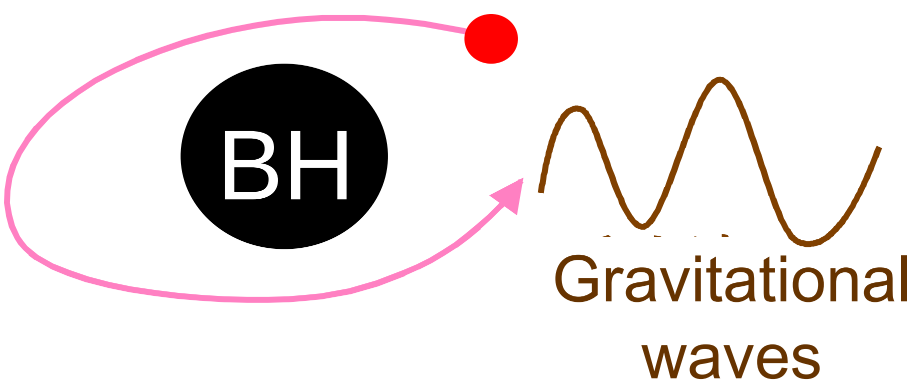
京都大学
重力波物理学研究センター
Gravitational Wave Research Center

重力波で探る修正重力



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Motivation for modified gravity

