Chiral liquid phase of simple quantum antiferromagnets

Oleg Starykh
University of Utah

arXiv:1708.02980

Editors' Suggestion

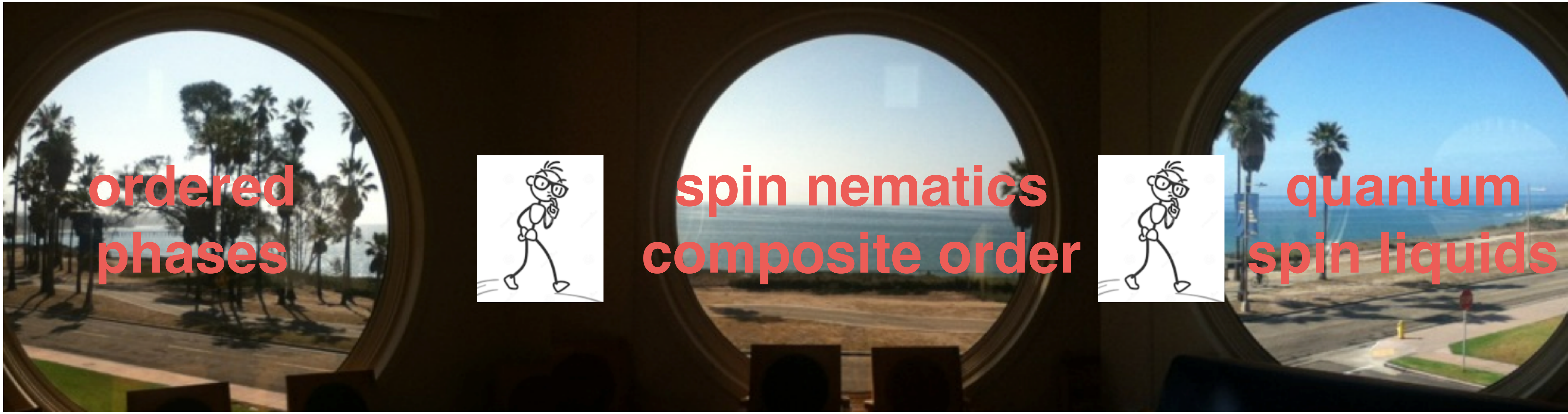
Andrey Chubukov (U Minnesota)
Zhentao Wang (U Tennessee)
Cristian Batista (U Tennessee)
Andrian Feigun (Northeastern U)
Wei Zhu (LANL)

YITP workshop "Novel quantum states in condensed matter", November 9, 2017

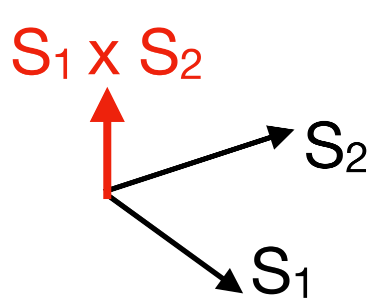
Outline

- ✳ Vector chirality
  Talks by Togawa, Nagaosa, Tokura
- ✳ 1/3 magnetization plateau and its instabilities:
- • spin-current phase
- ✳ Minimal s=1 XXZ model of spin-current phase
- ✳ Conclusions

Exotic ordered phases, emergent (Ising) orders



composite order parameter,
bilinear in spins



LE JOURNAL DE PHYSIQUE TOME 38, AVRIL 1977, PAGE 385

Classification
Physics Abstracts
7.480 — 8.514

A MAGNETIC ANALOGUE OF STEREOISOMERISM : APPLICATION TO HELIMAGNETISM IN TWO DIMENSIONS

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Centre d'Etudes Nucléaires de Grenoble, 85 X, 38041 Grenoble Cedex, France

(*Reçu le 13 juillet 1976, révisé le 8 novembre 1976, accepté le 4 janvier 1977*)

Annals of the Israel Physical Society,
vol.2, p. 565 (1978)

CHIRAL ORDER IN HELIMAGNETS

J. Villain

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ABSTRACT

We suggest the possibility of a new type of ordering in magnetic systems. This ordering has some similarity with quadrupolar ordering, since it has no effect on the spin pair correlation function.

Brief history

Journal of the Physical Society of Japan
Vol. 53, No. 3, March, 1984, pp. 1145–1154

Nature of the Phase Transition of the Two-Dimensional Antiferromagnetic Plane Rotator Model on the Triangular Lattice

Seiji MIYASHITA and Hiroyuki SHIBA†

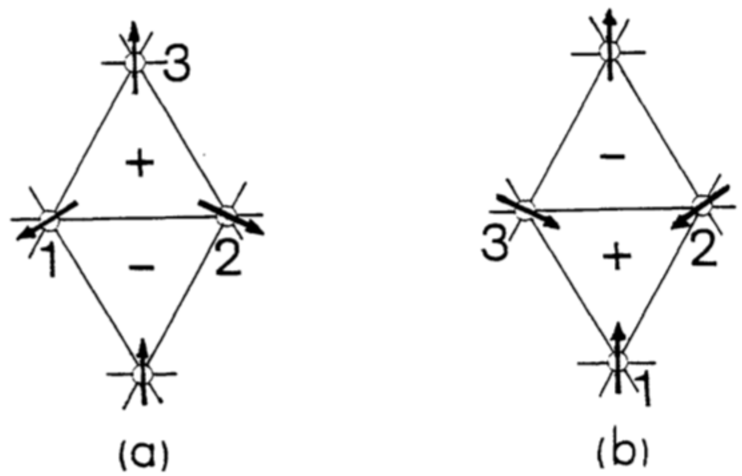


Fig. 1. Two degenerate ground states, +120° and −120° structures, are shown in (a) and (b). + and − denote the sign of chirality of the elementary triangle. In the definition of the chirality in (9) spins are numbered counterclockwise as shown here.

$$T_{\text{Ising}} = 0.513J > T_{\text{KT}} = 0.502J$$

Journal of the Physical Society of Japan **81** (2012) 054003 FULL PAPERS
DOI: 10.1143/JPSJ.81.054003

Spin and Chiral Orderings of the Antiferromagnetic *XY* Model on the Triangular Lattice and Their Critical Properties

Tomoyuki OBUCHI* and Hikaru KAWAMURA

$$T_{\text{Ising}} = 0.5125J > T_{\text{KT}} = 0.5046J$$

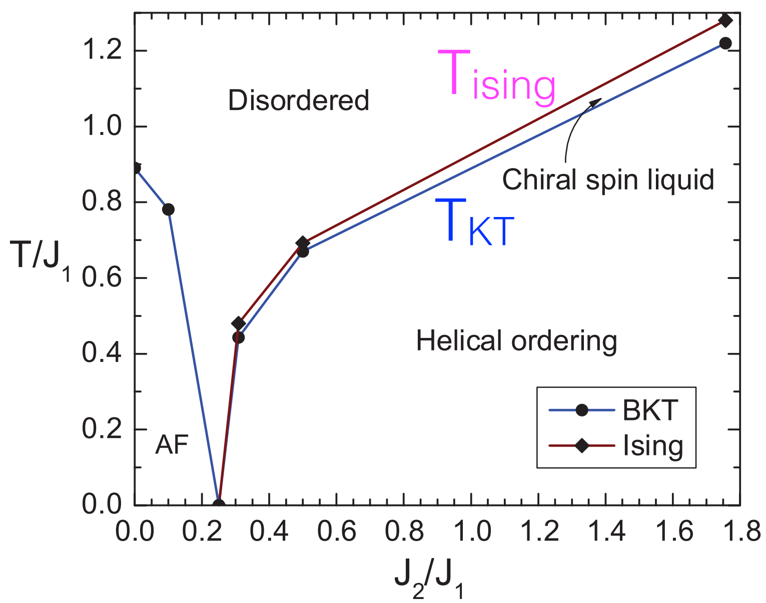
Emergent Ising order parameters (finite T)

PHYSICAL REVIEW B **85**, 174404 (2012)

Chiral spin liquid in two-dimensional XY helimagnets

A. O. Sorokin[1,*] and A. V. Syromyatnikov[1,2,†]

$$H = \sum_{\mathbf{x}} (J_1 \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x+a}}) + J_2 \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x+2a}}) - J_b \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x+b}})),$$



Vector spin chiral phase
is present,
but the temperature interval
is **tiny**.

Can be enhanced by
DM interaction + phonons,
Onoda, Nagaosa PRL 2007

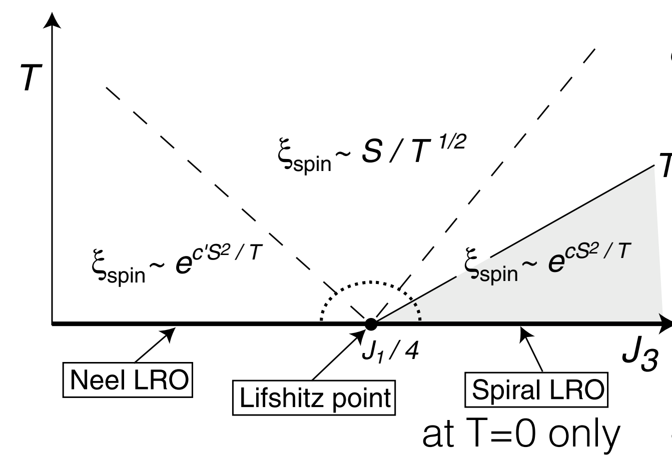
PRL **93,** 257206 (2004) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2004

Low-Temperature Broken-Symmetry Phases of Spiral Antiferromagnets

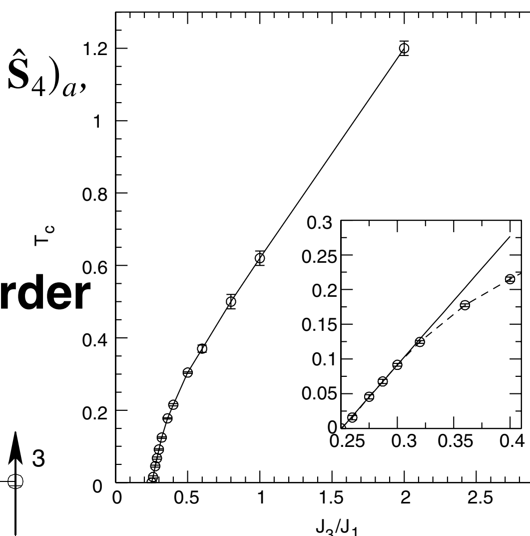
Luca Capriotti[1,2] and Subir Sachdev[2,3]

$$\hat{H} = J_1 \sum_{\langle i,j \rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j + J_3 \sum_{\langle\langle i,j \rangle\rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j,$$

$$\sigma_a = (\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_3 - \hat{\mathbf{S}}_2 \cdot \hat{\mathbf{S}}_4)_a,$$



Ising nematic order



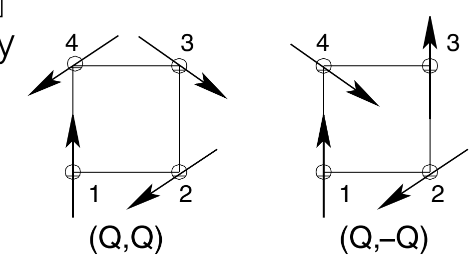


FIG. 2. The two different minimum energy configurations with magnetic wave vectors $\vec{Q} = (Q, Q)$ and $\vec{Q}^\star = (Q, -Q)$ with $Q = 2\pi/3$, corresponding to $J_3/J_1 = 0.5$.

PRL **95,** 137206 (2005) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2005

Two-Step Restoration of SU(2) Symmetry in a Frustrated Ring-Exchange Magnet

A. Läuchli,[1] J. C. Domenge,[2] C. Lhuillier,[2] P. Sindzingre,[2] and M. Troyer[3]
[1]*Institut Romand de Recherche Numérique en Physique des Matériaux (IRRMA), PPH-Ecublens, CH-1015 Lausanne, Switzerland*
[2] *Laboratoire de Physique Théorique des Liquides, Université P. et M. Curie, UMR 7600 of CNRS, case 121, 4 Place Jussieu, 75252 Paris Cedex, France*
[3]*Institut für Theoretische Physik, ETH Hönggerberg, CH-8093 Zürich, Switzerland*
(Received 23 December 2004; published 22 September 2005)

We demonstrate the existence of a spin-nematic, moment-free phase in a quantum four-spin ring-exchange model on the square lattice. This unusual quantum state is created by the interplay of frustration and quantum fluctuations that lead to a partial restoration of SU(2) symmetry when going from a four-sublattice orthogonal biaxial Néel order to this exotic uniaxial magnet. A further increase of frustration drives a transition to a fully gapped SU(2) symmetric valence bond crystal.

Vector chirality in 1d (T=0)

Chiral, nematic, and dimer states in quantum spin chains

Andrey V. Chubukov
Phys. Rev. B **44**, 4693(R) – Published 1 September 1991

VOLUME 81, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JULY 1998

Incommensurate Spin Correlations in Spin-1/2 Frustrated Two-Leg Heisenberg Ladders

Alexander A. Nersesyan,[1] Alexander O. Gogolin,[2] and Fabian H.L. Eßler[3]

PHYSICAL REVIEW B **72**, 094424 (2005)

Field-induced chiral phase in isotropic frustrated spin chains

Alexei Kolezhuk[1,*] and Temo Vekua[2,†]

PHYSICAL REVIEW B **78**, 144404 (2008)

Vector chiral and multipolar orders in the spin-$\frac{1}{2}$ frustrated ferromagnetic chain in magnetic field

Toshiya Hikihara,[1] Lars Kecke,[2,3] Tsutomu Momoi,[2] and Akira Furusaki[2]

PHYSICAL REVIEW B **81**, 224433 (2010)

Magnetic phase diagram of the spin-$\frac{1}{2}$ antiferromagnetic zigzag ladder

Toshiya Hikihara,[1] Tsutomu Momoi,[2] Akira Furusaki,[2] and Hikaru Kawamura[3]

PHYSICAL REVIEW B **86**, 094417 (2012)

Ground-state phase diagram of a spin-$\frac{1}{2}$ frustrated ferromagnetic XXZ chain: Haldane dimer phase and gapped/gapless chiral phases

Shunsuke Furukawa,[1] Masahiro Sato,[2] Shigeki Onoda,[3] and Akira Furusaki[3]

PHYSICAL REVIEW B **89**, 155142 (2014)

Chiral bosonic Mott insulator on the frustrated triangular lattice

Michael P. Zaletel,[1] S. A. Parameswaran,[1,2] Andreas Rüegg,[1,3] and Ehud Altman[1,4]

Today: Search for vector chirality without magnetic order in quantum 2d models

* Spin-current phase

Cheshire Cat's smile

Outline

- ✳ Vector chirality
- ✳ 1/3 magnetization plateau and its instabilities:
- • spin-current phase
- ✳ Minimal s=1 XXZ model of spin-current phase
- ✳ Conclusions

Phase diagram of the Heisenberg (XXX) model in the field

Journal of the Physical Society of Japan
Vol. 53, No. 12, December, 1984, pp. 4138–4154

