Quantum heterodyne Hall effect from oscillating magnetic fields

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"Nonequilibrium Quantum Matter"

1. Introduction
2. Classical heterodyne Hall effect
3. Quantum heterodyne Hall effect
4. Quantum Dirac heterodyne Hall effect

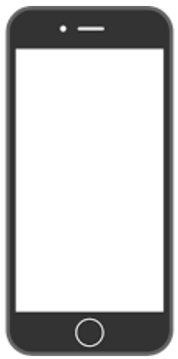
TO, Bucciantini, PRB'16
TO, Kitamura, Nag, Saha, Bucciantini *in prep*

What is a heterodyne?

Bluetooth

Wifi, TV, radio

What is a heterodyne?



2.40xGHz

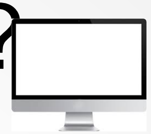
Bluetooth

Wifi, TV, radio

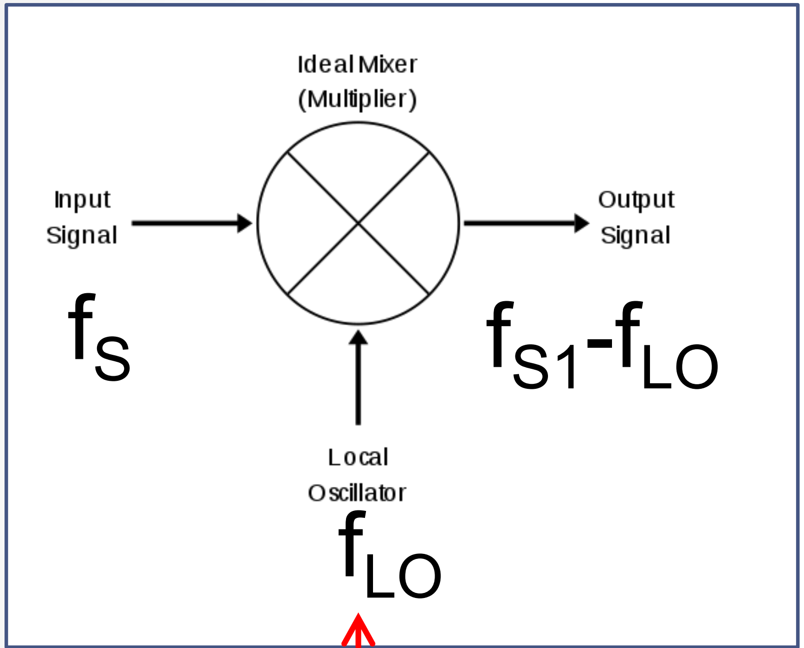
superheterodyne device

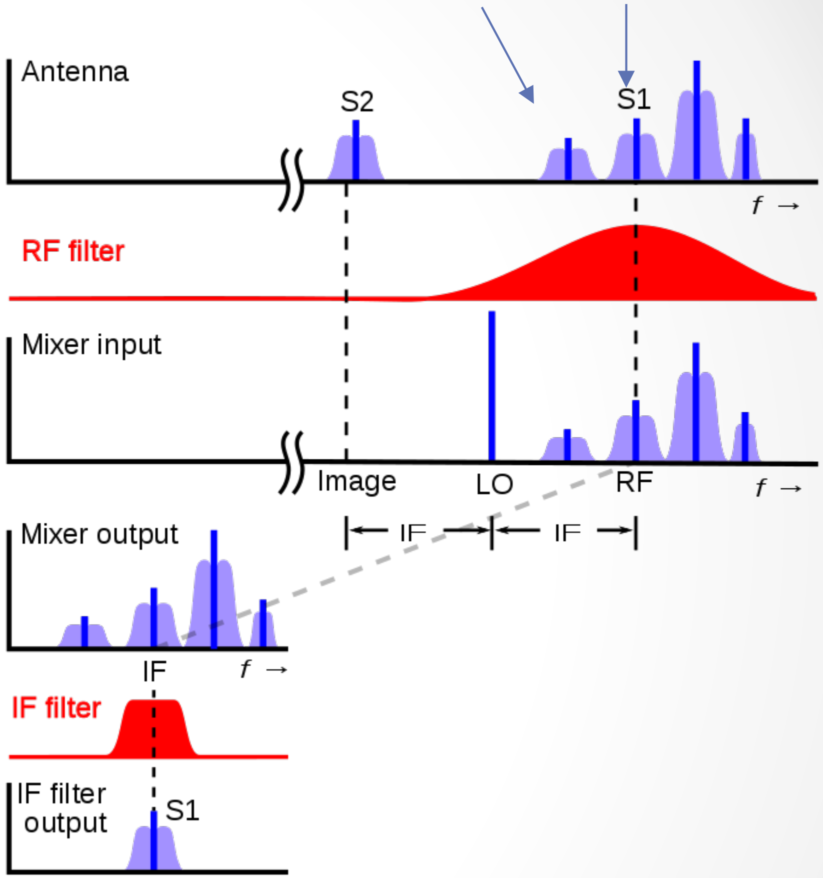
kHz

What is a heterodyne?




2.401GHz 2.402GHz





Local oscillator
= <u>periodically driven system</u>

f_{S1}-f_{LO}


from wikipedia “heterodyne”

Linear response theory

ω: signal frequency

$$j_a(t) = \int dt' \sigma_{ab}(t, t') E_b(\omega) e^{-i\omega t'}$$

Linear response theory

ω: signal frequency

$$j_a(t) = \int dt' \sigma_{ab}(t,t') E_b(\omega) e^{-i\omega t'}$$

If static, $\sigma(t-t')$

$$= \sigma_{ab}(\omega) E_b(\omega) e^{-i\omega t'}$$

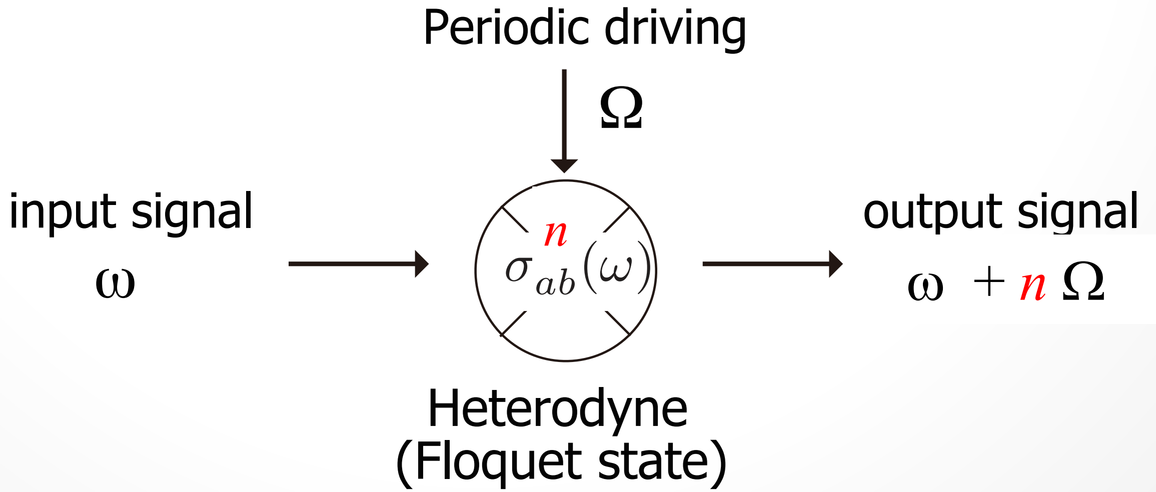
ac-conductivity

Linear response theory for periodically driven system

$$j_a(t) = \int dt' \sigma_{ab}(t,t') E_b(\omega) e^{-i\omega t'}$$

$$= \sum_{\boxed{n}} \sigma_{ab}^{\boxed{n}}(\omega) e^{-i(\omega + \boxed{n}\Omega)t} E_b(\omega)$$

ω: signal frequency
Ω: drive frequency

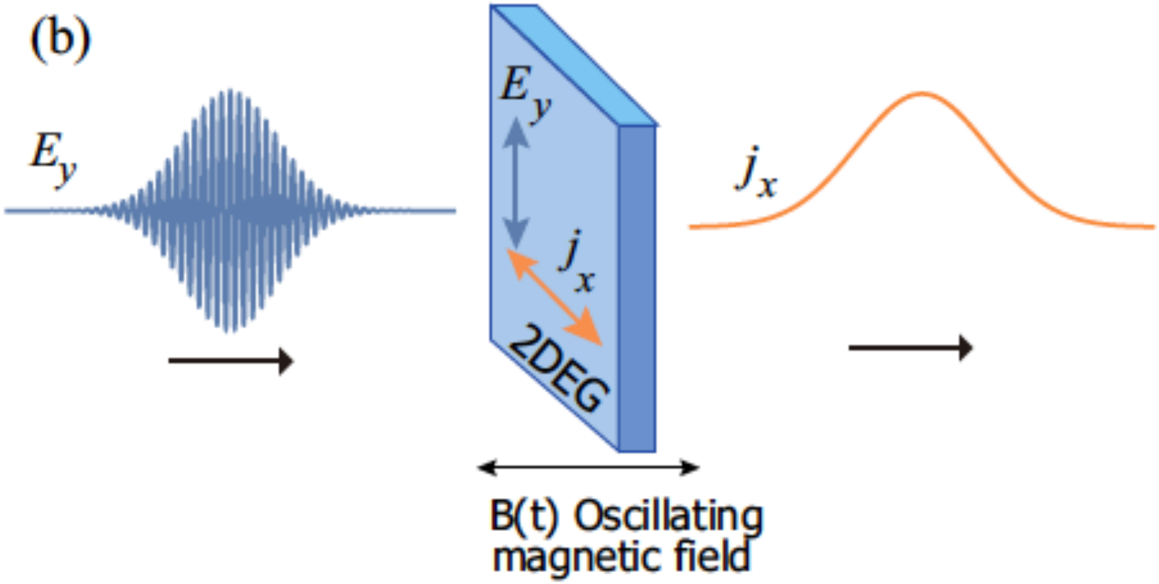


TO, Bucciantini, PRB'16

Heterodyne Hall effect

TO, Bucciantini, PRB'16

$$\sigma^n_{xy}(\omega)$$



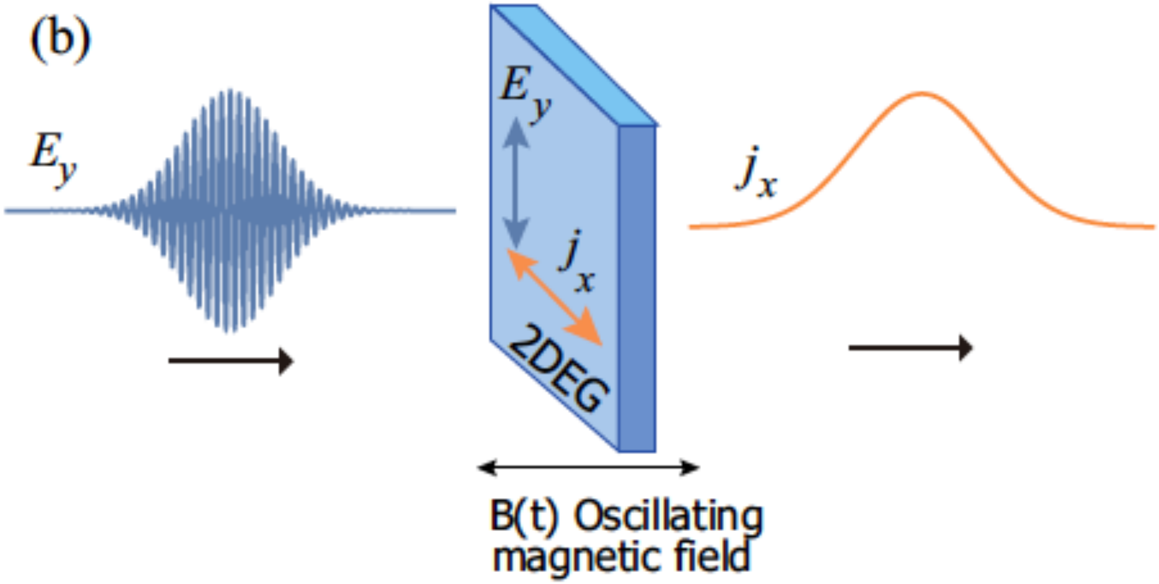
Application: Dissipationless frequency conversion

→ Ultra-low power consuming Bluetooth!?

