Skew scattering from spin chirality fluctuations in chiral magnets

Hiro Ishizuka

University of Tokyo

Collaborator:

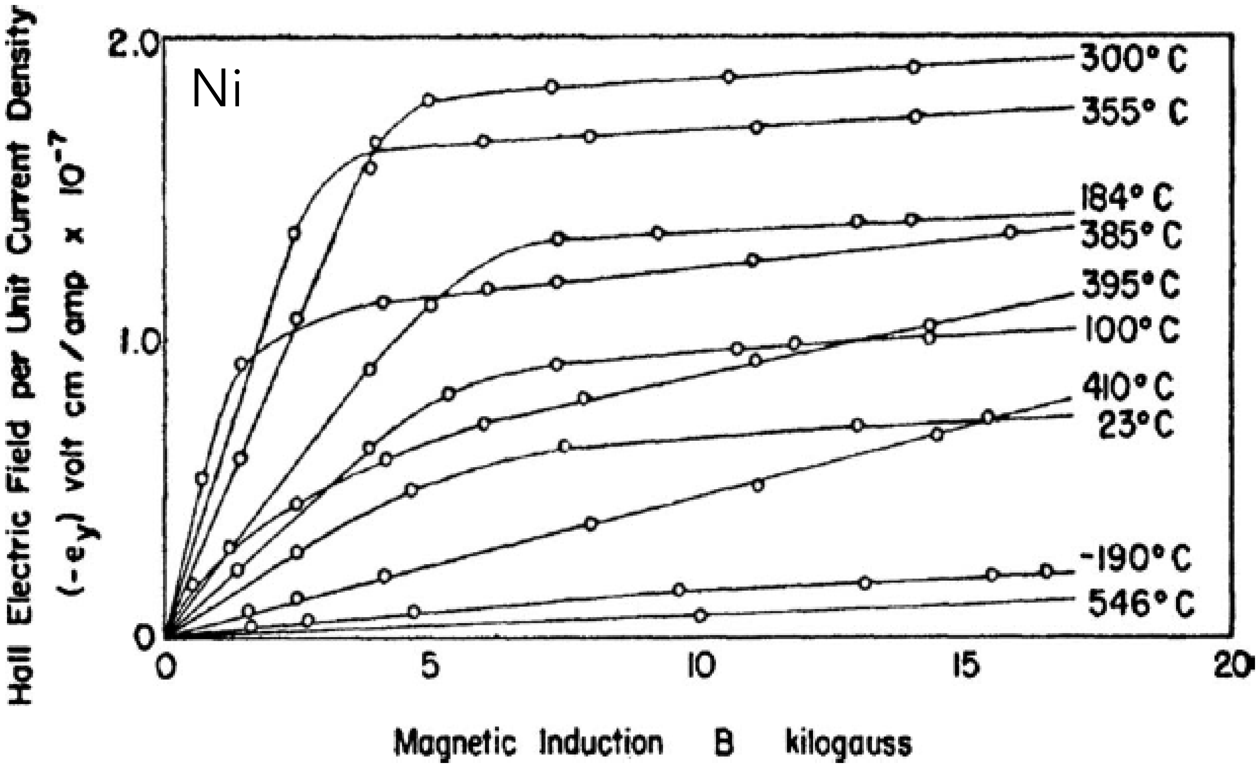
Naoto Nagaosa (UTokyo, RIKEN)

Outline:

1. Introduction
2. Chiral magnets under external magnetic field
3. Local inversion symmetry breaking by impurities

Anomalous Hall effect

Hall effect that is NOT proportional to the magnetic field H_z.



A W Smith, 1910

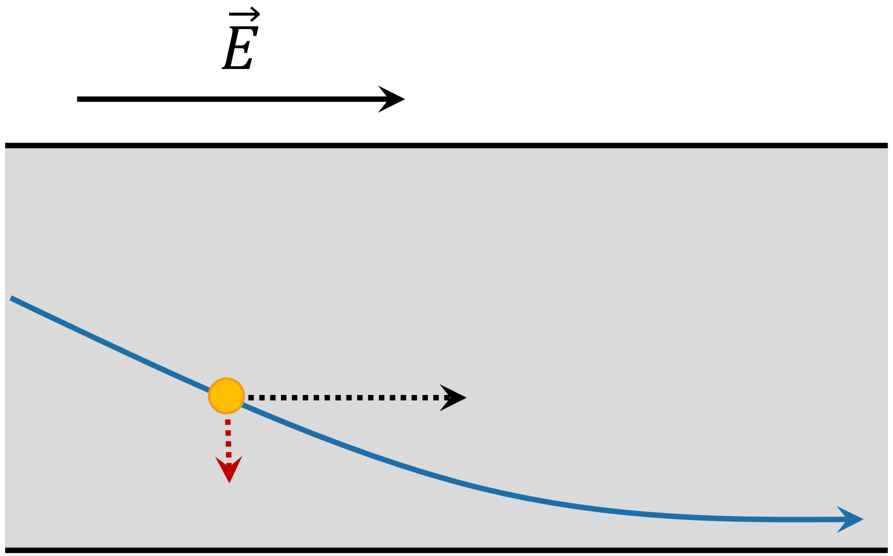
$$\rho_{xy} = R_0 H_z + R_s M_z$$

E M Pugh, 1930
E M Pugh *et al.*, 1932

Intrinsic vs extrinsic mechanisms

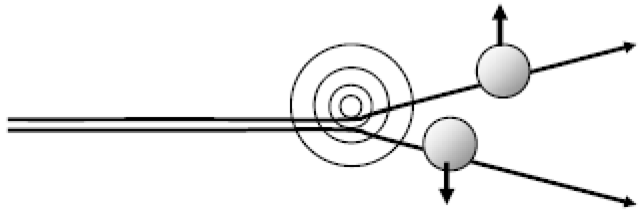
Intrinsic mechanism: R Karplus *et al.*, `55

$$\dot{r} = \vec{v}_k - \vec{b}(\vec{k}) \times \dot{k},$$
$$\dot{k} = \vec{E} - e\vec{v}_k \times \vec{B}.$$



Extrinsic mechanism (impurity scattering):

