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# Field theory for symmetry protected topological states in quantum antiferromagnets

University of Geneva

Shintaro Takayoshi

ST, P. Pujol, and A. Tanaka, Phys. Rev. B **94**, 235159 (2016).

ST, K. Totsuka and A. Tanaka, Phys. Rev. B **91**, 155136 (2015).

# Introduction

Gapped phases

Long range entangled state

anyonic exciation, intrinsic topological order
(e.g., FQHE)

Short range entangled state

$\xrightarrow[\text{Local unitary transformation}]{\text{w/o any symmetry}}$ Trivial (direct product) state

Short range entangled state can be nontrivial
by imposing some symmetry

Symmetry protected topological (SPT) phase

# Introduction

Ground state of S=1 Heisenberg antiferromagnet

Haldane phase (Affleck-Kennedy-Lieb-Tasaki state)

Haldane phase is protected by

A) $\pi$ rotation around $S^x, S^y, S^z$ spin axes (Dihedral symmetry)

B) Time-reversal symmetry

C) Parity symmetry (Bond-centered inversion)

Discussion by matrix product state (MPS)

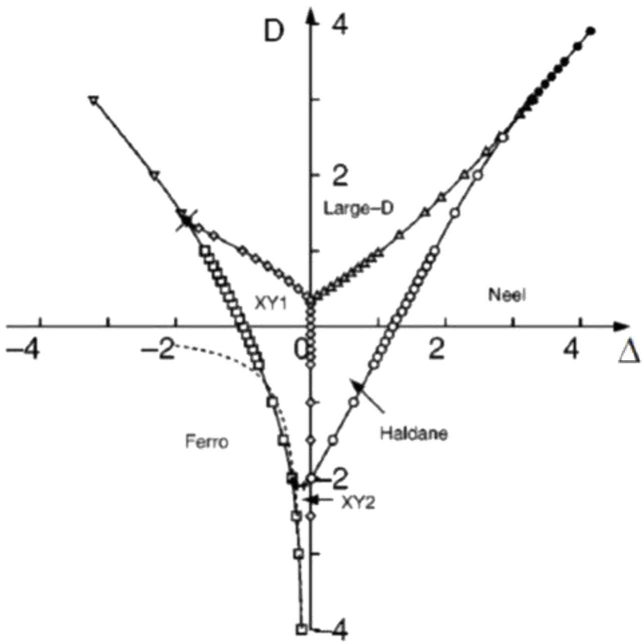
Pollmann, et al., 2010, 2012

# Example: Heisenberg antiferromagnets

$$\mathcal{H} = J\sum_j (S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z) + D\sum_j (S_j^z)^2$$

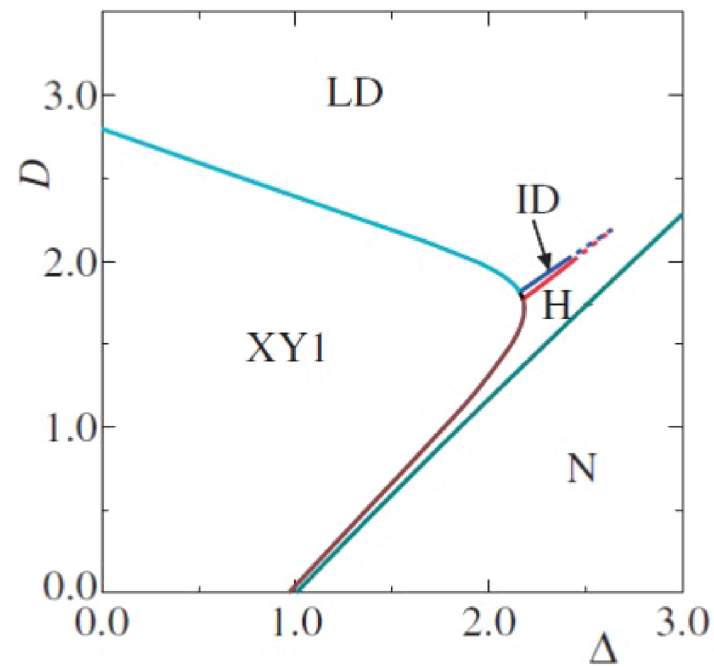
Large-D (trivial) phase $|000\cdots\rangle$ ( $S^z$-basis)