Heterodyne Hall effect

TO, Bucciantini, PRB'16

$$\sigma^n_{xy}(\omega)$$

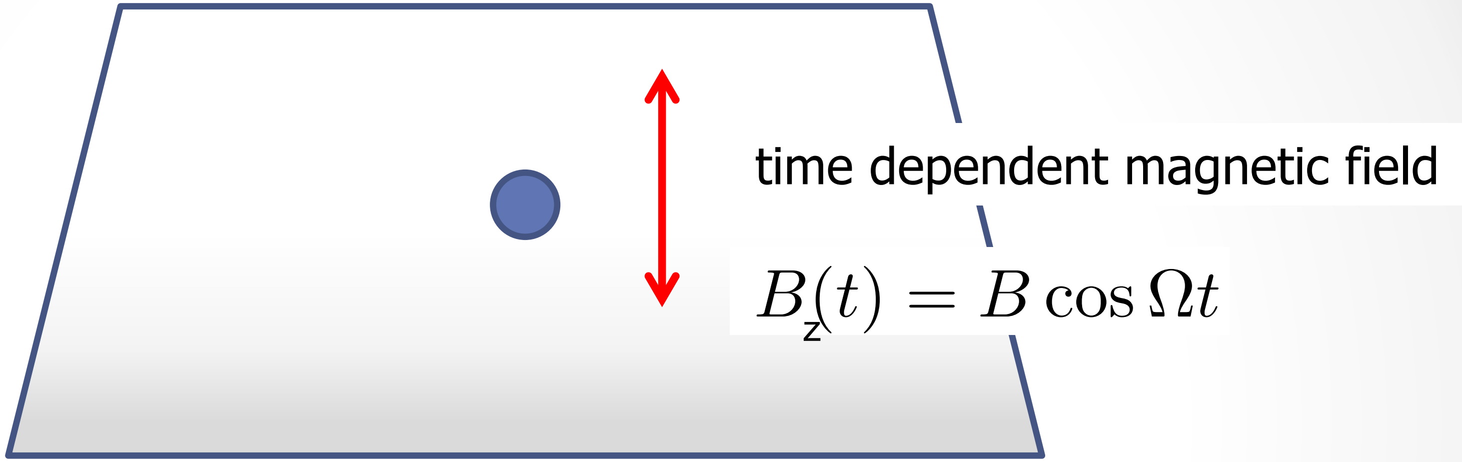


In the following, I will focus on the resonant case

$$\omega=\Omega$$

Example 1: Classical particle in an oscillating B field

TO, Bucciantini, PRB'16



Newton's equation

$$m\left(\frac{d}{dt}+\eta\right)\boldsymbol{v}(t) = e\left(\boldsymbol{E}(t)+\frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}(t)\right)$$

Example 1: Classical particle in an oscillating B field

TO, Bucciantini, PRB'16

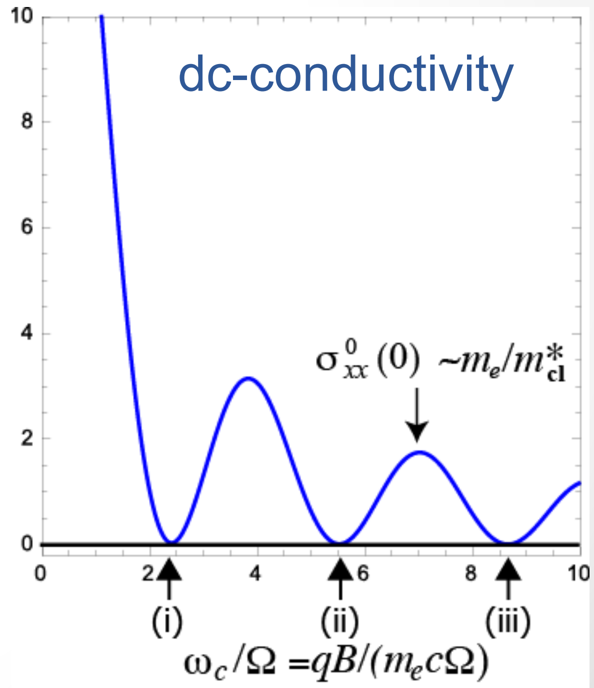
$$B(t) = B \cos \Omega t$$

no E-field

$\omega_c = qB/m_e c$

cyclotron frequency

$$j_a(t) = \int dt' \sigma_{ab}(t,t') E_b(\omega) e^{-i\omega t'}$$
$$= \sum_n \sigma^n_{ab}(\omega) e^{-i(\omega + n\Omega)t} E_b(\omega)$$



with static E_y-field

Out of plane
Magnetic field
$B_z = B \cos \Omega t$

In plane
Electric field
E_y

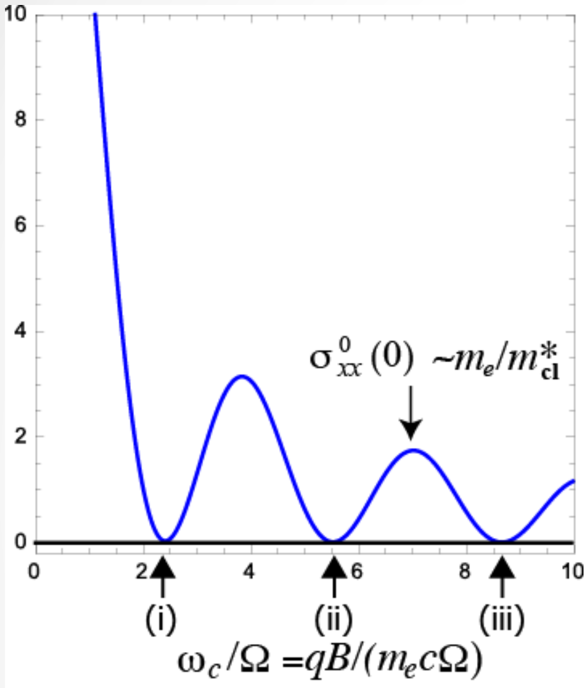
ω_c/Ω=3 5 6

(τ=0.05, E_y=1)

Periodic orbits

(τ=0.0, E_y=0)

Out of plane
Magnetic field
$B_z = B \cos \Omega t$



	ω_c/Ω=2.41	5.52	8.66
	(i)	(ii)	(iii)
winding per half cycle	α=1	α=2	α=3

Heterodyning Hall current

TO, Bucciantini, PRB'16

ω_c/Ω
$=3$

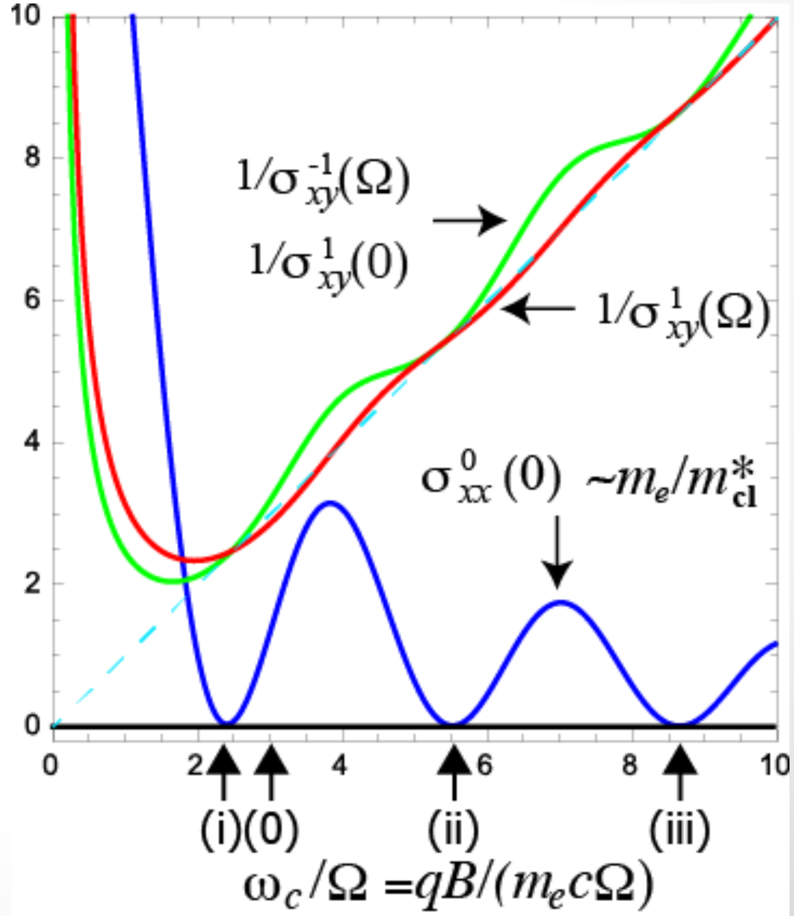
5

6

Out of plane
Magnetic field
$B_z = B\cos\Omega t$

In plane
Electric field
E_y

$$E_y^1 \cos\Omega t$$



exact results $\sigma_{ab}^{n,m} = \sigma_{ab}^{n-m}(m\Omega)$

$$\sigma_{xy}^{0,0} + i\sigma_{yy}^{0,0} = i\frac{e^2 n_e}{m_e}\sum_{n=-\infty}^{\infty}\frac{J_n(r)^2}{\eta - i\Omega n},$$

$$\sigma_{xy}^{1,0} + i\sigma_{yy}^{1,0} = i\frac{e^2 n_e}{m_e}\sum_{n=-\infty}^{\infty}\frac{J_n(r)J_{1-n}(r)(-1)^n}{\eta + i\Omega(1-n)}$$

$$\sigma_{xy}^{0,1} + i\sigma_{yy}^{0,1} = i\frac{e^2 n_e}{m_e}\sum_{n=-\infty}^{\infty}\frac{J_n(x)J_{n-1}(x)}{\eta + i\Omega n}$$

Summary: Example I

TO, Bucciantini, PRB'16

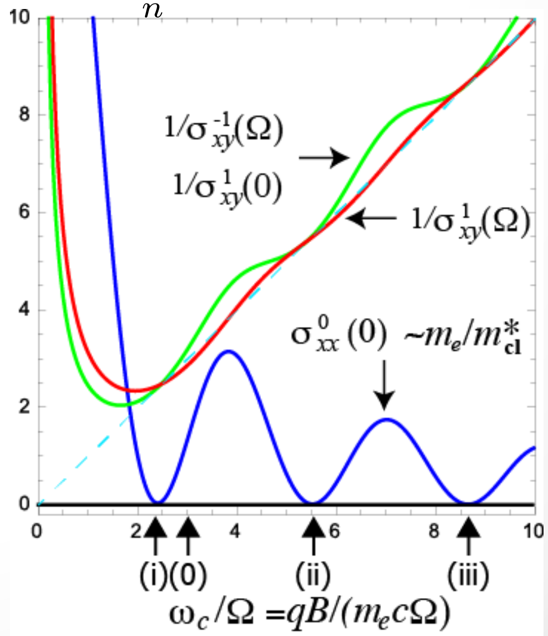
Classical particle in an oscillating B-field

heterodyne Hall ``insulator"

$$\sigma^0_{xx}(0) = 0 \qquad \sigma^{-1}_{xy}(\Omega) \sim \frac{1}{B}$$

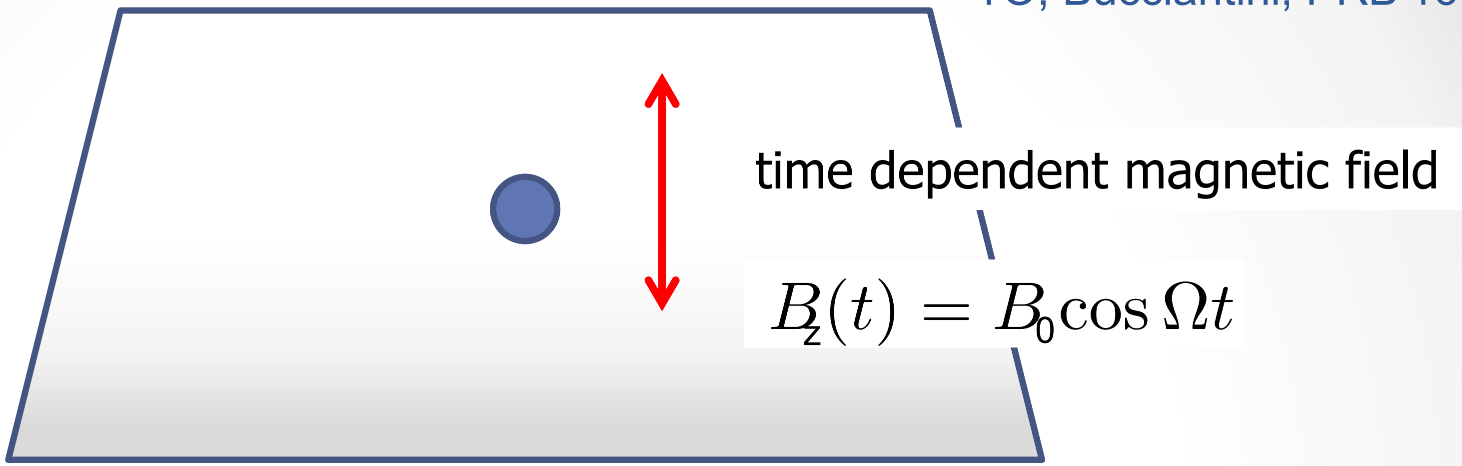
@ magic frequencies ((i), (ii), (iii),.. zeros of $J_0(B/\Omega)$)

$$j_a(t) = \sum_n \sigma^n_{ab}(\omega) e^{-i(\omega+n\Omega)t} E_b(\omega)$$



Example 2: Quantum particle in oscillating B field

TO, Bucciantini, PRB'16



Newton's equation

$$m\left(\frac{d}{dt}+\eta\right)\boldsymbol{v}(t) = e\left(\boldsymbol{E}(t)+\frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}(t)\right)$$

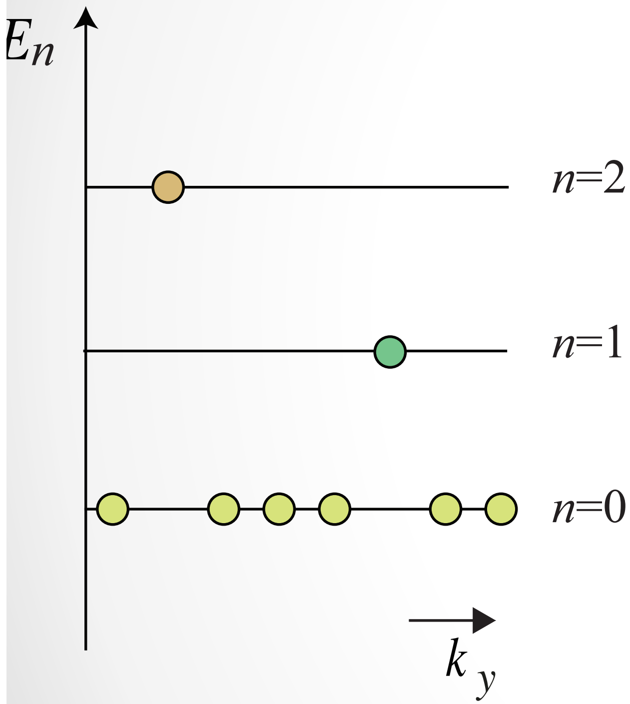
Schrodinger equation

$$H = \frac{1}{2m}\left[\hat{p_x}^2 + (p_y - \underbrace{B_0\cos(\Omega t)x}_{=A_y})^2\right]$$

