Kinetic SZ 効果

を通じた

宇宙論的 モーメンタム場

の理論モデルの構築

第四回観測論的宇宙論ワークショップ@京都大学 (11/18-20, 2015)

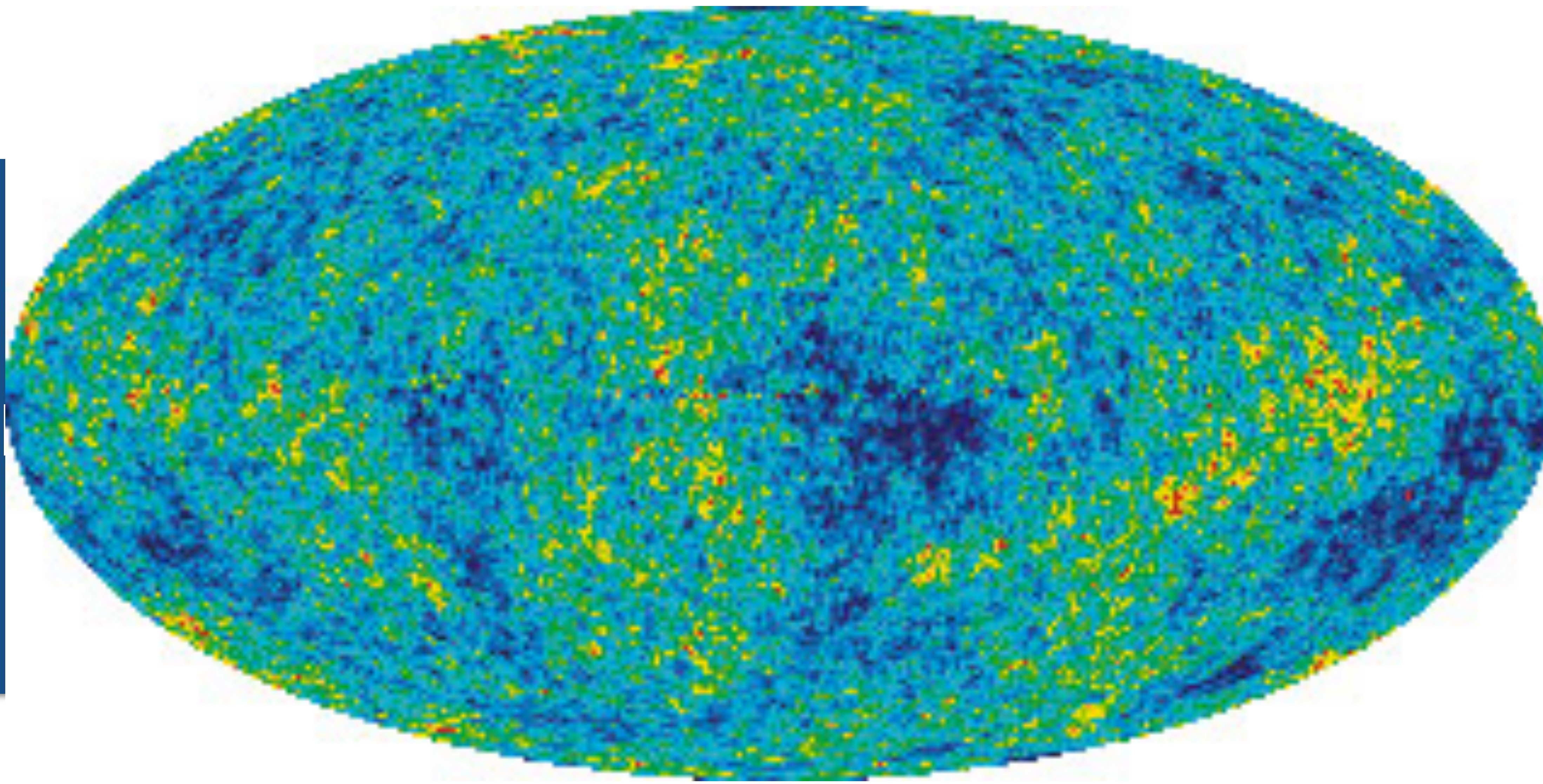
Naonori Sugiyama
Kavli IPMU (Yoshida lab.; CREST)
Collaborators： T.Okumura and D.N.Spergel

Cosmological Measurements

LSS	CMB	Lensing
Galaxy Number density	Temperature fluctuations	Gravitational potential

Cosmological Measurements

Ga



g

nal

l

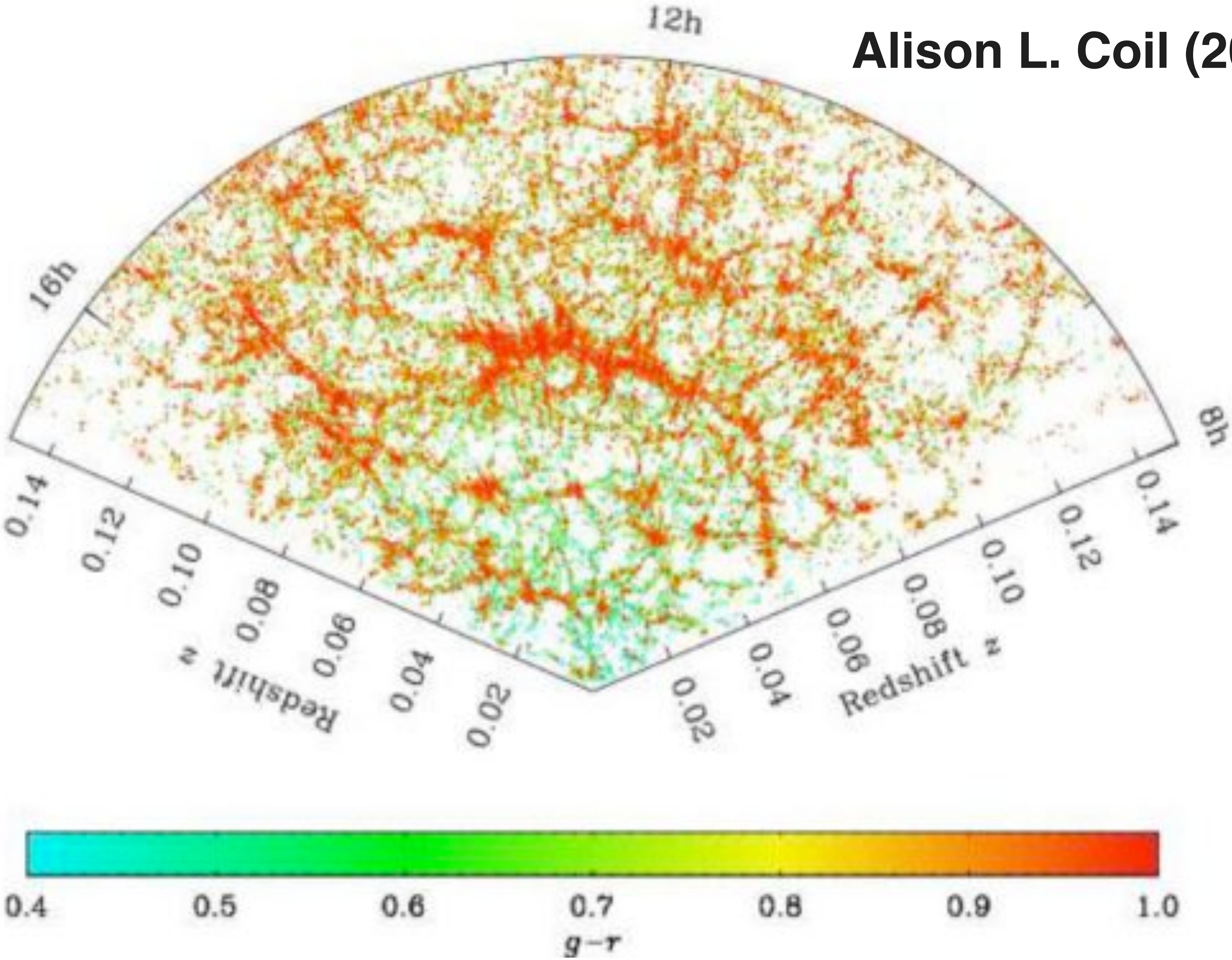
WMAP homepage

Cosmological Measurements

LSS	CMB	Lensing
Galaxy Number density	Temperature fluctuations	Gravitational potential

Cosmological Measurements

Alison L. Coil (2012)



Gala

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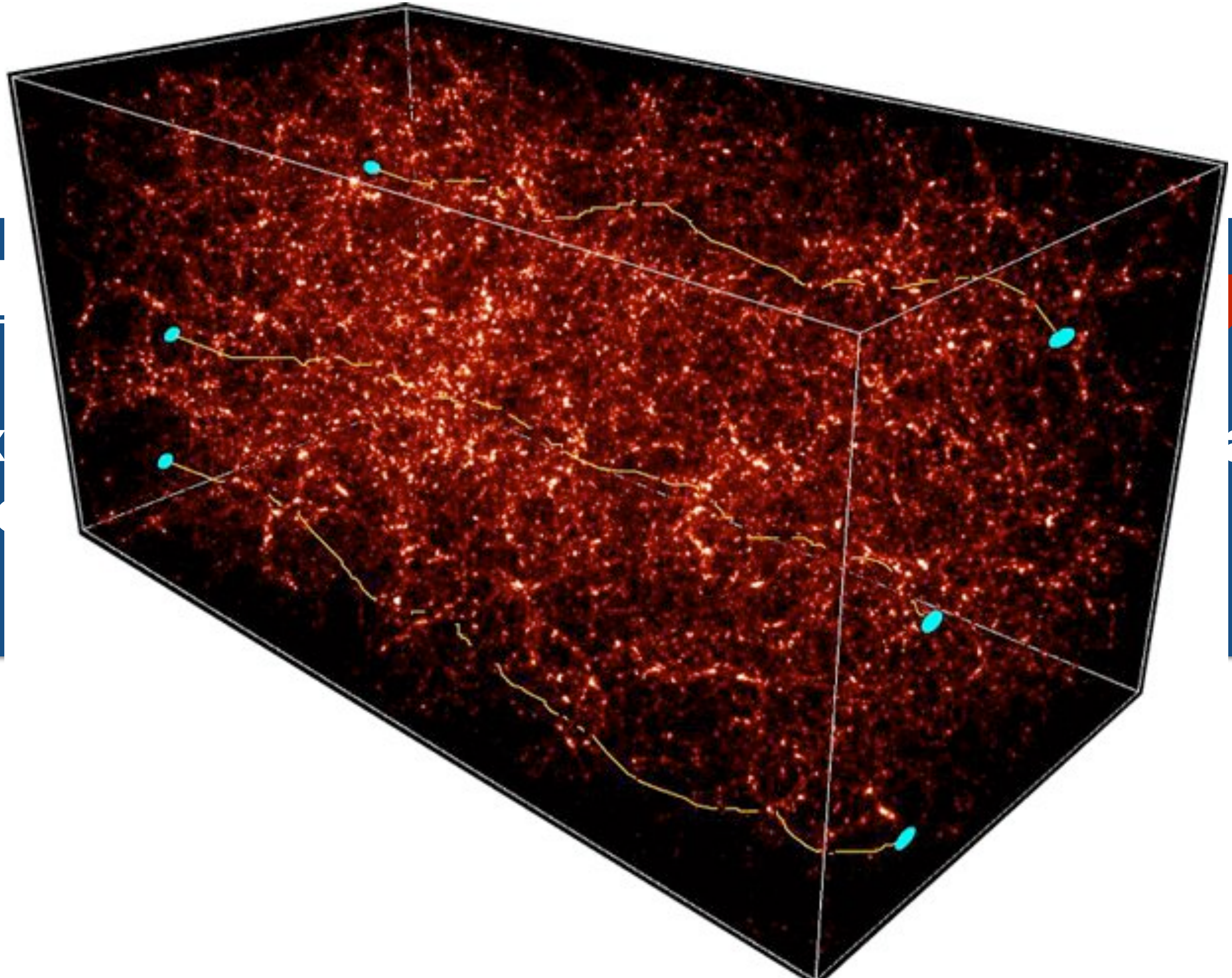
Cosmological Measurements

LSS	CMB	Lensing
Galaxy Number density	Temperature fluctuations	Gravitational potential

Cosmological Measurements



Cosmological Measurements

LSS

Galaxy Number density

CMB

Temperature fluctuations

Kinetic
Sunyaev
Zel'dovich
(KSZ) effect

Lensing

Gravitational potential

Cosmological Measurements

LSS

Gal ρ ber

CMB

Temperature fluctuations

Lensing

Gravitational potential

$$\frac{\delta T_{\mathrm{kSZ}}}{T} \propto \tau_{\mathrm{g}} \frac{\hat{n} \cdot \vec{v}}{c}$$

“Momentum Field”

$$\vec{p} = \vec{v}\rho$$

kSZ effect in CMB

Galaxy clustering

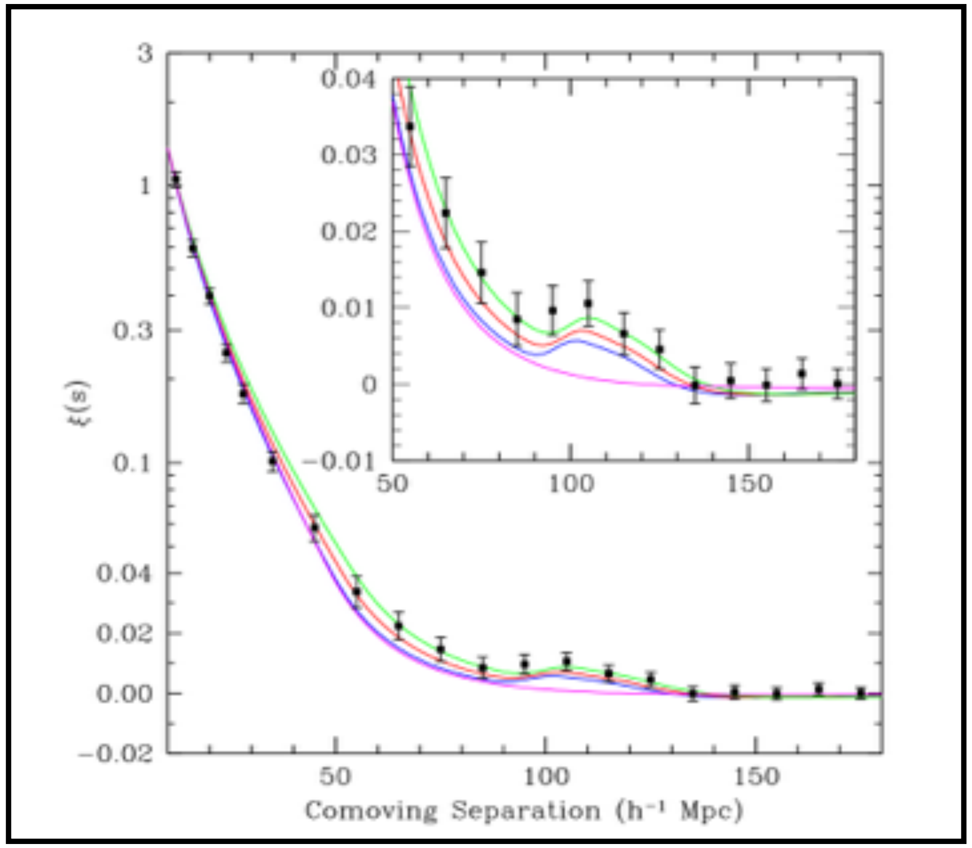
Importance of Velocity Information

- **Sensitive to gravity theories.**

$$\frac{d\vec{v}}{dt} \propto -\nabla \delta \phi$$

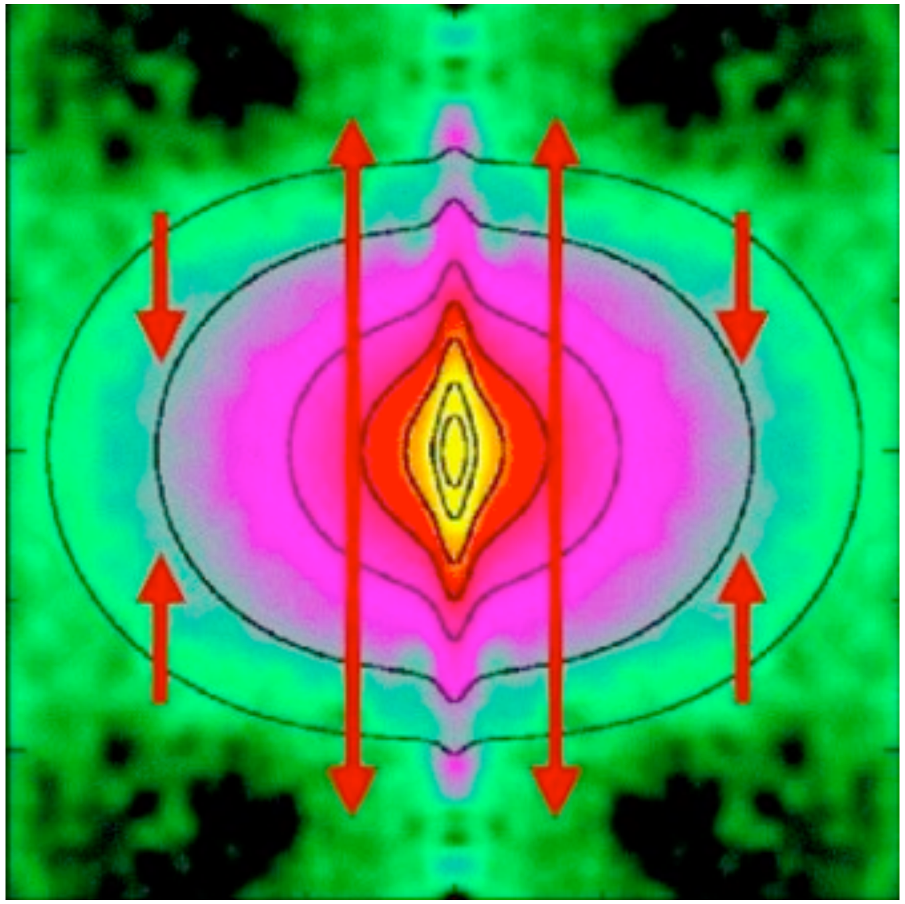
3D Galaxy Distributions

Baryon Acoustic Oscillation (BAO)



Eisenstein et al. (2005)

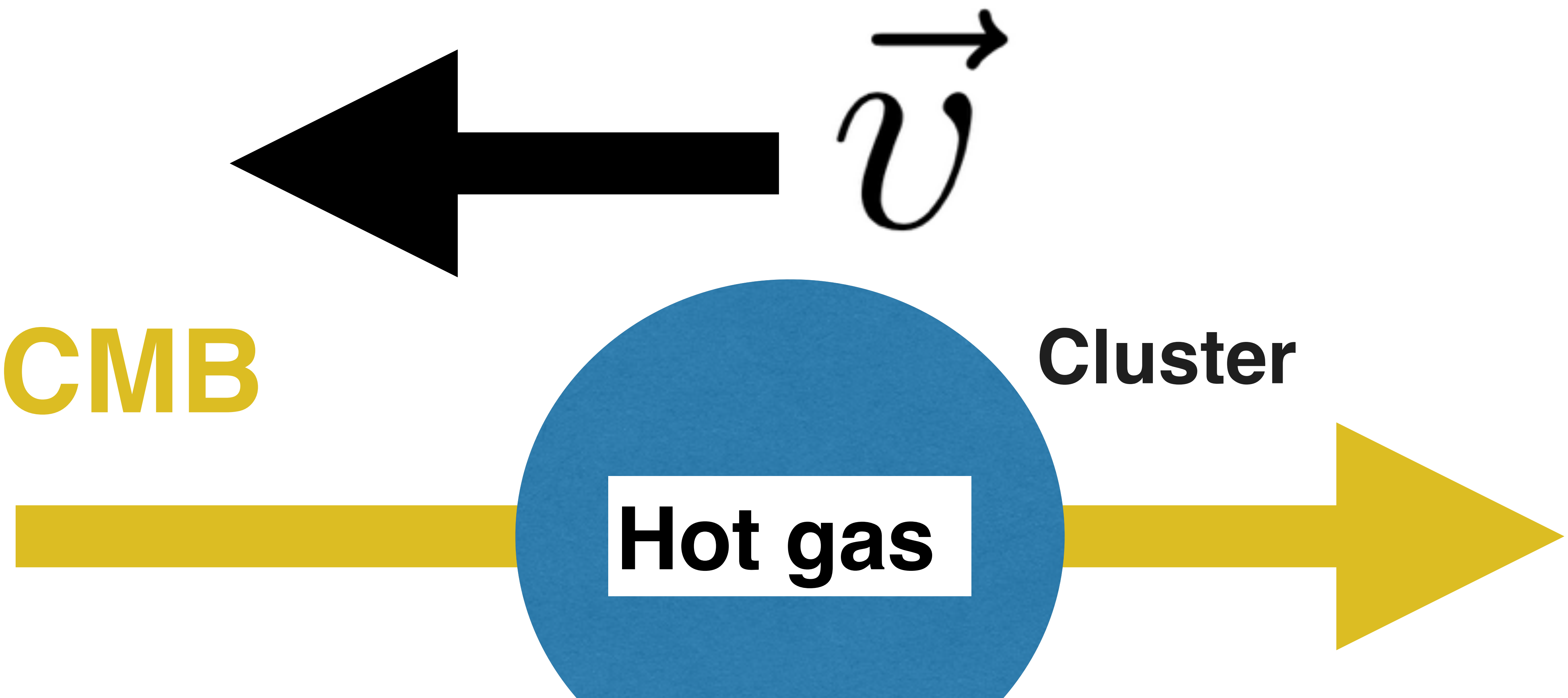
Redshift Space Distortions (RSD)



Alison L. Coil (2012)

Through RSD, the peculiar velocity correlation function can be measured.