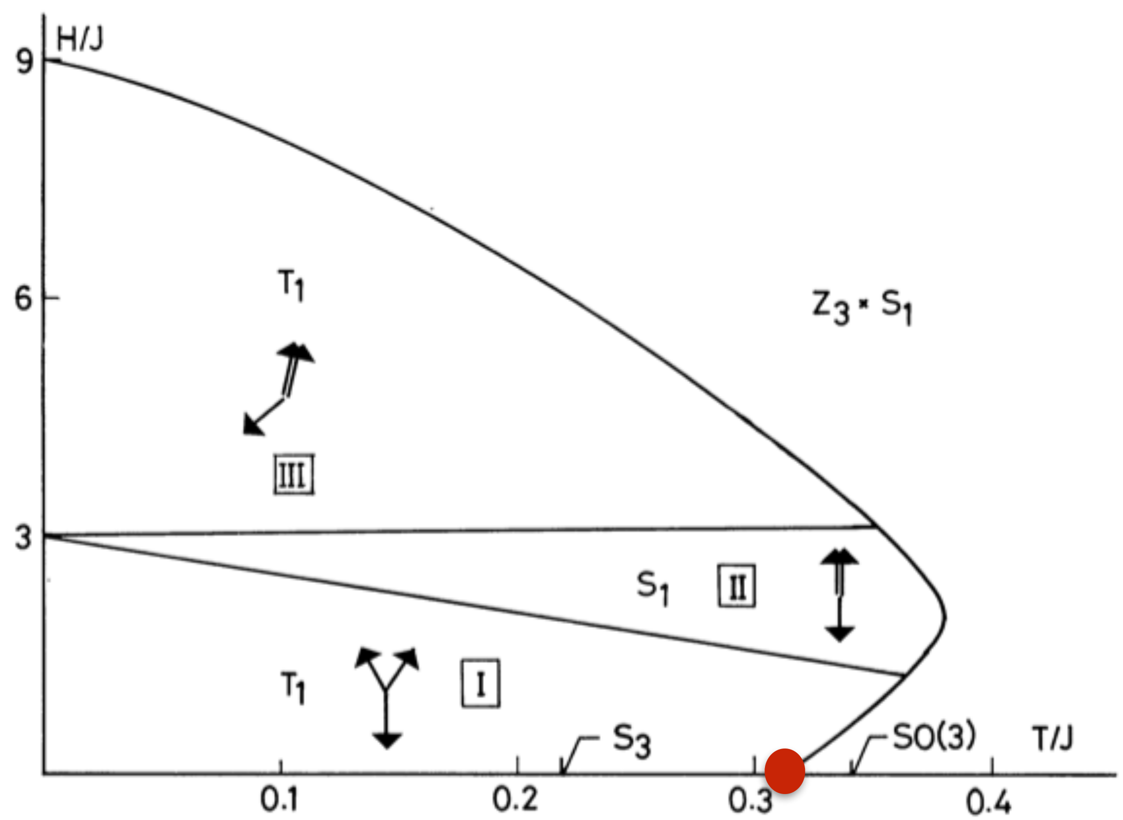
Phase Transition of the Two-Dimensional Heisenberg Antiferromagnet on the Triangular Lattice

Hikaru KAWAMURA and Seiji MIYASHITA†

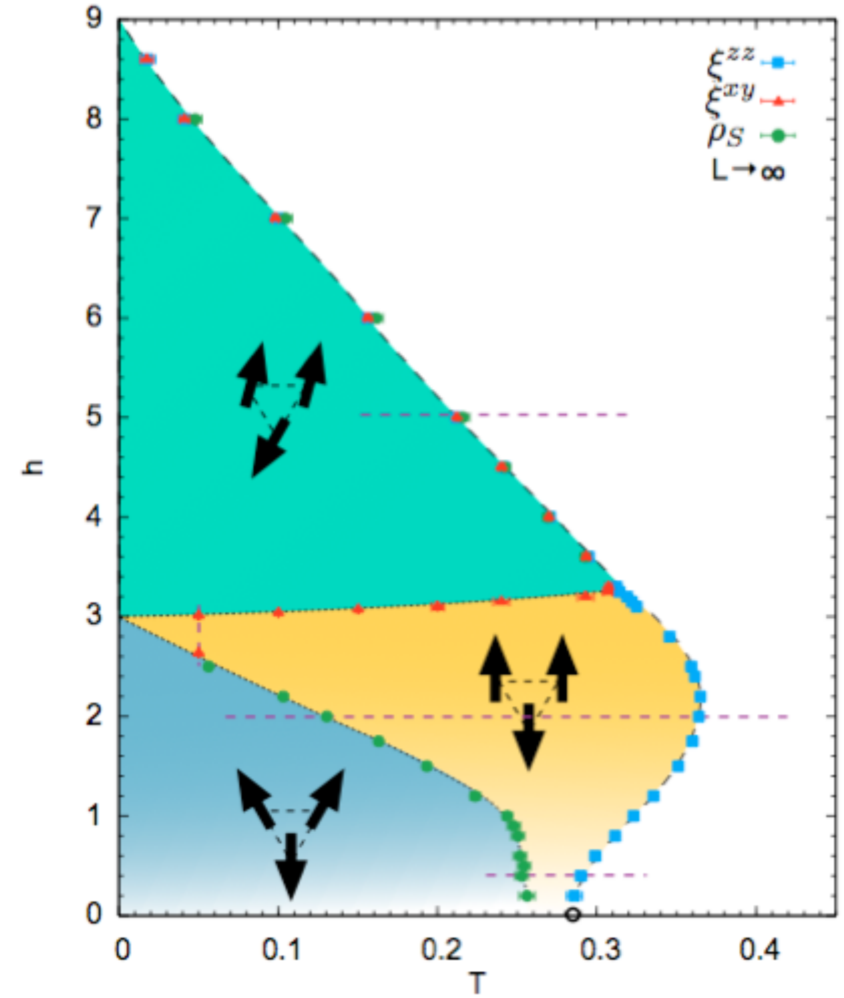
Journal of the Physical Society of Japan
Vol. 54, No. 12, December, 1985, pp. 4530–4538

Phase Transition of the Heisenberg Antiferromagnet on the Triangular Lattice in a Magnetic Field

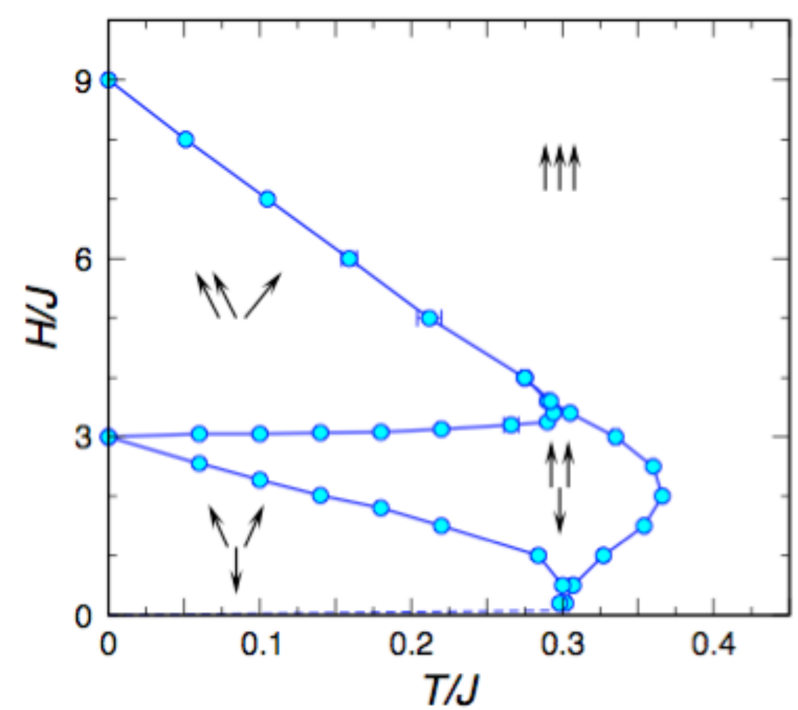
Hikaru KAWAMURA and Seiji MIYASHITA†



Z_2 vortex (chirality ordering) transition



Seabra, Momoi, Sindzingre, Shannon 2011



Gvozdikova, Melchy, Zhitomirsky 2010

Quantum fluctuations, S >> 1, T=0.

J' = J: Quantum fluctuations select co-planar and collinear phases

J. Phys.: Condens. Matter 3 (1991) 69–82. Printed in the UK

UUD plateau is due to interactions between spin waves

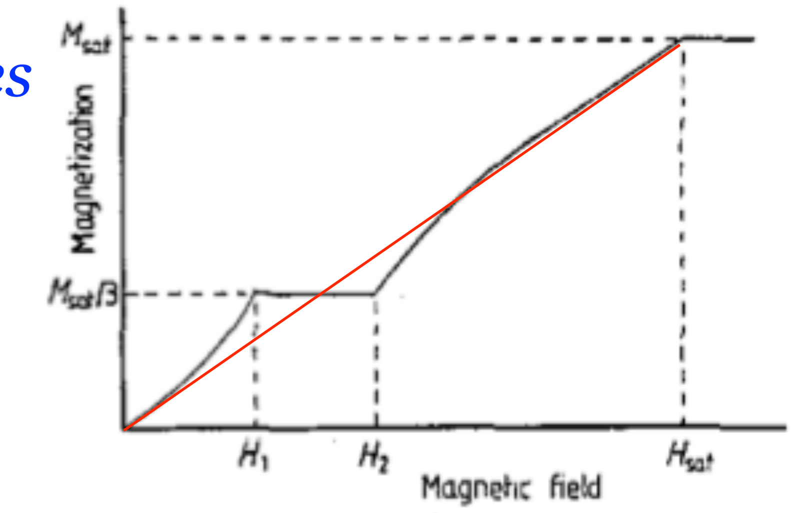


Figure 3. The anticipated behaviour of longitudinal magnetization in 2D Heisenberg AFM on a triangular lattice. The plateau on the magnetization curve results from the stabilization of the collinear phase in the finite region of magnetic fields due to zero-point motion.

Quantum theory of an antiferromagnet on a triangular lattice in a magnetic field

A V Chubukov and D I Golosov
Institute for Physical Problems, USSR Academy of Sciences, 117334 ul. Kosygina 2, Moscow, USSR

Received 9 February 1990

Abstract. The reorientation process in a magnetic field in two-dimensional isotropic and *XY* quantum Heisenberg antiferromagnets is shown to occur through the intermediate phase with unbroken continuous symmetry and constant magnetization equal to one third of the saturation value. The same reorientation process is also found in the more complicated classical models.

$$h_{c2} - h_{c1} = (0.6/2S)\ h_{sat}$$

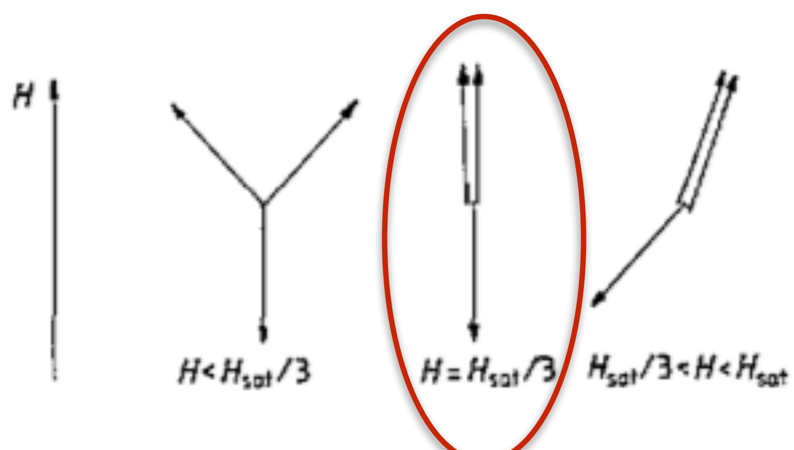


Figure 1. Reorientation process in the magnetic field in 2D Heisenberg AFM on a triangular lattice. Zero-point fluctuations stabilize the collinear phase in the finite region $H_1 < H < H_2$ in the vicinity of $H_{sat}/3$.

up-up-down collinear state

Spatially anisotropic model

Need to understand end-points

COLETTA, ZHITOMIRSKY, AND MILA

PRB 2013

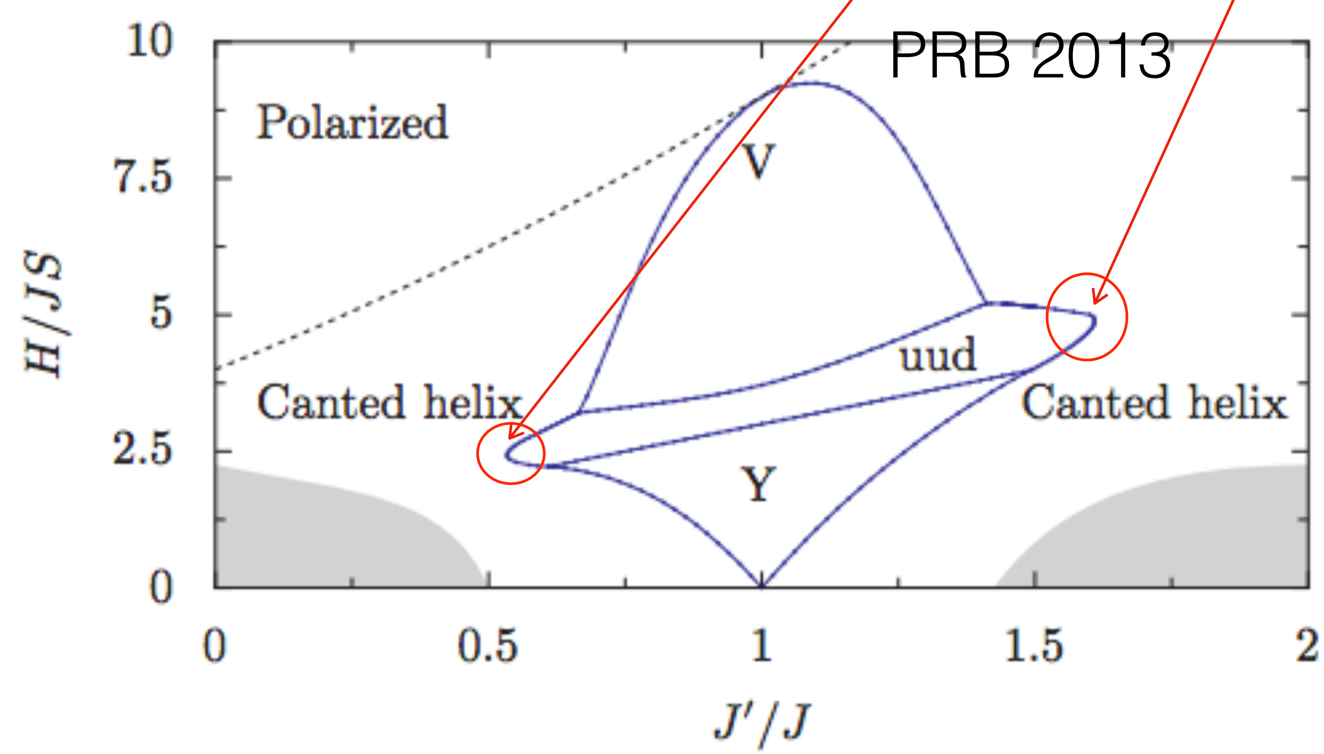


FIG. 4. (Color online) Phase diagram of the spin-1/2 anisotropic triangular lattice in magnetic field. Y and V regions denote three-sublattice planar states. The dashed line is the classical saturation field. The gray shading denotes regions where phases other than the canted helical states may be expected.

