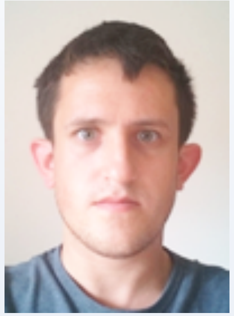
Topological
Materials
Science
トポロジーが紡ぐ物質科学のフロンティア

Chiral Anomaly Phenomena in Weyl Superconductors

Satoshi Fujimoto
Department of Materials Engineering Science,
Osaka University

Collaborators

T. Mizushima
(Osaka Univ.)



J. de Lisle
(Osaka Univ.)

T. Kobayashi
(Osaka Univ.)

Y. Ishihara
(Osaka Univ.)

Outline

- Introduction
- Torsional chiral magnetic effect in Weyl superconductors
- Thermal analogue of negative magnetoresistivity in Weyl superconductors
- Chiral spin-polarization effect in a ferromagnetic Weyl superconductor UCoGe

Weyl superconductor due to broken TRS

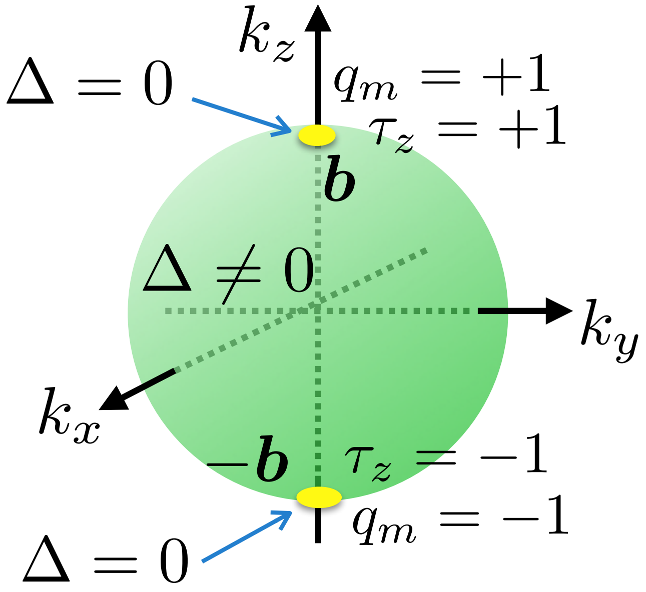
Weyl fermions = Bogoliubov quasiparticles from point-nodes

characterized by monopole charge of point nodes of SC gap

$$q_m = \pm 1$$

$$\nabla \cdot \underbrace{\Omega_{\boldsymbol{k}\boldsymbol{k}}}_{\text{Berry curvature}} = q_m \delta(\boldsymbol{k} \mp \boldsymbol{b})$$

e.g. chiral p_x+ip_y wave SC



Weyl SC: $H = \underbrace{\tau_z \boldsymbol{\sigma} \cdot \boldsymbol{k}}_{\text{TRSB}} - \underbrace{\boldsymbol{\sigma} \cdot \boldsymbol{b}}_{\text{TRSB}}$

for $b_y \neq 0$

$\boldsymbol{\sigma}$: particle-hole space

$\tau_z = \pm 1$ chirality

(sign of q_m)

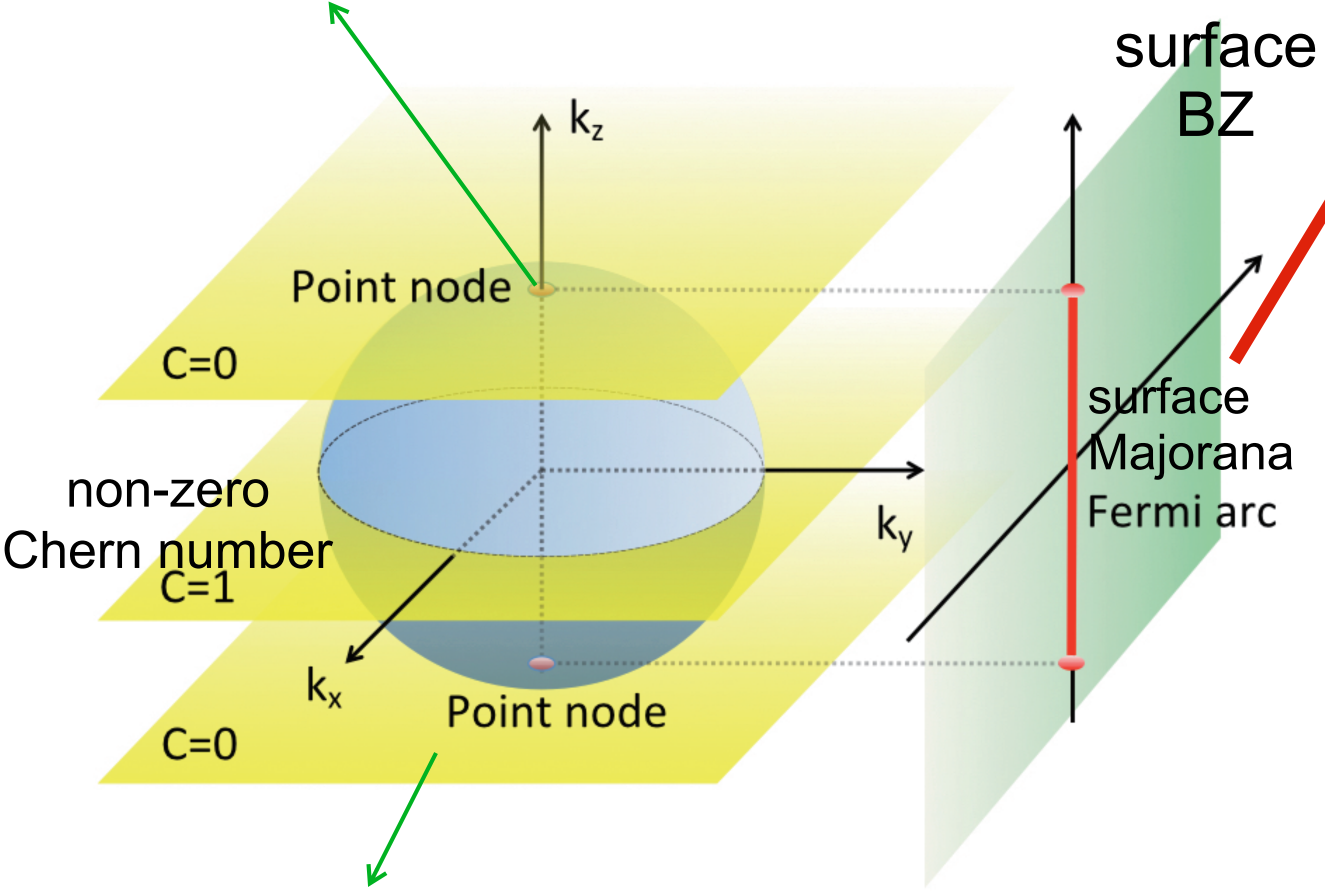
Time-reversal symmetry breaking is necessary for Weyl SC

Examples of Weyl superconductor/superfluid

example ABM phase of He3 $\Delta_{\boldsymbol{k}}^{\uparrow\uparrow} = \Delta_{\boldsymbol{k}}^{\downarrow\downarrow} = \Delta(k_x + ik_y)$

(Volovik)