Kinetic SZ effect



$$\frac{\delta T_{\mathrm{kSZ}}}{T} \propto \tau_{\mathrm{g}} \frac{\hat{n} \cdot \vec{v}}{c}$$

Observable

Density Correlation Function

$$\xi(r) \propto \sum_{i,j} \delta_{\mathrm{D}}\left(\vec{r} - \left(\vec{x}_i - \vec{x}_j \right) \right)$$

Particle positions

$$\Leftrightarrow$$

$$\langle \rho \, , \, \rho \rangle$$

Observable

kSZ Correlation Function

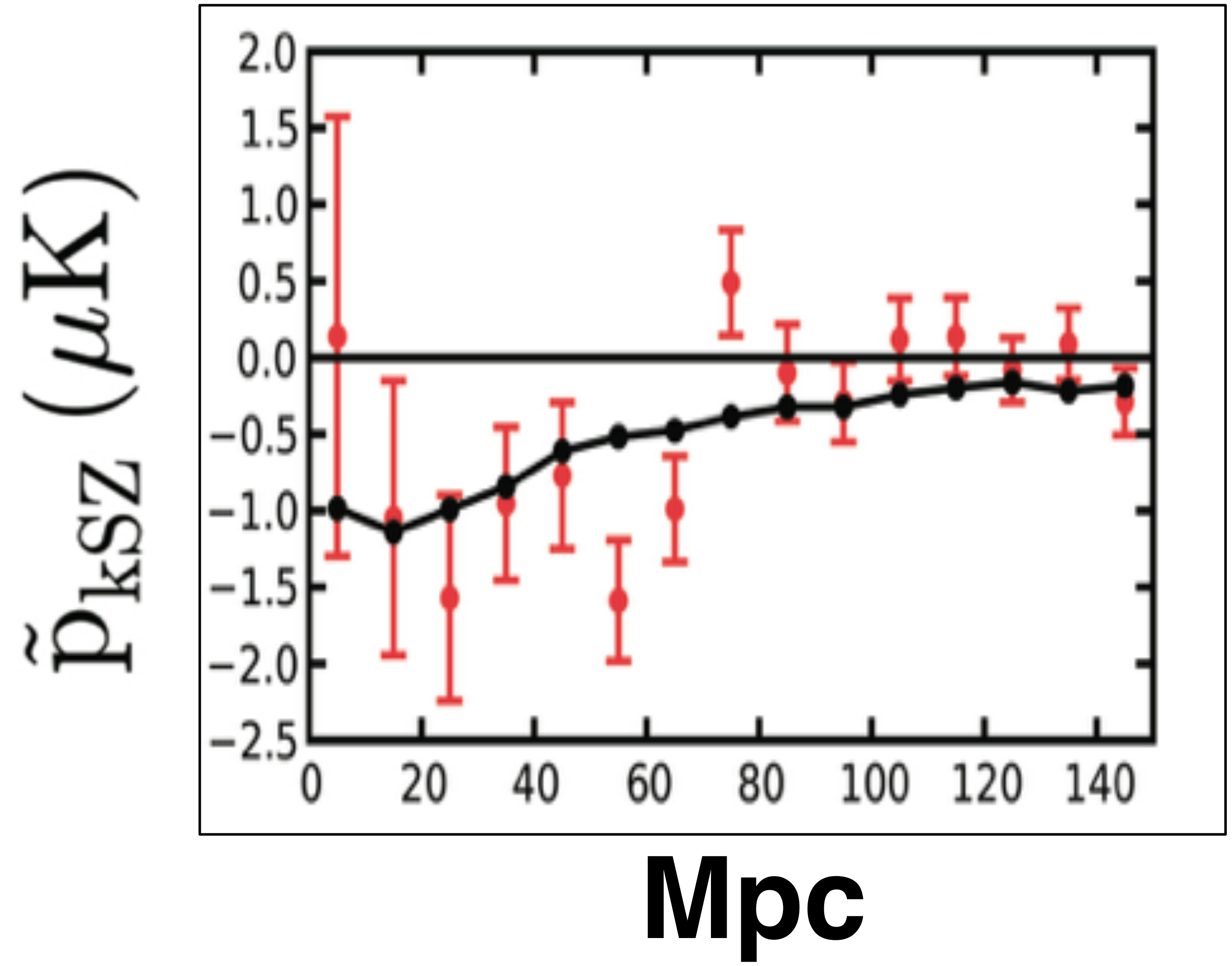
$$\xi_{\mathrm{kSZ}}(r) \propto \sum_{i,j} \underline{[\ \delta T_{\mathrm{kSZ,i}} - \delta T_{\mathrm{kSZ,j}}\ ]}\ \delta_{\mathrm{D}}(\ \vec{r} - (\ \vec{x}_i - \vec{x}_j\ )\ )$$

$$\propto \sum_{i,j} \underbrace{[\ \hat{n}\cdot\vec{v}_i - \hat{n}\cdot\vec{v}_j\ ]}_{\text{weight}}\ \delta_{\mathrm{D}}(\ \vec{r} - (\ \vec{x}_i - \vec{x}_j\ )\ )$$

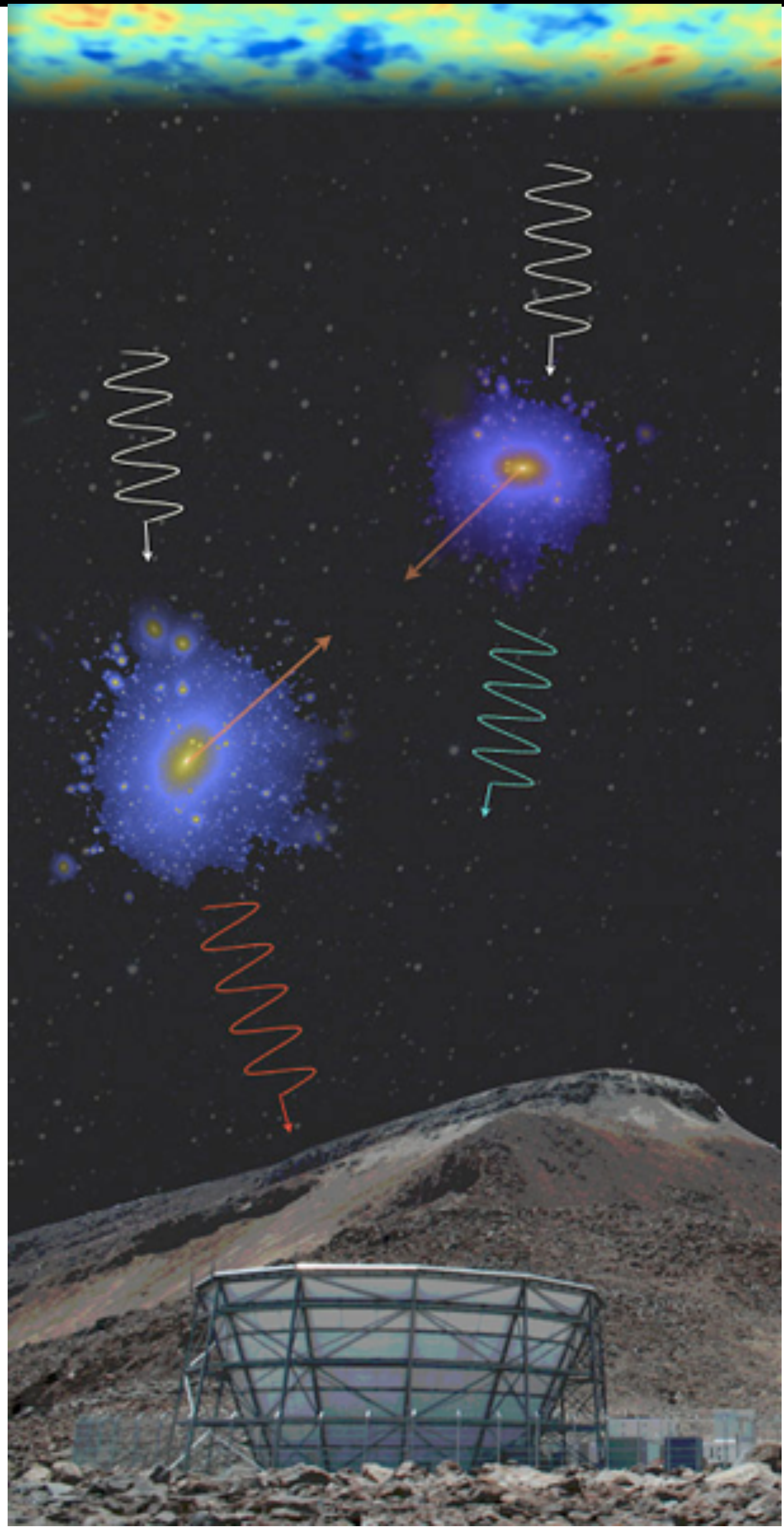
$$\langle\ \hat{n}\cdot\vec{p}\ ,\ \ \rho\ \rangle - \langle\ \rho\ ,\ \ \hat{n}\cdot\vec{p}\ \rangle$$

First Detection of kSZ

$$\hat{p}_{\mathrm{kSZ}}(r) = -\frac{\sum_{i<j}(\delta T_i - \delta T_j)\, c_{i,j}}{\sum_{i<j} c_{i,j}^2}$$



Hand et al. (2012) [See also Plank2015]

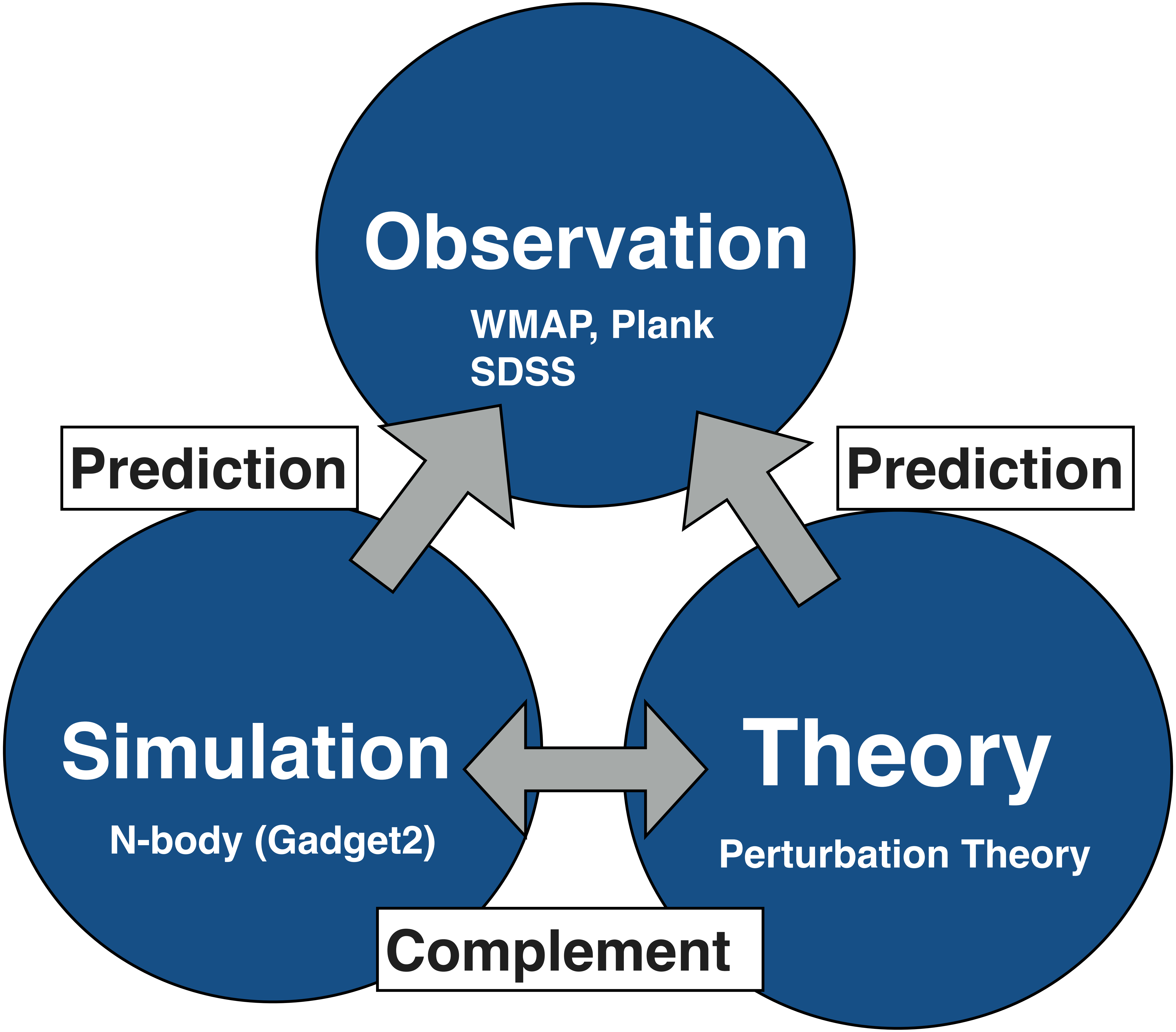


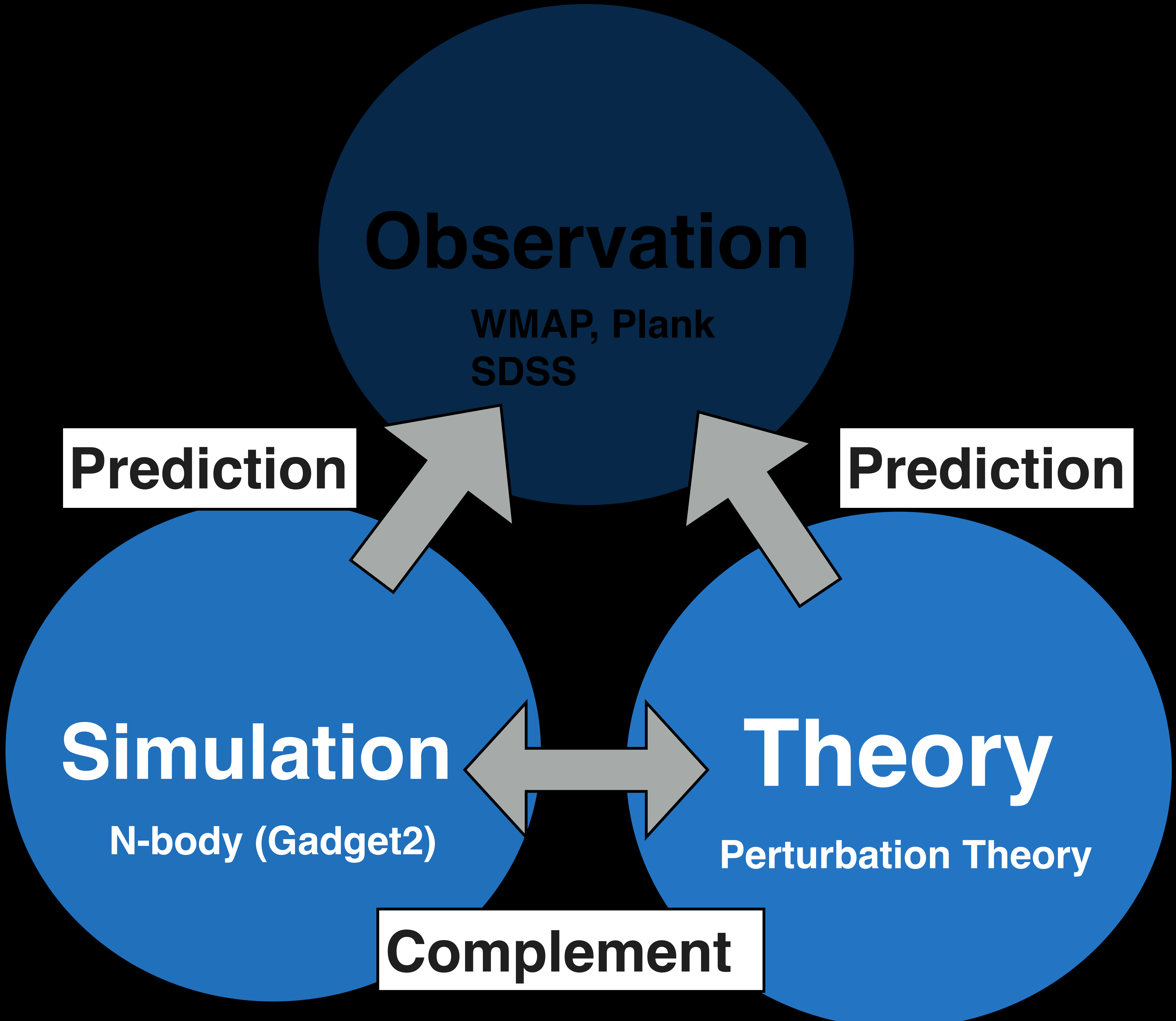
[Princeton University's Homepage]

