Application of relativistic energy density functional theory in description of stellar weak-interaction rates

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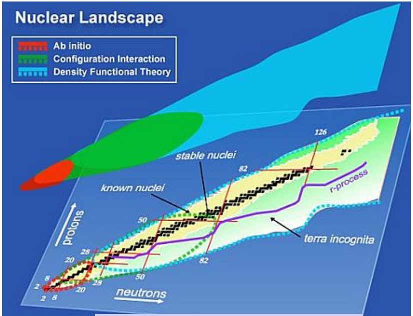
Introduction

- dynamics of core-collapse SNe
  - electron-to-baryon ratio (Y_e)
  - core entropy
- e^- capture (EC) $\Longrightarrow$ decreases Y_e and core entropy
- $M_{\text{Fe core}} > M_{ch} \sim Y_e^2 \Longrightarrow$ electron degeneracy pressure cannot hold gravity $\Longrightarrow$ collapse
- competition between β-decay and EC when collapse reaches $A \approx 60$ [H. T. Janka, Physics Reports, 442, 38-74 (2007), K. Langanke et. al., Rep. Prog. Phys. 84 066301 (2021)]

Models for rate calculation:

- F^2N: independent particle model: Fermi + Gamow-Teller (GT)
- Large-scale shell-model (LSSM): $45 < A < 65$
- Shell-model Monte-Carlo (SMMC)
- Hybrid approach: SMMC + RPA
- Skyrme Hartree-Fock + RPA
- ...



[https://unedf.mps.ohio-state.edu/]

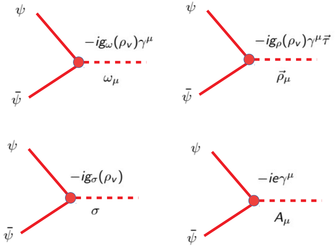
Relativistic mean-field theory

mean-field part

- Types of relativistic EDFs:
  - nonlinear (NL3) [Y.K Gambhir et al., *Annals of Physics*, 198, 132–179, (1990)]
  - meson-exchange (DD-ME) [G. A. Lalazissis et al., PRC, 71, 024312 (2005)]
  - derivative-coupling (D3C*) [S. Typel, PRC, 71, 064301 (2005)]
  - point-coupling (DD-PC) functionals [T. Niksic et al. PRC, 78, 034318 (2008)]

ME functionals:

- nucleons exchange σ, ω and ρ meson + EM field
- $\mathcal{L} = \mathcal{L}_N + \mathcal{L}_m + \mathcal{L}_{int}$
- $E_{RMF} = \int d^3 r \mathcal{H}(\boldsymbol{r})$



pairing correlations

- pairing treated within FT-(H)BCS theory
- $n_k = v_k^2(1 - f_k) + u_k^2 f_k$ ← $(1 - e^{-\beta E_k})^{-1}$
- Gap equation:

$$\Delta_k = \frac{1}{2} \sum_{k'>0} G_{kk'} \frac{\Delta_{k'}(1 - 2f_{k'})}{E_{k'}}$$

[A. L. Goodman, Nucl. Phys. A 352, 30 (1981)]

Finite-temperature proton-neutron relativistic (Q)RPA

- excitation operator [H. Sommermann, Ann. of Phys. 151, 163 (1983)]

$$\Gamma_\nu^\dagger = \sum_{pn} \left[X_{pn}^\nu a_p^\dagger a_n^\dagger - Y_{pn}^\nu a_n a_p \underbrace{+ P_{pn}^\nu a_p^\dagger a_n - Q_{pn}^\nu a_n^\dagger a_p}_{\text{only for T>0!}} \right]$$

a_p, a_n proton, neutron annihilation operator in q.p. basis

- equation of motion or linearization of denisty $\rightarrow$ matrix FT-PNRQRPA equation

