Mini-Symposium of
Novel Quantum States in Condensed Matter 2017

22 November 2017
YITP, Kyoto

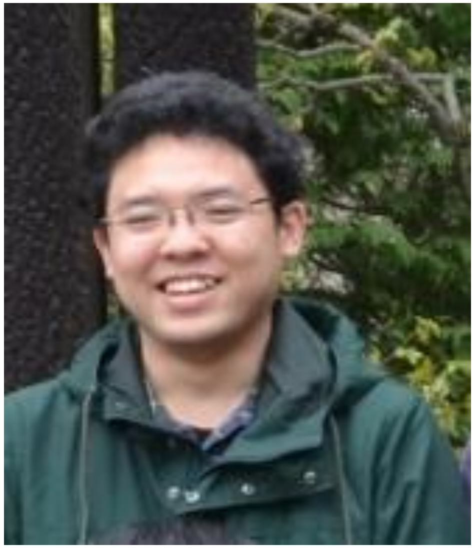
Topological Thouless pumping of ultracold fermions

Kyoto University

Yoshiro Takahashi

Collaborators

S. Nakajima （Kyoto Univ.）



Now in Hakubi

A. Sawada （Kyoto Univ.）

Now in industry

Y. Kuno （Kyoto Univ.）

L. Wang （IOP, CAS）

Also

T. Tomita, S. Taie, H. Ozawa, T. Ichinose (Kyoto Univ.)

M. Troyer (ETH)

Outline

I) Demonstration of Topological Thouless Pumping

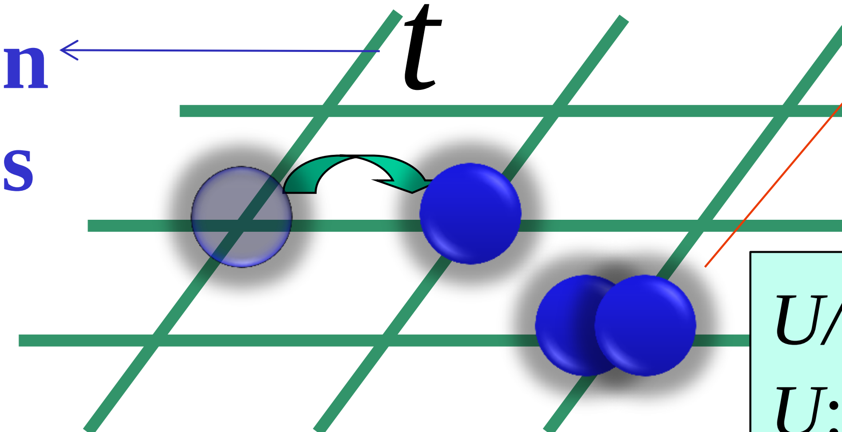
II) Effect of Disorder on Topological Thouless Pumping

III) Prospects on Topological Thouless Pumping

Ultracold Atoms in an Optical Lattice

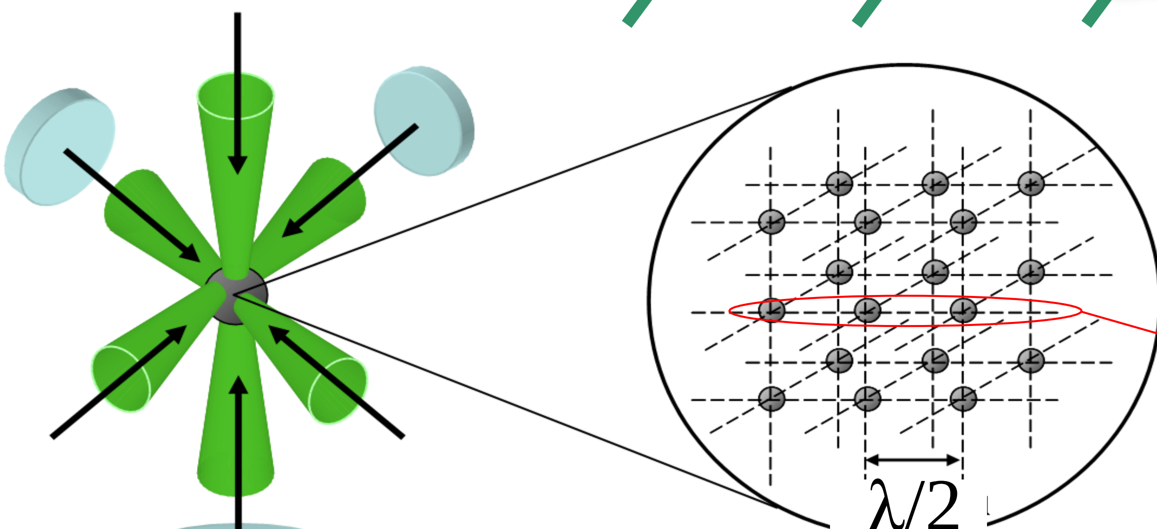
$$H = -t \sum_{<i,j>} c_i^+ c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

hopping between adjacent lattices ← t

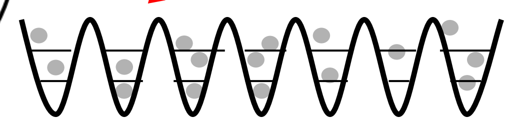


U: **on-site interaction**

U/t: laser intensity
U:Feshbach resonance
t: shaking/Raman



"Optical lattice"
= periodic potential for atoms

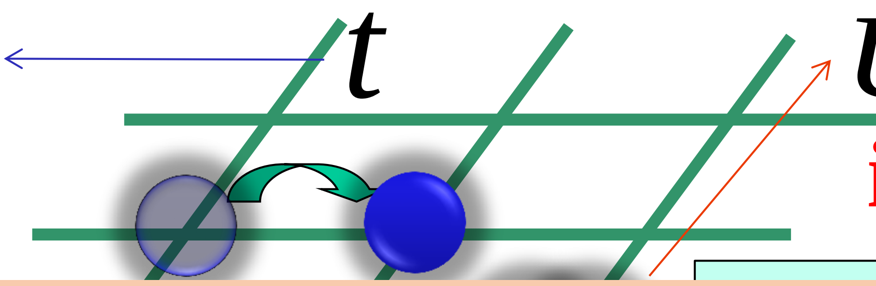


$$V = V_o \sin^2(kx)$$

Ultracold Atoms in an Optical Lattice

$$H = -t\sum_{<i,j>} c_i^{+} c_j + U\sum_{i} n_{i\uparrow} n_{i\downarrow}$$

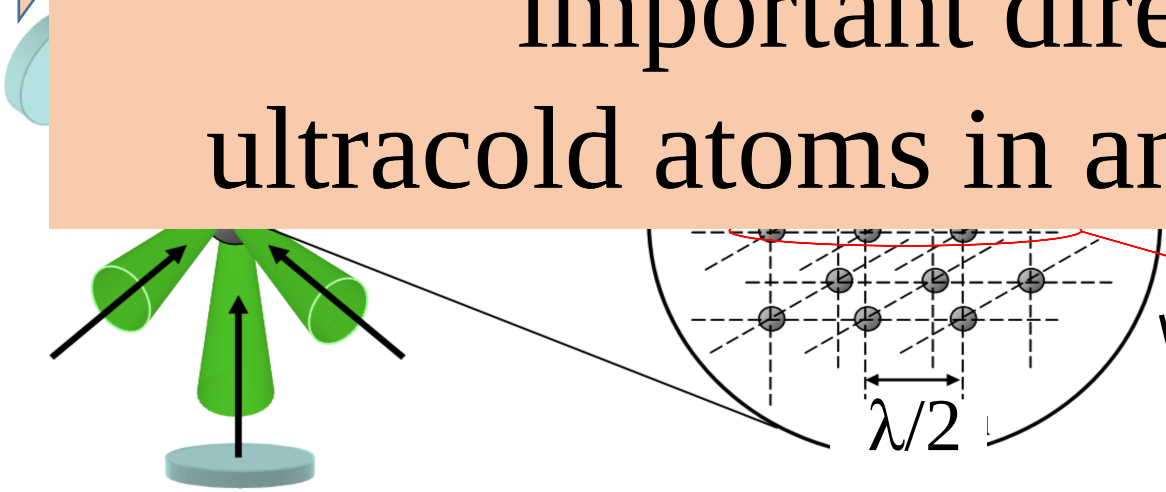
hopping between adjacent lattices

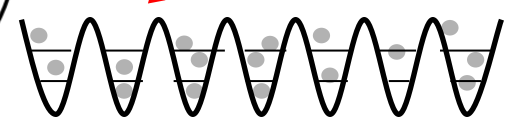


U: **on-site interaction**

Study of Topological Physics is an important direction of ultracold atoms in an optical lattice

– periodic potential for atoms





$$V = V_o \sin^2(kx)$$

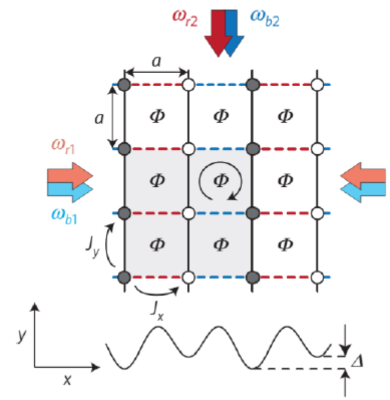
Exploring topological physics using optical lattice

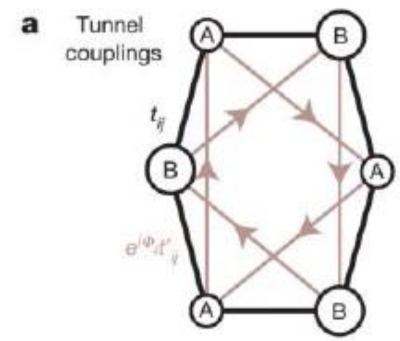
Spatial 2D:

Hofstadter Hamiltonian (MIT, MPQ)

topological Haldane model (ETH)