Theoretical Modering of Momentum Field

Motivation

- 観測量の説明
- kSZ パワースペクトルの予言
- kSZ 高次モーメンタムの予言
- Multi-pole 展開


Observation
WMAP, Plank
SDSS
Prediction
Prediction
Simulation
N-body (Gadget2)
Theory
Perturbation Theory
Complement


Observation
WMAP, Plank
SDSS
Prediction
Prediction
Simulation
N-body (Gadget2)
Theory
Perturbation Theory
Complement

Theory vs. N-body

	Gravity	Resolusion	Box	Realization	Speed
Theory	Perturbation	Infinity	Infinity (Ideally)	Infinity	Fast
N-body	Full	Finite	Finite	Finite	Slow

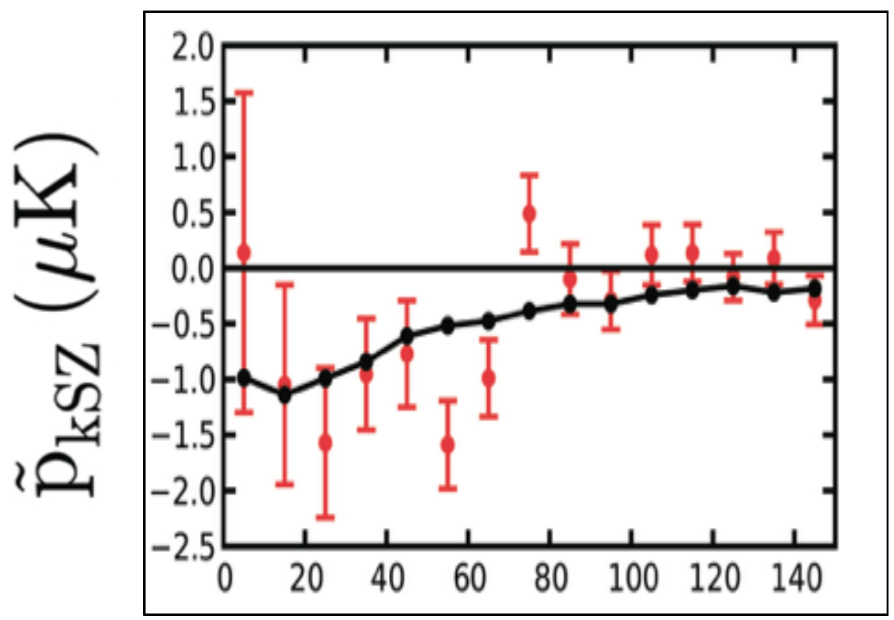
Theoretical Prediction: Linear Theory

$$
\begin{aligned}
\langle\, \hat{n}\cdot\vec{p}\;,\; \rho \,\rangle \;&=\; \bar{\rho}^2 \langle\, \hat{n}\cdot\vec{v}\;,\; \delta \,\rangle \\
&+\; \bar{\rho}^2 \langle\, \hat{n}\cdot\vec{v}\,\delta\;,\; \delta \,\rangle \\
&\sim\; \bar{\rho}^2 \langle\, \hat{n}\cdot\vec{v}\;,\; \delta \,\rangle
\end{aligned}
$$

線形理論では,

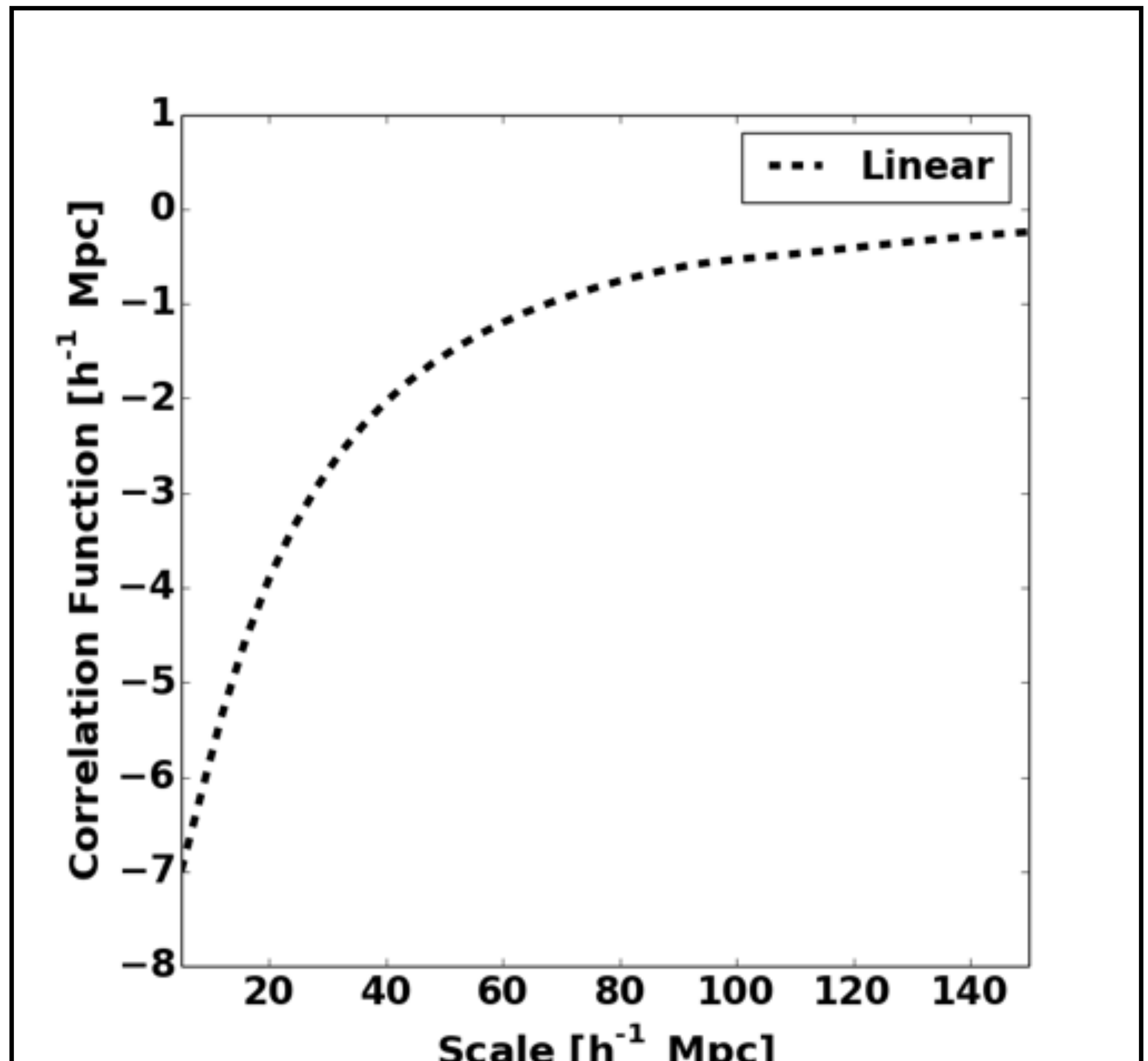
速度場と密度場の相関のみが残る。

Theoretical Prediction: Linear Theory



Mpc

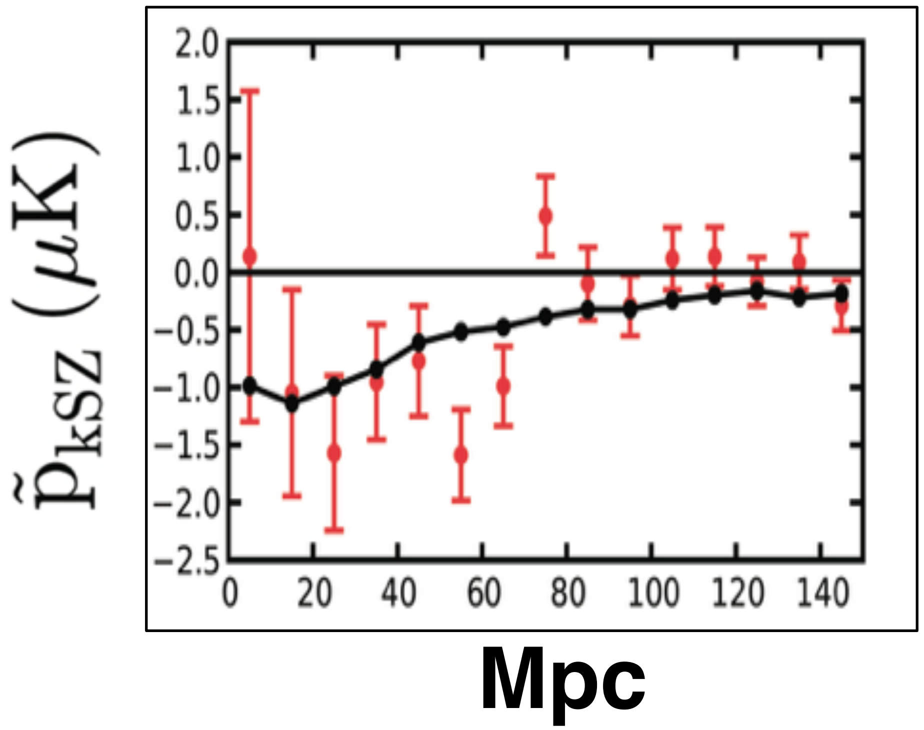
$\propto$



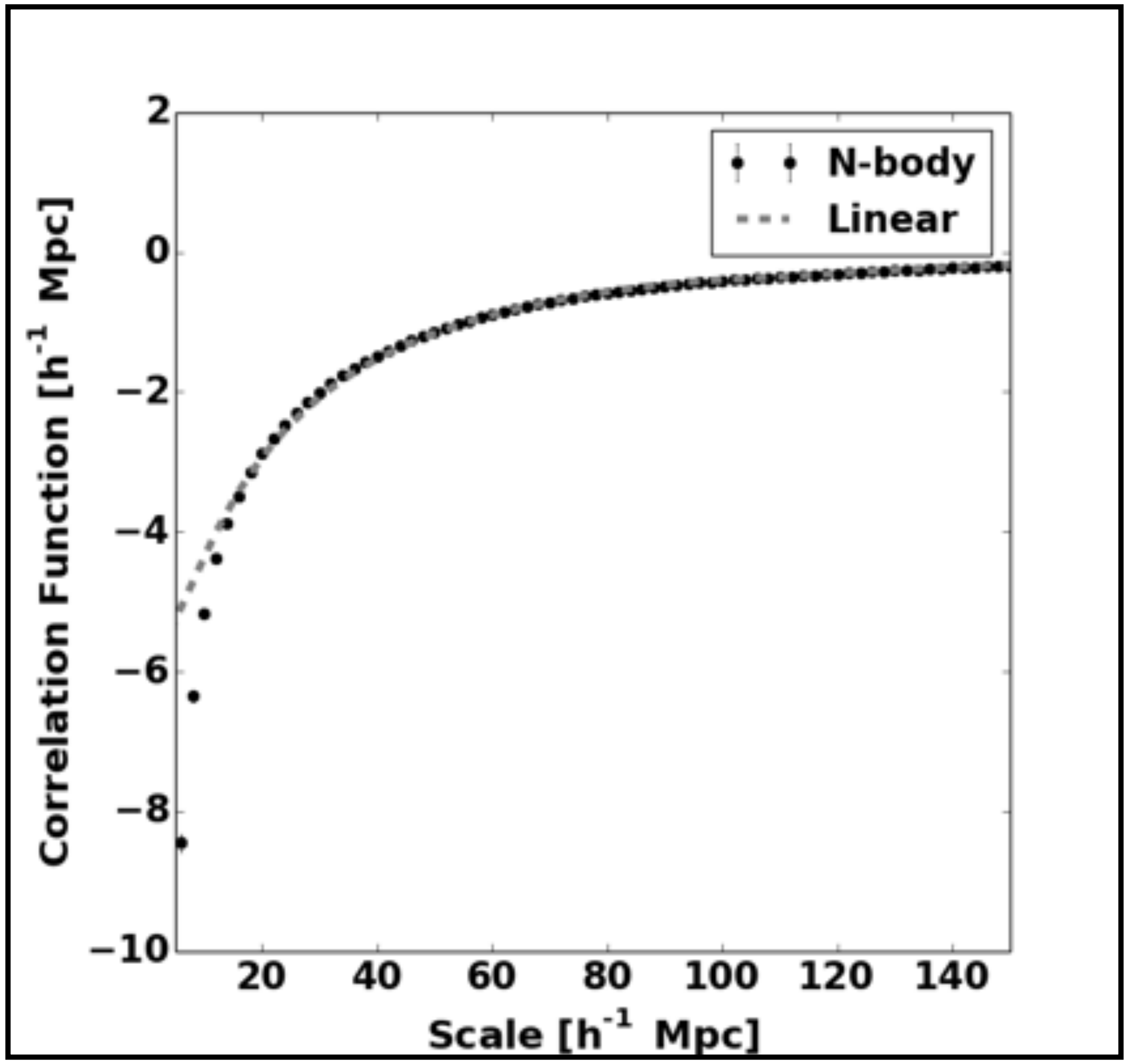
異なる規格化

Theoretical Prediction: N-body simulation

$$\sum_{i,j} [\hat{n} \cdot \vec{v}_i - \hat{n} \cdot \vec{v}_j] \; \delta_{\mathrm{D}} \left(\vec{r} - (\vec{x}_i - \vec{x}_j) \right)$$



$\propto$



Redshift Space Distortion

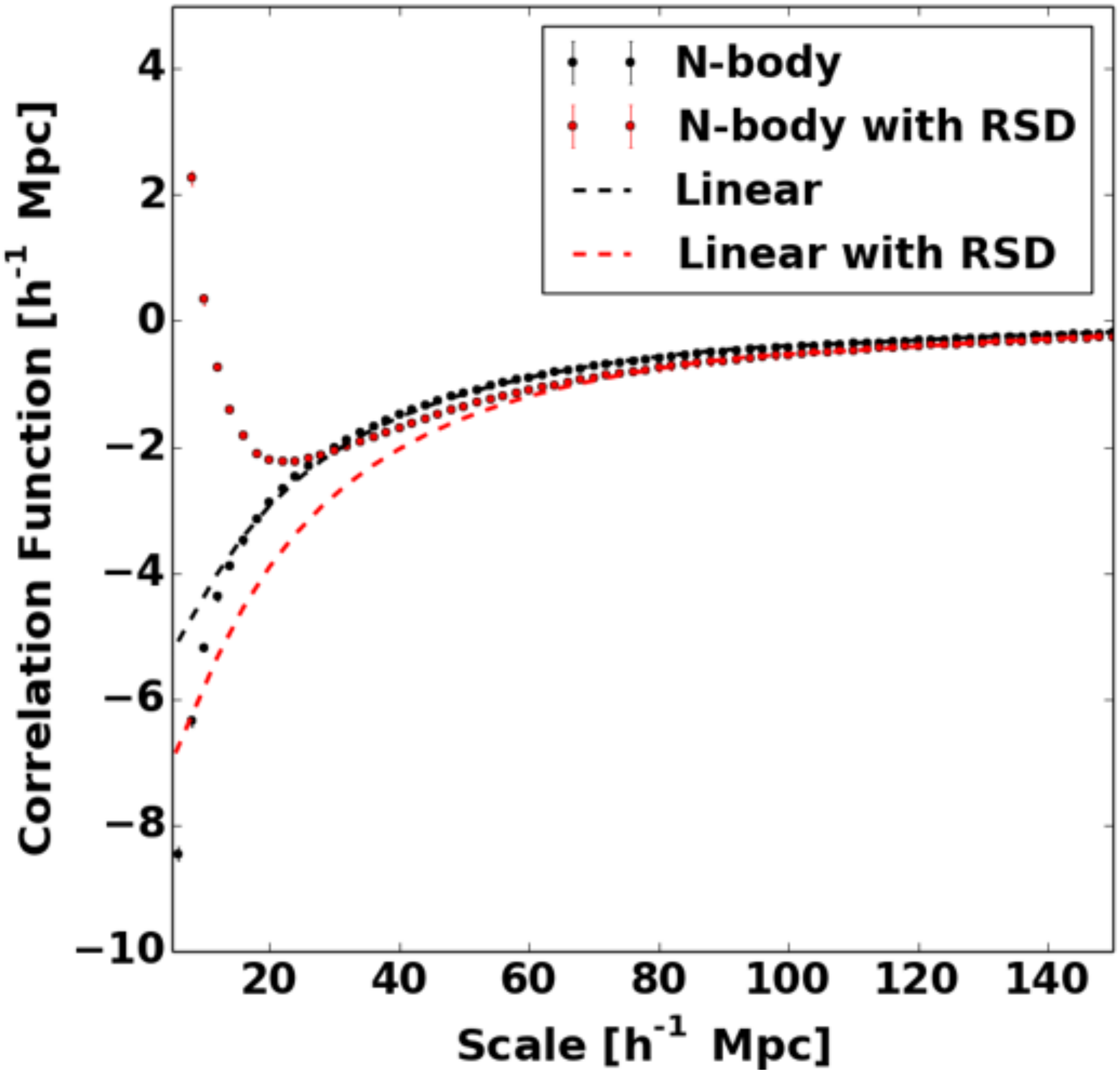
Coordinate transformation of particle positions

$$\vec{s} = \vec{x} + \underline{\frac{\hat{n} \cdot \vec{v}(\vec{x})}{aH}\hat{n},}$$

Coordinate transformation of correlation function

$$\sum_{i,j} [\hat{n} \cdot \vec{v}_i - \hat{n} \cdot \vec{v}_j]\, \delta_{\rm D} \left(\vec{r} - (\vec{x}_i - \vec{x}_j) - \underline{\left(\frac{\hat{n} \cdot \vec{v}_i}{aH} - \frac{\hat{n} \cdot \vec{v}_j}{aH} \right) \hat{n} \Bigg)} \right.$$

Simulation Result



RSD の重要性

Theoretical modering:
Non-linear Theory

$$
\begin{aligned}
\langle\, \hat{n}\cdot\vec{p}\,,\ \rho\,\rangle &= \bar{\rho}^2\langle\, \hat{n}\cdot\vec{v}\,,\ \delta\,\rangle \\
&+ \bar{\rho}^2\langle\, \hat{n}\cdot\vec{v}\,\delta\,,\ \delta\,\rangle
\end{aligned}
$$

RSD 込みの3点相関が必要。

Theoretical modering:
Non-linear Theory

$$\begin{aligned}\langle\, \hat{n}\cdot\vec{p}\,,\ \rho\,\rangle &= \bar{\rho}^2\langle\, \hat{n}\cdot\vec{v}\,,\ \delta\,\rangle \\ &+ \bar{\rho}^2\langle\, \hat{n}\cdot\vec{v}\,\delta\,,\ \delta\,\rangle\end{aligned}$$