$$\begin{pmatrix} \tilde{C} & \tilde{a} & \tilde{b} & \tilde{D} \\ \tilde{a}^+ & \tilde{A} & \tilde{B} & \tilde{b}^T \\ -\tilde{b}^+ & -\tilde{B}^* & -\tilde{A}^* & -\tilde{a}^T \\ -\tilde{D}^* & -\tilde{b}^* & -\tilde{a}^* & -\tilde{C}^* \end{pmatrix} \begin{pmatrix} \tilde{P} \\ \tilde{X} \\ \tilde{Y} \\ \tilde{Q} \end{pmatrix} = E_\nu \begin{pmatrix} \tilde{P} \\ \tilde{X} \\ \tilde{Y} \\ \tilde{Q} \end{pmatrix}$$

terms in red contribute for T > 0 !

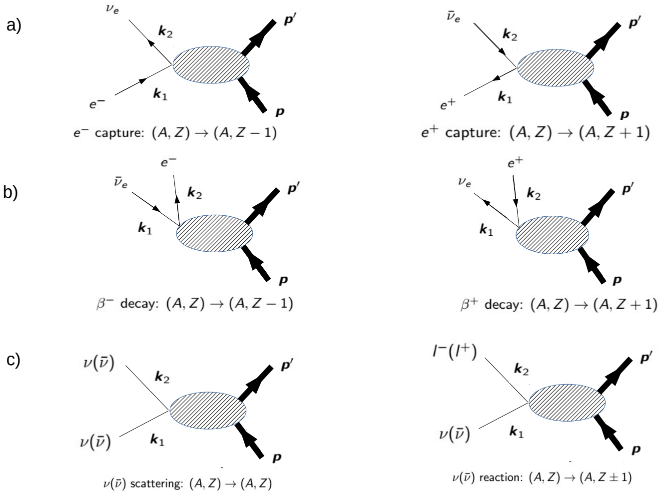
- strength function for external operator $\hat{F}$

Correlated QRPA state

$$B(\beta^+, \hat{F}) = |\langle \nu || \hat{F} || \tilde{0} \rangle|^2$$

defined for $\Delta T_z = \pm 1$ (isospin projection, $\beta^\mp$ direction).

Weak-force reactions



Electron capture rate

- expressions for weak-reaction rates can be derived using Walecka formalism [J. D. Walecka, *Theoretical Nuclear and Subnuclear Physics*, 2004]

$$\underbrace{\hat{H}_W = -\frac{G}{\sqrt{2}} \int d^3\mathbf{r}\, j_\mu^{lept.}(\mathbf{r}) \hat{\mathcal{J}}_\mu(\mathbf{r}),}_{\text{current-current Hamiltonian}}$$

  j_μ^{lept} - lepton current, $\hat{\mathcal{J}}_\mu(\mathbf{r})$ hadronic current

- weak-rate cross section can be calculated from the Fermi golden rule

$$\frac{d\sigma_{ec}}{d\Omega} = \frac{1}{(2\pi)^2}\Omega^2 E_\nu^2 \frac{1}{2} \sum_{lept.spin.} \underbrace{\frac{1}{2J_i+1} \sum_{M_i M_f} |\langle f|\hat{H}_W|i\rangle|^2}_{\text{matrix element of weak Hamiltonian}}$$

  E_ν - neutrino energy , $|i\rangle$ - initial nuclear state, $|f\rangle$ - final state

- final expression written in terms of charge $\hat{\mathcal{M}}_J$, longitudinal $\hat{\mathcal{L}}_J$, transverse electric $\hat{T}_J^{el}$ and transverse magnetic $\hat{T}_J^{mag}$ operators for multipole J^π

Folded with Lorentzian of 1 MeV

[A. Ravlić et al., *Phys. Rev. C*, 102, 065804, (2020)]

- Energy conservation:
  $E_\nu = E_e - E_{QRPA} - \Delta_{np} - (\lambda_n - \lambda_p)$
- EC rate calculated by

$$\lambda_{ec} = \frac{1}{\pi^2 \hbar^2} \int_{E_e^0}^{\infty} p_e E_e \sigma_{ec}(E_e) \underbrace{f(E_e, \mu_e, T)}_{(e^{\frac{E_e - \mu_e}{kT}} + 1)^{-1}} dE_e$$

