{k},t)\Gamma'^{-1} = H_S(\boldsymbol{k},t), \qquad \Gamma' = \Gamma \otimes \sigma_x,$$

$$H_S = \begin{pmatrix} H_S^+ & 0 \\ 0 & H_S^- \end{pmatrix} \qquad \tilde{\Gamma} = 1 \otimes \sigma_z.$$

	d											
Cartan	0	1	2	3	4	5	6	7	8	9	10	11
Complex case:												
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$

Floquet tenfold way

Class	T	C	Γ	$d=0$	$d=1$	$d=2$	$d=3$	$d=4$	$d=5$	$d=6$	$d=7$
A	0	0	0	$\mathbb{Z}^n$	0	$\mathbb{Z}^n$	0	$\mathbb{Z}^n$	0	$\mathbb{Z}^n$	0
AIII	0	0	1	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$
AI	$+1$	0	0	$\mathbb{Z}^n$	0	0	0	$\mathbb{Z}^n$	0	$\mathbb{Z}_2^n$	$\mathbb{Z}_2^n$
BDI	$+1$	$+1$	1	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$	0	0	0	$\mathbb{Z}^2$	0	$\mathbb{Z}_2^2$
D	0	$+1$	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$	0	0	0	$\mathbb{Z}^2$	0
DIII	-1	$+1$	1	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$	0	0	0	$\mathbb{Z}^2$
AII	-1	0	0	$\mathbb{Z}^n$	0	$\mathbb{Z}_2^n$	$\mathbb{Z}_2^n$	$\mathbb{Z}^n$	0	0	0
CII	-1	-1	1	0	$\mathbb{Z}^2$	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$	0	0
C	0	-1	0	0	0	$\mathbb{Z}^2$	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$	0
CI	$+1$	-1	1	0	0	0	$\mathbb{Z}^2$	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$

- The same types of topological numbers as in the equilibrium

Classification of time glide Floquet TIs

- Time glide symmetry gives an additional symmetry constraint on Hs:

$$M_T H(k_1, k_2, \ldots, k_d, t) M_T^{-1} = H\left(-k_1, k_2, \ldots, k_d, t + \frac{T}{2}\right).$$

$$\Rightarrow \quad M'_T H_S(k_1, k_2, \ldots, k_d, t) M'^{-1}_T = H_S(-k_1, k_2, \ldots, k_d, -t),$$
$$M'_T = M_T \otimes \sigma_x.$$

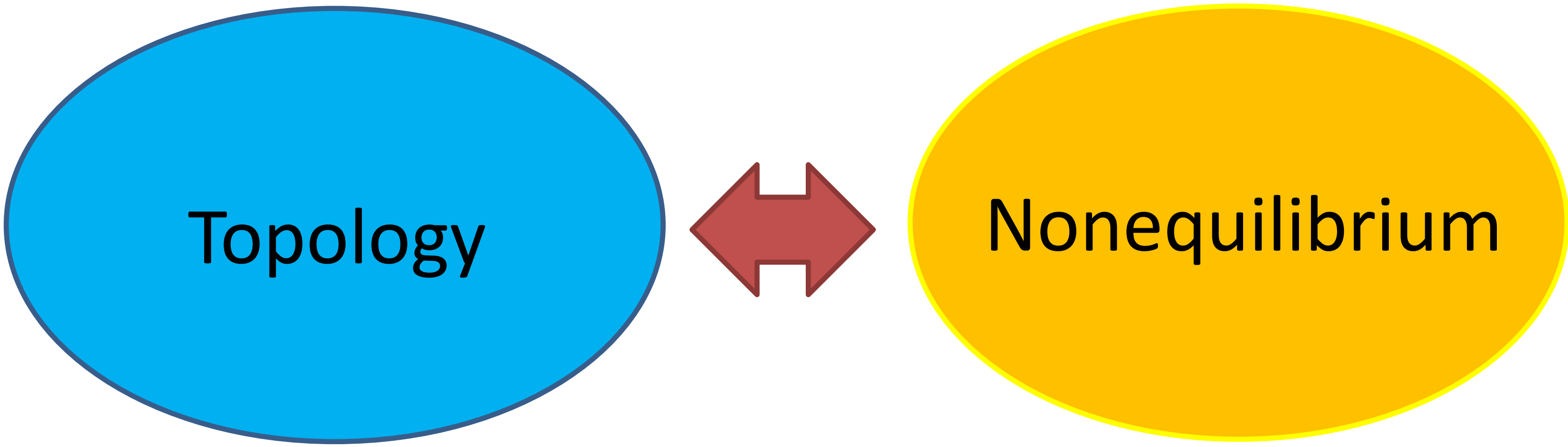
- We apply classification method for topological crystalline insulator with reflection