Skew scattering: J Smit, `55; `58



N Nagaosa *et al.*, `10

$$W^A_{k\to k'} = -\frac{\vec{M}_s}{\tau_A} \cdot \vec{k} \times \vec{k'}$$

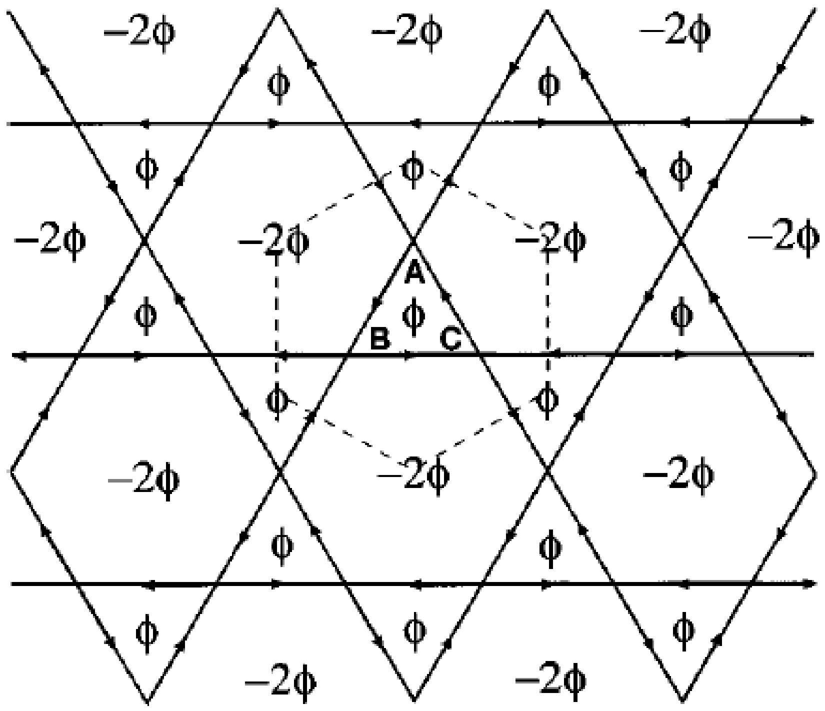
(Pseudo-)spin dependent scattering by impurities.

Side jump: L Berger, `70

Spin scattering: J Kondo, `62

Berry phase and fictitious magnetic field in strong coupling limit

D Loss *et al.*, `92; J Ye *et al.*, `99; K Ohgushi *et al.* `00; Y Taguchi *et al.*, `01; R Shindou *et al.* `01; I Martin *et al.*, `08; …

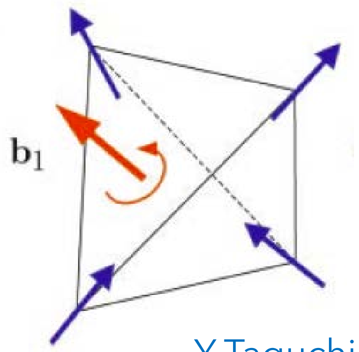


K Ohgushi *et al.*, `00

$$H = -t \sum_{\langle i,j \rangle, \sigma} c_{i\sigma}^{+} c_{j\sigma} + h.c. - J_H \sum_{i,\alpha,\beta} \left(c_{i\alpha}^{+} \boldsymbol{\sigma}_{\alpha\beta} c_{i\beta} \right) \cdot \boldsymbol{S}_i$$

$$\tilde{t}_{ij} = \cos\left(\frac{\theta_i}{2}\right) \cos\left(\frac{\theta_j}{2}\right) + \sin\left(\frac{\theta_i}{2}\right) \sin\left(\frac{\theta_j}{2}\right) e^{i(\phi_j - \phi_i)}$$

Zener `50; P. W. Anderson *et al.*, `56; E Mueller-Hartmann *et al.*, `96



Y Taguchi *et al.*, `01

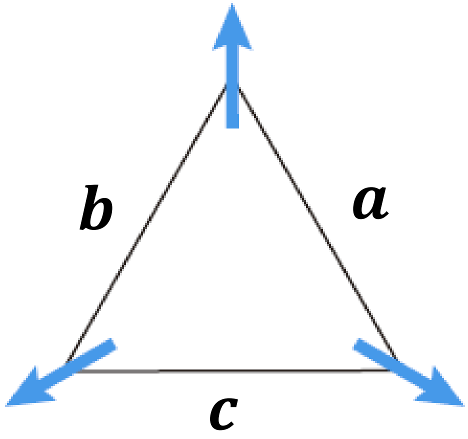
$$\chi = S_1 \cdot S_2 \times S_3$$

Non-coplanar magnetic states often gives rise to a fictitious net magnetic field.

Spin chirality related anomalous Hall effect in weak coupling limit

G Tatara *et al.*, `02

1. Anomalous Hall effect also appears in the weak-coupling limit, where the scattering by localized moments can be treated as a perturbation.
2. The Hall conductivity in this limit is linearly proportional to the spin scalar chirality.

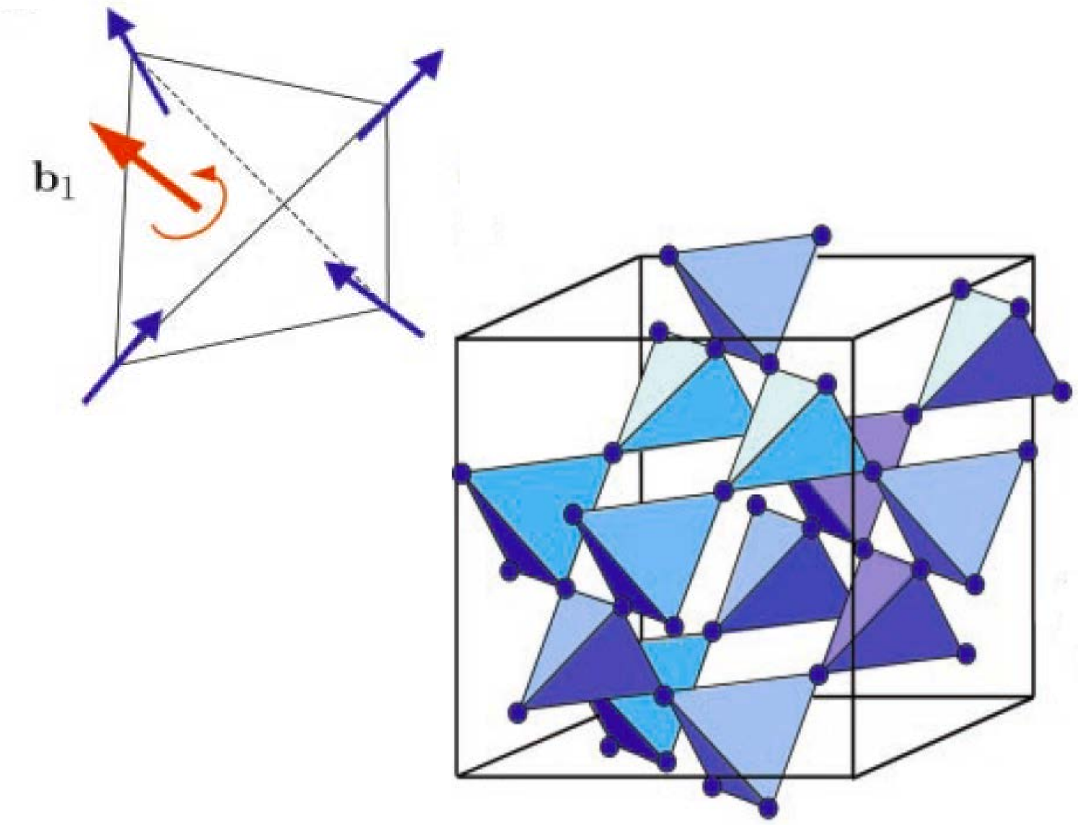


$$\sigma_{xy}^{(3)} = \frac{N}{\pi V}\left(\frac{e}{m}\right)^2 (2\pi\nu J)^3 \tau^2 \chi_0 = (4\pi)^2 \sigma_0 J^3 \nu^2 \tau \chi_0,$$