…

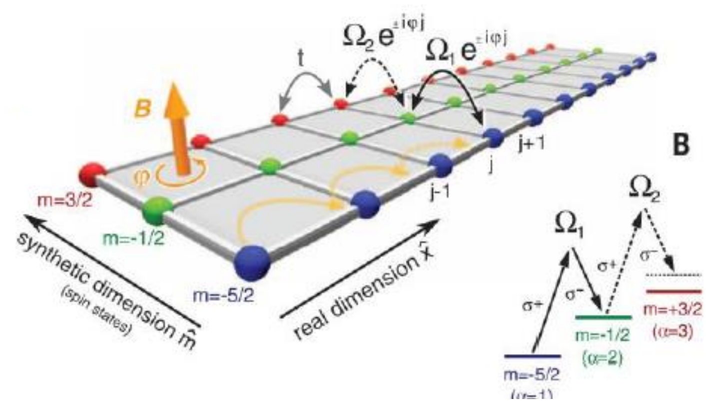




Spatial 1D + Synthetic dimension:

chiral edge state (NIST, LENS)

…

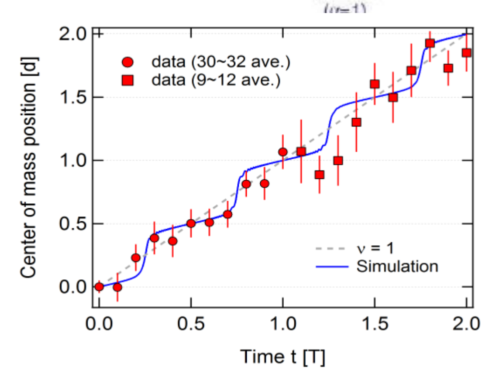


Spatial 1D + Temporal 1D (1+1):

Thouless topological charge pumping

fermion (Kyoto)

boson (MPQ)



Thoulesss Topological Charge Pumping

Thouless PRB 27, 6083(1983**) "Quantization of Particle Transport"**

Particle Transport in a 1D Periodically-driven lattice system:

H(k+2π/L, t) = H(k, t) : spatially periodic 1D potential
H(k, t+T) = H(k, t) : temporally cyclic adiabatic evolution

pumped charge for one cycle *T*:

$$C = \frac{i}{2\pi}\sum_{\lambda} f_{\lambda}\int_0^T dt \int_0^{2\pi/L} dk \left[\left\langle \frac{\partial\psi_{\lambda k}}{\partial t}\middle|\frac{\partial\psi_{\lambda k}}{\partial k}\right\rangle - \left\langle \frac{\partial\psi_{\lambda k}}{\partial k}\middle|\frac{\partial\psi_{\lambda k}}{\partial t}\right\rangle\right]$$

topological invariant
: Chern number

f_λ =1 for filled bands
=0 for empty bands

λ: band index

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Hall conductivity for Interger Quantum Hall Effect (TKNNformula)

$$\sigma_{xy} = \frac{e^2}{h}\sum_{m\le m_0}\int_{\mathrm{BZ}} \frac{d^2\tilde{k}}{2\pi i}\left[\left\langle \frac{\partial u_{m\vec{\tilde{k}}}}{\partial \tilde{k}_x}\middle|\frac{\partial u_{m\vec{\tilde{k}}}}{\partial \tilde{k}_y}\right\rangle - \left\langle \frac{\partial u_{m\vec{\tilde{k}}}}{\partial \tilde{k}_y}\middle|\frac{\partial u_{m\vec{\tilde{k}}}}{\partial \tilde{k}_x}\right\rangle\right]$$

$$= \frac{e^2}{h}N_{\mathrm{ch}} \longleftarrow$$ topological invariant: Chern Number

Thoulesss Topological Charge Pumping

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pumped charge for one cycle *T*:

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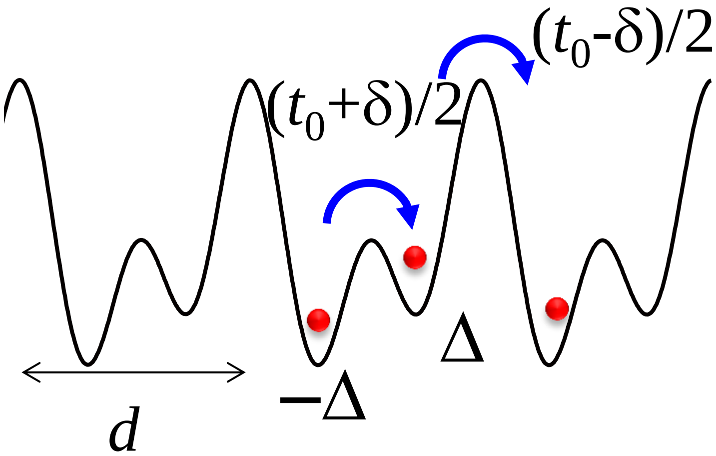
"1D quantum charge pumping" shares the same topological origin as "2D IQHE"

Topological Charge Pumping has never been realized in any system

Quantum Rice-Mele Charge Pumping

Rice & Mele, PRL (1982): One-Dimensional Dimerized Lattice Model



Su-Schrieffer-Heeger model

$$\mathcal{H} = \sum_i \left(\frac{t_0}{2} + (-1)^i \frac{\delta}{2} \right) (c_i^\dagger c_{i+1} + \text{h.c.})$$

$$+ \Delta \sum_i (-1)^i c_i^\dagger c_i$$

Staggered Potential

"two-level description"

$$a_k = \frac{1}{\sqrt{N}} \sum_{j:even} c_i e^{-ikdj}$$

$$b_k = \frac{1}{\sqrt{N}} \sum_{j:odd} c_i e^{-ikdj}$$

$$\sigma_x = a_k{}^\dagger b_k + b_k{}^\dagger a_k$$

$$\sigma_y = -i(a_k{}^\dagger b_k - b_k{}^\dagger a_k)$$

$$\sigma_z = a_k{}^\dagger a_k - b_k{}^\dagger b_k$$

$$H = \sum_k \mathbf{h}(k) \cdot \boldsymbol{\sigma}(k)$$

$$\mathbf{h}(\mathrm{k}) = (t_0 \cos(kd/2),\ -\delta \sin(kd/2),\ \Delta)$$

$$\mathbf{E}(\mathrm{k}) = \pm\sqrt{t_0^2 \cos^2(kd/2) + \delta^2 \sin^2(kd/2) + \Delta^2})$$

Quantum Rice-Mele Charge Pumping

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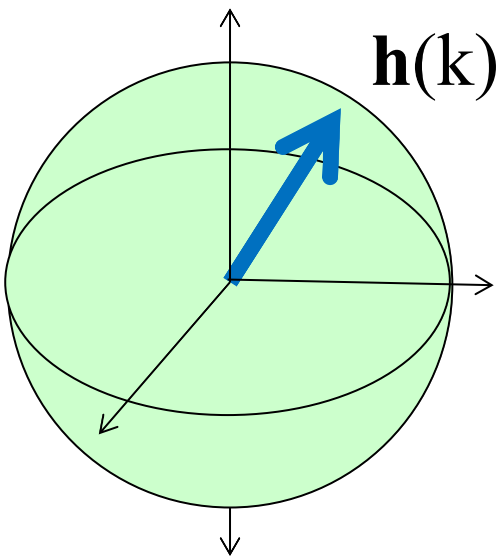
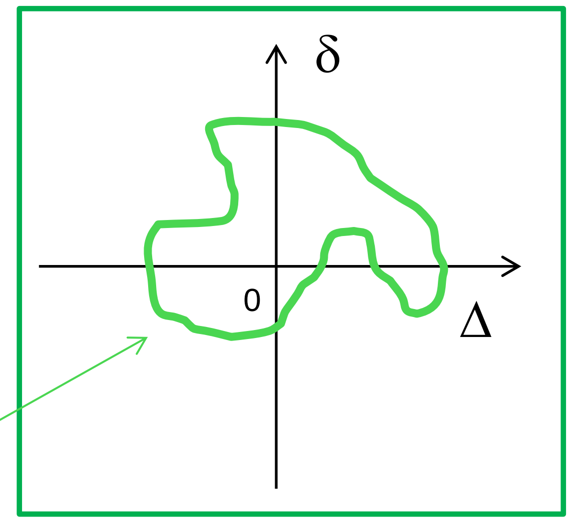


$$\mathbf{h}(\mathrm{k})=(t_0\cos(kd/2),\ -\delta(t)\sin(kd/2),\ \Delta(t))$$

degeneracy point: $\delta(t)=0,\ \Delta(t)=0$



"charge pumping sequence"
↔"trajectory in $\delta-\Delta$ parameter space"

Quantum Rice-Mele Charge Pumping

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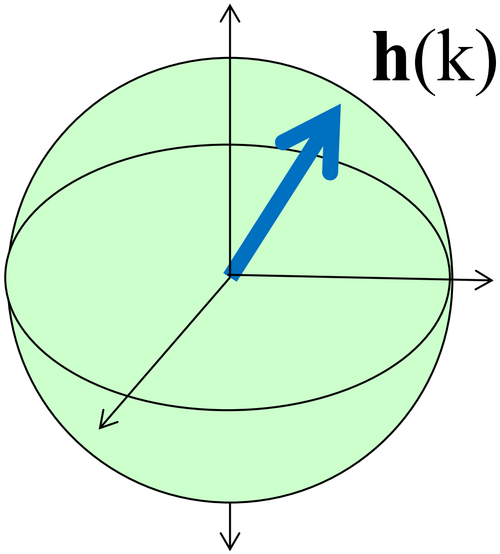
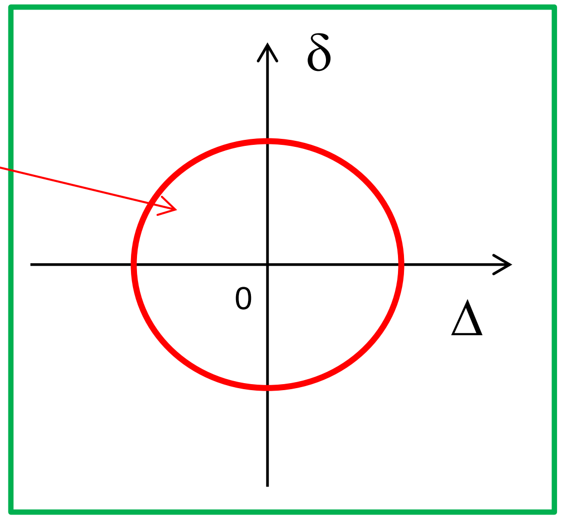


$$\mathbf{h}(k) = (t_0\cos(kd/2),\ -\delta(t)\sin(kd/2),\ \Delta(t))$$

degeneracy point: $\delta(t) = 0,\ \Delta(t) = 0$

$$\begin{cases} \delta(t) = \delta_0 \cos(\frac{2\pi t}{T}) \\ \Delta(t) = \Delta_0 \sin(\frac{2\pi t}{T}) \end{cases}$$



