Color superfluidity of neutral ultracold fermions in the presence of color-orbit and color-flip fields

Carlos A. R. Sa de Melo
Georgia Institute of Technology

Yukawa Institute for Theoretical Physics

Kyoto, November 8th, 2017

謝辞

この素晴らしいシンポジウムの主催者の皆さま、発表させていただきましてありがとうございます。

私は日本語話せませんし、今日たくさんの外国人の方がいらっしゃいますし、申し訳ございませんが英語で発表をさせていただきます。

Acknowledgement

I would like to thank the organizers for the opportunity to speak at this Symposium.

Since I can not speak Japanese and many people here are from overseas, I will have to speak in English.

Acknowledgements

Doga Kurkcuoglu

Ian Spielman

Outline of talk

1) Motivation: color superfluidity and ultracold fermions

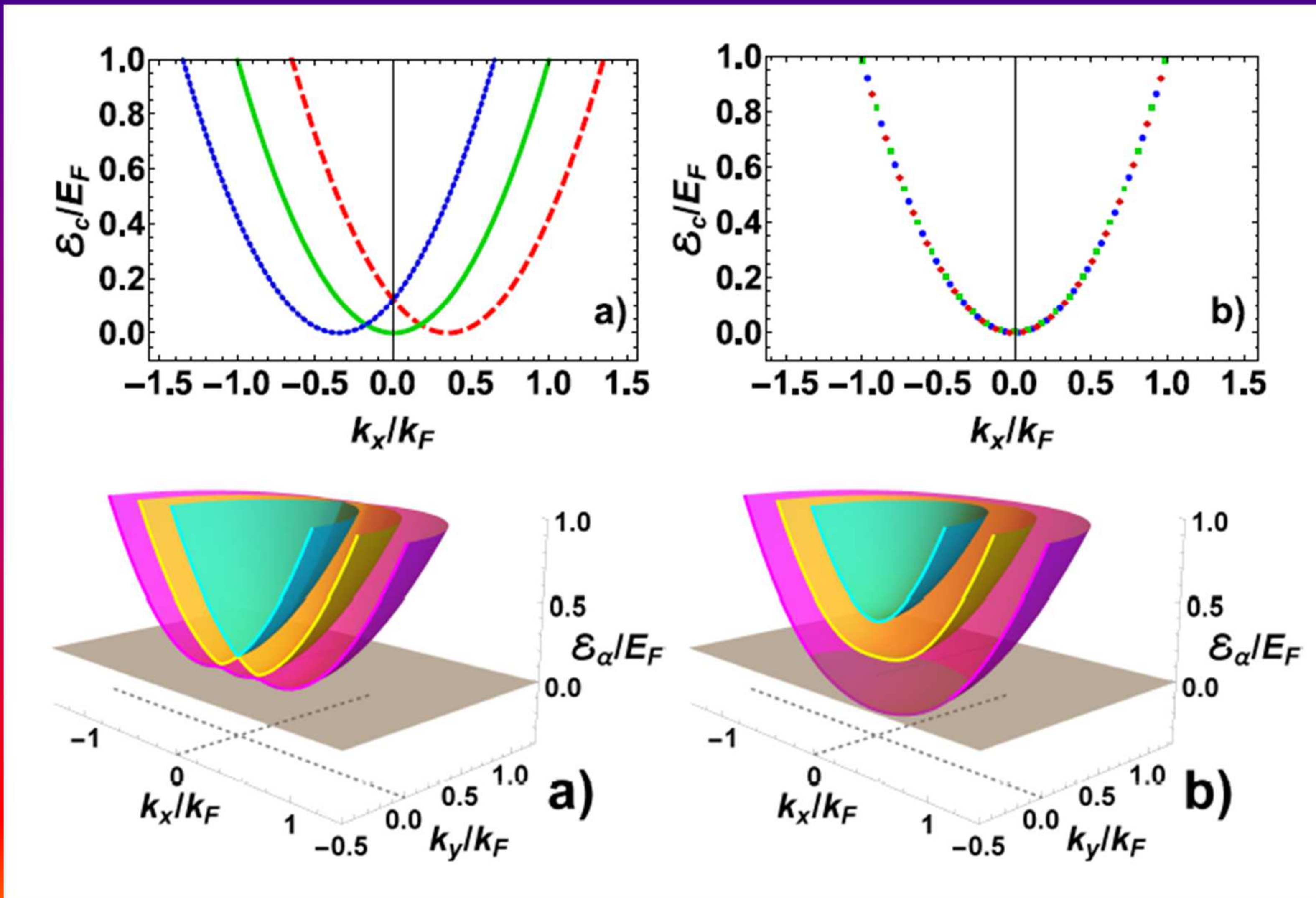
2) Introduction to spin-orbit and color-orbit coupling

3) Interacting fermions with color-orbit and color-flip fields

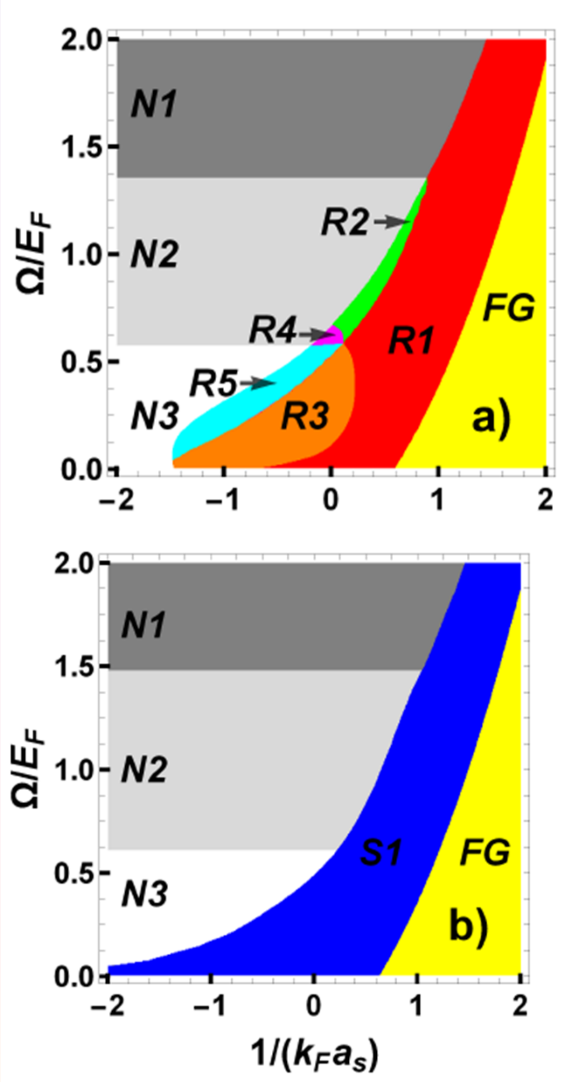
4) Spectroscopic and thermodynamic properties

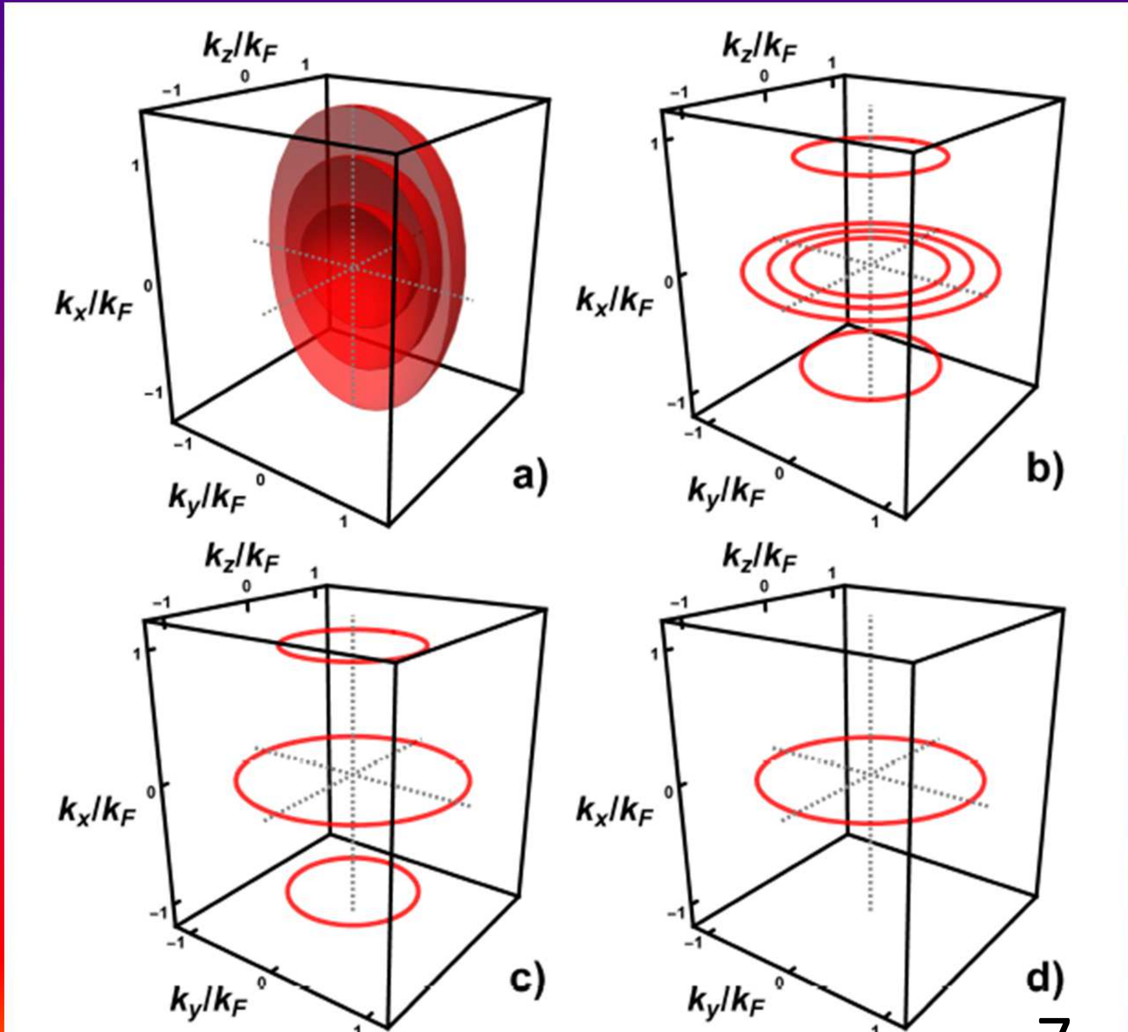
5) Conclusions

Conclusions in pictures: color-orbit and color flip fields



Conclusions in pictures: color-orbit and color flip fields





Conclusions in words

Ultracold fermions with three internal states can exhibit very unusual color superfluidity in the presence of color-orbit and color-flip fields, where SU(3) symmetry is explicitly broken.

The phase diagram of color-flip versus interaction parameter for fixed color-orbit coupling exhibits several topological phases associated with the nodal structure of the quasiparticle excitation spectrum. The phase diagram exhibits a pentacritical point where five nodal superfluid phases merge.

Even for interactions that occur only in the color s-wave channel, the order parameter for superfluidity exhibits singlet, triplet and quintuplet components due to the presence of color-orbit and color-flip fields.

These topological phases can be probed through measurements of spectroscopic properties such as excitation spectra, momentum distributions and density of states.

References for today's talk

Color superfluidity of neutral ultra-cold fermions in the presence of color-flip and color-orbit fields

Doga Murat Kurkcuoglu[1,2] and C. A. R. Sá de Melo[2]

arXiv:1707.09923v1 To appear in PRA

Quantum phases of interacting three-component fermions under the influence of spin-orbit coupling and Zeeman fields

Doga Murat Kurkcuoglu and C. A. R. Sá de Melo

arXiv:1612.02365v1 To appear in PRL

Creating spin-one fermions in the presence of artificial spin-orbit fields: Emergent spinor physics and spectroscopic properties

Doga Murat Kurkcuoglu and C. A. R. Sá de Melo

arXiv:1609.06607v1

Outline of Talk

1) Motivation: color superfluidity and ultracold fermions

2) Introduction to spin-orbit and color-orbit coupling

3) Interacting fermions with color-orbit and color-flip fields

4) Spectroscopic and thermodynamic properties

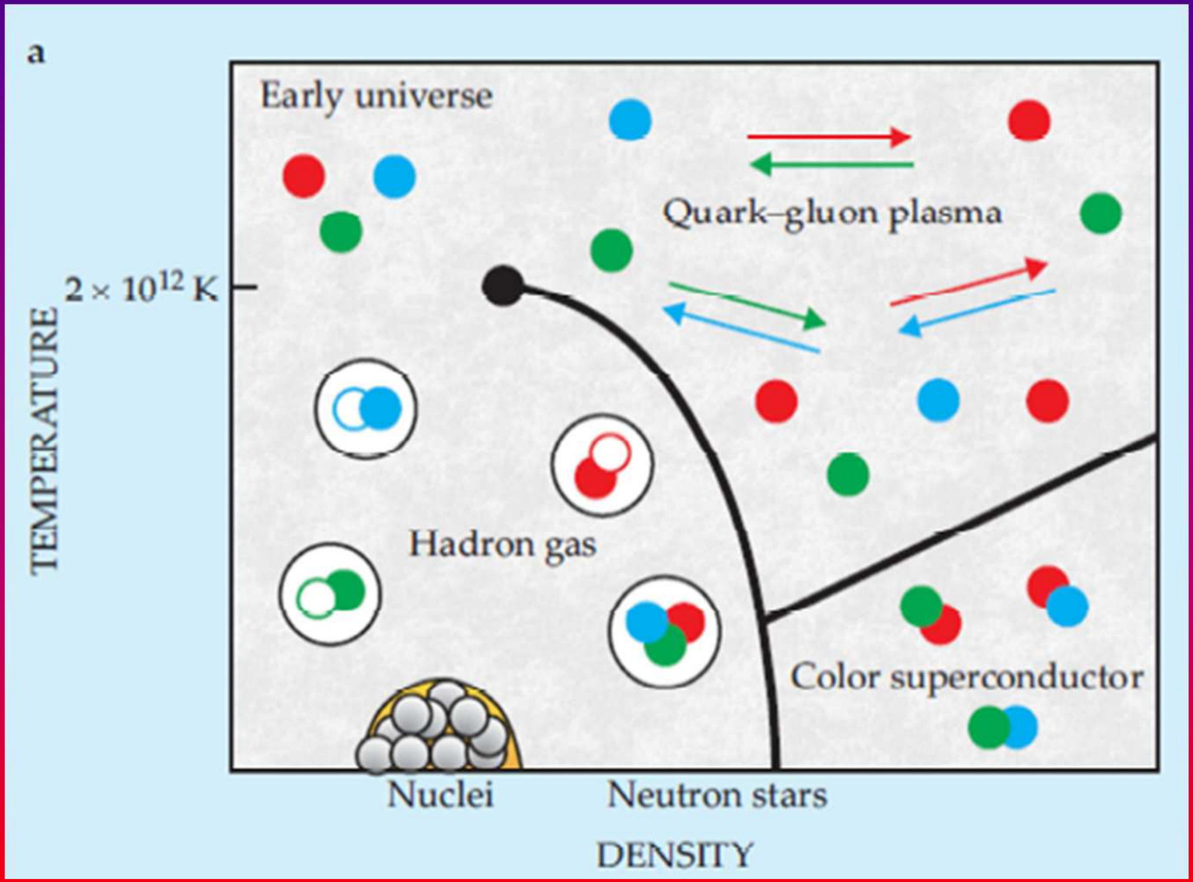
5) Conclusions

Motivation: color superfluidity and ultracold fermions

- Why studying ultracold fermions is important?

- Because it allows for the exploration of several fundamental properties of matter, such as superfluidity, which is encountered in atomic, condensed matter, nuclear and astrophysics.