PHYSICAL REVIEW B **94**, 075136 (2016)

Semiclassical theory of the magnetization process of the triangular lattice Heisenberg model

Tommaso Coletta,[1] Tamás A. Tóth,[2] Karlo Penc,[3,4] and Frédéric Mila[5]

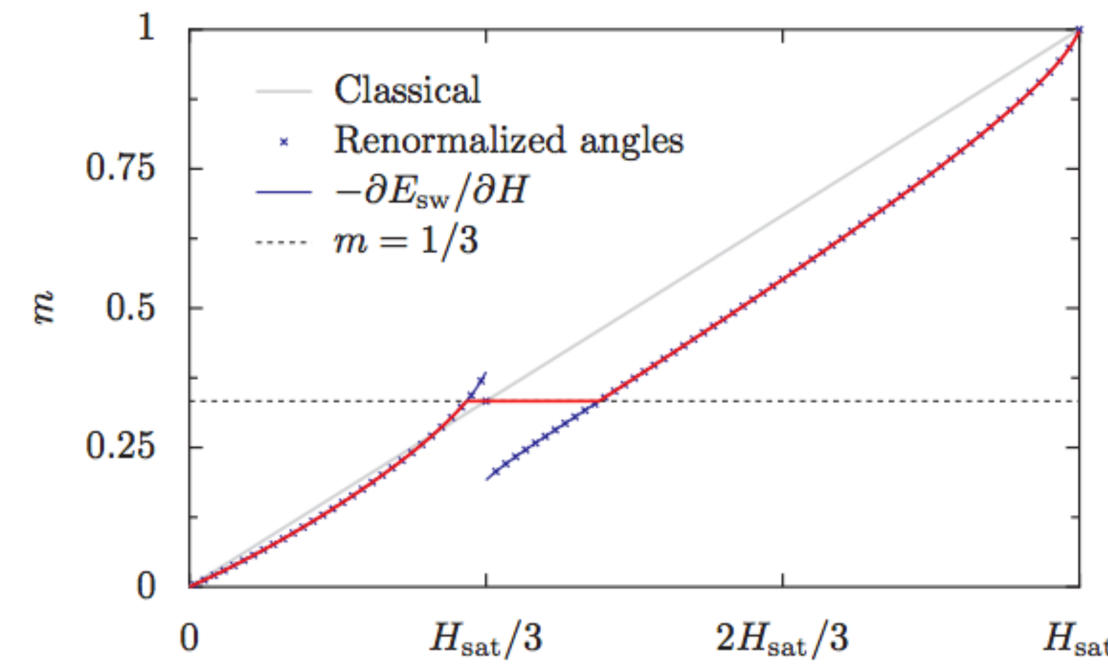
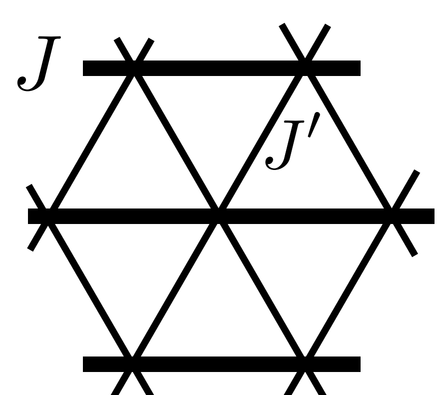
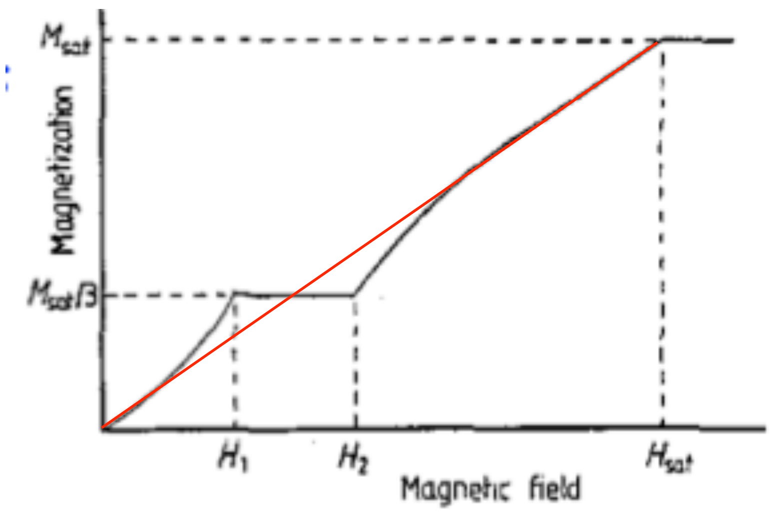


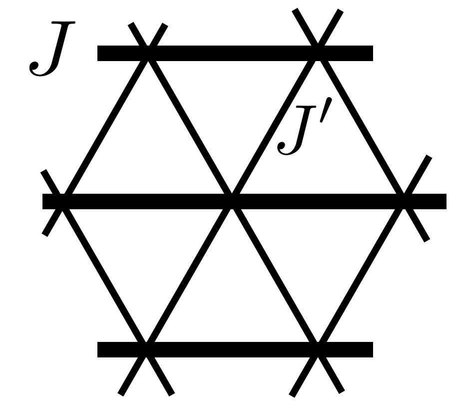
FIG. 3. Plots of the classical magnetization (gray solid line) and of the magnetization including corrections to first order in $1/S$ for $S = 1/2$ (blue curve). The $1/S$ corrections to the magnetization are computed in two equivalent ways: either as the derivative of the energy with respect to the magnetic field (blue curve) or by direct calculation taking into account the renormalization of the spin orientations (crosses). The overall $1/S$ magnetization curve obtained from our phenomenological approach is shown in red.



Spatially anisotropic model: classical vs *quantum*



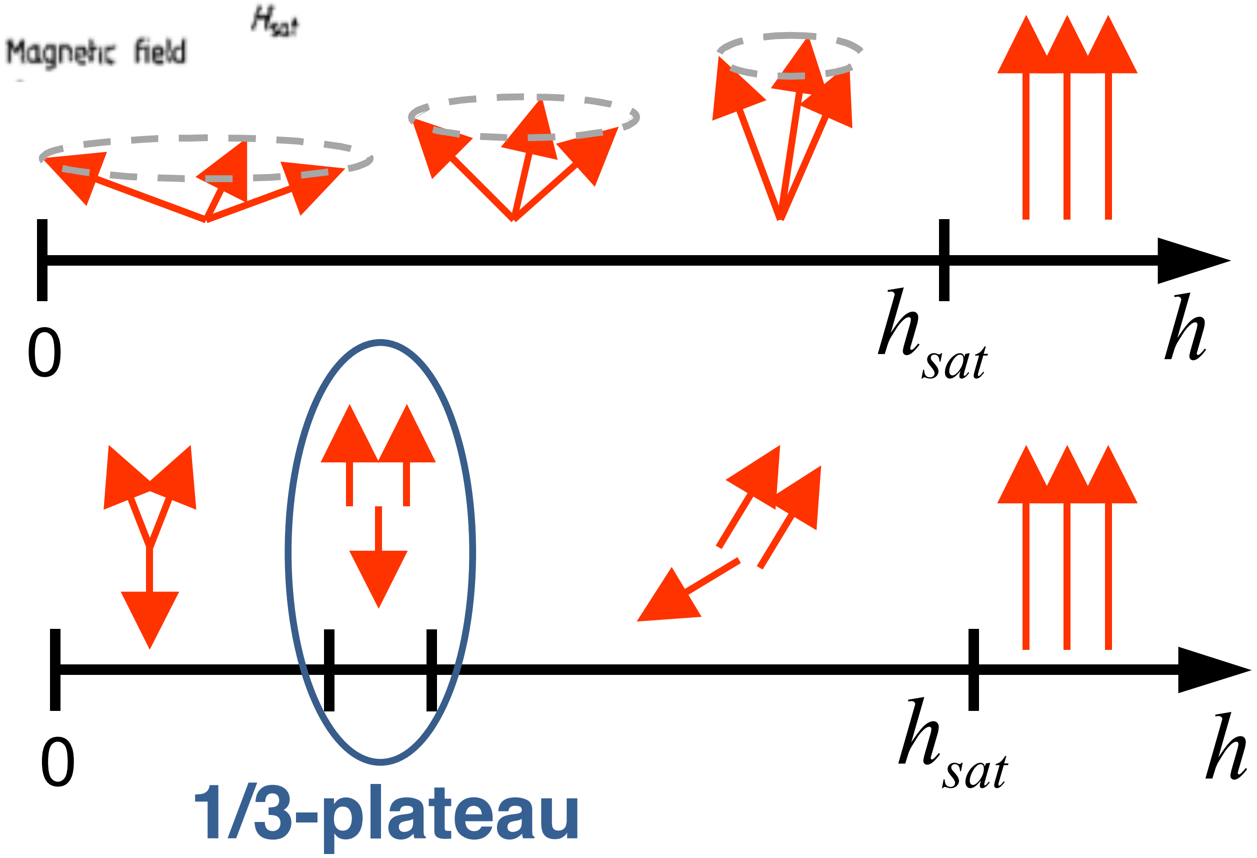
$$H = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - h \sum_i \mathbf{S}_i^z$$



$S = \infty$

$J' \neq J$

$S = \frac{1}{2}$



Umbrella state:
favored classically;
energy gain $(J-J')^2/J$

Planar states: favored by quantum fluctuations;
energy gain J/S

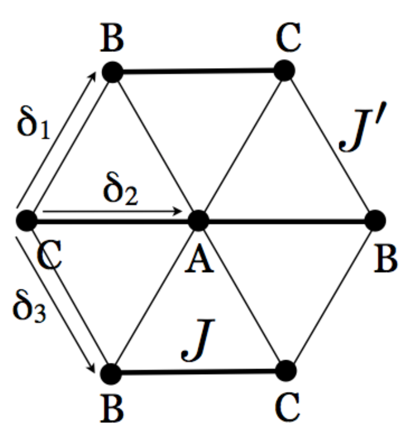
The competition is controlled by dimensionless parameter

$$\delta = S(J - J')^2 / J^2$$

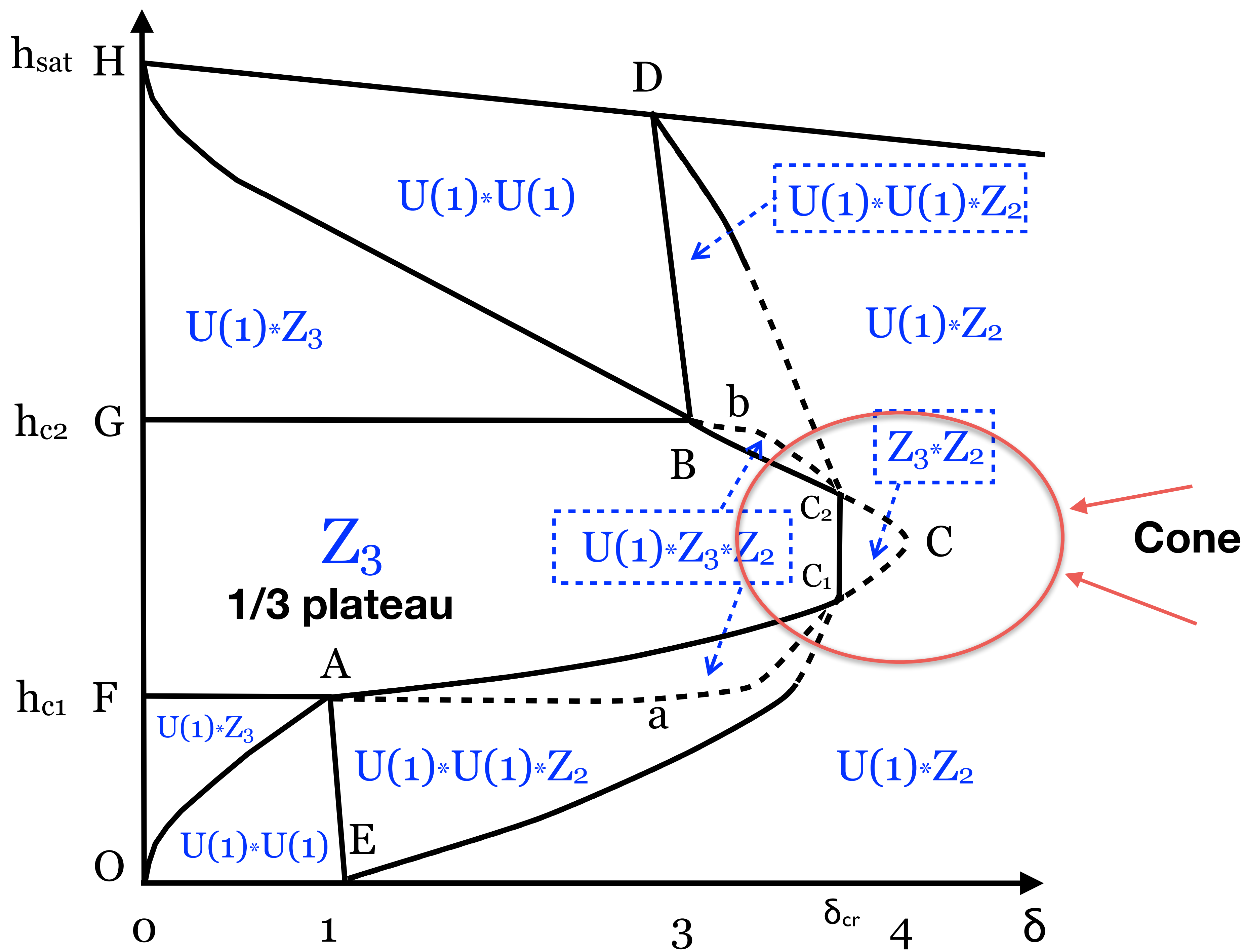
Alicea, Chubukov, OS PRL 2009

Emergent Ising order near the end-point of the 1/3 magnetization plateau

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



$$\delta = \frac{40}{3} S \left(\frac{J - J'}{J} \right)^2$$

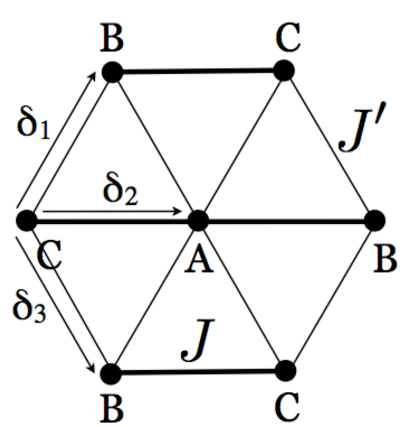


OAS, Reports on Progress in Physics 78, 052502 (2015),
OAS, Wen Jin, Chubukov, Phys. Rev. Lett. 113, 087204 (2014)

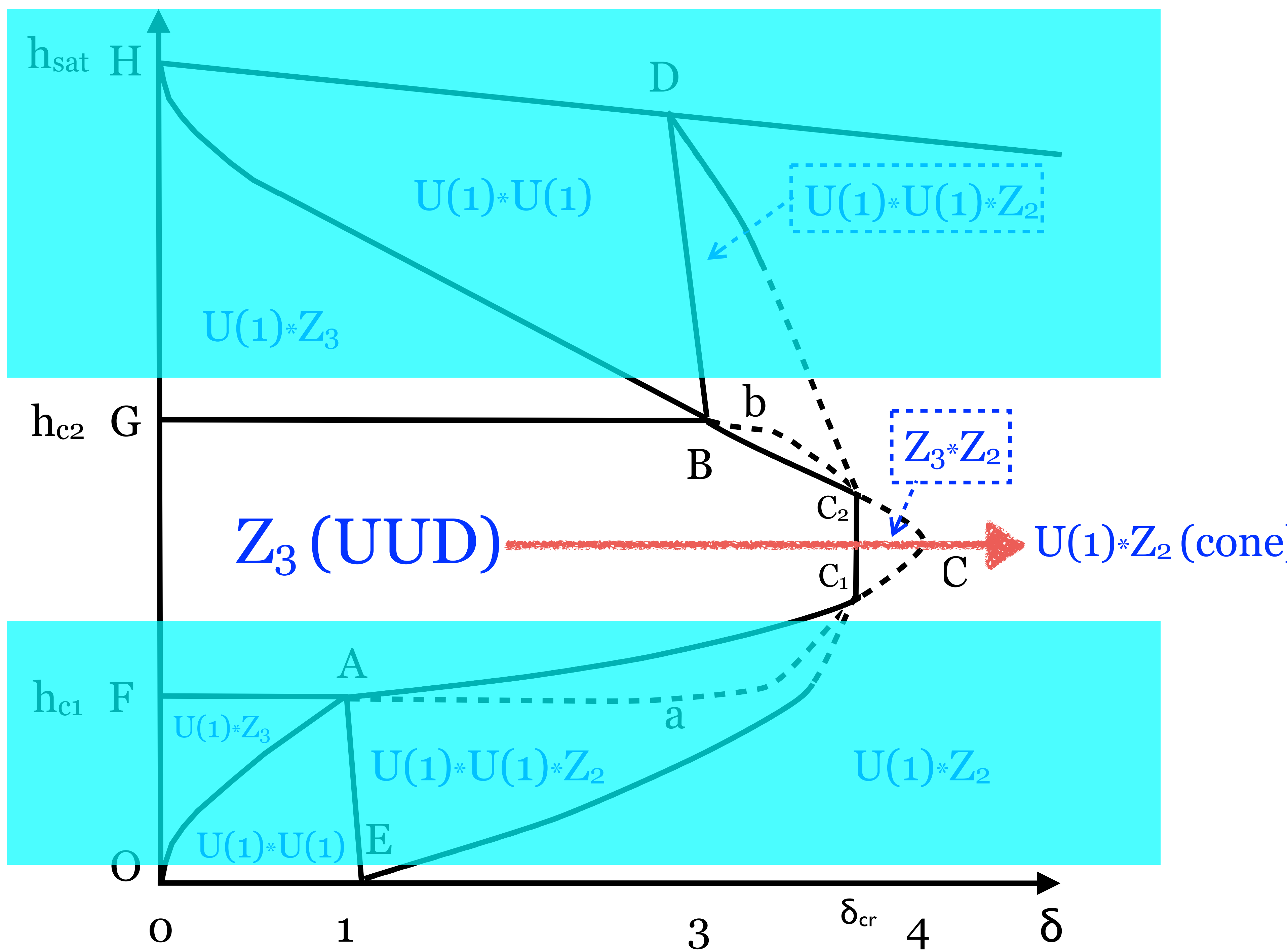
UUD-to-cone phase transition

$$Z_3 \to U(1) \times Z_2 \text{ or } Z_3 \to \text{smth else} \to U(1) \times Z_2?$$