Pumped Charge: $n = -\mathrm{sgn}(t_0\delta_0\Delta_0)$

S. –Q, Shen, "Topological Insulators: Dirac Equation in Condensed matter"
D. Xiao *et.al*. RMP (2010)

"charge pumping sequence"
↔"trajectory in $\delta - \Delta$ parameter space"

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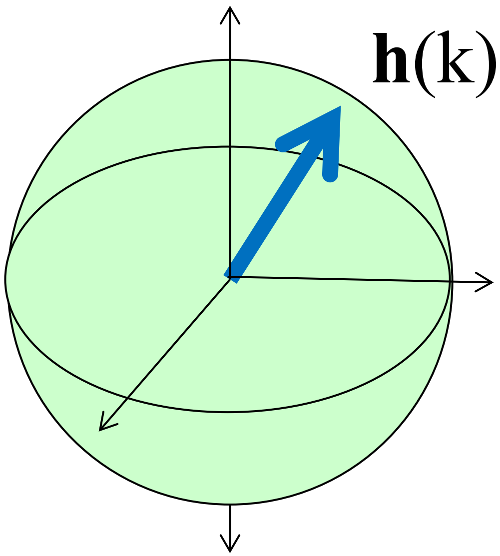
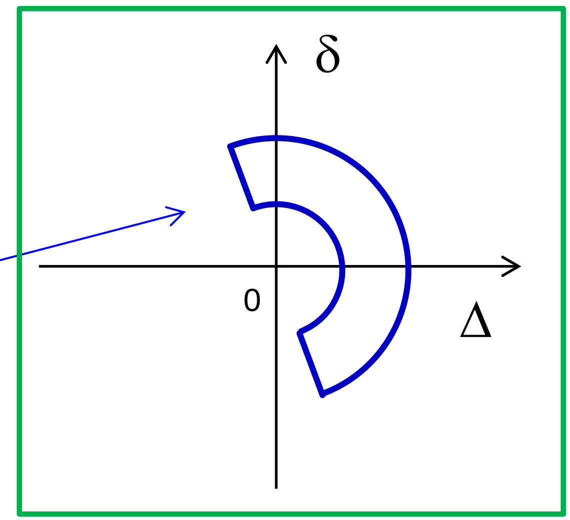


$$\mathbf{h}(k) = (t_0\cos(kd/2), \; -\delta(t)\sin(kd/2), \; \Delta(t))$$

degeneracy point: $\delta(t) = 0, \; \Delta(t) = 0$

Pumped Charge: $n = 0$



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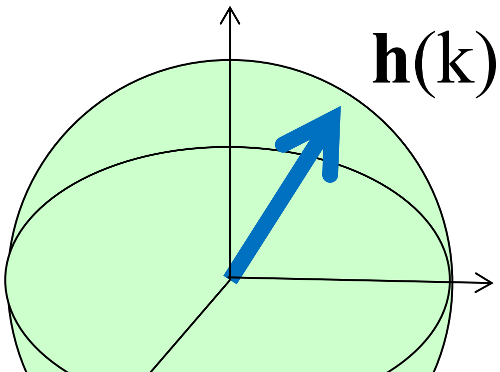

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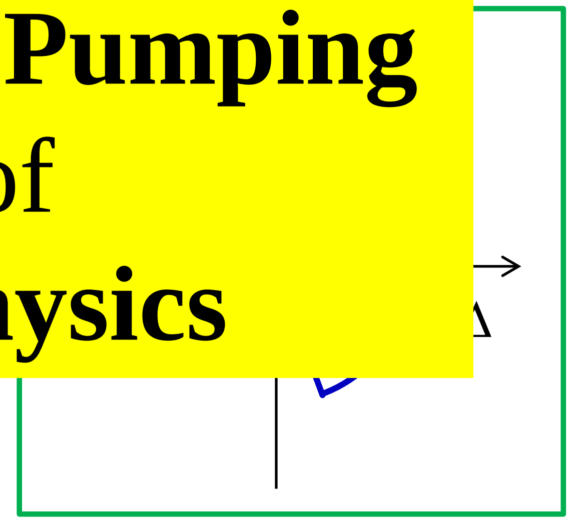
degeneracy point: $\delta(t) = 0,\ \Delta(t)=0$

Quantum Rice-Mele Charge Pumping

captures the essence of

Topological Quantum Physics



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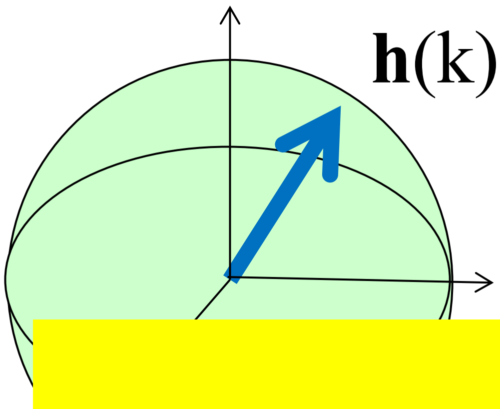

$$\mathbf{h}(k)=(t_0\cos(kd/2),\ -\delta(t)\sin(kd/2),\ \Delta(t))$$

degeneracy point: $\delta(t)=0,\ \Delta(t)=0$

How can we realize

Quantum Rice-Mele Charge Pumping

by using

Ultracold Atoms in an Optical Lattice ?

S. –Q, Shen, "Topological Insulators: Dirac Equation in Condensed matter"
D. Xiao *et.al.* RMP (2010)

"charge pumping sequence"
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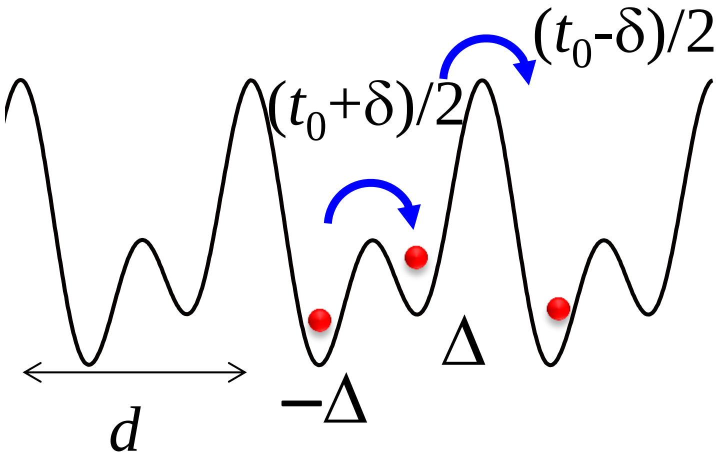
Quantum Rice-Mele Charge Pumping

Rice & Mele, PRL (1982): One-Dimensional Dimerized Lattice Model



Su-Schrieffer-Heeger model

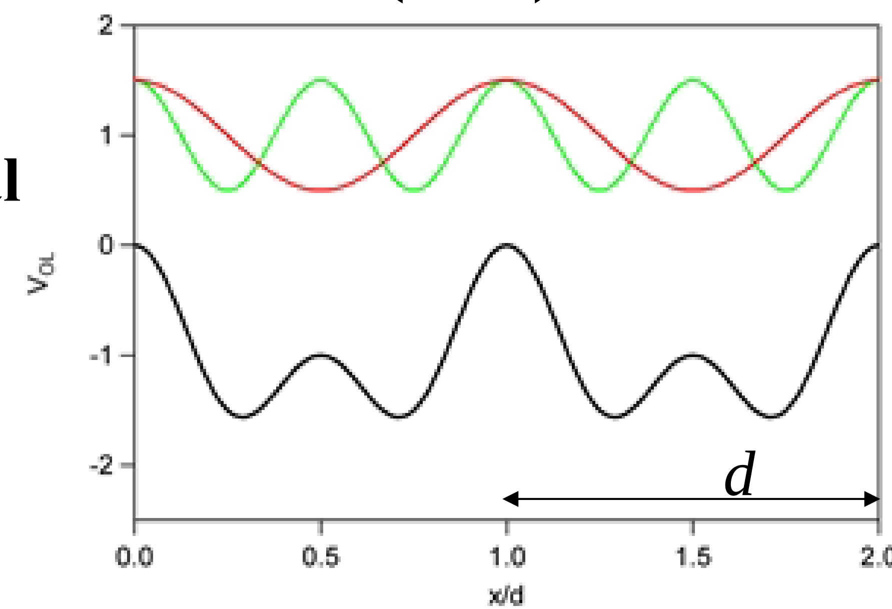
$$\mathcal{H} = \sum_i \left(\frac{t_0}{2} + (-1)^i \frac{\delta}{2} \right) (c_i^\dagger c_{i+1} + \text{h.c.})$$

$$+ \Delta \sum_i (-1)^i c_i^\dagger c_i$$

Staggered Potential

L. Wang *et al*, PRL111, 026802(2013)

proposed
"dynamical optical super-lattice"
scheme



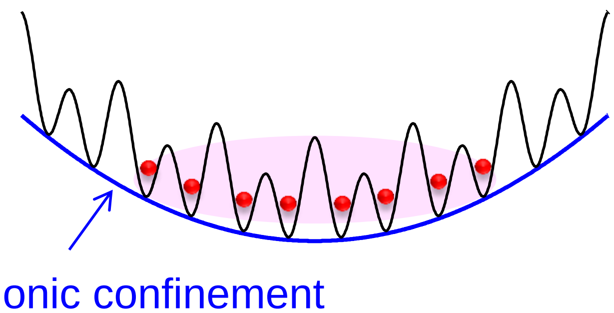
V_S: static short lattice
lattice constant $d/2$