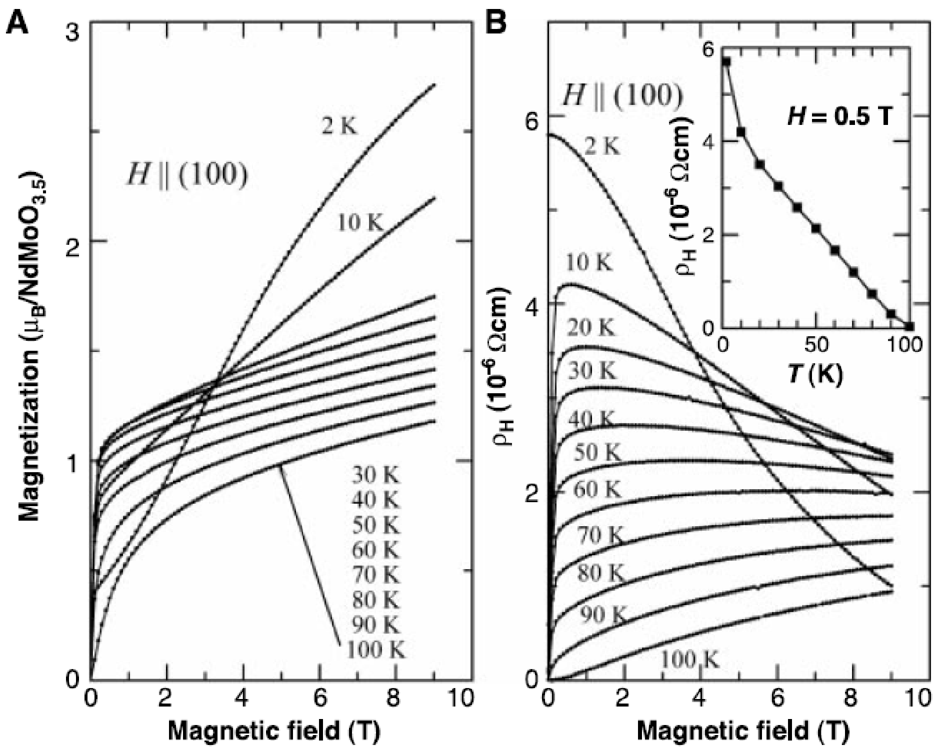
$$\chi_0 \equiv \frac{1}{6N}\sum_{X_i} \boldsymbol{S}_{X_1}\cdot(\boldsymbol{S}_{X_2}\times\boldsymbol{S}_{X_3})\left[\frac{(\boldsymbol{a}\times\boldsymbol{b})_z}{ab}I'(a)I'(b)I(c) + \frac{(\boldsymbol{b}\times\boldsymbol{c})_z}{bc}I(a)I'(b)I'(c) + \frac{(\boldsymbol{c}\times\boldsymbol{a})_z}{ca}I'(a)I(b)I'(c)\right]$$

$$I(r) \equiv \frac{1}{2\pi N\nu\tau}\sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} G_{\boldsymbol{k}}^R G_{\boldsymbol{k}}^A$$

Topological Hall effect due to non-coplanar spin texture



$Nd_2Mo_2O_7$

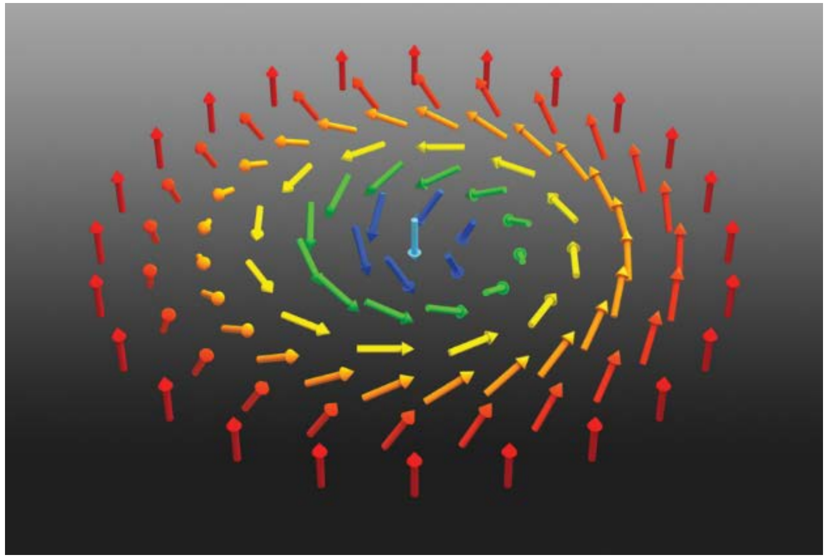


Y Taguchi *et al.*, `01

Topological Hall effect observed as an unusual magnetic field dependence of σ_H.

Anomalous Hall effect by magnetic skyrmions

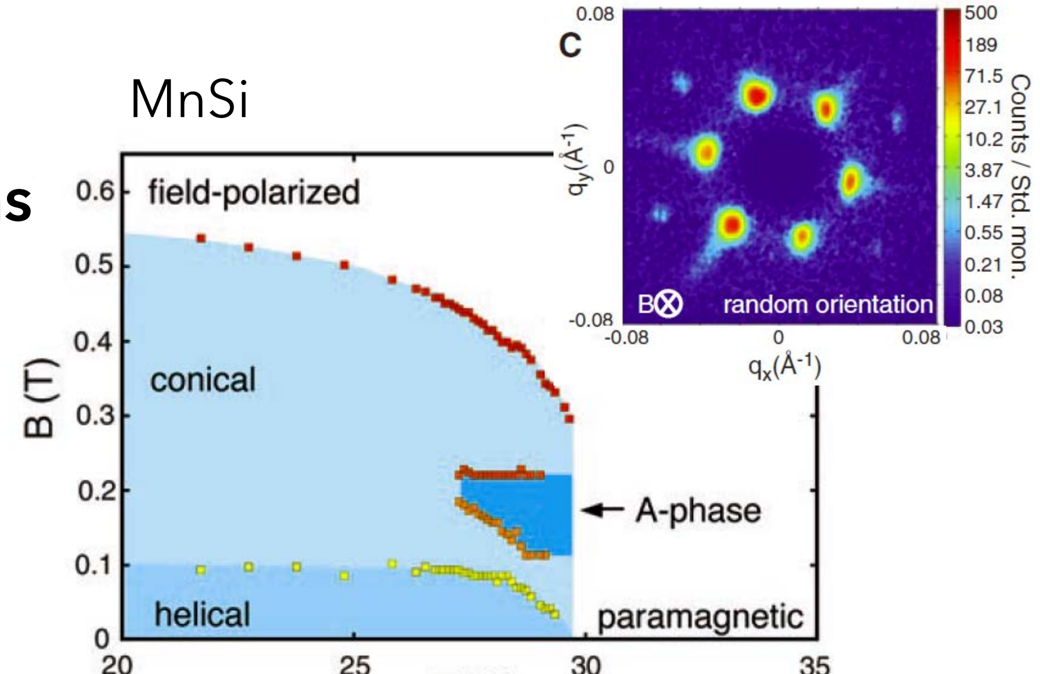
http://www.riken.jp



- Topological defect in magnets.
- Topological number (skyrmion number):