Morimoto, Furusaki, PRB (2013)

Classification of time glide Floquet TIs

$\eta_T,\eta_C,\eta_\Gamma$	Class	C_q or R_q	$d=0$	$d=1$	$d=2$	$d=3$	$d=4$	$d=5$	$d=6$	$d=7$
-	A	C_{d+3}	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\eta_\Gamma=+$	AIII	$C_{d+3}\times C_{d+3}$	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$
$\eta_\Gamma=-$	AIII	C_{d+4}	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
	AI	R_{1-d}	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
	BDI	R_{2-d}	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
$\eta_T=+$ (AI,AII)	D	R_{3-d}	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
$\eta_C=-$ (D,C)	DIII	R_{4-d}	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
$(\eta_T,\eta_C)=(+,-)$ (BDI,DIII,CII,CI)	AII	R_{5-d}	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
	CII	R_{6-d}	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
	C	R_{7-d}	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	R_{-d}	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
	AI	R_{-1-d}	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	BDI	R_{-d}	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\eta_T=-$ (AI,AII)	D	R_{1-d}	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
$\eta_C=+$ (D,C)	DIII	R_{2-d}	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
$(\eta_T,\eta_C)=(-,+)$ (BDI,DIII,CII,CI)	AII	R_{3-d}	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
	CII	R_{4-d}	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
	C	R_{5-d}	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
	CI	R_{6-d}	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
$(\eta_T,\eta_C)=(+,+)$	BDI	$R_{1-d}\times R_{1-d}$	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$	0	0	0	$\mathbb{Z}^2$	0	$\mathbb{Z}_2^2$
$(\eta_T,\eta_C)=(-,-)$	DIII	$R_{3-d}\times R_{3-d}$	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$	0	0	0	$\mathbb{Z}^2$
[illegible]	CII	[illegible]	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$	$\mathbb{Z}^2$	$\mathbb{Z}^2$	0	0

Similar to, but different from classification for TCIs



- Floquet symmetry protected topological phases
  - Classification and 1D/2D realizations