より簡単な計算方法を提案。

Theoretical modering: Non-Linear Theory

$$\hat{\xi}(r) = \frac{V}{N_{\rm p}^2} \sum_{i,j} \underline{[\hat{n}\cdot\vec{v}_i - \hat{n}\cdot\vec{v}_j]}\, \delta_{\rm D}\left(\vec{r} - (\vec{x}_i - \vec{x}_j) - \underline{\left(\frac{\hat{n}\cdot\vec{v}_i}{aH} - \frac{\hat{n}\cdot\vec{v}_j}{aH}\right)\hat{n}}\right)$$

Fourier transformation

$$\hat{P}(k) = \frac{V}{N_{\rm p}^2} \sum_{i,j} \underline{[\hat{n}\cdot\vec{v}_i - \hat{n}\cdot\vec{v}_j]}\, e^{-i\vec{k}\cdot(\vec{x}_i - \vec{x}_j) \underline{- i\vec{k}\cdot\left(\frac{\hat{n}\cdot\vec{v}_i}{aH} - \frac{\hat{n}\cdot\vec{v}_j}{aH}\right)\hat{n}}}$$

Theoretical modering:
Non-Linear Theory

Momentum Power Spectrum

$$\hat{P}_{\rm p}^{(n)}(\vec{k}) = \left(i \frac{aH}{\vec{k}\cdot\hat{n}} \right)^n \left[\frac{d^n}{d^n\gamma} \hat{P}_{\rm p}^{(0)}(\vec{k};\gamma) \right] \Bigg|_{\gamma=1}$$

Density Power Spectrum (Generating Function)

$$\hat{P}_{\rm p}^{(0)}\left(\vec{k};\gamma\right) \equiv \frac{V}{N_{\rm p}^2} \sum_{i,j} \left[e^{-i\vec{k}\cdot\vec{x}_{ij} - i\gamma \frac{\vec{k}\cdot\hat{n}}{aH}(\hat{n}\cdot\vec{v}_i - \hat{n}\cdot\vec{v}_j)} \right]$$

Theoretical modering:
Non-linear Theory

Main result

$$\hat{P}_{\mathrm{p}}^{(n)}(\vec{k},\hat{n}) = \left(i\frac{aHf}{\vec{k}\cdot\hat{n}}\right)^n \frac{\partial^n}{\partial^n f}\hat{P}_{\mathrm{m}}(D,f,\vec{k},\hat{n}).$$

From

$$\vec{v} \propto f = \frac{d\ln D}{d\ln a} \quad \text{for} \quad f = \Omega_{\mathrm{m}}^{0.5}$$

モーメントパワースペクトルの理論予言は,
密度パワースペクトルから求まる。

Theoretical modering:
Non-linear Theory

Main result

$$\hat{P}_{\mathrm{p}}^{(n)}(\vec{k},\hat{n}) = \left(i\frac{aHf}{\vec{k}\cdot\hat{n}}\right)^n \frac{\partial^n}{\partial^n f}\hat{P}_{\mathrm{m}}(D,f,\vec{k},\hat{n}).$$

- 摂動展開とは関係なく，一般的に成り立つ。
- どんな摂動論やfitting formula にも成り立つ。
- ハローでも成立する。
- 密度パワースペクトル理論の妥当性のチェックにも使えるかも

摂動論のお話

Infinite Mode-Coupling

Power Spectrum

$$P(\vec{k}) = \left\langle \frac{V}{N_{\mathrm{p}}^2} \sum_{i,j} e^{-i\vec{k}\cdot(\vec{x}_i - \vec{x}_j)} \right\rangle$$

$$= \int d^3q\, e^{-i\vec{k}\cdot(\vec{q}_1 - \vec{q}_2)} \left\langle e^{-i\vec{k}\cdot\left(\vec{\Psi}(\vec{q}_1) - \vec{\Psi}(\vec{q}_2)\right)} \right\rangle \quad \text{, where} \quad \vec{q} = \vec{q}_1 - \vec{q}_2$$

連続極限

パワースペクトルを計算する際には,
必ず空間積分が生じる。

Infinite Mode-Coupling

パワースペクトルを分解すると。。。

$$= \int d^3q e^{-i\vec{k}\cdot(\vec{q}_1-\vec{q}_2)} \left\langle 1 + \left(-i\vec{k}\cdot\left(\vec{\Psi}(\vec{q}_1)-\vec{\Psi}(\vec{q}_2)\right)\right) + \frac{1}{2}\left(-i\vec{k}\cdot\left(\vec{\Psi}(\vec{q}_1)-\vec{\Psi}(\vec{q}_2)\right)^2 + \dots\right)\right\rangle$$

$$= \Gamma^{(1)}(k)P_{\rm lin}(k) + \frac{1}{2}\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}(2\pi)^3\delta_{\rm D}\left(\vec{k}-\vec{k}_1-\vec{k}_2\right)\left[\Gamma^{(2)}(\vec{k}_1,\vec{k}_2)\right]^2 P_{\rm lin}(k_1)P_{\rm lin}(k_2) + \dots$$

重力とは関係なく,

パワースペクトルを計算するために,

無限のモードカップリング積分が必要。

Infinite Mode-Coupling

Power Spectrum

$$\begin{aligned} P(\vec{k}) &= \left\langle \frac{V}{N_{\mathrm{p}}^2} \sum_{i,j} e^{-i\vec{k}\cdot(\vec{x}_i - \vec{x}_j)} \right\rangle \\ &= \int d^3q e^{-i\vec{k}\cdot(\vec{q}_1 - \vec{q}_2)} \left\langle e^{-i\vec{k}\cdot\left(\vec{\Psi}(\vec{q}_1) - \vec{\Psi}(\vec{q}_2)\right)} \right\rangle \quad \text{, where} \quad \vec{q} = \vec{q}_1 - \vec{q}_2 \end{aligned}$$

計算方法:

- 可能な限りパワースペクトルを展開せずに, 空間積分を直接計算する。
- Displacement Vector を摂動展開する。

Infinite Mode-Coupling

Power Spectrum

$$P(\vec{k}) = \left\langle \frac{V}{N_{\mathrm{p}}^2} \sum_{i,j} e^{-i\vec{k}\cdot(\vec{x}_i - \vec{x}_j)} \right\rangle$$
$$= \int d^3q e^{-i\vec{k}\cdot(\vec{q}_1 - \vec{q}_2)} \left\langle e^{-i\vec{k}\cdot\left(\vec{\Psi}(\vec{q}_1) - \vec{\Psi}(\vec{q}_2)\right)} \right\rangle \quad \text{, where} \quad \vec{q} = \vec{q}_1 - \vec{q}_2$$

$$\vec{s} = \vec{x} + \frac{\hat{n}\cdot\vec{v}(\vec{x})}{aH}\hat{n},$$

RSD における座標変換を

テイラー展開をせずに計算することに対応。

Analogy to δN formalism

Curvature Perturbation

$$N\left(\bar{\rho}, \bar{\varphi}_*^a\right) + \zeta(\vec{x}) = N\left(\bar{\rho}, \varphi_*^a(\vec{x})\right)$$

通常はスカラー場で展開するところを。。。

$$N\left(\bar{\rho}, \bar{\varphi}_*^a\right) + \zeta(\vec{x}) = \int \frac{d\alpha}{2\pi} \left[e^{i\alpha\varphi_*(\vec{x})}\right] N\left[\bar{\rho}, \alpha\right]$$

無限のモードカップリングが計算可能？

Perturbation Theory

密度 ρ

重力 $\vec{v} \sim \dot{\vec{\Psi}} \sim \vec{\Psi}$

	1	3	5	7	~~	Full
1	Linear					ZA
3		3SPT (1-loop)				3LPT
5			5SPT (2-loop)			5LPT
7				7SPT (3-loop)		
~~						
Full						N-body

Perturbation Theory

密度 ρ

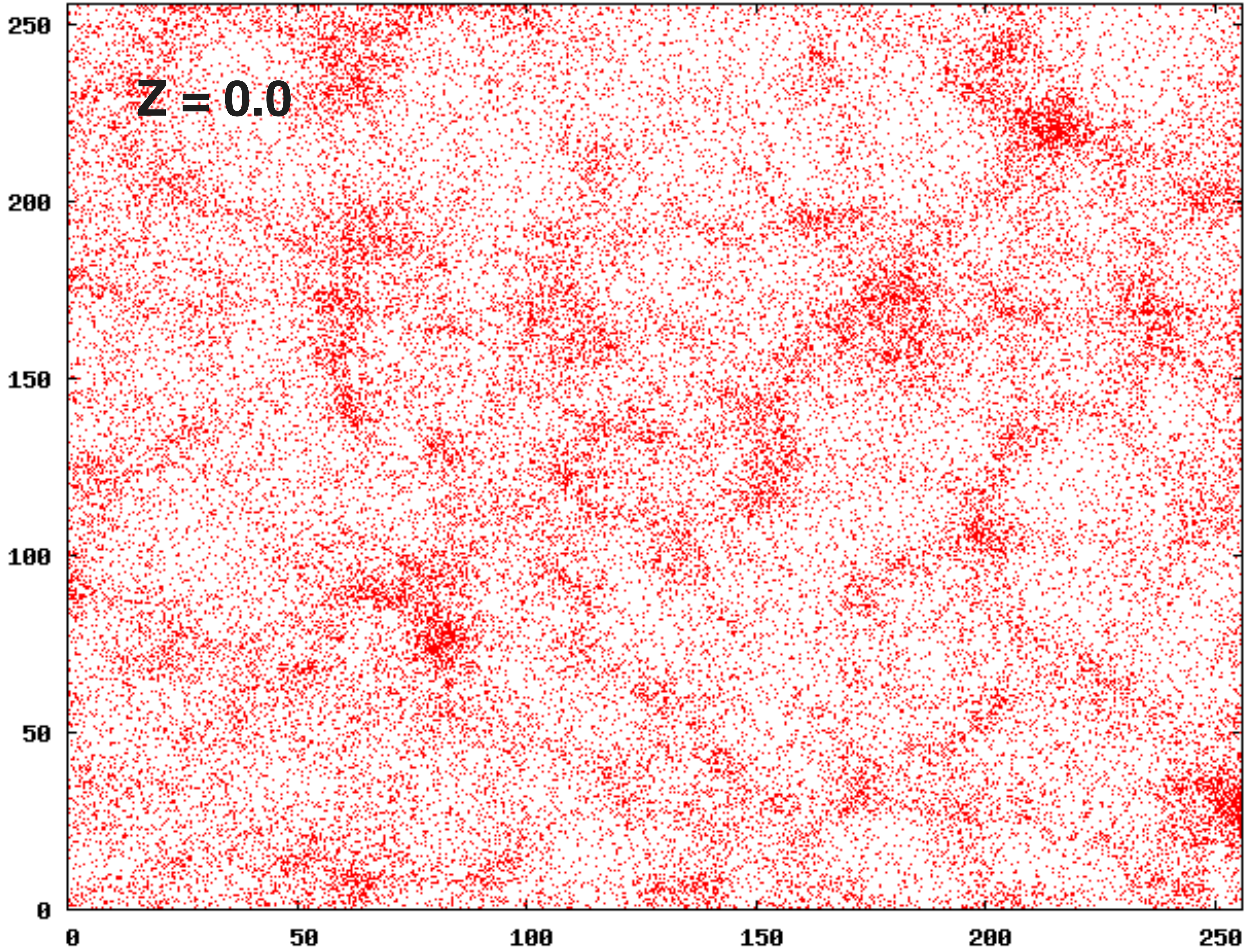
重力 $\vec{v} \sim \dot{\vec{\Psi}} \sim \vec{\Psi}$

	1	3	5	7	~~	Full
1	Linear					ZA
3		3SPT (1-loop)				3LPT
5			5SPT (2-loop)			5LPT
7				7SPT (3-loop)		
~~						
Full						N-body

Improved PT

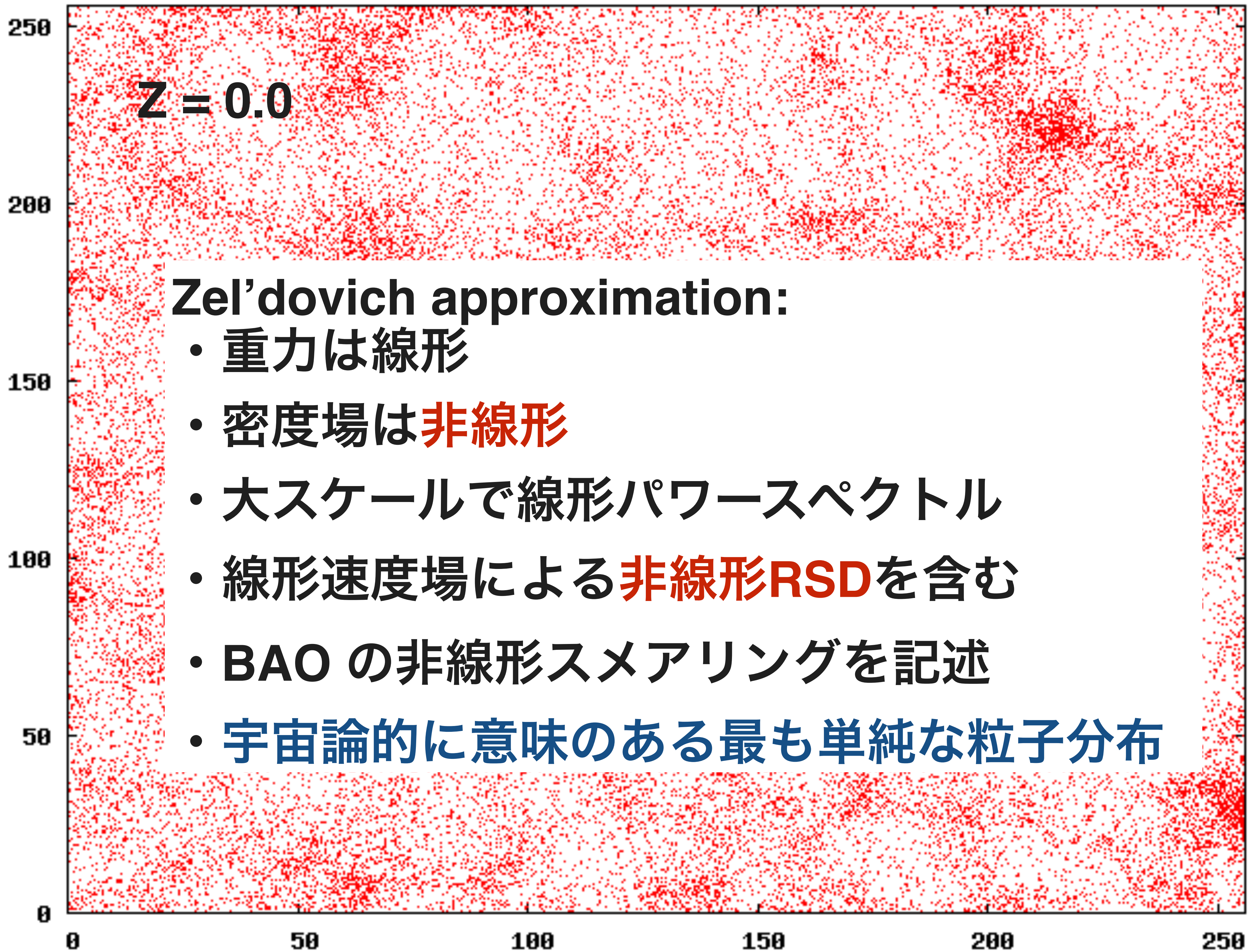
Zel'dovich Approximation

ZA



Zel’dovich Approximation

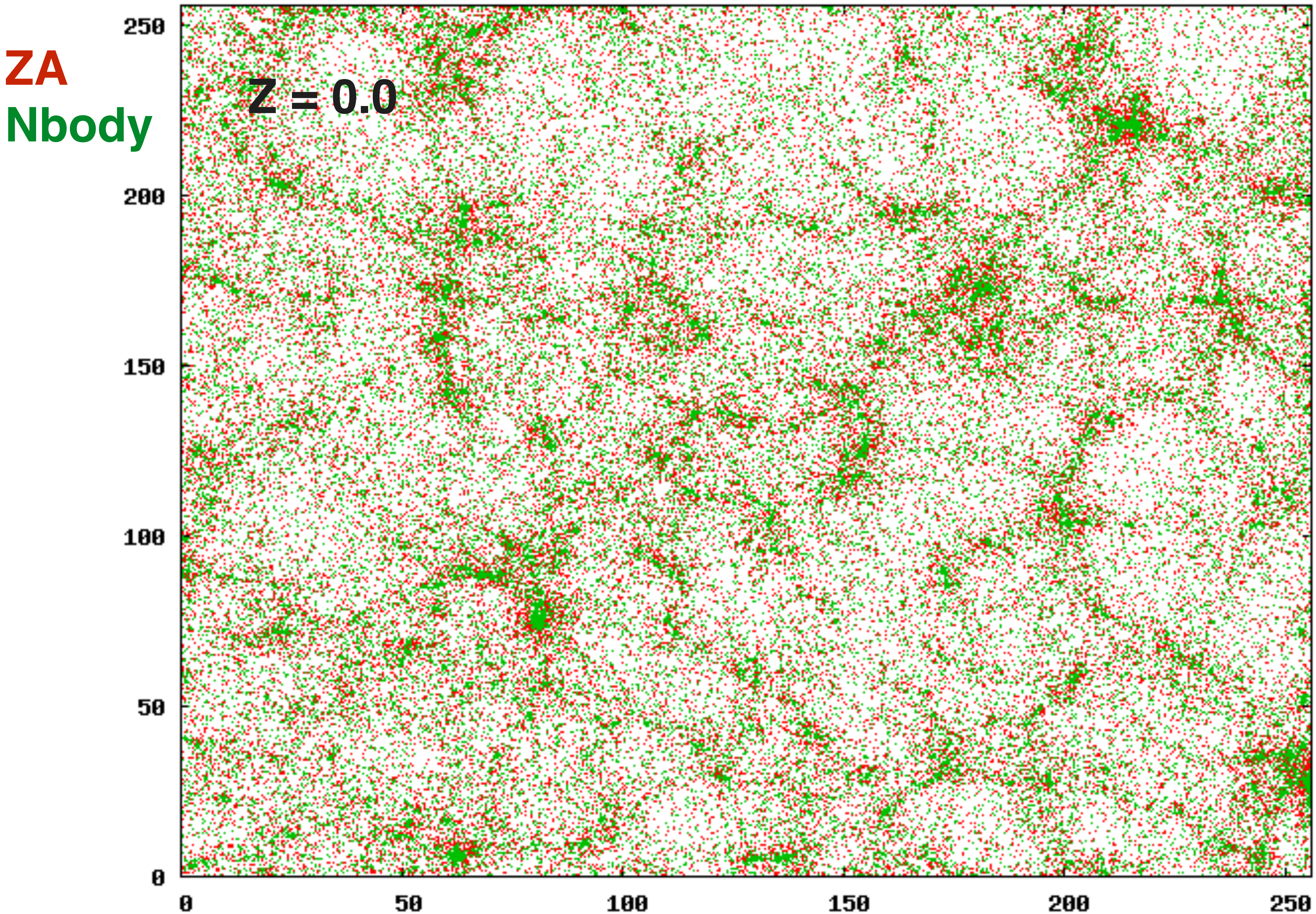
ZA



Zel’dovich approximation:

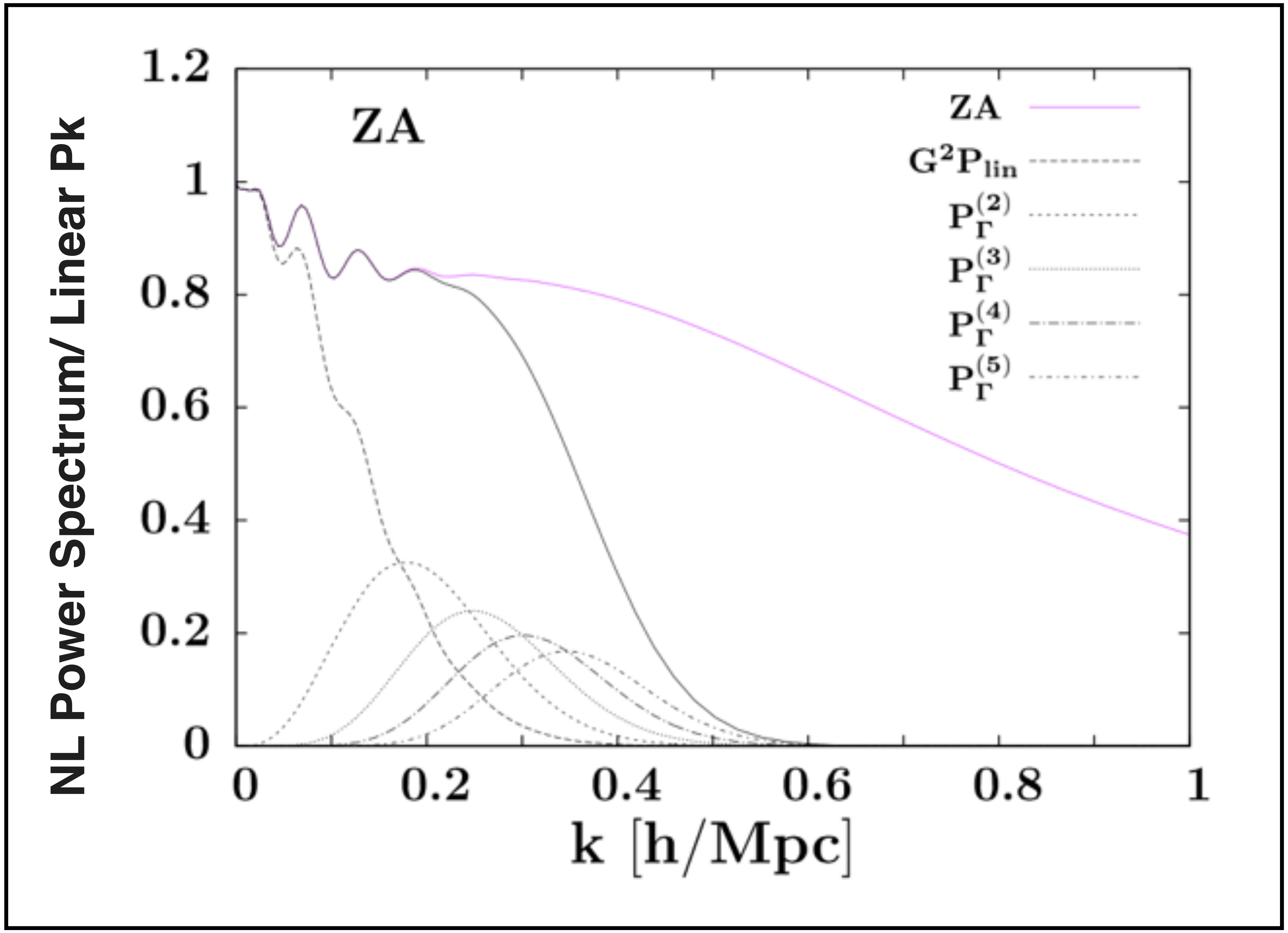
- 重力は線形
- 密度場は**非線形**
- 大スケールで線形パワースペクトル
- 線形速度場による**非線形RSD**を含む
- BAO の非線形スメアリングを記述
- 宇宙論的に意味のある最も単純な粒子分布

Zel’dovich Approximation



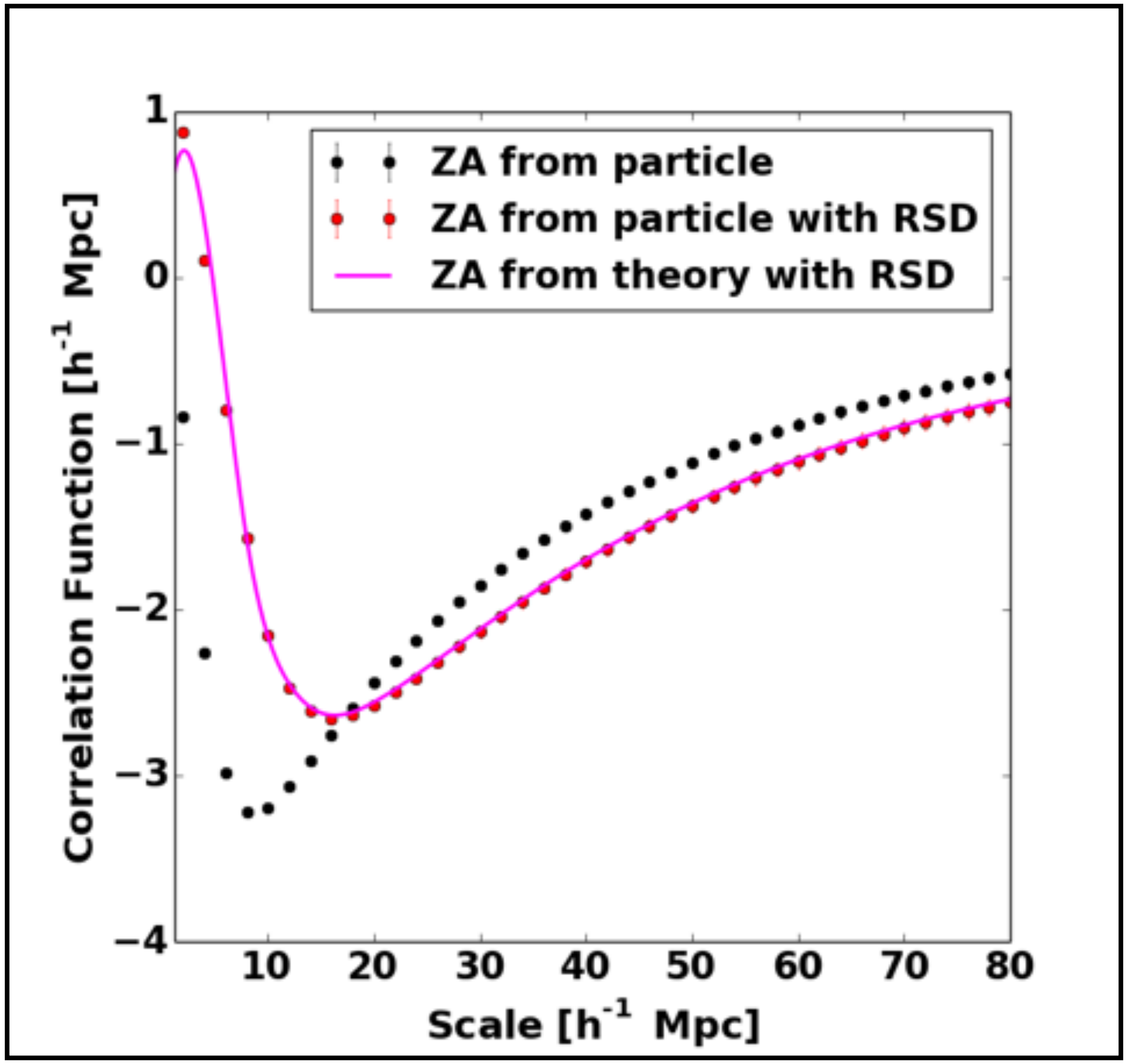
Zel’dovich Approximation

Density Power Spectrum in
Gamma-Expansion method
(Wiener Hermite expansion (Sugiyama and Futamase), iPT, or etc.)