Possible phase diagram for Quantum Chromodynamics (QCD)



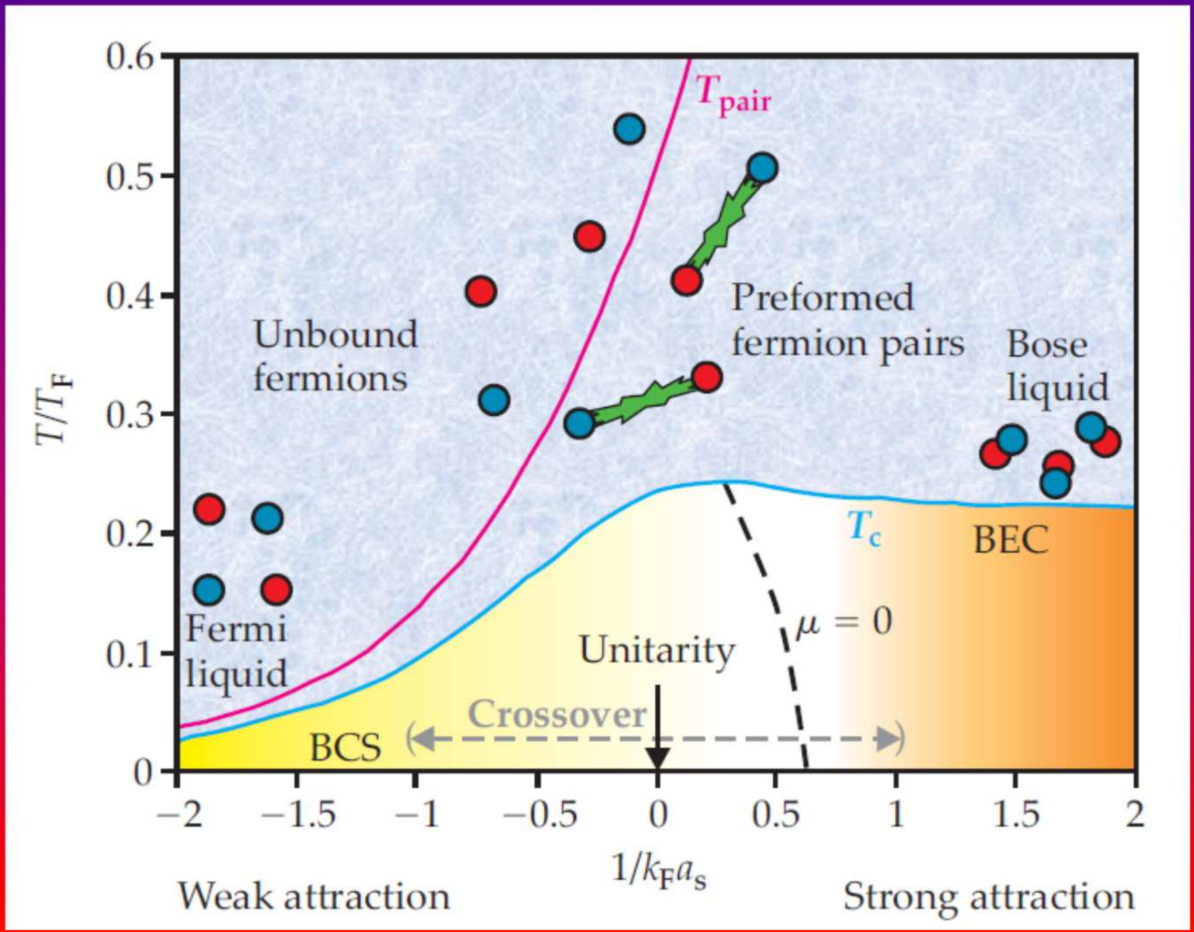
SdM – Physics Today, October (2008)

QCD and ultracold fermions (UCF) with three internal states: SU(3) case

- QCD – gluons mediate interactions
- QCD – s-wave interactions are not controllable
- QCD - quark masses are different
- QCD – quarks are charged
- QCD – quarks have three colors (internal states)

- UCF – contact interactions
- UCF – s-wave interactions are controllable
- UCF – Fermi atoms masses are the same
- UCF – Fermi atoms are neutral
- UCF – Fermi atoms can have three internal states

Ultracold fermions (UCF) with two internal states: SU(2) case



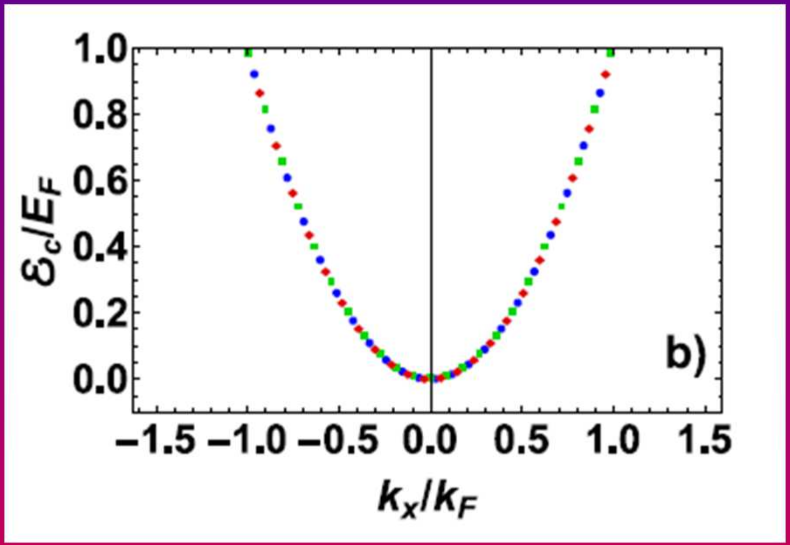
F = 5/2

^{6}Li, ^{40}K

F = 9/2

SdM - Physics Today, October (2008)

Simplest example: colored fermions and single interaction channel

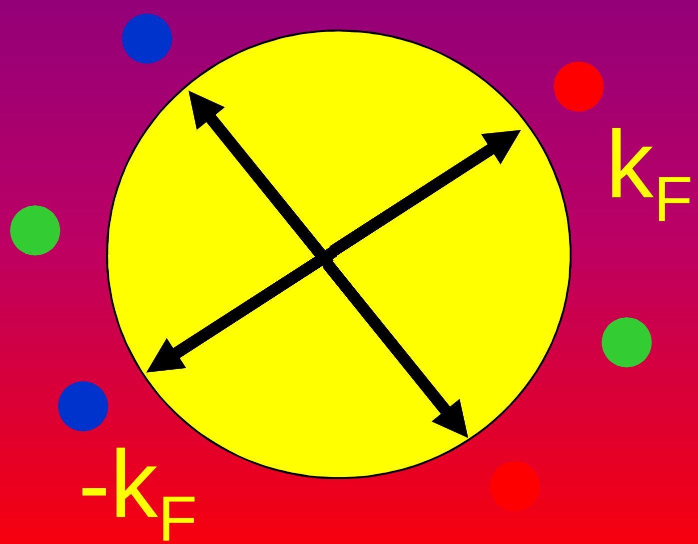


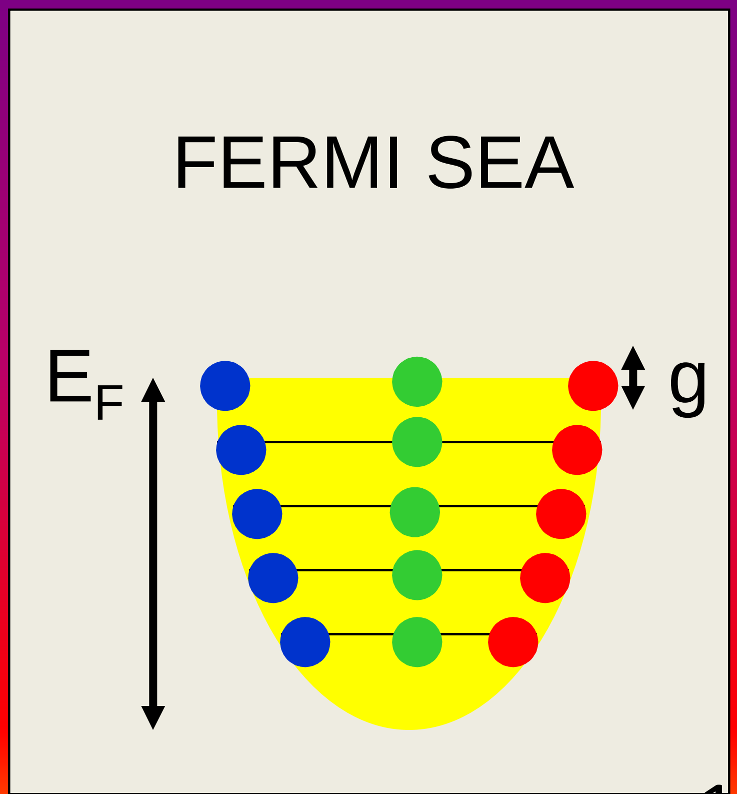
Single channel
only Red and Blue
have contact interactions

Green band is inert: non-interacting

BCS Pairing ($g \ll E_F$ or $k_F a_s \rightarrow 0^-$)

$\mu = E_F > 0$

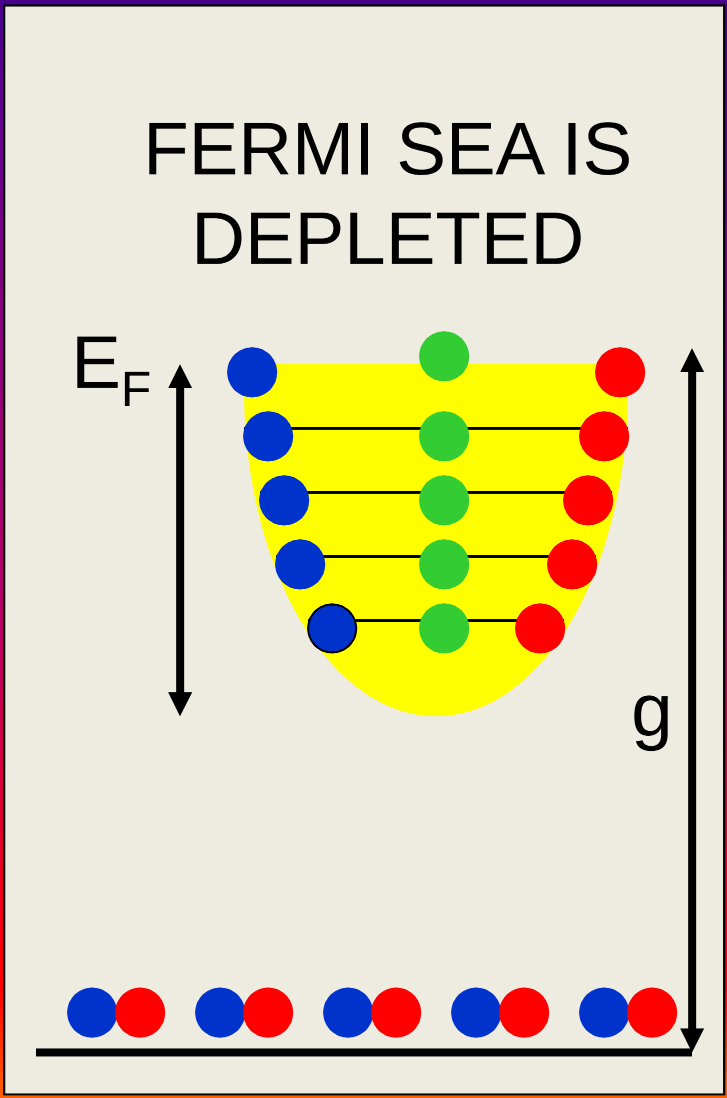




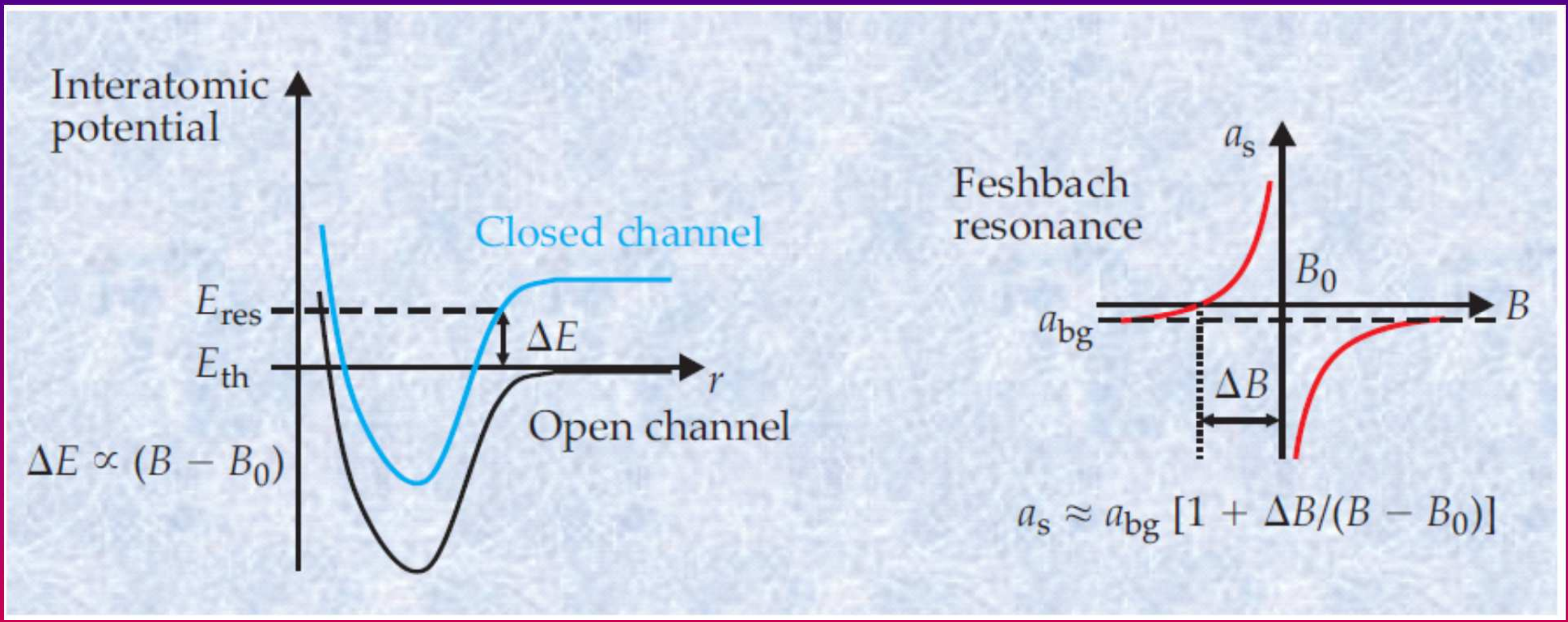
BEC Pairing ($g \gg E_F$ or $k_F a_s \rightarrow 0^+$)