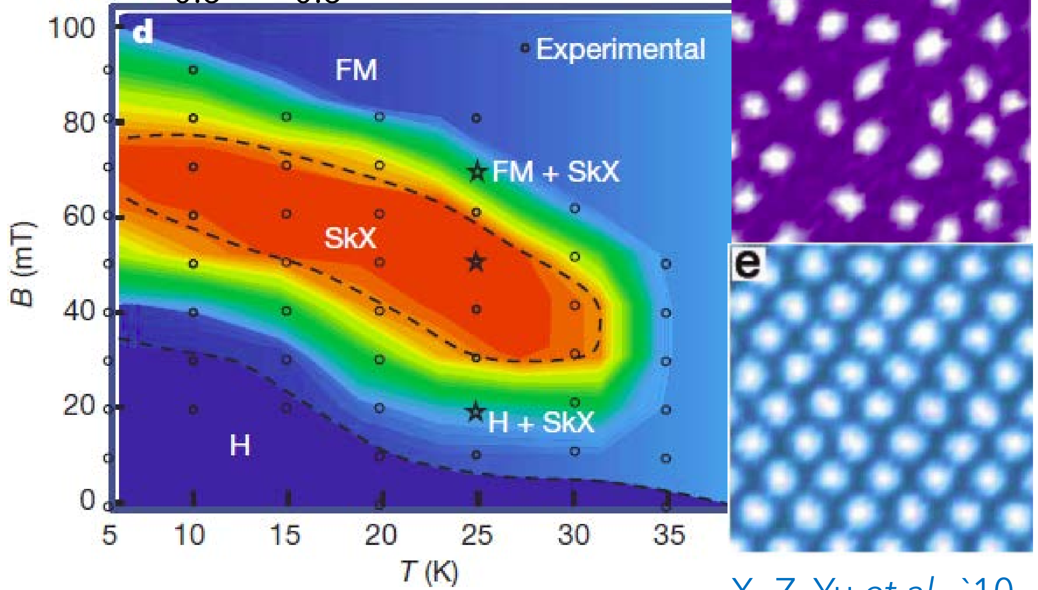
$$\mathcal{N} = \frac{1}{4\pi}\int d^2r\, \boldsymbol{n}(\boldsymbol{r})\cdot[\partial_x \boldsymbol{n}(\boldsymbol{r})\times\partial_y \boldsymbol{n}(\boldsymbol{r})]$$

MnSi



S Muhlbauer *et al.*, `09

$Fe_{0.5}Co_{0.5}Si$



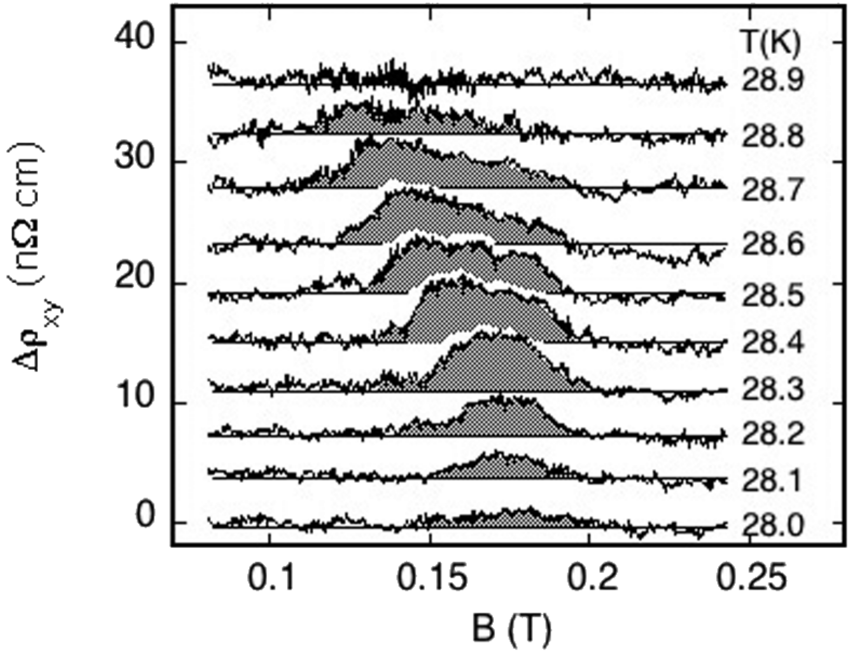
X.-Z. Yu *et al.*, `10

Topological Hall effect by magnetic skyrmions

$b_z \propto \vec{n}(\vec{r}) \cdot \partial_x \vec{n}(\vec{r}) \times \partial_y \vec{n}(\vec{r})$