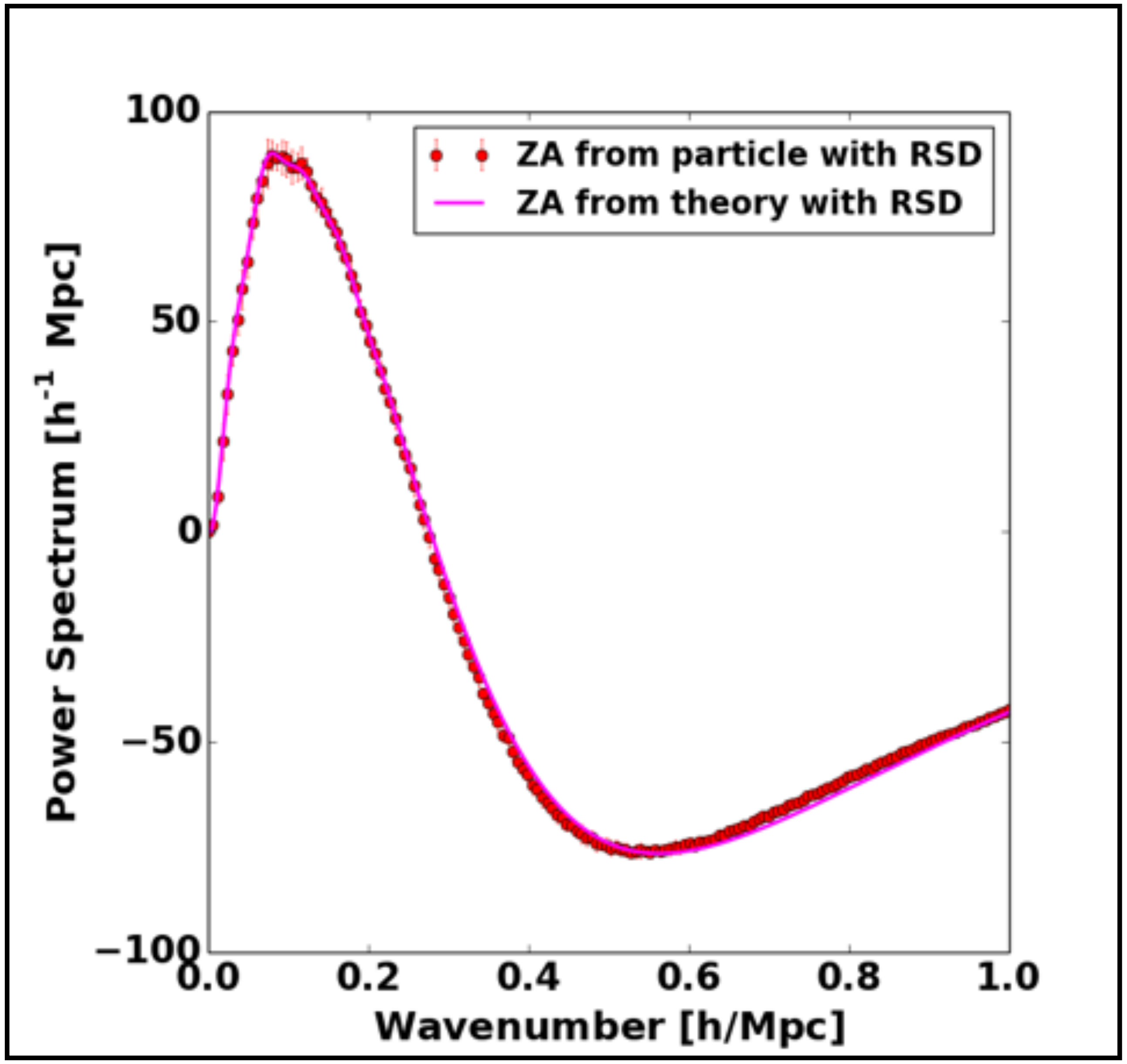


Zel'dovich Approximation

Momentum Correlation Function

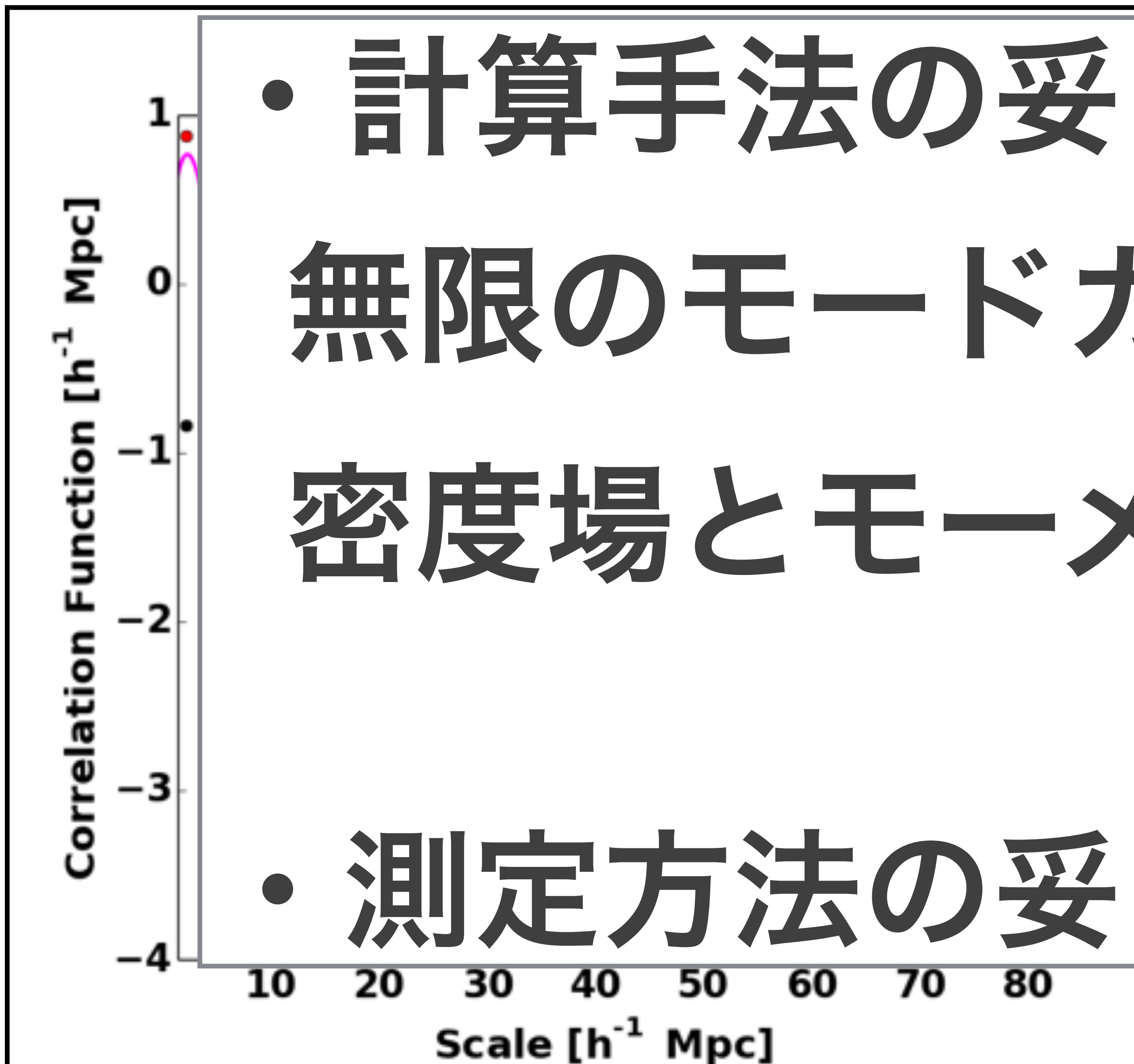


Momentum Power Spectrum

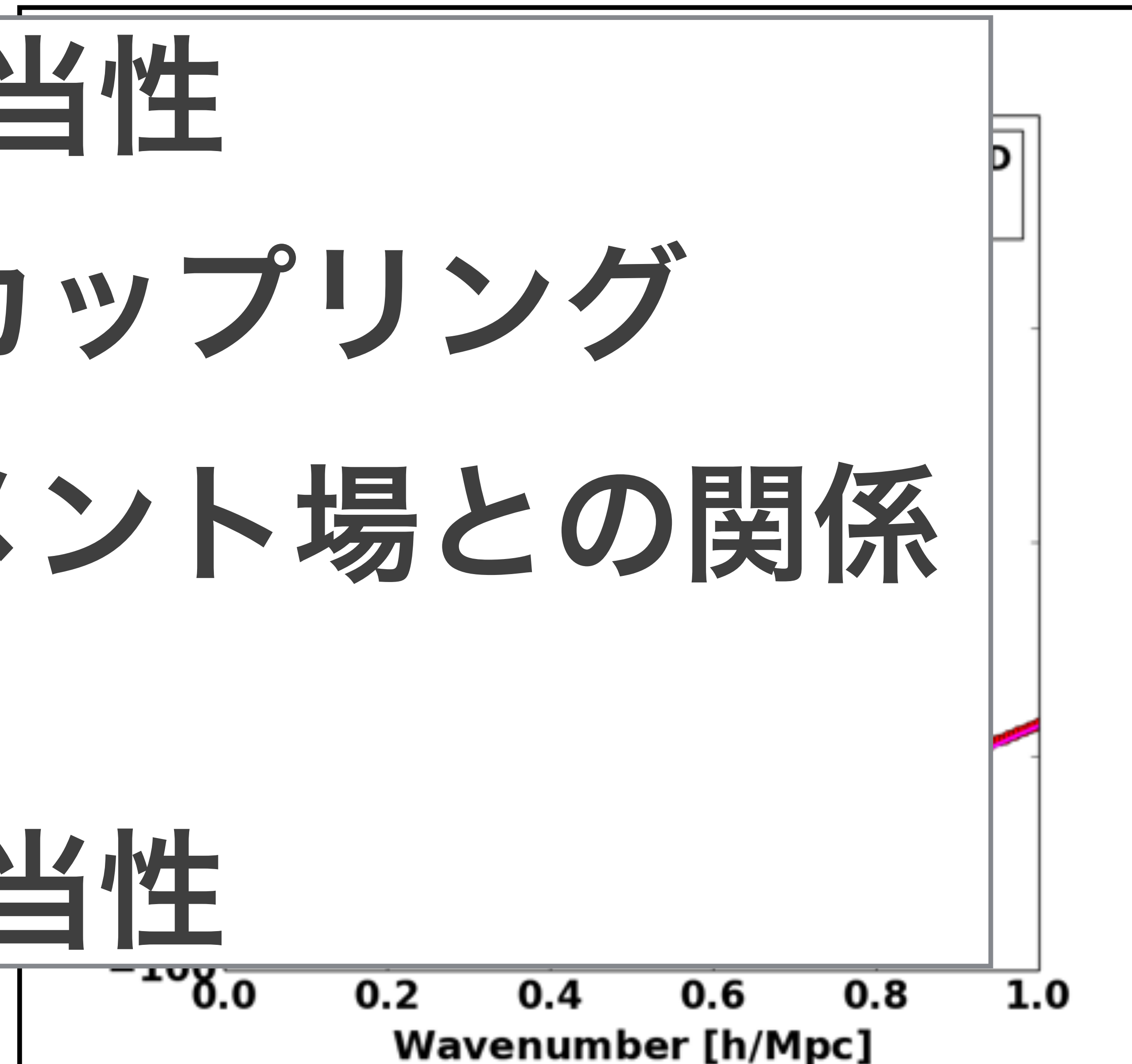


Zel’dovich Approximation

Momentum Correlation Function



Momentum Power Spectrum



- 計算手法の妥当性

  無限のモードカップリング

  密度場とモーメント場との関係

- 測定方法の妥当性

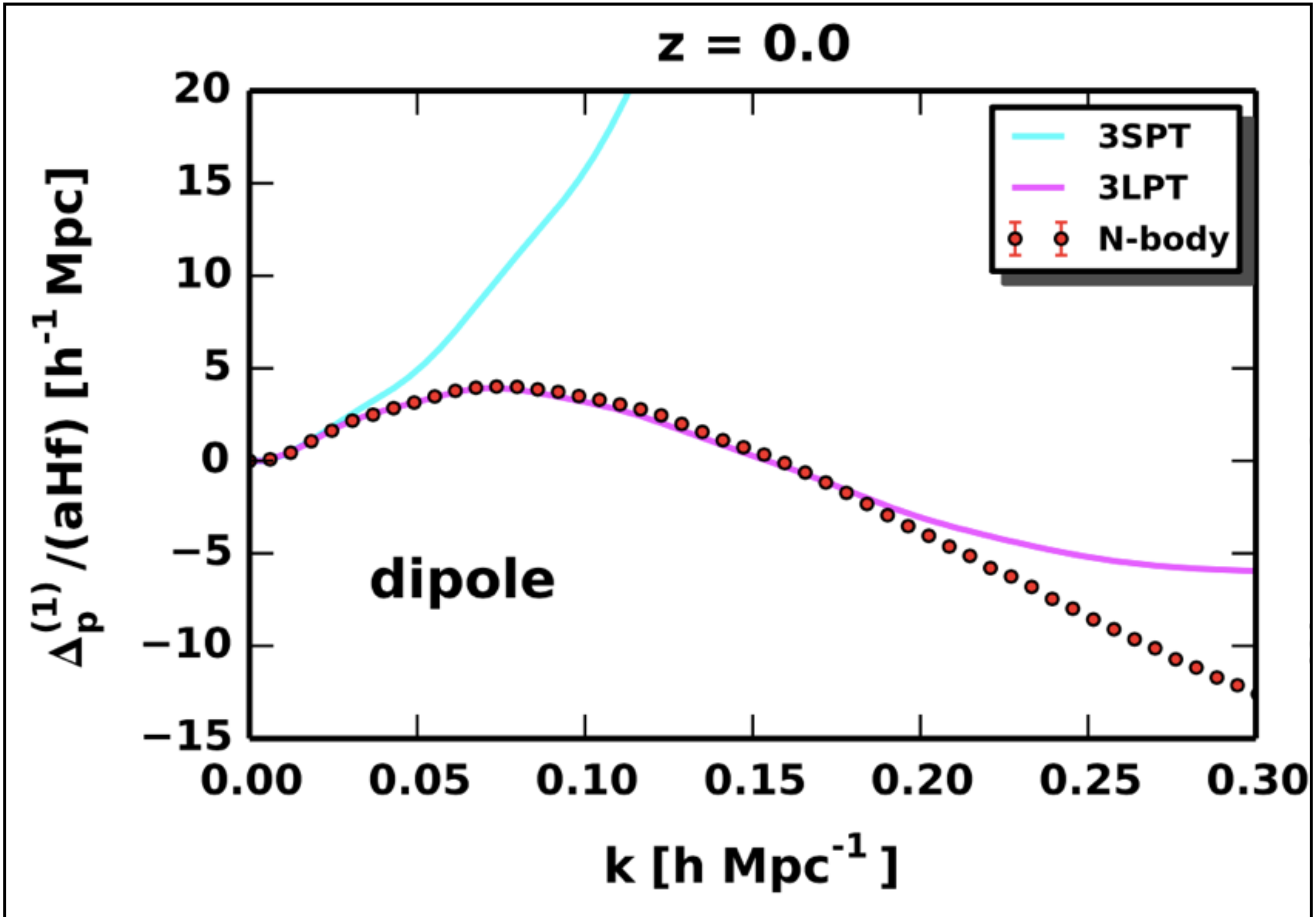
Perturbation Theory

密度 ρ

重力 $\vec{v} \sim \dot{\vec{\Psi}} \sim \vec{\Psi}$

	1	3	5	7	~~	Full
1	Linear					ZA
3		3SPT (1-loop)				3LPT
5			5SPT (2-loop)			5LPT
7				7SPT (3-loop)		
~~						
Full						N-body

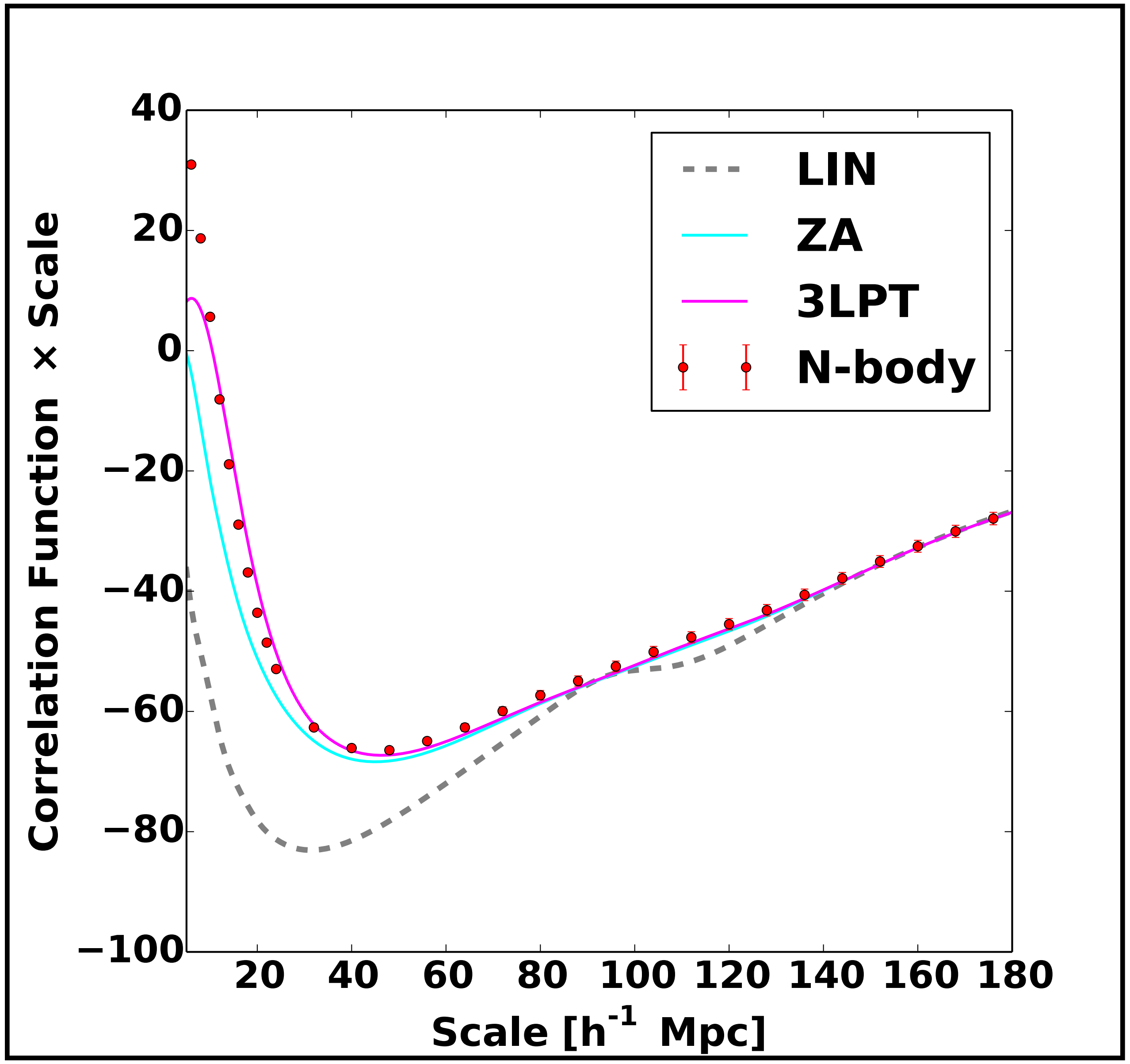
vs. SPT



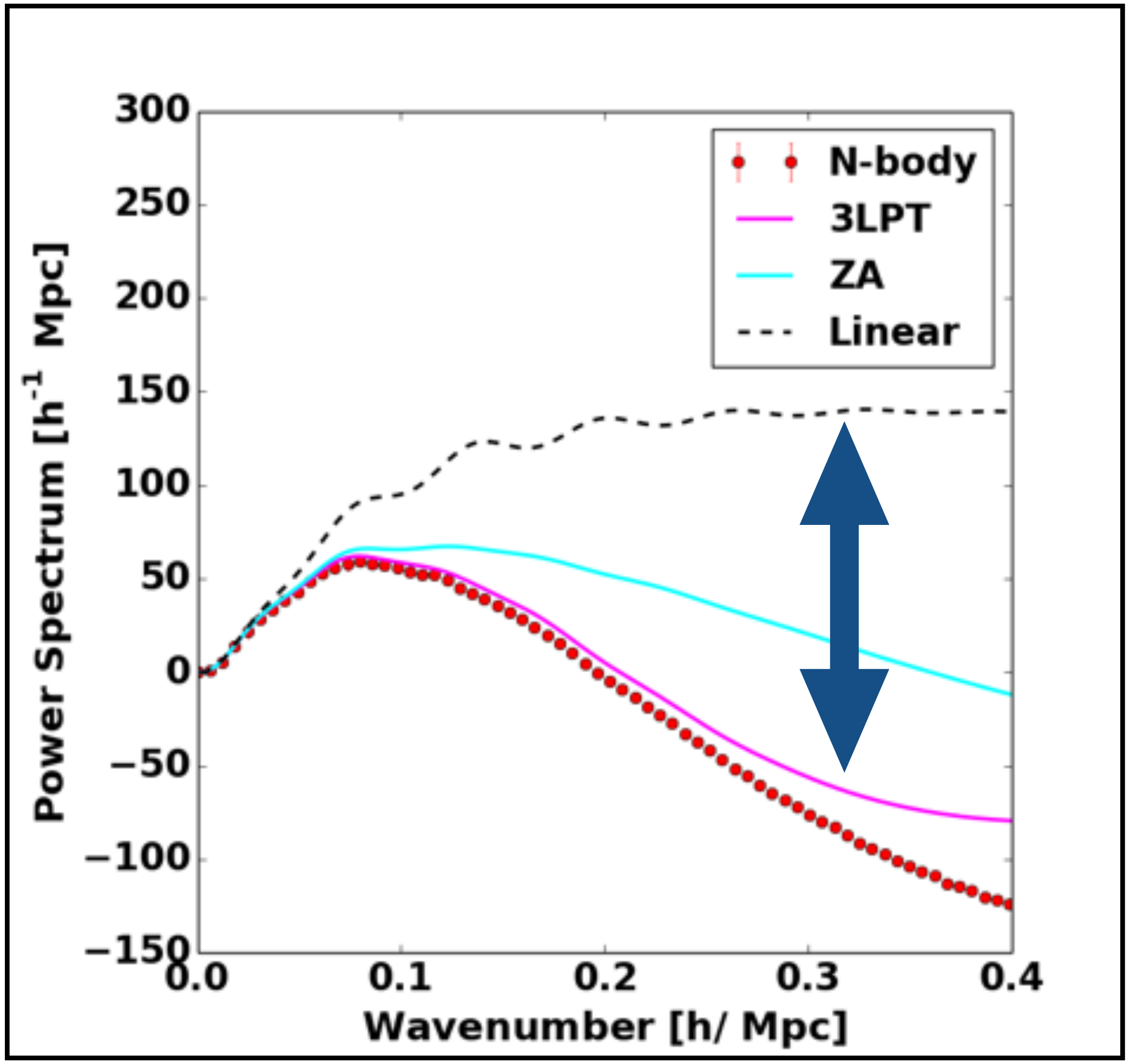
z = 0.0
3SPT
3LPT
N-body
dipole
$\Delta_p^{(1)}/(aHf)$ [h^{-1} Mpc]
k [h Mpc^{-1}]
20
15
10
5
0
−5
−10
−15
0.00
0.05
0.10
0.15
0.20
0.25
0.30

Third Order PT

Momentum Correlation Function



Momentum Power Spectrum



Higher order of Momentum Field

$$\hat{\xi}(r) = \frac{V}{N_{\rm p}^2} \sum_{i,j} \underline{[\hat{n}\cdot\vec{v}_i - \hat{n}\cdot\vec{v}_j]^2}\, \delta_{\rm D}\left(\vec{r} - (\vec{x}_i - \vec{x}_j) - \left(\frac{\hat{n}\cdot\vec{v}_i}{aH} - \frac{\hat{n}\cdot\vec{v}_j}{aH}\right)\hat{n}\right)$$