Weakly interacting gas of tightly bound Molecules with inert Green fermions

$2\mu = -E_b < 0$



Feshbach Resonances

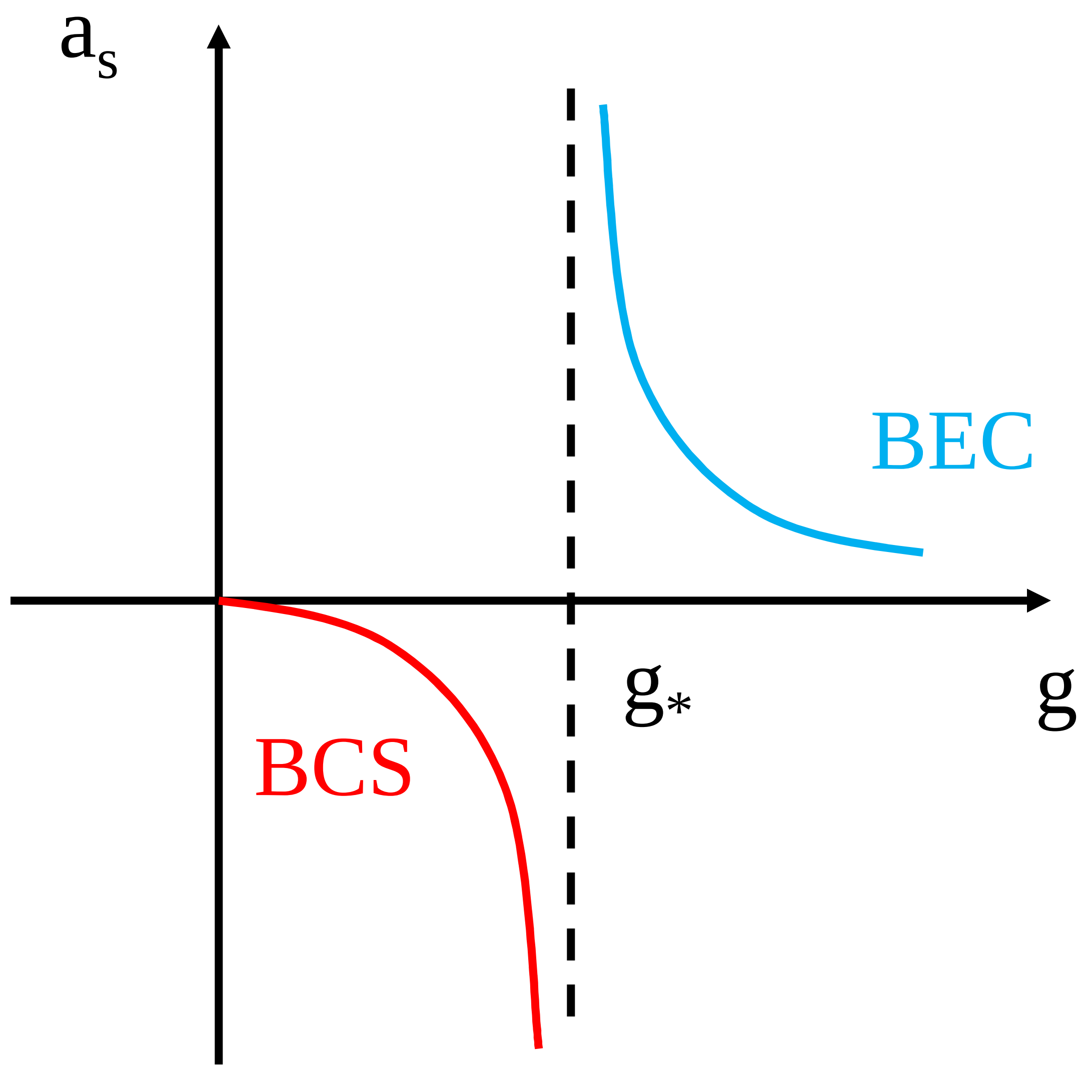


Contact interaction

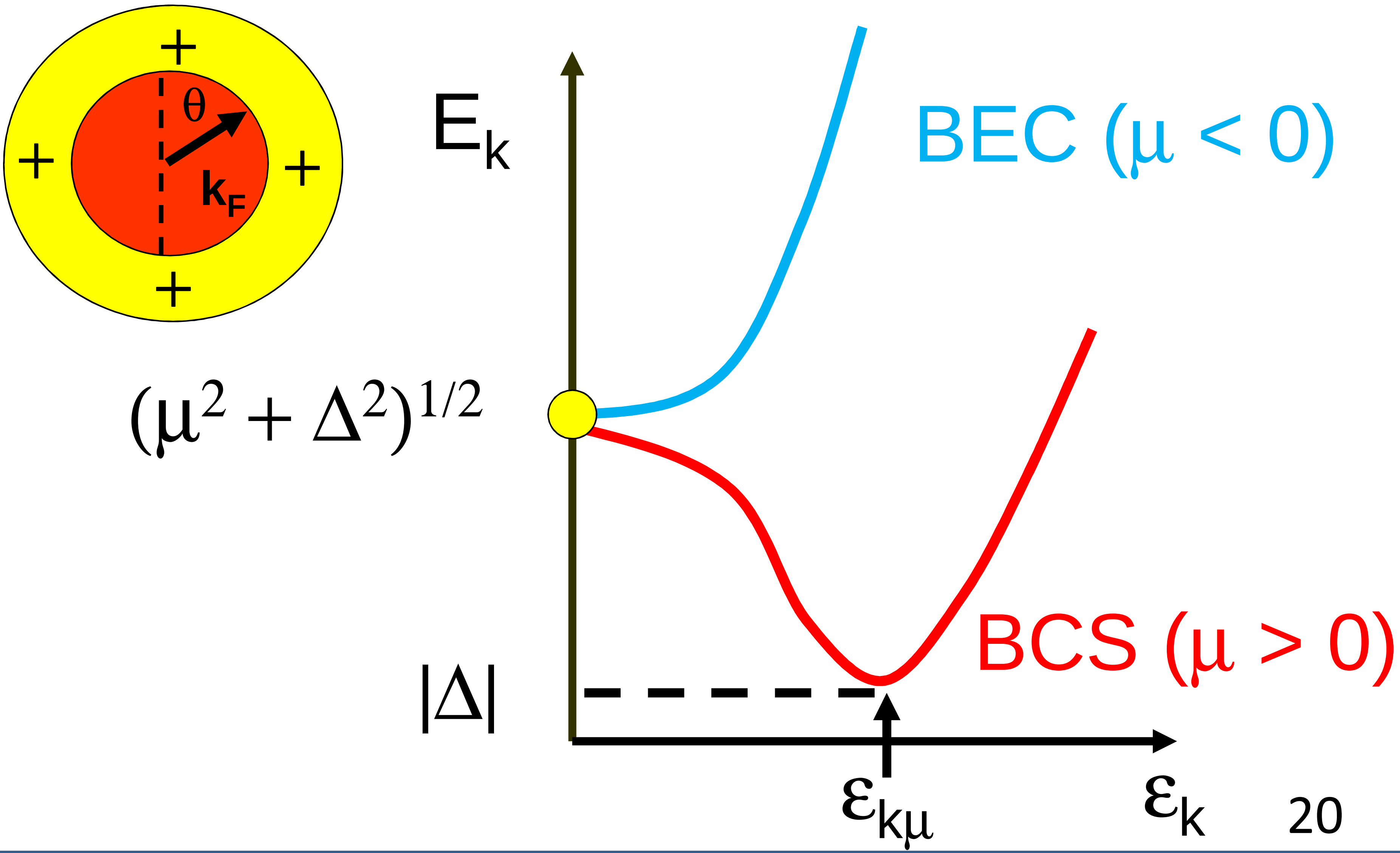
$$g \to a_s \to a_s(B)$$

B-dependent scattering length

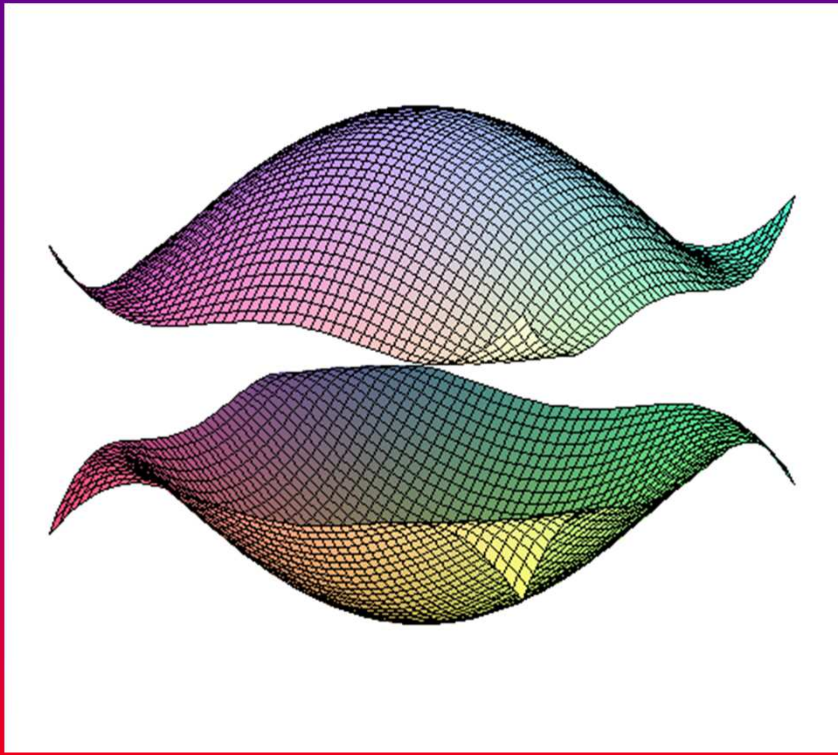
Scattering Length



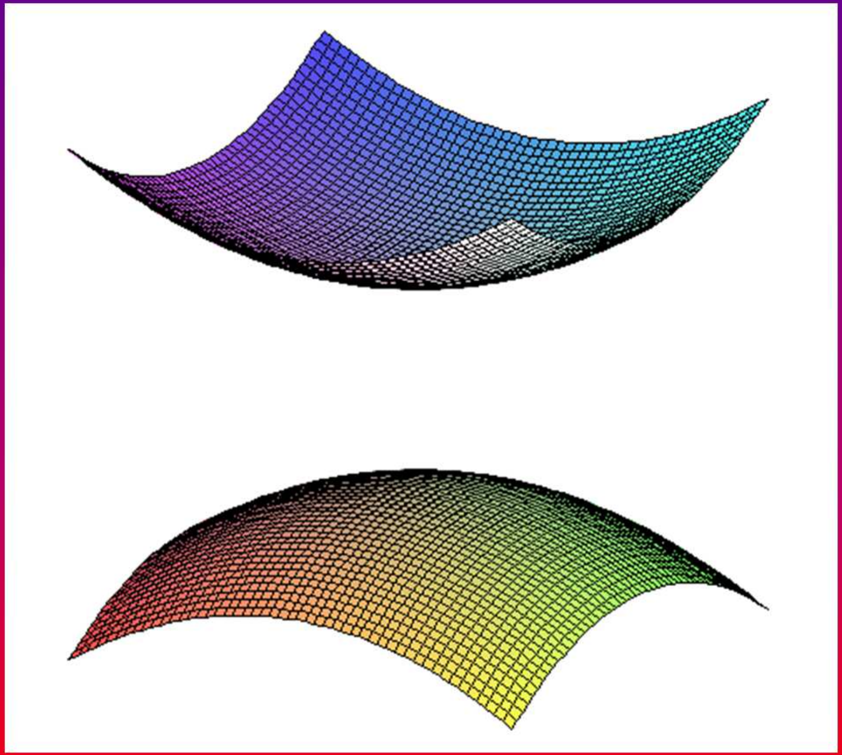
$$E(k) = [(\varepsilon_k - \mu)^2 + \Delta^2]^{1/2}$$



E(k) at T = 0 and k_x = 0 (S-wave)



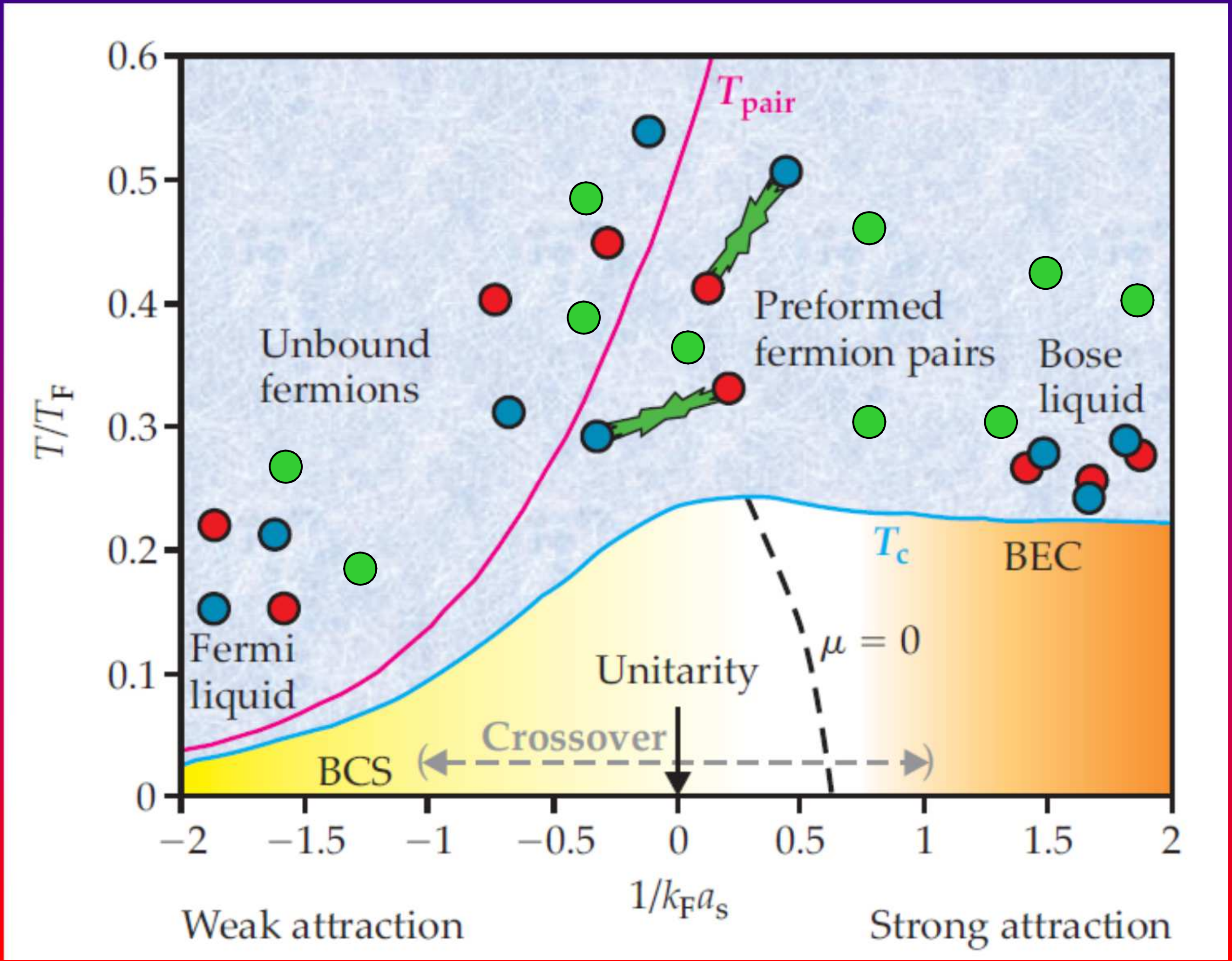
$\mu > 0$

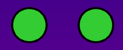


$\mu < 0$

Same Topology

QCD-like color superfluidity nearly identical to BCS-BEC crossover of SU(2) case





Inert fermions

Change of scale

$$n_{2c} = k_{F2c}^3 / 3\pi^2$$

$$n_{3c} = k_{F3c}^3 / 2\pi^2$$

$$n_{2c} = n_{3c}$$

$$k_{F2c} = (3/2)^{1/3} k_{F3c}$$

$$T_{F2c} = k_{F2c}^2 / 2m$$

$$T_{F3c} = k_{F3c}^2 / 2m$$

$$T_{F2c} = (3/2)^{2/3} T_{F3c}$$

SdM, Physics Today (2008) + inert Green fermions

Outline of Talk

1) Motivation: color superfluidity and ultracold fermions

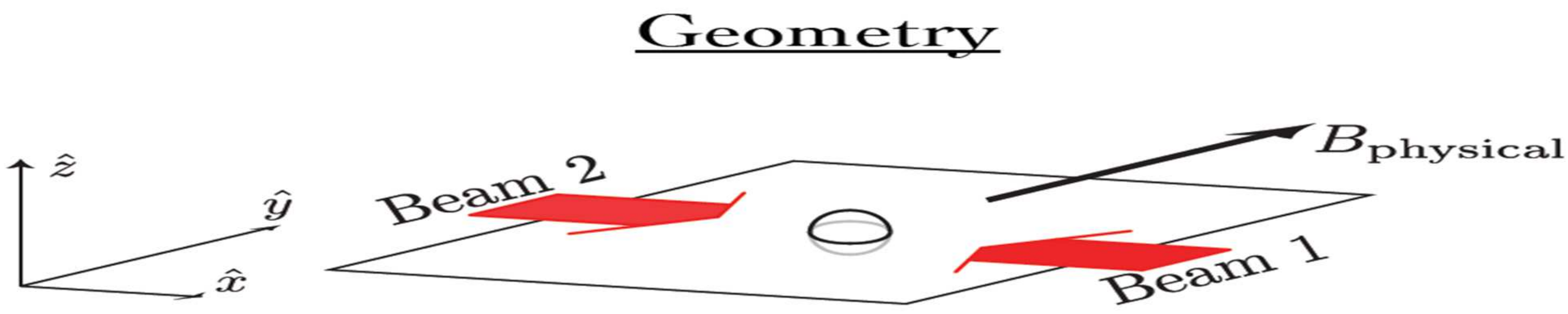
2) Introduction to spin-orbit and color-orbit coupling