Andrew Potter

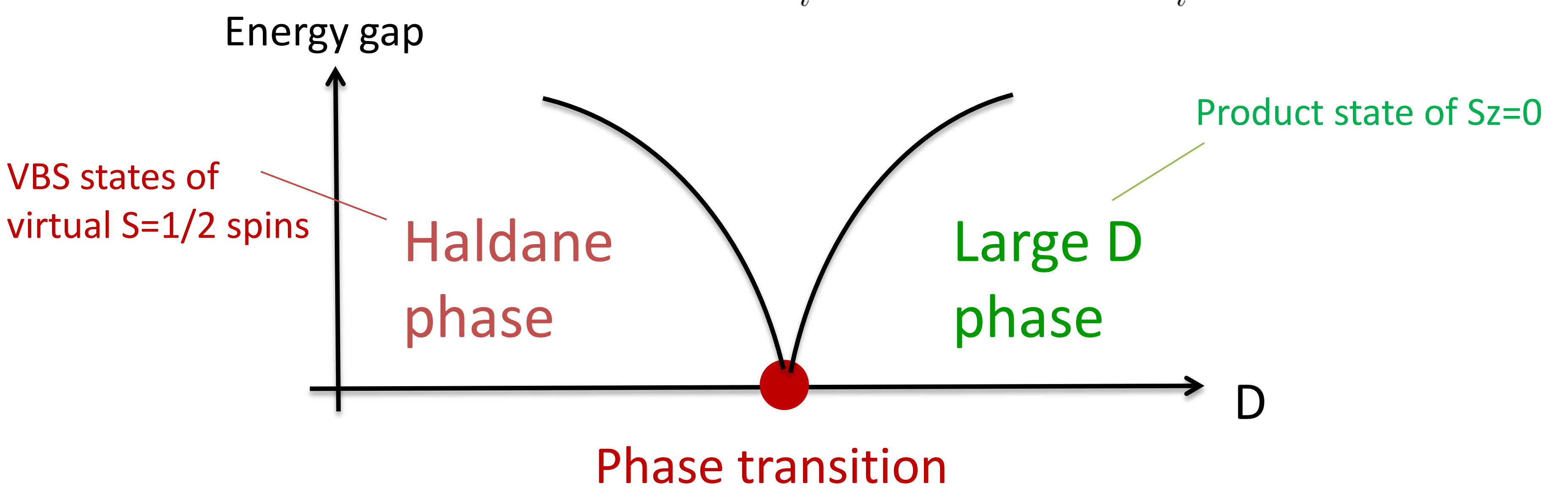
Ashvin Vishwanath

Potter, Morimoto, Vishwanath, PRX (2016)

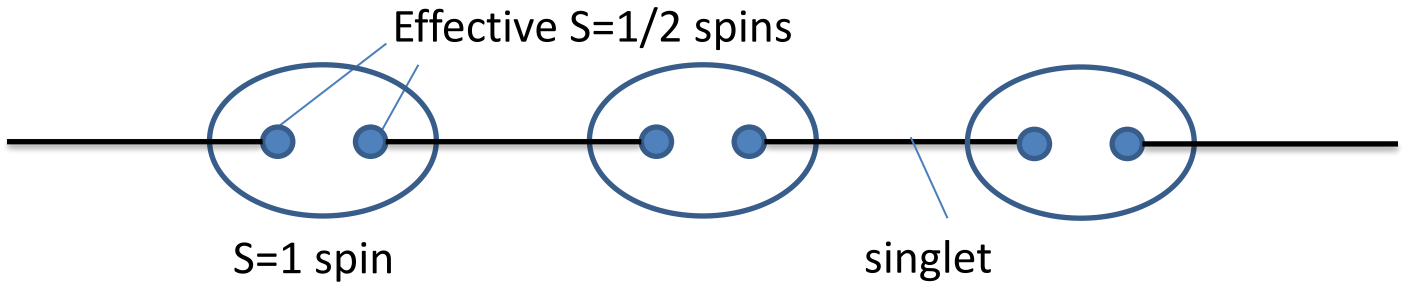
Haldane phase

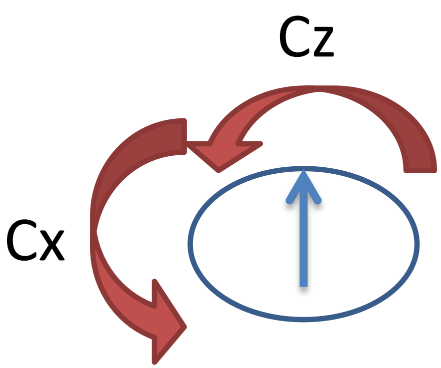
- Topological phase of interacting bosons
  "Symmetry-protected topological phases"

S=1 spin chain: $$H = J\sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + D\sum_i (S^z)^2$$



Characterization by projective representation





$$C_x = e^{i\pi S_x} = \begin{pmatrix} & & -1 \\ & -1 & \\ -1 & & \end{pmatrix} \qquad C_z = e^{i\pi S_z} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$$

Cx Cz = Cz Cx

For edge effective ½ spins: $(U_{C_x}, U_{C_z}, U_{C_xC_z}) = (\sigma_x, \sigma_z, i\sigma_y),$