E_e^0 - threshold energy, μ_e - electron chemical potential, T - temperature, $p_e = \sqrt{E_e^2 - m_e^2}$

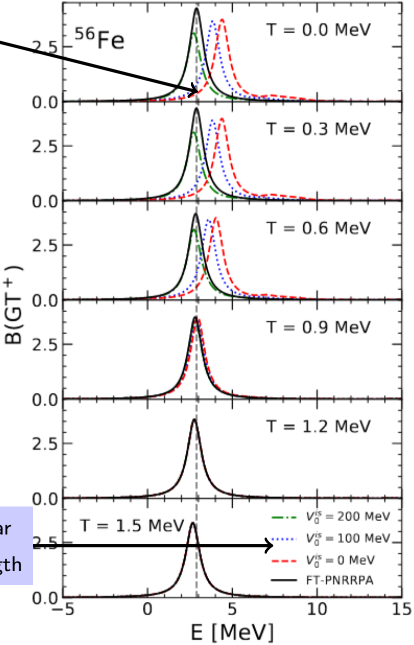
- μ_e determined by inverting the relation

$$\rho Y_e = \frac{1}{\pi^2 N_A} \left(\frac{m_e c}{\hbar} \right)^3 \int_0^{\infty} [f_e - f_{e^+}] \, p^2 dp$$

- Gamow-Teller (GT) strength:

$$B(\mathrm{GT}^+) = g_A^2 \frac{|\langle f || \boldsymbol{\sigma \tau}_+ || i \rangle|^2}{2J_i + 1}$$

V_0^{is}: isoscalar pairing strength



DD-ME2

[A. Ravlić et al., *Phys. Rev. C*, 102, 065804, (2020)]

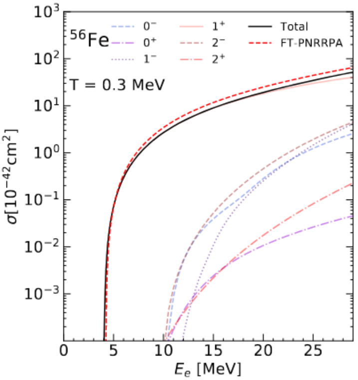


Figure: Electron capture cross-section σ_{ec} on ^{56}Fe for $J^{\pi} = 0^{\pm}, 1^{\pm}, 2^{\pm}$ multipoles at T $=$ 0.3 MeV. Calculated using FT-PNRQRPA (black full line + multipoles) and FT-PNRRPA (no pairing, red dashed line)

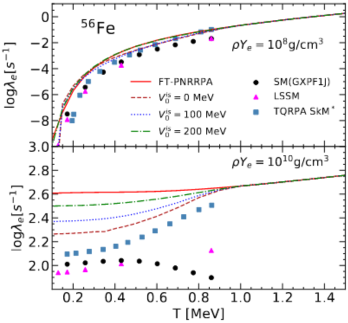


Figure: Electron capture rates λ_{ec} on ^{56}Fe as calculated with FT-PNRQRPA for various values of isoscalar pairing strength V_0^{is} at $\rho Y_e = 10^8$ g/cm^3 (upper panel) and $\rho Y_e = 10^{10}$ g/cm^3.

$\beta-$decay rates

[A. Ravlić et al., arXiv: 2010.06394]

- general form of reaction rate [T. Marketin et al., *Phys. Rev. C*, 93, 025805, (2016)]