3) Interacting fermions with color-orbit and color-flip fields

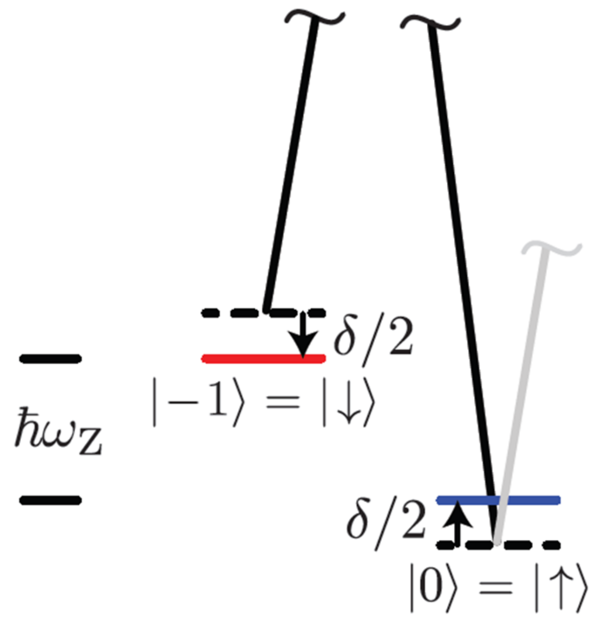
4) Spectroscopic and thermodynamic properties

5) Conclusions

Raman process and spin-orbit coupling



$$\begin{pmatrix} \dfrac{(\mathbf{k}-\mathbf{k}_R)^2}{2m}+\dfrac{\delta}{2} & \dfrac{\Omega}{2} \\ \dfrac{\Omega}{2} & \dfrac{(\mathbf{k}+\mathbf{k}_R)^2}{2m}-\dfrac{\delta}{2} \end{pmatrix}$$



SU(2) rotation to new spin basis:

$\sigma_x \rightarrow \sigma_z\,;\ \ \sigma_z \rightarrow \sigma_y\,;\ \ \sigma_y \rightarrow \sigma_x$

LETTER 3 MARCH 2011 | VOL 471 | NATURE | 83

doi:10.1038/nature09887

Spin–orbit-coupled Bose–Einstein condensates

Y.-J. Lin[1], K. Jiménez-García[1,2] & I. B. Spielman[1]

^{87}Rb

$$\begin{pmatrix} \dfrac{\mathbf{k}^2 + k_R^2}{2m} + \dfrac{\Omega}{2} & -i\left(\dfrac{\delta}{2} - \dfrac{k_R}{m}k_x\right) \\ i\left(\dfrac{\delta}{2} - \dfrac{k_R}{m}k_x\right) & \dfrac{\mathbf{k}^2 + k_R^2}{2m} - \dfrac{\Omega}{2} \end{pmatrix}$$

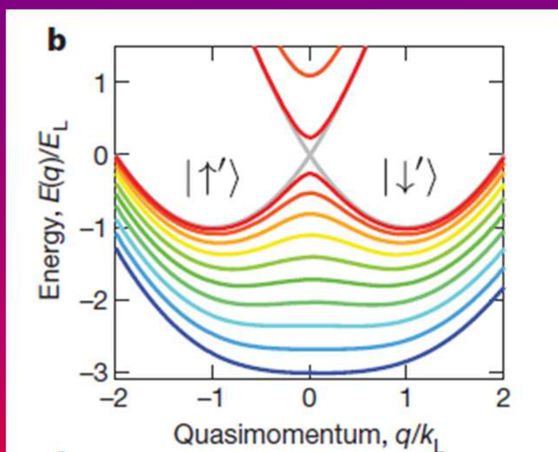
spin-orbit

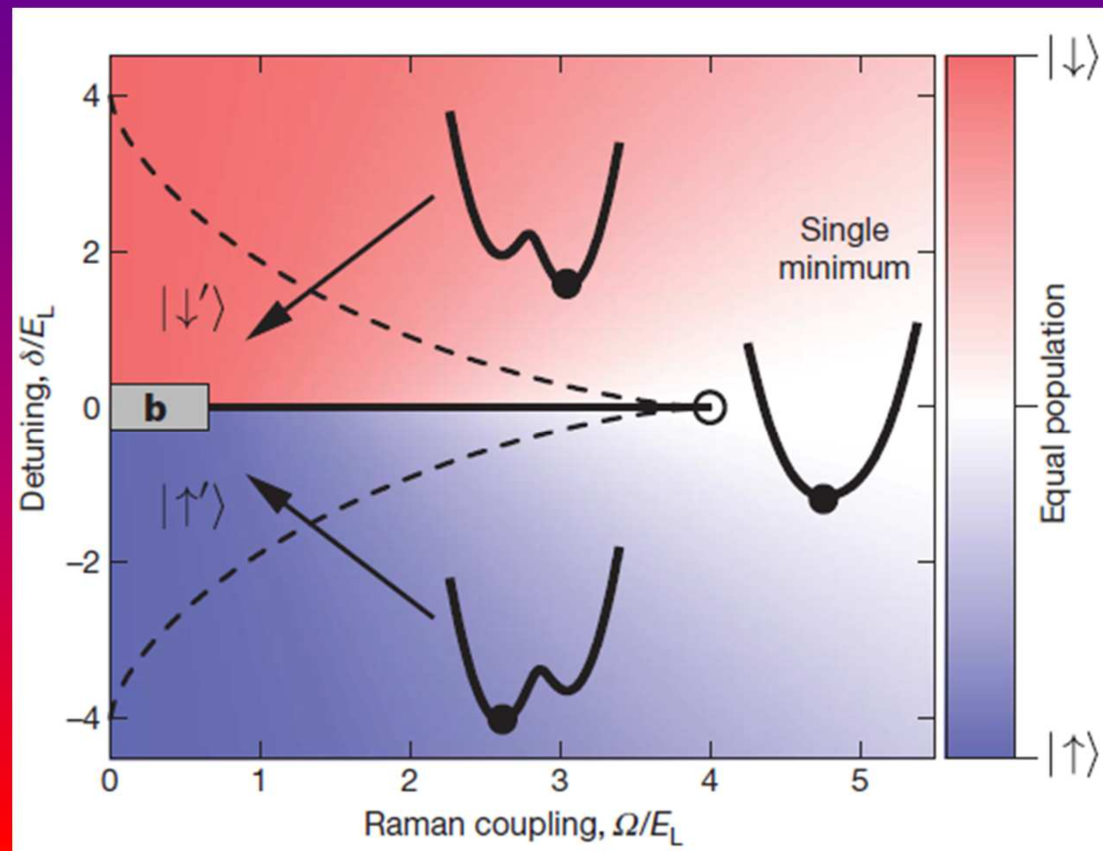
detuning

Raman coupling

Experimental phase diagram for ^{87}Rb: bosons with two internal states (spin-1/2)

3 MARCH 2011 | VOL 471 | NATURE | 83





Y.-J. Lin[1], K. Jiménez-García[1,2] & I. B. Spielman[1]

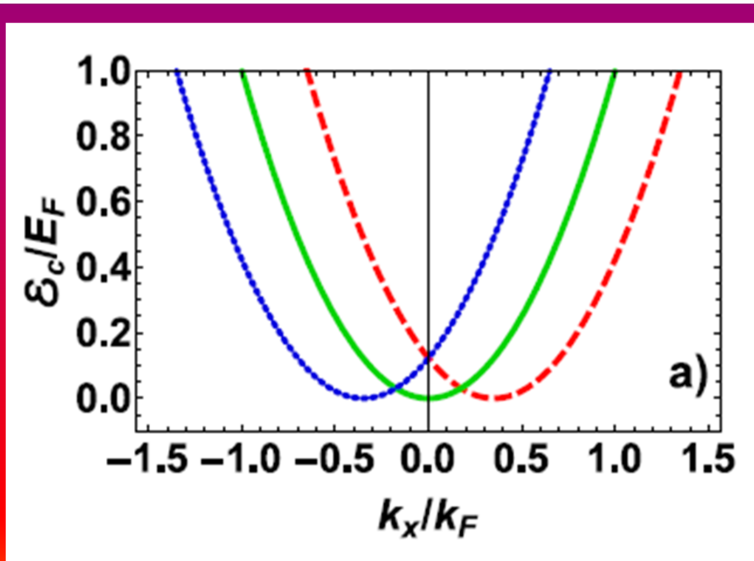
Case with three internal states: color-orbit and color flip fields

$$\mathbf{H}_0(\mathbf{k}) = \begin{pmatrix} \varepsilon_R(\mathbf{k}) & \Omega_{RG} & \cancel{\Omega_{RB}} \\ \Omega^*_{RG} & \varepsilon_G(\mathbf{k}) & \Omega_{GB} \\ \cancel{\Omega^*_{RB}} & \Omega^*_{GB} & \varepsilon_B(\mathbf{k}) \end{pmatrix}$$

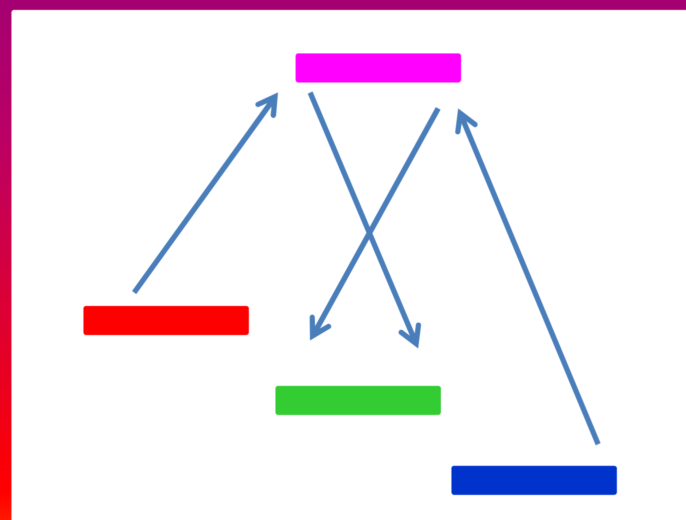
$$c = \{R, G, B\}$$

^{6}Li, ^{40}K, ^{173}Yb

$$\varepsilon_c(\mathbf{k}) = (\mathbf{k} - \mathbf{k}_c)^2/(2m) + \eta_c$$



Raman Process



Case with three internal states color-orbit and color-flip fields

$$\mathbf{H}_{\mathrm{IP}}(\mathbf{k}) = \begin{pmatrix} \varepsilon_{\mathrm{R}}(\mathbf{k}) & -h_x(\mathbf{k})/\sqrt{2} & 0 \\ -h_x(\mathbf{k})/\sqrt{2} & \varepsilon_{\mathrm{G}}(\mathbf{k}) & -h_x(\mathbf{k})/\sqrt{2} \\ 0 & -h_x(\mathbf{k})/\sqrt{2} & \varepsilon_{\mathrm{B}}(\mathbf{k}) \end{pmatrix}$$

$$\varepsilon_{\mathrm{R}}(\mathbf{k}) = \varepsilon(\mathbf{k}) - h_z(\mathbf{k}) + b_z$$

$$\varepsilon_{\mathrm{G}}(\mathbf{k}) = \varepsilon(\mathbf{k})$$

$$\varepsilon_{\mathrm{B}}(\mathbf{k}) = \varepsilon(\mathbf{k}) + h_z(\mathbf{k}) + b_z$$

Kinetic energies of
Red, Green and Blue fermions

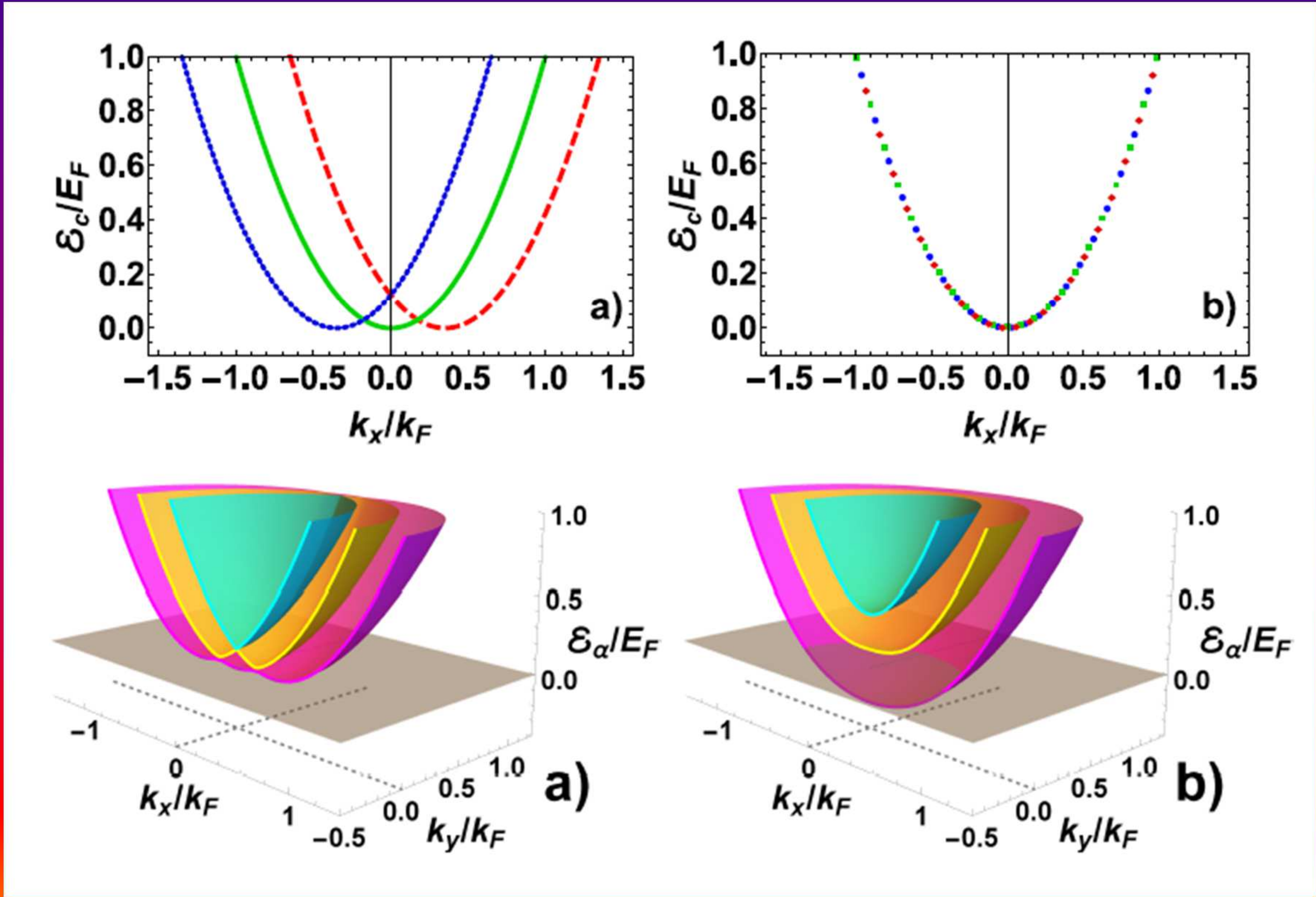
$$h_z(\mathbf{k}) = 2k_T k_x/(2m) + \delta$$