S=1




Chen et al., 2003

S=2




Tonegawa et al., 2011

# (1+1)D antiferromagnets and NLSM

$$\mathcal{H} = J \sum_j \boldsymbol{S}_j \cdot \boldsymbol{S}_{j+1}$$

Effective field theory:

O(3) nonlinear sigma model + theta term Haldane, 1983

$$\mathcal{S}_{\text{eff}}^{\text{1d}} = \frac{1}{2g} \int d\tau dx (\partial_\mu \boldsymbol{n})^2 + i\Theta Q_{\tau x} \qquad \Theta = 2\pi S$$

$$\boldsymbol{n} = (n_x, n_y, n_z) \qquad Q_{\tau x} = \frac{1}{4\pi} \int d\tau dx \boldsymbol{n} \cdot \partial_\tau \boldsymbol{n} \times \partial_x \boldsymbol{n} \in \mathbb{Z}$$

$$|\boldsymbol{n}| = 1$$

- S: half-odd integer $\Theta \equiv \pi \pmod{2\pi}$ gapless
- S: integer $\Theta \equiv 0 \pmod{2\pi}$ gapped
  - S=odd: SPT
  - S=even: trivial

Matrix product state discussion

Pollmann et al.,2010, 2012

What is the field theoretical difference (S=odd/even)?

-See the ground state wave functional.

# Planar limit of NLSM

Take the easy-plane config. (planar limit) $\boldsymbol{n} \to \boldsymbol{n}^{\mathrm{pl}} \equiv (\cos\phi, \sin\phi, 0)$

Theta term vanishes? -> No.

$$\mathcal{H} = J\sum_j \boldsymbol{S}_j \cdot \boldsymbol{S}_{j+1} + D\sum_j (S_j^z)^2$$

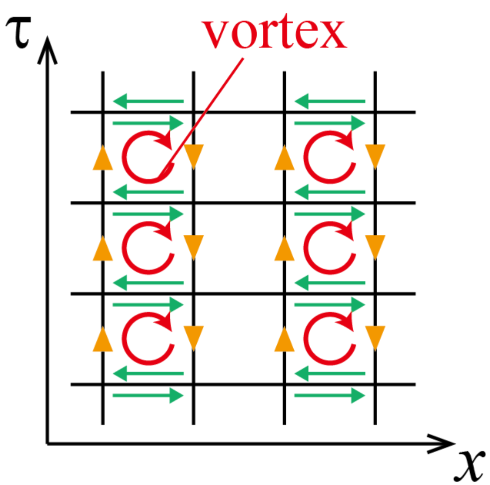
Origin: staggered (AF) summation over spin Berry phase.

$$i\frac{2\pi S}{4\pi}\int d\tau dx \boldsymbol{n}\cdot\partial_\tau\boldsymbol{n}\times\partial_x\boldsymbol{n} \longleftarrow i\sum_j(-1)^j S\frac{1}{4\pi}\int_0^1 du dx \boldsymbol{n}\cdot\partial_u\boldsymbol{n}\times\partial_\tau\boldsymbol{n}$$

Planar limit -> space-time vortex contributes

Sachdev, 2002

$$\mathcal{S}_{\mathrm{BP}}^{\mathrm{tot}} = iS\sum_j(-1)^j\int d\tau\partial_\tau\phi_j(\tau) = i2\pi S\sum_{\bar{j}} Y_{\bar{j}} Q_{\mathrm{v}}(\bar{j})$$




$$Y_{\bar{j}} = 1 \quad (\bar{j}:\ \text{odd})$$
$$Y_{\bar{j}} = 0 \quad (\bar{j}:\ \text{even})$$

Average weight of vortex
(coarse grained) $\langle Y\rangle = 1/2$

$$\mathcal{S}_{\mathrm{BP}}^{\mathrm{tot}} = i2\pi S\langle Y\rangle Q_{\mathrm{v}} = i\pi S Q_{\mathrm{v}}$$