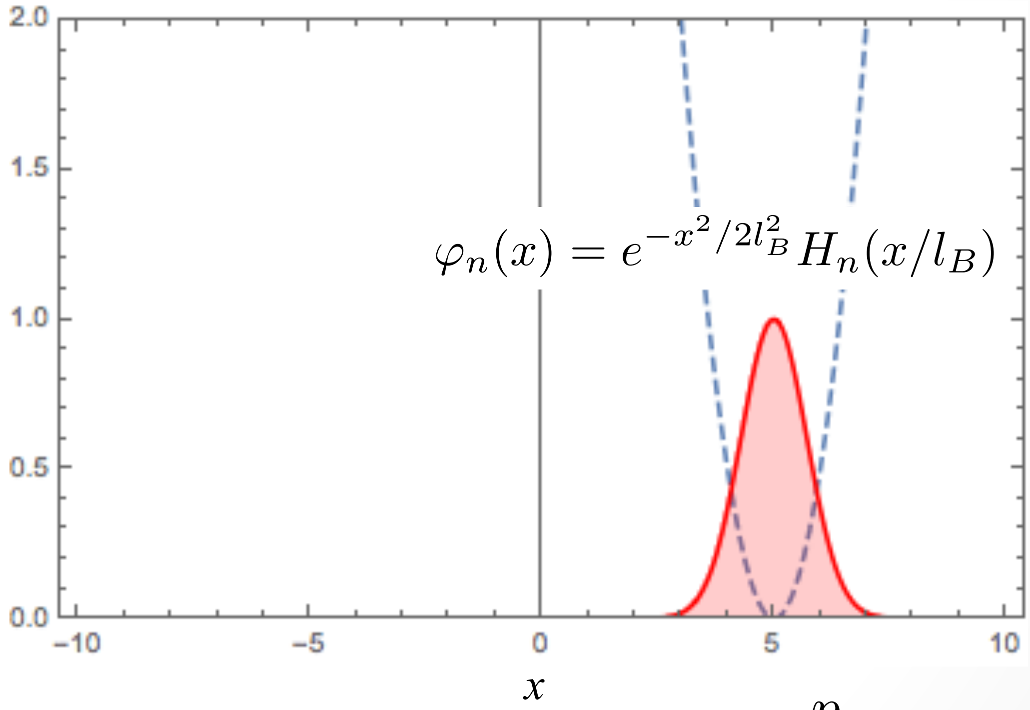
Solvable by Husimi transformation

Quantization: static B

$$H = \frac{1}{2m}\left[\hat{p_x}^2 + (p_y - \underset{\substack{\shortparallel \\ A_y}}{Bx})^2\right]$$



level spacing
$\omega_c = qB/m_e c$
cyclotron frequency



$l_B = B^{-1/2}$

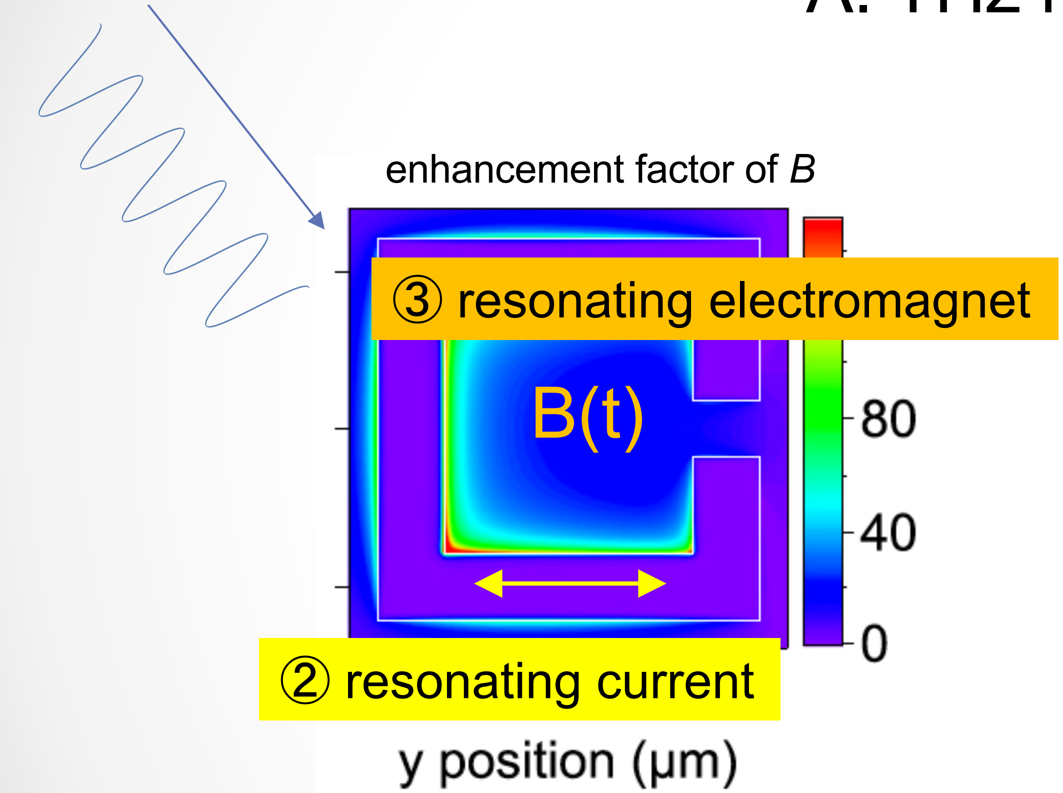
$$X = \frac{p_y}{B}$$

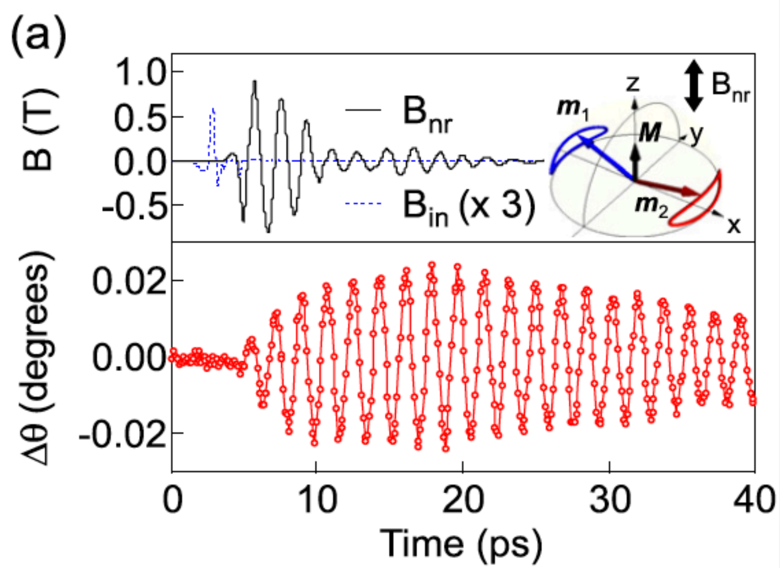
momentum-position locking

How to realize $A_y = B\cos(\Omega t)x$?

① incoming laser

A. THz metamaterial



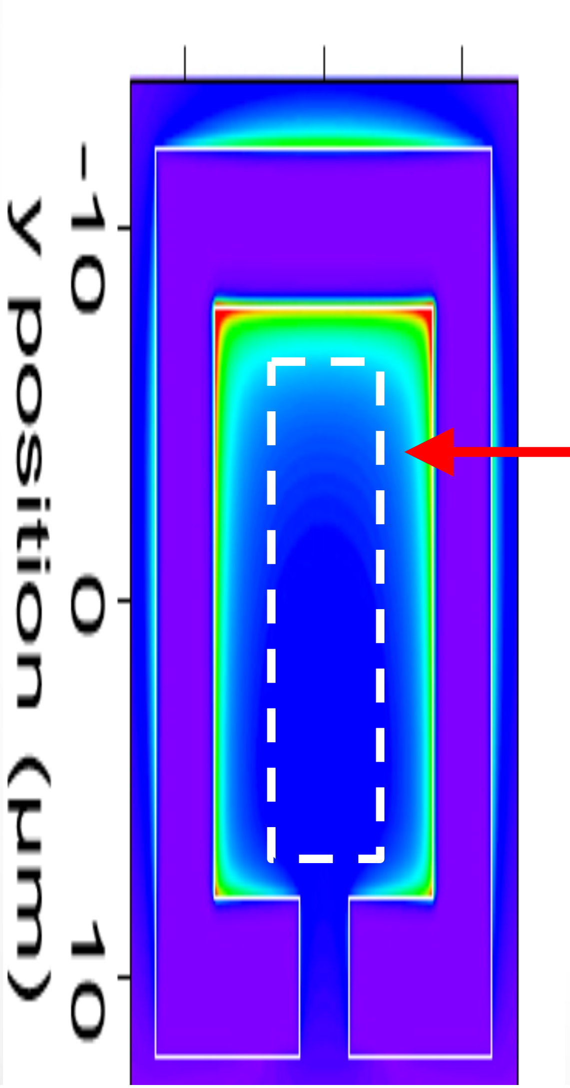


Y. Mukai, K. Tanaka, *et al.* (Kyoto grp.) New J. Phys.'16

<u>1 Tesla, 1 THz!!</u>

cf) E-field enhancement (Liu, Nelson, Averitt, *et al.* Nature 12)

$$H = \frac{1}{2m}\left[\hat{p_x}^2 + (p_y - \underbrace{B_0\cos(\Omega t)x}_{= A_y})^2\right]$$



EM-fields

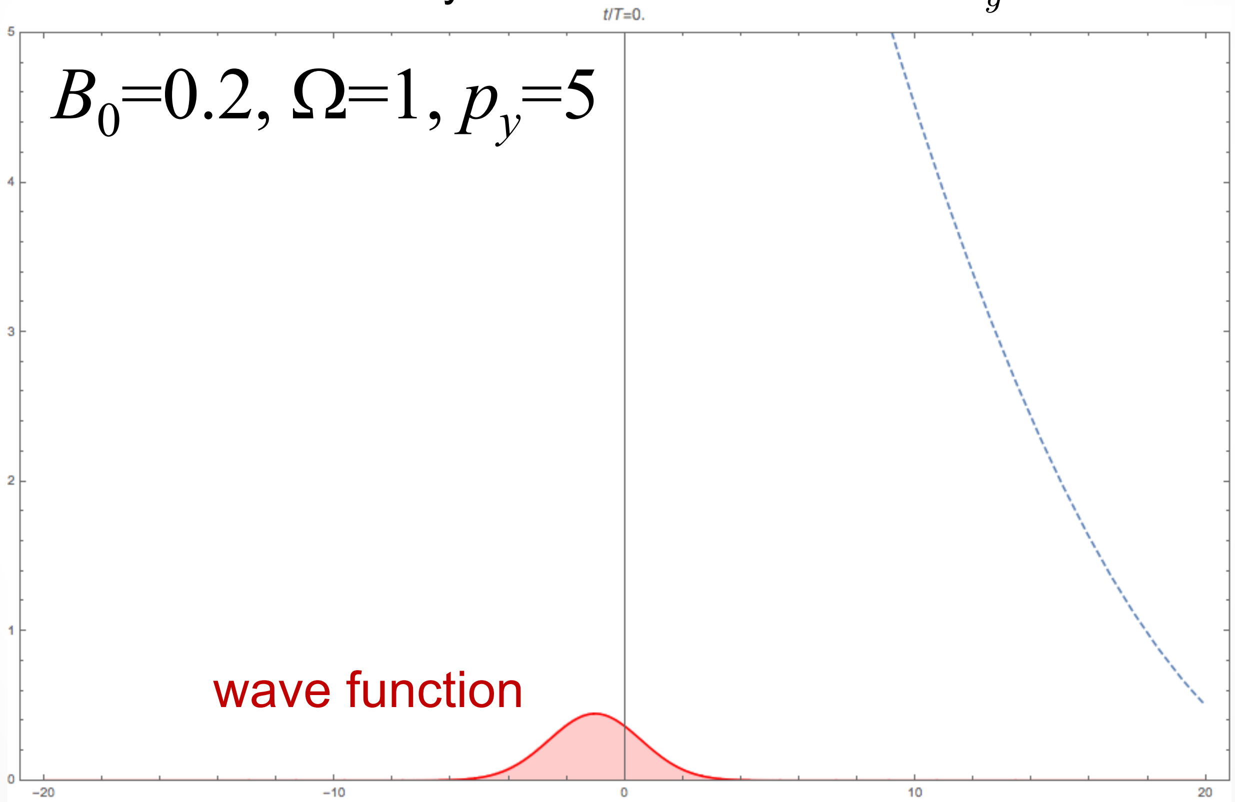
$$B_z = B\cos(\Omega t)$$
$$E_y = B\Omega\sin(\Omega t)x$$