Projective representation: $U_{C_x}U_{C_z} = -U_{C_z}U_{C_x}$

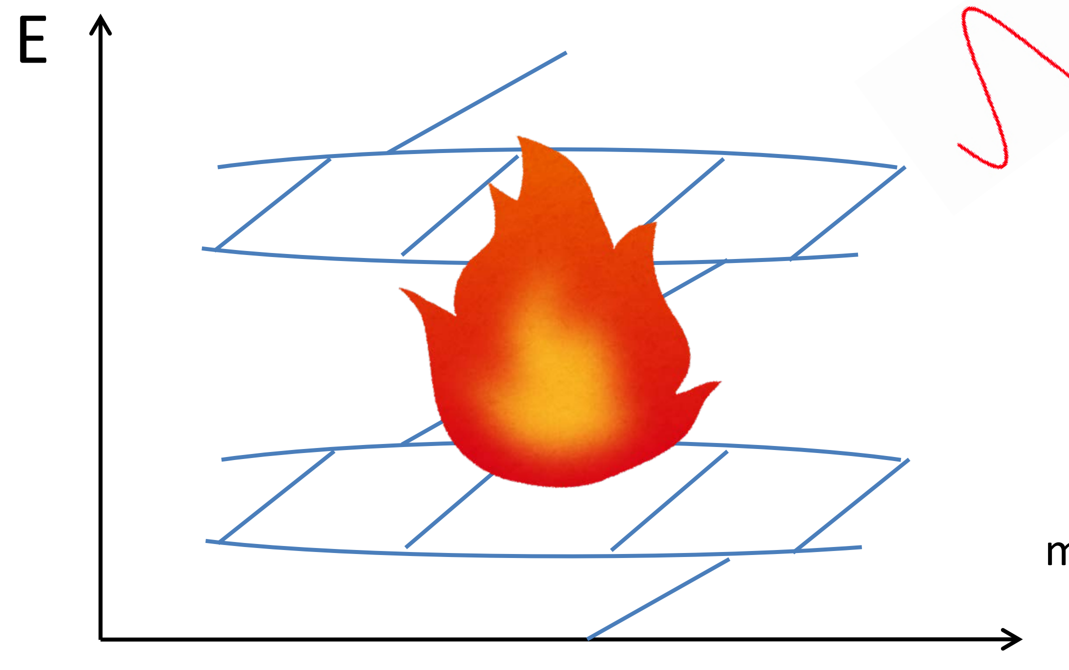
$\Cap$

Group cohomology: $H^2(Z_2 \times Z_2, U(1)) = \mathbb{Z}_2$

Floquet version?

Symmetry protected topological phases in periodically driven systems

Avoid heating!

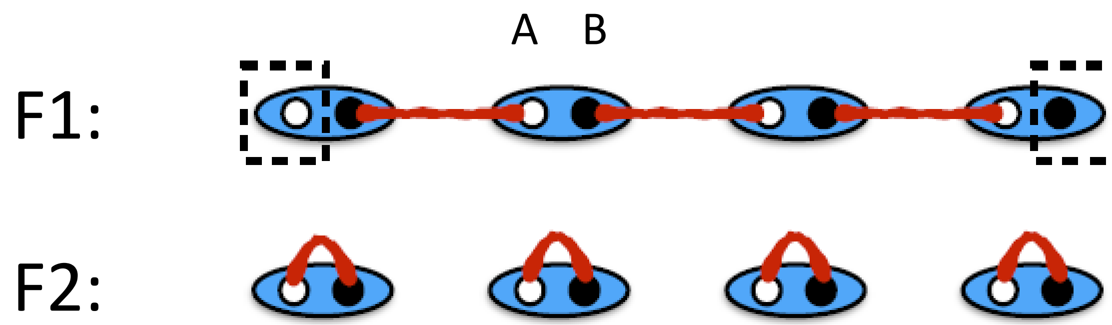


many body localization

- Compatibility with many body localization excludes:
  - Fermions with antiunitary symmetry (T)
  - Bosons with non-Abelian symmetry (SU(2))
- Bosons with Abelian discrete symmetry (Z_N)

Potter, Vasseur, PRB (2016)

1D model with Z2 symmetry



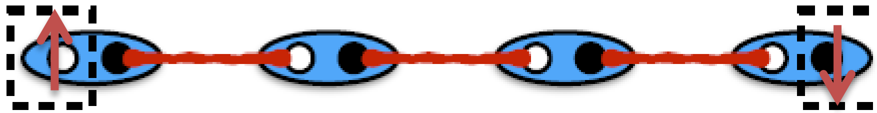
Two step drive:

$$F_1 = e^{-iH_{\mathrm{AKLT}}} \qquad H_{\mathrm{AKLT}} = \lambda \sum_{j=1}^{L-1} \boldsymbol{\sigma}_{B,j} \cdot \boldsymbol{\sigma}_{A,j+1}$$