# Ground state wave functional

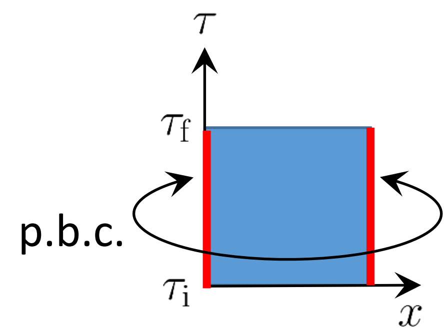
$$\mathcal{S} = \int d\tau dx \Big[\frac{1}{2g}(\partial_\mu \phi)^2 + i\frac{\pi S}{2\pi}(\partial_\tau\partial_x - \partial_x\partial_\tau)\phi\Big]$$

Strong coupling limit $g \to \infty$

$$|\Psi\rangle = \sum_{\{\phi(x)\}} \Psi[\phi(x)] |\phi(x)\rangle$$ Spin config. $\{\phi(x)\}$

## Path integral formalism Xu-Senthil, 2013

$$\Psi[\phi(x)] \propto \int_{\phi_{\rm i}}^{\phi_{\rm f}=\phi(x)} \mathcal{D}\phi'(\tau,x) e^{-\mathcal{S}[\phi'(\tau,x)]}$$




Initial and final imaginary time $\tau_{\rm i(f)}$

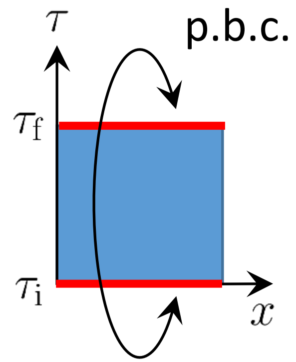
$$\longrightarrow \boxed{\Psi[\phi(x)] \propto e^{-iS\pi W}}$$

$$= \begin{cases} (-1)^W & \text{if } S = \text{odd} \\ 1 & \text{if } S = \text{even} \end{cases}$$

$$W \equiv \frac{1}{2\pi}\int dx \partial_x \phi \in \mathbb{Z}$$

Winding number of the planar spin config.

# Edge state




$$\mathcal{S} = \int d\tau dx \Big[\frac{1}{2g}(\partial_\mu \phi)^2 + i\frac{S}{2}(\partial_\tau\partial_x - \partial_x\partial_\tau)\phi\Big]$$

$$\mathcal{S}_{\mathrm{edge}} = \pm i\frac{S}{2}\int d\tau\partial_\tau\phi = \pm i\frac{\Theta}{2\pi}\int d\tau\partial_\tau\phi$$

$$\Theta = \pi S$$

$$\begin{cases} \Theta \equiv \pi \pmod{2\pi}\ (S = \mathrm{odd}) \\ \Theta \equiv 0 \pmod{2\pi}\ (S = \mathrm{even}) \end{cases}$$

# Dual field theory and SPT breaking

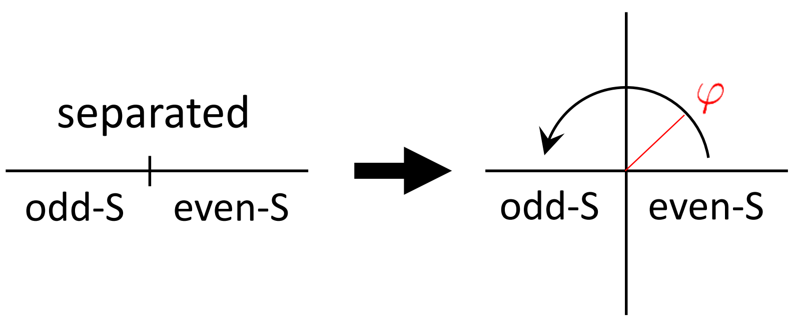
Dual vortex field theory (and low fugacity expansion)
-> sine-Gordon model

$$\mathcal{L}^{\rm 1d}_{\rm dual}[\varphi(\tau,x)] = \frac{g}{8\pi^2}(\partial_\mu\varphi)^2 - 4z\cos(\pi S)\cos\varphi \qquad z = e^{-\mu}$$

staggered field -> staggered mag. $\delta m$

$$\mathcal{L}'^{\rm 1d}_{\rm dual}[\varphi(\tau,x)] = \frac{g}{8\pi^2}(\partial_\mu\varphi)^2 - 2z\cos(\varphi - \pi(S-\delta m))$$