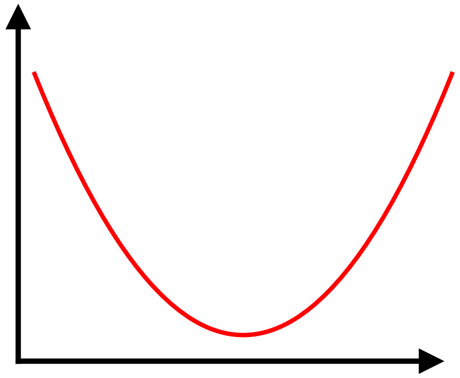
$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



$$\delta = \frac{40}{3} S \left(\frac{J - J'}{J} \right)^2$$



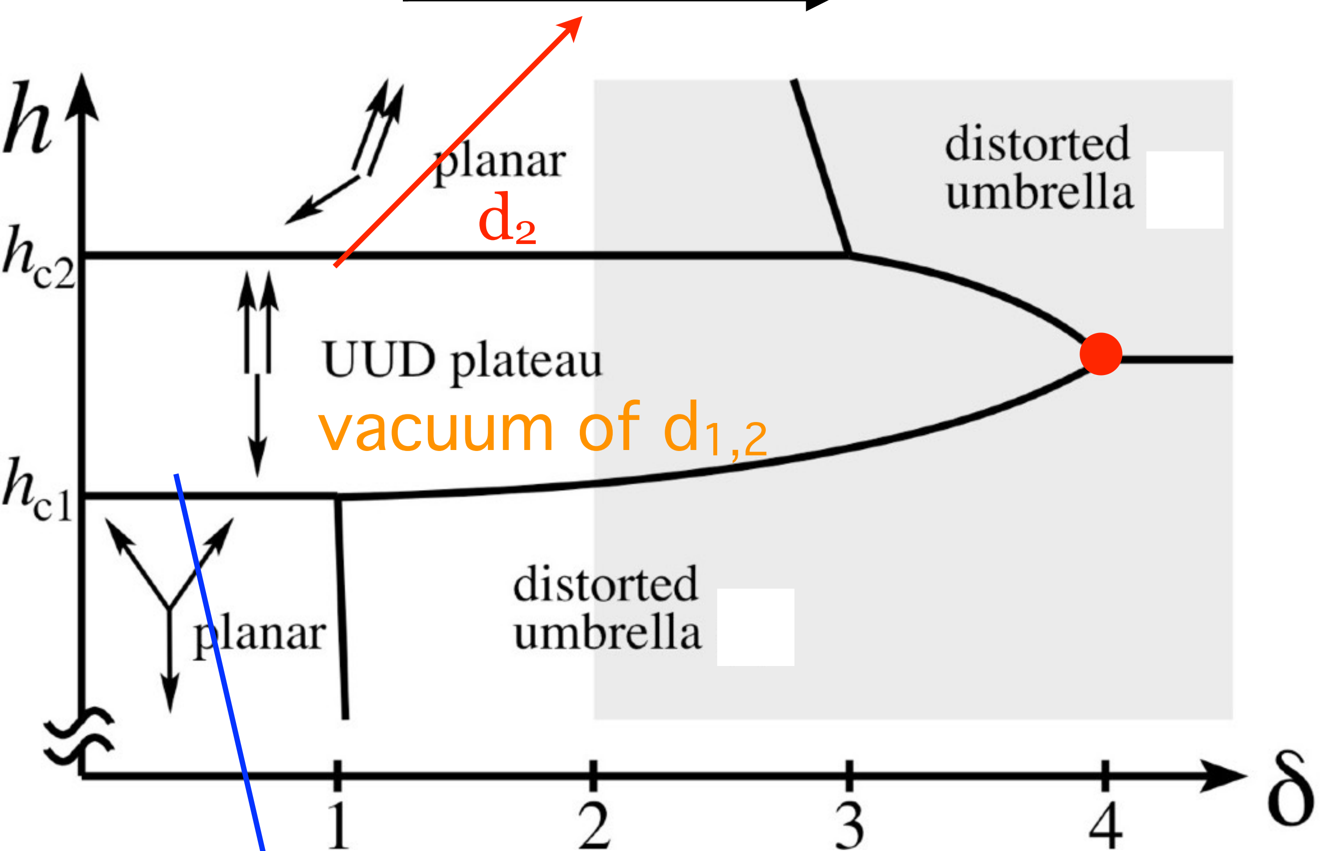
Low-energy excitation spectra



$$\epsilon_{d2} = h_{c2} - h + \frac{9Jk^2}{4}$$

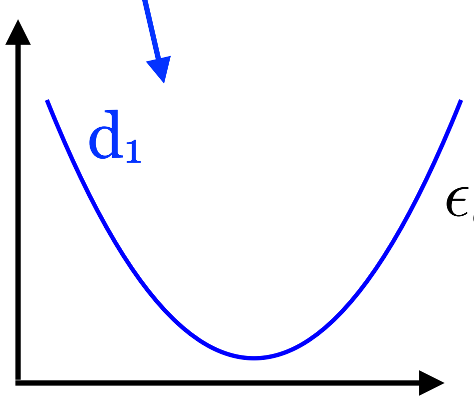
for δ < 3

Magnetization plateau is **collinear** phase: preserves O(2) rotations about magnetic field -- no gapless spin waves. Breaks only discrete Z_3. Hence, ***very stable***.



$$\delta = \frac{40\ S}{3}(1 - J'/J)^2$$

$$h_{c2} - h_{c1} = \frac{0.6}{2S} h_{\text{sat}} = \frac{0.6}{2S}(9JS)$$



$$\epsilon_{d1} = h - h_{c1} + \frac{3Jk^2}{4}$$

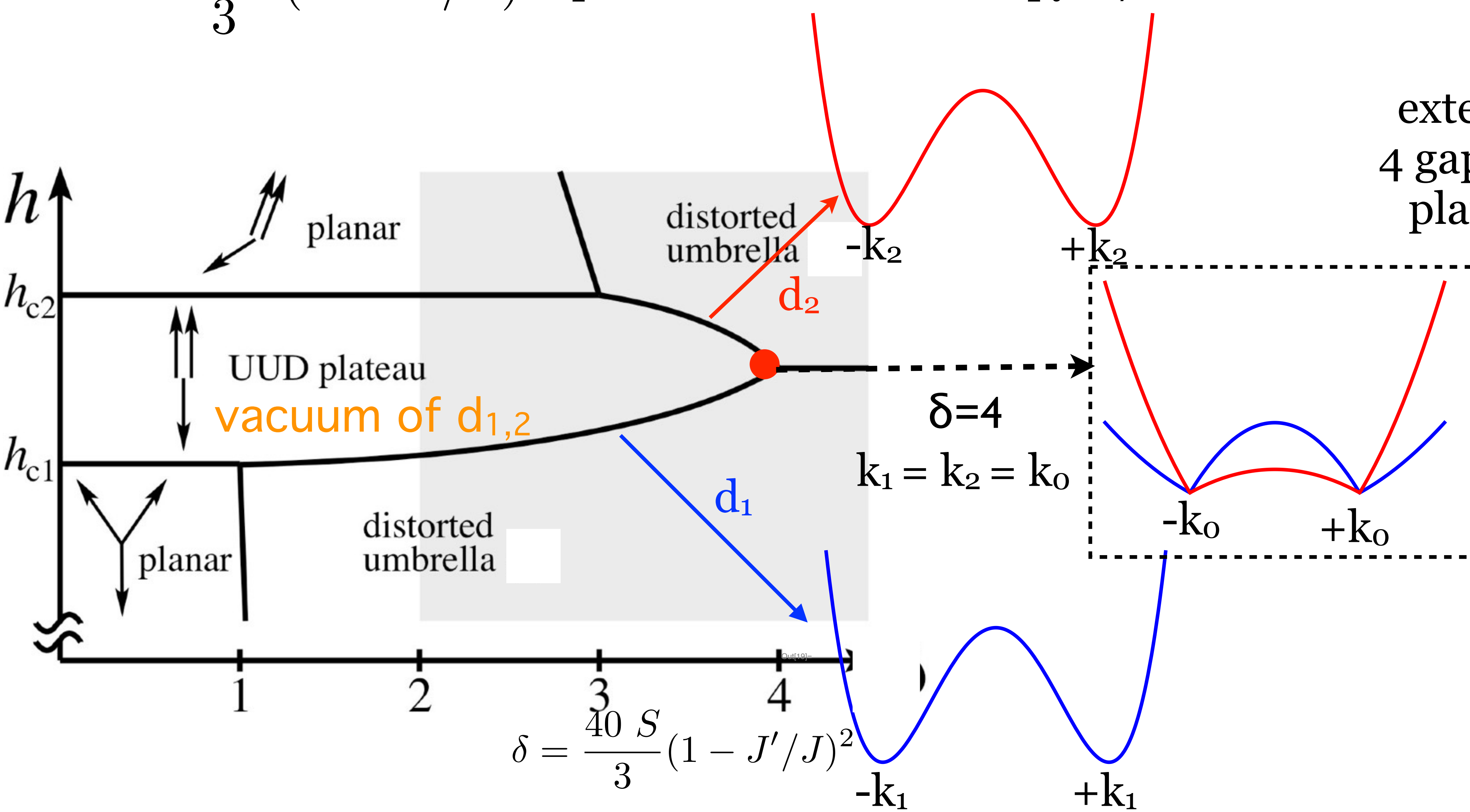
for δ < 1

Bose-Einstein condensation of d_1 (d_2) mode at k =0 leads to lower (upper) co-planar phase

Alicea, Chubukov, OS PRL 2009

Low-energy excitation spectra near the plateau’s end-point

$$\delta = \frac{40\ S}{3}(1 - J'/J)^2$$ parameterizes anisotropy J’/J



extended symmetry:
4 gapless modes at the
plateau’s end-point

$$k_0 = \sqrt{\frac{3}{10S}}$$

S>>1

Magnetization plateau is **collinear** phase: preserves O(2) rotations about magnetic field -- no gapless spin waves. Breaks only discrete Z_3.

Alicea, Chubukov, OS PRL 2009

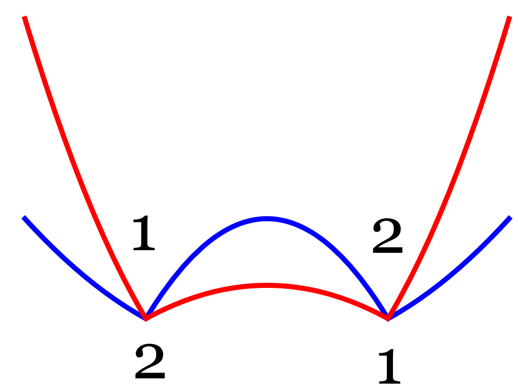
Interaction between low-energy magnons

$$\mathcal{H}^{(4)}_{d_1 d_2} = \frac{3}{N}\sum_{p,q}\Phi(p,q)\Big(d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d^\dagger_{2,-\mathbf{k}_0-\mathbf{p}}d_{1,-\mathbf{k}_0+\mathbf{q}}d_{2,\mathbf{k}_0-\mathbf{q}} - d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d^\dagger_{2,-\mathbf{k}_0-\mathbf{p}}d^\dagger_{1,-\mathbf{k}_0+\mathbf{q}}d^\dagger_{2,\mathbf{k}_0-\mathbf{q}}\Big) + \text{h.c.}$$

$$\Phi(p,q) \sim \frac{(-3J)k_0^2}{|\mathbf{p}||\mathbf{q}|}$$

singular magnon interaction

$$\Psi^\dagger_{1,p}\Psi_{2,q} \qquad \Psi^\dagger_{1,p}\Psi^\dagger_{2,q}$$



magnon **pair operators**

$$\begin{cases}\Psi_{1,p} = d_{1,k_0+p}d_{2,-k_0-p}\\ \Psi_{2,p} = d_{1,-k_0+p}d_{2,k_0-p}\end{cases}$$

Obey canonical Bose commutation relations in the UUD ground state

$$[\Psi_{1,\mathbf{p}},\Psi_{2,\mathbf{q}}] = \delta_{1,2}\delta_{\mathbf{p},\mathbf{q}}\Big(1 + d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d_{1,\mathbf{k}_0+\mathbf{p}} + d^\dagger_{2,\mathbf{k}_0+\mathbf{p}}d_{2,\mathbf{k}_0+\mathbf{p}}\Big) \to \delta_{1,2}\delta_{\mathbf{p},\mathbf{q}}$$

In the UUD ground state $\langle d_1^\dagger d_1\rangle_{\text{uud}} = \langle d_2^\dagger d_2\rangle_{\text{uud}} = 0$

★ Interacting magnon Hamiltonian in terms of $\mathbf{d}_{1,2}$ bosons = non-interacting Hamiltonian in terms of $\Psi_{1,2}$ magnon pairs

Chubukov, OS PRL 2013

Two-magnon instability

Magnon pairs $\Psi_{1,2}$ condense *before* single magnons $d_{1,2}$

Equations of motion for Ψ - Hamiltonian

$$\langle \Psi^\dagger_{1,\mathbf{p}} - \Psi_{1,\mathbf{p}} \rangle = \frac{6Jf_p^2}{\Omega_p} \frac{3}{N} \sum_q f_q^2 \langle \Psi^\dagger_{2,\mathbf{q}} - \Psi_{2,\mathbf{q}} \rangle$$

$$\langle \Psi^\dagger_{2,\mathbf{p}} - \Psi_{2,\mathbf{p}} \rangle = \frac{6Jf_p^2}{\Omega_p} \frac{3}{N} \sum_q f_q^2 \langle \Psi^\dagger_{1,\mathbf{q}} - \Psi_{1,\mathbf{q}} \rangle$$

\`Superconducting' solution with *imaginary* order parameter

$$\langle \Psi_{1,p} \rangle = \langle \Psi_{2,p} \rangle \sim i \frac{\Upsilon}{\mathbf{p}^2}$$

Instability = softening of two-magnon mode @ $\delta_{cr} = 4 - O(1/S^2)$

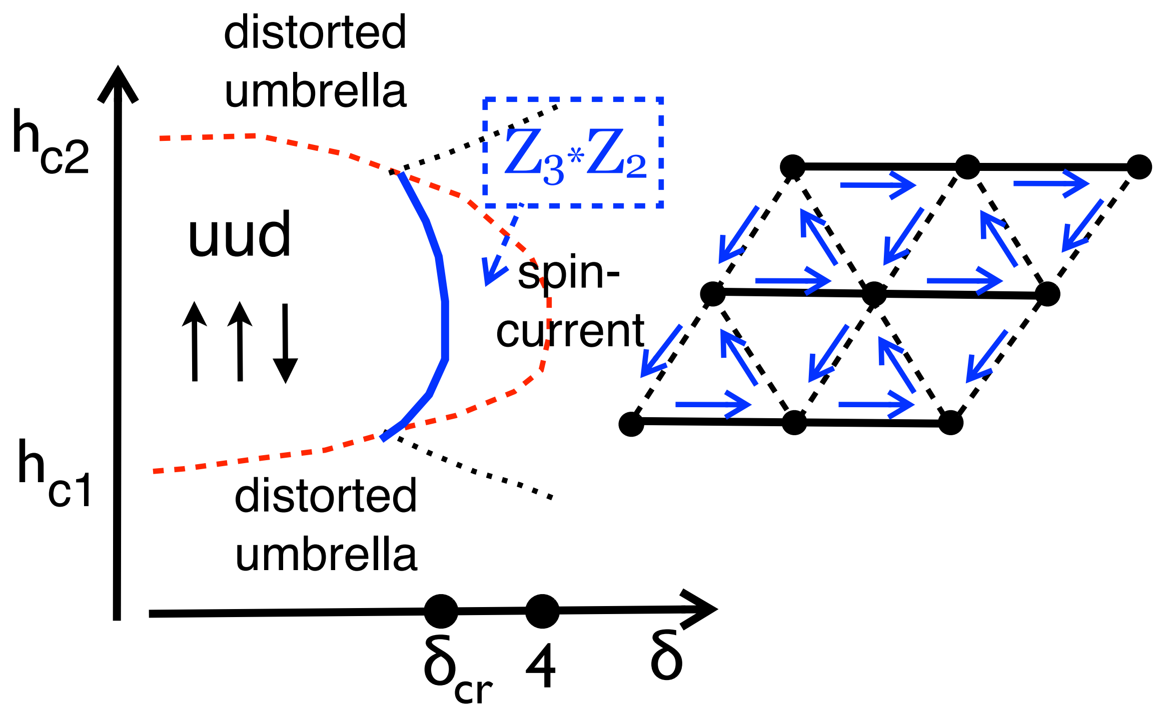
$$1 = \frac{1}{S} \frac{1}{N} \sum_p \frac{k_0}{\sqrt{|\mathbf{p}|^2 + (1 - \delta/4) k_0^2}}$$