1) Incompleteness of General relativity

GR is non-renormalizabile
Singularity formation after gravitational collapse

2) Dark matter/dark energy problem

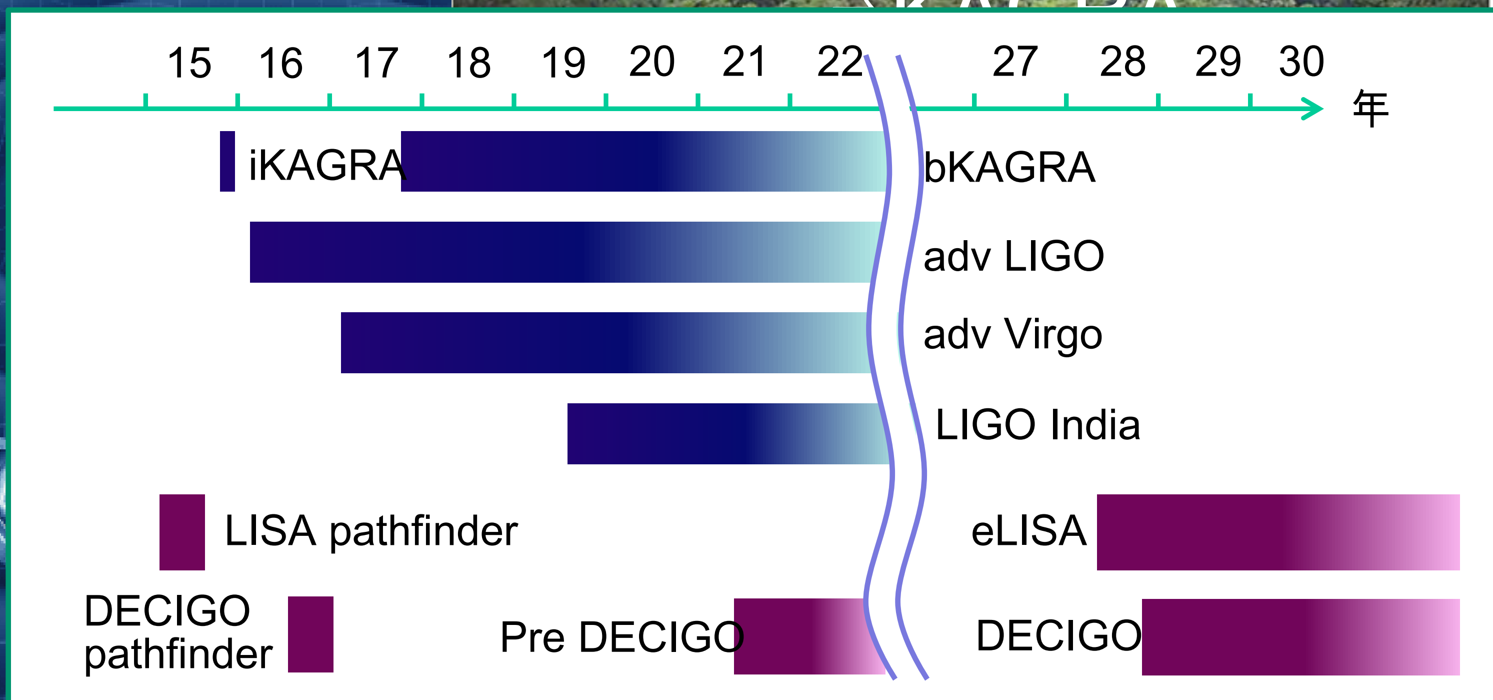
3) To test General relativity

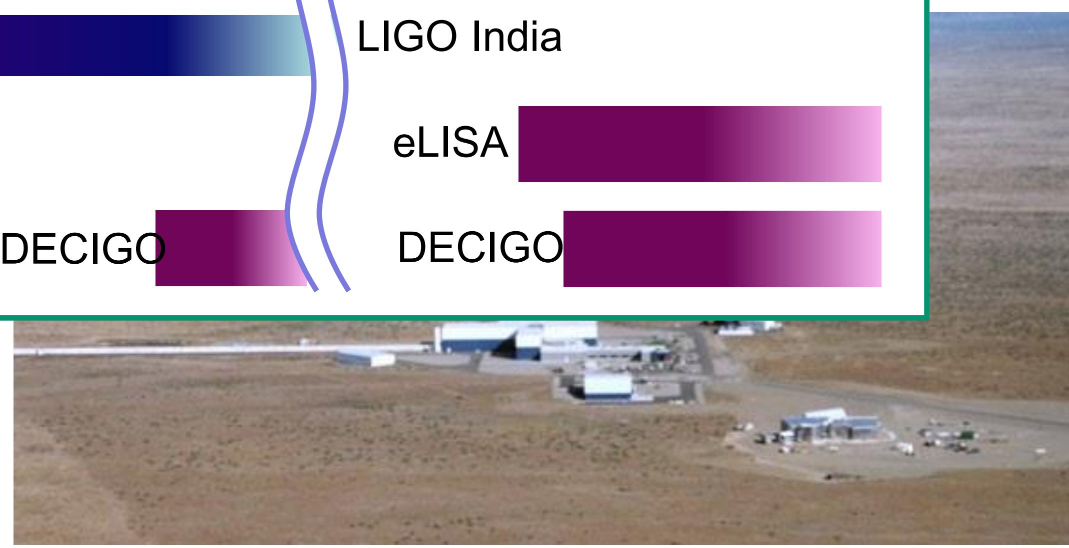
GR has been repeatedly tested since its first proposal.
The precision of the test is getting higher and higher.

⇒ Do we need to understand what kind of modification is theoretically possible before experimental test?

Yes, especially in the era of gravitational wave observation!