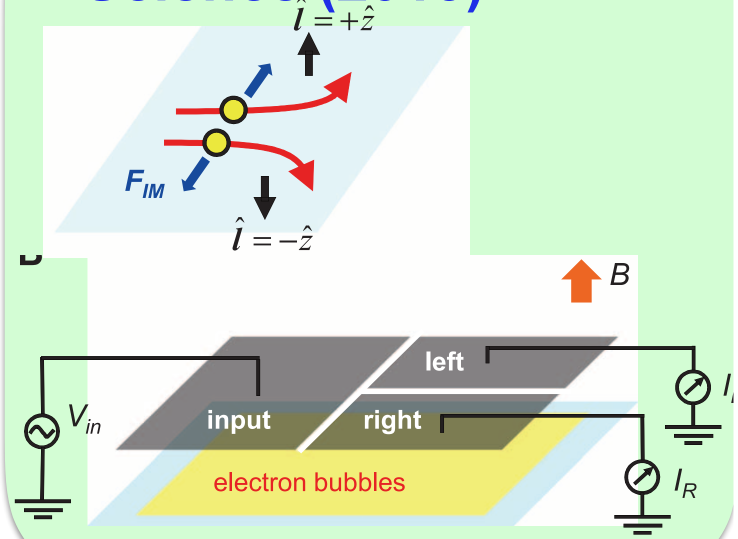
Weyl points (monopole charge -1)



Weyl points(monopole charge +1)

(thermal) Anomalous Hall effect

c.f. H. Ikegami et al, Science (2013)

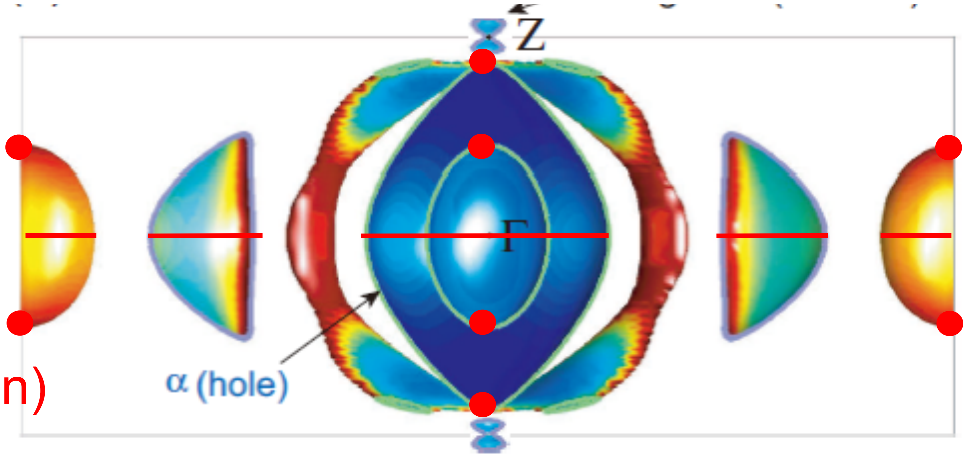


N.B. Weyl points have spin-degeneracy

Examples of Weyl superconductor

example 2: URu_2Si_2 Chiral $d_{zx}+id_{yz}$ SC

$$\Delta_{\boldsymbol{k}} = \Delta k_z(k_x + ik_y)$$



Line nodes

point nodes
(linear dispersion)

Thermal conductivity,
specific heat, …
(Kasahara et al.;
Yano et al.)

- **Kerr effect** (Kapitulnik’s group, 2015)
- **Giant Nernst effect due to chiral SC fluctuation**

(exp. : Matsuda’s group, 2016 ;

theory : H. Sumiyoshi and S. F. , 2015)

N.B. Weyl points have spin-degeneracy

Examples of Weyl superconductor

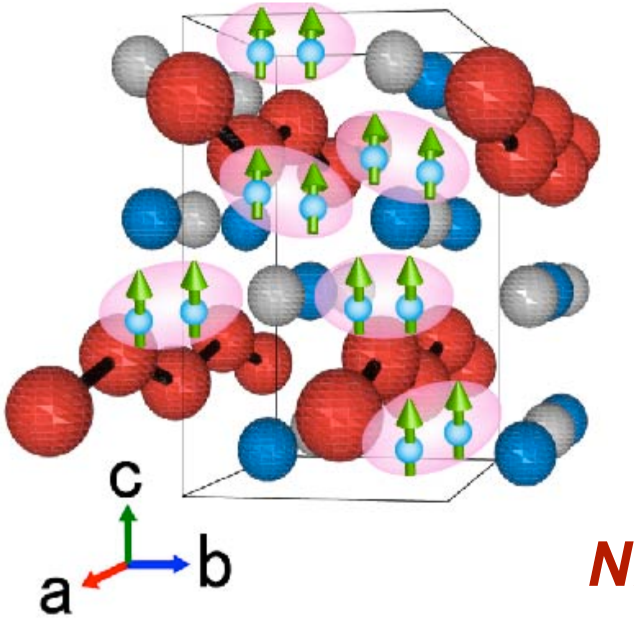
example 3: UCoGe Ferromagnetic SC non-unitary spin-triplet SC

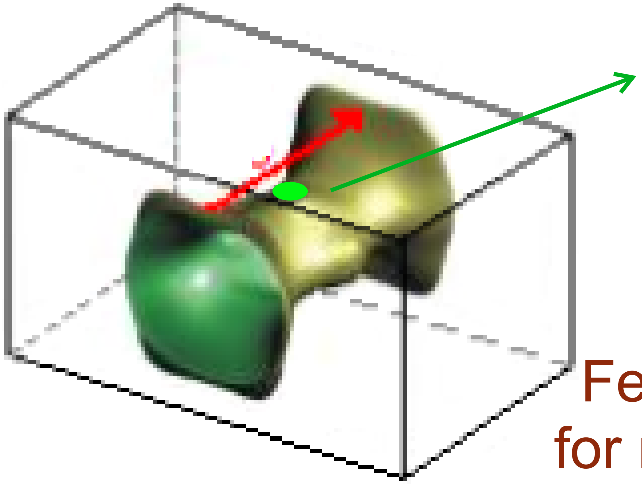
$$\boldsymbol{d} = (a_1 k_a + i a_2 k_b, a_3 k_b + i a_4 k_a, 0)$$

$$\boldsymbol{d} \times \boldsymbol{d}^* \neq 0$$

$$\Delta_{\uparrow\uparrow} \neq \Delta_{\downarrow\downarrow}$$

(Mineev, PRB66, 134504;
Hattori, Tada et al., PRL108, 066403)





point node
Weyl point

Fermi surface
for majority spin
(Samsel-Czekala et al.)

N.B. Weyl points have no spin-degeneracy !!

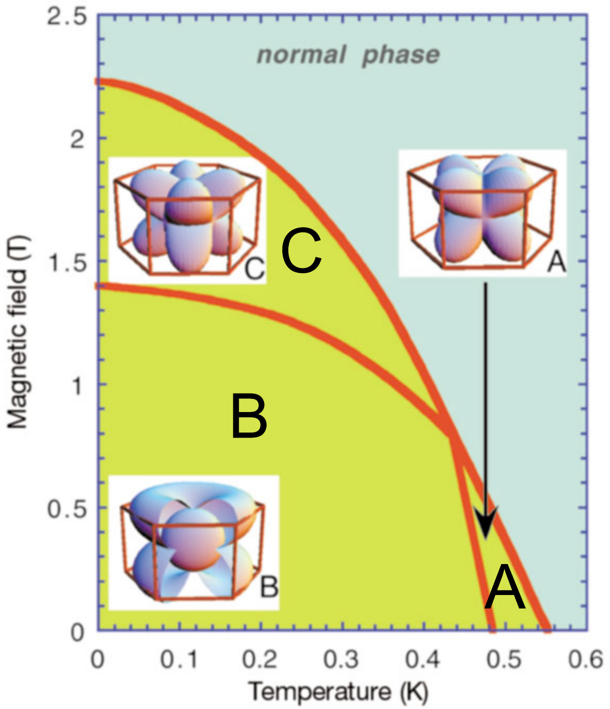
Genuine Weyl superconductor !!