Quantization: time-oscillating *B*

$$H = \frac{1}{2m}\left[\hat{p_x}^2 + \boxed{(p_y - \underbrace{B_0\cos(\Omega t)x}_{=A_y})^2}\right]$$

dotted line

Solvable by Husimi transformation



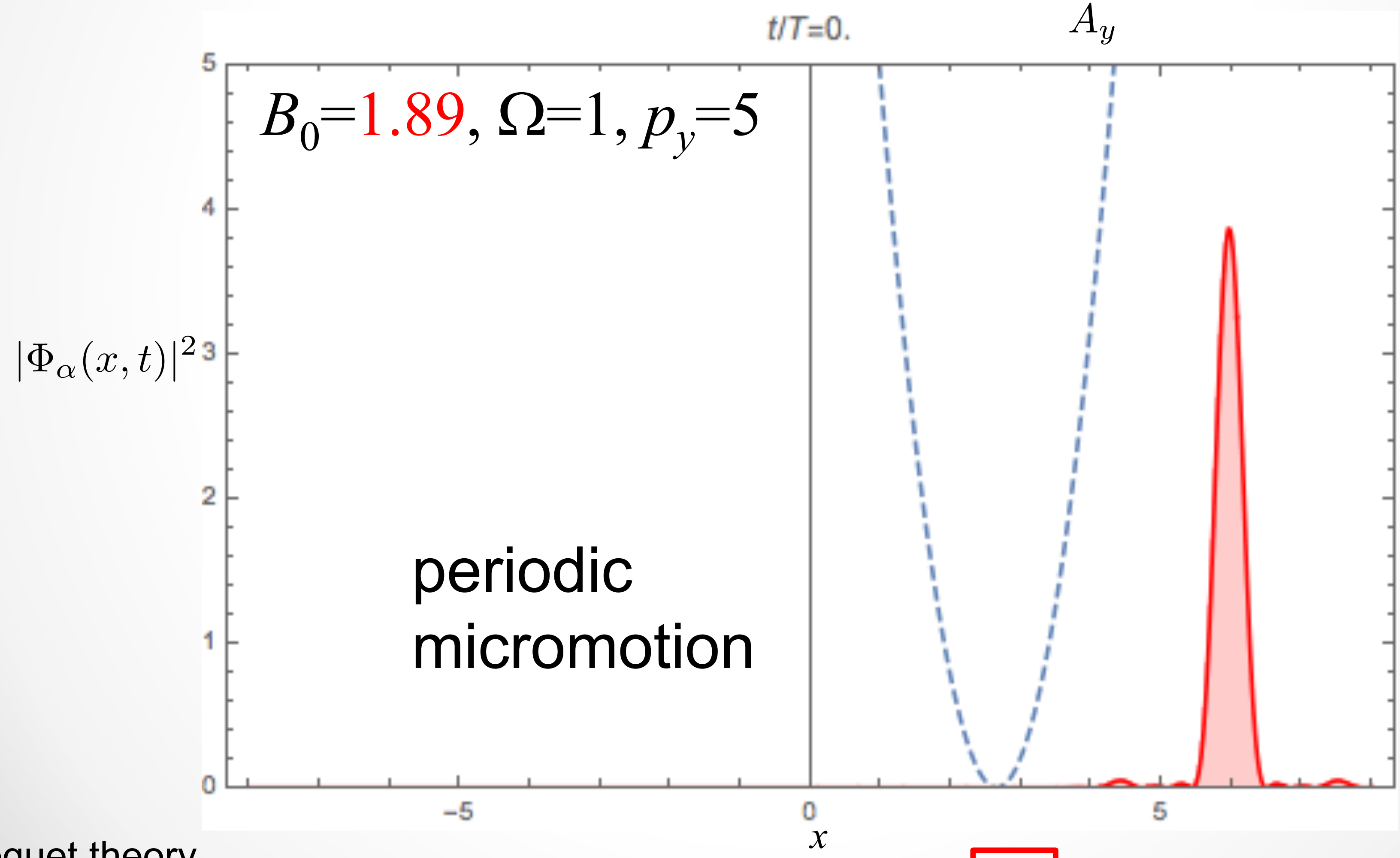
Floquet theory
see A. Eckardt RMP'16

Floquet state $[H - i\partial_t]\,|\Phi_\alpha\rangle = \varepsilon_\alpha|\Phi_\alpha\rangle$

Quantization: time-oscillating *B*

$$H = \frac{1}{2m}\left[\hat{p_x}^2 + \left(p_y - \underbrace{B_0 \cos(\Omega t) x}_{A_y}\right)^2\right]$$

dotted line



Floquet theory
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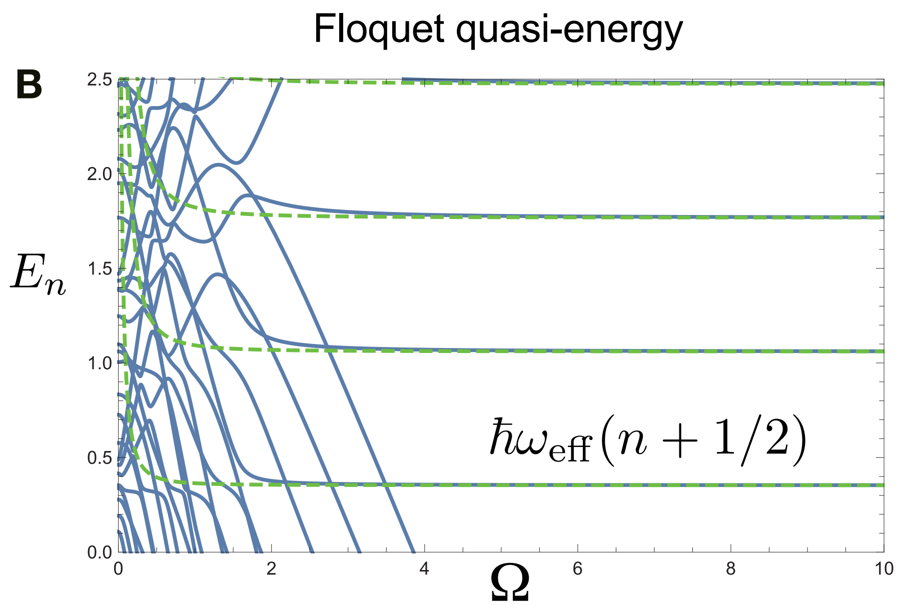
Floquet state $[H - i\partial_t]\,|\Phi_\alpha\rangle = \boxed{\varepsilon_\alpha}|\Phi_\alpha\rangle$

Floquet quasi-energy Spectrum

$$\varepsilon_n(p_y) = \boxed{E_n} + \frac{p_y^2}{2m_e^*}$$

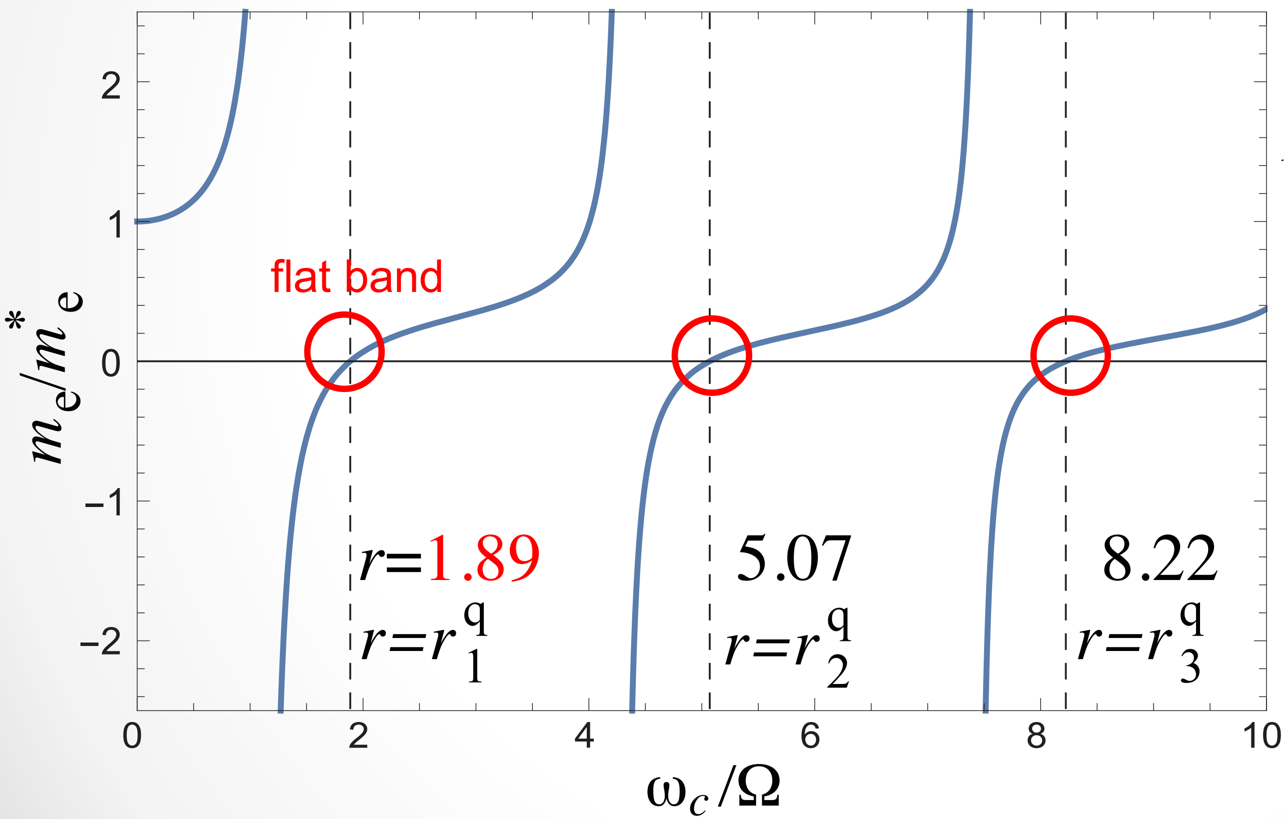


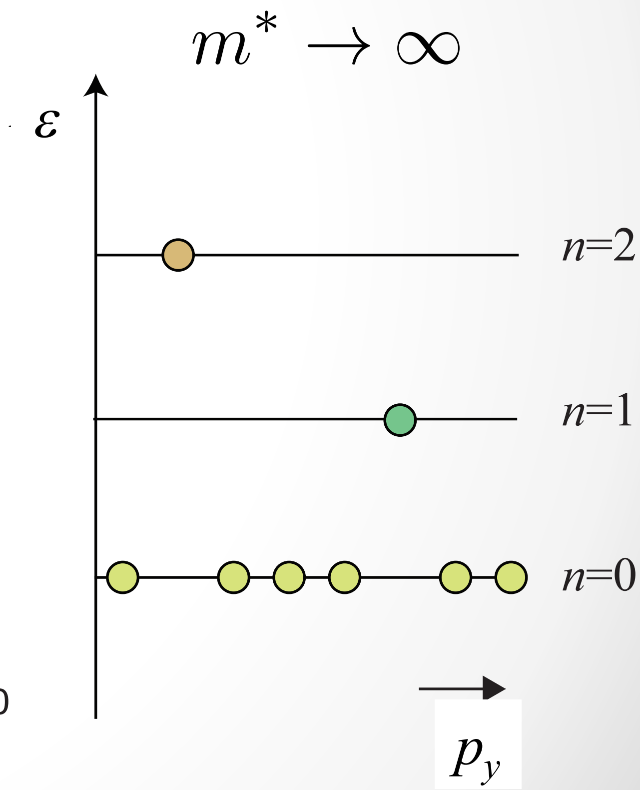
temporal mixture *n*-th Landau levels

Floquet quasi-energy Spectrum

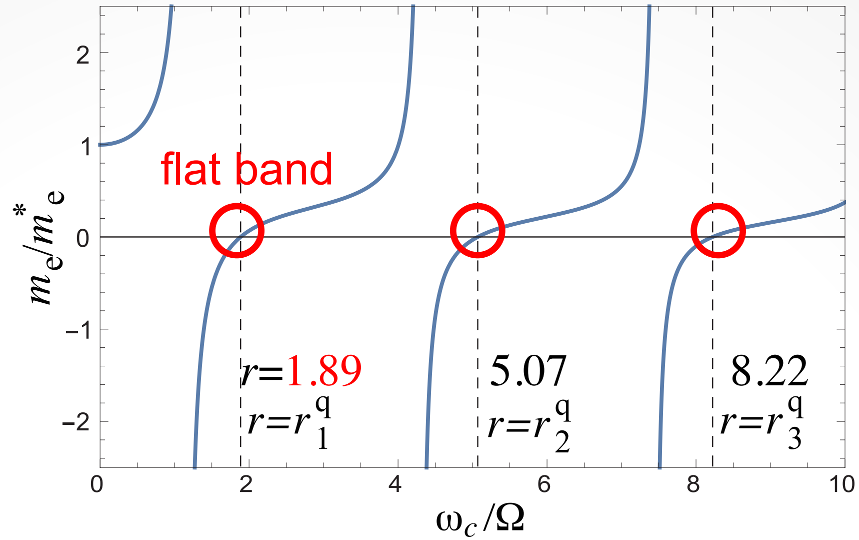
$$\varepsilon_n(p_y) = E_n + \frac{p_y^2}{2\boxed{m_e^*}}$$