V_L:dynamic long lattice
lattice constant d

$V_{OL}=V_S+V_L$

Experimental Observable: shift of atom cloud

L. Wang *et al*, PRL**111**, 026802(2013)



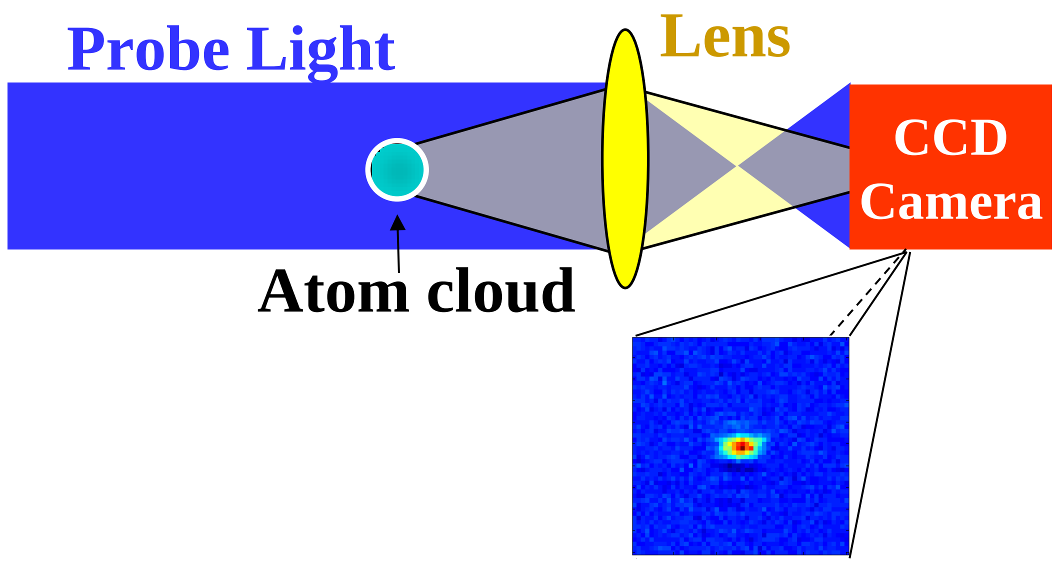
$$\langle x\rangle/d=\Delta n$$

d:lattice constant

Shift of Center of Mass (a finite system)

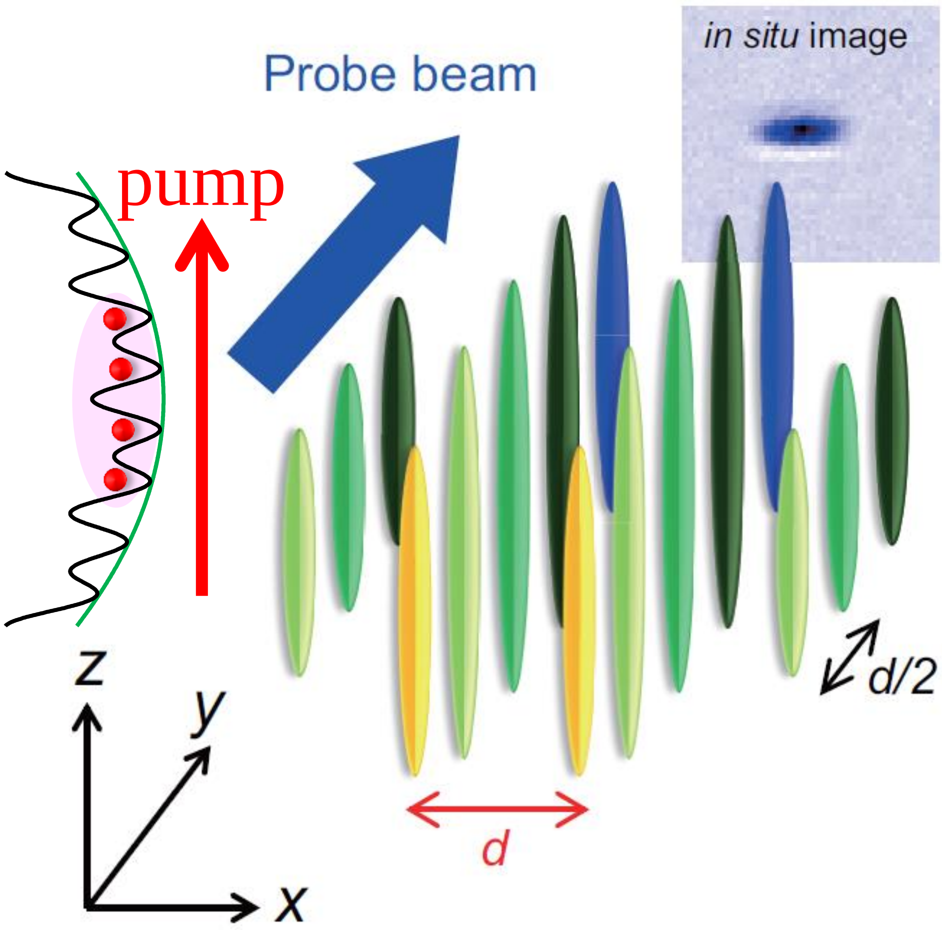
Pumped Charge (an infinite system)

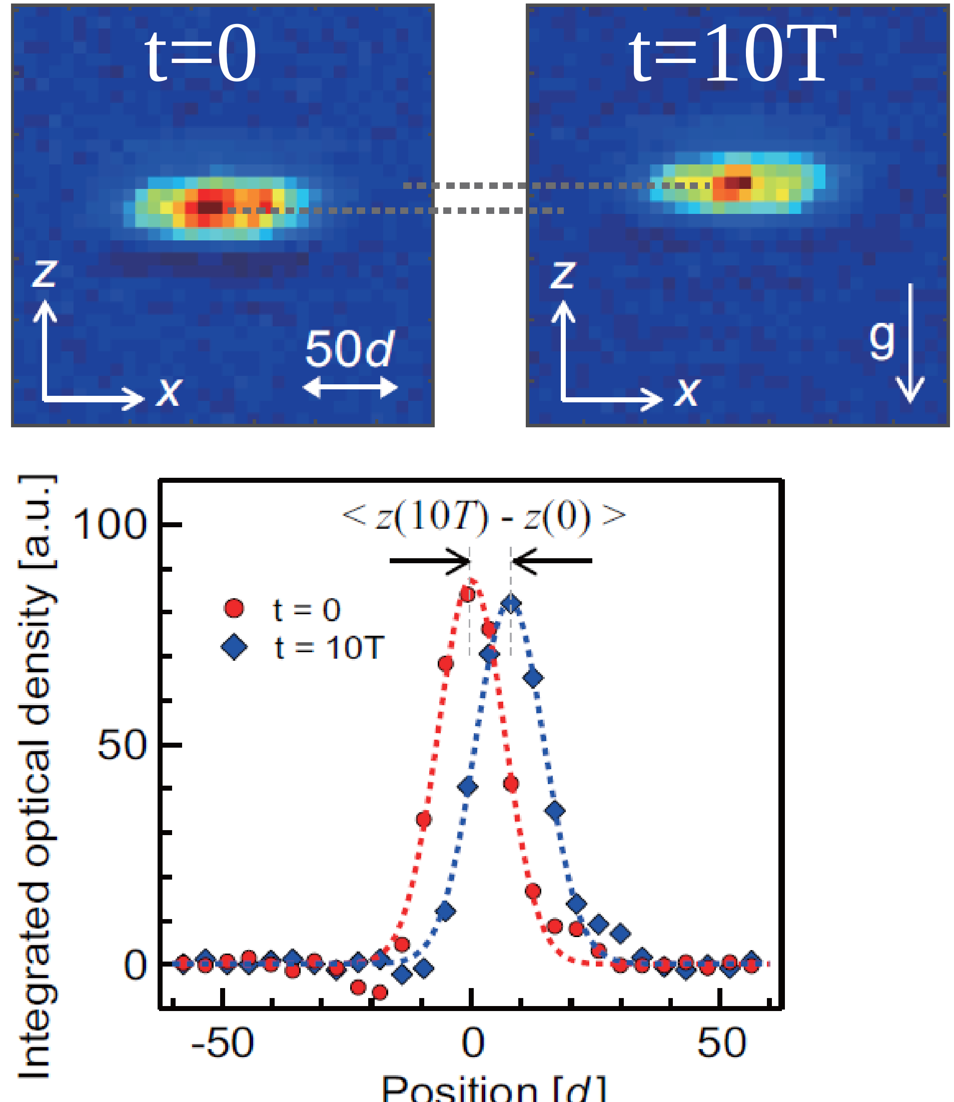
"*In Situ* Absorption Imaging"