Phase is locked at $\varphi = \pi(S-\delta m)$



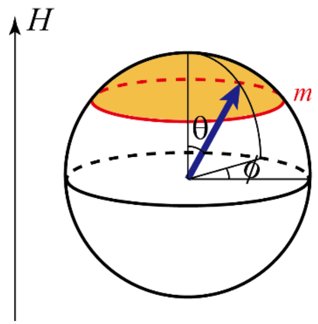

$\varphi = 0$ $\qquad$ $\varphi = \pi$

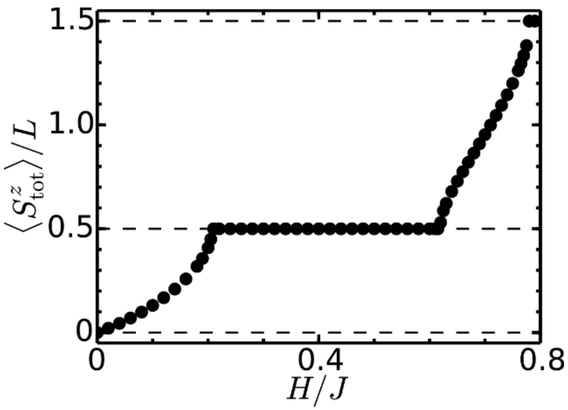
S = even and odd are continuously connected by changing $\delta m$. Staggered field should be prohibited.

# Magnetization plateau

ST, K. Totsuka and A. Tanaka, Phys. Rev. B 91, 155136 (2015).

The above discussion is also valid for magnetization plateaus just by replacing $S \to S - m$.

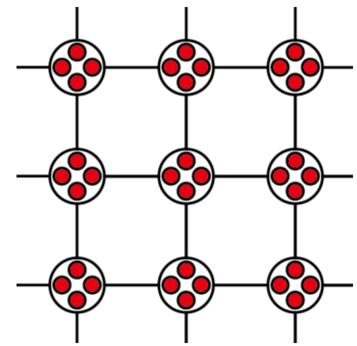



$$\mathcal{S} = \int d\tau dx \left[ \frac{1}{2g} (\partial_\mu \phi)^2 + i \frac{S-m}{2} (\partial_\tau \partial_x - \partial_x \partial_\tau) \phi \right]$$

$$\Psi[\phi(x)] \propto e^{-i(S-m)\pi W} = \begin{cases} (-1)^W & \text{if } S-m = \text{odd} \\ 1 & \text{if } S-m = \text{even} \end{cases}$$

# (2+1)D AKLT states

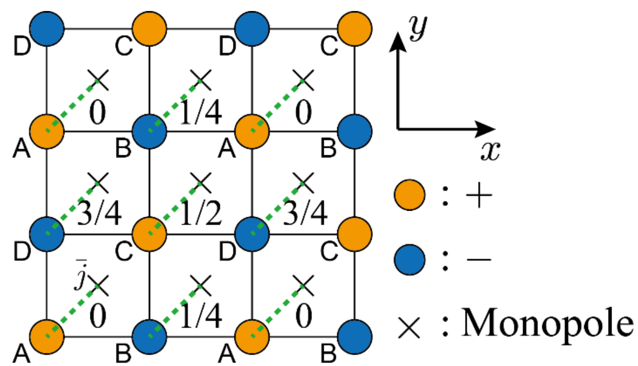


Spatially isotropic VBS state (S=even).