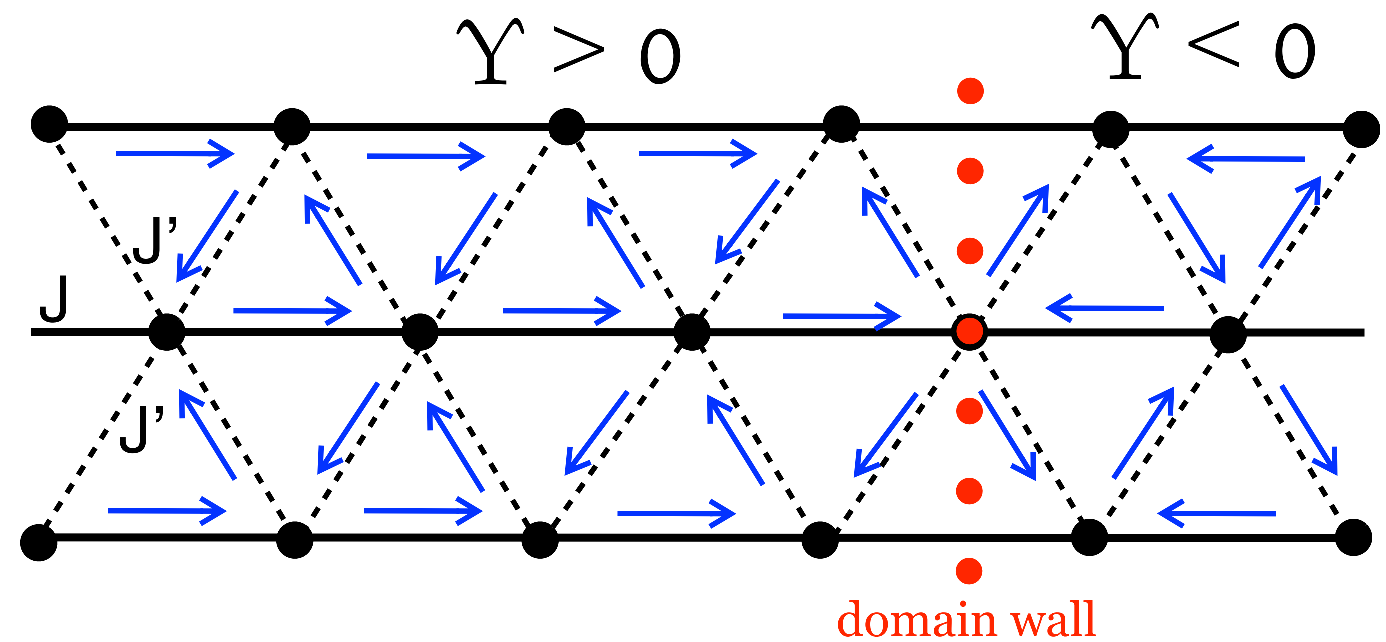
no single particle condensate

$$\langle d_1 \rangle = \langle d_2 \rangle = 0$$

Chubukov, OS PRL 2013

Spin-current nematic state near the end-point of the 1/3 magnetization plateau (large-S analysis)





no transverse magnetic order $\langle \mathbf{S}_r^{x,y} \rangle = 0$ $\langle \mathbf{S}_r \cdot \mathbf{S}_{r'} \rangle$ is not affected

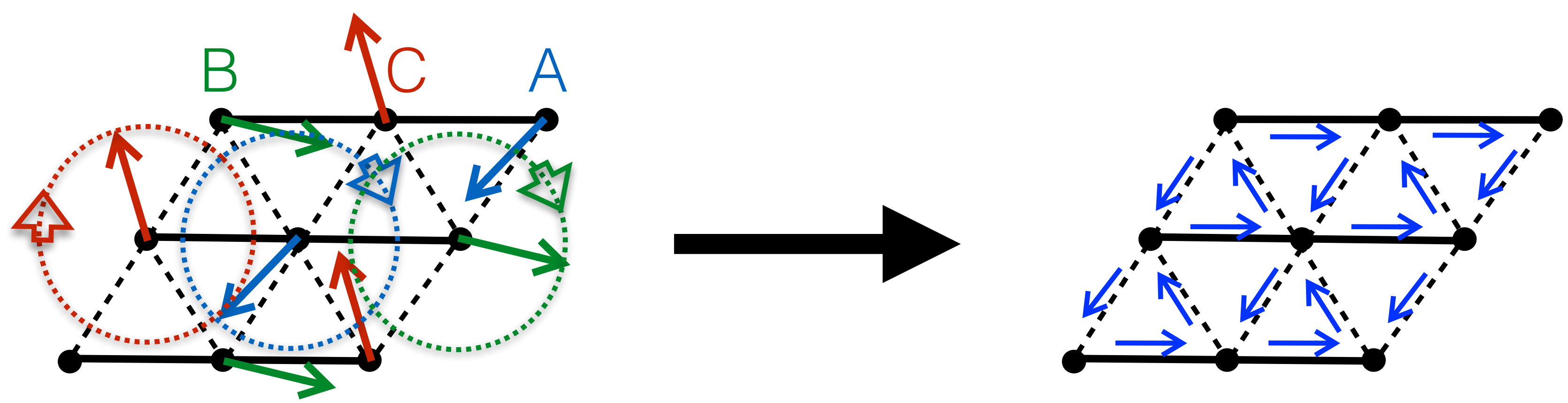
Finite **vector chirality**

$$\langle \hat{z} \cdot \mathbf{S}_A \times \mathbf{S}_C \rangle = \langle \hat{z} \cdot \mathbf{S}_C \times \mathbf{S}_B \rangle = \langle \hat{z} \cdot \mathbf{S}_B \times \mathbf{S}_A \rangle \propto \Upsilon$$

Spontaneously broken Z_2 -- spatial inversion [in addition to broken Z_3 inherited from the UUD state]

Chubukov, OS PRL 2013

Spin current visualization



Precessing spins on sub lattices A, B, C are phase shifted by 2π/3:

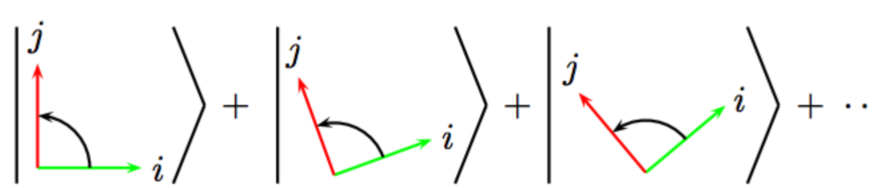
$$\mathbf{S}_A = (\cos[\omega t], \sin[\omega t], M_A), \mathbf{S}_B = (\cos[\omega t \pm \frac{4\pi}{3}], \sin[\omega t \pm \frac{4\pi}{3}], M_B), \mathbf{S}_C = (\cos[\omega t \pm \frac{2\pi}{3}], \sin[\omega t \pm \frac{2\pi}{3}], M_C)$$

Then no dipolar transverse order:

$$\langle \mathbf{S}^{x,y}_{\mathbf{r}} \rangle = 0 \text{ and } \langle \mathbf{S}_A \cdot \mathbf{S}_C \rangle = \langle \mathbf{S}_C \cdot \mathbf{S}_B \rangle = \langle \mathbf{S}_B \cdot \mathbf{S}_A \rangle = \cos[\frac{2\pi}{3}]$$

But finite **chirality**, determined by the sign of 2π/3 shift between the sublattices:

$$\langle \mathbf{S}_A \times \mathbf{S}_C \rangle = \langle \mathbf{S}_C \times \mathbf{S}_B \rangle = \langle \mathbf{S}_B \times \mathbf{S}_A \rangle = \pm \sin[\frac{2\pi}{3}]$$



End-point of the plateau on kagome lattice

Semiclassical analysis of a magnetization plateau in a 2D frustrated ferrimagnet

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(Dated: November 9, 2016)

PRB 2017

Kagome geometry

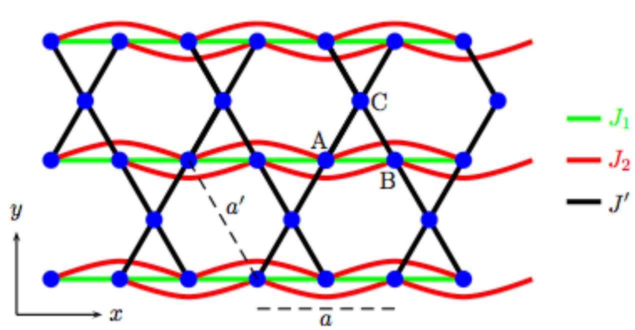


FIG. 1. Proposed Hamiltonian for volborthite. The blue dots represent spin-1/2 copper ions and the line segments represent Heisenberg couplings. $J_1 < 0$ is ferromagnetic while $J_2 > 0$ and $J' > 0$ are antiferromagnetic. The distances between adjacent unit cells is slightly anisotropic, with $a = 5.84$ Å and $a' = 6.07$ Å [10]. Capital letters label the sublattices.

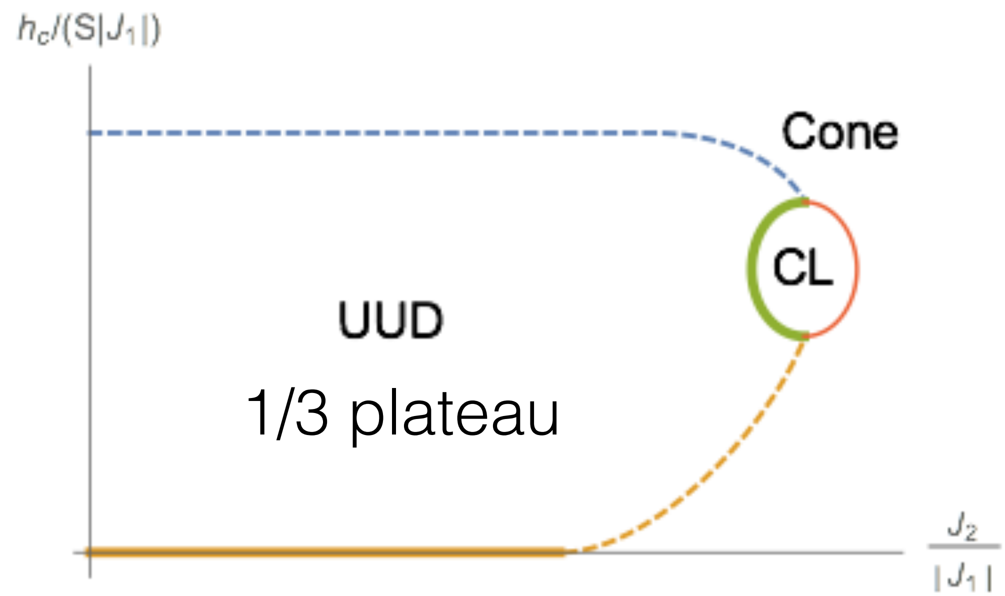


FIG. 6. Schematic quantum phase diagram at $J' = 0.5|J_1|$. The UUD state breaks no symmetries, the gapped chiral liquid (CL) phase only breaks chiral symmetry, and the gapless cone state breaks both chiral and a $U(1)$ symmetry combining translation and spin rotation. The thick solid lines and dashed lines represent first- and second-order transitions respectively. We did not investigate the nature of the transition between the chiral liquid and cone phases represented by the red line. In a 3D phase diagram like that of Fig. 4 that includes the applied field, the chiral liquid phase would appear as a thin tube around the stabilization curve where the two sheets meet.

Spin-current pattern

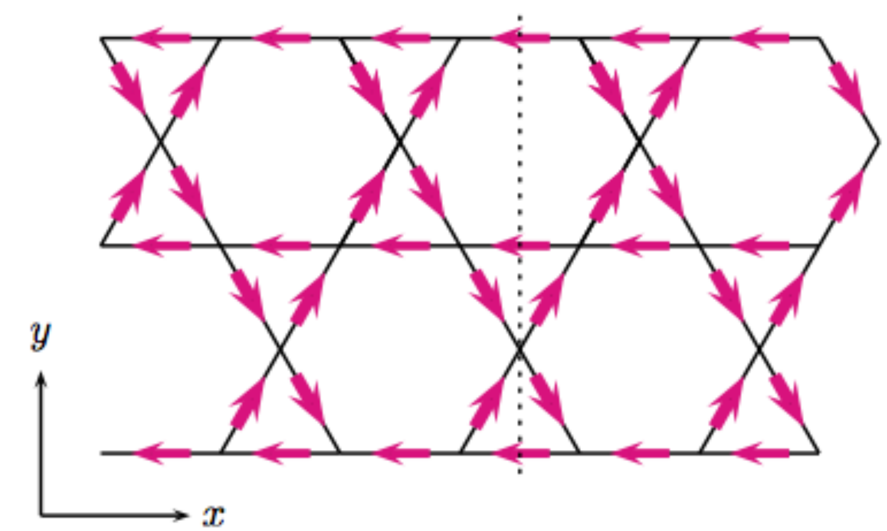


FIG. 7. Spin-current configuration in the chiral liquid phase. The magenta arrows indicating the direction of spin current flow; in the orthogonal ground state the flow is reversed. The spin current on the diagonal bonds, represented by thicker arrows, is larger by a factor of $\sqrt{2}$ and determines the net current flow. The ground state is chiral and spontaneously breaks the lattice symmetry of reflection about the dotted line.