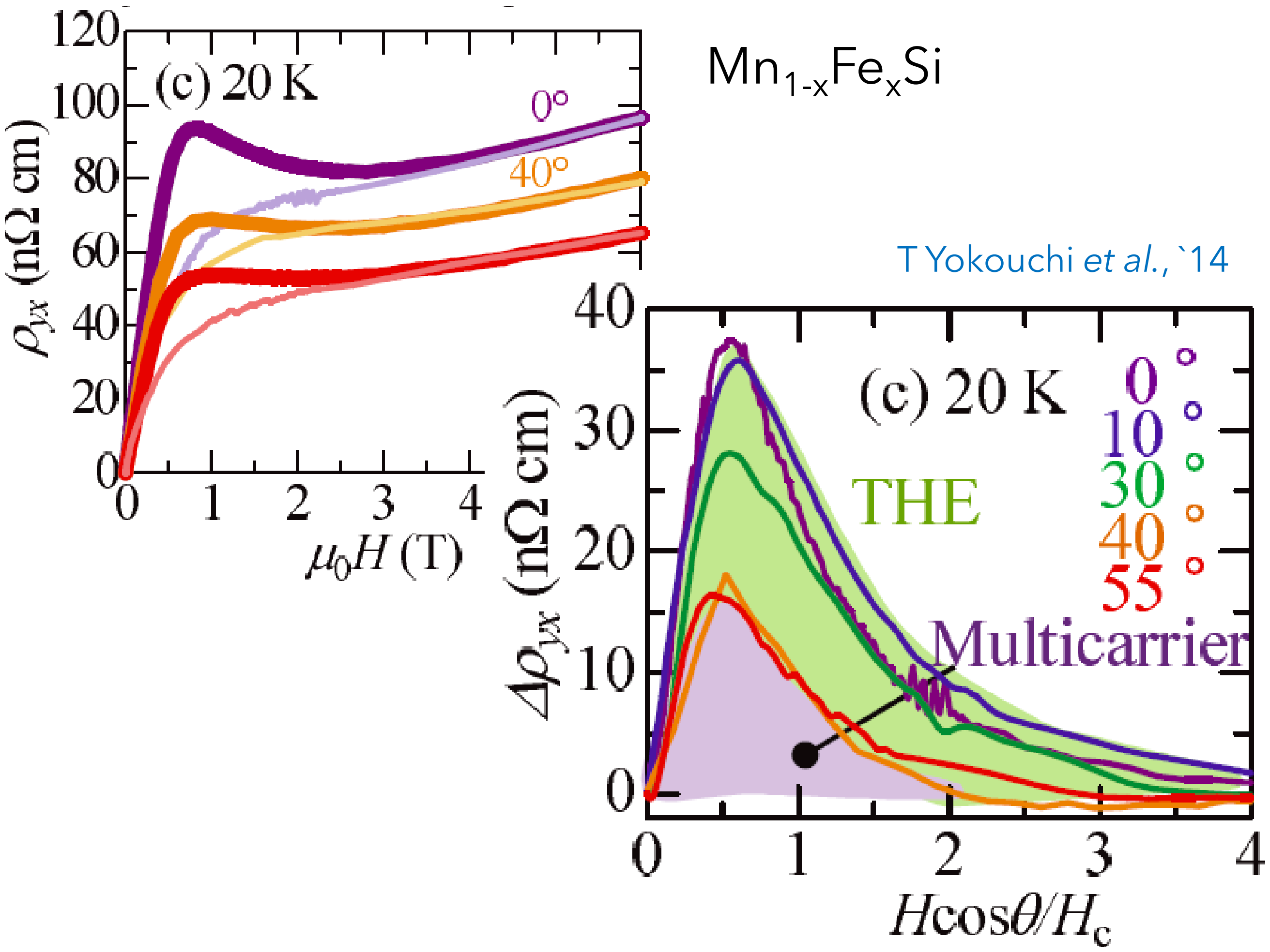
MnSi

A Neubauer *et al.*, `09



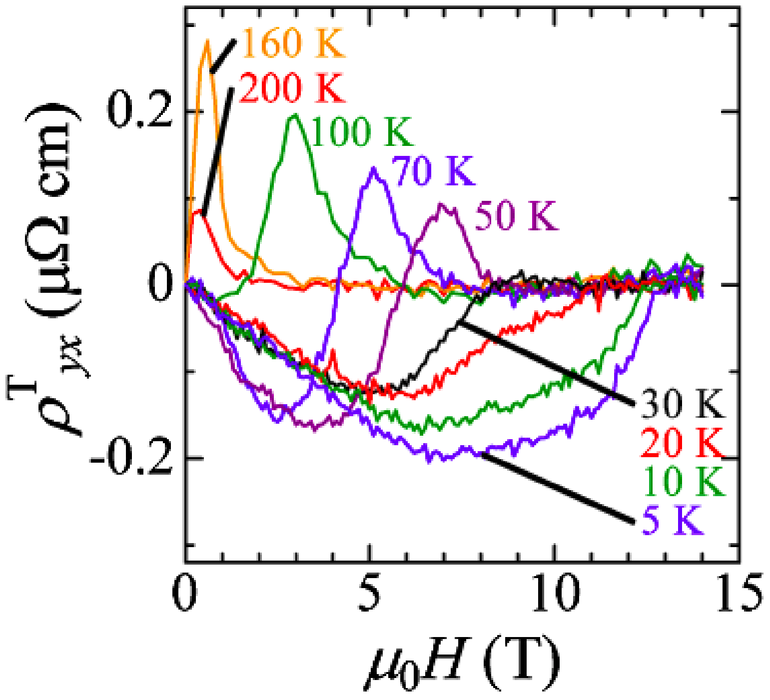
$Mn_{1-x}Fe_xSi$

T Yokouchi *et al.*, `14



Topological Hall effect observed in skyrmion crystal phases, consistently with neutron scattering/Lorenz TEM experiments.

Topological Hall conductivity in MnGe



N Kanazawa *et al.*, `11

It seems the Berry phase mechanism alone cannot explain the AHE in MnGe…

Is the scattering by the fluctuating spins similar to that by the Berry phase effect?

Model: Classical spin Kondo lattice model

We consider classical spin Kondo lattice model with the quadratic dispersion.

$$H = H_0 + H_K$$

$$H_0 = \sum_{\boldsymbol{k},\sigma} \varepsilon_{\boldsymbol{k}\sigma} c_\sigma(\boldsymbol{k})^\dagger c_\sigma(\boldsymbol{k})$$

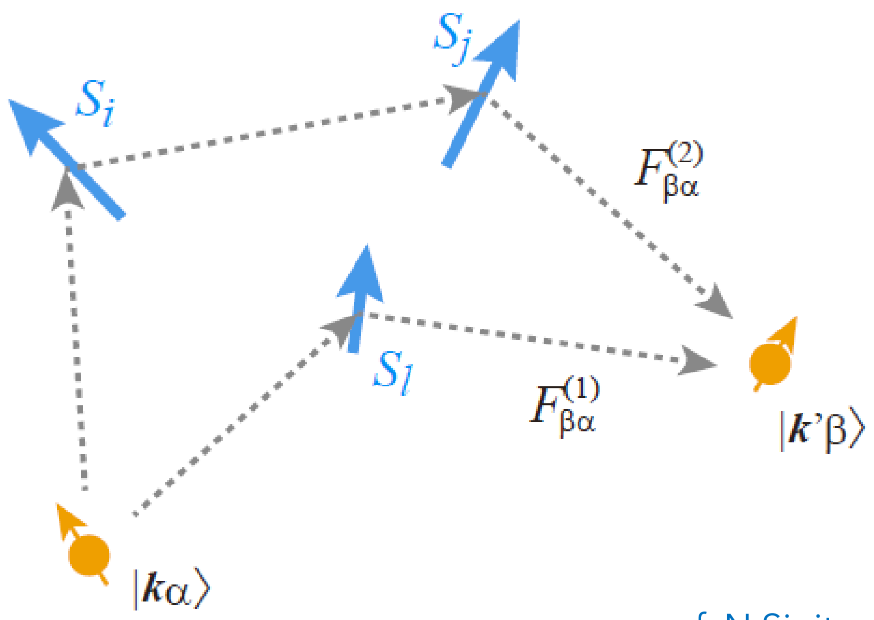
$$H_K = J \sum_i \boldsymbol{S}_i \cdot c_\sigma(\boldsymbol{R}_i)^\dagger \boldsymbol{\sigma}_{\sigma\sigma'} c_{\sigma'}(\boldsymbol{R}_i)$$

$$\varepsilon_{k\sigma} = \frac{k^2}{2m}$$

$$\boldsymbol{S} = (S_x, S_y, S_z)$$

Second Born approximation

We study the scattering by localized moment using second Born approximation. In particular, we consider the term that comes from the interference of single spin scattering and two spin scattering terms (see the right figure).