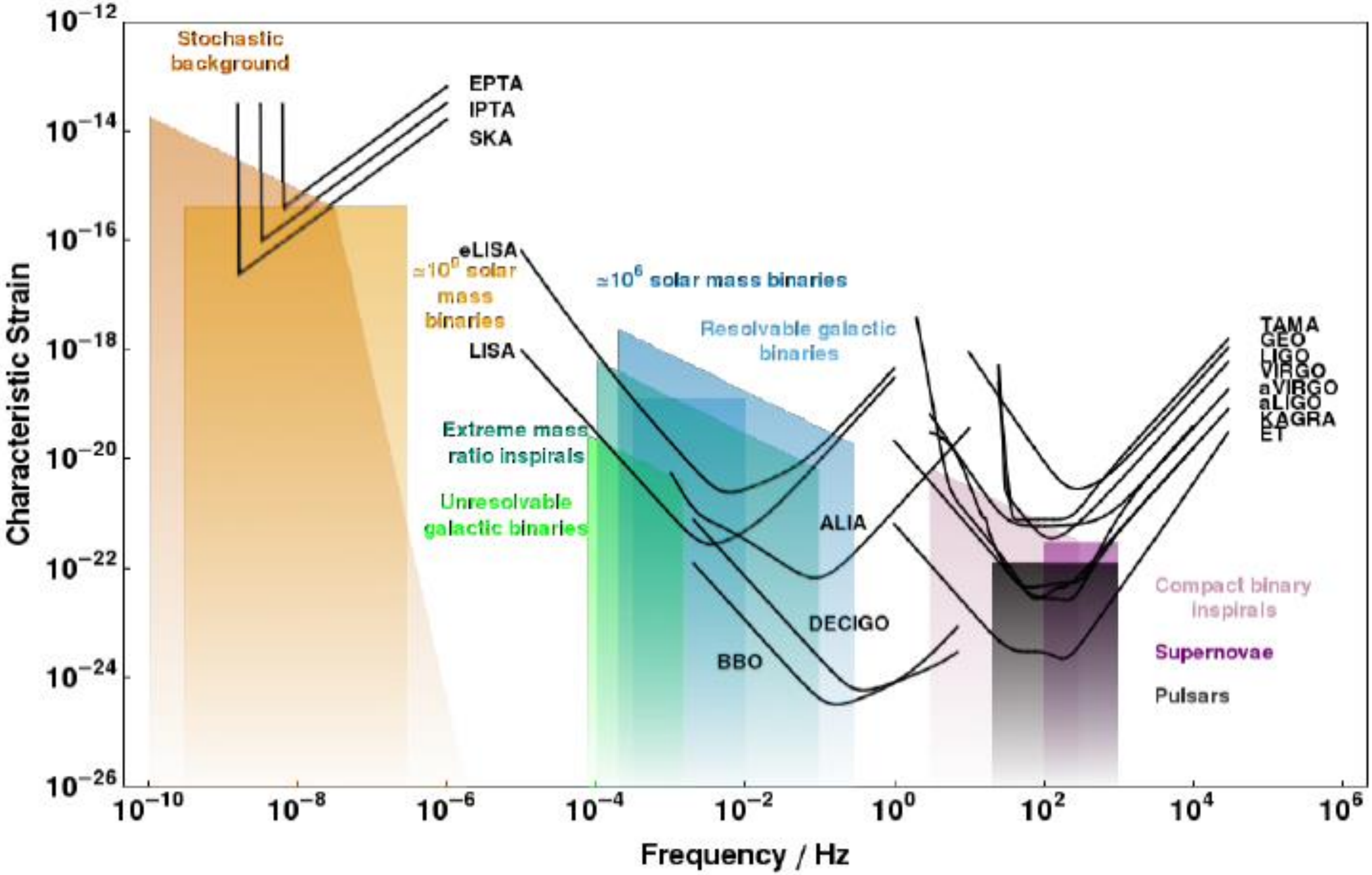
Gravitation wave detectors





eLISA(NGO)
⇒DECIGO/BBO

LIGO⇒adv LIGO



(Moore, Cole, Berry
http://www.ast.cam.ac.uk/~rhc26/sources/)

Inspiraling-coalescing binaries

- Inspiral phase (large separation)

  (Cutler et al, PRL **70** 2984(1993))

  Clean system: ~ point particles

  Internal structure of stars is not so important

  Accurate theoretical prediction of waveform is possible.

  - for detection
  - for parameter extraction(direction, mass, spin,…)
  - for precision test of general relativity

- Merging phase

  Numerical relativity

  - EOS of nuclear matter
  - Electromagnetic counterpart

- Ringing tail - quasi-normal oscillation of BH

Prediction of the event rate for binary NS mergers

BINARY SYSTEMS CONTAINING RADIO PULSARS THAT COALESCE IN LESS THAN 10^{10} yr

PSR	P (ms)	P_b (hr)	e	Total Mass ($M_\odot$)	τ_c (Myr)	τ_{GW} (Myr)	Reference
J0737−3039A	22.70	2.45	0.088	2.58	210	87	Burgay et al. 2003 } double pulsar
J0737−3039B	2773	2.45	0.088	2.58	50	87	Lyne et al. 2004 } double pulsar
B1534+12	37.90	10.10	0.274	2.75	248	2690	Wolszczan 1990
J1756−2251	28.46	7.67	0.181	2.57	444	1690	This Letter
B1913+16	59.03	7.75	0.617	2.83	108	310	Hulse & Taylor 1975
B2127+11C	30.53	8.04	0.681	2.71	969	220	Anderson et al. 1990
J1141−6545†	393.90	4.74	0.172	2.30	1.4	590	Kaspi et al. 2000 ← NS-WD

NOTES.—One NS-WD (†) and five DNS systems. PSR B2127+11C is in a globular cluster, implying a different formation history to the Galactic DNS systems. Here τ_c is the pulsars' characteristic age and τ_{GW} is the time remaining to coalesce due to emission of gravitational radiation. The total coalescence time is $\tau_c + \tau_{GW}$.

(Faulkner et al ApJ 618 L119 (2005))

total coalescence time

$$\tau(i) = \tau_c + \tau_{GW}$$

Time to spin-down to the current spin velocity
+ time to elapse before coalescence

event rate per Milky way galaxy

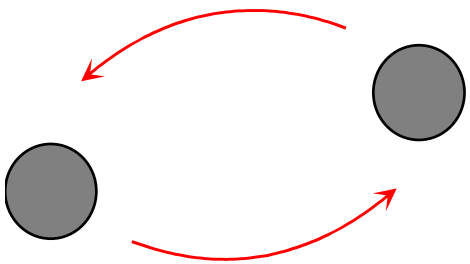
$$R = \sum_i \frac{V_{gal}}{V_{max}(i)\,\tau(i)}$$

$V_{max}(i)$ the volume in which we can detect an observed binary NS when it is placed there.