Outline

✳ Vector chirality

✳ 1/3 magnetization plateau and its instabilities:
• spin-current phase

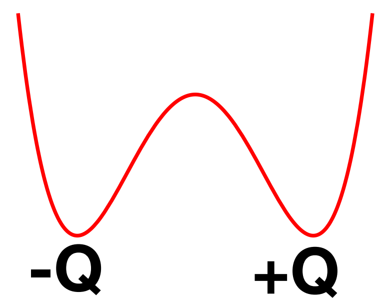
✳ Minimal s=1 XXZ model of spin-current phase

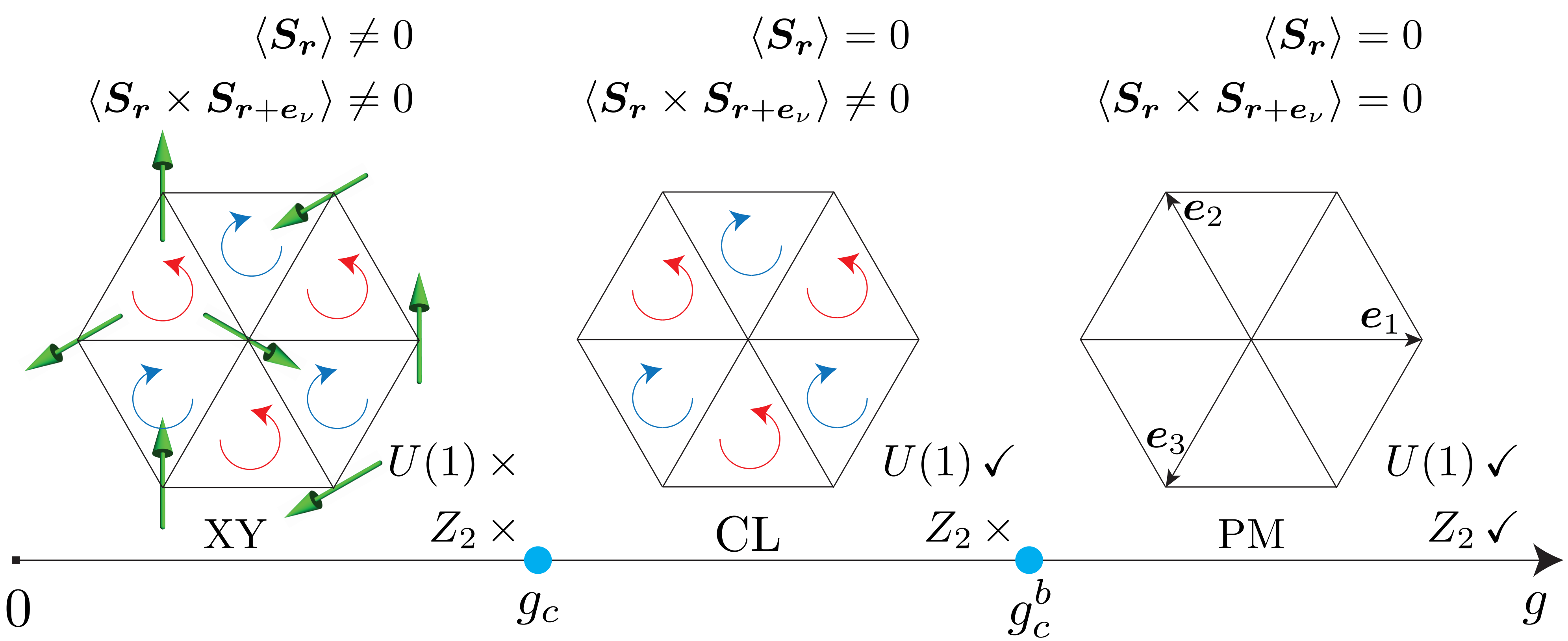
✳ Conclusions

The minimal 2d quantum spin model

- Spin-1 model with featureless Mott ground state at large D > 0 $[S_r^z = 0]$
- Triangular lattice: two-fold degenerate spectrum, at +**Q** and -**Q**

$$H = \sum_{\langle r, r' \rangle} J(S_r^x S_{r'}^x + S_r^y S_{r'}^y + \Delta S_r^z S_{r'}^z) + D \sum_r (S_r^z)^2$$





The minimal 2d quantum spin model

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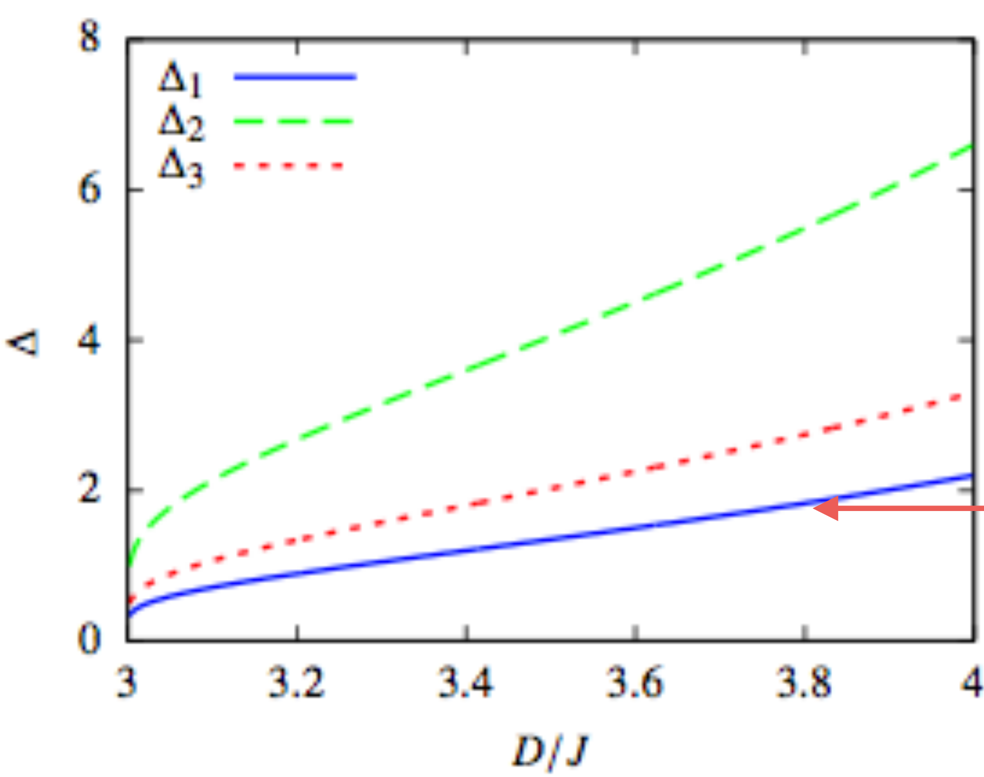
1. Toy problem of two-spin exciton. Derive Schrodinger eqn for the pair wave function ψ

$$|\mathrm{ex}\rangle = \sum_{n \neq m} \psi_{n,m} |n, m\rangle \text{ where } |n, m\rangle = \frac{1}{2} S_n^+ S_m^- \Pi_j |0\rangle_j \longrightarrow B_{\mathbf{g}} = \Delta J \sum_{\mathbf{g}'} M_{\mathbf{g}\mathbf{g}'} B_{\mathbf{g}'}$$

\+ charge, − charge

where

$$M_{\mathbf{g}\mathbf{g}'} = \frac{1}{N} \sum_{\mathbf{q}} \frac{e^{i\mathbf{q}\cdot(\mathbf{g}'-\mathbf{g})}}{4J \sum_{j=1}^{3} \cos[\frac{K_j}{2}] \cos[q_j] + 2D - E}$$



Solution which is **odd** under inversion
is the first instability when approaching from large-D limit.
Indicates chiral Mott phase.
[Single-particle condensation occurs at D=3J.]

Schwinger boson representation of S=1

$$S^z_{\mathbf{r}} = \mathbf{b}^\dagger_{\mathbf{r}} \mathcal{S}^z \mathbf{b}_{\mathbf{r}} = b^\dagger_{\mathbf{r}\uparrow} b_{\mathbf{r}\uparrow} - b^\dagger_{\mathbf{r}\downarrow} b_{\mathbf{r}\downarrow},$$
$$S^+_{\mathbf{r}} = \mathbf{b}^\dagger_{\mathbf{r}} \mathcal{S}^+ \mathbf{b}_{\mathbf{r}} = \sqrt{2}(b^\dagger_{\mathbf{r}\uparrow} b_{\mathbf{r}0} + b^\dagger_{\mathbf{r}0} b_{\mathbf{r}\downarrow}),$$
$$S^-_{\mathbf{r}} = \mathbf{b}^\dagger_{\mathbf{r}} \mathcal{S}^- \mathbf{b}_{\mathbf{r}} = \sqrt{2}(b^\dagger_{\mathbf{r}\downarrow} b_{\mathbf{r}0} + b^\dagger_{\mathbf{r}0} b_{\mathbf{r}\uparrow}).$$

Large-**D** limit: b_0 is condensed, $\langle b_{\mathbf{r}0} \rangle = s$, $b_{\mathbf{r}\uparrow,\downarrow}$ are excitations about the vacuum.

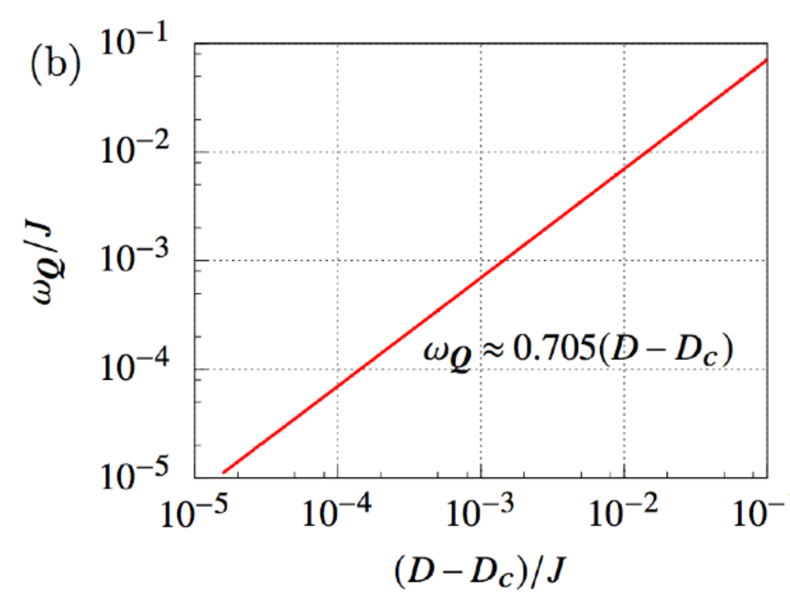
$$\bar{\mathcal{H}}_{sw} = \sum_{\boldsymbol{k},\sigma} (\mu + s^2 \epsilon_{\boldsymbol{k}}) b^\dagger_{\boldsymbol{k}\sigma} b_{\boldsymbol{k}\sigma} + N(\mu - D)(s^2 - 1)$$
$$+ \sum_{\boldsymbol{k},\sigma} \frac{s^2 \epsilon_{\boldsymbol{k}}}{2} (b^\dagger_{\boldsymbol{k}\sigma} b^\dagger_{-\boldsymbol{k}\bar{\sigma}} + h.c.),$$

$$b_{\boldsymbol{k}\sigma} = u_{\boldsymbol{k}} \gamma_{\boldsymbol{k}\sigma} + v_{\boldsymbol{k}} \gamma^\dagger_{-\boldsymbol{k}\bar{\sigma}},$$

$$u_{\boldsymbol{k}} = (\mu + \omega_{\boldsymbol{k}}) / \left(2\sqrt{\mu \omega_{\boldsymbol{k}}}\right),$$
$$v_{\boldsymbol{k}} = (\mu - \omega_{\boldsymbol{k}}) / \left(2\sqrt{\mu \omega_{\boldsymbol{k}}}\right),$$
$$\omega_{\boldsymbol{k}} = \sqrt{\mu^2 + 2\mu s^2 \epsilon_{\boldsymbol{k}}}.$$

This accounts for quantum fluctuations

$$\bar{\mathcal{H}}_{sw} = N(\mu - D)(s^2 - 1) + \sum_{\boldsymbol{k}\sigma} \left[\omega_{\boldsymbol{k}} (\gamma^\dagger_{\boldsymbol{k}\sigma} \gamma_{\boldsymbol{k}\sigma} + \frac{1}{2}) - \frac{\mu}{2} \right]$$



Magnon interaction comes from Ising part of the exchange

$$\mathcal{H}^{(4)}_{\mathrm{I}} = \zeta J \sum_{\boldsymbol{r},\nu} (n_{\boldsymbol{r}\uparrow} - n_{\boldsymbol{r}\downarrow})(n_{\boldsymbol{r}+\boldsymbol{e}_\nu\uparrow} - n_{\boldsymbol{r}+\boldsymbol{e}_\nu\downarrow})$$

$$\zeta = J_z / J$$

Interaction between magnons

$$\mathcal{H}_{\mathrm{I}}^{(4)} = \frac{1}{N}\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{q}} V_{\boldsymbol{q}}^{22o}(\boldsymbol{k}_1,\boldsymbol{k}_2)\gamma^{\dagger}_{\boldsymbol{k}_1+\boldsymbol{q}\uparrow}\gamma^{\dagger}_{\boldsymbol{k}_2-\boldsymbol{q}\downarrow}\gamma_{\boldsymbol{k}_2\downarrow}\gamma_{\boldsymbol{k}_1\uparrow}$$

$$+\frac{1}{N}\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{q},\sigma} V_{\boldsymbol{q}}^{22s}(\boldsymbol{k}_1,\boldsymbol{k}_2)\gamma^{\dagger}_{\boldsymbol{k}_1+\boldsymbol{q}\sigma}\gamma^{\dagger}_{\boldsymbol{k}_2-\boldsymbol{q}\sigma}\gamma_{\boldsymbol{k}_2\sigma}\gamma_{\boldsymbol{k}_1\sigma}$$

Number conserving
2 -> 2

$$+\frac{1}{N}\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{q},\sigma}\Big[V_{\boldsymbol{q}}^{31}(\boldsymbol{k}_1,\boldsymbol{k}_2)\gamma^{\dagger}_{\boldsymbol{k}_1+\boldsymbol{q}\sigma}\gamma_{\boldsymbol{k}_1\sigma}\gamma_{\boldsymbol{k}_2\sigma}\gamma_{-\boldsymbol{k}_2+\boldsymbol{q}\bar{\sigma}}$$

$$+V_{\boldsymbol{q}}^{40}(\boldsymbol{k}_1,\boldsymbol{k}_2)\gamma_{-\boldsymbol{k}_1-\boldsymbol{q}\uparrow}\gamma_{-\boldsymbol{k}_2+\boldsymbol{q}\downarrow}\gamma_{\boldsymbol{k}_2\downarrow}\gamma_{\boldsymbol{k}_1\uparrow}+h.c.\Big]$$

Non-conserving
3 -> 1, 1 -> 3
4 -> 0, 0 -> 4

Chiral order parameter

$$\kappa = \frac{1}{N}\sum_{\boldsymbol{r}\in\nabla} \hat{z}\cdot\sum_{j=1}^{3}\langle \boldsymbol{S}_{\boldsymbol{r}} \times \boldsymbol{S}_{\boldsymbol{r}+\boldsymbol{e}_j}\rangle$$

Vector chirality

$$\kappa = -\frac{1}{N}\sum_{\boldsymbol{q}}\sum_{j=1}^{3}\sin[\boldsymbol{q}\cdot\boldsymbol{e}_j]\langle S^{+}_{\boldsymbol{q}} S^{-}_{-\boldsymbol{q}}\rangle$$

q-space

Total spin S^z=0,
Odd under **Q** -> - **Q** ,
Odd under ↑ ↔ ↓

$$\kappa = -\frac{3\sqrt{3}\mu s^2}{N\omega_{\boldsymbol{Q}}}\langle \gamma^{\dagger}_{-\boldsymbol{Q}\uparrow}\gamma^{\dagger}_{\boldsymbol{Q}\downarrow} - \gamma^{\dagger}_{\boldsymbol{Q}\uparrow}\gamma^{\dagger}_{-\boldsymbol{Q}\downarrow} + \text{h.c.}\rangle$$

Low-energy approximation

$$\phi_L(\boldsymbol{k}) \equiv \gamma_{\boldsymbol{Q}-\boldsymbol{k}\uparrow}\gamma_{\bar{\boldsymbol{Q}}+\boldsymbol{k}\downarrow}$$
$$\phi_R(\boldsymbol{k}) \equiv \gamma_{\bar{\boldsymbol{Q}}+\boldsymbol{k}\uparrow}\gamma_{\boldsymbol{Q}-\boldsymbol{k}\downarrow}$$