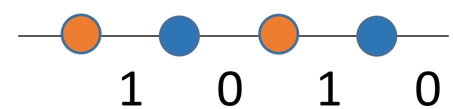
Berry phase from monopoles
(tunneling of skyrmion).

$$Q_{xy} = \frac{1}{4\pi}\int dxdy\, \boldsymbol{n}\cdot\partial_x\boldsymbol{n}\times\partial_y\boldsymbol{n} \in \mathbb{Z}$$

$Q_{\mathrm{mon}}(\bar{j}) = \Delta_\tau Q_{xy}(\bar{j})$ monopole number at dual site $\bar{j}$




cf. (1+1)D



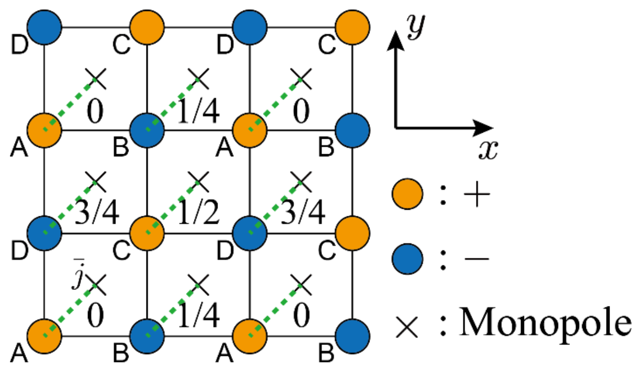

$$\mathcal{S}_{\mathrm{BP}} = \sum_{x,y}(-1)^{x+y}\mathcal{S}_{\mathrm{BP}}(x,y) \;\to\; i4\pi S\sum_{\bar{j}} Y_{\bar{j}} Q_{\mathrm{mon}}(\bar{j})$$

Haldane, 1988
Sachdev-Vojta, 2000

$$Y_{\bar{j}} = 0, 1/4, 1/2, 3/4$$

$$\mathcal{S}_{\mathrm{BP}}(x,y) = iS\int_0^1 du\int d\tau\, \boldsymbol{n}\cdot\partial_u\boldsymbol{n}\times\partial_\tau\boldsymbol{n}$$ at site(x,y)

# (2+1)D AKLT states




$$\mathcal{S}_{\mathrm{BP}} = i4\pi S \sum_{\bar{j}} Y_{\bar{j}} Q_{\mathrm{mon}}(\bar{j})$$

Shift $\tilde{Y}_{\bar{j}} = Y_{\bar{j}} - 1/4$

Average weight of vortex (coarse grained) $\langle \tilde{Y} \rangle = 1/8$

$$\mathcal{S}_{\mathrm{BP}} \overset{\mathrm{cont.}}{=} i4\pi S \langle \tilde{Y} \rangle Q_{\mathrm{mon}}^{\mathrm{tot}} = i\frac{S}{4} \int d\tau d^2\boldsymbol{r} \epsilon_{\mu\nu\lambda}\partial_\mu\partial_\nu a_\lambda$$

$$\mathcal{S}_{\mathrm{eff}}^{\mathrm{2d}} = \int d\tau d^2\boldsymbol{r} \left\{ \frac{1}{2K}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 + i\frac{S}{4}\epsilon_{\mu\nu\lambda}\partial_\mu\partial_\nu a_\lambda \right\}$$

$$= \frac{1}{2K}\int d\tau d^2\boldsymbol{r} (\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 + i\frac{\pi S}{2} Q_{\mathrm{mon}}^{\mathrm{tot}}$$

# (2+1)D AKLT states

## Ground state wave functional

$$\mathcal{S}_{\text{eff}}^{\text{2d}} = \frac{1}{2K}\int d\tau d^2\boldsymbol{r}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 + i\frac{\pi S}{2}Q_{\text{mon}}^{\text{tot}}$$

Path integral formalism

$$\Psi[\boldsymbol{n}(\boldsymbol{r})] = \int_{\boldsymbol{n}_{\text{i}}(\boldsymbol{r})}^{\boldsymbol{n}(\boldsymbol{r})}\mathcal{D}\boldsymbol{n}(\tau,\boldsymbol{r})e^{-\mathcal{S}_{\text{eff}}^{\text{2d}}} \propto e^{-i\frac{\pi S}{2}Q_{xy}} = (-1)^{\frac{S}{2}Q_{xy}}$$

$$= \begin{cases} (-1)^{Q_{xy}} \text{ if } S\equiv 2 \pmod 4 \\ \quad 1 \quad \text{ if } S\equiv 0 \pmod 4 \end{cases}$$

$$Q_{xy} = \frac{1}{4\pi}\int dxdy\boldsymbol{n}\cdot\partial_x\boldsymbol{n}\times\partial_y\boldsymbol{n} = \frac{1}{2\pi}\int dxdy(\partial_x a_y - \partial_y a_x) \in \mathbb{Z}$$