Examples of Weyl superconductor

example 4: B phase of $\mathrm{UPt_3}$ Chiral *f-wave* SC ? (controversial)

$$\Delta_{\boldsymbol{k}} = \Delta k_z (k_x + i k_y)^2$$

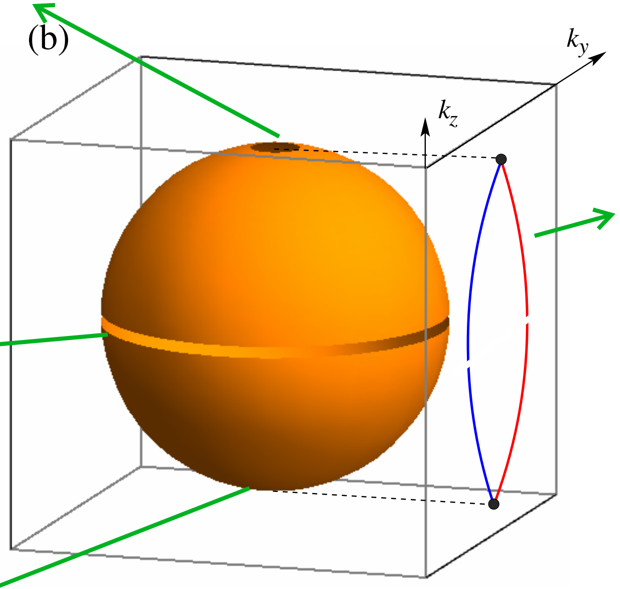
(Schemm et al.)



(Huxley et al.)

Point node
(k-quadratic)
double-Weyl
points
(monopole
charge +2)

line node



Majorana arc
anomalous
thermal
Hall effect

Point node
(k-quadratic)
double-Weyl points
(monopole
charge -2)

(Goswami-Nevidomskyy)

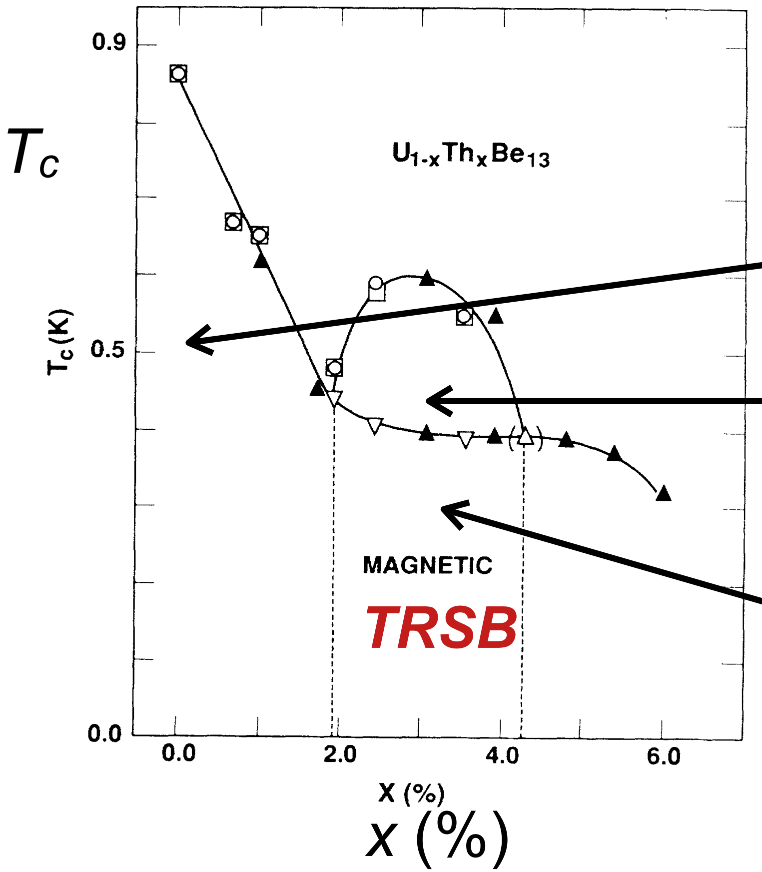
N.B. Weyl points have spin-degeneracy

Examples of Weyl superconductor

example 5: $U_{1-x}Th_xBe_{13}$ odd parity pairing state ?

possible d-vector (Shimizu et al.(2017))

(Mizushima and Nitta (2017))



(Heffner et al.(1990))

E_u representation (cubic symmetry)

$$\left.\begin{array}{l} \boldsymbol{l}_2(k) = 2\hat{\boldsymbol{z}}k_z - \hat{\boldsymbol{x}}k_x - \hat{\boldsymbol{y}}k_y \\ \boldsymbol{l}_1(k) = \sqrt{3}(\hat{\boldsymbol{x}}k_x - \hat{\boldsymbol{y}}k_y) \end{array}\right\} \textit{degenerate}$$

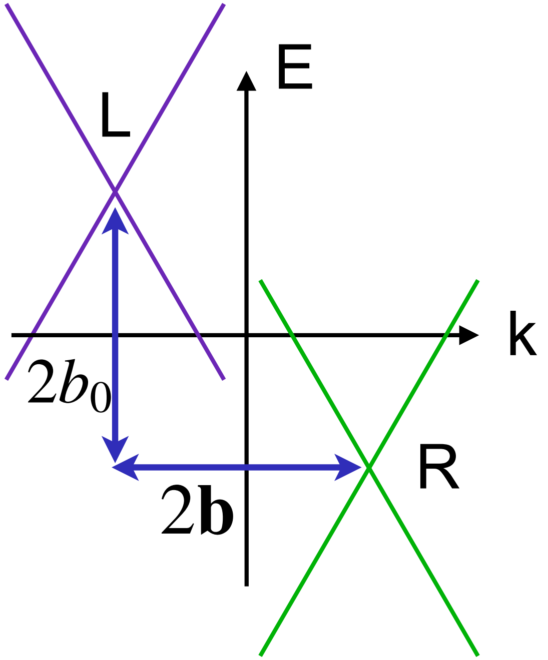
$$\boldsymbol{d}(\boldsymbol{k}) = \boldsymbol{l}_1 + i\boldsymbol{l}_2 = \hat{\boldsymbol{x}}k_x + \epsilon\hat{\boldsymbol{y}}k_y + \epsilon^2\hat{\boldsymbol{z}}k_z$$

non-unitary state

Weyl SC !!

Chiral Anomaly of Weyl semimetal

violation of conservation law of axial current



$$\partial_\mu j_5^\mu = \frac{e^3}{4\pi^2}\vec{E}\cdot\vec{B}$$

$$j_5^\mu = j_L^\mu - j_R^\mu$$

$$\mu = 0,1,2,3$$
$$(t,x,y,z)$$

$$\mathcal{L}_{\rm eff} = \frac{\theta e^2}{2\pi h}\boldsymbol{E}\cdot\boldsymbol{B}$$

$$\theta(\mathbf{r},t) = 2\mathbf{b}\cdot\mathbf{r} - 2b_0 t$$

Anomalous Hall effect $\boldsymbol{J} = \frac{e^2}{\pi h}\mathbf{b}\times\boldsymbol{E}$

Chiral magnetic effect $\boldsymbol{J} = \frac{e^2 b_0}{\pi h}\boldsymbol{B}$

negative magnetoresistance $\sigma_{zz} \propto B^2\tau$ small *B*

$\boldsymbol{E}\parallel\boldsymbol{B}$ $\propto B\tau$ large *B*

(Nielsen,Ninomiya,
Burkov, Son,
Spivak)