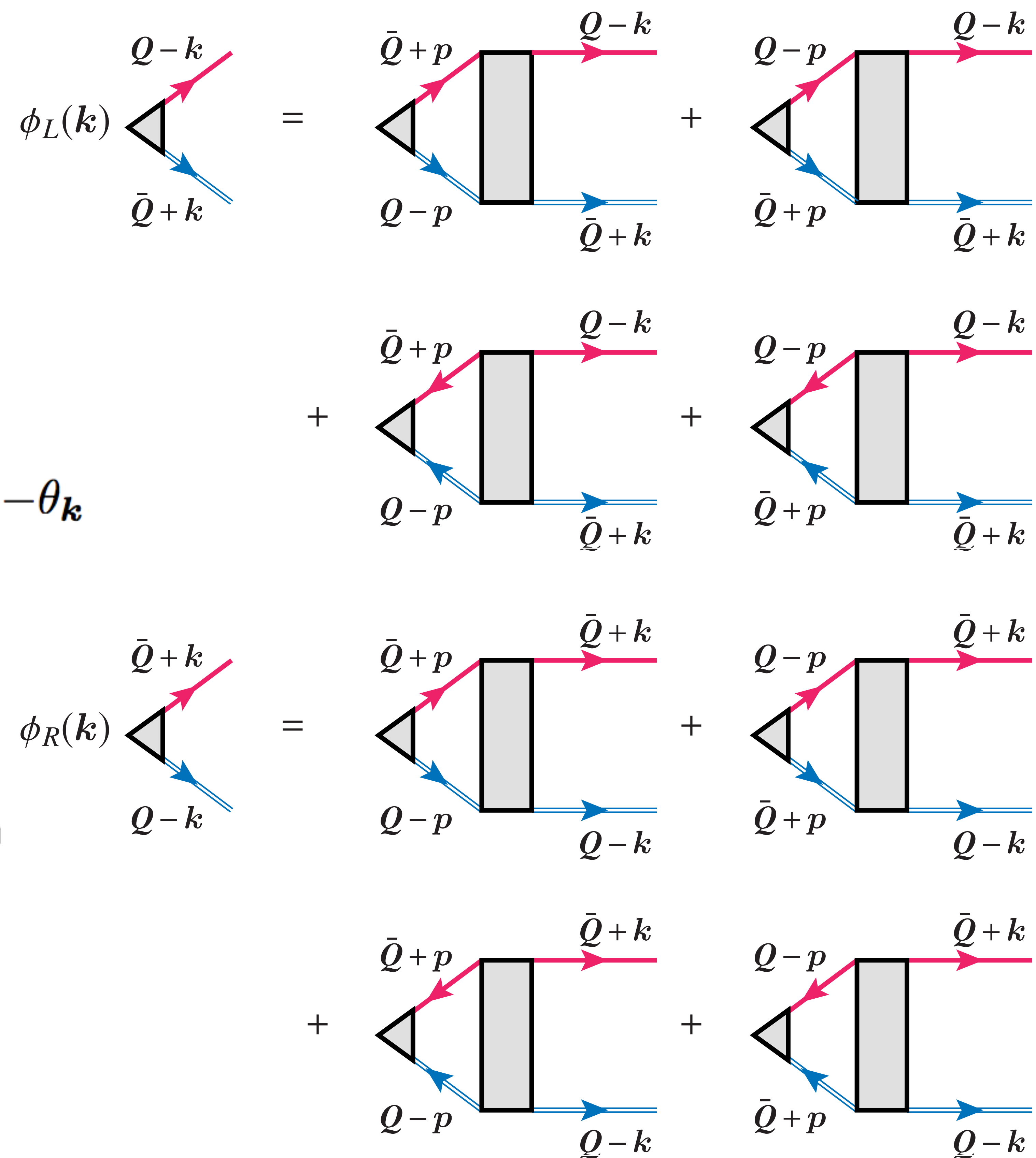
Boson pair operators

$$\kappa = -\frac{3\sqrt{3}\mu s^2}{N\omega_{\boldsymbol{Q}}}\left(\phi^*_R + \phi_R - \phi^*_L - \phi_L\right)$$

$$\theta_{\boldsymbol{k}} \equiv 2\omega_{\boldsymbol{Q}-\boldsymbol{k}}\langle \phi_R(\boldsymbol{k}) - \phi_L(\boldsymbol{k})\rangle$$

Convenient parameterization

Integral equation for pair vertices



$$\frac{1}{N}\sum_{\boldsymbol{p}} \frac{F^{22o}_{\boldsymbol{k},\boldsymbol{p}}\,\theta_{\boldsymbol{p}} - 4F^{04}_{\boldsymbol{k},\boldsymbol{p}}\,\theta^*_{\boldsymbol{p}}}{2\omega_{\boldsymbol{Q}-\boldsymbol{p}}} = -\theta_{\boldsymbol{k}}$$

Shaded rectangles denote
fully dressed
Irreducible interactions between
Low-energy magnons

First order in $J_z = \zeta J$

Interaction is given by bare vertices $F^{22o}_{\boldsymbol{k},\boldsymbol{p}} = -4F^{04}_{\boldsymbol{k},\boldsymbol{p}} \approx -\frac{9}{4}J_z\frac{(\omega_{\boldsymbol{Q}-\boldsymbol{p}}+\omega_{\boldsymbol{Q}-\boldsymbol{k}})^2}{\omega_{\boldsymbol{Q}-\boldsymbol{p}}\omega_{\boldsymbol{Q}-\boldsymbol{k}}}.$

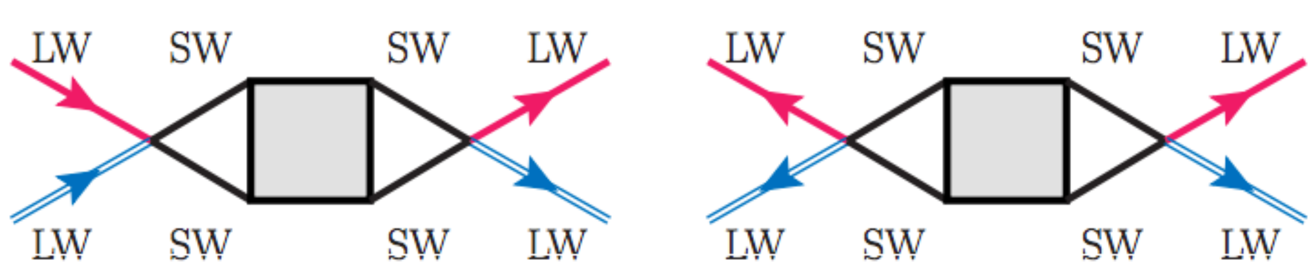
Obtain for the 2-magnon instability $1 = a\frac{J_z}{N}\sum_{\boldsymbol{p}}\frac{1}{\omega_{\boldsymbol{Q}-\boldsymbol{p}}}.$

No weak-coupling instability !
Interaction vertices are of order 1 (in units of J_z) and are not singular :-(

$$A_{\boldsymbol{k}_1,\boldsymbol{k}_2} \equiv u_{\boldsymbol{k}_1}u_{\boldsymbol{k}_2} - v_{\boldsymbol{k}_1}v_{\boldsymbol{k}_2} = \frac{\omega_{\boldsymbol{k}_1}+\omega_{\boldsymbol{k}_2}}{2\sqrt{\omega_{\boldsymbol{k}_1}\omega_{\boldsymbol{k}_2}}},$$
$$B_{\boldsymbol{k}_1,\boldsymbol{k}_2} \equiv u_{\boldsymbol{k}_1}v_{\boldsymbol{k}_2} - v_{\boldsymbol{k}_1}u_{\boldsymbol{k}_2} = \frac{\omega_{\boldsymbol{k}_1}-\omega_{\boldsymbol{k}_2}}{2\sqrt{\omega_{\boldsymbol{k}_1}\omega_{\boldsymbol{k}_2}}}.$$

$\mathbf{k}_{1,2}$ near $\pm\mathbf{Q}$

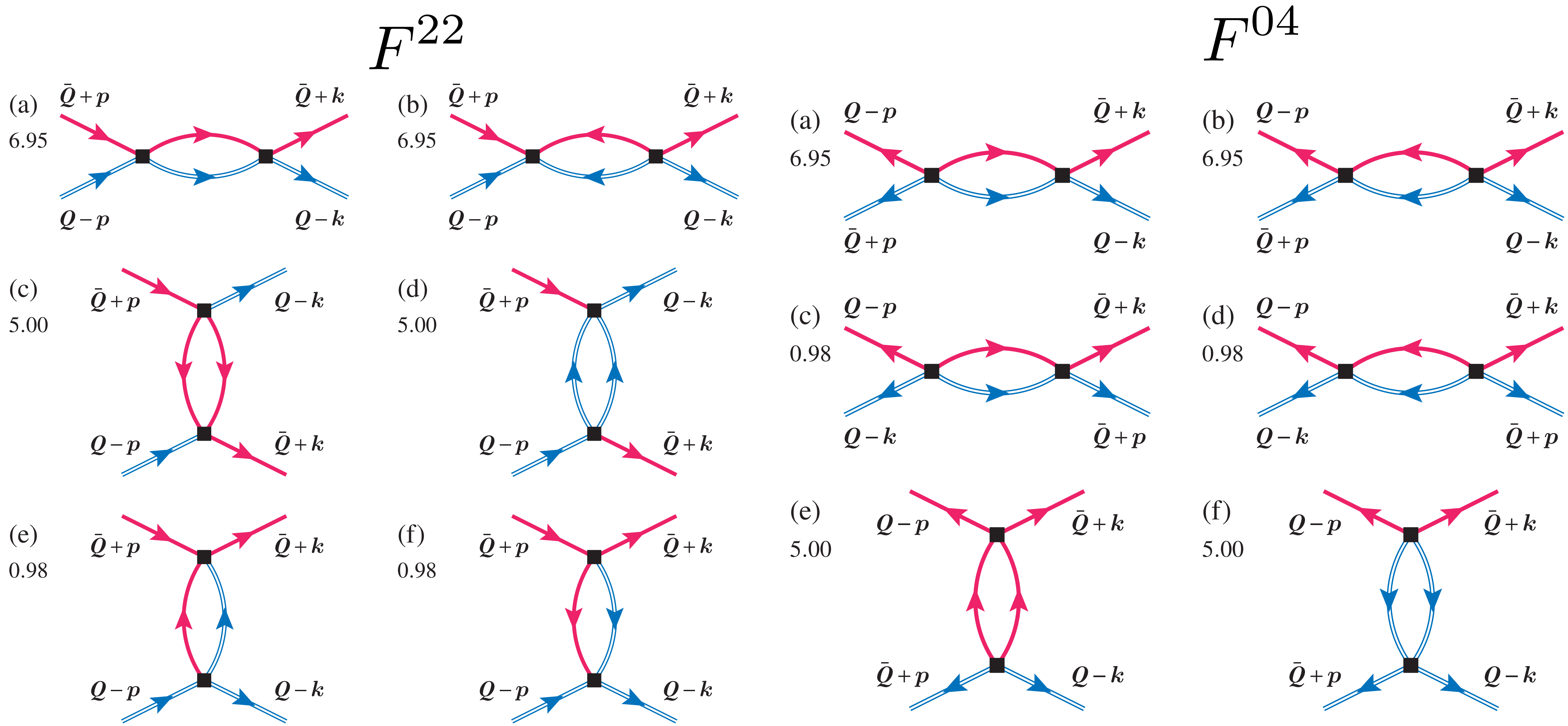
Need to renormalize it!

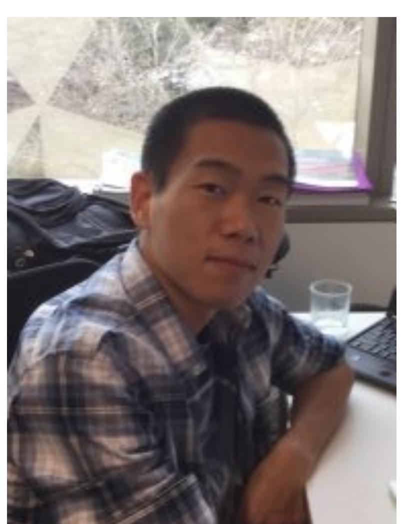


LW = Long wavelength, SW = short wavelength

To find the dressed interaction, we have to go to the 2nd order in J_z …

Kohn-Luttinger like mechanism but for bosons





The result

Pair vertex is **real**, *renormalized* interaction is **singular**

$$F^{22o}_{\mathbf{k},\mathbf{p}} = -4F^{04}_{\mathbf{k},\mathbf{p}} = \frac{-\alpha\zeta^2 J^3}{\omega_{\mathbf{Q}-\mathbf{p}}\omega_{\mathbf{Q}-\mathbf{k}}}$$

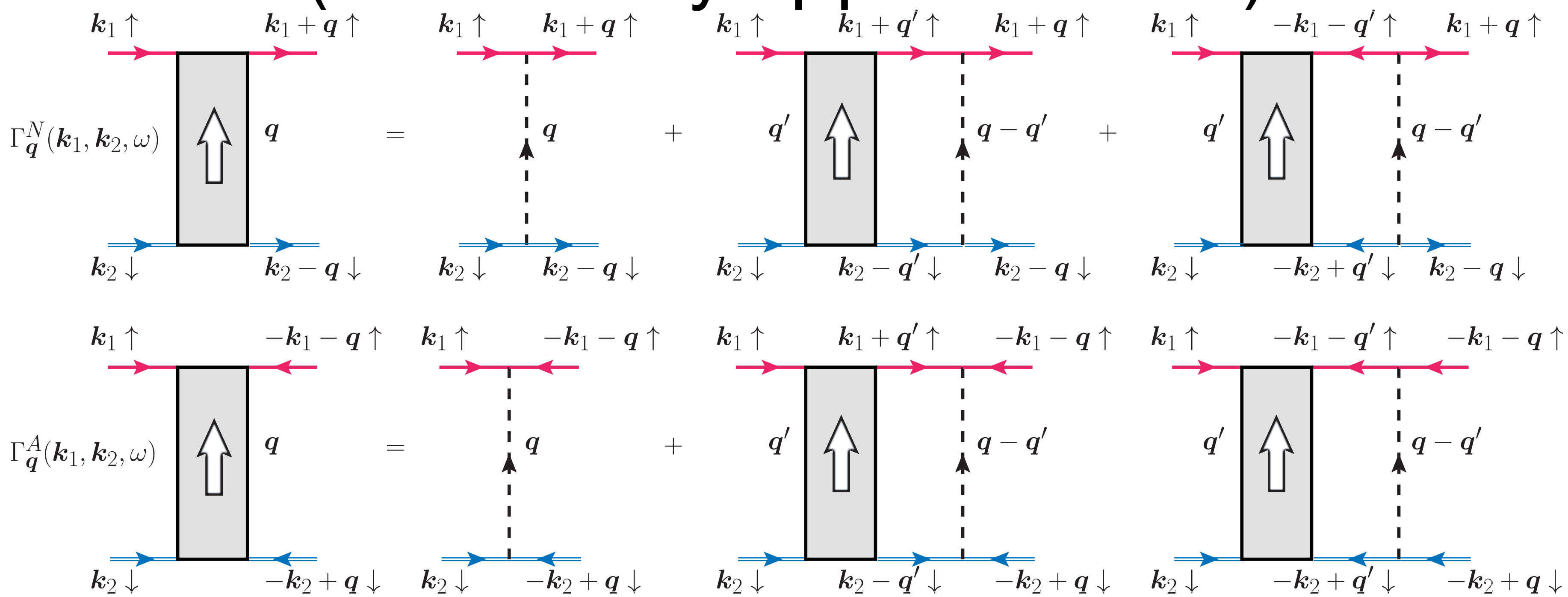
$$\frac{1}{N}\sum_{\boldsymbol{p}} \frac{\alpha\zeta^2 J^3}{\omega^2_{\boldsymbol{Q}-\boldsymbol{p}}\omega_{\boldsymbol{Q}-\boldsymbol{k}}}\theta_{\boldsymbol{p}} = \theta_{\boldsymbol{k}} \qquad \alpha = 2.49$$

$$\frac{1}{\alpha\zeta^2 J^3} = \frac{1}{N}\sum_{\boldsymbol{p}} \frac{1}{\omega^3_{\boldsymbol{Q}-\boldsymbol{p}}} \approx \frac{1}{N}\sum_{\boldsymbol{p}} \frac{1}{(\omega^2_{\boldsymbol{Q}} + 9J^2 s^4 p^2)^{3/2}}$$