Color-orbit and
Color-Zeeman fields

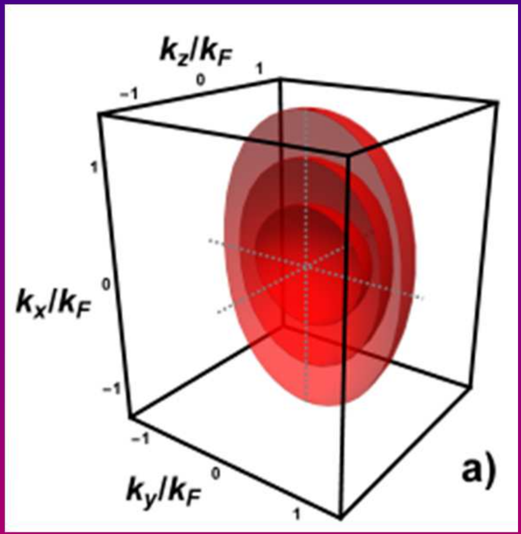
$$h_x(\mathbf{k}) = -\sqrt{2}\Omega$$

Color-flip field

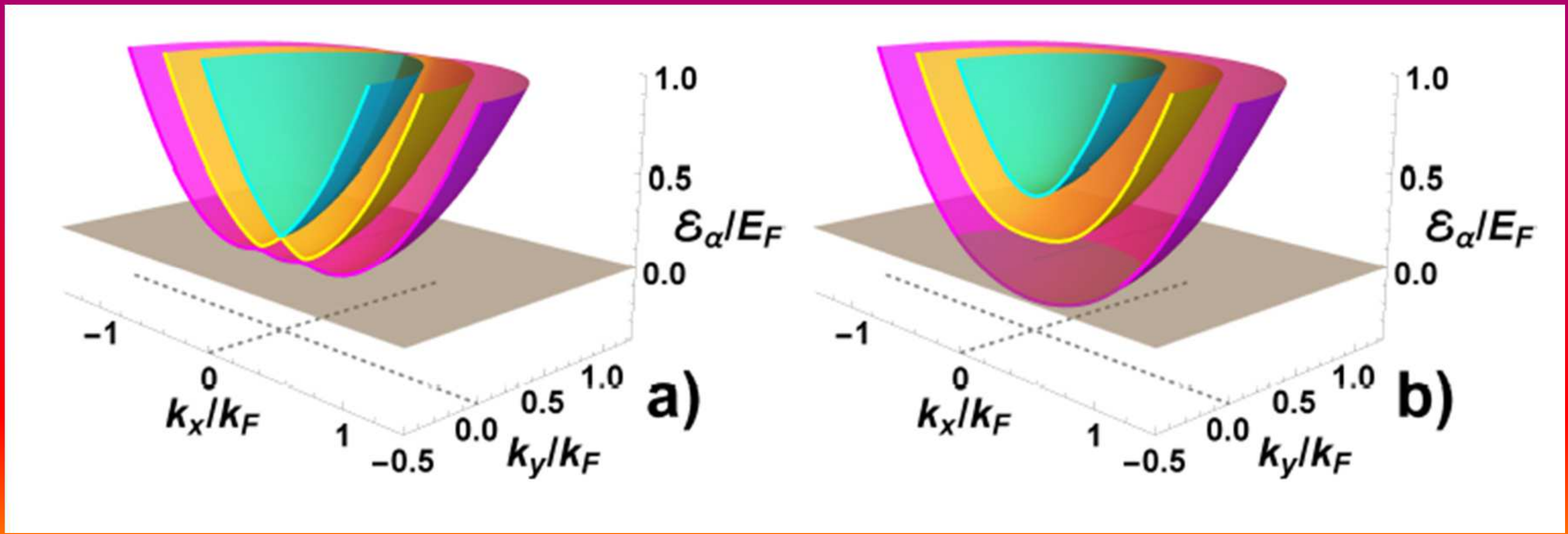
Case with three internal states: color-orbit and color flip fields



Colored fermions are a correlated three band system



Example of Fermi Surface



Outline of Talk

1) Motivation: color superfluidity and ultracold fermions

2) Introduction to spin-orbit and color-orbit coupling

3) Interacting fermions with color-orbit and color-flip fields

4) Spectroscopic and thermodynamic properties

5) Conclusions

Start with SU(2) case

- For simplicity and to gain insight let me start first with the SU(2) case: two colors or simple peudospin-1/2 fermions.

- How spin-orbit and Zeeman fields change the crossover from BCS to BEC as interactions are tuned?

3 MARCH 2011 | VOL 471 | NATURE | 83

LETTER

^{87}Rb

doi:10.1038/nature09887

Spin–orbit-coupled Bose–Einstein condensates

Y.-J. Lin[1], K. Jiménez-García[1,2] & I. B. Spielman[1]

$$\begin{pmatrix} \dfrac{\mathbf{k}^2 + k_R^2}{2m} + \dfrac{\Omega}{2} & -i\left(\dfrac{\delta}{2} - \dfrac{k_R}{m}k_x\right) \\ i\left(\dfrac{\delta}{2} - \dfrac{k_R}{m}k_x\right) & \dfrac{\mathbf{k}^2 + k_R^2}{2m} - \dfrac{\Omega}{2} \end{pmatrix}$$

spin-orbit

detuning

Raman coupling

Zeeman and Spin-Orbit Hamiltonian

Hamiltonian Matrix

$$\mathbf{H}_0(\mathbf{k}) = \varepsilon(\mathbf{k})\mathbf{1} - h_x(\mathbf{k})\boldsymbol{\sigma}_x - h_y(\mathbf{k})\boldsymbol{\sigma}_y - h_z(\mathbf{k})\boldsymbol{\sigma}_z$$

$$\varepsilon_{\Uparrow}(\mathbf{k}) = \varepsilon(\mathbf{k}) - \left|h_{\text{eff}}(\mathbf{k})\right|$$

$$\varepsilon_{\Downarrow}(\mathbf{k}) = \varepsilon(\mathbf{k}) + \left|h_{\text{eff}}(\mathbf{k})\right|$$

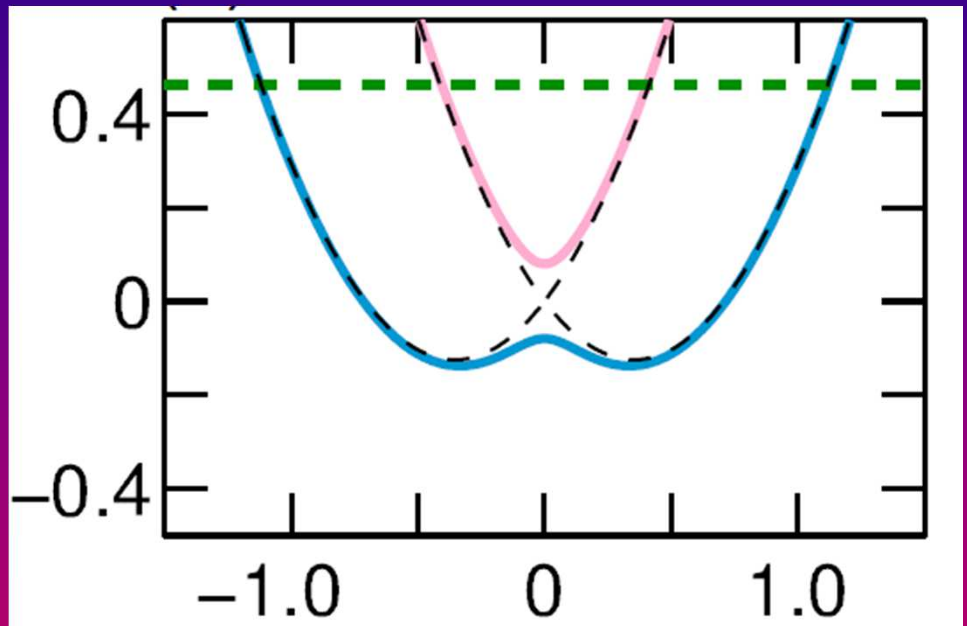
$$\left|h_{\text{eff}}(\mathbf{k})\right| = \sqrt{\left|h_x(\mathbf{k})\right|^2 + \left|h_y(\mathbf{k})\right|^2 + \left|h_z(\mathbf{k})\right|^2}$$

Energy Dispersions in the ERD case

$$h_x(\mathbf{k}) = 0$$

$$\frac{h_y(\mathbf{k})}{\varepsilon_F} = 0.71\frac{k_x}{k_F}$$

$$\frac{h_z(\mathbf{k})}{\varepsilon_F} = 0.05$$



$$\varepsilon_{\Uparrow}(\mathbf{k}) = \varepsilon(\mathbf{k}) - \sqrt{h_z^2 + |vk_x|^2}$$

$$\varepsilon_{\Downarrow}(\mathbf{k}) = \varepsilon(\mathbf{k}) + \sqrt{h_z^2 + |vk_x|^2}$$

Can have intra- and inter-helicity pairing.

Bring Interactions Back (real space)

$$\mathcal{H}(\mathbf{r}) = \mathcal{H}_0(\mathbf{r}) + \mathcal{H}_I(\mathbf{r})$$

$$\mathcal{H}_0(\mathbf{r}) = \sum_{\alpha\beta} \psi^\dagger_\alpha(\mathbf{r}) \left[\hat{K}_\alpha \delta_{\alpha\beta} - h_i(\mathbf{r}) \sigma_{i,\alpha\beta} \right] \psi_\beta(\mathbf{r})$$

Kinetic Energy　　　Spin-orbit and Zeeman

$$\mathcal{H}_I(\mathbf{r}) \;=\; -g \psi^\dagger_\uparrow(\mathbf{r}) \psi^\dagger_\downarrow(\mathbf{r}) \psi_\downarrow(\mathbf{r}) \psi_\uparrow(\mathbf{r})$$

Contact Interaction

Bring interactions back: Hamiltonian in initial spin basis

$$
\mathrm{H}_0 = \begin{array}{c} \\ \psi^{+}_{k\uparrow} \\ \psi^{+}_{k\downarrow} \\ \psi_{-k\uparrow} \\ \psi_{-k\downarrow} \end{array}
\begin{array}{c}
\begin{array}{cccc} \psi_{k\uparrow} & \psi_{k\downarrow} & \psi^{+}_{-k\uparrow} & \psi^{+}_{-k\downarrow} \end{array} \\
\begin{pmatrix}
\widetilde{K}_{\uparrow}(\mathbf{k}) & -h_{\perp}(\mathbf{k}) & 0 & -\Delta_0 \\
-h^{*}_{\perp}(\mathbf{k}) & \widetilde{K}_{\downarrow}(\mathbf{k}) & \Delta_0 & 0 \\
0 & \Delta_0^{\dagger} & -\widetilde{K}_{\uparrow}(-\mathbf{k}) & h^{*}_{\perp}(-\mathbf{k}) \\
-\Delta_0^{\dagger} & 0 & h_{\perp}(-\mathbf{k}) & -\widetilde{K}_{\downarrow}(-\mathbf{k})
\end{pmatrix}
\end{array}
$$

$$\widetilde{K}_{\uparrow}(\mathbf{k}) = \xi(\mathbf{k}) - h_z \qquad \widetilde{K}_{\downarrow}(\mathbf{k}) = \xi(\mathbf{k}) + h_z$$

Bring interactions back: Hamiltonian in the helicity basis

$$
\begin{array}{cc}
 & \begin{matrix} \Phi_{k\Uparrow} & \Phi_{k\Downarrow} & \Phi^{+}_{-k\Uparrow} & \Phi^{+}_{-k\Downarrow} \end{matrix} \\
\begin{matrix} \Phi^{+}_{k\Uparrow} \\ \Phi^{+}_{k\Downarrow} \\ \Phi_{-k\Uparrow} \\ \Phi_{-k\Downarrow} \end{matrix} & \tilde{\mathrm{H}}_0 = \begin{pmatrix}
\xi_{\Uparrow}(\mathrm{k}) & 0 & \boxed{\Delta_T(\mathrm{k})e^{-i\varphi_{\mathrm{k}}}} & -\Delta_S(\mathrm{k}) \\
0 & \xi_{\Downarrow}(\mathrm{k}) & \Delta_S(\mathrm{k}) & \boxed{-\Delta_T e^{i\varphi_{\mathrm{k}}}} \\
\boxed{\Delta_T^*(\mathrm{k})e^{i\varphi_{\mathrm{k}}}} & -\Delta_S^*(\mathrm{k}) & -\xi_{\Uparrow}(\mathrm{k}) & 0 \\
\Delta_S^*(\mathrm{k}) & \boxed{-\Delta_T^*(\mathrm{k})e^{-i\varphi_{\mathrm{k}}}} & 0 & -\xi_{\Downarrow}(\mathrm{k})
\end{pmatrix}
\end{array}
$$

$$\varphi_{\mathbf{k}} = \mathrm{Arg}\left[h_{\perp}(\mathbf{k})\right]$$

Excitation Spectrum

$$E_1(\mathbf{k}) = \sqrt{\left[\left(\frac{\xi_\Uparrow - \xi_\Downarrow}{2}\right) - \sqrt{\left(\frac{\xi_\Uparrow + \xi_\Downarrow}{2}\right)^2 + |\Delta_S(\mathbf{k})|^2}\right]^2 + |\Delta_T(\mathbf{k})|^2},$$

★

$$E_2(\mathbf{k}) = \sqrt{\left[\left(\frac{\xi_\Uparrow - \xi_\Downarrow}{2}\right) + \sqrt{\left(\frac{\xi_\Uparrow + \xi_\Downarrow}{2}\right)^2 + |\Delta_S(\mathbf{k})|^2}\right]^2 + |\Delta_T(\mathbf{k})|^2},$$