Chiral Anomaly of Weyl superconductor

Weyl fermions in p-h space → Axial current does not couple to E and B

Chiral anomaly due to geometrical distortion (gravitational fields)

$$\partial_\mu j_5^\mu = \frac{1}{32\pi^2 \ell^2}\epsilon^{\mu\nu\rho\sigma}(\eta_{ab}T^a_{\mu\nu}T^b_{\rho\sigma} - 2R_{ab;\mu\nu}e^a_\rho e^b_\sigma) + \frac{1}{192\pi^2}\epsilon^{\mu\nu\rho\sigma}\frac{1}{4}R^{ab}_{\mu\nu}R^{cd}_{\rho\sigma}\eta_{ad}\eta_{bc}$$

(Parrikar, Hughes, Leigh,Shopurian, Ryu)

$$j_5^\mu = v_L n_L - v_R n_R$$

$R^{ab}_{\mu\nu}$: Riemann curvature

ℓ : non-universal cutoff

$T^a_{\mu\nu} = \partial_\mu e^a_\nu - \partial_\nu e^a_\mu$: torsion

e^a_μ : vielbein

e.g. T^0_{0j} : temperature gradient (Shitade,Bradlyn,Read,Gromov,Abanov)

T^i_{0j} : spatial rotation

T^i_{jk} : vortex, topological texture of SC order parameter (l-vector, d-vector)

$i, j, k = x, y, z$

Emergent “gauge field” and “magnetic field” due to torsion

(Hughes, Leigh, Parrikar)

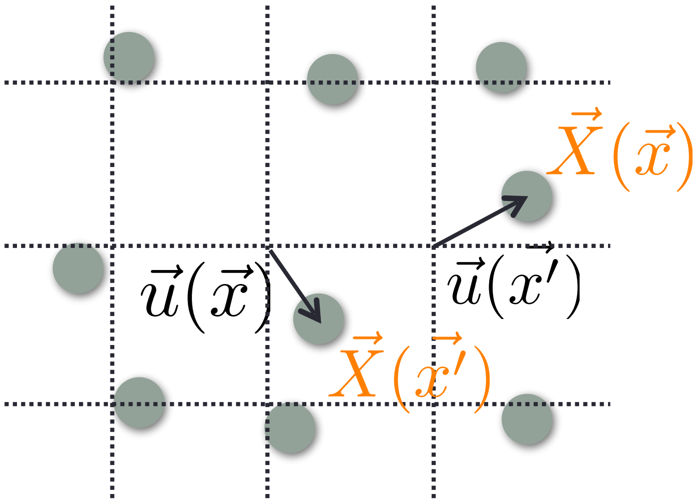
spatial inhomogeneity: $p_\mu \to e^\mu_a p_\mu$

$$e^\mu_a \approx \delta^\mu_a - \partial u^\mu / \partial x^a$$

$$e^\mu_a p_\mu \approx p_a - \underline{\frac{\partial u^\mu}{\partial x^a} p_\mu}$$

emergent
“U(1) gauge field” $\mathcal{A}_a$

Spatial distortion



Emergent “magnetic field”:

$$\mathcal{B}^\mu = \frac{\epsilon^{\mu\nu\lambda}}{2} T^a_{\nu\lambda} p_a$$

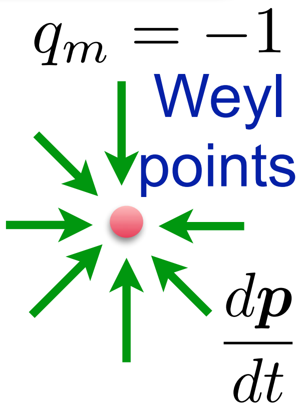
depending on momentum !

Torsion:

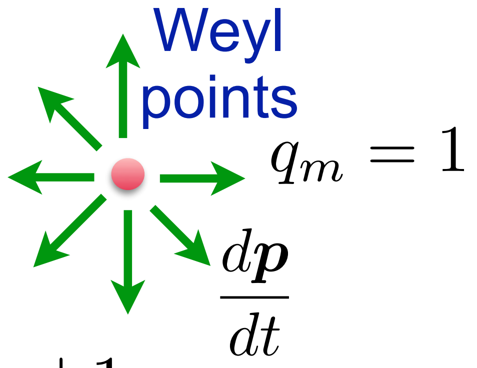
$$T^a_{\mu\nu} = \partial_\mu e^a_\nu - \partial_\nu e^a_\mu$$

Semiclassical EOM of Bogoliubov quasiparticles in Weyl SC

$$\frac{d\boldsymbol{r}}{dt} = \frac{\partial \mathcal{E}_{\boldsymbol{p}s}}{\partial \boldsymbol{p}} + \underbrace{\frac{\partial U(\boldsymbol{r})}{\partial \boldsymbol{r}} \times \boldsymbol{\Omega}_{\boldsymbol{p}\boldsymbol{p}s}}_{\text{(thermal) AHE}} - \underbrace{\left(\frac{\partial \mathcal{E}_{\boldsymbol{p}s}}{\partial \boldsymbol{p}} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}\boldsymbol{p}s}\right)\underline{\boldsymbol{\mathcal{B}}}}_{\text{torsional chiral magnetic effect}}$$



$$\frac{d\boldsymbol{p}}{dt} = -\frac{\partial U(\boldsymbol{r})}{\partial \boldsymbol{r}} + \frac{d\boldsymbol{r}}{dt} \times \underline{\boldsymbol{\mathcal{B}}} + \underbrace{\left(\frac{\partial U(\boldsymbol{r})}{\partial \boldsymbol{r}} \cdot \underline{\boldsymbol{\mathcal{B}}}\right)\boldsymbol{\Omega}_{\boldsymbol{p}\boldsymbol{p}s}}_{\text{chiral anomaly}}$$



$\boldsymbol{\Omega}_{\boldsymbol{p}\boldsymbol{p}s}$ Berry curvature of Weyl band with chirality $s = \pm 1$

$\boldsymbol{\mathcal{B}} = \boldsymbol{T}^{\mu} p_{\mu}$ emergent magnetic field due to torsion $(\boldsymbol{T}^{\mu})^{\nu} = \frac{1}{2}\epsilon^{\nu\lambda\rho}T^{\mu}_{\lambda\rho}$

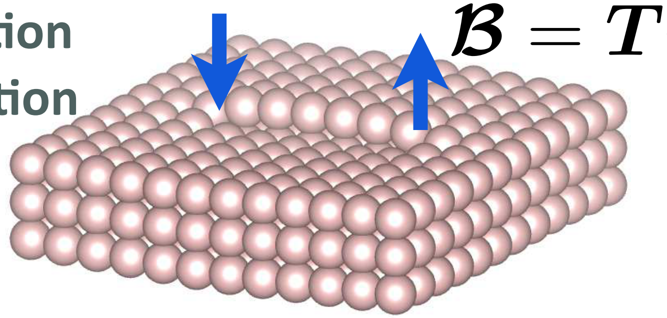
$\mathcal{E}_{\boldsymbol{p}s}$ band energy of Bogoliubov-Weyl quasiparticle