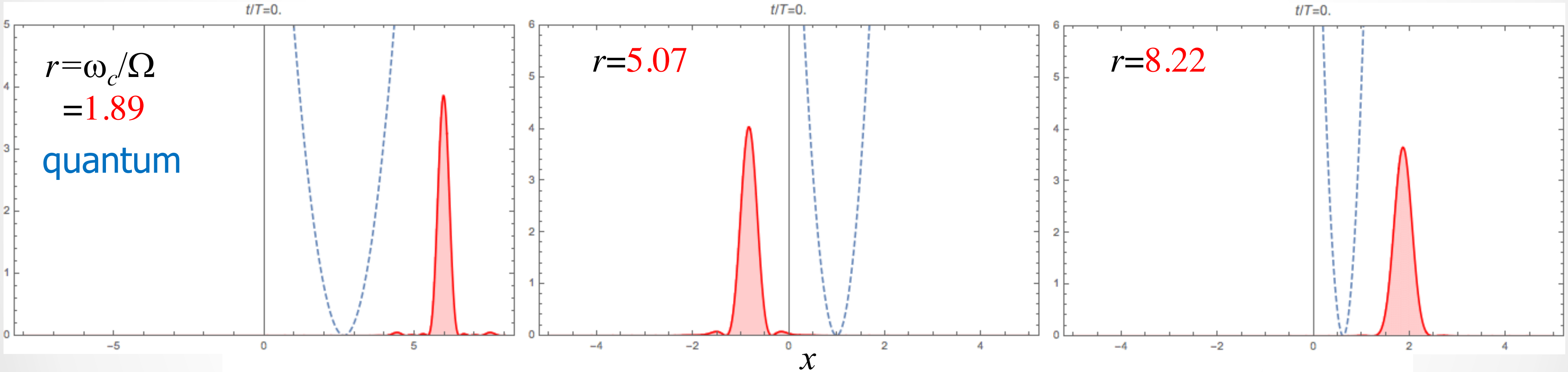
inverse effective mass





new Landau quantization





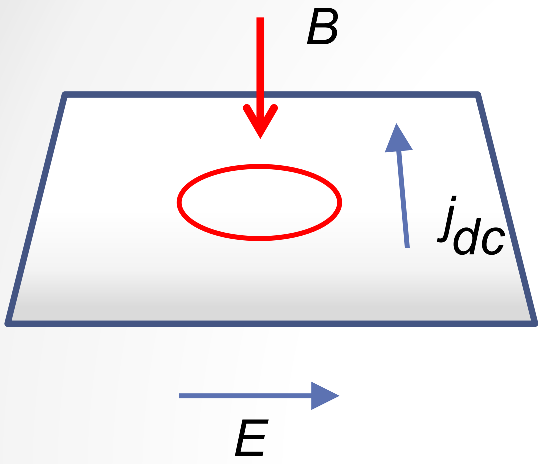
classical

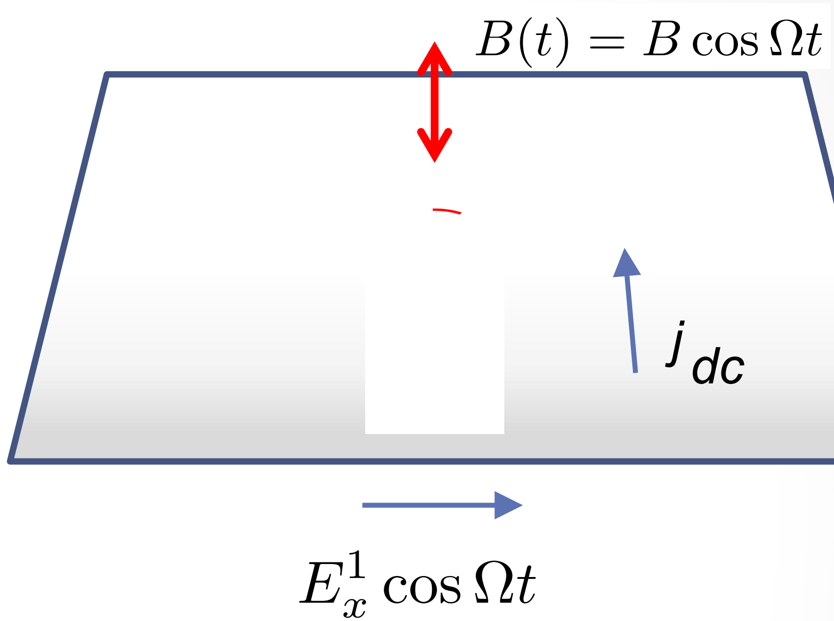
$r=\omega_c/\Omega$
=2.40

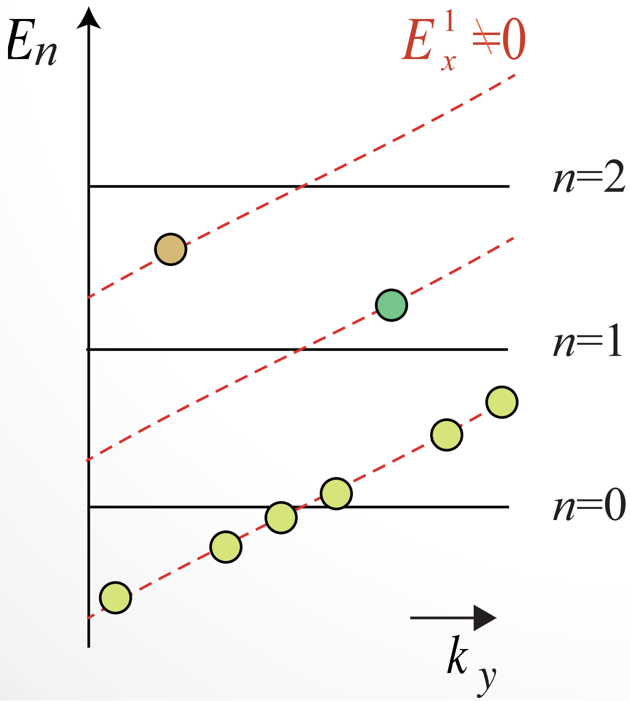
r=5.52

r=8.66

Dissipationless Heterodyne Hall current







$$j_y^{\mathrm{dc}} = \sigma_{yx}^{0,1} E_x^1$$

LL filling

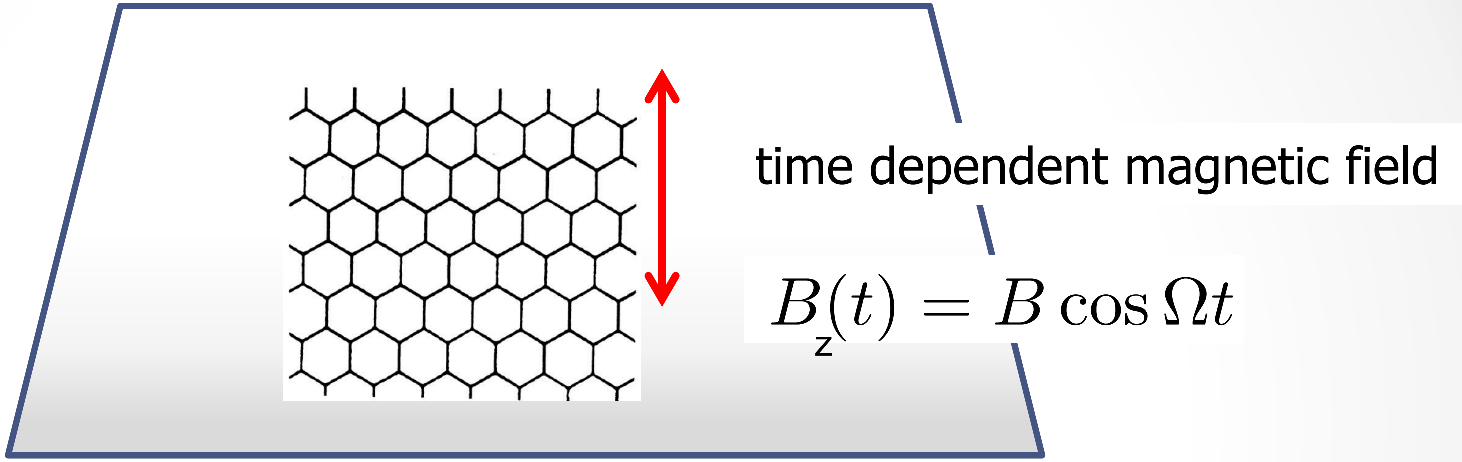
$$\sigma_{yx}^{0,1} = \frac{e^2}{h} Q\nu \qquad \nu = N_e/N_\Phi$$

Integer heterodyne Hall effect

pre-factor Q *is non-universal*

Example3: 2D Dirac electron in oscillating B field

TO, Kitamura, Nag, Saha, Bucciantini, *in prep*

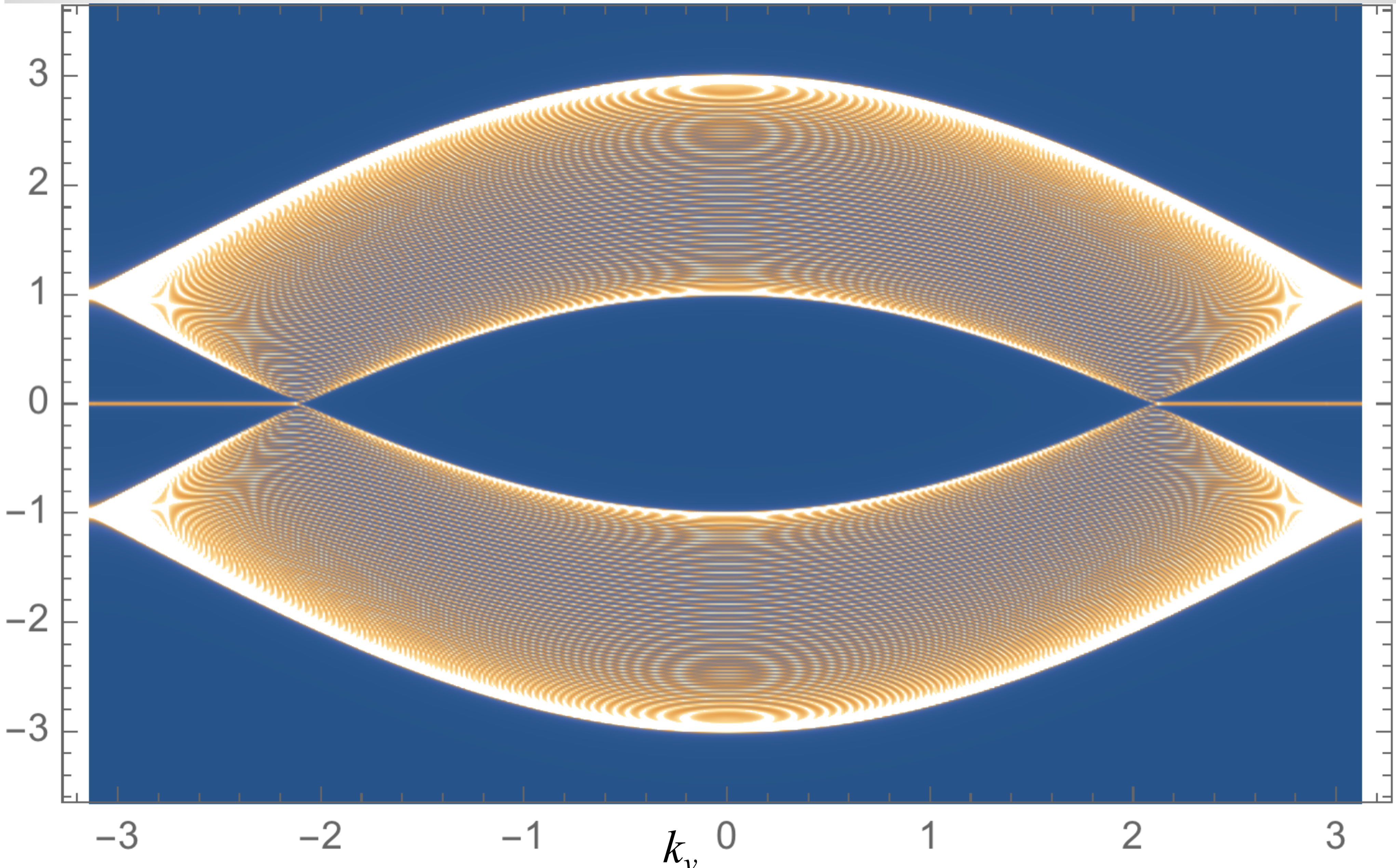


graphene, surface of 3D TI, …

Dirac equation

$$H_{\mathrm{Dirac}} = \sigma_x \hat{p}_x + \sigma_y (p_y - \underbrace{B\cos\Omega t x}_{\shortparallel \atop A_y})$$

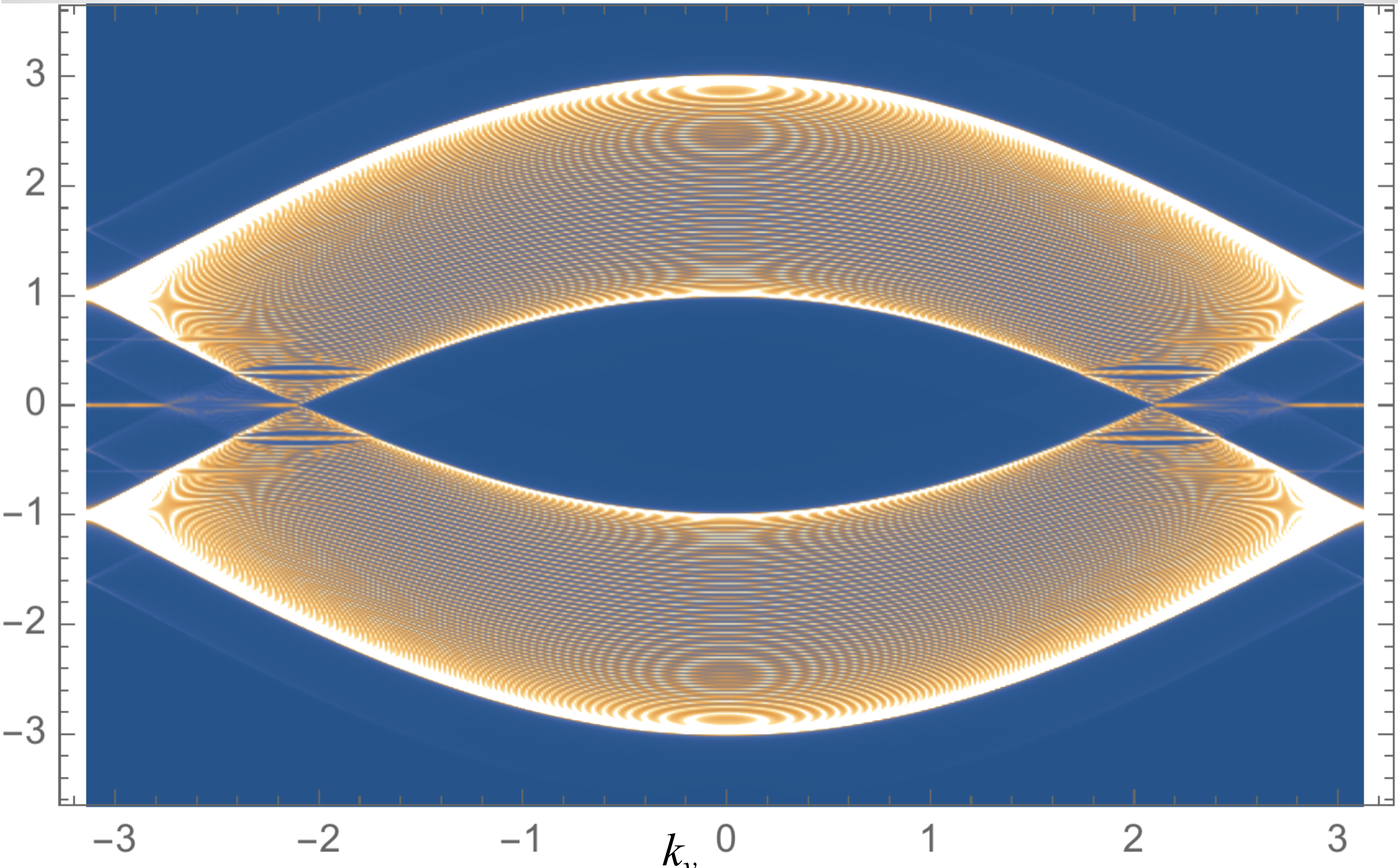
Spectrum $A(k,\omega) = \frac{-1}{\pi}\mathrm{Im}\mathrm{Tr}\hat{\mathrm{P}}_{\mathrm{static}}\frac{1}{\omega - \hat{H}_k + i\delta}$