0.4 ~ 400yr^{-1} for advLIGO/Virgo → $8^{+10}_{-5}\, yr^{-1}$
(Abadie et al. 2010) (Kim et al. 2013)

If short γ-ray bursts are binary NS mergers,
>1.5yr^{-1} for advanced detector network (Yonetoku et al. 1402.5463)

Theoretical prediction of GW waveform



Standard post Newtonian approximation

~ (v/c)expansion

3.5PN=$(v/c)^7$ computation is ready

(Blanchet, Living Rev.Rel.17:2)

Waveform in Fourier space

for quasi-circular inspiral

$$h(f) \approx A f^{-7/6} e^{i\Psi(f)}$$

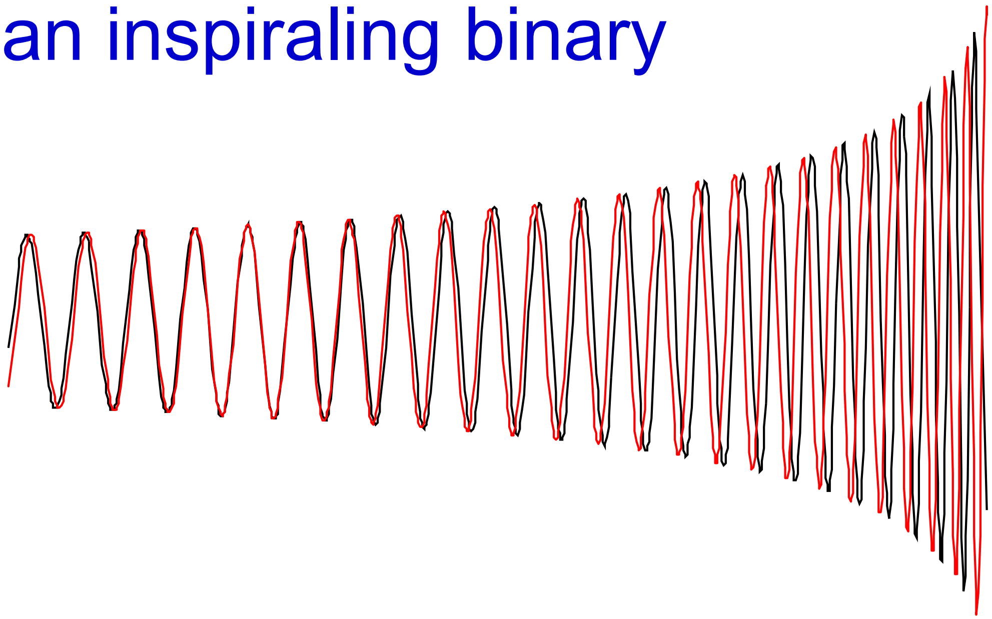
$$A = \frac{1}{\sqrt{20\pi^3}} \frac{\mathcal{M}^{5/6}}{D_L}, \quad \mathcal{M} = \mu^{3/5} M^{2/5}, \quad \eta = \frac{\mu}{M}$$

$$\Psi = 2\pi f t_c - \phi_c + \frac{3}{128}(\pi \mathcal{M} f)^{-5/3}\left[1 + \underbrace{\frac{20}{9}\left(\frac{743}{331} + \frac{11}{4}\eta\right)u^{2/3}}_{\text{1PN}} \underbrace{-(16\pi - \beta)u}_{\text{1.5PN}} + \cdots\right]$$

$$u \equiv \pi M f = O(v^3)$$

GR is correct in strong gravity regime?