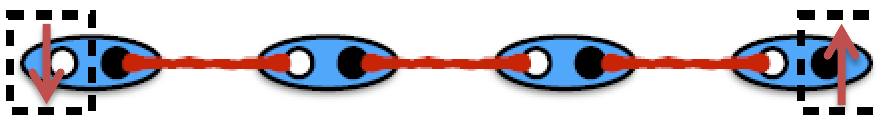
$$F_2 = e^{i\pi/2 \sum_j \sigma^x_{A,j} \sigma^x_{B,j}} = i^L \prod_{j=1}^{L} \sigma^x_{A,j} \sigma^x_{B,j}$$

Z2 symmetry: $\quad g = \prod_j \sigma^z_{A,j} \sigma^z_{B,j}$

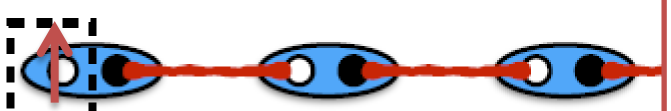
Pumping at the edge

F1: 

$$\mathrm{F2} = i^L \prod_{j=1}^{L} \sigma^x_{A,j}\sigma^x_{B,j}$$

F1: 

$$\mathrm{F2} = i^L \prod_{j=1}^{L} \sigma^x_{A,j}\sigma^x_{B,j}$$

F1: 

Z2 charge is pumped at the edge each cycle

$$g = \prod_j \sigma^z_{A,j}\sigma^z_{B,j}$$

Classification of FSPT, Kunneth formula

- Assumption: Time translation over the period can be regarded as an additional Z symmetry

$$H^2(G \times Z, U(1))$$

Von Keyserlingk, Sondhi, PRB (2016)
Else, Nayak, PRB (2016)
Potter, Morimoto, Vishwanath, PRX (2016)

- Kunneth formula

$$H^2(G \times Z, U(1)) = \underbrace{H^2(G, U(1))}_{\text{Equilibrium SPT}} \times \underbrace{H^1(G, U(1))}_{\text{Floquet SPT}}$$

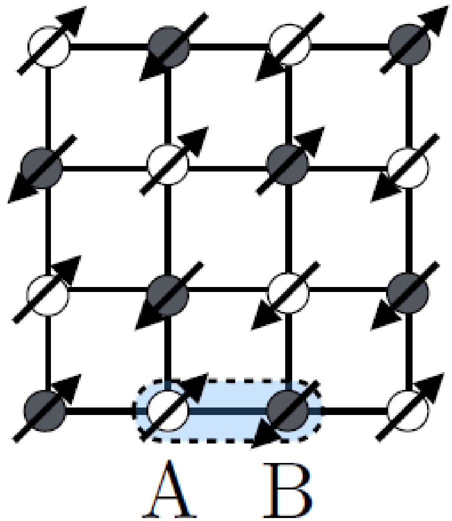
- Symmetry charge pumping at the edge

Higher dimensions

- Also classified by group cohomology
- Assumption: Low energy effective theory is given by a G-gauge theory
- Classification of SPTs is obtained from that for G-gauge theories

$$H^{d+1}(G, U(1))$$

2D model



$$G = \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$g_{A,B} = \prod_{i \in A,B} \sigma_i^x .$$

Target Floquet topological phase:

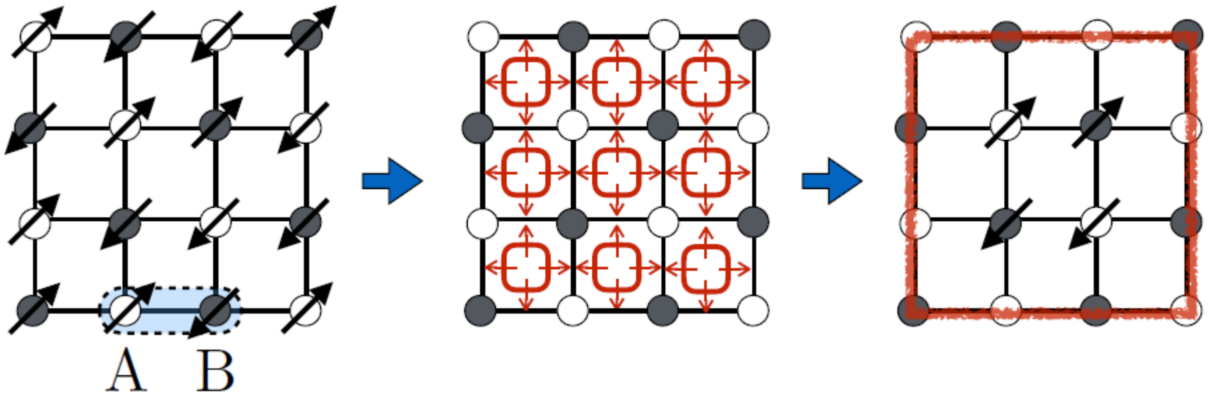
$H^3(G \times Z, U(1)) = H^3(G, U(1)) \times H^2(G, U(1))$

$H^2(Z2 \times Z2, U(1)) = Z2$

Pump Haldane phase at the boundary every cycle!

Reverse engineering

- Pump 1D SPT to boundary every cycle



Z2 x Z2 1D-SPT: $$H_{\text{1D-SPT}} = \sum_i \lambda_i \sigma^z_{i-1}\sigma^x_i\sigma^z_{i+1}$$

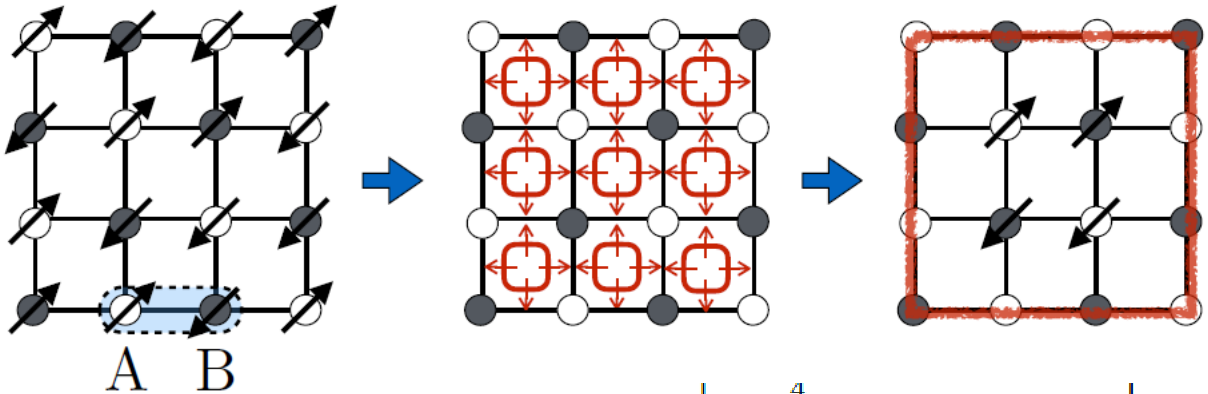
$$U_{\text{SPT}} = e^{i\frac{\pi}{4}\sum_i (-1)^i \sigma^z_i \sigma^z_{i+1}}$$

Trivial PM: $$H_{\text{1D-PM}} = \sum_i \lambda_i \sigma^x_i$$

Potter, Morimoto, PRB (2017)

Reverse engineering

- Pump 1D SPT to boundary every cycle



$$U_{\mathrm{SPT}}^{(1234)} = \exp\left[i\frac{\pi}{4}\sum_{i=1}^{4}(-1)^i\sigma_i^z\sigma_{i+1\mathrm{mod}4}^z\right]$$

Unitary for each plaquette:

$$= \frac{1}{2}\left(1+\sigma_1^z\sigma_2^z\sigma_3^z\sigma_4^z - \sigma_1^z\sigma_3^z - \sigma_2^z\sigma_4^z\right)$$

$$= \exp\left[i\pi\underbrace{\left(\frac{1-\sigma_1^z\sigma_3^z}{2}\right)\left(\frac{1-\sigma_2^z\sigma_4^z}{2}\right)}\right]$$