Can be zero

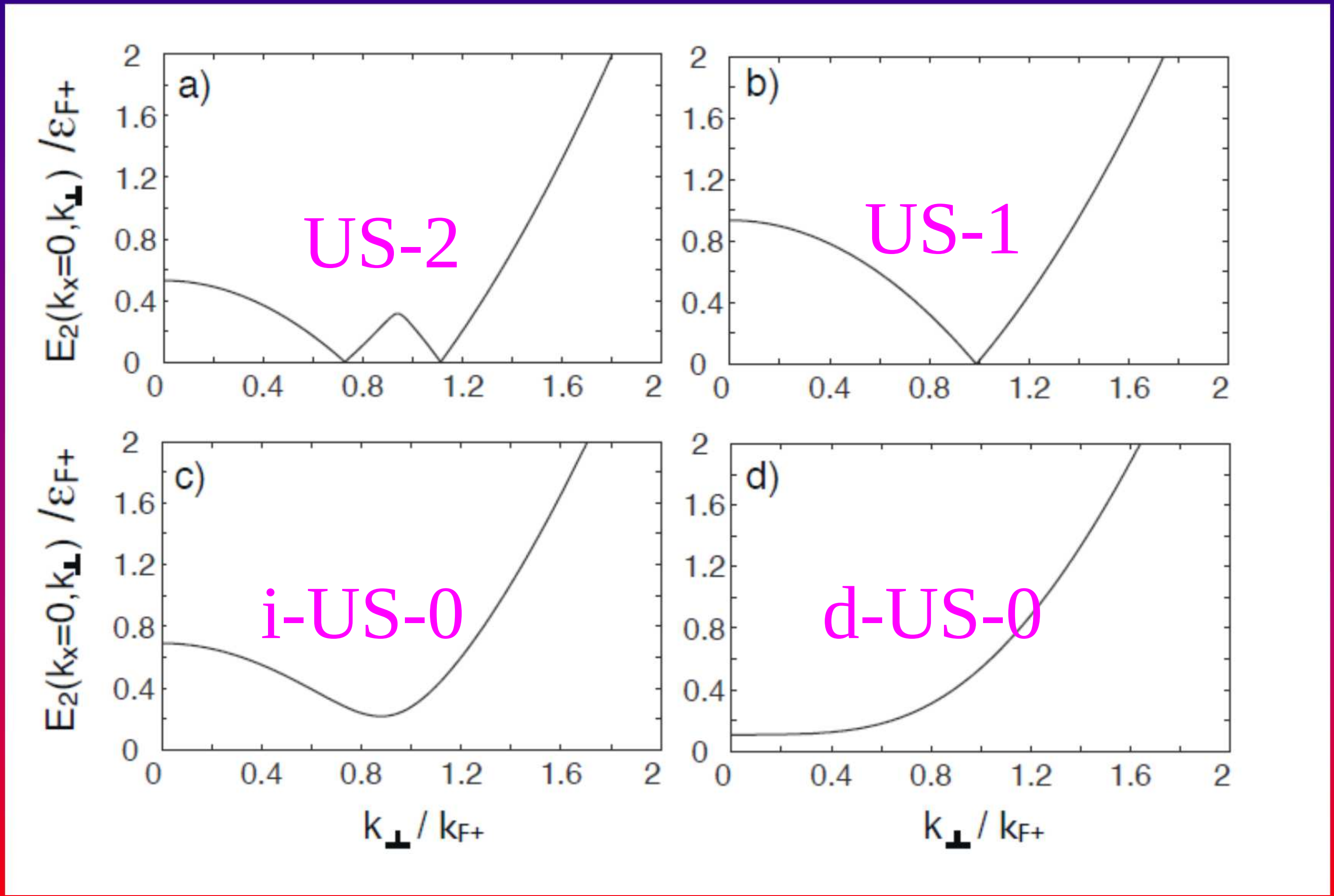
$$E_3(\mathbf{k}) = -E_2(\mathbf{k})$$

$$\xi_\Uparrow(\mathbf{k}) = \xi(\mathbf{k}) - |h_{\text{eff}}(\mathbf{k})|$$

$$E_4(\mathbf{k}) = -E_1(\mathbf{k})$$

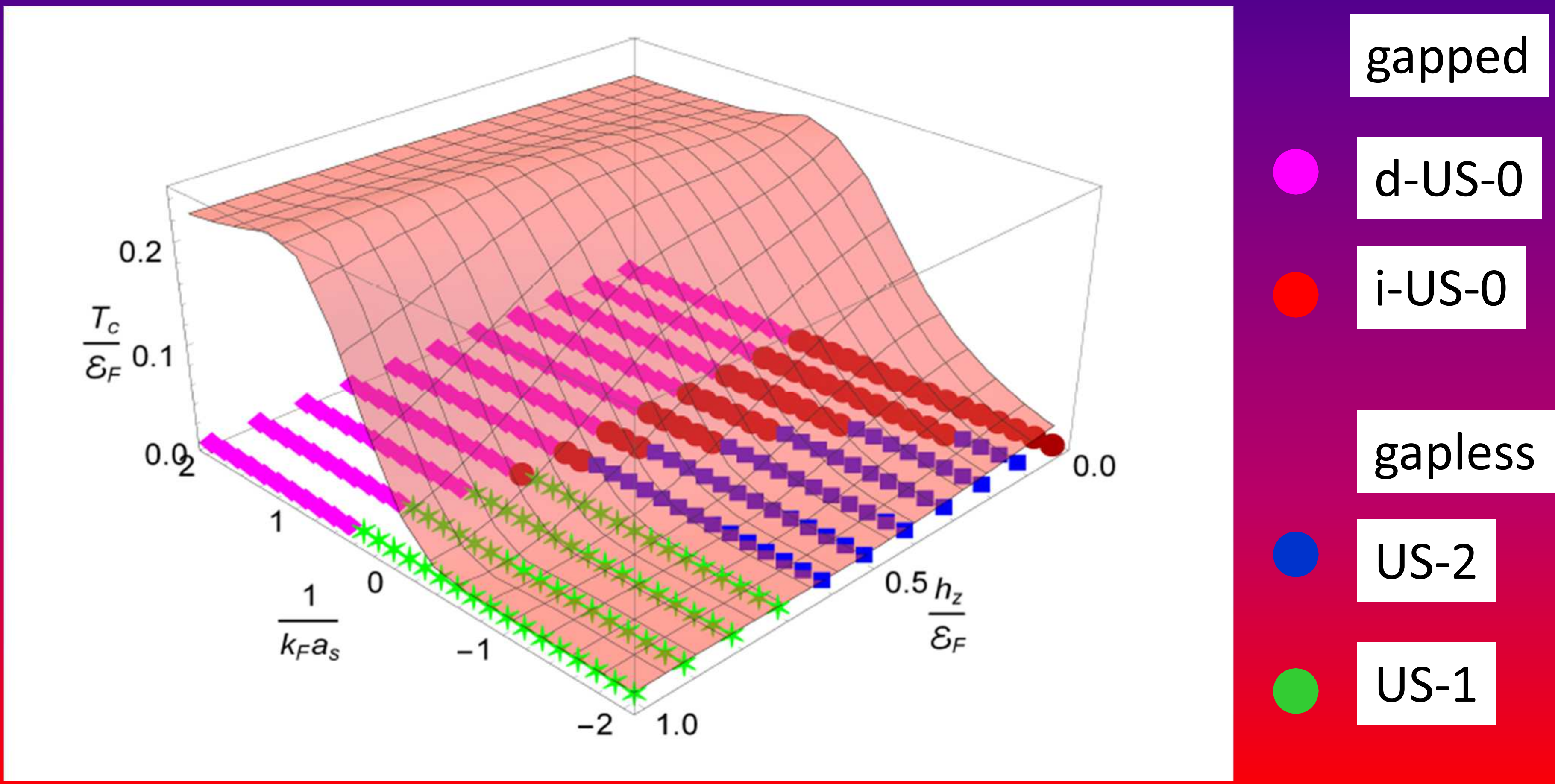
$$\xi_\Downarrow(\mathbf{k}) = \xi(\mathbf{k}) + |h_{\text{eff}}(\mathbf{k})|$$

Excitation Spectrum (ERD)



$$\Delta_T(\mathbf{k}) = \Delta_0|h_\perp(\mathbf{k})|/|\mathbf{h}_{\text{eff}}(\mathbf{k})| \quad = 0$$

Phase diagram for finite spin-orbit coupling and changing Zeeman field



Triple-point: US-0/US-1/US-2

Now look at SU(3) case

- Let me analyze the SU(3) case: three colors or pseudo-spin-1 fermions.
- How color-orbit and color-flip fields change the crossover from BCS to BEC as interactions are tuned?

SU(3) invariant kinetic energy and three identical interaction channels

$$\hat{H}_{\mathrm{IP}} = \sum_{\mathbf{k}} \mathbf{F}^{\dagger}(\mathbf{k}) \mathbf{H}_{\mathrm{IP}}(\mathbf{k}) \mathbf{F}(\mathbf{k})$$

$$\varepsilon_R(\mathbf{k}) = \varepsilon_G(\mathbf{k}) = \varepsilon_B(\mathbf{k}) = \varepsilon(\mathbf{k})$$

$$\mathbf{F}^{\dagger}(\mathbf{k}) \quad = \quad \left[f_R^{\dagger}(\mathbf{k}), f_G^{\dagger}(\mathbf{k}), f_B^{\dagger}(\mathbf{k}) \right]$$

$$\hat{H}_{\mathrm{INT}} = -\frac{1}{V} \sum_{\mathbf{Q},\{c \neq c'\}} g_{cc'} a_{cc'}^{\dagger}(\mathbf{Q}) a_{cc'}(\mathbf{Q}),$$

$$a_{cc'}^{\dagger}(\mathbf{Q}) = \sum_{\mathbf{k}} f_c^{\dagger}(\mathbf{k} + \mathbf{Q}/2) f_{c'}^{\dagger}(-\mathbf{k} - \mathbf{Q}/2)$$

Pair operator

$$g_{RG} = g_{RB} = g_{GB} = g \Rightarrow a_{RG} = a_{RB} = a_{GB} = a_s$$

No color-orbit and no color-flip fields

$$\hat{H}_{\mathrm{IP}} = \sum_{\mathbf{k}} \mathbf{F}^{\dagger}(\mathbf{k})\mathbf{H}_{\mathrm{IP}}(\mathbf{k})\mathbf{F}(\mathbf{k})$$

$$\mathbf{H}_{\mathrm{IP}}(\mathbf{k}) = \varepsilon(\mathbf{k})\mathbf{1}$$

KE is SU(3) invariant

$$\hat{H}_{\mathrm{INT}} = -\frac{1}{V} \sum_{\mathbf{Q},\{c \neq c'\}} g_{cc'} a^{\dagger}_{cc'}(\mathbf{Q}) a_{cc'}(\mathbf{Q}),$$

$$g_{RG} = g_{RB} = g_{GB} = g$$

NOT VERY INTERESTING, JUST CROSSOVER!

Can go to a mixed color basis where only two mixed colors pair and the third one is inert as a result of SU(3) invariance!

Add color-orbit and color-flip fields (near zero temperature)

$$\hat{H}_0 = \frac{1}{2}\sum_{\mathbf{k}} \mathbf{f}_N^\dagger(\mathbf{k})\mathbf{H}_0(\mathbf{k})\mathbf{f}_N(\mathbf{k}) + V\sum_{c\neq c'} \frac{|\Delta_{cc'}|^2}{g_{cc'}} + \mathcal{C}(\mu),$$

$$\mathbf{f}_N^\dagger(\mathbf{k}) \quad = \quad \left[f_{\mathrm{R}}^\dagger(\mathbf{k}), f_{\mathrm{G}}^\dagger(\mathbf{k}), f_{\mathrm{B}}^\dagger(\mathbf{k}), f_{\mathrm{R}}(-\mathbf{k}), f_{\mathrm{G}}(-\mathbf{k}), f_{\mathrm{B}}(-\mathbf{k})\right]$$

$$\mathbf{H}_0(\mathbf{k}) = \begin{pmatrix} \overline{\mathbf{H}}_{\mathrm{IP}}(\mathbf{k}) & \boldsymbol{\Delta} \\ \boldsymbol{\Delta}^\dagger & -\overline{\mathbf{H}}_{\mathrm{IP}}^*(-\mathbf{k}) \end{pmatrix},$$

Spectrum has 3 quasiparticle and 3 quasihole bands

Hamiltonian Blocks

$$\varepsilon_c(\mathbf{k}) \;=\; (\mathbf{k}-\mathbf{k}_c)^2/(2m)+\eta_c$$

$$h_x(\mathbf{k}) = -\sqrt{2}\Omega$$

$$\mathbf{H}_{\rm IP}(\mathbf{k}) = \begin{pmatrix} \varepsilon_{\rm R}(\mathbf{k}) & -h_x(\mathbf{k})/\sqrt{2} & 0 \\ -h_x(\mathbf{k})/\sqrt{2} & \varepsilon_{\rm G}(\mathbf{k}) & -h_x(\mathbf{k})/\sqrt{2} \\ 0 & -h_x(\mathbf{k})/\sqrt{2} & \varepsilon_{\rm B}(\mathbf{k}) \end{pmatrix},$$

$$\mathbf{\Delta} = \begin{pmatrix} 0 & \Delta_{RG} & \Delta_{RB} \\ -\Delta_{RG} & 0 & \Delta_{GB} \\ -\Delta_{RB} & -\Delta_{GB} & 0 \end{pmatrix}$$

Mixed (rotated) color basis

$$\widetilde{\mathrm{H}}_0(\mathbf{k}) = \begin{pmatrix} \mathbf{H}_M(\mathbf{k}) & \mathbf{\Delta}_M \\ \mathbf{\Delta}_M^\dagger & -\mathbf{H}_M^*(-\mathbf{k}) \end{pmatrix}$$

$$\mathbf{H}_{M,\alpha\beta}(\mathbf{k}) = \xi_\alpha(\mathbf{k})\delta_{\alpha\beta} \qquad \Delta_{M,\alpha\beta}(\mathbf{k}) = \Delta_{\alpha\beta}(\mathbf{k})$$

$$\mathbf{R}(\mathbf{k}) = \begin{pmatrix} R_{\Uparrow R}(\mathbf{k}) & R_{\Uparrow G}(\mathbf{k}) & R_{\Uparrow B}(\mathbf{k}) \\ R_{0R}(\mathbf{k}) & R_{0G}(\mathbf{k}) & R_{0B}(\mathbf{k}) \\ R_{\Downarrow R}(\mathbf{k}) & R_{\Downarrow G}(\mathbf{k}) & R_{\Downarrow B}(\mathbf{k}) \end{pmatrix}$$

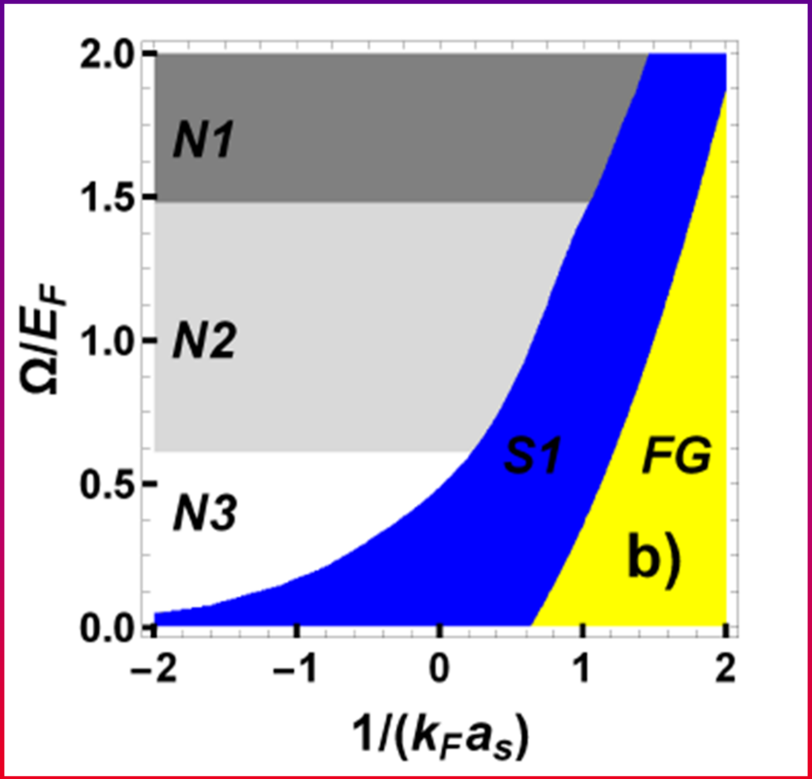
$$\Delta_{\alpha\beta}(\mathbf{k}) = R_{\alpha c}(\mathbf{k})\Delta_{cc'}R_{c'\beta}(-\mathbf{k}),$$

Zero color-orbit coupling

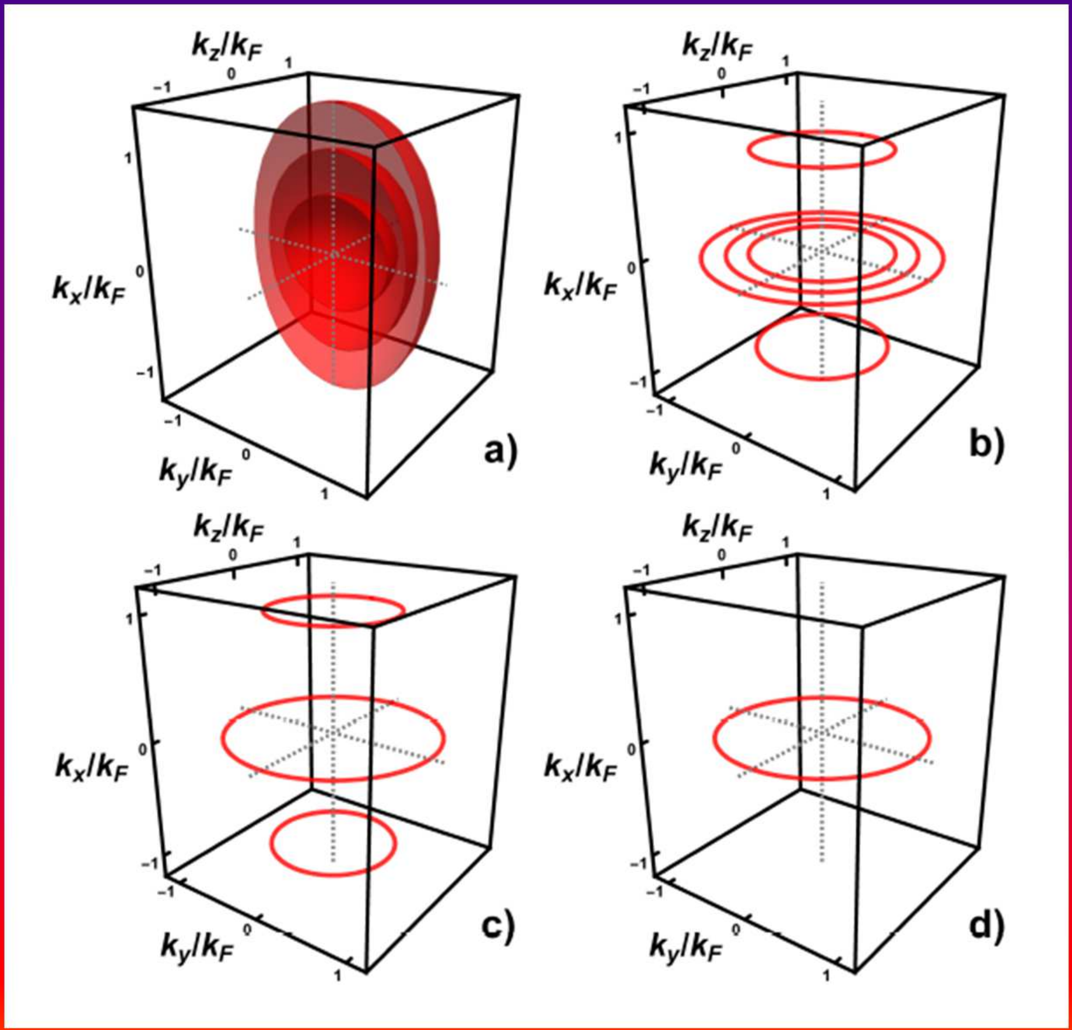
Color - orbit coupling is zero, but color - flip field Ω is not!