Wrapping (skyrmion) # of the spin snapshot configuration

## Edge state

$$\mathcal{S}_{\text{eff}}^{\text{2d}} = \int d\tau d^2\boldsymbol{r}\left\{\frac{1}{2K}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 + i\frac{S}{4}\epsilon_{\mu\nu\lambda}\partial_\mu\partial_\nu a_\lambda\right\}$$

$$\mathcal{S}_{y\text{-edge}} = \pm i\frac{S}{4}\int d\tau dx(\partial_\tau a_x - \partial_x a_\tau) = \pm i\frac{\pi S}{2}Q_{\tau x}$$

theta term

$$\Theta = \frac{\pi S}{2}$$

$$\begin{cases} \Theta \equiv \pi \pmod{2\pi}\ (S = 2, 6, \ldots) \\ \Theta \equiv 0 \pmod{2\pi}\ (S = 4, 8, \ldots) \end{cases}$$

# 1D-2D analogy

| | (1+1)D easy plane Haldane state | (2+1)D VBS states |
|---|---|---|
| target manifold | $S^1$ (planar) | $S^2$ (spherical) |
| singular space-time event | vortex (phase-slip) $\Delta_\tau Q_x \neq 0$ | monopole $\Delta_\tau Q_{xy} \neq 0$ |
| GS wave functional | $\Psi[\phi(x)] \propto e^{-i\pi S Q_x}$ | $\Psi[\boldsymbol{n}(x,y)] \propto e^{-i\pi \frac{S}{2} Q_{xy}}$ |
| topo. # of spin config. | $Q_x \equiv \frac{1}{2\pi}\int dx \partial_x \phi$ | $Q_{xy} \equiv \frac{1}{4\pi}\int dx dy \boldsymbol{n}\cdot\partial_x \boldsymbol{n} \times \partial_y \boldsymbol{n}$ |
| Distinction | $S$ = even vs odd | $S = 2,6,\ldots$ vs $4,8,\ldots$ |

S = 2,6,… : SPT
S = 4,8,… : trivial
Dual monopole theory:
x- and y- dimerization breaks SPT.

SO(3) + translational symmetry would protect SPT.

# Strange Correlator

- Definition Y.-Z. You et al., 2014

$$C_{\mathrm{S}}(\boldsymbol{R},0) \equiv \frac{\langle \Psi_0 | \hat{\boldsymbol{n}}(\boldsymbol{R}) \cdot \hat{\boldsymbol{n}}(0) | \Psi \rangle}{\langle \Psi_0 | \Psi \rangle}$$

$|\Psi\rangle$ : Ground state

$|\Psi_0\rangle$ : Trivial (direct product) state

$R \to \infty$
- nonzero or power-law decay: SPT
- Exp. decay: trivial

- Idea

$$\Psi[\boldsymbol{n}(\boldsymbol{r})] = N e^{-W[\boldsymbol{n}(\boldsymbol{r})]}$$

$$W[\boldsymbol{n}(\boldsymbol{r})] \equiv \int d^2\boldsymbol{r} \left[ \frac{1}{2\tilde{g}} (\partial_\alpha \boldsymbol{n})^2 + i \frac{\Theta}{4\pi} \boldsymbol{n} \cdot \partial_x \boldsymbol{n} \times \partial_y \boldsymbol{n} \right]$$

Usual two-point correlator

$$C(\boldsymbol{R},0) = \langle \Psi | \hat{\boldsymbol{n}}(\boldsymbol{R}) \cdot \hat{\boldsymbol{n}}(0) | \Psi \rangle = \int \mathcal{D}\boldsymbol{n}(\boldsymbol{r}) |\Psi[\boldsymbol{n}(\boldsymbol{r})]|^2 \boldsymbol{n}(\boldsymbol{R}) \cdot \boldsymbol{n}(0)$$