Many cycles of gravitational waves from an inspiraling binary



1 cycle phase difference is detectable

- Precise determination of orbital parameters
- Mapping of the strong gravity region of BH spacetime

Typical modification of GR

often discussed in the context of test by GWs

Scalar-tensor gravity

$$S=\frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\phi R-\omega_{BD}\phi^{-1}\phi_{,\alpha}\phi^{,\alpha}\right)-\sum_a\int d\tau_a m_a(\phi)$$

$$G=\frac{4+2\omega_{BD}}{\phi(3+2\omega_{BD})}$$

scalar charge:

$$s_a=-\left[\partial(\ln m_a)/\partial(\ln G)\right]_0$$

G-dependence of the gravitational binding energy

$$\Psi=\cdots+\frac{3}{128}(\pi\mathcal{M}f)^{-5/3}\left[\alpha u^{-2/3}+1+\left(\frac{3715}{756}+\frac{55}{9}\eta\right)u^{2/3}-(16\pi-\beta)u+\cdots\right]$$

Dipole radiation＝－1 PN frequency dependence

$$u=\pi M f=O(v^3)$$

$$\alpha=-\frac{5(s_1-s_2)^2}{64\omega_{BD}}$$

For binaries composed of similar NSs, $(s_1-s_2)^2\ll 1$

Einstein Æther

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(R - M^{\alpha\beta}{}_{\mu\nu} \nabla_\alpha U^\mu \nabla_\beta U^\nu \right)$$

U is not coupled to matter field directly.

$$M^{\alpha\beta}{}_{\mu\nu} = c_1 g^{\alpha\beta} g_{\mu\nu} + c_2 \delta^\alpha_\mu \delta^\beta_\nu + c_3 \delta^\alpha_\nu \delta^\beta_\mu + c_4 U^\alpha U^\beta g_{\mu\nu}$$

with $U^\alpha U_\alpha = -1$

- At the lowest order in the weak field approximation, there is no correction to the metric if U^α // u^α (≡the four momentum of the star).
- The Lorentz violating effects should be suppressed.

⇨ two constraints among the four coefficients

Nevertheless, compact self-gravitating bodies can have significant scalar charge due to the strong gravity effect.

⇨ Constraint from dipole radiation.

Scalar-tensor gravity (conti)

Current constraint on dipole radiation:

$\omega_{BD} > 2.4 \times 10^4$ J1141-6545

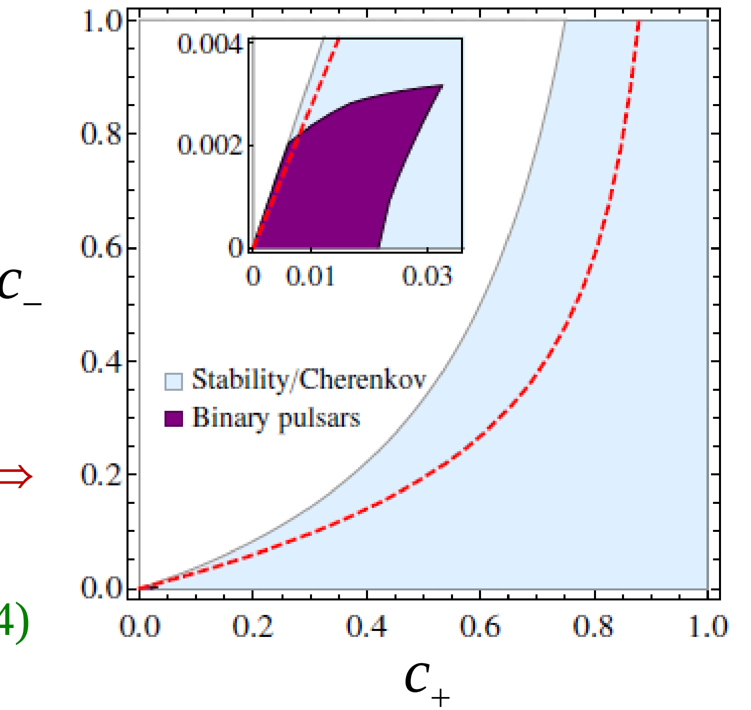
(NS(young pulsar)-WD)

(Bhat et al. arXiv:0804.0956)

The case of Einstein Æther ⇒

$c_{\pm} = c_1 \pm c_3$

(Yagi et al. arXiv:1311.7144)



Constraint from future observations:

(Yagi & TT, arXiv:0908.3283)

LISA– $1.4 M_{\odot}$NS$+1000 M_{\odot}$BH: $\omega_{BD} > 5 \times 10^3$

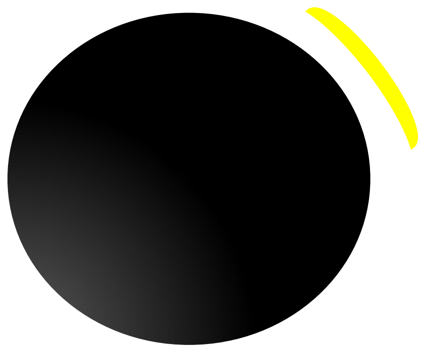
at 40Mpc corresponding to $SNR = \sqrt{200}$