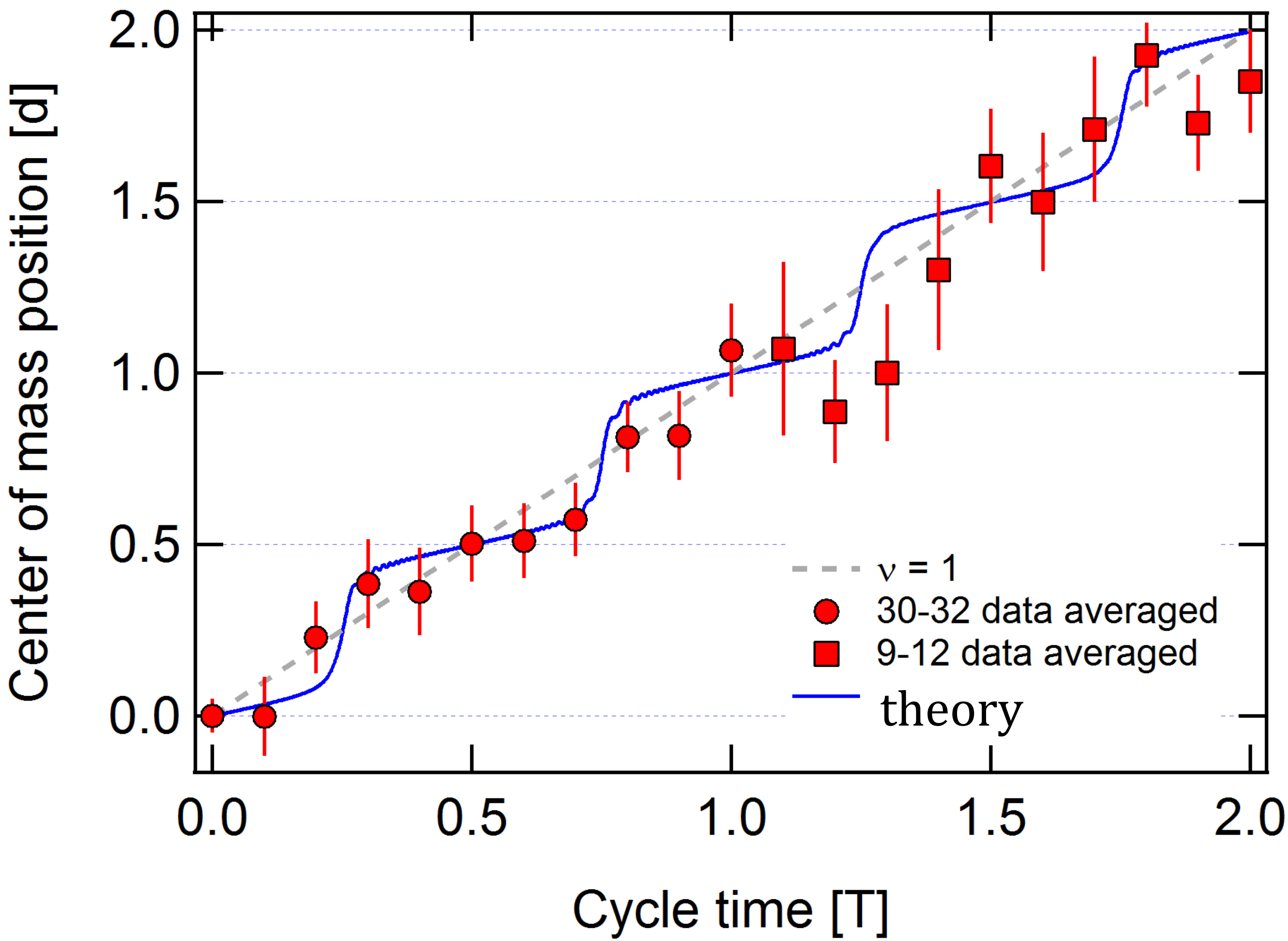
Experimental Observation of Charge Pumping

Atomic Species : ^{171}Yb(Fermion,m_I=±1/2)
Number of atoms : 5×10^3 ～ 2×10^3
Temperature : 30 nK