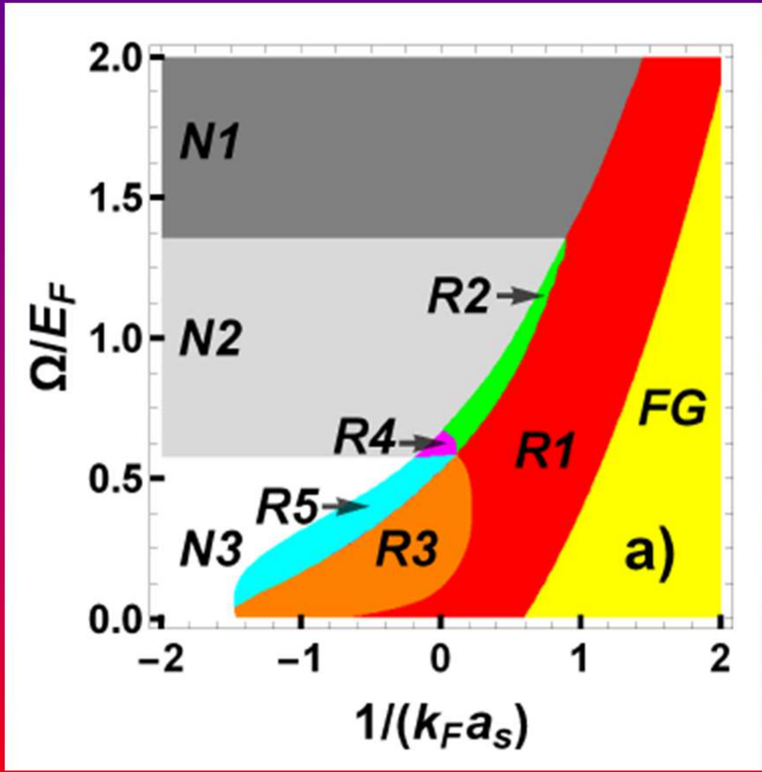
One of the three quasiparticle bands has a surface of nodes, the other two are fully gapped.

When color - flip field Ω is zero one mixed color band is completely inert.



Non-zero color-orbit coupling





Outline of Talk

1) Motivation: color superfluidity and ultracold fermions

2) Introduction to spin-orbit and color-orbit coupling

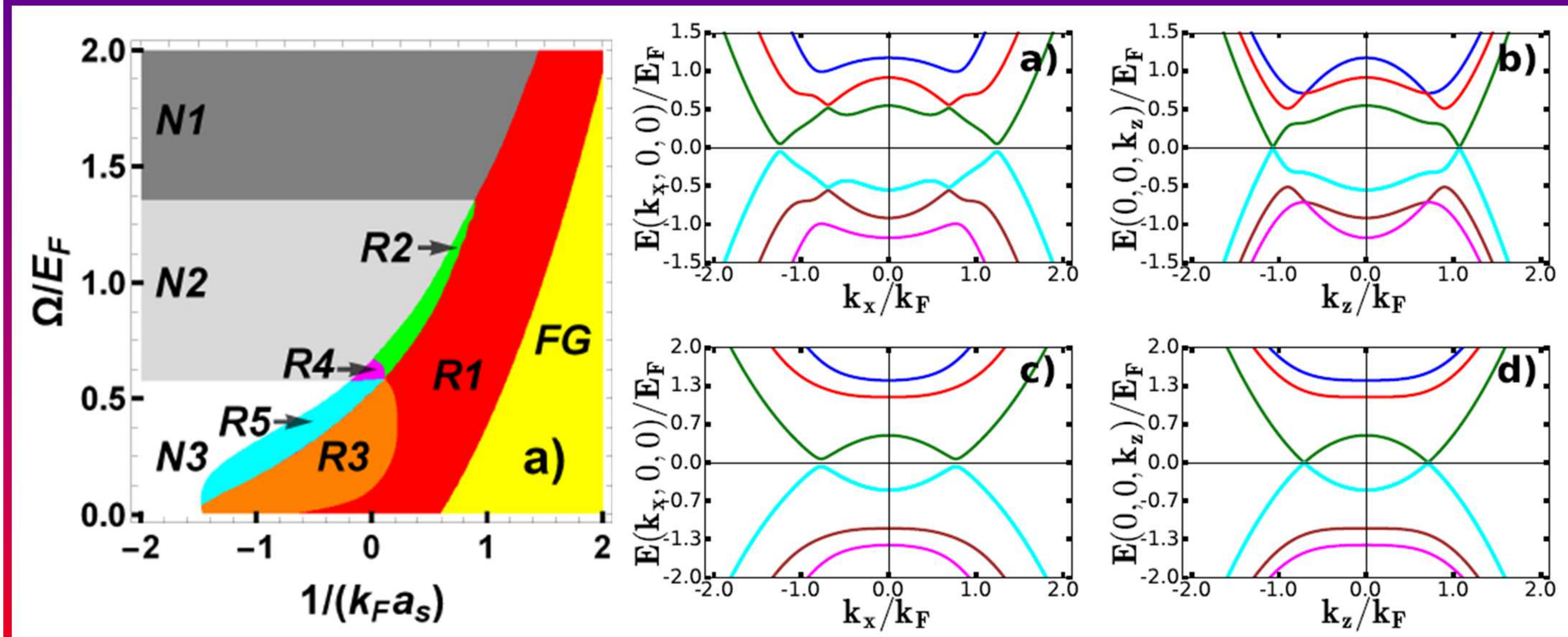
3) Interacting fermions with color-orbit and color-flip fields

4) Spectroscopic and thermodynamic properties

5) Conclusions

Non-zero color-orbit coupling

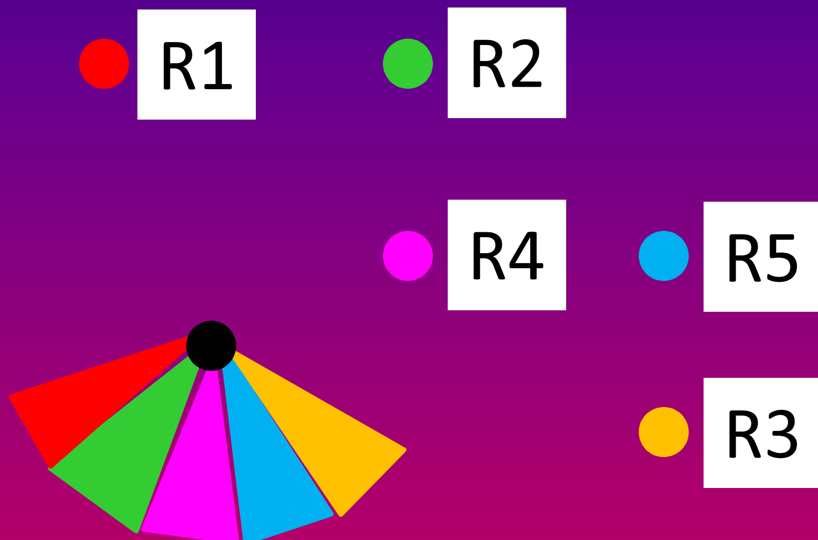
R3

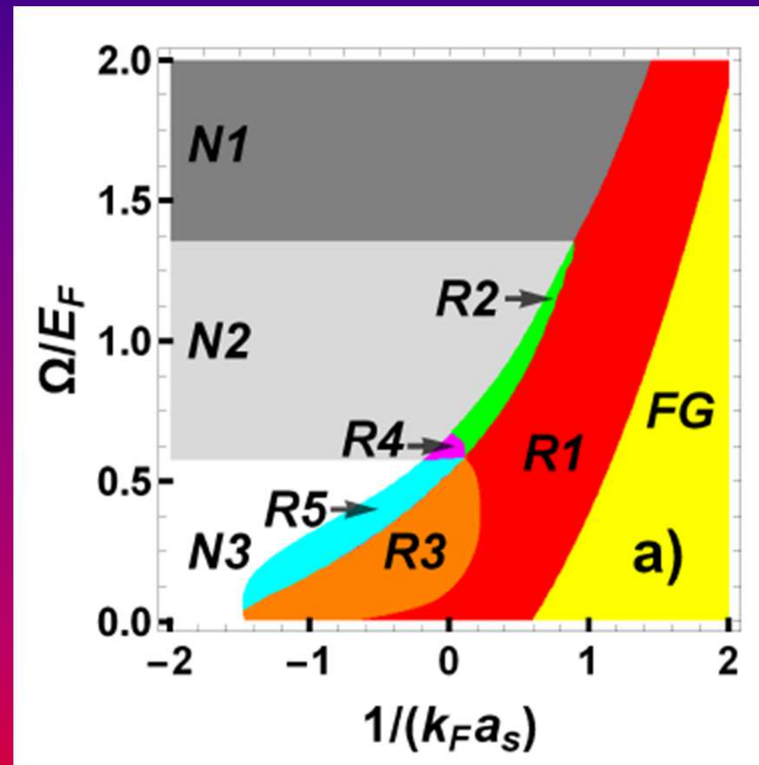


Quintuple and pentacritical point

R1

Color compressibility near quintuple point





$$\kappa = n^{-2}[\partial n / \partial \mu]_{T,V}$$

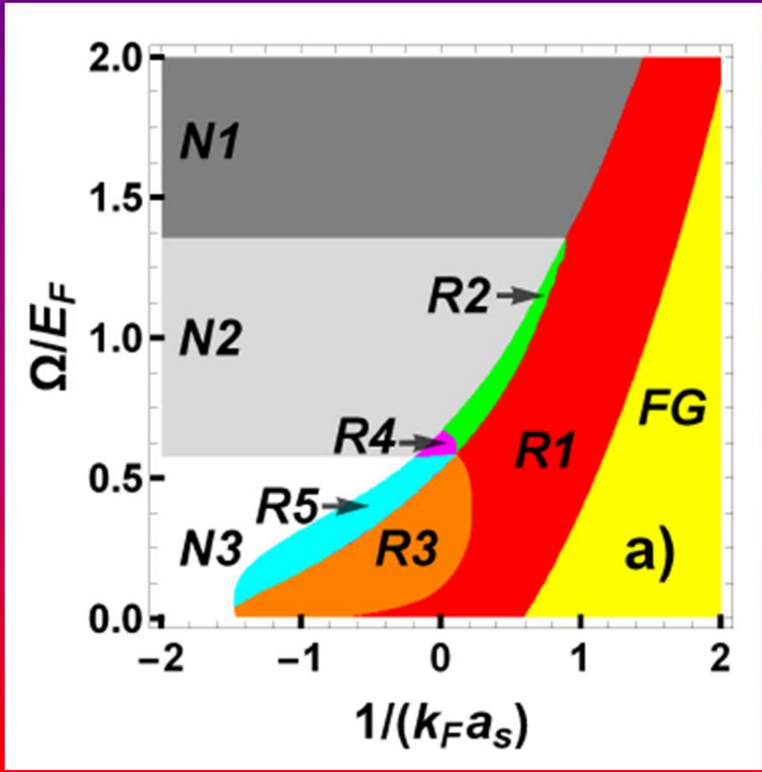
$$\kappa = \kappa_R - \alpha_{Rm} \left| \Omega - \Omega_c(\lambda) \right|^{1/2}$$

Color compressibility near gapless R1 to fully gapped FG line

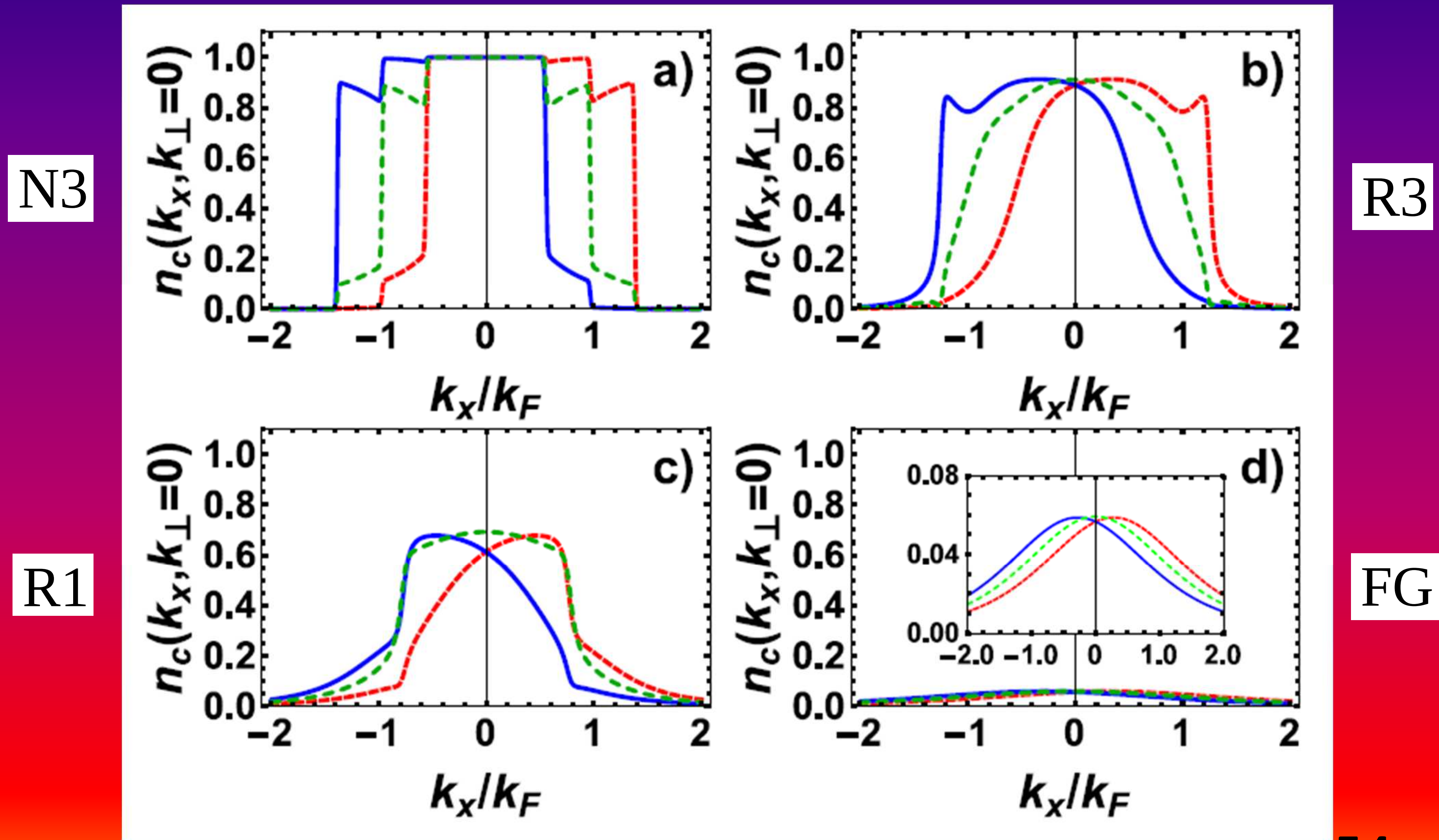
R1 FG

$$\kappa = n^{-2}[\partial n / \partial \mu]_{T,V}$$

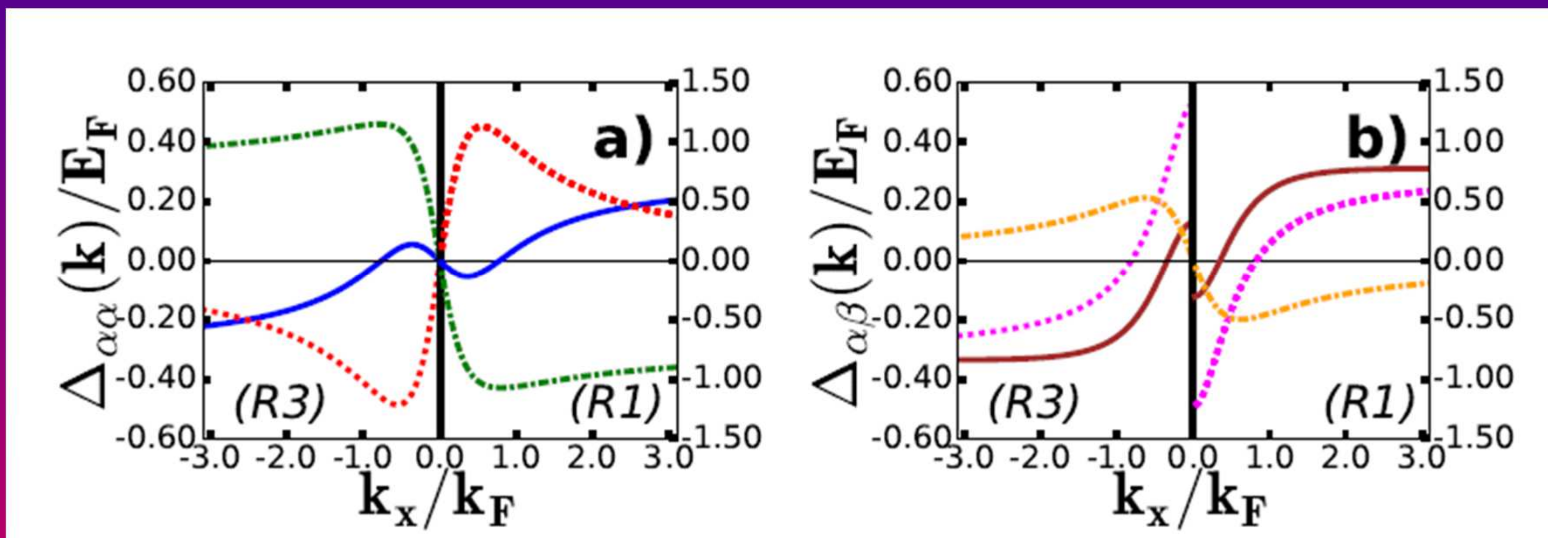
$$\kappa = \kappa_R \left| \ln \left| \Omega - \Omega_c(\lambda) \right| \right|$$