Ω=*0.6*, *B/a*=0.000, E_x=0.0

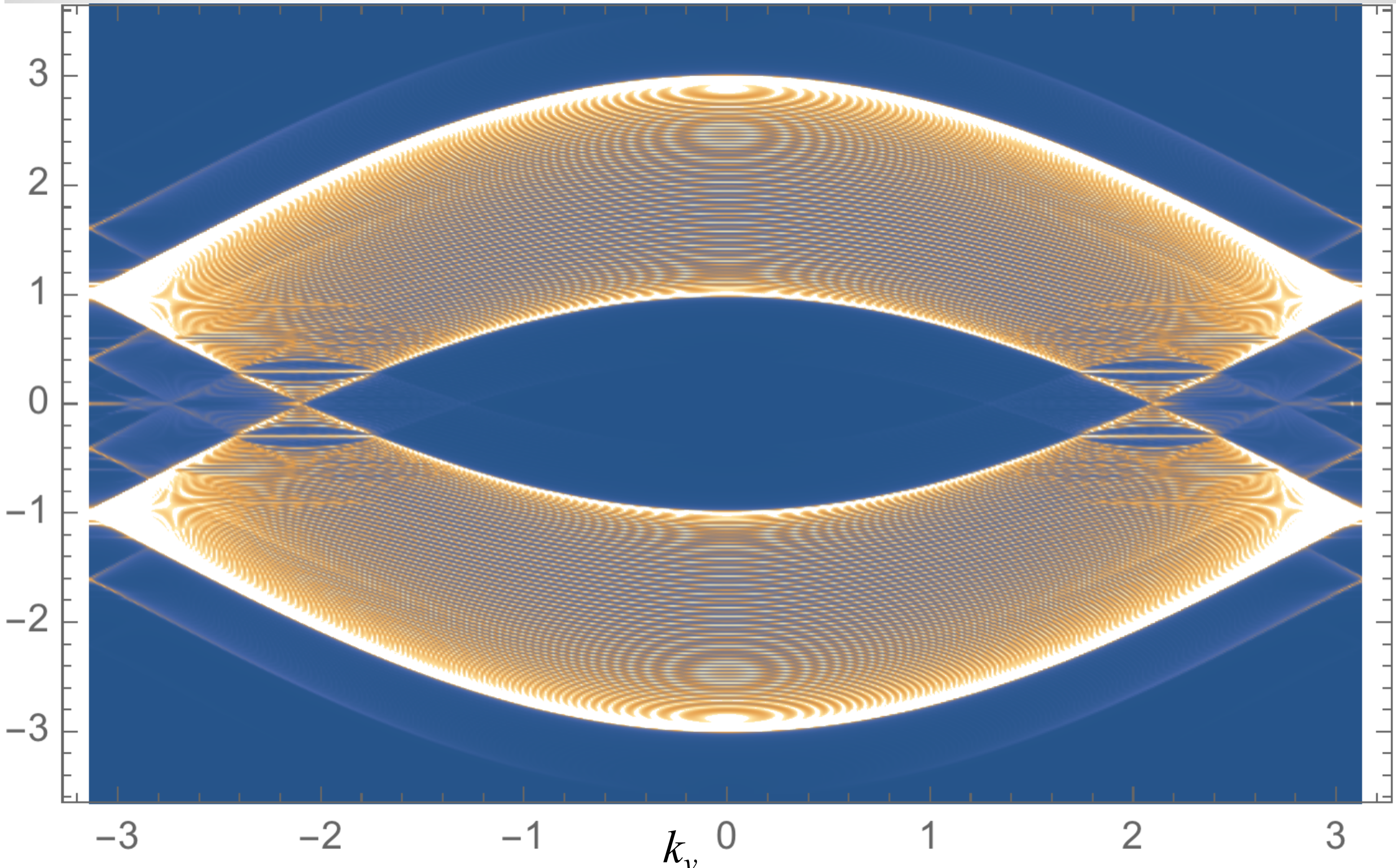
honeycomb, zigzag edge

Spectrum $A(k,\omega) = \frac{-1}{\pi} \mathrm{Im}\mathrm{Tr}\hat{\mathrm{P}}_{\mathrm{static}} \frac{1}{\omega - \hat{H}_k + i\delta}$



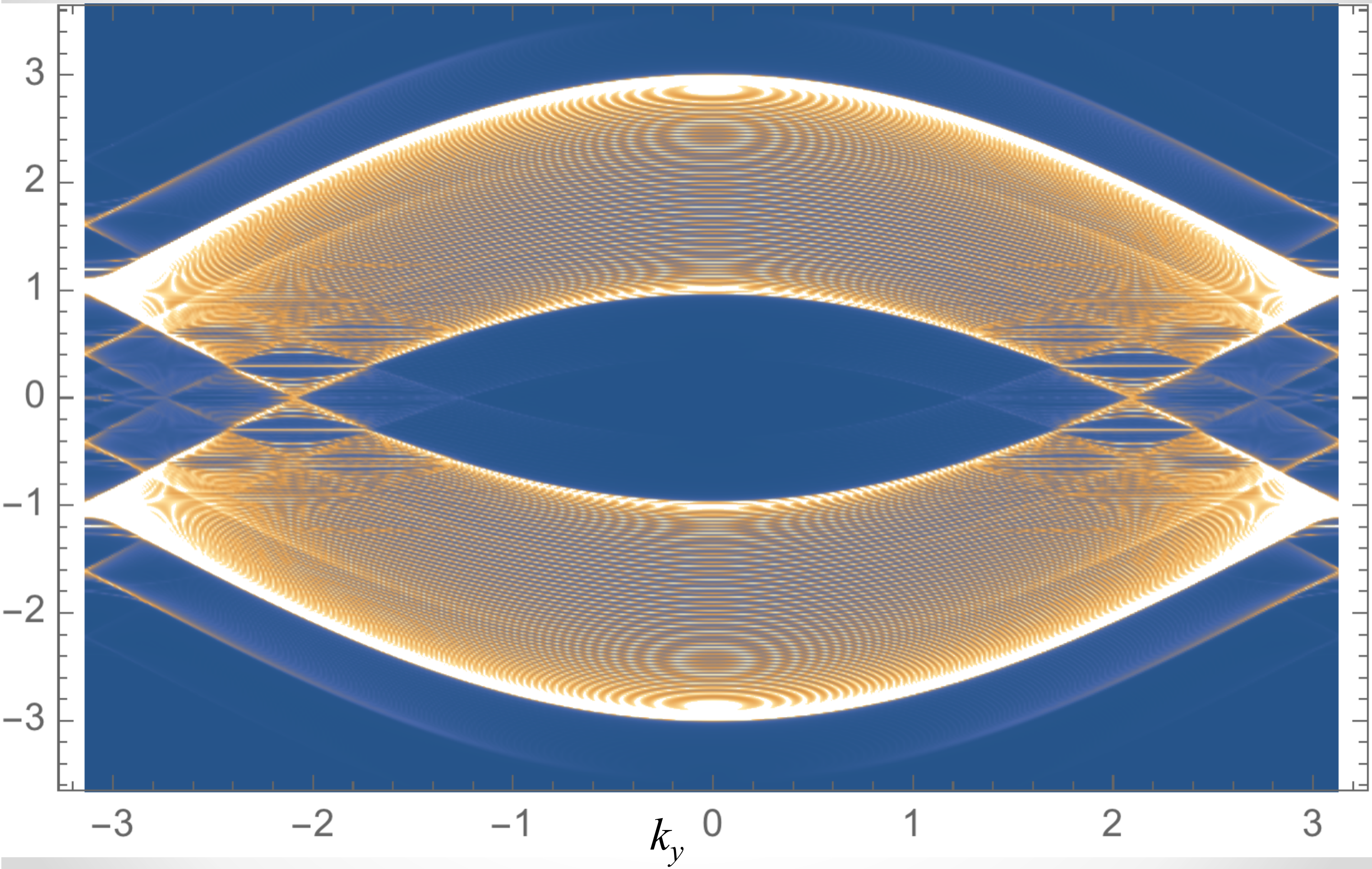
Ω=*0.6*, *B/a*=0.0010, E_x=0.0 honeycomb, zigzag edge

Spectrum $A(k,\omega) = \frac{-1}{\pi}\mathrm{Im}\mathrm{Tr}\hat{\mathrm{P}}_{\mathrm{static}}\frac{1}{\omega - \hat{H}_k + i\delta}$



Ω=*0.6, B/a*=0.0020, E_x=0.0 honeycomb, zigzag edge

Spectrum $A(k,\omega) = \frac{-1}{\pi}\mathrm{Im}\mathrm{Tr}\hat{\mathrm{P}}_{\mathrm{static}}\frac{1}{\omega - \hat{H}_k + i\delta}$



Ω=*0.6*, *B/a*=0.0030, E_x=0.0

honeycomb, zigzag edge

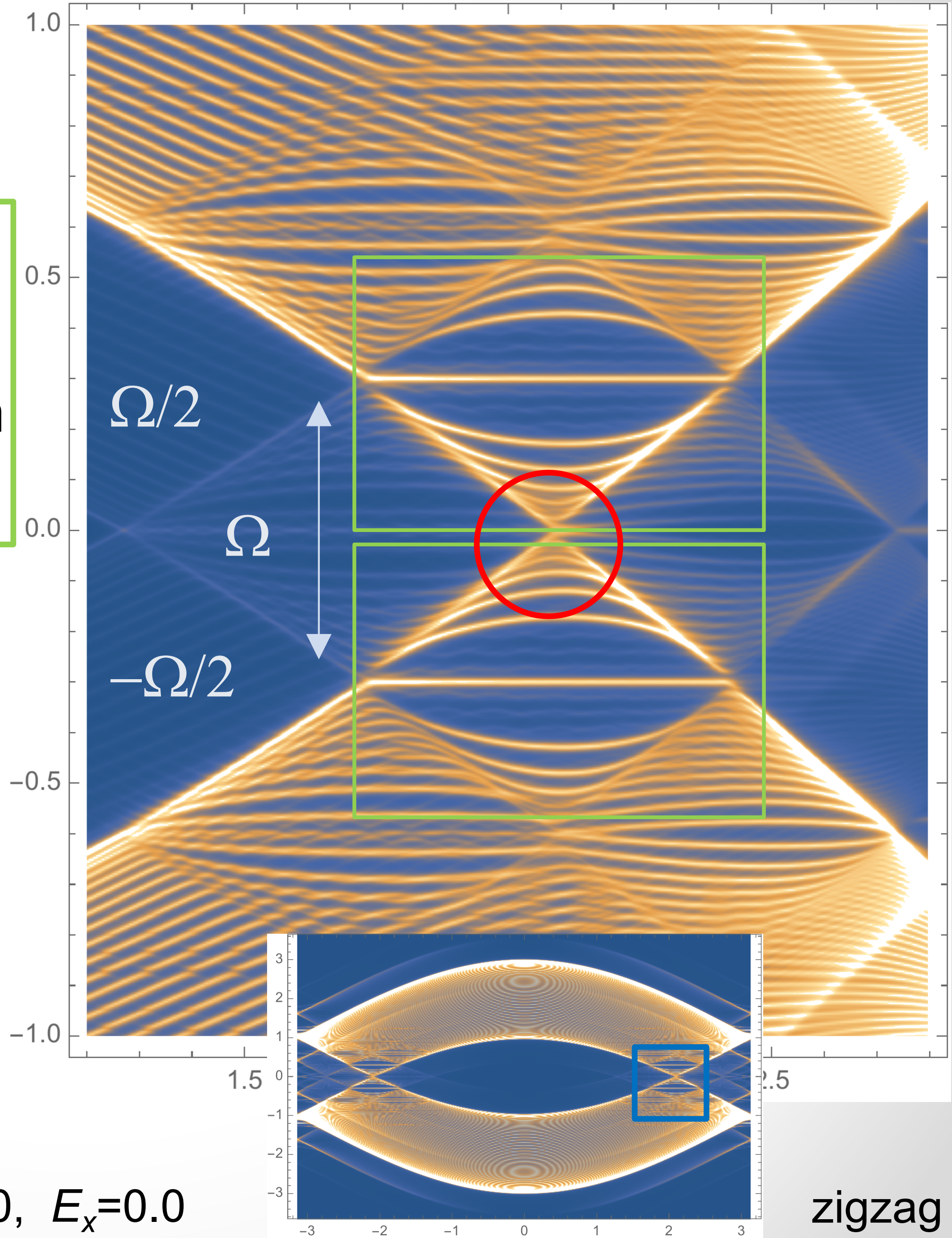
Spectrum $A(k,\omega) = \frac{-1}{\pi}\mathrm{Im}\mathrm{Tr}\hat{\mathrm{P}}_{\mathrm{static}}\frac{1}{\omega - \hat{H}_k + i\delta}$

"π-Landau levels"

- π-Flat bands at $\varepsilon = \pm\Omega/2$
- A series of bands around them
- electron-hole resonant state

Dirac node

preserved



Ω=*0.6*, *B/a*=0.0030, E_x=0.0

zigzag

Effective Hamiltonian

"π-Landau levels"

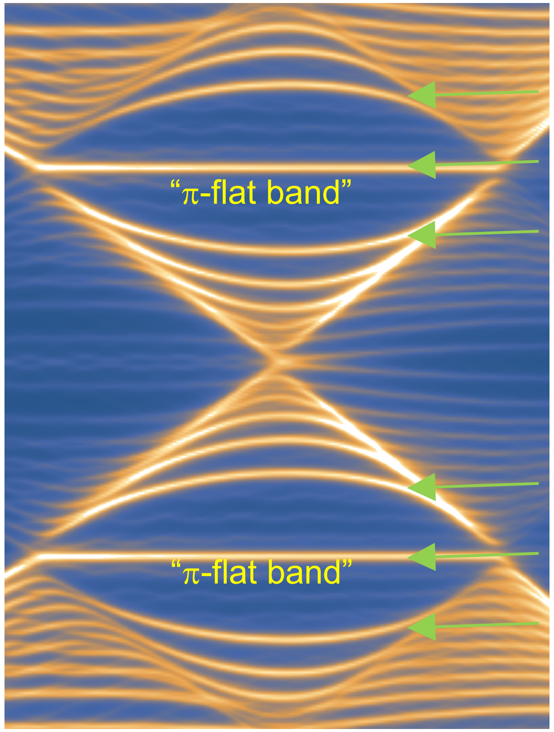
$$H_{\mathrm{Dirac}} = \sigma_x \hat{p}_x + \sigma_y (p_y - B\cos\Omega t x)$$