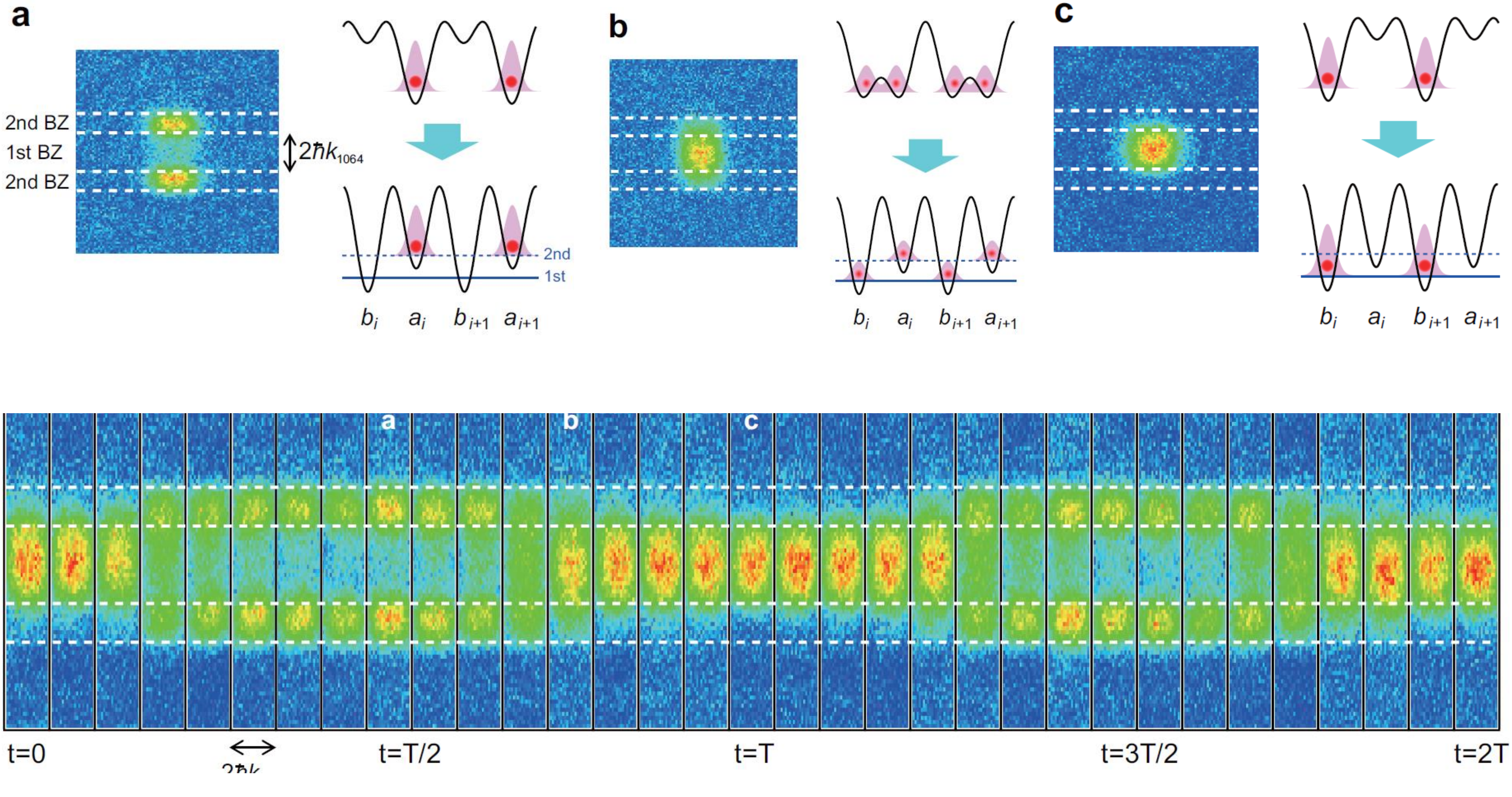


Experimental Observation of Charge Pumping

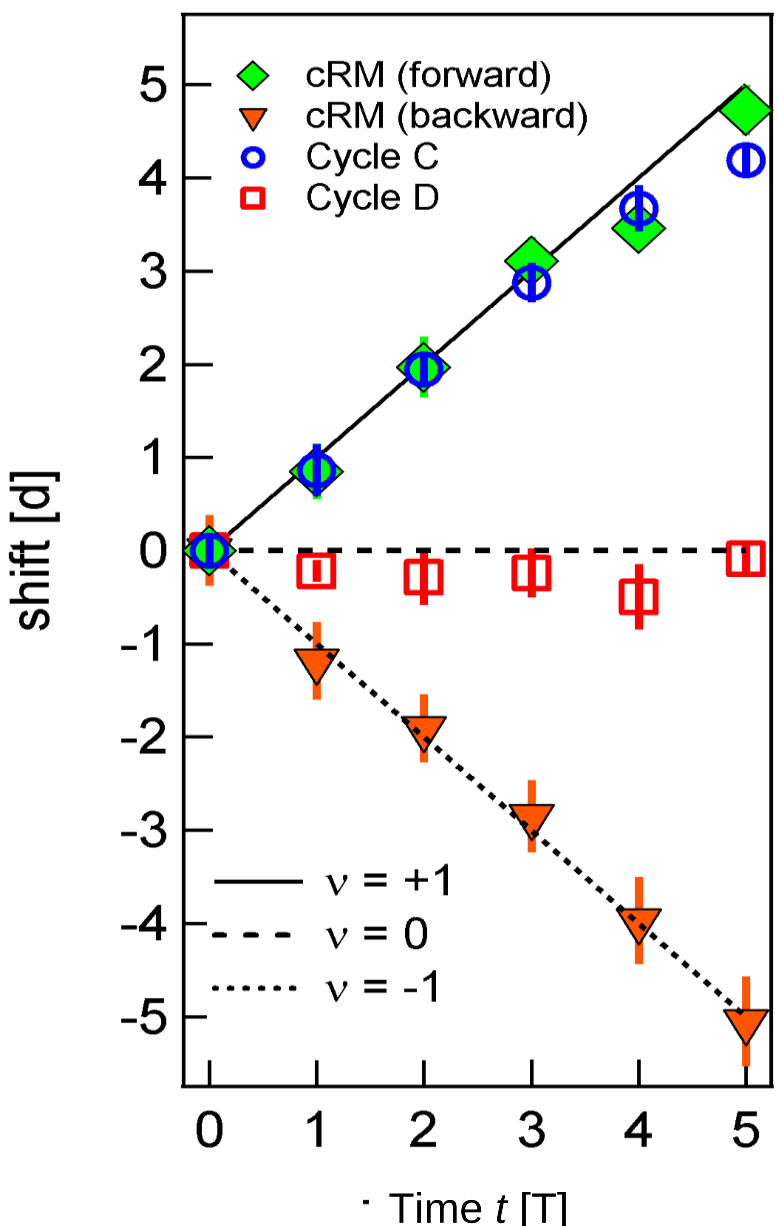


Sublattice Mapping

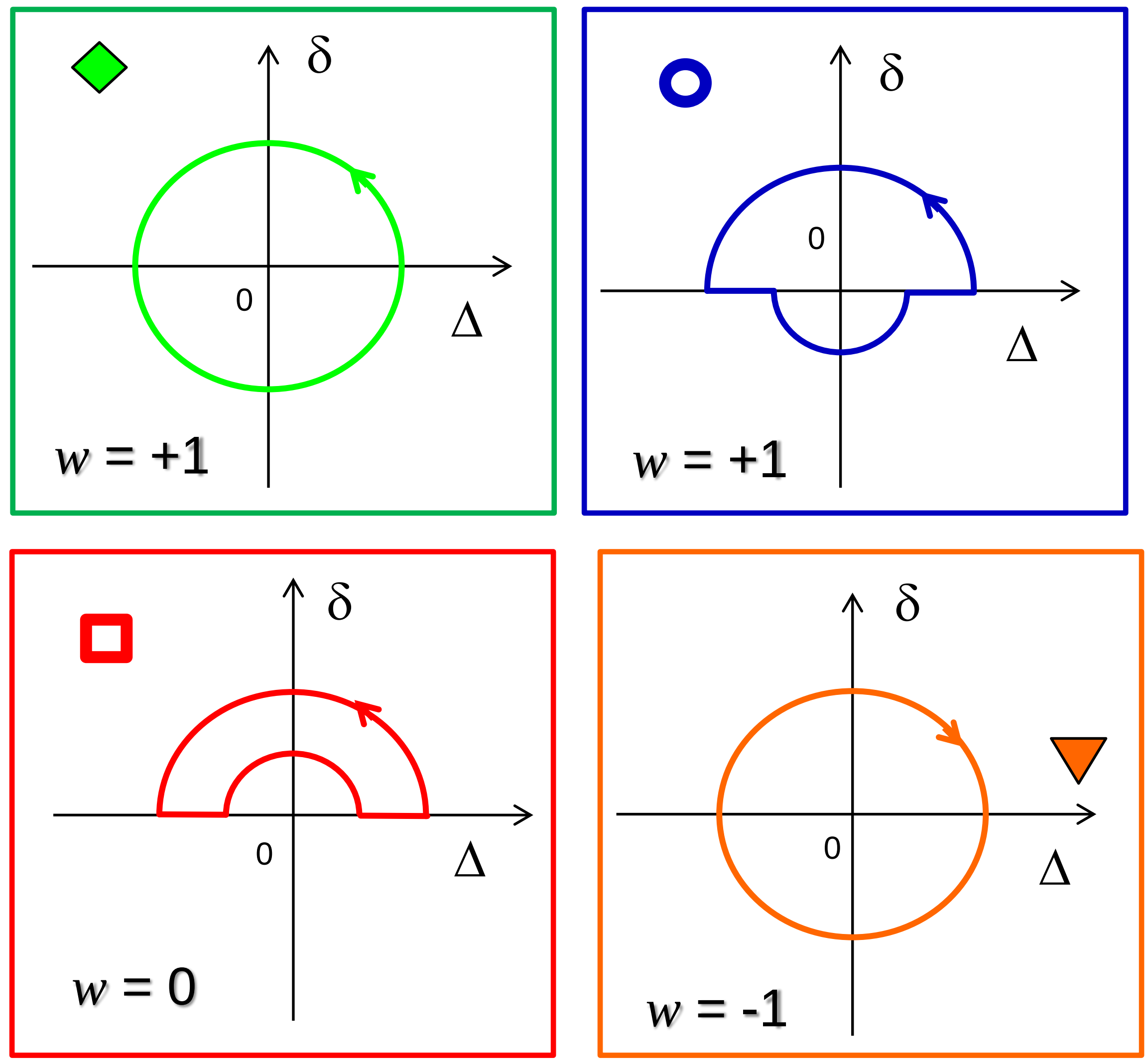
"Right" in a double well | "superposition" in a double well | "Left" in a double well



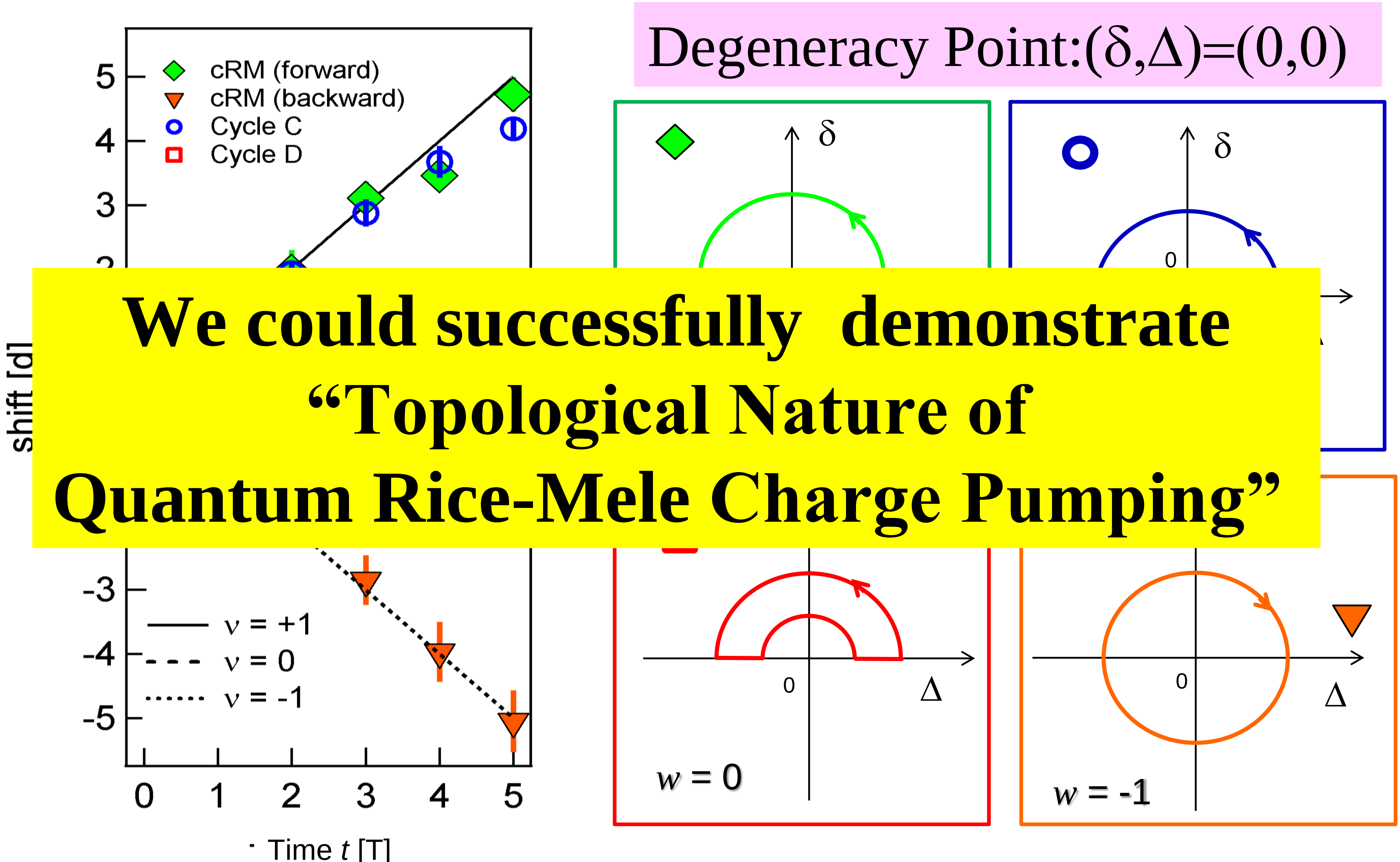
Demonstrating “Topological” Charge Pumping



Degeneracy Point:(δ,Δ)=(0,0)



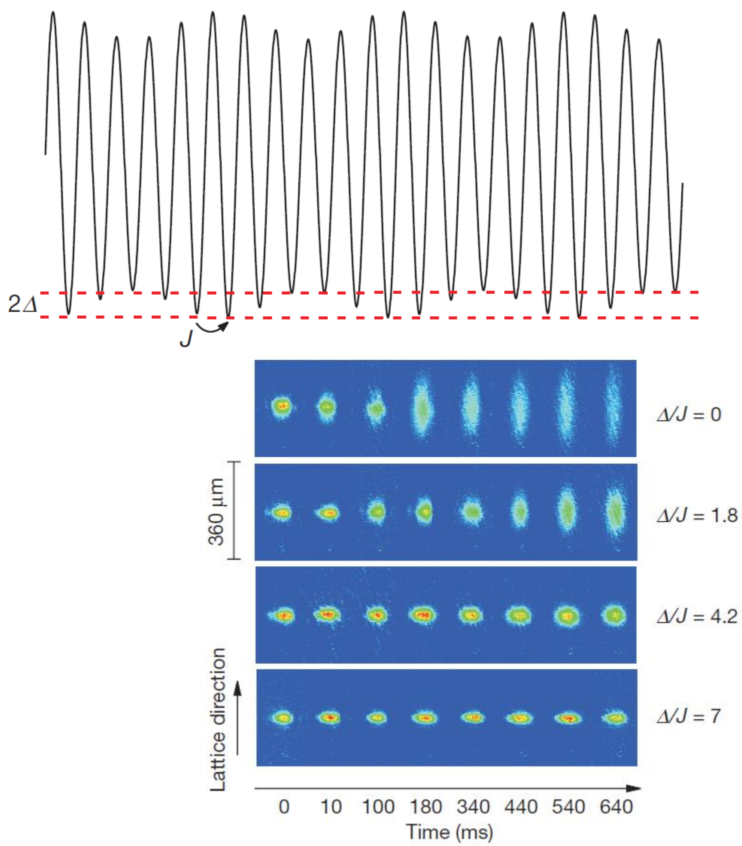
Demonstrating "Topological" Charge Pumping



We could successfully demonstrate "Topological Nature of Quantum Rice-Mele Charge Pumping"

transport is suppressed by disorder

Anderson localization



G. Roati *et al.*, Nature **453**, 895 (2008)

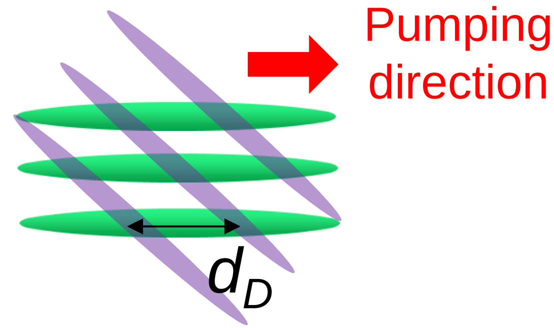
⇒

Does Disorder Suppress Thouless Topological Pumping?