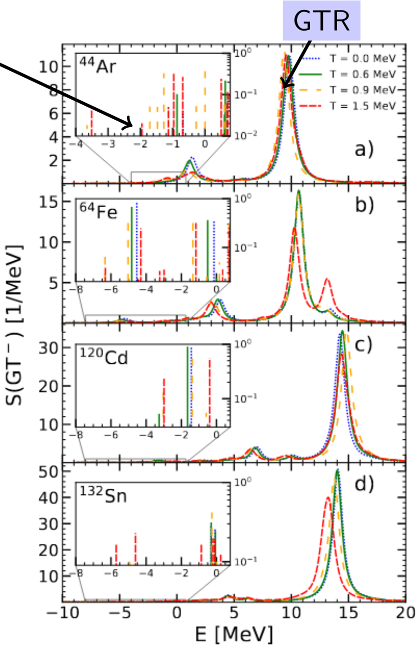
$$\lambda_\beta = \frac{\ln 2}{K} \int_0^{p_0} p_e^2 (W_0 - W)^2 F(Z, W) C(W) dp_e$$

W_0 - maximum electron energy
$C(W)$ - shape factor, $F(Z, W)$ - Fermi function, $K \approx 6147$ s
$T_{1/2} = \ln(2)/\lambda_\beta$

- $W_0 \approx \lambda_n - \lambda_p + \Delta_{np} - E_{QRPA}$ ($\lambda_n - \lambda_p$: neutron-proton chem. pot. diff.)
- shape-factor for allowed GT transitions

$$B(\mathrm{GT}^-) = g_A^2 \frac{|\langle f || \boldsymbol{\sigma\tau}_- || i \rangle|^2}{2J_i + 1}$$

axial coupling
$g_A = -1.26 \to -1.0$
"quenching"



Large-scale calculation of $\beta-$decay rates

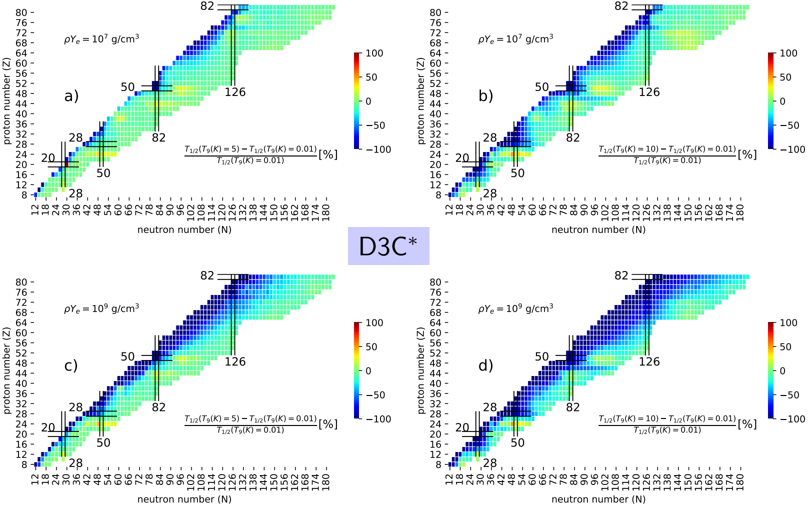


Figure: Percentage change of $\beta-$decay half-lives at $T_9(K) = 5$ and $T_9(K) = 10$ w.r.t zero-temperature at $\rho Y_e = 10^7$ g/cm^3 (a)-(b) and $\rho Y_e = 10^7$ g/cm^9 (c)-(d) for even-even nuclei in the range $8 \leq Z \leq 82$ [paper in preparation].

Conclusion

- developed self-consistent FT-HBCS + FT-PNRQRPA framework for description of weak-force reactions $\Longrightarrow$ **temperature** + **pairing** effects
- use of relativistic EDFs (DD-ME2, D3C* ..) $\rightarrow$ excellent predictions for ground-state observables throughout nuclide chart
- finite-temperature effects $\Longrightarrow$ **thermal unblocking** + dependence on **stellar density** **(ρY_e)** + electron screening
- Model instrumental to provide large-scale data:
  - ✓ $e^{\pm}$ capture rates (CCSN)
  - ✓ $\beta^{\pm}$ decay rates (CCSN, r-process(?), rp-process(?))
  - $\nu\bar{\nu}$ scattering and reactions (in development)
- in development: implementation of deformation

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