Momentum distributions of original colors

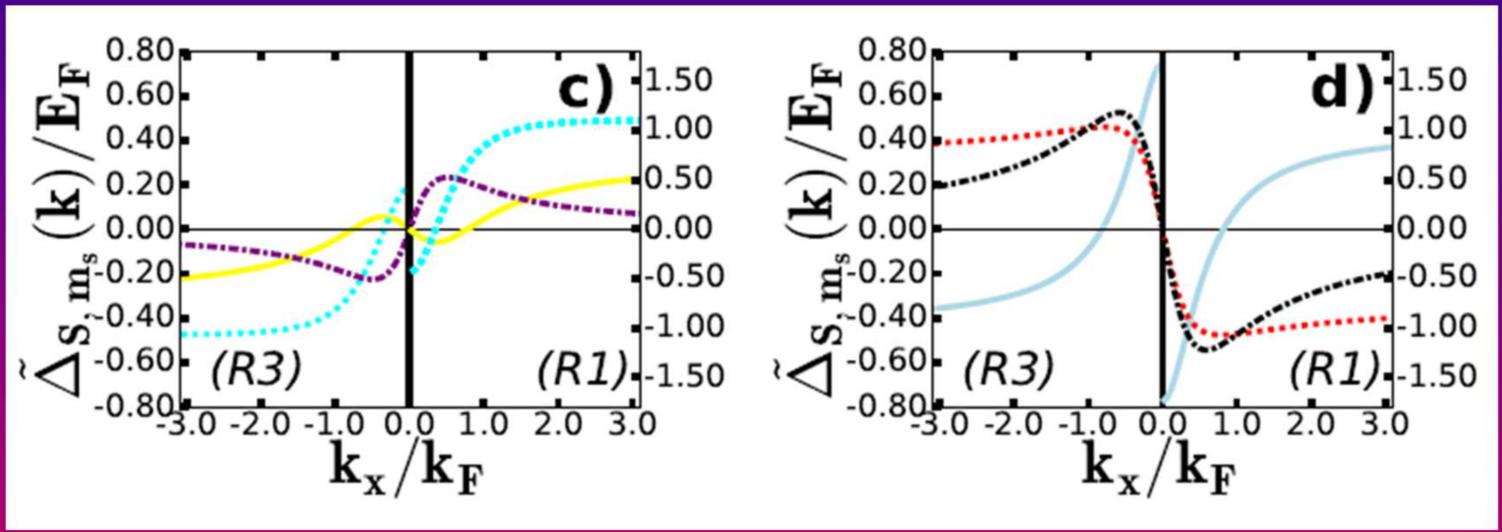


Order parameter tensor (mixed color basis)



In a) the solid blue curve describes $\Delta_{\Uparrow\Uparrow}(\mathbf{k})$, the dashed red curve describes $\Delta_{00}(\mathbf{k})$, and the dot-dashed green curve describes $\Delta_{\Downarrow\Downarrow}(\mathbf{k})$. In b) the solid brown line represents $\Delta_{\Uparrow 0}(\mathbf{k})$, the dashed magenta line represents $\Delta_{\Downarrow 0}(\mathbf{k})$, the dot-dashed orange line represents $\Delta_{\Uparrow\Downarrow}(\mathbf{k})$.

Order parameter tensor (total pseudo-spin basis)



In c) the solid yellow curve corresponds to $\widetilde{\Delta}_{22}(\mathbf{k})$, the dashed cyan curve corresponds to $\widetilde{\Delta}_{21}(\mathbf{k})$, the dot-dashed purple curve corresponds to $\widetilde{\Delta}_{20}(\mathbf{k})$. In d) the solid light-blue line indicates $\widetilde{\Delta}_{2\bar{1}}(\mathbf{k})$, the dashed red line indicates $\widetilde{\Delta}_{2\bar{2}}(\mathbf{k})$, the dot-dashed black line indicates $\widetilde{\Delta}_{00}(\mathbf{k})$. SINGLET AND QUINTET PAIRING

Outline of talk

1) Motivation: color superfluidity and ultracold fermions

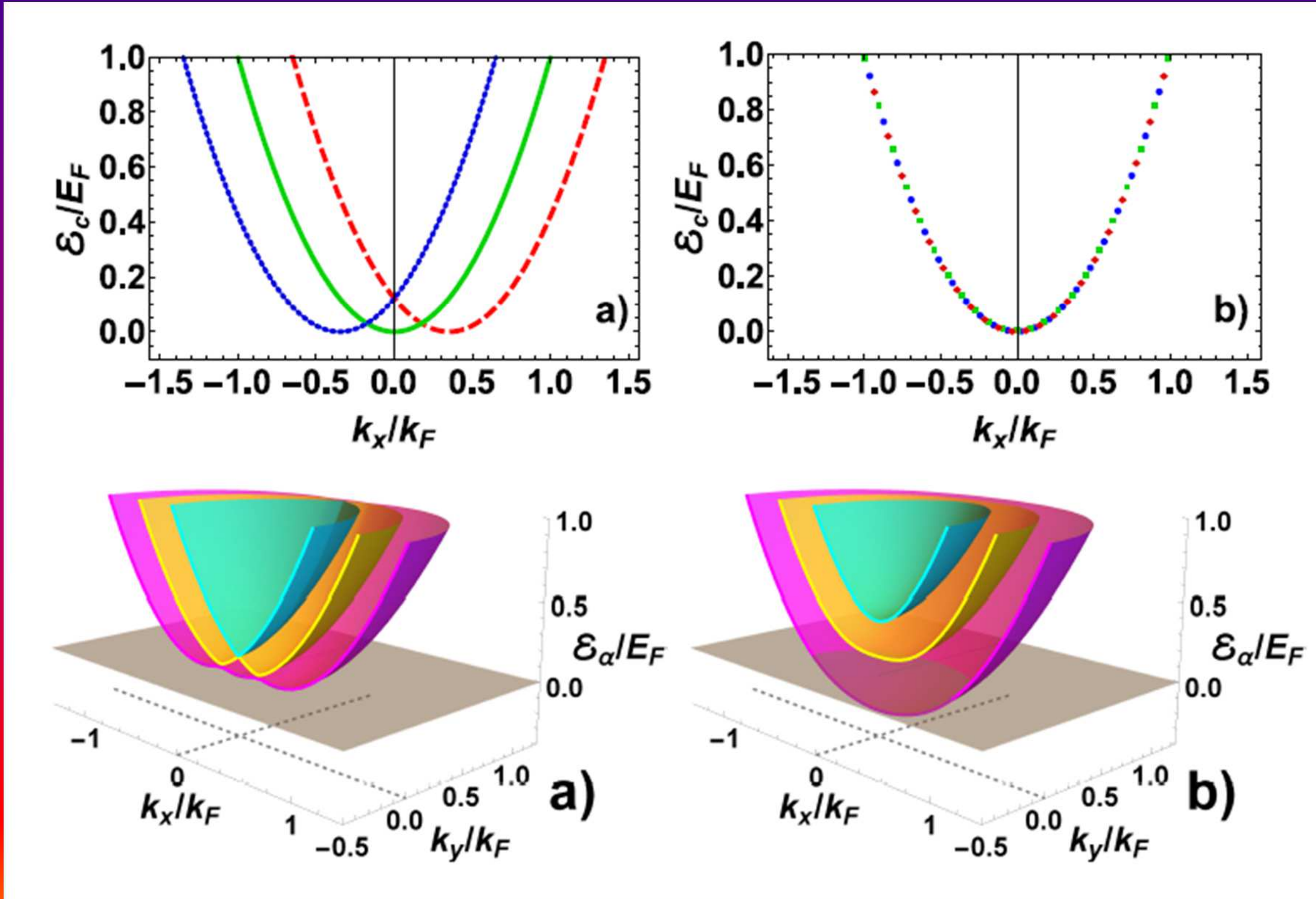
2) Introduction to spin-orbit and color-orbit coupling

3) Interacting fermions with color-orbit and color-flip fields

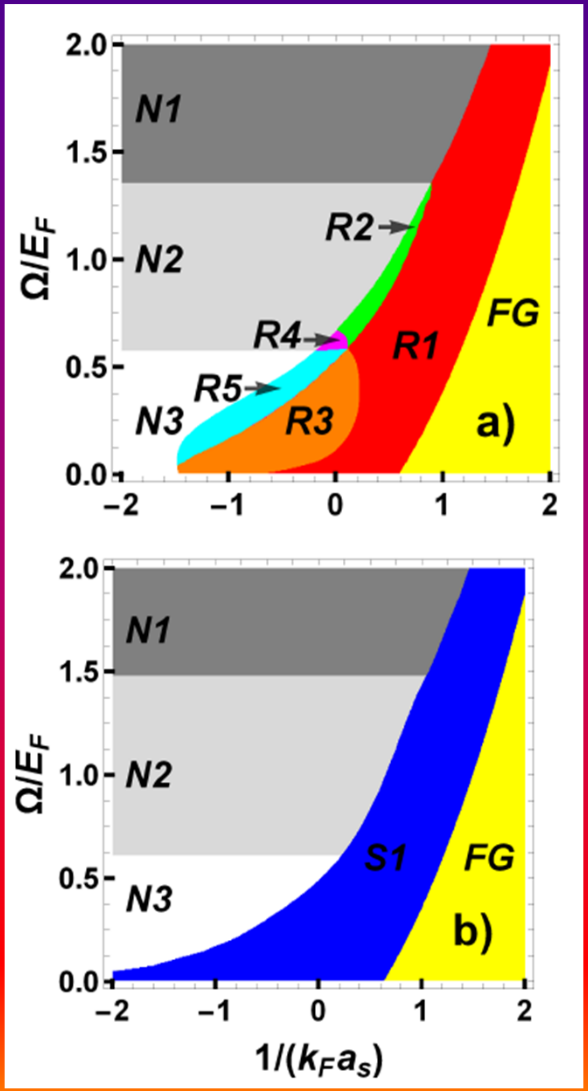
4) Spectroscopic and thermodynamic properties

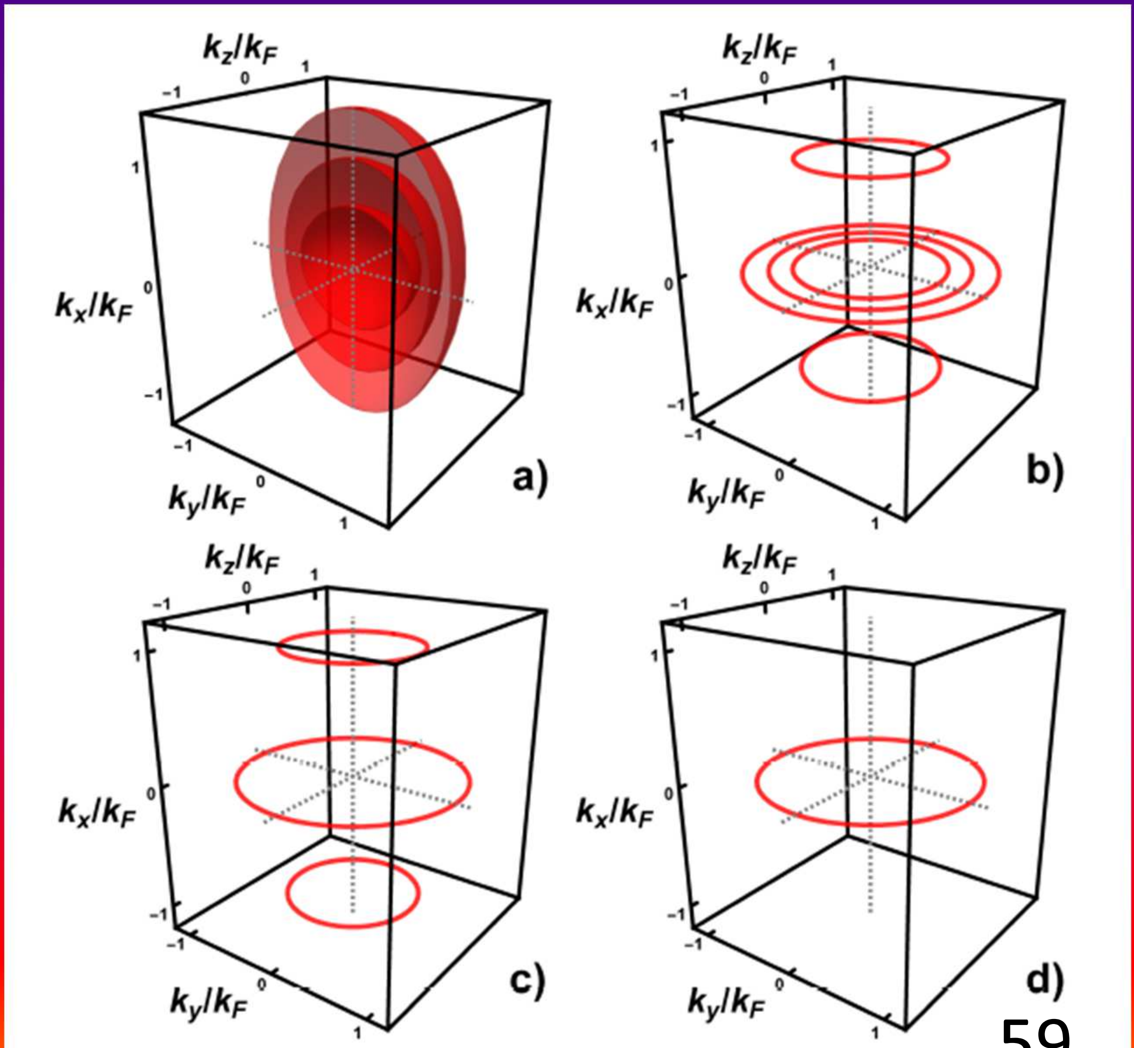
5) Conclusions

Conclusions in pictures: color-orbit and color flip fields



Conclusions in pictures: color-orbit and color flip fields





Conclusions in words

Ultracold fermions with three internal states can exhibit very unusual color superfluidity in the presence of color-orbit and color-flip fields, where SU(3) symmetry is explicitly broken.

The phase diagram of color-flip versus interaction parameter for fixed color-orbit coupling exhibits several topological phases associated with the nodal structure of the quasiparticle excitation spectrum. The phase diagram exhibits a pentacritical point where five nodal superfluid phases merge.

Even for interactions that occur only in the color s-wave channel, the order parameter for superfluidity exhibits singlet, triplet and quintuplet components due to the presence of color-orbit and color-flip fields.

These topological phases can be probed through measurements of spectroscopic properties such as excitation spectra, momentum distributions and density of states.

THE END