$U(\boldsymbol{r})$ applied potential, or gravitational potential

Torsional chiral magnetic effect in Weyl superconductors

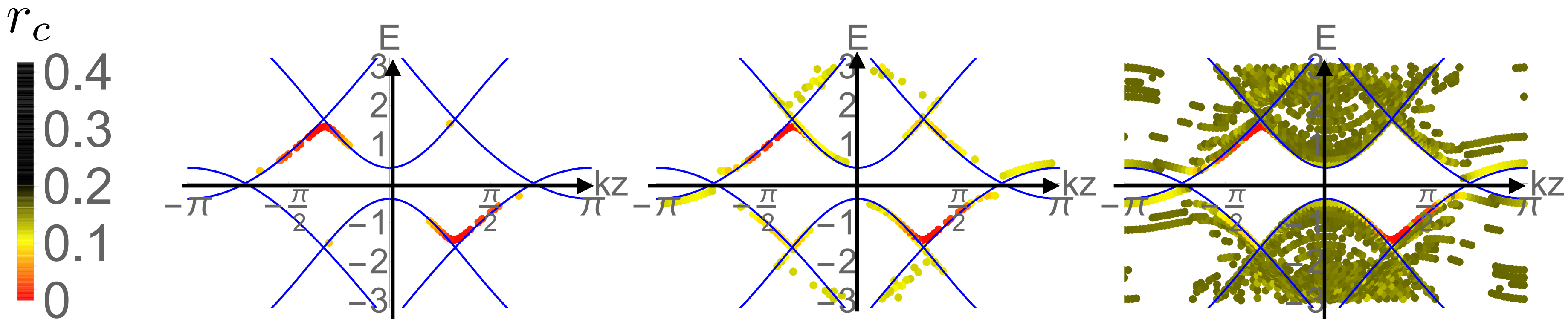
Torsional Chiral Magnetic Effect in Weyl (semi)metals with Broken Time-Reversal Symmetry and Lattice Defect

Screw dislocation
+ anti-dislocation



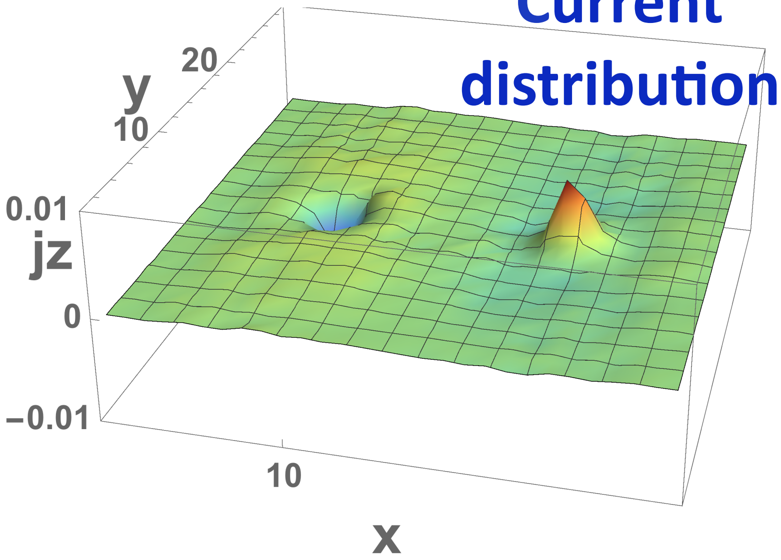
$$\boldsymbol{\mathcal{B}} = \boldsymbol{T}^{\mu} p_{\mu}$$

H. Sumiyoshi and S. F.,
Phys.Rev. Lett.116, 166601 (2016)



Spectrum asymmetry !!

Current distribution



Local equilibrium **current flowing along screw dislocation**

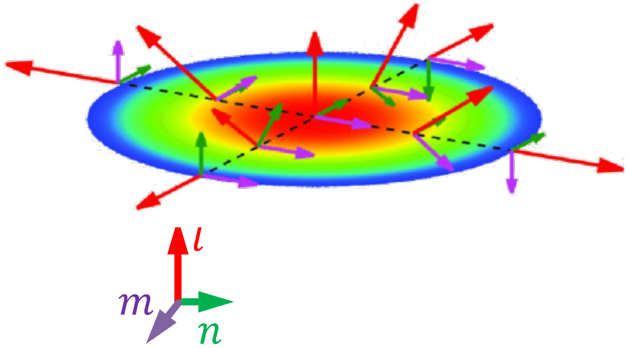
not breaking Bloch theorem !

Torsional CME in Weyl superconductor

Vortex of *l*-vector of A phase of Helium 3 : $\Delta = \Delta_0(\boldsymbol{m} + i\boldsymbol{n}) \cdot \boldsymbol{k}$

(Skyrmion-like textures)

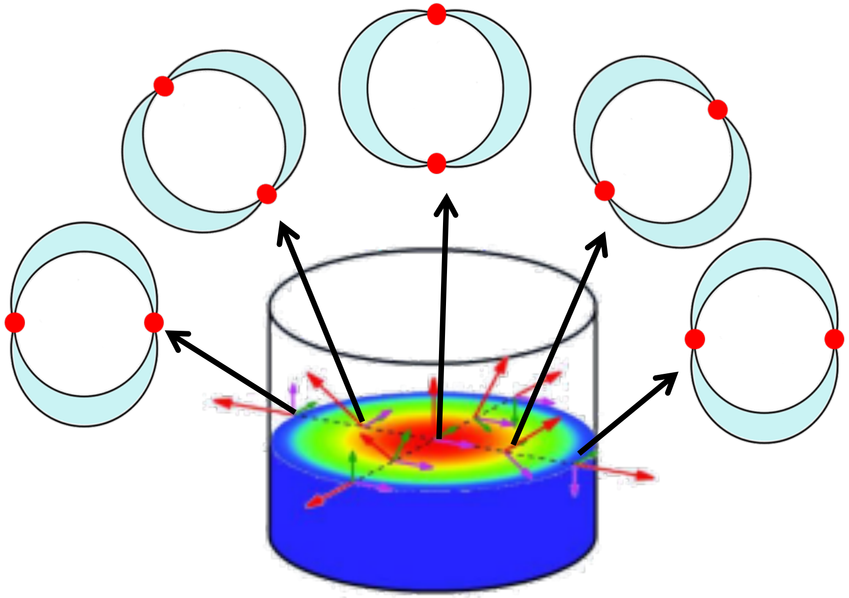
$$\boldsymbol{\ell} = \boldsymbol{m} \times \boldsymbol{n}$$



- Anderson-Toulouse vortex
- Mermin-Ho vortex

$$\mathcal{H} = e_a^{\mu} \tau^a (p_\mu - p_F \ell_\mu)$$

chiral anomaly due to $\nabla \times \boldsymbol{\ell}$ and $\partial \boldsymbol{\ell} / \partial t$ (Volovik)



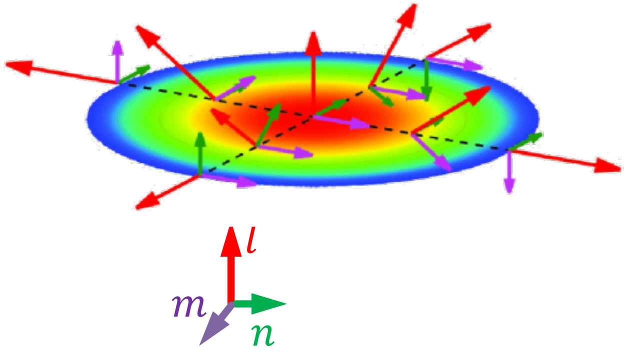
Spatially varying
Weyl points

Torsional CME in Weyl superconductor

Vortex of *l*-vector of A phase of Helium 3 : $\Delta = \Delta_0(\boldsymbol{m} + i\boldsymbol{n}) \cdot \boldsymbol{k}$

$\boldsymbol{\ell} = \boldsymbol{m} \times \boldsymbol{n}$

- Anderson-Toulouse vortex
- Mermin-Ho vortex



Torsion: $T^a_{\mu\nu} = \partial_\mu e^a_\nu - \partial_\nu e^a_\mu$

$$T^3_{23} = \partial_2 \ell_3 - \partial_3 \ell_2 \qquad T^3_{31} = \partial_3 \ell_1 - \partial_1 \ell_3 \quad \text{etc.}$$

in-plane torsional magnetic field (parallel to xy-plane)