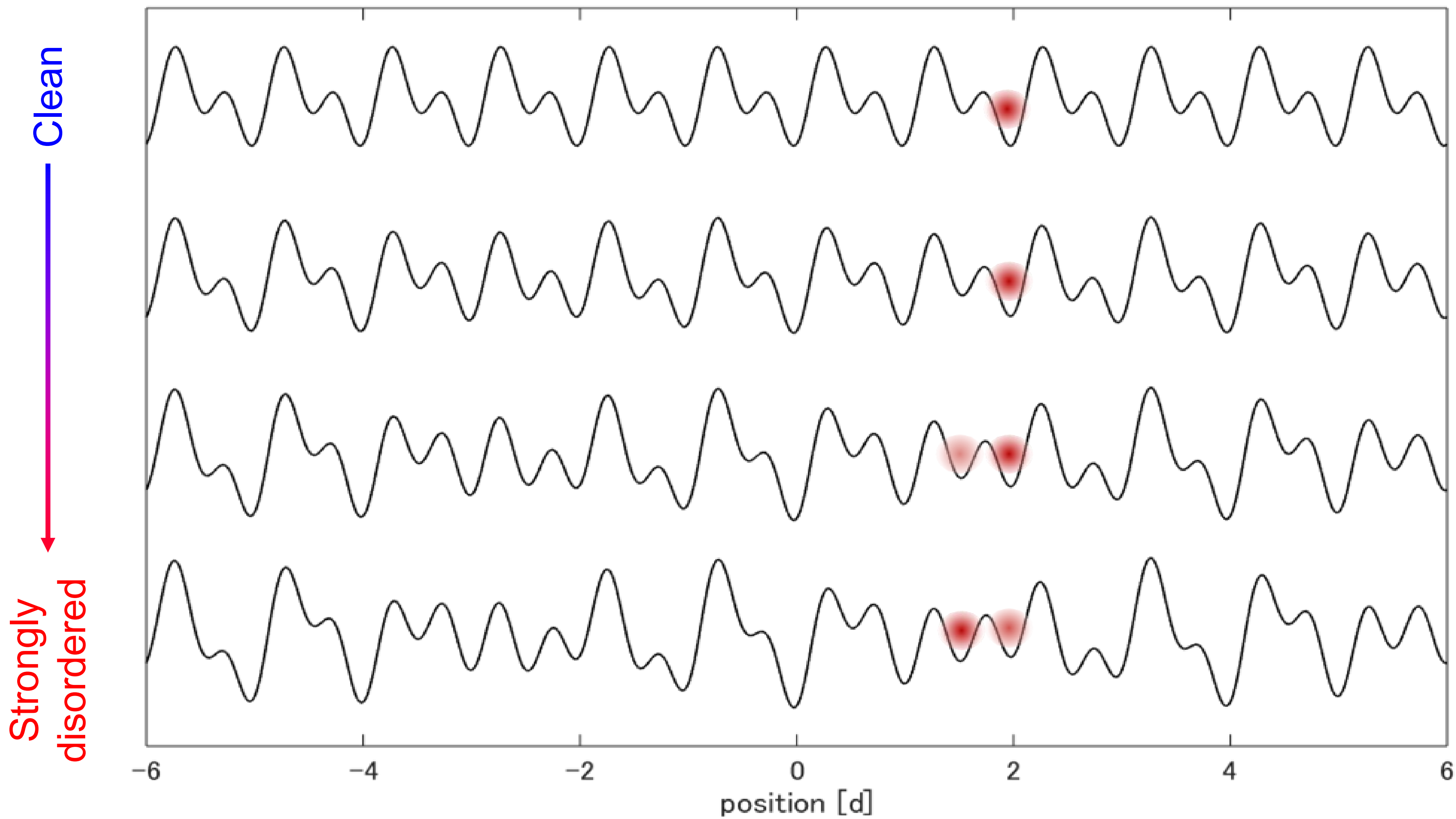
Additional Lattice : V_D

λ=798nm, tilted by 45° $\Rightarrow$ d_D:~$564nm$~$3\sqrt{2}d/4$

(d=532nm)



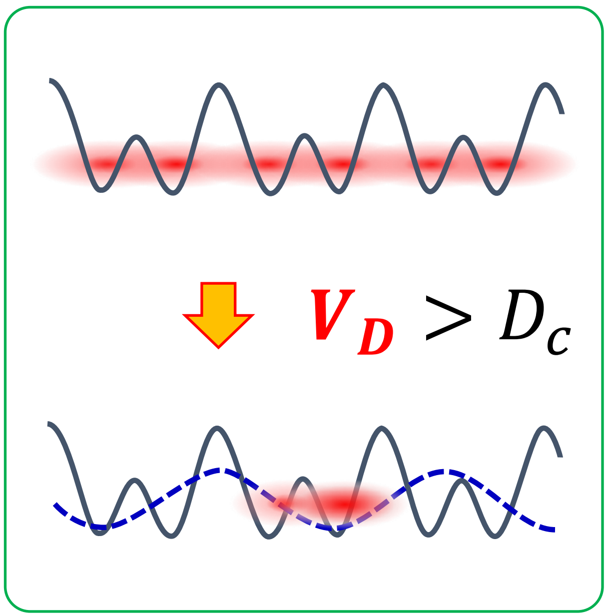
"Quasi-periodic disorder"



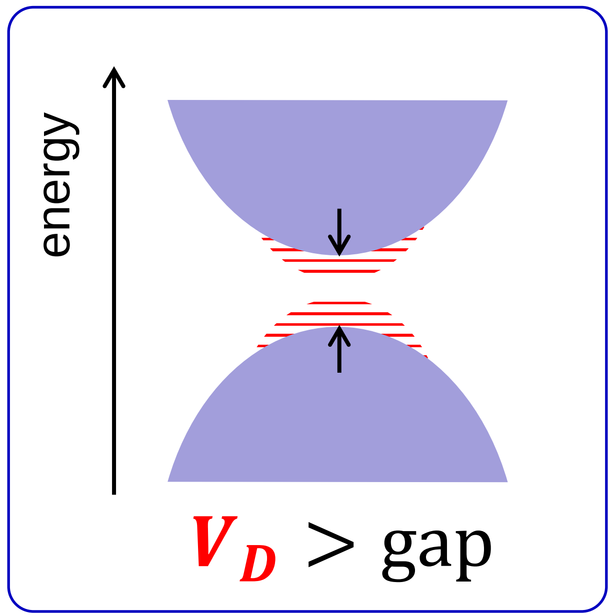
Effect of Disorder *for Topological Charge Pump*

What is the most relevant energy scale for V_D to influence the TCP?

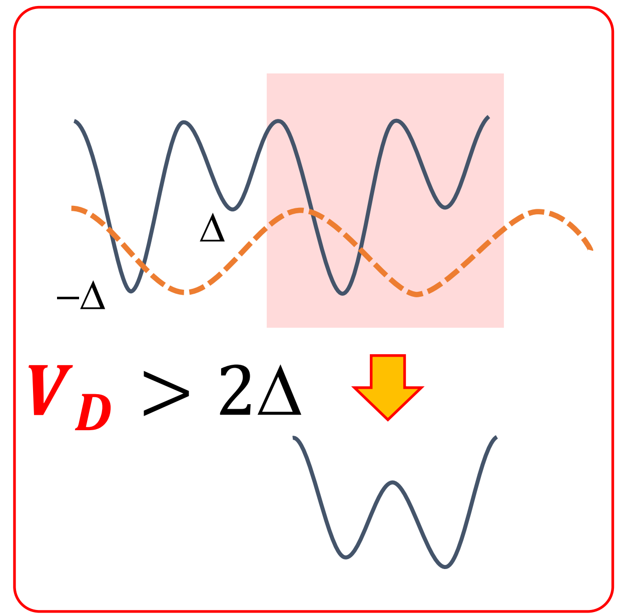
Anderson Localization transition point D_c



Band gap



On-site offset 2Δ



Effect of Disorder *for* *Topological Charge Pump*

Theory

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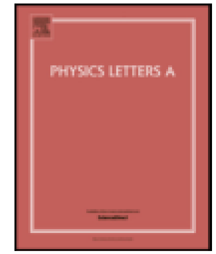
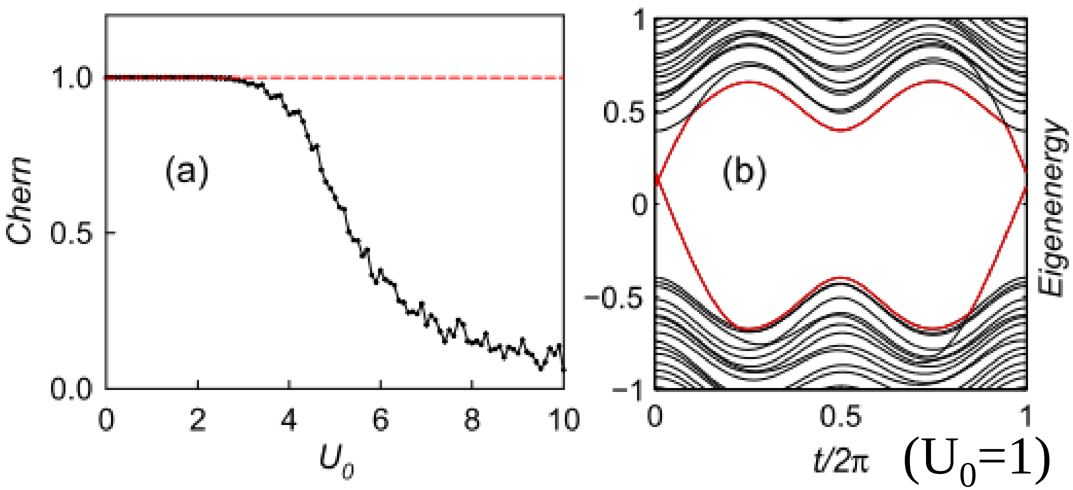