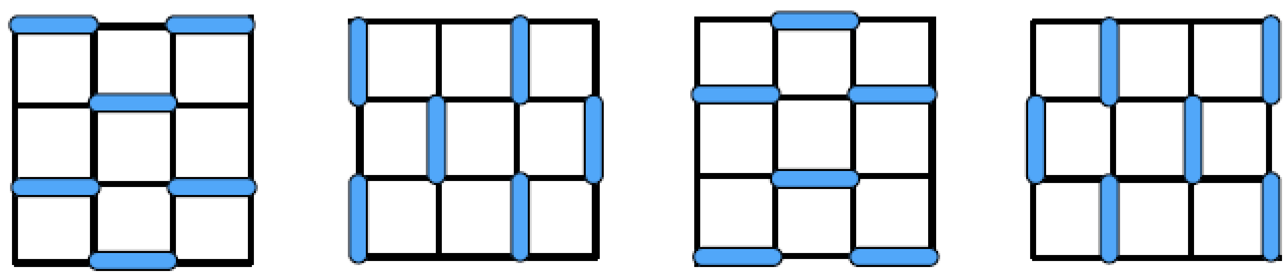
Stroboscopic drive:

$$H(t) = \begin{cases} 2H_1 = 2\sum_P H_{\mathrm{pump}}^{(P)} & 0 \le t < 1/2 \\ 2H_2 = 2\sum_i h_i\sigma_i^x & 1/2 \le t < 1 \end{cases}$$

Potter, Morimoto, PRB (2017)

FSPT beyond cohomology

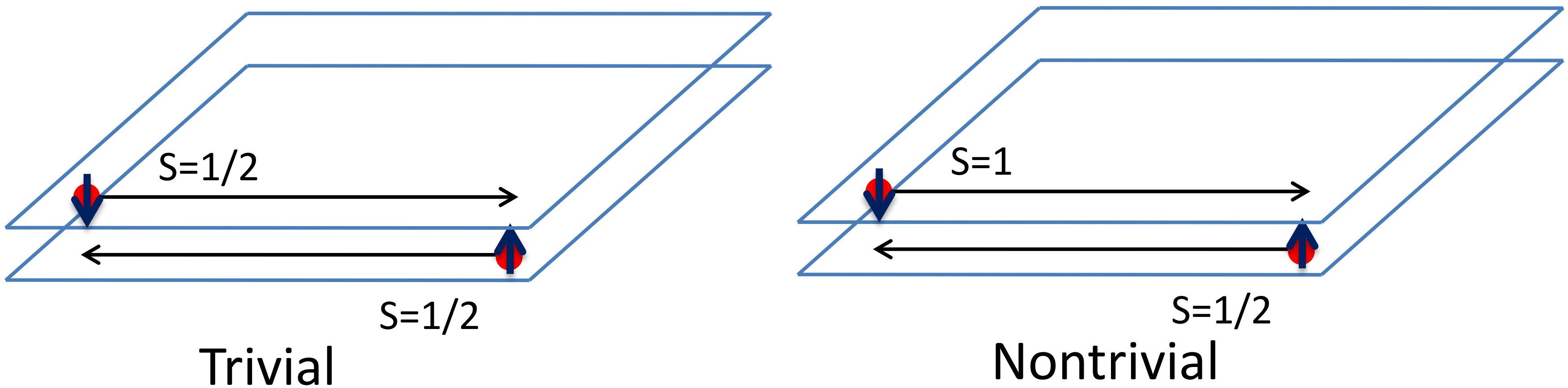
- Bosons with chiral driving



- Single mode of chiral bosons (e.g., S=1/2)
  - cf. at least 8 modes in the equilibrium (E8 state)

Po, Fidkowski, Morimoto, Potter, Vishwanath, PRX (2016)

Rational topological number



Topological number=
Dim. of Hilbert space of right mover / Dim. of Hilbert space of left mover
$\in Q$

Summary

- Noninteracing Floquet topolgical phases
  - Tenfold way classification
  - Time glide symmetry

Morimoto, Po, Vishwanath, PRB (2017)

- Interacting Floquet topological phases
  - Floquet SPT phases ~ SPT phases pumped to the boundary every cycle
  - 1D and 2D spin models

Potter, Morimoto, Vishwanath, PRX (2016)