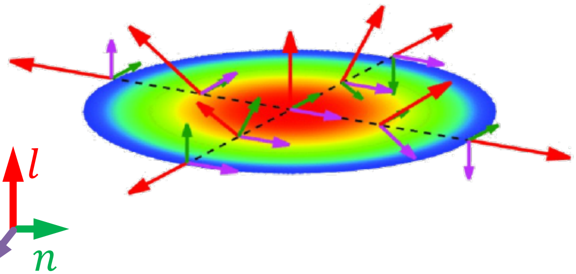
→ ***Torsional chiral magnetic effect of mass current***

$$\boldsymbol{J} = \frac{v_F \Lambda}{2\pi^2} \boldsymbol{T}^3 k_{F3} \qquad (\boldsymbol{T}^a)^\mu = \frac{\epsilon^{\mu\nu\lambda}}{2} T^a_{\nu\lambda}$$

$$\Lambda \sim \frac{\Delta}{E_F} k_F$$: UV momentum cutoff

Torsional CME in Weyl superconductor

- Mermin-Ho vortex



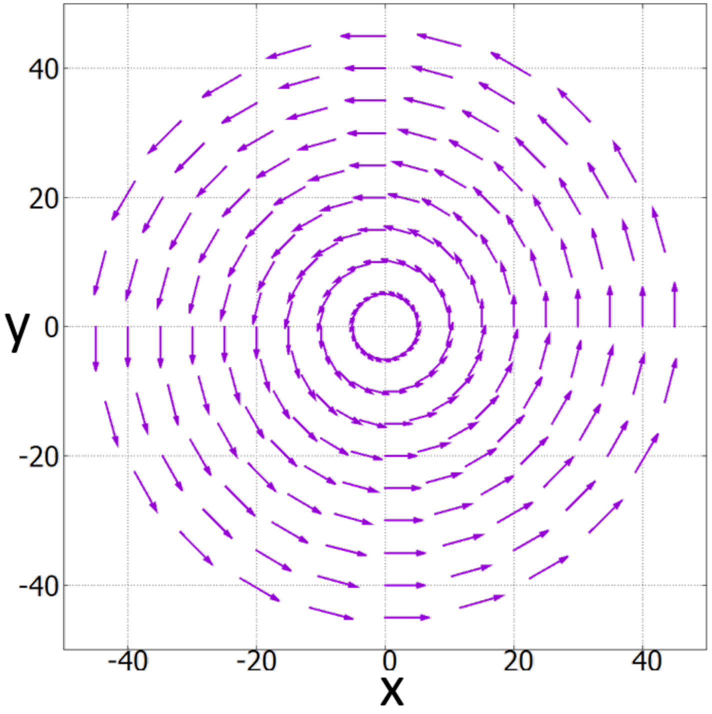
in-plane torsional magnetic field

$$T^3_{23} = \partial_2 \ell_3 - \partial_3 \ell_2 \quad T^3_{31} = \partial_3 \ell_1 - \partial_1 \ell_3$$

$$\longrightarrow \boldsymbol{J} = \frac{v_F \Lambda}{2\pi^2} \boldsymbol{T}^3 k_{F3} \qquad (\boldsymbol{T}^a)^\mu = \frac{\epsilon^{\mu\nu\lambda}}{2} T^a_{\nu\lambda}$$

Numerical Results from BdG equation

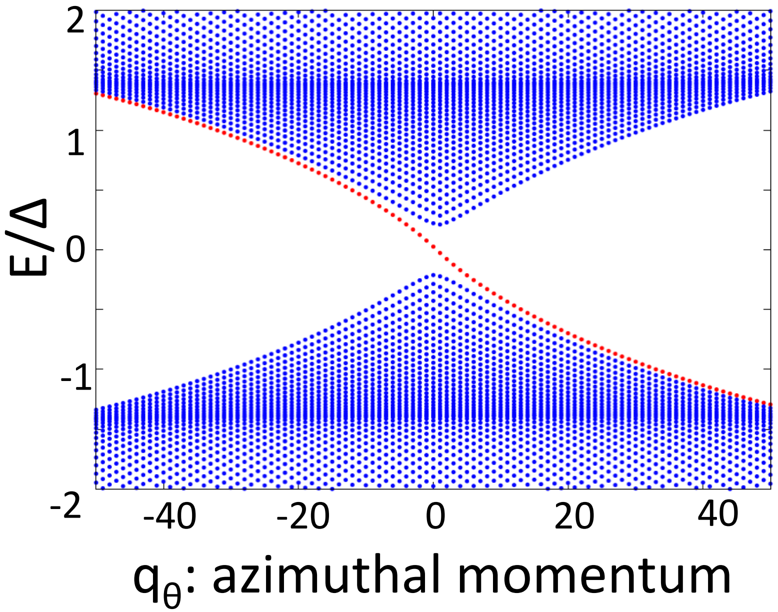
current distribution



Spectrum asymmetry

suppress backward scattering

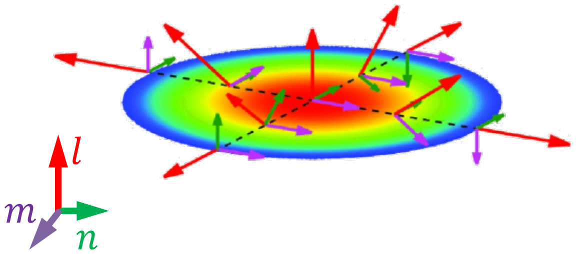
→ ***dissipationless current***



Comparison between torsional CME and supercurrent induced by l-vector textures

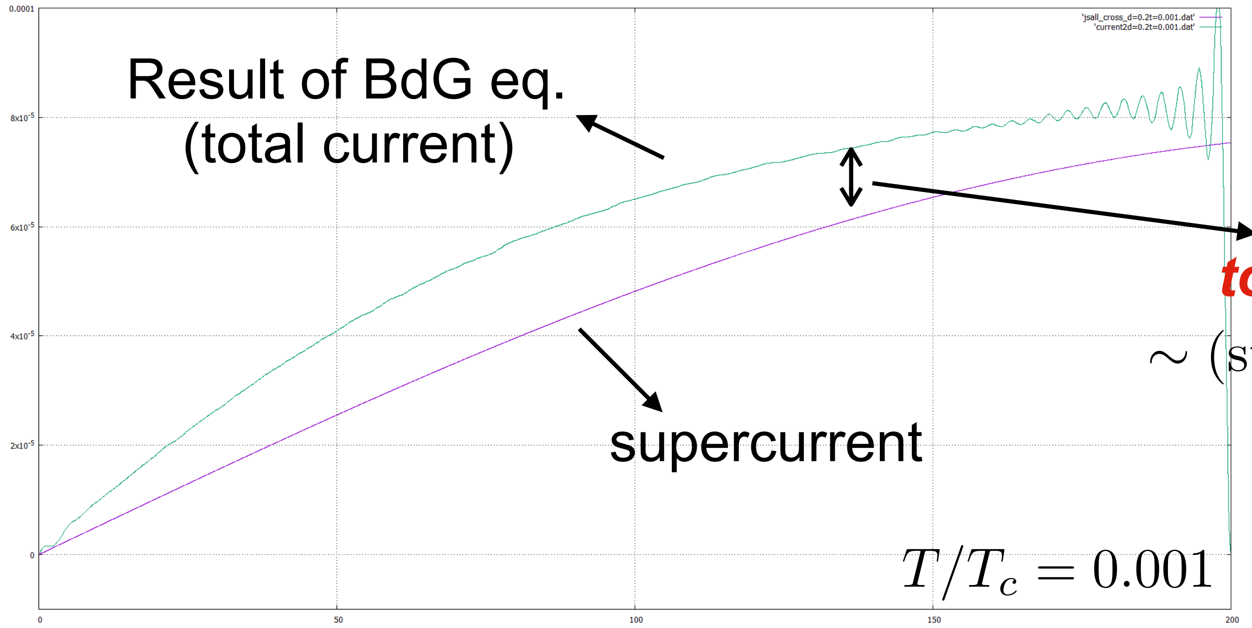
supercurrent : [Cross(1975) , Ishikawa et al.(1978), Mermin-Muzikar(1980)]

$$\mathbf{J}^0 = \rho_s^0 \frac{1}{2m} \nabla\phi + \mathbf{C}^0 \frac{1}{2m} \operatorname{curl} \hat{l}$$



Weyl quasiparticle current (torsional CME) : $\boldsymbol{J} = \frac{v_F \Lambda}{2\pi^2} \boldsymbol{T}^3 k_{F3} \sim J^0 \times \frac{\Delta}{E_F}$ $\quad \boldsymbol{T}^3 = \nabla \times \boldsymbol{\ell}$

current density



Result of BdG eq. (total current)

supercurrent

$T/T_c = 0.001$

due to torsional CME !!