Decigo–$1.4 M_{\odot}$NS$+10 M_{\odot}$BH: $\omega_{BD} > 8 \times 10^7$

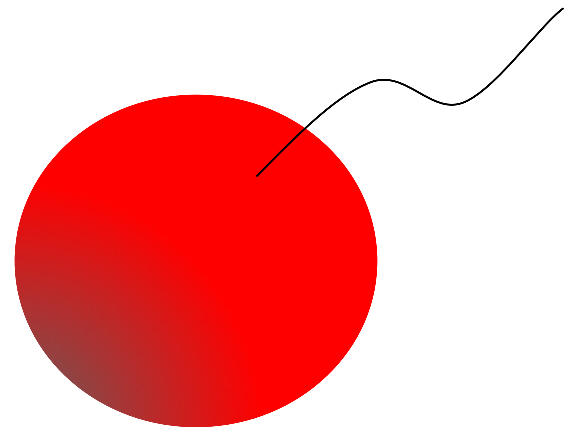
collecting 10^4events at cosmological distances

Scalar-tensor theory

BH no hair



Turu-turu



NS can have a scalar hair

Einstein dilaton Gauss-Bonnet, Chern-Simons gravity

$$S \supset \frac{\alpha}{G_N}\int d^4x\sqrt{-g}\,\theta\begin{pmatrix} R_{GB} \\ {}^*R\,R \end{pmatrix} - \frac{1}{2G_N}\int d^4x\sqrt{-g}\left[(\partial\theta)^2 + 2V(\theta)\right]$$

$\theta\times$(higher curvature)

$$R_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R^{\alpha\beta}{}_{\mu\nu}R^{\mu\nu}{}_{\alpha\beta} \qquad {}^*R\,R = \varepsilon^{\alpha\beta}{}_{\sigma\chi}R^{\sigma\chi}{}_{\mu\nu}R^{\mu\nu}{}_{\alpha\beta}$$

- For constnat θ, these higher curvature terms are topological invariant. Hence, no effect on EOM.
- Higher derivative becomes effective only in strong field.

Hairy BH - bold NS

- NS in EDGB and CS do not have any scalar charge.

$$\Box\theta \approx "R^2" \Rightarrow Q = \int d^3x "R^2" = \frac{1}{T}\int \underline{d^4x "R^2"}$$

topological invariant, which vanishes on topologically trivial spacetime.

- By contrast, BH solutions in EDGB and CS have scalar monopole and dipole, respectively.

EDGB : monopole charge ⇒ dipole radiation (-1PN order)

CS : dipole charge ⇒ 2PN order corrections

(Yagi, Stein, Yunes, Tanaka (2012))

Observational bounds

- **EDGB**

Cassini $\alpha_{EDGB}^{1/2} < 1.3 \times 10^{12}\,\mathrm{cm}$ (Amendola, Charmousis, Davis (2007))

Low mass X-ray binary, A0620-00, orbital decay

$$\alpha_{EDGB}^{1/2} < 1.9 \times 10^{5}\,\mathrm{cm}$$ (Yagi, arXiv:1204.4524)

Future Ground-based GW observation
SNR=20, 6Msol + 12Msol

$$\alpha_{EDGB}^{1/2} < 4 \times 10^{5}\,\mathrm{cm}$$ (Yagi, Stein, Yunes, TT, arXiv:1110.5950)

- **CS**

Gravity Probe B, LAGEOS (Ali-Haimound, Chen (2011))

$$\alpha_{CS}^{1/2} < 10^{13}\,\mathrm{cm}$$

Future Ground-based GW observation with favorable spin alignment: 100Mpc, $a \sim 0.4$M (This must be corrected…)

$$\alpha_{CS}^{1/2} < 10^{6-7}\,\mathrm{cm}$$ (Yagi, Yunes, TT, arXiv:1208.5102)

Simple addition of mass to graviton

phase velocity of massive graviton

$$c_{phase}(f) = \frac{k}{\omega} \approx 1 - \frac{m^2}{2\omega^2} = 1 - \frac{1}{2\lambda_g^2 f^2}$$

$$D = \int d\eta\, a^2$$

$$\Rightarrow \Delta\Psi = 2\pi f \Delta t = 2\pi f D \Delta c_{phase}(f) \approx -\frac{\pi D}{\lambda_g^2 f}$$