Quantum pumping induced by disorder in one dimension

Jihong Qin [a], Huaiming Guo [b,*]

[a] Department of Physics, University of Science and Technology Beijing, Beijing 100083, China
[b] Department of Physics, Beihang University, Beijing 100191, China



(U_0=1)

1D Wilson-Dirac Model

$$H_0 = \sum_i (M + 2B)\Psi_i^\dagger \sigma_z \Psi_i - \sum_{i,\hat{x}} B\Psi_i^\dagger \sigma_z \Psi_{i+\hat{x}} - \sum_{i,\hat{x}} sgn(\hat{x}) I A \Psi_i^\dagger \sigma_x \Psi_{i+\hat{x}}$$

Disorder Term

$$H_{dis} = \sum_i U_i (c_i^\dagger c_i + d_i^\dagger d_i),$$

U_i uniformly distributed in $(-\frac{U_0}{2}, \frac{U_0}{2})$

Prospects : effect of disorder

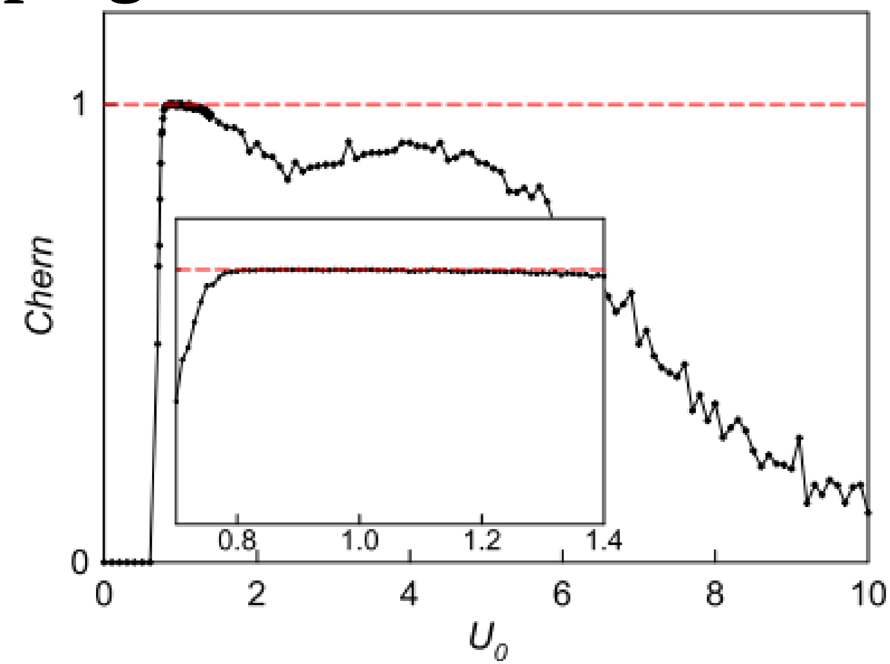
Disorder induced Topological Pumping

1D Wilson-Dirac Model

$$H_0 = \sum_i (M+2B)\Psi_i^\dagger \sigma_z \Psi_i - \sum_{i,\hat{x}} B\Psi_i^\dagger \sigma_z \Psi_{i+\hat{x}} - \sum_{i,\hat{x}} sgn(\hat{x}) IA\Psi_i^\dagger \sigma_x \Psi_{i+\hat{x}}$$

Disorder Term $H_{dis} = \sum_i U_i (c_i^\dagger c_i + d_i^\dagger d_i),$

U_i uniformly distributed in $(-\frac{U_0}{2}, \frac{U_0}{2})$



[Phys. Lett. A 380, 2317(2016)

Time-dependent Disorder

Q Niu and D J Thouless J. Phys. A **17**, 2453 (1984)

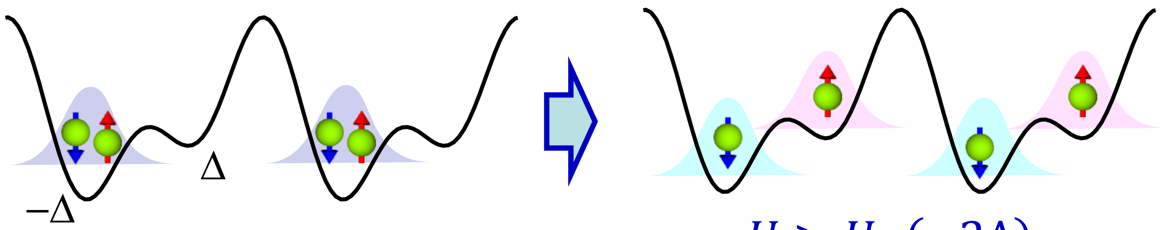
"***a weak disorder***, *which does not close a finite Fermi gap*, ***is irrelevant to the charge transport*** *no matter* ***whether it is static or varies periodically in time***"

Non-Quasi-Periodic Disorder

Prospects :effect of interaction

Mott transition in Ionic Hubbard model (M. Nakagawa, private commun.)

(G. Ortiz et al., PRB 54, 13515 (1996))



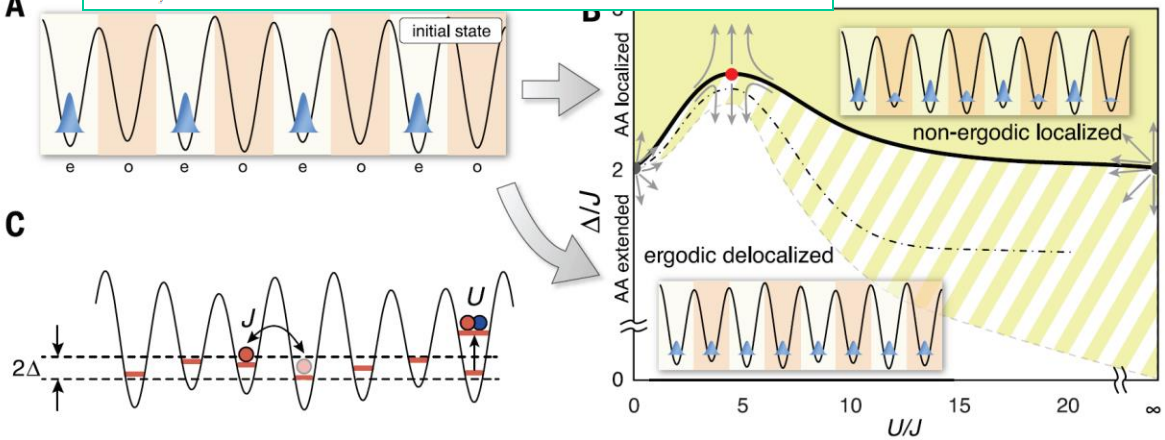
$U < U_c\ (\sim 2\Delta)$

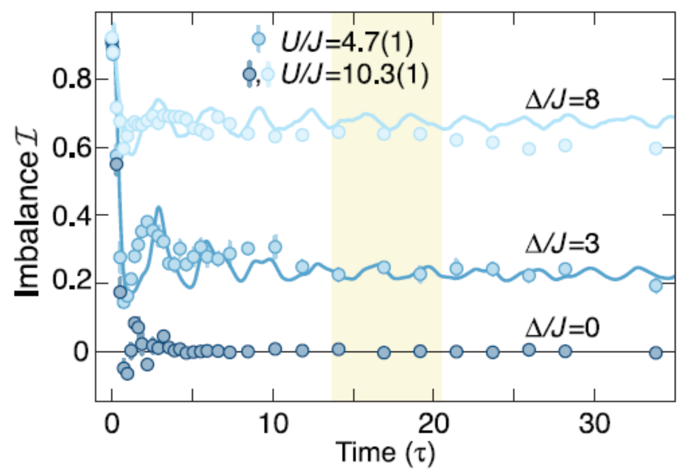
$U > U_c\ (\sim 2\Delta)$:
Formation of Mott state

Many-body Localization (M. Schreiber, *et al*, Science 349,842(2015))

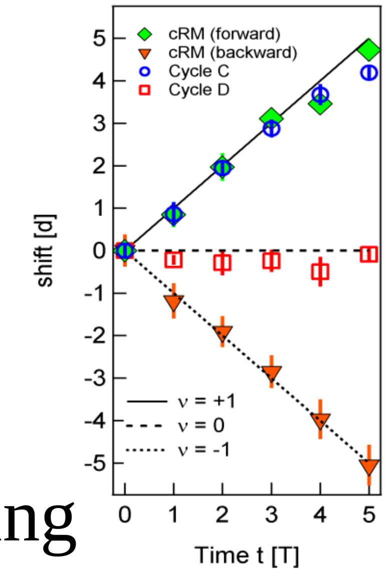
$$\hat{H} = -J\sum_{i,\sigma}(\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i+1,\sigma} + \text{h.c.}) + \Delta\sum_{i,\sigma}\cos(2\pi\beta i + \phi)\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma} + U\sum_{i}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

1D, Fermion, Interaction, Disorder





Summary



I) Demonstration of Topological Thouless Pumping

[S. Nakajima, *et al*, Nature Physics, **12**, 296(2016)]

Realization of Rice-Mele model by optical super-lattice

Revealing the topological nature

II) Effect of Disorder on Topological Thouless Pumping

Successful implementation of quasi-periodic disorder potential

Demonstrating the robustness against the disorder

Suggesting the topological phase transition

III) Prospects on Topological Thouless Pumping

Various topics on disorder

Effect of interaction on topological Thouless pumping

Thank you very much for attention

16 August   Mount Daimonji at Kyoto