⇓ rotating frame transformation

$$H_{\mathrm{eff}} = \cos\theta \begin{pmatrix} 0 & -i\partial_x + i\frac{B}{2}x \\ -i\partial_x - i\frac{B}{2}x & 0 \end{pmatrix}$$

Landau levels of 2D Dirac system

$$\varepsilon_n = \sqrt{\Omega^2 - p_z^2}\sqrt{Bn} \pm \Omega/2$$

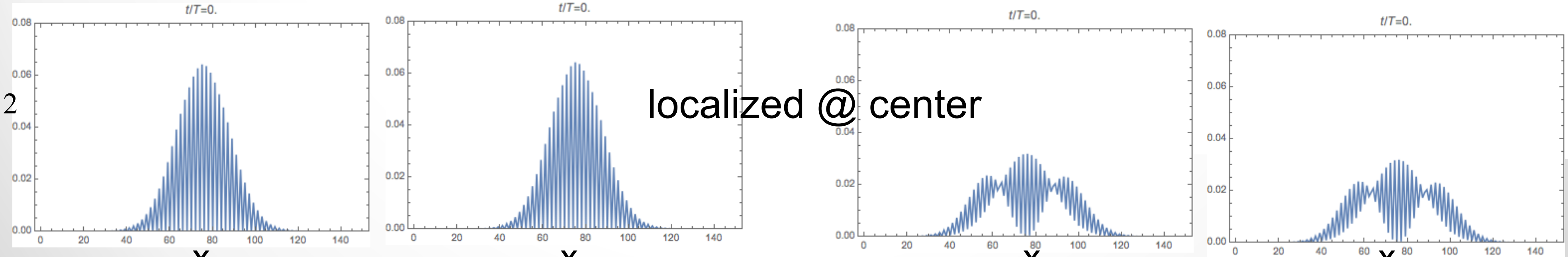


n=1
n=0
n=-1
n=1
n=0
n=-1

two n=0 states

two n=1 states

$|\psi|^2$

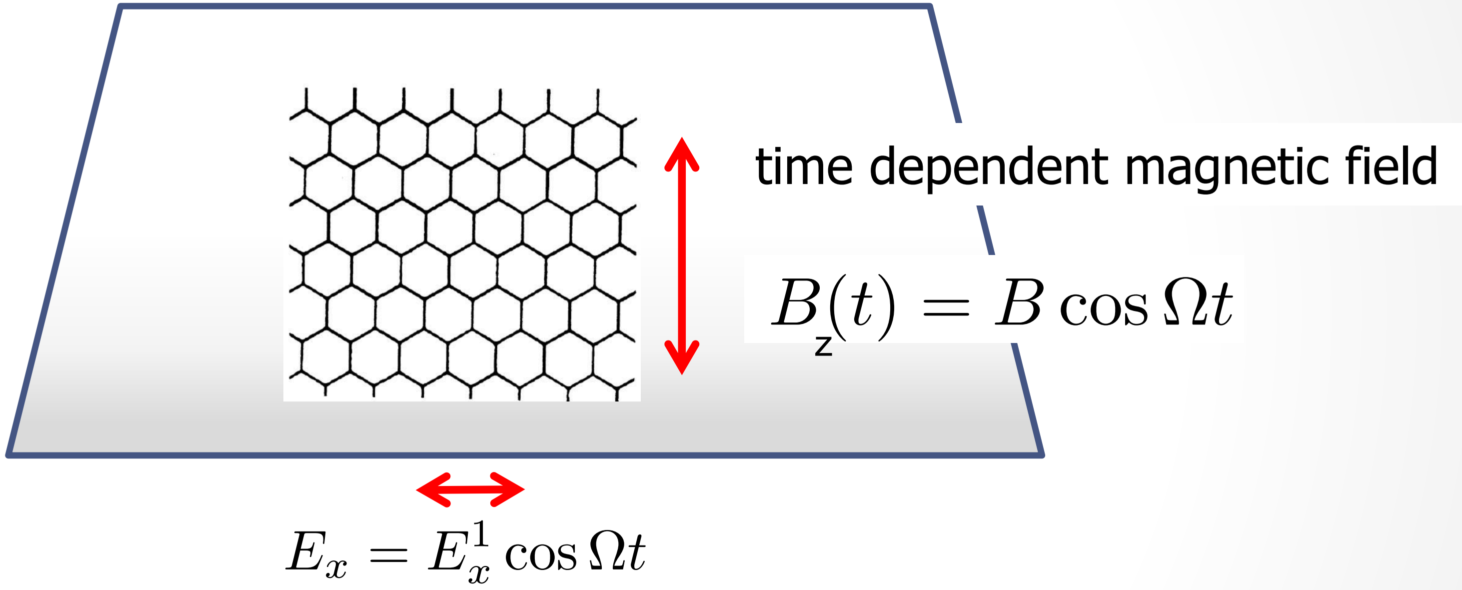


x x x x

localized @ center

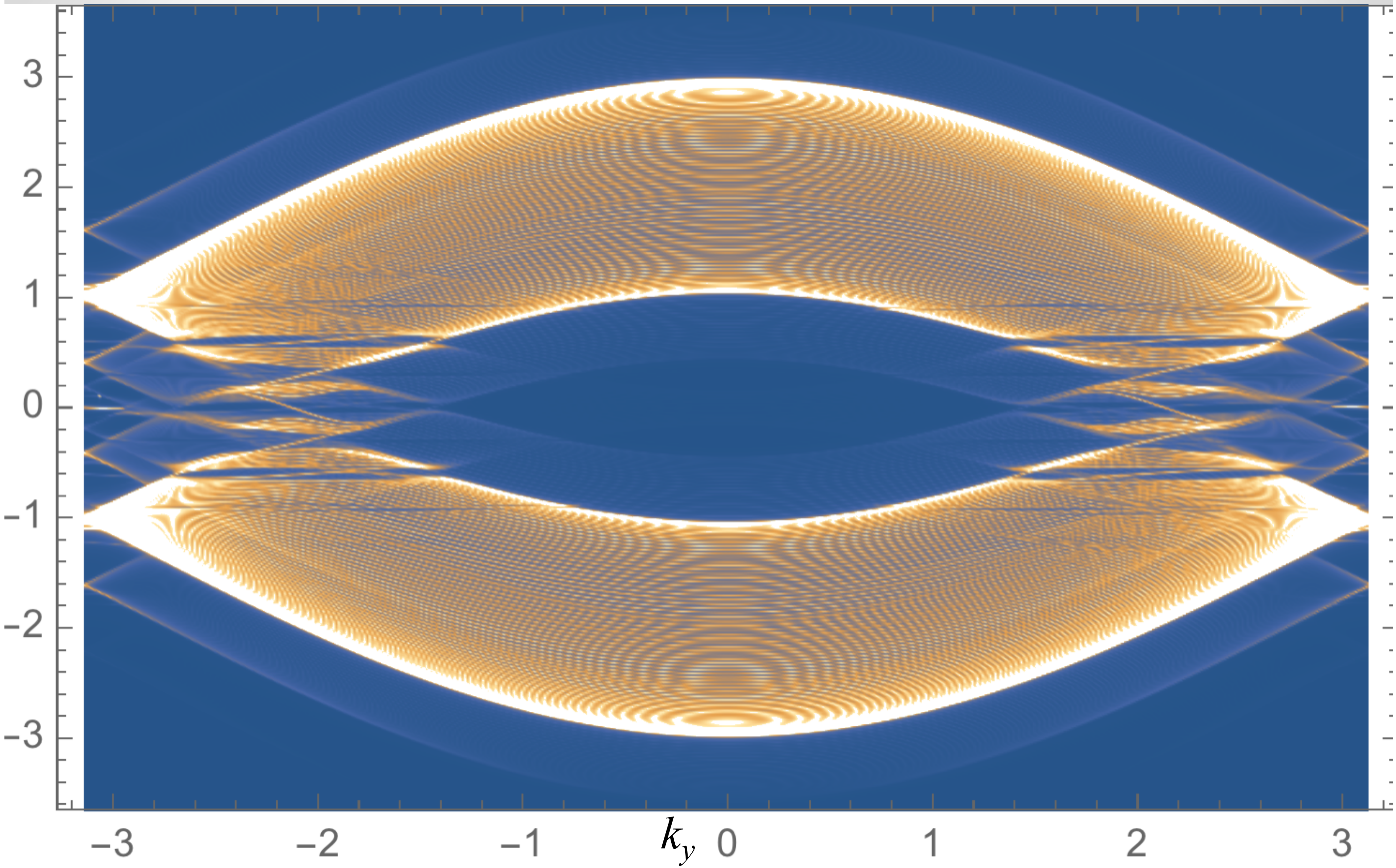
The flat band is protected by time-glide symmetry (Morimoto-Po-Vishwanath'17)

Heterodyne Hall effect



additional ac-electric field

Heterodyne Hall effect (add B and E)



Ω=*0.6*, *B/a*=0.0020, E_x=0.20 honeycomb, zigzag edge

Heterodyne Hall effect (add B and E)

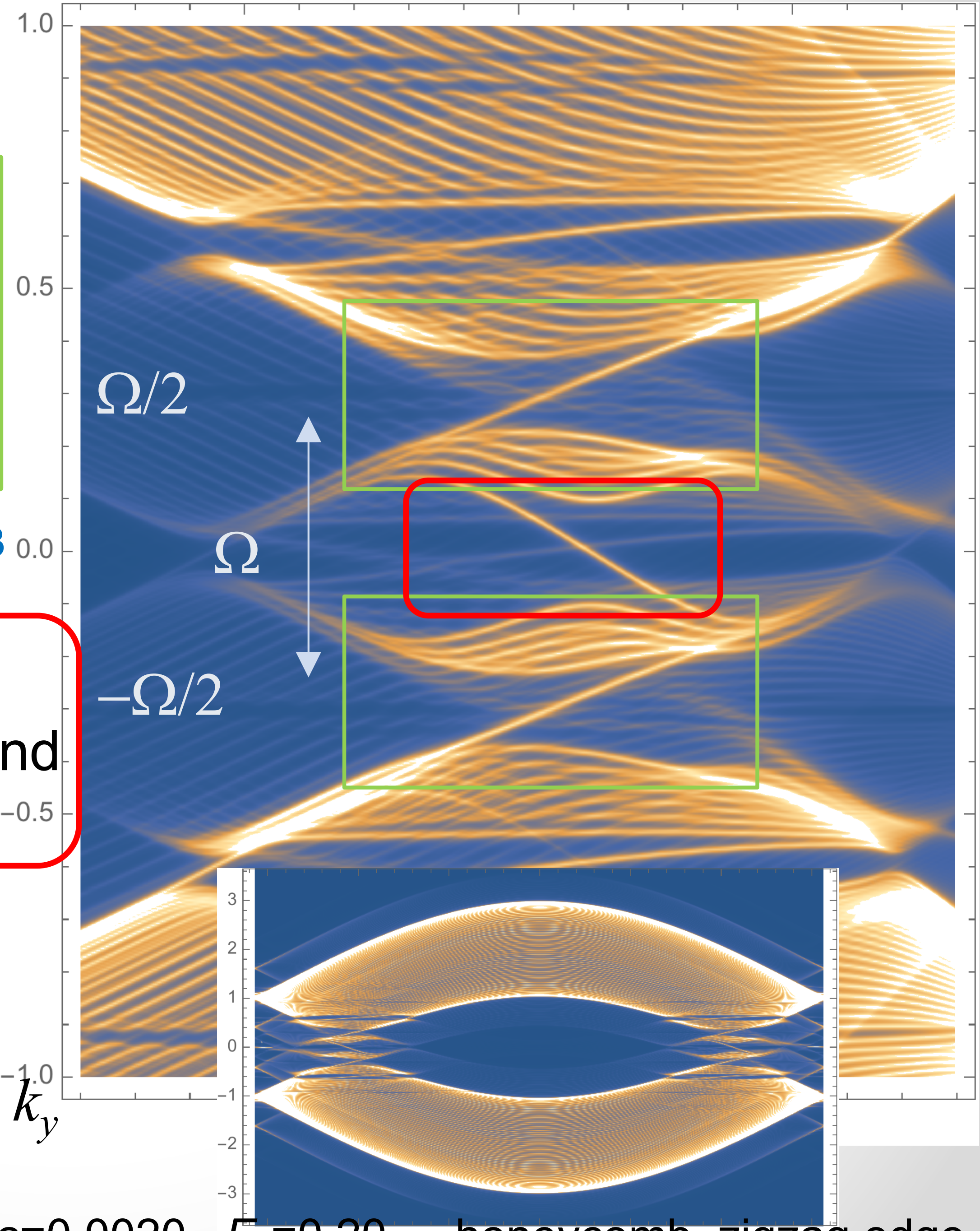
“π-Landau levels”