No topological effect

Strange correlator

$$C_{\mathrm{S}}(\boldsymbol{R},0) \equiv \frac{\langle \Psi_0 | \hat{\boldsymbol{n}}(\boldsymbol{R}) \cdot \hat{\boldsymbol{n}}(0) | \Psi \rangle}{\langle \Psi_0 | \Psi \rangle}$$

Effects from the topo. term

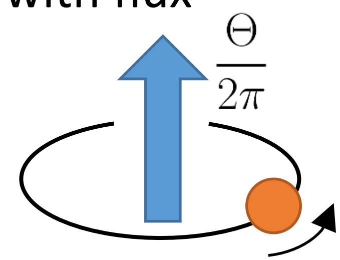
# 1D strange correlator

$$C_{\mathrm{S}}(X,0) \equiv \frac{1}{Z}\int_{\mathrm{pbc}} \mathcal{D}\phi(x)\cos\phi(X)\cos\phi(0)e^{-\frac{1}{\hbar}\int dx[\frac{\hbar^2}{\tilde{g}}(\partial_x\phi)^2 \underline{+i\hbar\frac{\Theta}{2\pi}\partial_x\phi]}}$$

Aharonov-Bohm phase

Relabel $x \to \tau$

Imaginary time correlator of a particle on a ring with flux




$$\hat{\mathcal{H}} = \frac{\tilde{g}}{4}\left(\hat{N} - \frac{\Theta}{2\pi}\right)^2 \qquad \psi_n(\phi) = \frac{1}{\sqrt{2\pi}}e^{-in\phi}$$

$$\Theta = \pi S$$

$$C_{\mathrm{S}}(\tau,0) \equiv \langle \cos\hat{\phi}(\tau)\cos\hat{\phi}(0)\rangle \qquad \langle \hat{O}(\tau)\hat{O}(0)\rangle = \sum_n e^{-\frac{\tau}{\hbar}(E_n - E_0)}|\langle n|\hat{O}|G\rangle|^2$$

(i) $\Theta = 0$ case (S=even)

$$C_{\mathrm{S}}(\tau,0) = \frac{1}{2}e^{-\frac{\tilde{g}\tau}{4\hbar}}$$ exp. decay: trivial

(ii) $\Theta = \pi$ case (S=even)

$$C_{\mathrm{S}}(\tau,0) = \frac{1}{4}(1 + e^{-\frac{\tilde{g}\tau}{2\hbar}})$$ Nonzero at $\tau \to \infty$ : SPT

# 2D strange correlator

$$C_{\mathrm{S}}(\boldsymbol{R}) = \frac{\int \mathcal{D}\boldsymbol{n}(\boldsymbol{r}) e^{-W[\boldsymbol{n}(\boldsymbol{r})]} \boldsymbol{n}(\boldsymbol{R}) \cdot \boldsymbol{n}(0)}{\int \mathcal{D}\boldsymbol{n}(\boldsymbol{r}) e^{-W[\boldsymbol{n}(\boldsymbol{r})]}}$$

$$W[\boldsymbol{n}(\boldsymbol{r})] \equiv \int d^2\boldsymbol{r} \left[ \frac{1}{2\tilde{g}} (\partial_\alpha \boldsymbol{n})^2 + i\frac{\Theta}{4\pi} \boldsymbol{n} \cdot \partial_x \boldsymbol{n} \times \partial_y \boldsymbol{n} \right] \qquad \Theta = \frac{\pi S}{2}$$

Relabeling of coordinate $y \to \tau$

Strange correlator
-> two point correlator in (1+1)d NLSM + theta term

| | | |
|---|---|---|
| S=2,6,… | $\Theta \equiv \pi \pmod{2\pi}$ | half-odd integer spin chain (gapless)<br>power-law decay: SPT |
| S=4,8,… | $\Theta \equiv 0 \pmod{2\pi}$ | integer spin chain (gapped)<br>exp. decay: trivial |

Strange correlator correctly detects SPT states.

# Summary

- We described the SPT properties in AKLT-VBS states using an effective field theory, especially NLSM + topo. Term.
- SPT can be distinguished by looking at the ground state wave functional. In (2+1)d, monopole Berry phase is important.
- We calculate the strange correlator in one and two dimensions, and confirm that SPT phase can be detected.