Phase shift depending on frequencies

$$\Psi = \cdots + \frac{3}{128}(\pi \mathcal{M} f)^{-5/3}\left[1 + \left(\frac{3715}{756} + \frac{55}{9}\eta - \frac{128}{3}\eta\beta_g\right)u^{2/3} - (16\pi - \beta)u + \cdots\right]$$

$$u = \pi M f = O(v^3)$$

Graviton mass effect

$$\beta_g = \frac{\pi^2 DM}{\lambda_g^2}$$

Constraint from future observations:

LISA– $10^7 M_\odot$ BH+$10^6 M_\odot$ BH at 3Gpc:

graviton compton wavelength

$\lambda_g > 4\text{kpc}$

(Yagi & TT, arXiv:0908.3283)

Parametorized post–Einstein

$$h(f) \approx A f^{-7/6} e^{i\Psi(f)}$$ GW waveform for Quasi-circular orbits

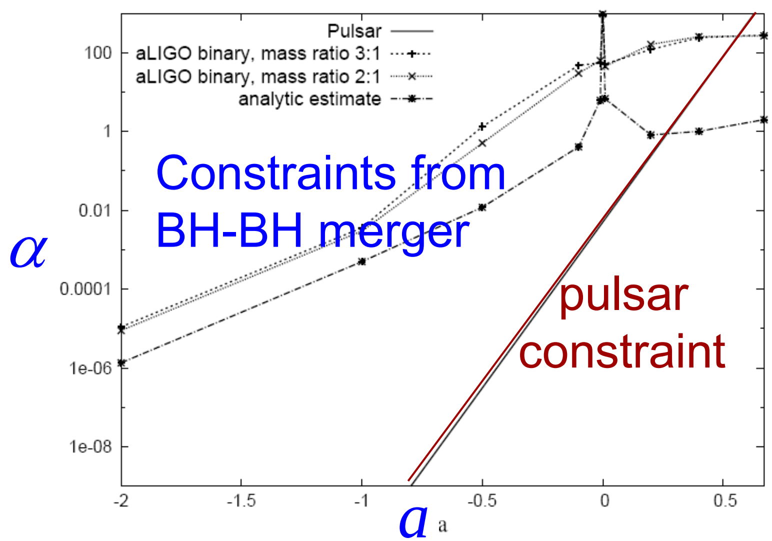
$$\Rightarrow \begin{cases} A(f) \to \left(1+\sum_i \alpha_i u^{a_i}\right) A_{GR}(f) \\ \Psi(f) \to \left(\Psi_{GR}(f) + \sum_i \beta_i u^{b_i}\right) \end{cases}$$

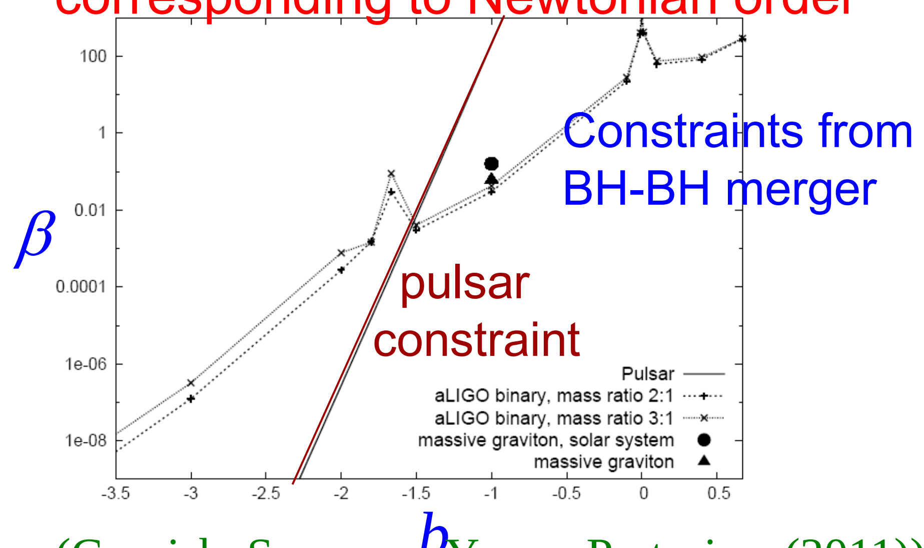
Theory	a	α	b	β
Brans-Dicke [9, 10, 14–16]	–	0	-7/3	β
Parity-Violation [22, 34–37]	1	α	0	–
Variable $G(t)$ [38]	-8/3	α	-13/3	β
Massive Graviton [8–14]	–	0	-1	β
Quadratic Curvature [23, 44]	–	0	-1/3	β
Extra Dimensions [45]	–	0	-13/3	β
Dynamical Chern-Simons [46]	+3	α	+4/3	β