π-Flat bands at ε=±Ω/2 tilts
= π-chiral “center” mode
→ current in y-direction

cf) π-edge state: Rudner-Lindner-Berg-Levin ‘13

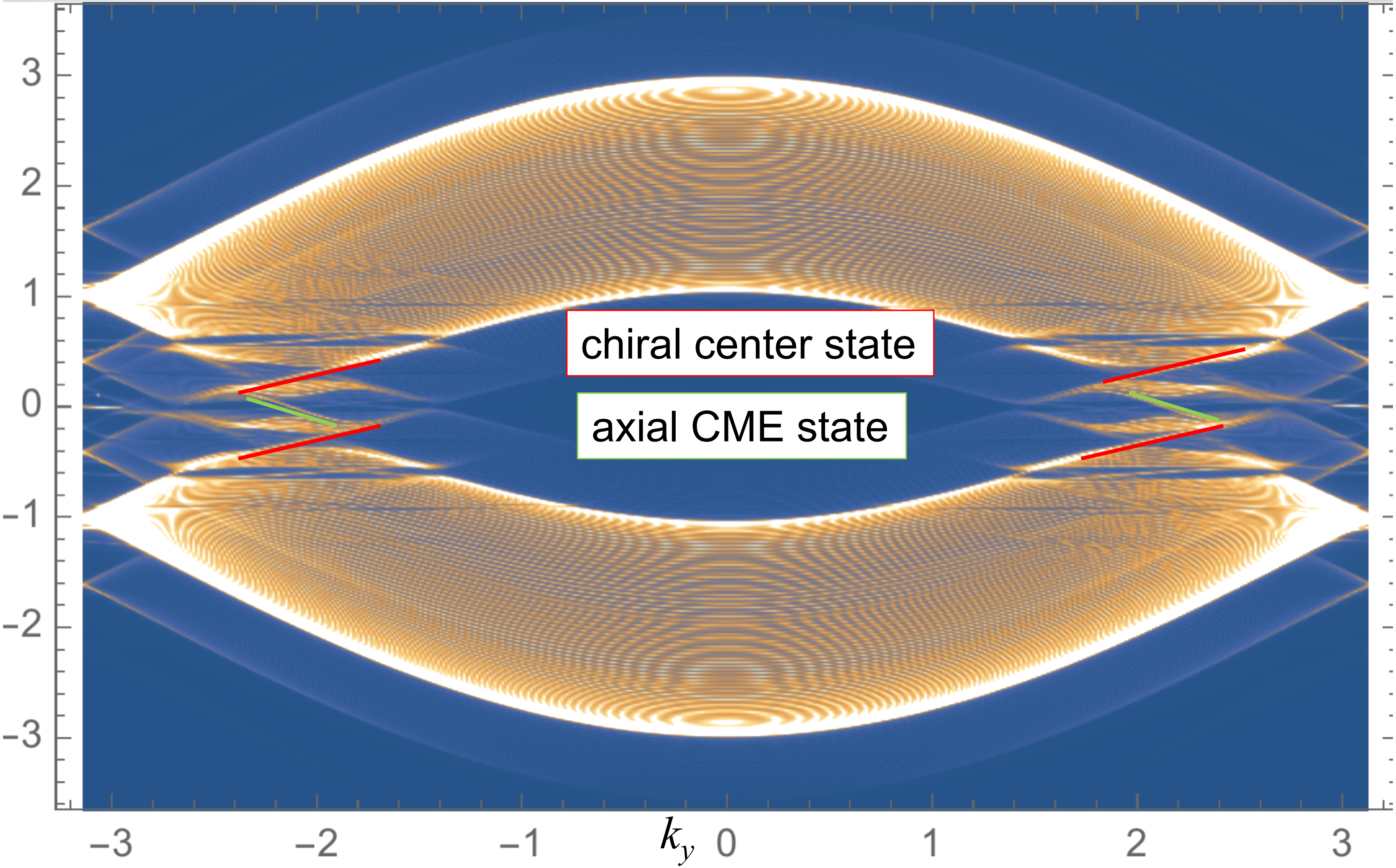
Dirac node
axial chiral magnetic effect-like band
→ current in (-y)-direction

CME: 3D Weyl in E,B fields
Fukushima-Kharzeev-Warringa’08



Ω=0.6, B/a=0.0020, E_x=0.20 honeycomb, zigzag edge

Heterodyne Hall effect



Ω=*0.6*, *B/a*=0.0020, E_x=0.20 honeycomb, zigzag edge

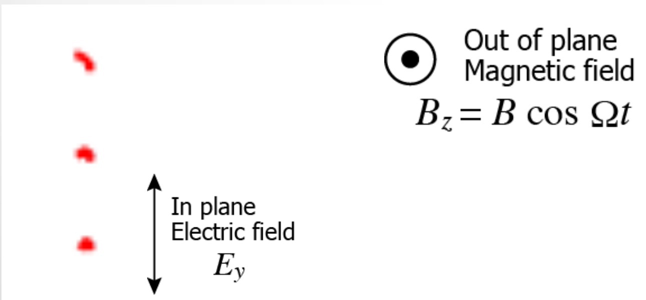
Summary

- Heterodyne Hall effect in three examples was studied
- They are characterized by the heterodyne response functions

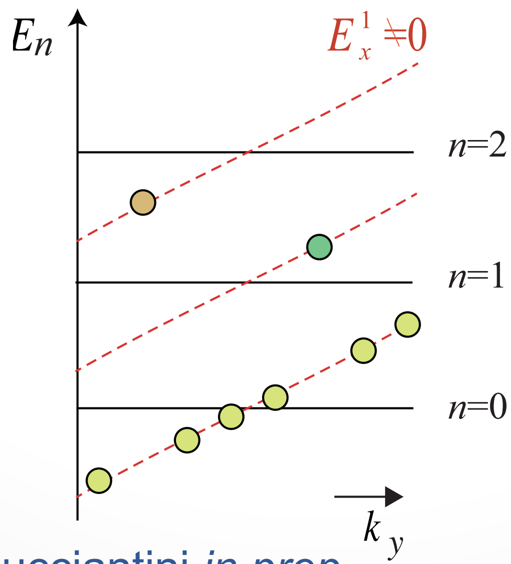
$$\sigma_{xy}^{n}(\omega)$$

ongoing: Relation with topology, interaction (fractional state)

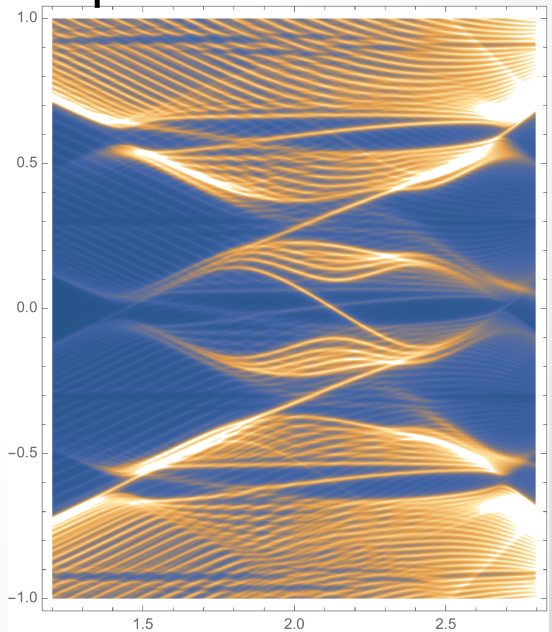
classical particle



quantum 2DEG



quantum 2DDirac



TO, Bucciantini, PRB'16
TO, Kitamura, Nag, Saha, Bucciantini *in prep*

Heterodyne Kubo formula

$$J_i^n(\omega) = \sigma_{ij}^n(\omega) E_j(\omega)$$

$$\sigma_{ij}^n(\omega) = \frac{1}{i\omega} \sum_{k,\alpha,\beta,m} f_\beta \left\{ \frac{[\varepsilon_{k\alpha} - \varepsilon_{k\beta} - m\Omega][(\varepsilon_{k\alpha} - \varepsilon_{k\beta}) + (n-m)\Omega]}{(\varepsilon_{k\alpha} - \varepsilon_{k\beta}) + (n-m)\Omega - \omega - i\delta} \mathcal{A}_{\beta i \alpha}^m \mathcal{A}_{\alpha j \beta}^{(n-m)} \right.$$
$$\left. - \frac{[\varepsilon_{k\beta} - \varepsilon_{k\alpha} - m\Omega][(\varepsilon_{k\beta} - \varepsilon_{k\alpha}) + (n-m)\Omega]}{(\varepsilon_{k\beta} - \varepsilon_{k\alpha}) + (n-m)\Omega - \omega - i\delta} \mathcal{A}_{\alpha i \beta}^m \mathcal{A}_{\beta j \alpha}^{(n-m)} \right\}$$

$$\langle \phi_{k\beta}(t) | \partial_{k_i} \phi_{k\alpha}(t) \rangle = \sum_m e^{im\Omega t} \mathcal{A}_{\beta i \alpha}^m$$

Floquet theory (non-perturbative in driving)

review: A. Eckardt, *RMP'16*

time dependent problem

$$i\partial_t \psi = H(t)\psi$$

$$H(t) = H(t+T)$$

$$\Omega = 2\pi/T$$

$$\psi(t) = e^{-i\varepsilon t}\phi(t)$$

$$\phi(t+T) = \phi(t)$$

Floquet state

eigenvalue problem

$$\mathcal{H}\phi_\alpha = \varepsilon_\alpha \phi_\alpha$$

$$\mathcal{H} = H(t) - i\partial_t$$

ε: Floquet quasi-energy

Fourier transformation

Floquet Hamiltonian

$$\sum_{m=-\infty}^{\infty} \mathcal{H}^{mn}\phi_\alpha^m = \varepsilon_\alpha \phi_\alpha^n \qquad \phi(t) = \sum_m \phi^m e^{-im\Omega t}$$

$$(\mathcal{H})^{mn} = \frac{1}{T}\int_0^T dt H(t) e^{i(m-n)\Omega t} + m\delta_{mn}\Omega I$$

$$H_m = \mathcal{H}^{m0}$$

~ absorption of m “photons”

How to construct the effective Hamiltonian?

$$H_{\text{eff}} = i \ln U(T)/T$$

- Mathematically ill defined in many-body systems
- Many expansion schemes (non-convergent)

(i) 2nd order perturbation Pershan, van der Ziel, Malmstrom Phys. Rev. 1966

$$\mathcal{H}_{\text{eff}}(t)_{ab} = -h^{-1} \sum_n \left[\frac{v_{an}(t) v_{nb}^*(t)}{\omega_{nb} - \omega} + \frac{v_{an}^*(t) v_{nb}(t)}{\omega_{nb} + \omega} \right]$$

(ii) $1/\Omega$ expansions (van Vleck, Floquet-Magnus, Brillouin-Wigner)

relations between schemes: Mikami, *et al.* PRB '16

$$H_{\text{eff}} = H_0 + \sum_{m>0} \frac{[H_{-m}, H_m]}{m\Omega} + \dots$$

(iii) $1/(\Delta E_{ab}$-n$\Omega)$, $1/(U$-n$\Omega)$ expansions (Brillouin-Wigner, ...)

Brillouin-Wigner expansion

$$\sum_{m=-\infty}^{\infty} \mathcal{H}^{mn}\phi_\alpha^m = \varepsilon_\alpha \phi_\alpha^n$$

$$\begin{pmatrix} \ddots & & & & & & \\ & H_0-2\Omega & H_{+1} & 0 & 0 & 0 & \\ & H_{-1} & H_0-\Omega & H_{+1} & 0 & 0 & \\ & 0 & H_{-1} & H_0 & H_{+1} & 0 & \\ & 0 & 0 & H_{-1} & H_0+\Omega & H_{+1} & \\ & 0 & 0 & 0 & H_{-1} & H_0+2\Omega & \\ & & & & & & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ |\Phi^2\rangle \\ |\Phi^1\rangle \\ |\Phi^0\rangle \\ |\Phi^{-1}\rangle \\ |\Phi^{-2}\rangle \\ \vdots \end{pmatrix} = \epsilon \begin{pmatrix} \vdots \\ |\Phi^2\rangle \\ |\Phi^1\rangle \\ |\Phi^0\rangle \\ |\Phi^{-1}\rangle \\ |\Phi^{-2}\rangle \\ \vdots \end{pmatrix}$$

P=

P

$$Q|\boldsymbol{u}\rangle = Q\frac{1}{\varepsilon + \mathcal{N}\Omega - \mathcal{H}_0} Q\tilde{\mathcal{H}}|\boldsymbol{u}\rangle$$ Q=1-P

Projection I 1/Ω expansions Mikami *et al.* PRB’16

$$H_{\text{eff}} = H_0 + \sum_{m>0} \frac{[H_{-m}, H_m]}{m\Omega} + \cdots$$ different from Magnus expansion

Projection II Bucciantini, Roy, Kitamura, Oka, arXiv’16 (appendix)

higher order monopoles in the Floquet Weyl semimetal

$$H_{\text{eff}}^{W,\pm} = \left(|\mathbf{k}| - \Omega + A^2 \frac{|\mathbf{k}|^2 + k_3^2 \pm \Omega k_3}{|\mathbf{k}|(4|\mathbf{k}|^2 - \Omega^2)} \right) \sigma^3 - \frac{A^2(|\mathbf{k}| + k_3)^{\pm 1}}{2|\mathbf{k}|\Omega(2|\mathbf{k}| - \Omega)} (k_+^{2\mp 1}\sigma^+ + \text{h.c.})$$

Husimi transformation

Husimi (Taniuti) PTP '53

$$H(p_x, p_y - qA_y; t) = \frac{1}{2m_e}\left[p_x^2 + \left(p_y - qA_y - \frac{e}{c}B_z(t)x)^2\right)^2\right]$$

= driven Harmonic oscillator with an oscillating potential

(i) Quantum oscillator without driving

$$H(t) = \frac{p_x^2}{2m_e} + \frac{m_e\omega^2(t)}{2}x^2$$

$$\omega(t) = \omega_c \cos \Omega t$$

+

(ii) Classical driven oscillator

$$m_e\ddot{X} + m_e\omega(t)^2 X = S(t)$$

$$S(t) = \omega(t)(\hbar k_y - qA_y)$$

solution

$$\Psi_n(\boldsymbol{x}, t) = e^{-\frac{i}{\hbar}E_n t} e^{ik_y y}\varphi_n(x - X(t), t)\exp\left[\frac{i}{\hbar}\{m_e\dot{X}(t)(x - X(t)) + \int_0^t dt' L(t') - L_0 t\}\right]$$

$$L = \frac{1}{2}m_e\dot{X}^2 - \frac{1}{2}m_e\omega(t)^2X^2 + XF(t)$$

pseudo-energy

$$E_n = \varepsilon_n + \frac{\hbar^2 k_y^2}{2m^*}$$

$$E_n = \varepsilon_n + \frac{\hbar^2 k_y^2}{2m_e} - \frac{1}{T}\int_0^T L(t')dt'$$