$\sim (\text{supercurrent}) \times \frac{\Delta}{E_F}$

distance from the center of MH vortex

Thermal analogue of negative magnetoresistivity

in Weyl superconductors

Thermal negative magnetoresistivity in Weyl superconductor with vortices

Vortex of chiral p-waveSC : $\Delta = \Delta_0 e^{i\phi}(k_x + ik_y)$

Torsion : $T^2_{12} = \dfrac{k_F}{\Delta_0 r}$ $\quad r = \sqrt{x^2+y^2}$

Torsional CME does not occur $T^3_{\mu\nu} = 0$

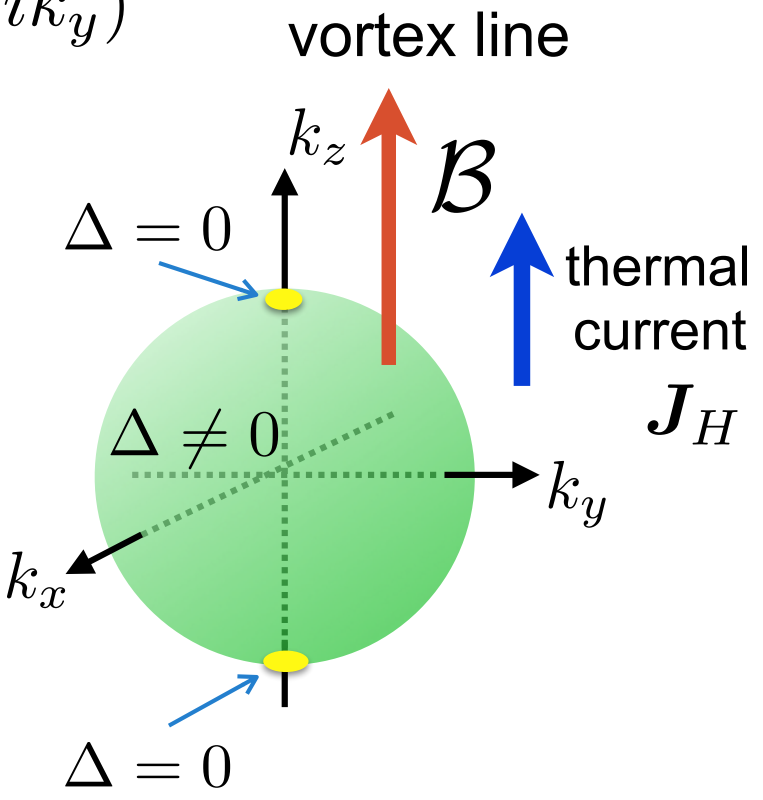
$$\boldsymbol{J} = \frac{v_F \Lambda}{2\pi^2}\boldsymbol{T}^3 k_{F3} = 0$$

However

Emergent magnetic field due to torsion :

$$(\boldsymbol{\mathcal{B}})_z = \frac{1}{r}(k_y \cos\phi - k_x \sin\phi)$$

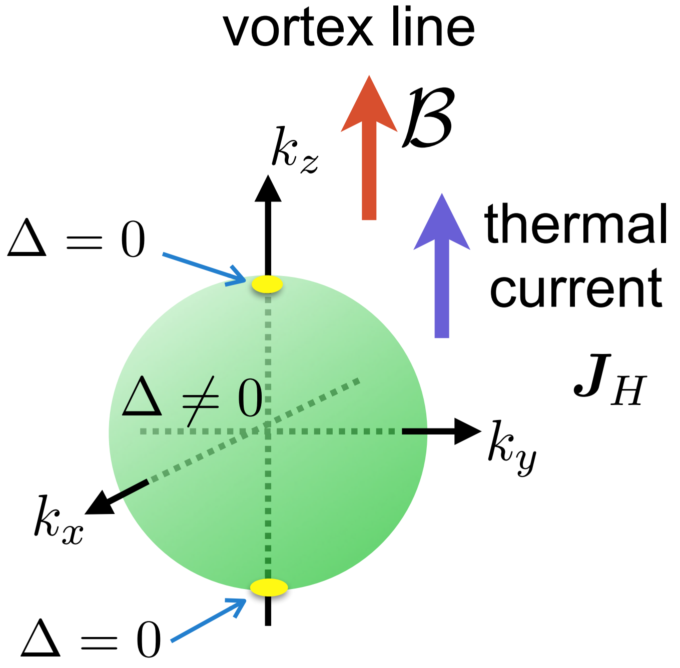
→ ***Negative magnetoresistivity of thermal current for*** $\boldsymbol{J}_H \parallel \boldsymbol{B}$



Thermal negative magnetoresistivity in Weyl superconductor with vortices

Vortex of SC order : $\Delta = \Delta_0 e^{i\phi}(k_x + ik_y)$

Torsion : $T^2_{12} = \dfrac{k_F}{\Delta_0 r}$ $\qquad r = \sqrt{x^2+y^2}$



Emergent magnetic field due to torsion :

$$(\boldsymbol{\mathcal{B}})_z = \frac{1}{r}(k_y \cos\phi - k_x \sin\phi)$$

→ ***Negative thermal magnetoresistivity***

$$\boldsymbol{J}_H = \sum_{s=\pm1}\sum_{\boldsymbol{k}} (\boldsymbol{v}_{\boldsymbol{p}s} \cdot \boldsymbol{\Omega}_{\boldsymbol{k}\boldsymbol{k}s})^2 \varepsilon^2_{\boldsymbol{p}s} \left(\frac{\partial f}{\partial \varepsilon_{\boldsymbol{p}s}}\right) \tau_{\boldsymbol{p}s}(\frac{\nabla T}{T} \cdot \boldsymbol{\mathcal{B}})\boldsymbol{\mathcal{B}},$$

$\Omega_{\boldsymbol{k}\boldsymbol{k}s}$: Berry curvature due to Weyl points

Thermal negative magnetoresistivity
in Weyl superconductor with vortices

Negative thermal magnetoresistivity

$$\boldsymbol{J}_H = \sum_{s=\pm1}\sum_{\boldsymbol{k}}(\boldsymbol{v}_{\boldsymbol{p}s}\cdot\underbrace{\boldsymbol{\Omega}_{\boldsymbol{k}\boldsymbol{k}s}}_{\downarrow})^2\varepsilon^2_{\boldsymbol{p}s}\left(\frac{\partial f}{\partial\varepsilon_{\boldsymbol{p}s}}\right)\tau_{\boldsymbol{p}s}\left(\frac{\nabla T}{T}\cdot\boldsymbol{\mathcal{B}}\right)\boldsymbol{\mathcal{B}}, = \kappa_{\mathcal{B}}\nabla T$$

$\sim 1/|\boldsymbol{k}-\boldsymbol{k}_F|^2$ $\boldsymbol{k}\sim\boldsymbol{k}_F$ **at Weyl points**

Thermal conductivity (Born approx.)

$$\kappa = \kappa_0 + \kappa_{\mathcal{B}}$$

usual contribution $\propto T$

due to torsion fields $\propto \frac{1}{T}$

Chiral anomaly effect is enhanced as temperature is lowered

However, Berry phase formula is not applicable to zero temperature limit, because of singularity at Weyl points !