cf. N Sinitsyn, `08

$$\pi\left(F^{(1)}_{\beta\alpha}(\boldsymbol{k}',\boldsymbol{k})\left[F^{(2)}_{\beta\alpha}(\boldsymbol{k}',\boldsymbol{k})\right]^{*}-F^{(1)}_{\alpha\beta}(\boldsymbol{k},\boldsymbol{k}')\left[F^{(2)}_{\alpha\beta}(\boldsymbol{k},\boldsymbol{k}')\right]^{*}+\mathrm{h.c.}\right)\delta(\varepsilon_{\boldsymbol{k}\alpha}-\varepsilon_{\boldsymbol{k}'\beta}),$$

$$F^{(1)}_{\beta\alpha}(\boldsymbol{k}',\boldsymbol{k})=-\frac{J}{(2\pi)^3}\sum_i \boldsymbol{S}_i\cdot\boldsymbol{\sigma}_{\beta\alpha}e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{R}_i},$$

$$F^{(2)}_{\beta\alpha}(\boldsymbol{k}',\boldsymbol{k})=-i\frac{J^2m}{(2\pi)^4}\sum_{i\neq j}\boldsymbol{\sigma}_{\beta\alpha}\cdot\boldsymbol{S}_i\times\boldsymbol{S}_j\frac{e^{ik\delta_{ij}}}{\delta_{ij}}e^{i\boldsymbol{k}\cdot\boldsymbol{R}_j-i\boldsymbol{k}'\cdot\boldsymbol{R}_i}.$$

Second Born approximation

We focus on the asymmetric part of scattering: $w^{\pm}_{\boldsymbol{k}'\beta\to\boldsymbol{k}\alpha} = \frac{1}{2}\left(W_{\boldsymbol{k}\alpha\to\boldsymbol{k}'\beta} \pm W_{\boldsymbol{k}'\beta\to\boldsymbol{k}\alpha}\right)$

$$\begin{aligned} w^{-}_{\boldsymbol{k}\alpha\to\boldsymbol{k}'\beta} &= \frac{J^3 m a^4}{(2\pi L)^3} 4k\epsilon_{\mu\nu z}\left(\boldsymbol{k}\times\boldsymbol{k}'\right)_z \sum_l \Re\langle[\boldsymbol{S}_l\cdot\boldsymbol{\sigma}_{\alpha\beta}\rangle\left[\boldsymbol{\sigma}_{\beta\alpha}\cdot\partial_\mu\boldsymbol{S}_l\times\partial_\nu\boldsymbol{S}_l\right]\}\,, \\ &\sim \frac{J^3 mka}{(\pi L)^3}\left(\boldsymbol{k}\times\boldsymbol{k}'\right)_z \Re\int d^3x\, \langle(\boldsymbol{S}(x)\cdot\boldsymbol{\sigma}_{\alpha\beta})(\boldsymbol{\sigma}_{\beta\alpha}\cdot\partial_x\boldsymbol{S}(x)\times\partial_y\boldsymbol{S}(x))\rangle \end{aligned}$$

(for $ka \ll 1$)

The scattering due to non-coplanar magnetic textures gives rise to skew scattering (scattering term proportional to $k \times k'$).

Boltzmann theory and Hall conductivity

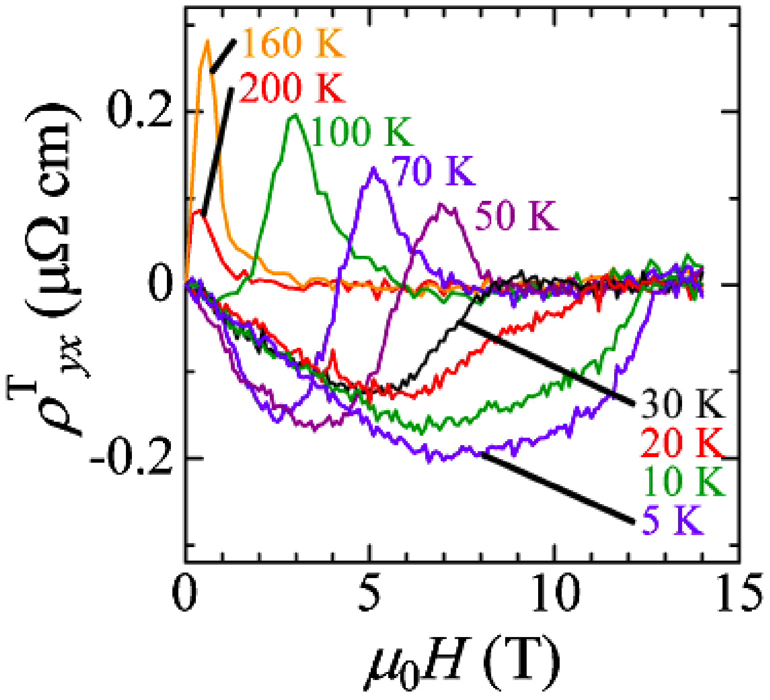
$$q\boldsymbol{v}_{\boldsymbol{k}} \cdot \boldsymbol{E} f_0'(\varepsilon_{\boldsymbol{k}}) = -\frac{g_{\boldsymbol{k}\alpha}}{\tau} + \sum_{\beta} \int d\phi' d\theta' \sin\theta' \frac{\rho(k)}{4\pi} w^{-}_{\boldsymbol{k}'\beta \to \boldsymbol{k}\alpha} g_{\boldsymbol{k}'\beta} \qquad w^{-}_{\boldsymbol{k}'\beta \to \boldsymbol{k}\alpha} = \tilde{\boldsymbol{V}}_{\alpha\beta}(k) \cdot \frac{\boldsymbol{k} \times \boldsymbol{k}'}{k^2}$$

$$\sigma_{xy} = \frac{2n_e q^2 \tau^2}{m} \rho(k_F) \left\{ \tilde{V}_0(k_F) + \tilde{V}_1(k_F) \right\}$$

$$\tilde{V}_0(k_F) + \tilde{V}_1(k_F) = \left(\frac{J k_F}{\pi L} \right)^3 ma \int d^3x \left[\boldsymbol{S}(\boldsymbol{x}) \cdot \partial_x \boldsymbol{S}(\boldsymbol{x}) \times \partial_y \boldsymbol{S}(\boldsymbol{x}) \right]$$

As $J < 0$, the Hall conductivity due to spin chirality has opposite sign to the Hall conductivity by the Berry phase (topological Hall effect).