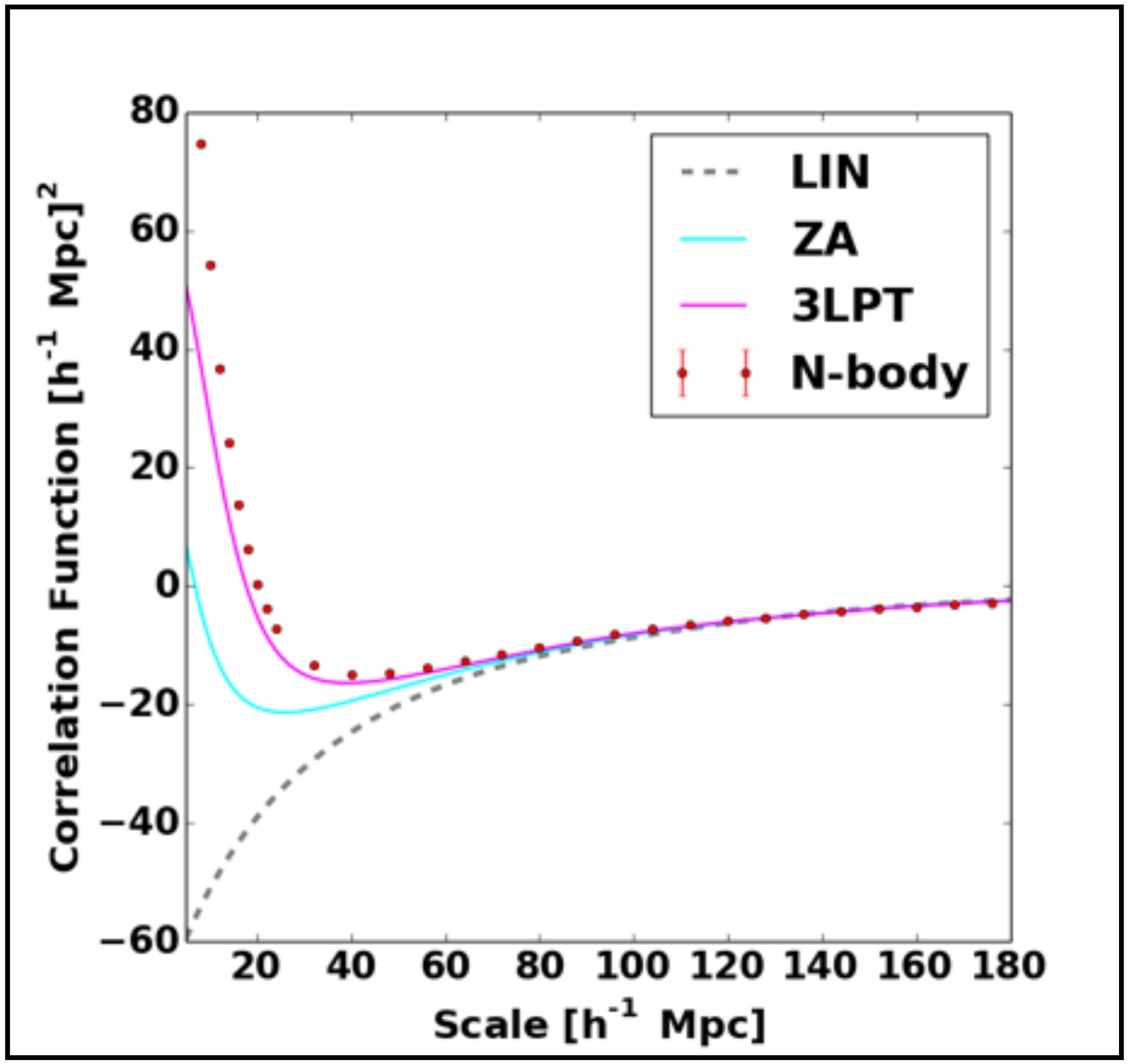
Linear Theory

$$\langle\ \hat{n}\cdot\vec{v},\ \hat{n}\cdot\vec{v}\ \rangle$$

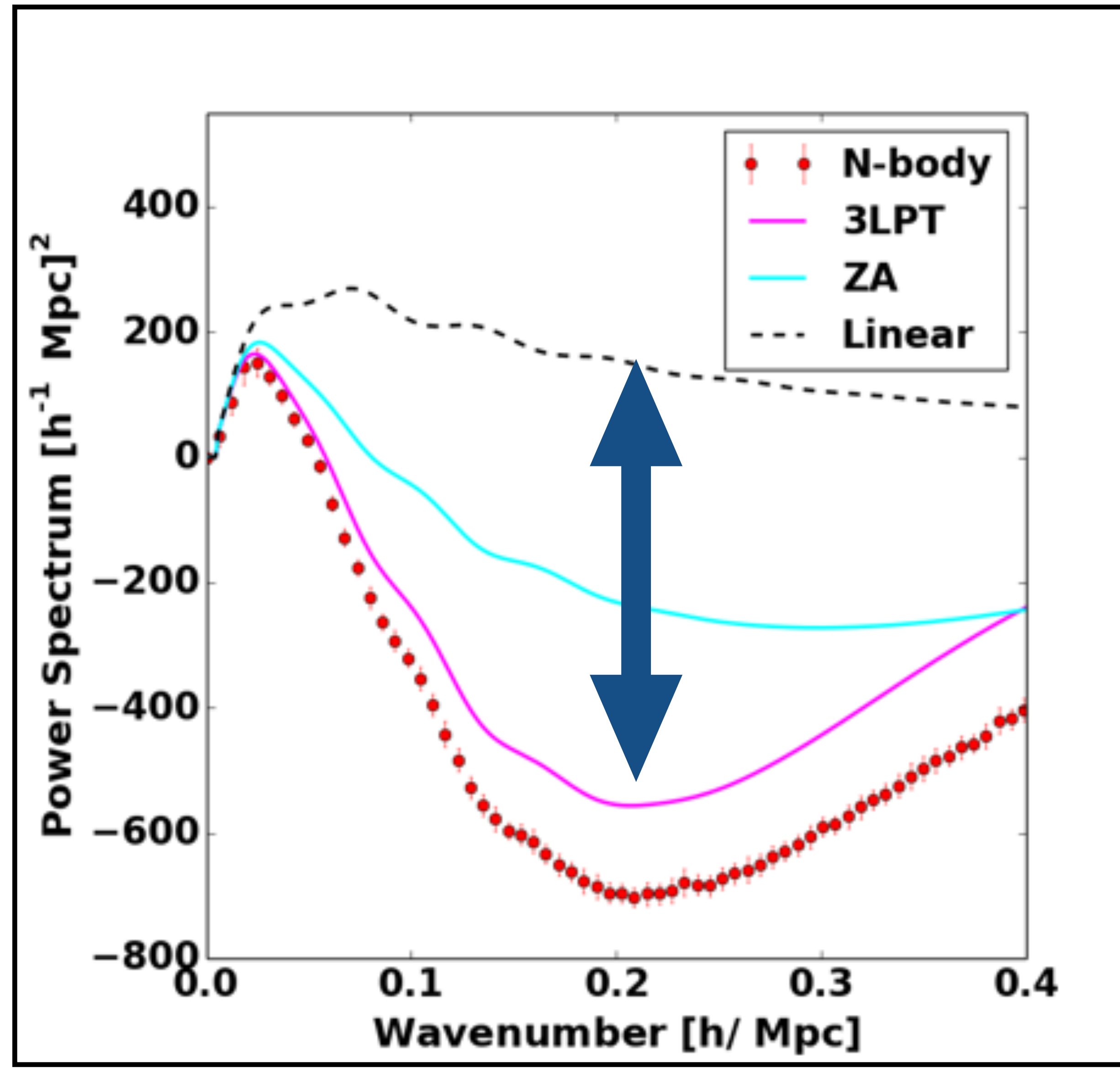
Halo Bias Free

Higher order of Momentum Field

Momentum Correlation Function



Momentum Power Spectrum

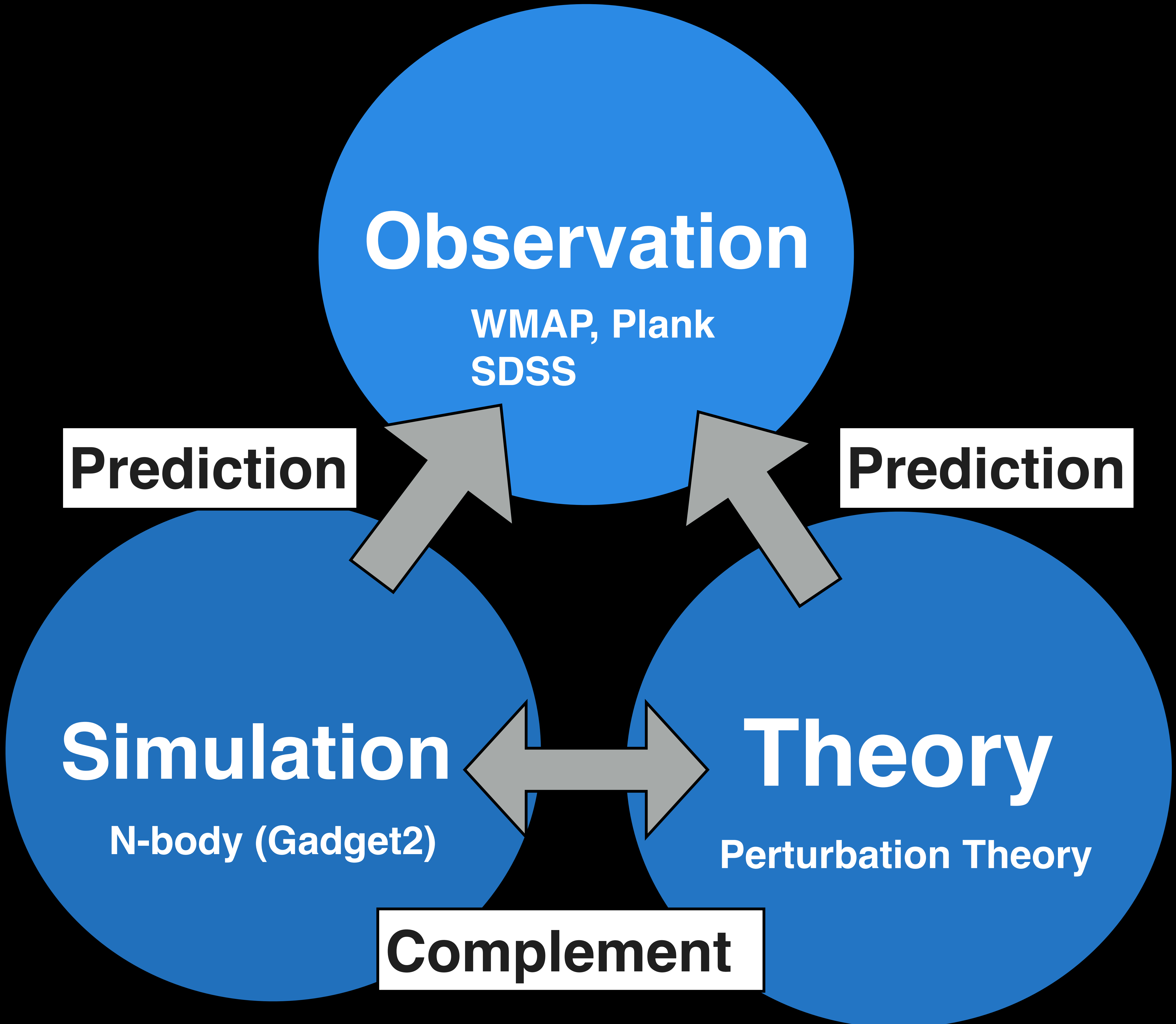


Summary

$$\hat{P}_{\mathrm{p}}^{(n)}(\vec{k},\hat{n}) = \left(i\frac{aHf}{\vec{k}\cdot\hat{n}}\right)^{n}\frac{\partial^{n}}{\partial^{n}f}\hat{P}_{\mathrm{m}}(D,f,\vec{k},\hat{n}).$$

Future Work

- **Covariance matrix (computing)**
- **Halo (computing)**
- **Measurement of kSZ power spectrum**


Observation
WMAP, Plank
SDSS
Prediction
Prediction
Simulation
N-body (Gadget2)
Theory
Perturbation Theory
Complement

Extra Slides

Fisher Analysis

$$\frac{\partial P_{\rm s}^{(n)}(\vec{k})}{\partial \ln f} = nP_{\rm s}^{(n)}(\vec{k}) + \left(i\frac{aH}{\vec{k}\cdot\hat{n}}\right)^{-1} P_{\rm s}^{(n+1)}(\vec{k})$$

Measurable in simulations

Covariance Matrix

$$\mathrm{Cov}\left(\hat{P}^{(n_1)}_{\ell_1}(k_1), \hat{P}^{(n_2)}_{\ell_2}(k_2)\right) = \frac{\delta^{(\mathrm{K})}_{k_1,k_2}}{N_{\mathrm{mode}}(k_1)} C^{(n_1)(n_2)}_{\ell_1\ell_2}(k_1) + T^{(n_1)(n_2)}_{\ell_1\ell_2}(k_1, k_2)$$

Gaussian term　non-Gaussian term

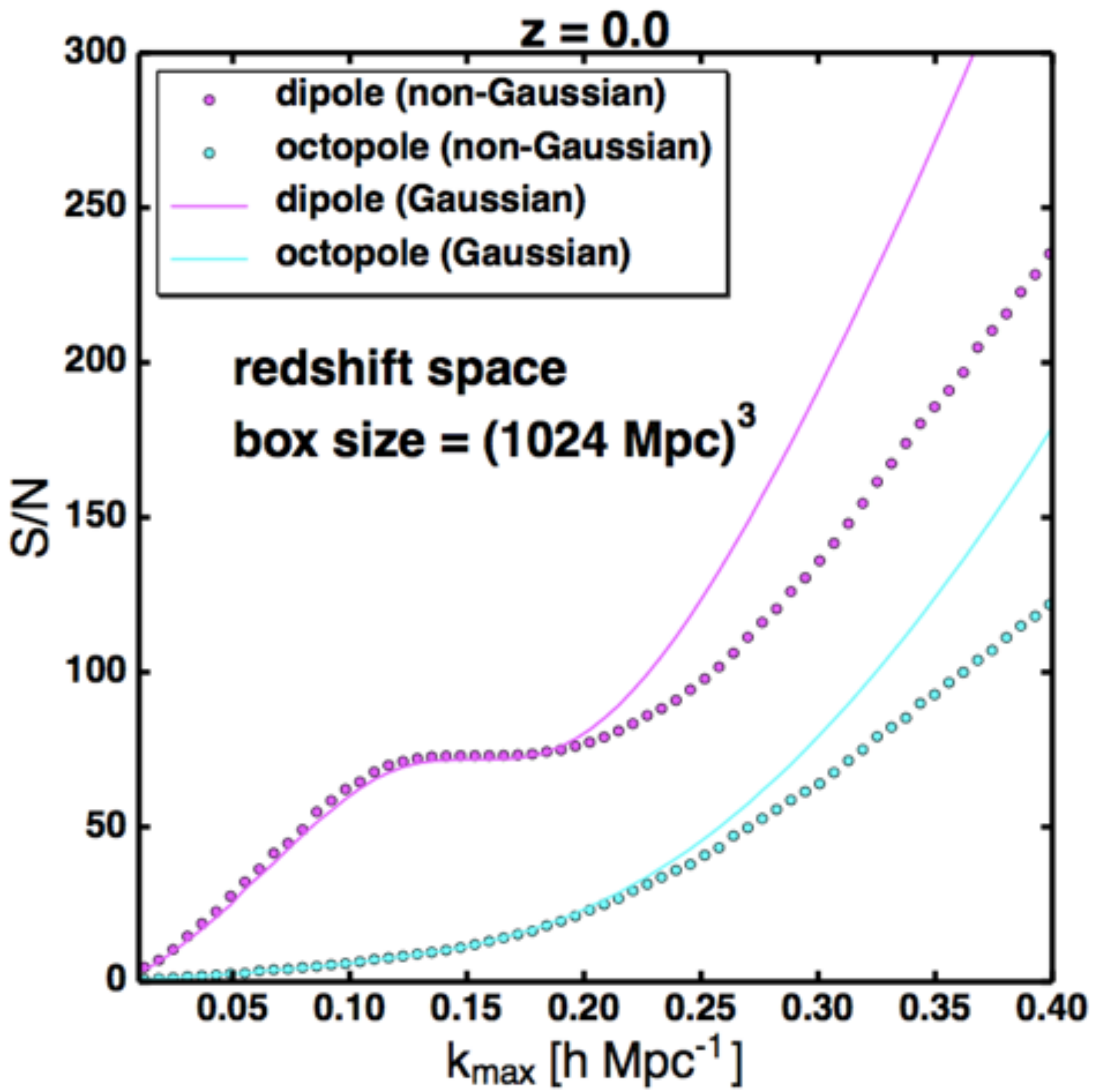
Gaussian term

$$C^{(1)(1)}_{\ell_1\ell_2}(k) = \left[1 + (-1)^{\ell_2+1}\right] \frac{(2\ell_1+1)(2\ell_2+1)}{2} \int d\mu \mathcal{L}_{\ell_1}(\mu)\mathcal{L}_{\ell_2}(\mu) \times \left[-2P^{(0)(0)}(\vec{k})P^{(1)(1)}(\vec{k}) + 2P^{(1)(0)}(\vec{k})P^{(1)(0)}(\vec{k})\right].$$

Gaussian limit でも複雑なスケール依存性を持つ。

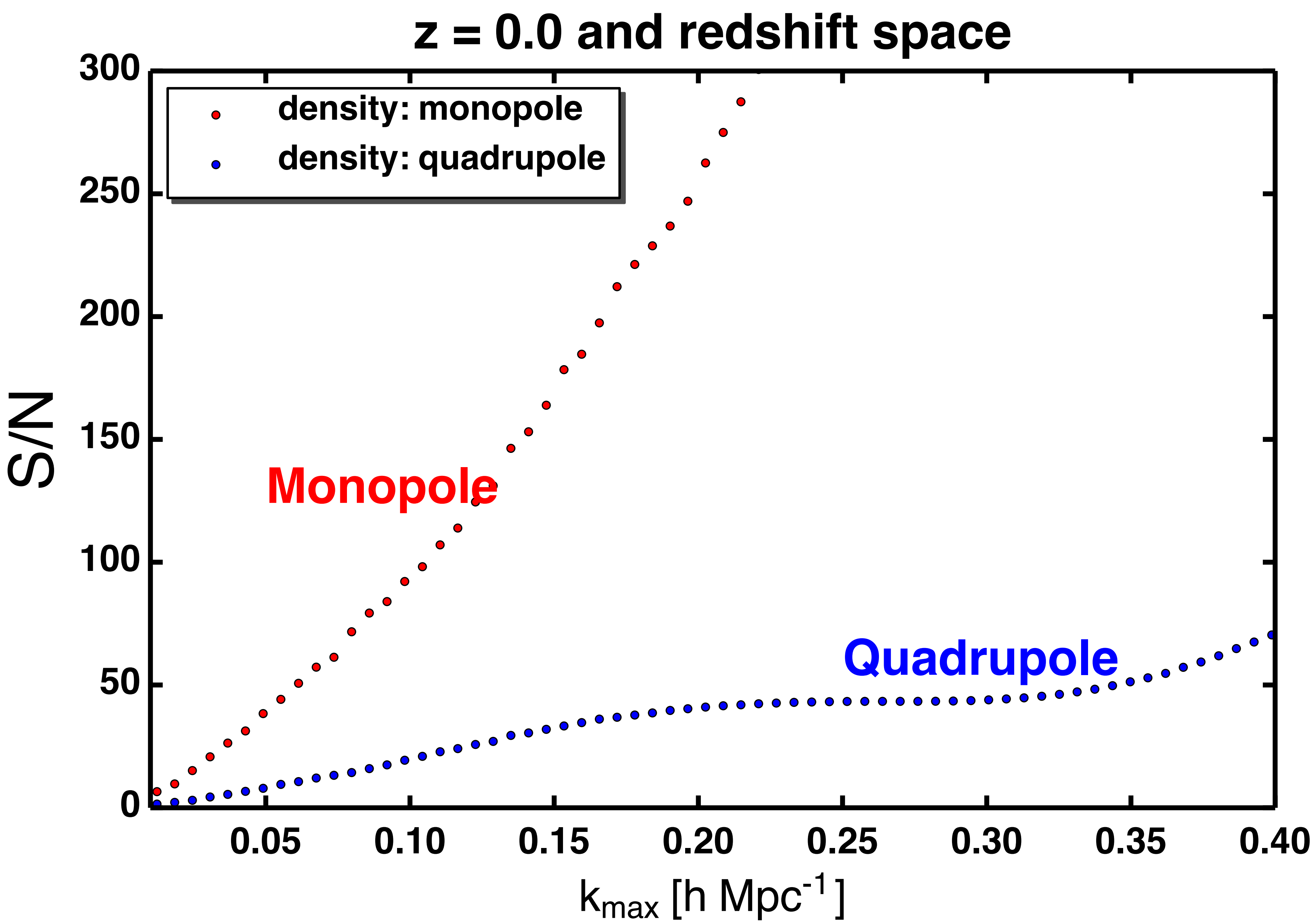
(単純にpower の２乗ではない。)