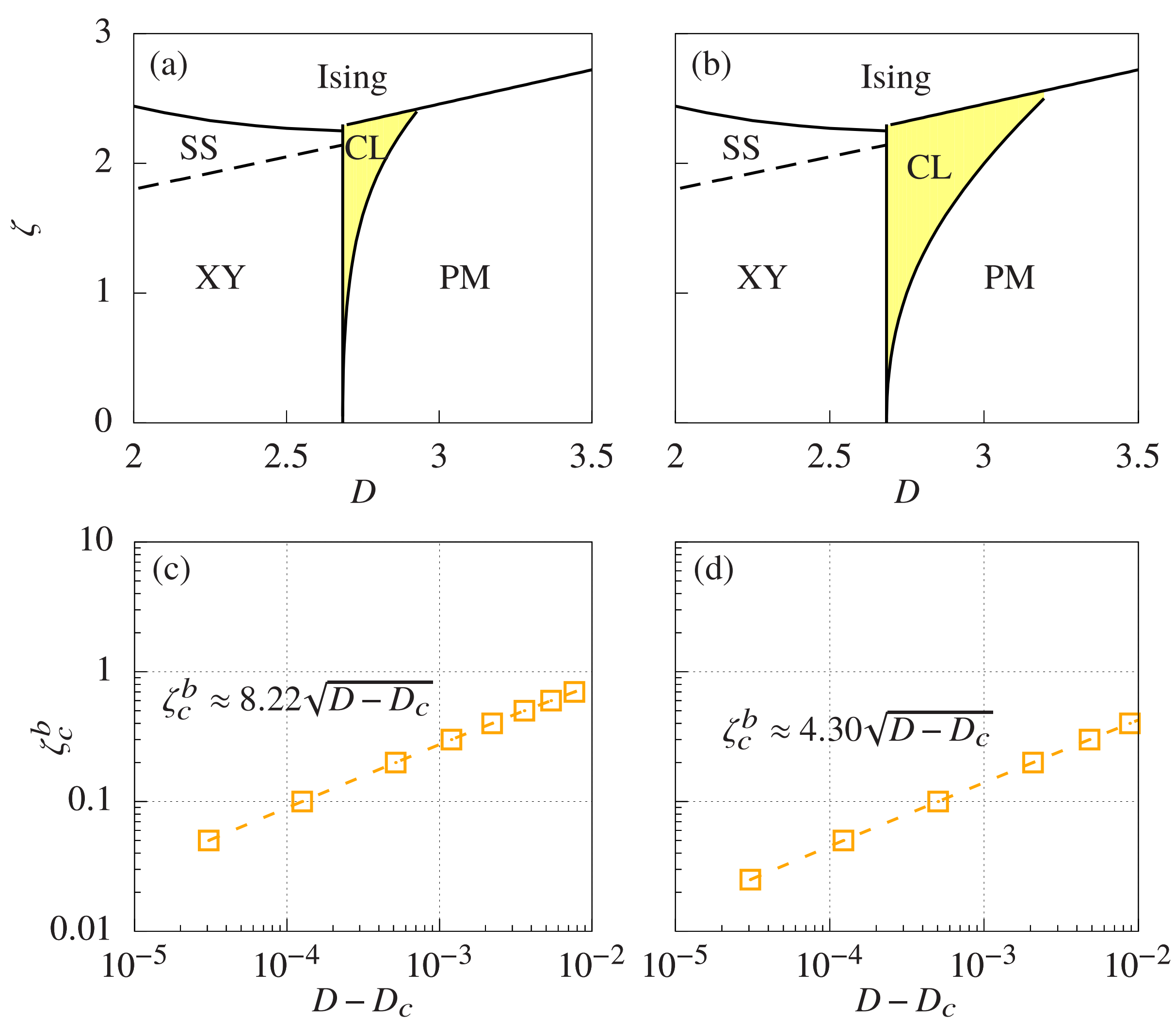
Two magnon condensation takes place **before** the single magnon one:

$$D^b_c = D_c + 0.042\alpha\zeta^2 J \qquad \zeta = \frac{J_z}{J} = \Delta$$

More checks: Bethe-Salpeter equation (low-density approximation)

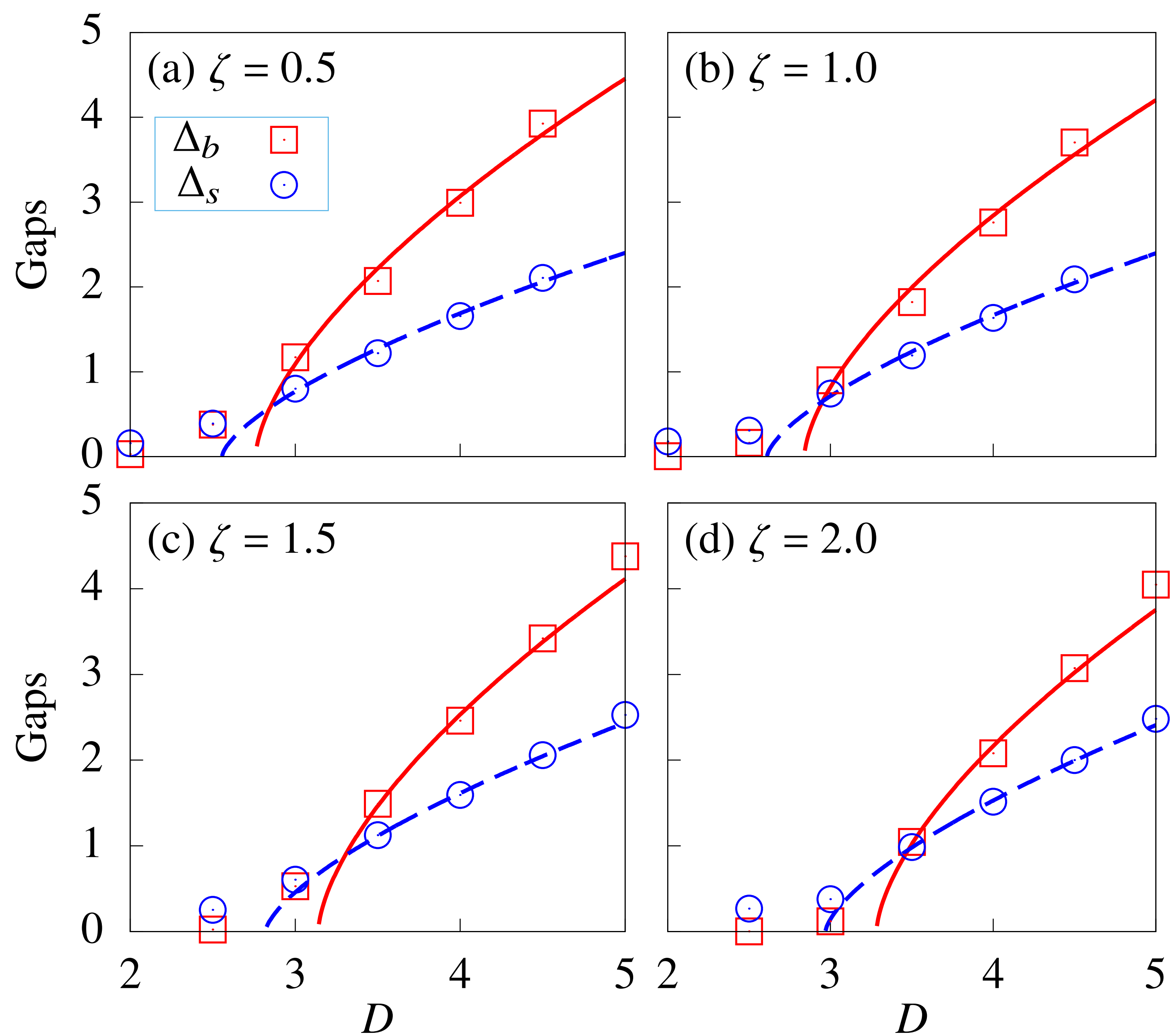


Only normal
(2 -> 2) vertices



Full problem

DMRG on 6x6 triangular lattice



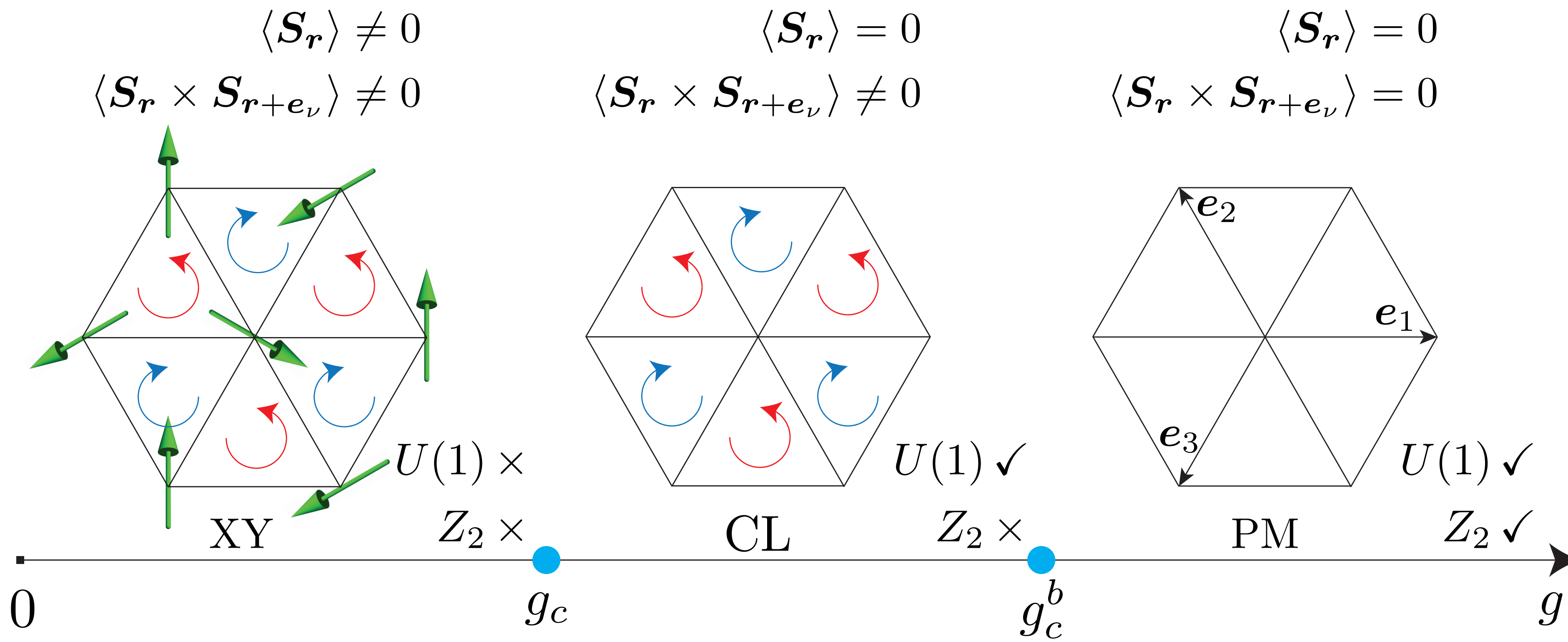
Gap crossing:
VC order via Ising transition
before
U(1) order via XY transition

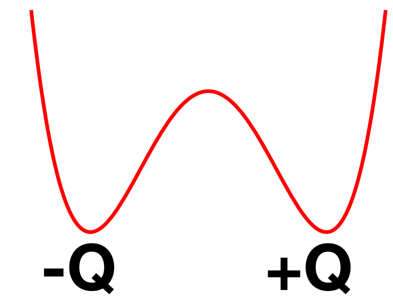
Two-magnon gap $\Delta_b = E^{(1)}_{S_z=0} - E^{(0)}_{S_z=0} = c_b(D - D_b)^{\nu_{\text{Ising}}},$

Single magnon gap $\Delta_s = E^{(0)}_{S_z=1} - E^{(0)}_{S_z=0} = c_s(D - D_s)^{\nu_{\text{XY}}},$

$$\nu_{\text{Ising}} = 0.63, \nu_{\text{XY}} = 0.67$$

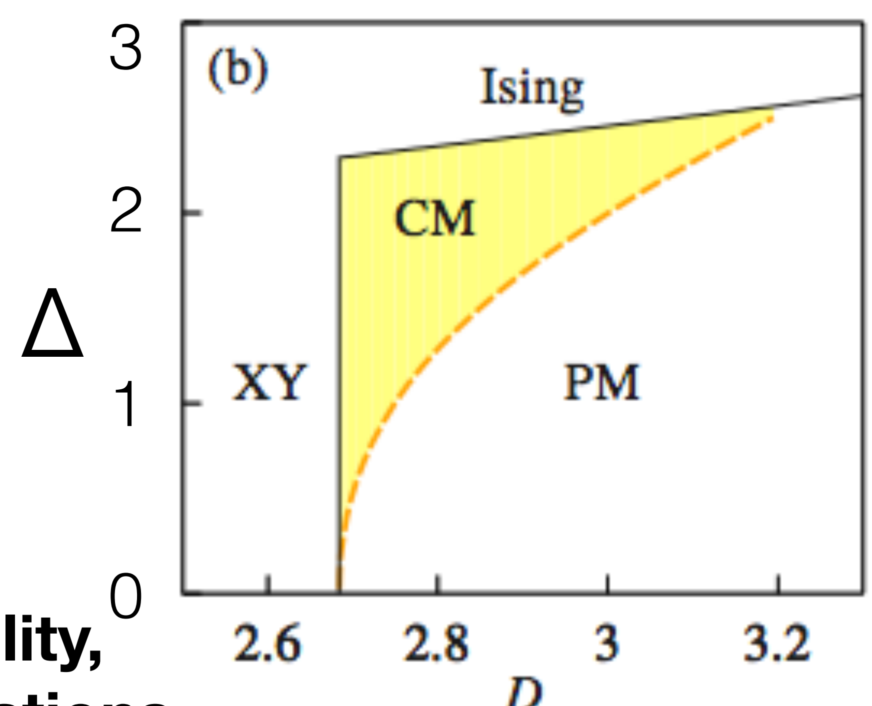
Summary





CM = Chiral Mott = Chiral Liquid = CL

Magneto-electric effect!



Chiral spin liquid appears naturally in the vicinity of magnetic quantum critical point!

Broken inversion,
Spontaneous vector chirality,
Gapped single particle excitations.

Conclusions

Paramagnet XY ordered

Mott -> **superfluid** transition on a frustrated lattice requires U(1) x Z_2 breaking.

This proceeds via intermediate **spin-current** (**chiral Mott**) phase (breaking Z_2 only).

$$|\Psi_{\rm CL}\rangle \sim e^{u\sum_{\boldsymbol{k}} \phi(\boldsymbol{k}) S^+_{\boldsymbol{k}} S^-_{\boldsymbol{k}}} |0\rangle$$

Spontaneously breaks spatial inversion.

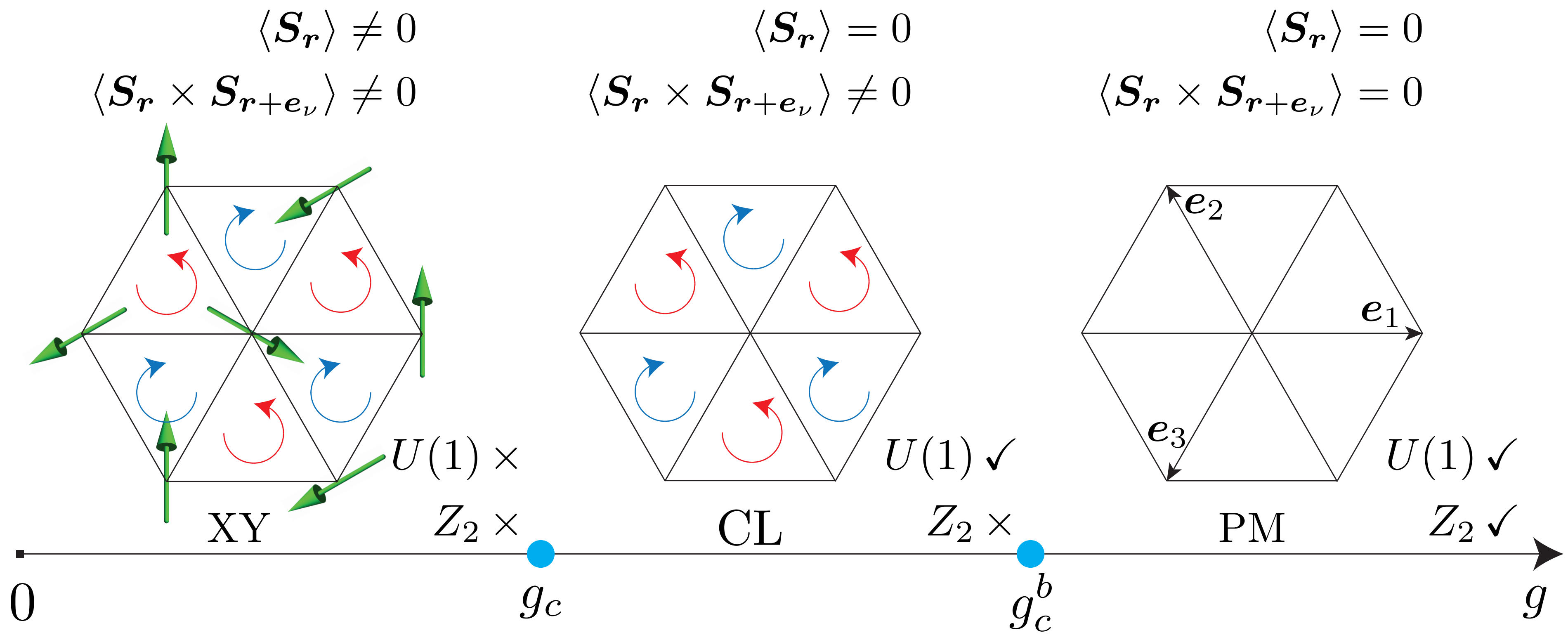
$$\phi(\mathbf{k}) = -\phi(-\mathbf{k})$$

But preserves time-reversal $u \in \mathbb{R}$

All single particle excitations are gapped.

Thank you!

Summary



Chiral liquid can be detected via inverse Dzyaloshinskii-Moriya effect:
Leads to charge density wave of O^{2-} anions

Magneto-electric phenomena