Topological Hall conductivity in MnGe



N Kanazawa *et al.*, `11

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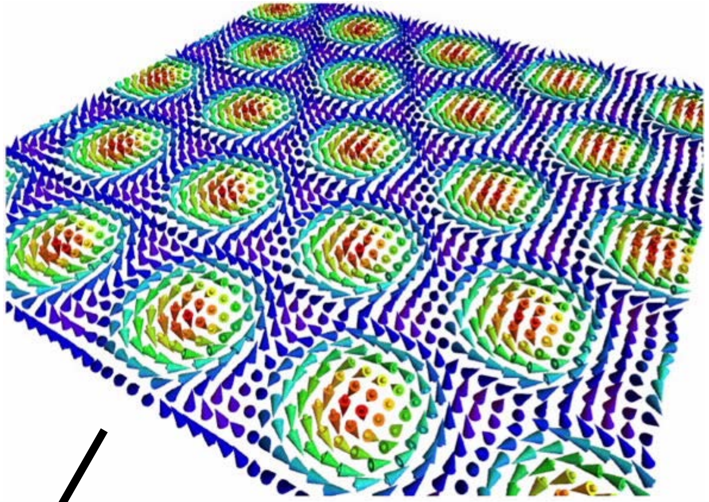
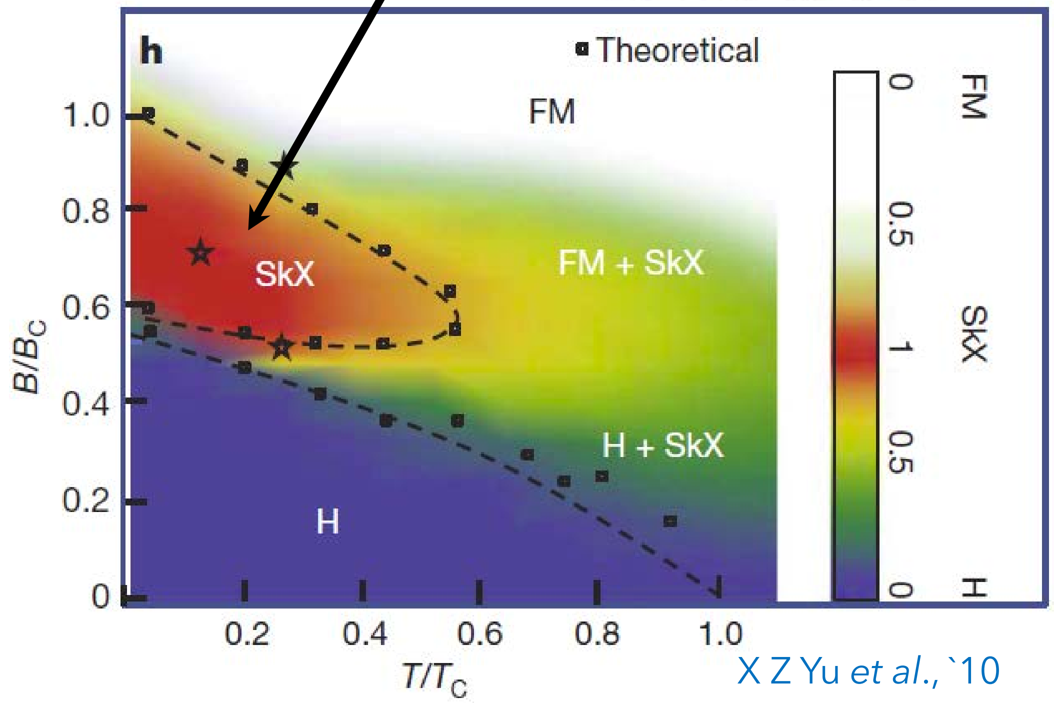
2d Heisenberg model with Dzyaloshinskii-Moriya interaction

http://www.ansto.gov.au/



$$H_S = -J_H \sum_{\boldsymbol{r}} \boldsymbol{S}(\boldsymbol{r}) \cdot \boldsymbol{S}(\boldsymbol{r}+\hat{\boldsymbol{x}}) + \boldsymbol{S}(\boldsymbol{r}) \cdot \boldsymbol{S}(\boldsymbol{r}+\hat{\boldsymbol{y}})$$
$$-K \sum_{\boldsymbol{r}} [\hat{\boldsymbol{x}} \cdot \boldsymbol{S}(\boldsymbol{r}) \times \boldsymbol{S}(\boldsymbol{r}+\hat{\boldsymbol{x}})$$
$$+\hat{\boldsymbol{y}} \cdot \boldsymbol{S}(\boldsymbol{r}) \times \boldsymbol{S}(\boldsymbol{r}+\hat{\boldsymbol{y}})]$$
$$-h \sum_{\boldsymbol{r}} S^z(\boldsymbol{r}).$$