S/N



Gaussian vs. non-Gaussian

S/N

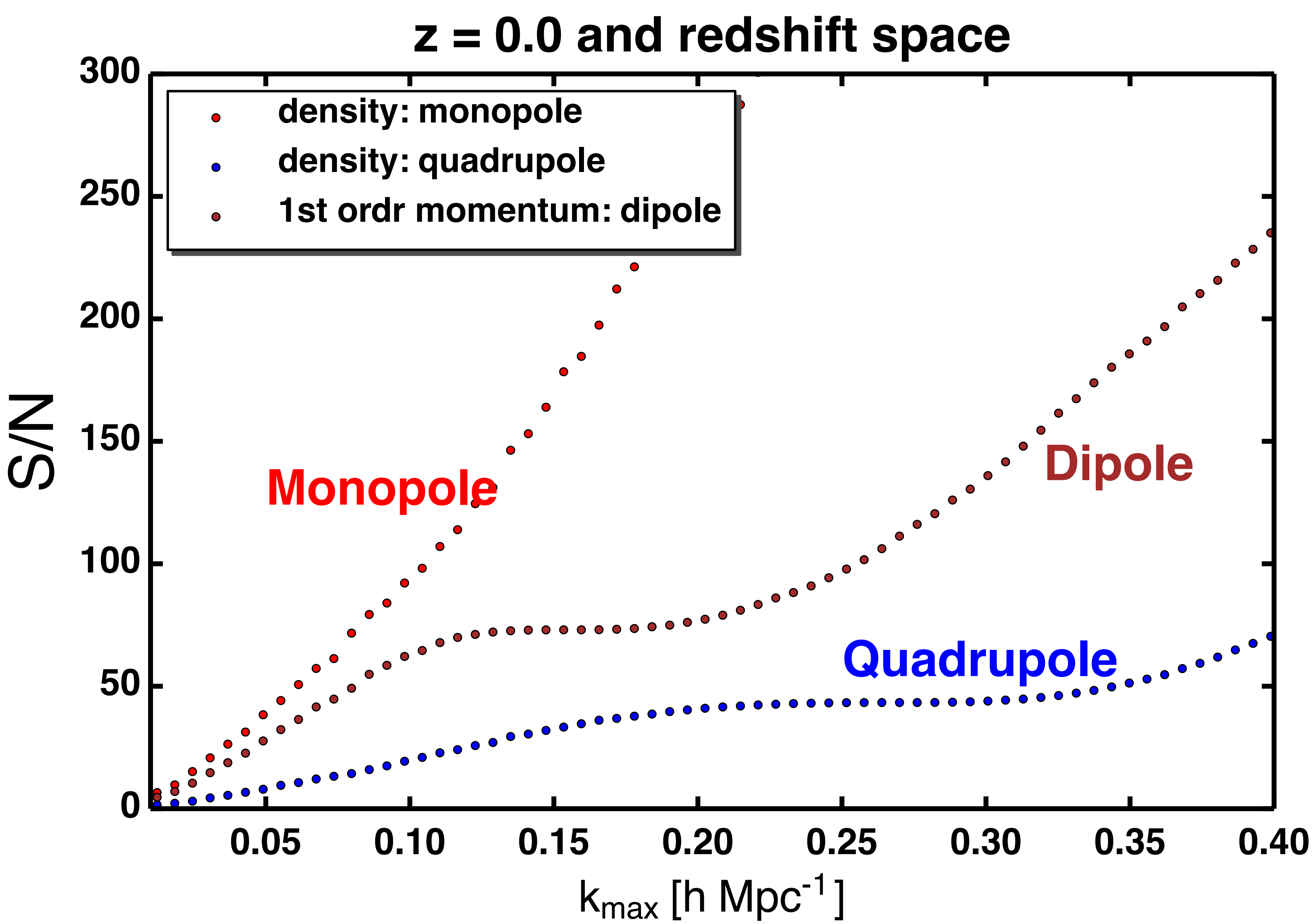


Density Field

<δ δ> : Monopole

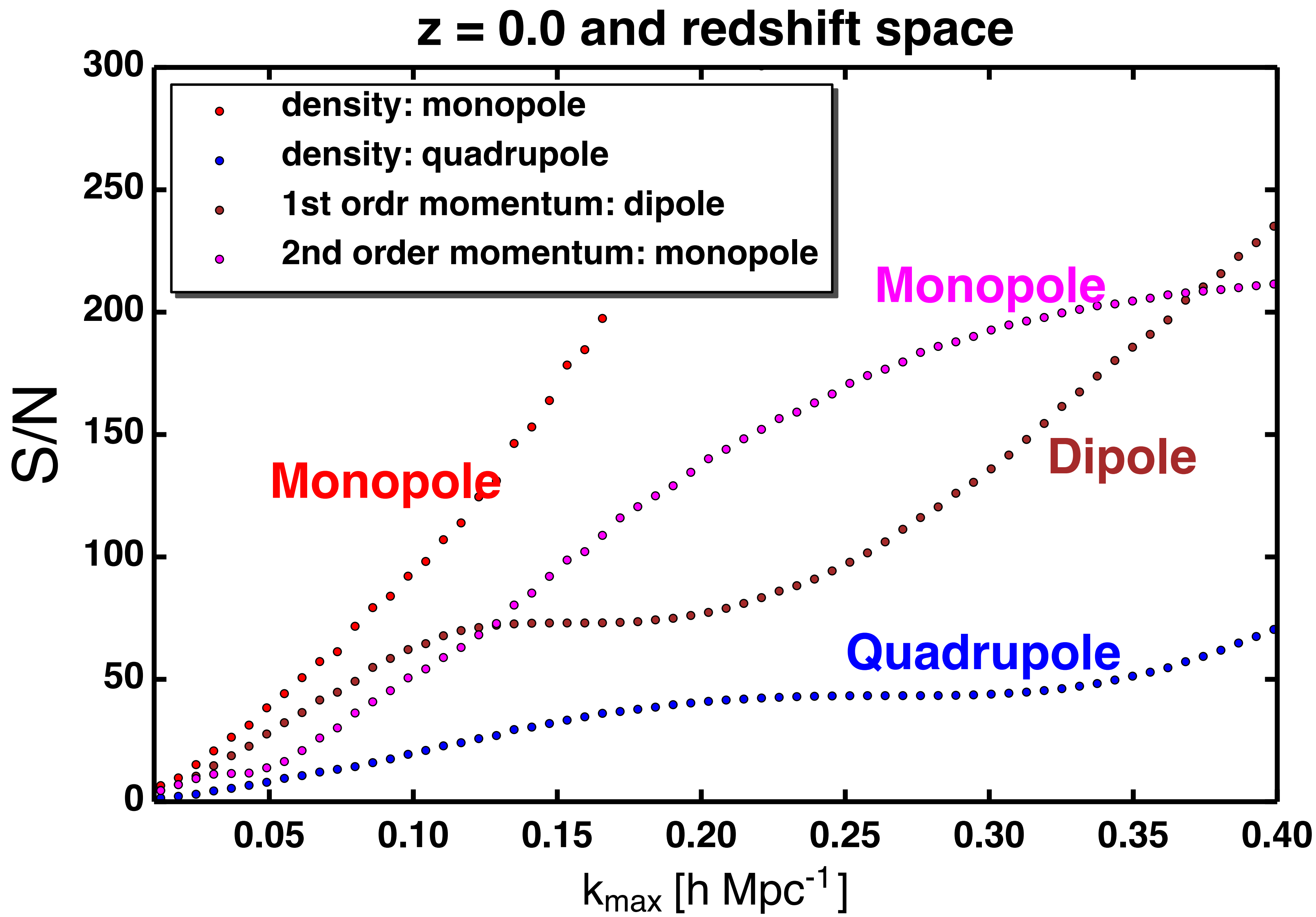
<δ v> : Quadrupole

S/N



1st order Momentum Field
<δ v> : Dipole

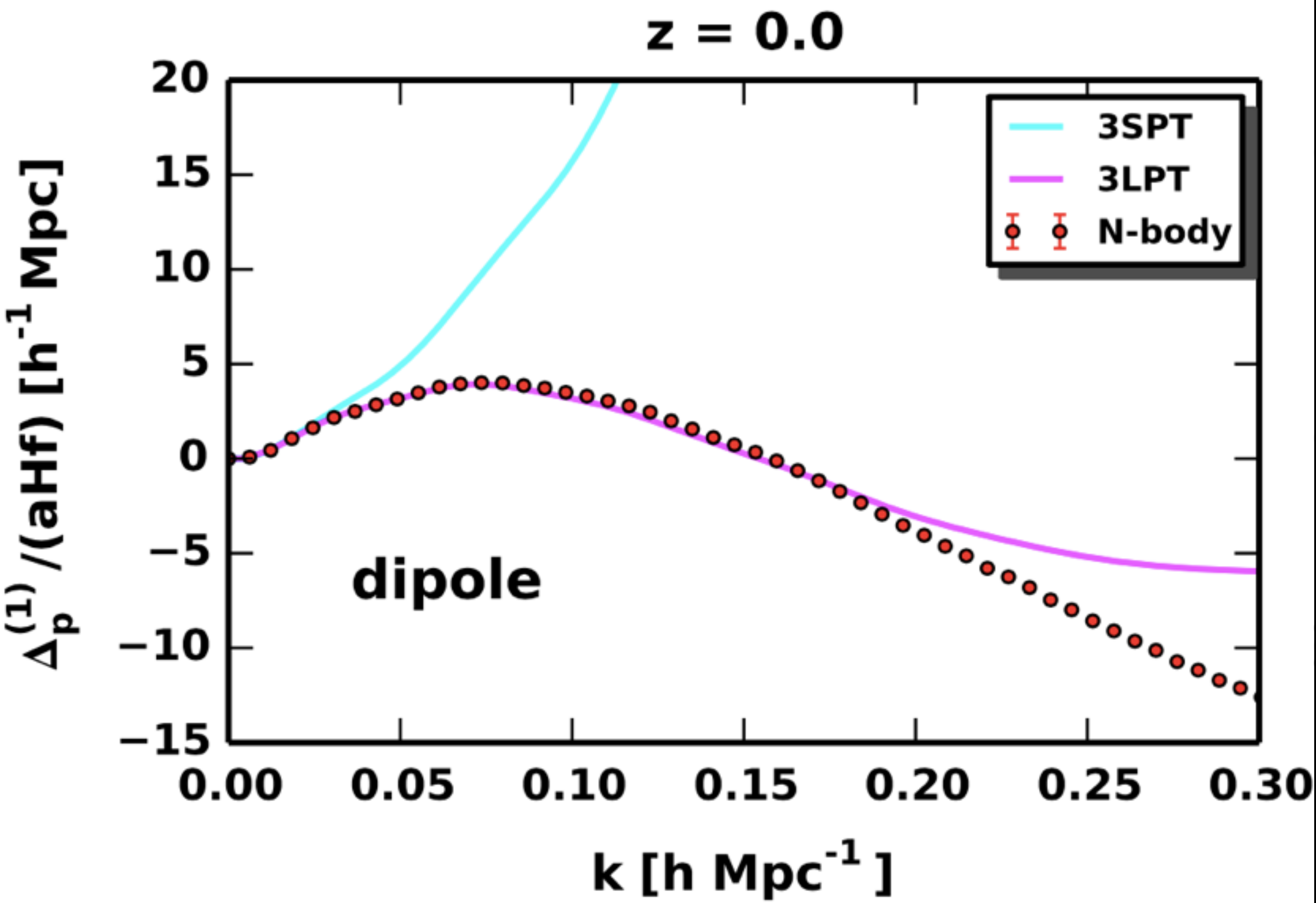
S/N



2nd order Momentum Field

<v v> : Monopole

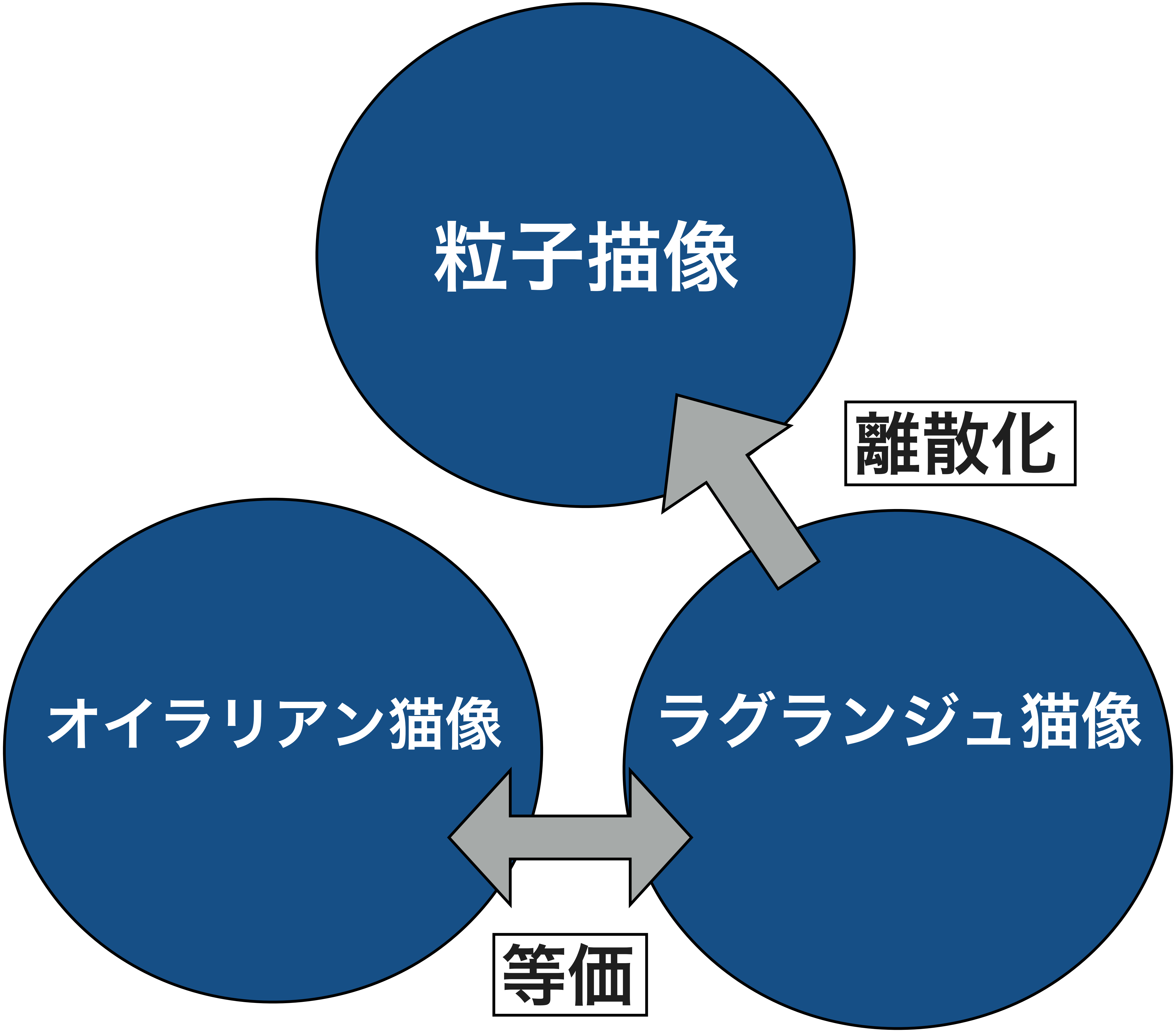
Bias free

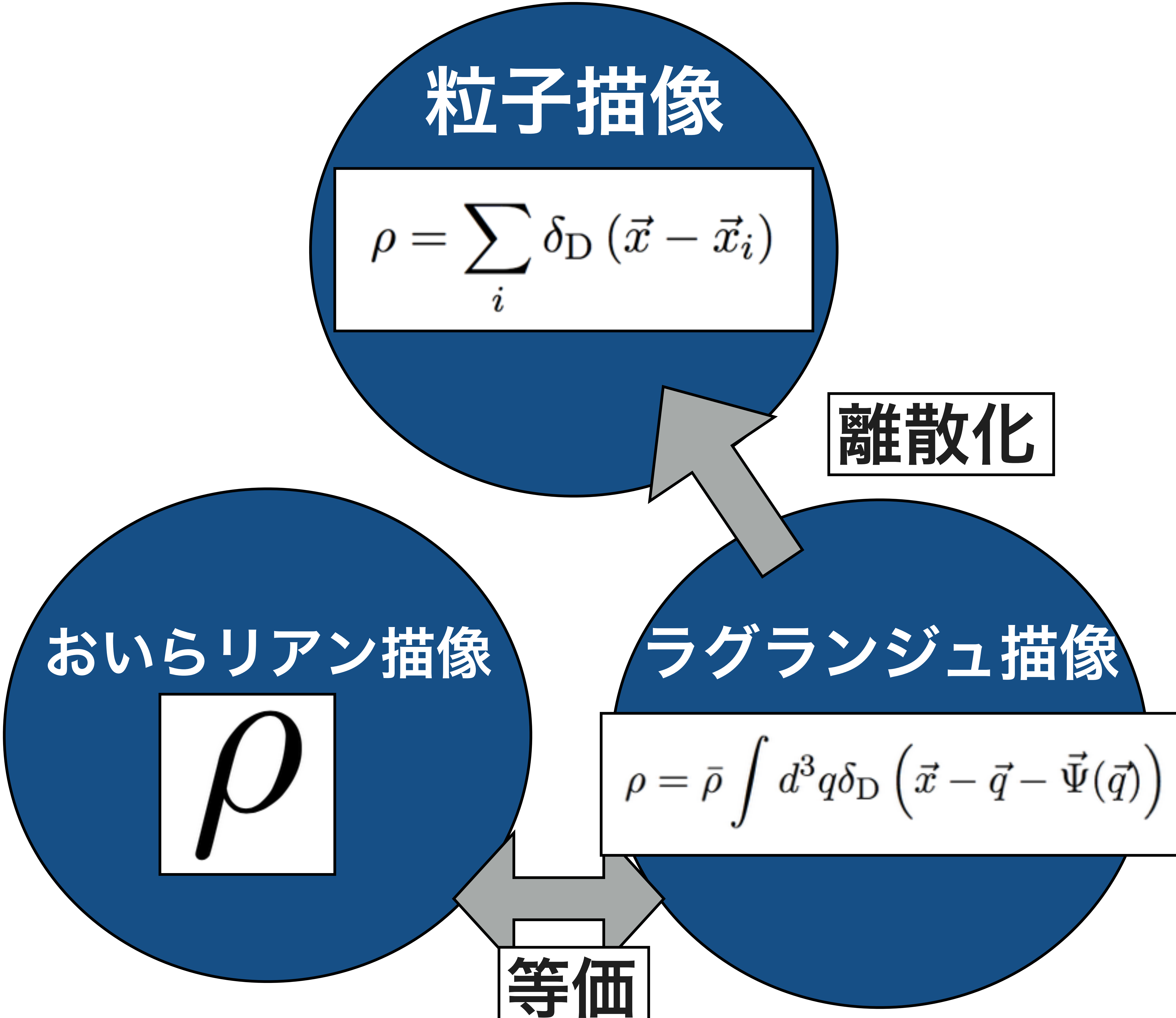


z = 0.0
3SPT
3LPT
N-body
dipole
$\Delta_p^{(1)}/(aHf)$ [h^{-1} Mpc]
k [h Mpc^{-1}]
20
15
10
5
0
−5
−10
−15
0.00
0.05
0.10
0.15
0.20
0.25
0.30

運動方程式の非線形性

$$\frac{d^2\vec{x}_i}{dt^2} \propto \sum_j \frac{\vec{x}_i - \vec{x}_j}{|\vec{x}_i - \vec{x}_j|^3}$$


粒子描像
離散化
オイラリアン描像
ラグランジュ描像
等価



粒子描像
$\rho = \sum_i \delta_{\rm D}\left(\vec{x} - \vec{x}_i\right)$
離散化
おいらリアン描像
ρ
ラグランジュ描像
$\rho = \bar{\rho} \int d^3q \delta_{\rm D}\left(\vec{x} - \vec{q} - \vec{\Psi}(\vec{q})\right)$
等価

Eulerian vs. Lagrangian Perturbation Theory

Eulerian PT

$$\begin{aligned} \rho &= \mathcal{O}(1) + \mathcal{O}(\delta_{\mathrm{L}}) + \mathcal{O}\left(\delta_{\mathrm{L}}^2\right) + \ldots, \\ \vec{v} &= \mathcal{O}(\delta_{\mathrm{L}}) + \mathcal{O}\left(\delta_{\mathrm{L}}^2\right) + \ldots \end{aligned}$$

Lagrangian PT

$$\begin{aligned} \rho &= \mathrm{Full}, \\ \vec{\Psi} &= \dot{\vec{\Psi}} = \vec{v} = \mathcal{O}(\delta_{\mathrm{L}}) + \mathcal{O}\left(\delta_{\mathrm{L}}^2\right) + \ldots \end{aligned}$$

展開するものが違う。

Euleriain vs. Lagrangian Perturbation Theory

Improved Eulerian PT

$$\rho = \mathcal{O}(1) + \mathcal{O}(\delta_{\rm L}) + \ldots$$

$$\vec{v} = \mathcal{O}(\delta_{\rm L}) + \mathcal{O}(\delta_{\rm L}^2) + \ldots$$

Full

Resummation of Lagrangian PT

$$\rho = \ldots$$

$$\vec{\Psi} = \dot{\vec{\Psi}} = \vec{v} = \mathcal{O}(\delta_{\rm L}) + \mathcal{O}(\delta_{\rm L}^2) + \ldots$$

Expanding

Perturbation Theory

$\vec{v} \sim \dot{\vec{\Psi}} \sim \vec{\Psi}$

Density field
(Continuity Equation) ρ

Displacement vector (Velocity field)
(Equation of Motion, Gravity)

	1	3	5	7	~~	Full
1	Linear					ZA
3		3SPT (1-loop)				3LPT
5			5SPT (2-loop)			5LPT
7				7SPT (3-loop)		
~~						
Full						N-body

Improved PT

Perturbation Theory

$\vec{v} \sim \dot{\vec{\Psi}} \sim \vec{\Psi}$

Density field
(Continuity Equation)

ρ

Displacement vector (Velocity field)
(Equation of Motion, Gravity)

	1	3	5	7	~~	Full
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3		3SPT (1-loop)				3LPT
5			5SPT (2-loop)			5LPT
7				7SPT (3-loop)		
~~						
Full						N-body

Zel'dovich Approximation

Density Power Spectrum in
Gamma-Expansion method
(Wiener Hermite expansion (Sugiyama and Futamase), iPT, or etc.)