Thermal negative magnetoresistivity
in Weyl superconductor with vortices

Results from Keldysh formalism of Eilenberger equation

$$[\epsilon\hat{\tau}_z - \hat{h}, \hat{g}] + i\hbar\boldsymbol{p}_F \cdot \nabla_R \hat{g} = \underline{\frac{i}{2}\left(\boldsymbol{\nabla_R}\hat{h} \cdot \boldsymbol{\nabla_p}\hat{g} - \boldsymbol{\nabla_p}\hat{h} \cdot \boldsymbol{\nabla_R}\hat{g}\right) - \frac{i}{2}\left(\boldsymbol{\nabla_R}\hat{g} \cdot \boldsymbol{\nabla_P}\hat{h} - \boldsymbol{\nabla_p}\hat{g} \cdot \boldsymbol{\nabla_R}\hat{h}\right)}$$

quantum corrections (torsional magnetic fields)

$\hat{h} = \hat{\sigma} + \hat{\Delta}$ $\hat{\sigma}$: self-energy(Born approx.) $\hat{\Delta}$: SC gap

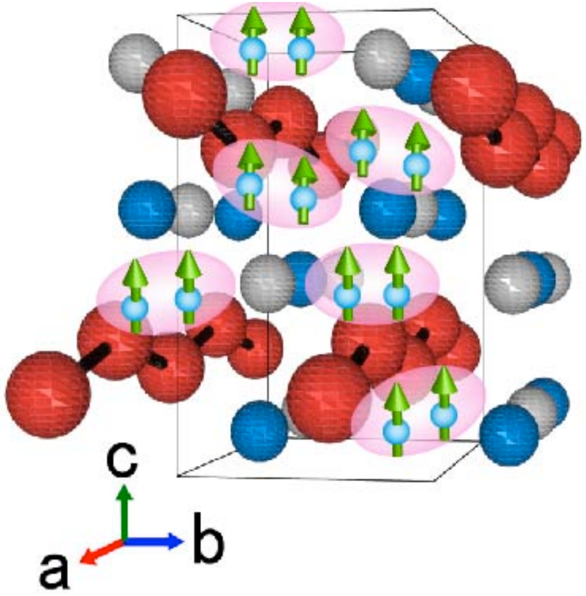
Quasi-classical Green function $\hat{g} = \begin{pmatrix} \hat{g}^R & \hat{g}^K \\ 0 & \hat{g}^A \end{pmatrix}$

Heat current $\boldsymbol{J}_H = N(0) \int dp \int \frac{d\varepsilon}{4\pi i} \varepsilon \boldsymbol{v} \mathrm{tr}[\delta\hat{g}^K]$

We consider the case with single vortex line

Chiral spin-polarization effect in ferromagnetic Weyl superconductors

Chiral Anomaly
in Ferromagnetic Weyl superconductor



UCoGe Ferromagnetic SC

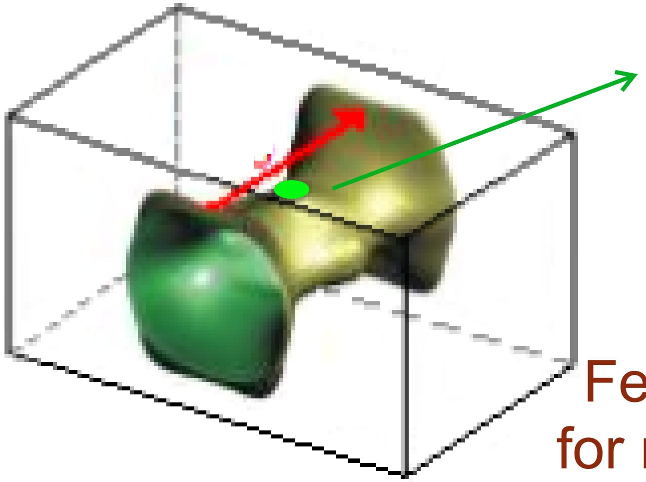
- non-unitary spin-triplet SC $\Delta_{\uparrow\uparrow} \neq \Delta_{\downarrow\downarrow}$
- strong Ising-type magnetic anisotropy
- close to quantum criticality

$T_{\mathrm{Curie}} \sim 2.5\ \mathrm{K} \qquad T_{\mathrm{SC}} \sim 0.6\ \mathrm{K}$

→ large longitudinal spin fluctuation

(NMR-exp, Hattori et al.)

fluctuating Weyl point position



point node
Weyl point

Fermi surface
for majority spin
(Samsel-Czekala et al.)

Chiral Anomaly
in Ferromagnetic Weyl superconductor

Effective Lagrangian for emergent electrodynamcis (Axion electrodynamcis)

$$\mathcal{L} = \frac{\epsilon^{\mu\nu\lambda\xi}}{4\pi^2} A^{em5}_{\mu} A^{em}_{\nu} \partial_{\xi} A^{em}_{\xi} + \frac{\epsilon^{\mu\nu\lambda\xi}}{4\pi^2} A^{em5}_{\mu} A^{em5}_{\nu} \partial_{\xi} A^{em5}_{\xi}$$

A^{em5}_{μ} : emergent chiral gauge field

(e.g. strain, twist distortion of lattice)

A^{em}_{μ} : emergent gauge field

(e.g. vortex of SC gap)

$$\boldsymbol{A}^{em5} \longleftrightarrow \boldsymbol{k}_F = \boldsymbol{k}_{F0} + \delta \boldsymbol{k}_F$$

position of Weyl points in k-space

$\delta \boldsymbol{k}_F \propto \delta \boldsymbol{m}$ fluctuation due to
ferromagnetic spin fluctuation

FM spin fluctuation couples to emergent electromagnetic fields !

Chiral Spin-Polarization Effect
in Ferromagnetic Weyl superconductor UCoGe

Effective Lagrangean

$$\mathcal{L}_\sigma = \frac{\Phi^{em}}{2\pi} \boldsymbol{A}_\sigma^{em5} \cdot \boldsymbol{B}^{em} \qquad \boldsymbol{E}^{em} = -\nabla\Phi^{em}$$

e.g. temperature gradient

$$\boldsymbol{A}_\sigma^5 = \boldsymbol{k}_{F\sigma} + \delta\boldsymbol{k}_{F\sigma} \qquad \delta\boldsymbol{k}_{F\sigma} = (0, 0, c_\sigma \delta m)$$

δm longitudinal FM spin fluctuation along z-axis

➡ Total action of spin fluctuation $S_{\mathrm{tot}} = S_{\mathrm{spin}} + \int d\boldsymbol{r}dt(\mathcal{L}_\uparrow + \mathcal{L}_\downarrow)$

$$S_{\mathrm{spin}} = \sum_{\omega,\boldsymbol{q}} \left[\frac{i\omega}{vq} - q^2 - \kappa\right] \delta m(\boldsymbol{q},\omega)\delta m(-\boldsymbol{q},-\omega)$$

➡ $\delta m = \frac{c_\uparrow + c_\downarrow}{4\pi\kappa}\Phi^{em}B_z^{em}$ Increase of spin magnetization !!

SUMMARY

- Torsional chiral magnetic effect in Weyl superconductors can be induced by skyrmion-like vortex textures of SC order parameter

- Negative thermal magnetoresistivity as a signature of chiral anomaly is realized by a vortex in Weyl superconductors

- In FM Weyl superconductors near FM quantum criticality, chiral anomaly can be detected as the increase of spin magnetization due to torsional magnetic fields induced by, e.g., twist deformation of a sample around c-axis.