$$\hat{x} = (1,0), \qquad \hat{y} = (0,1)$$

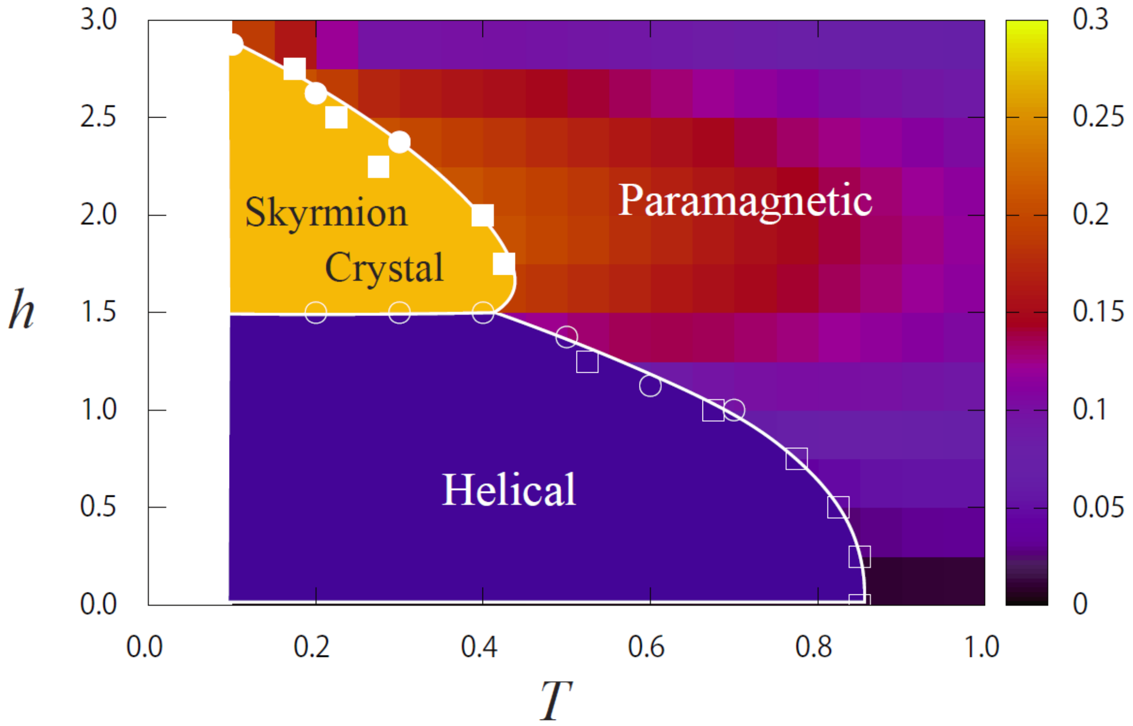


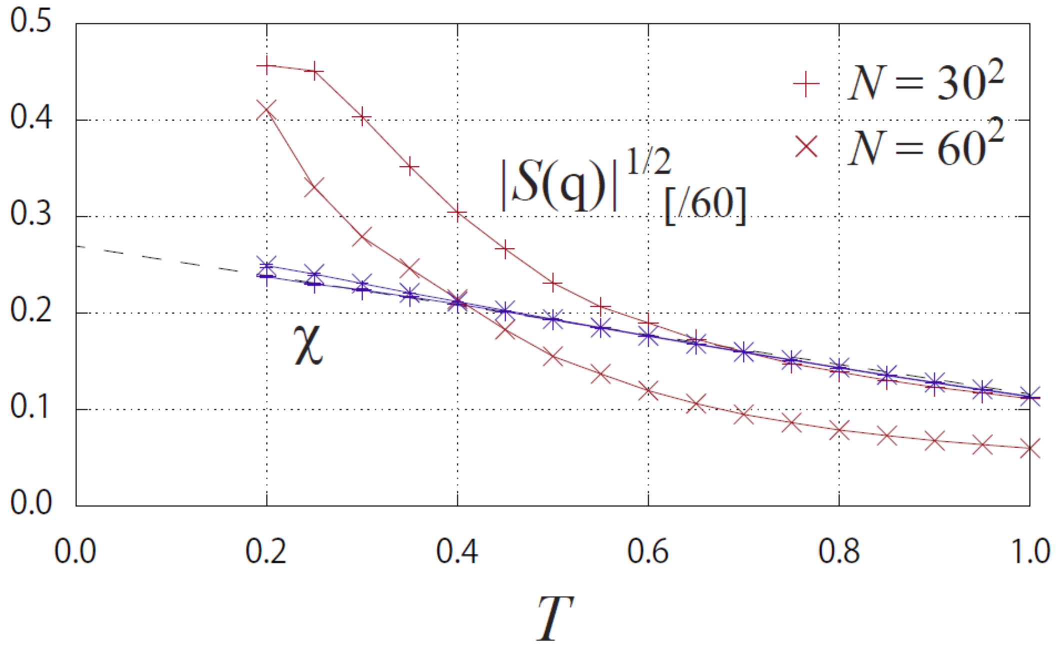
X Z Yu *et al.*, \`10

An spin model that reproduces the phase diagram of MnSi thin films under magnetic field.

S D Yi *et al.*, \`09, X Z Yu *et al.*, \`10

Phase diagram and scalar spin chirality

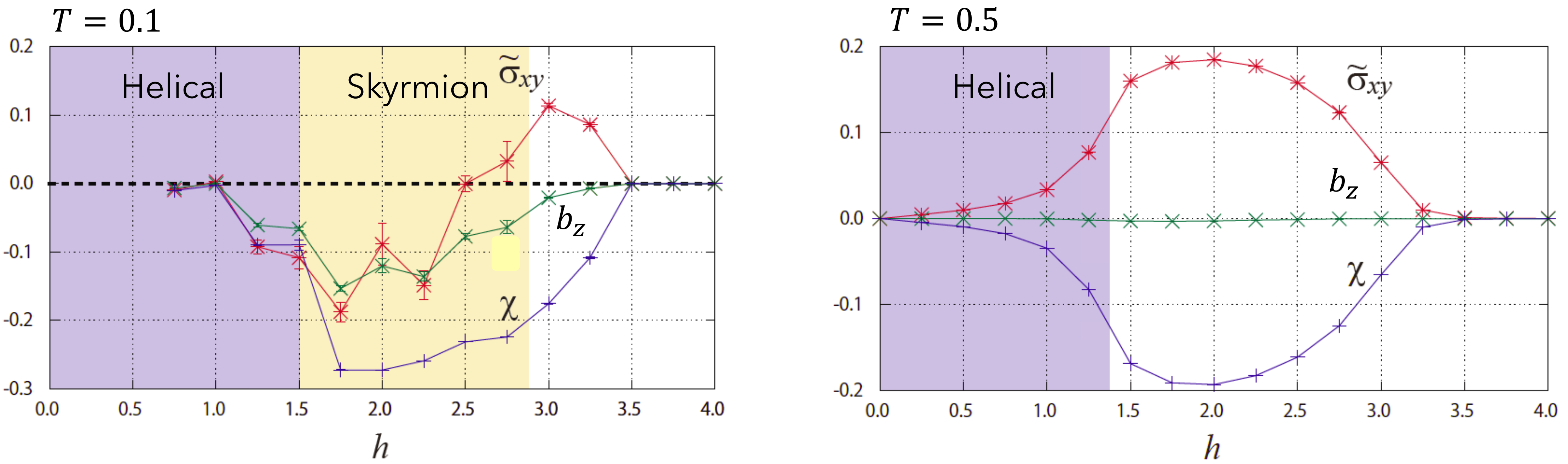




Monte Carlo simulation shows large scalar chirality χ even above the magnetic transition temperature.

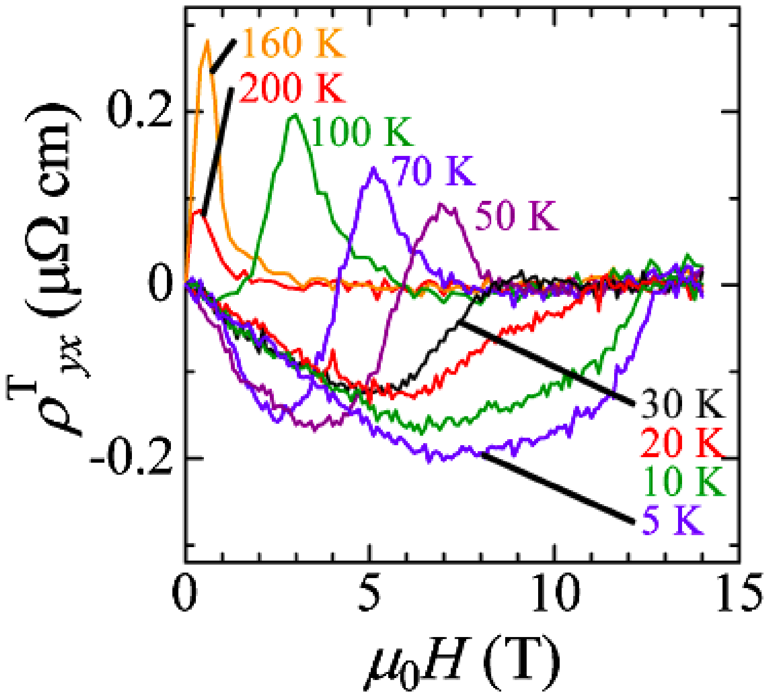
Anomalous Hall effect

$$\tilde{\sigma}_{xy} = \chi - (1+\gamma) b_z \qquad b_z = \frac{1}{4} \sum_{\boldsymbol{r},i} \langle \boldsymbol{S}(\boldsymbol{r}) \rangle \cdot \langle \boldsymbol{S}(\boldsymbol{r}+\boldsymbol{\delta}_i) \rangle \times \langle \boldsymbol{S}(\boldsymbol{r}+\boldsymbol{\delta}_{i+1}) \rangle$$



Non-monotonic behavior of anomalous Hall conductivity appears due to competition of the emergent magnetic field and skew scattering mechanisms.

Topological Hall conductivity in MnGe



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Summary: skew scattering induced by spin fluctuations

1. Skew scattering appears in the second Born approximation, which is related to the non-coplanar spin texture.
2. The skew scattering term gives rise to anomalous Hall conductivity, which is different from the Berry phase contribution.
3. In presence of impurities, the local inversion symmetry breaking can lead to a similar phenomenon.