(Yunes & Pretorius (2009))

Better constraint than pulsar timing for a_i>0 or b_i>-5/3.

corresponding to Newtonian order



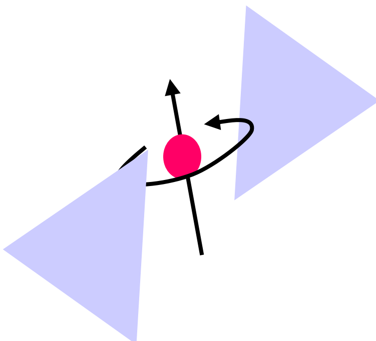


(Cornish, Sampson, Yunes, Pretorius. (2011))

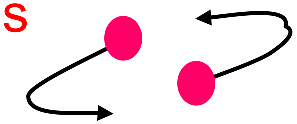
12$M_\odot$BH-6$M_\odot$BH and 18$M_\odot$BH-6$M_\odot$BH mergers

Test of GW generation

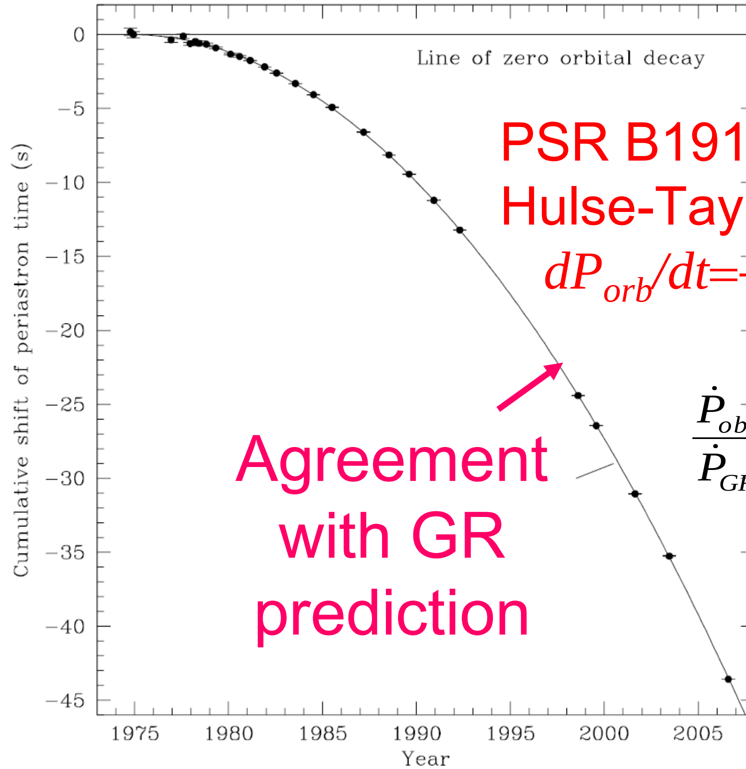
Pulsar : ideal clock



Test of GR by pulsar binaries



Periastron advance due to GW emission



PSR B1913+16

Hulse-Taylor binary

$dP_{orb}/dt = -2.423\times10^{-12}$

$$\frac{\dot{P}_{obs}}{\dot{P}_{GR}} = 0.997 \pm 0.002$$

Agreement with GR prediction

(J.M. Weisberg, Nice and J.H. Taylor, arXiv:1011.0718)

We know that GWs are emitted from binaries.

But, then
what can be a big surprise
when we first detect GWs?

Is there any possibility that
gravitons disappear during its propagation
over a cosmological distance?

Just fast propagation of GWs can be realized in Lorentz violating models such as Einstein Æther theory.

Graviton Oscillation in Bi-gravity

(De Felice, Nakamura, TT arXiv:1304.3920)

Massive gravity

$$\Box h_{\mu\nu} = 0 \quad \Rightarrow \quad \left(\Box - m^2\right) h_{\mu\nu} = 0$$

Simple graviton mass term is theoretically inconsistent → ghost, instability, etc.

⇒ Bi-gravity

$$\frac{L}{M_G^2} = \frac{\sqrt{-g}R}{16\pi} + \frac{\sqrt{-\tilde{g}}\tilde{R}}{16\pi\kappa} + \frac{L_{matter}(g,\phi)}{M_G^2} + \cdots$$

Both massive and massless gravitons exist.

→ ν oscillation-like phenomena?

First question is whether or not we can construct a viable cosmological model.

1) Ghost-free bigravity model exists.

2) It has a FLRW background very similar to the GR case at low energy.

3) The non-linear mechanism seems to work to pass the solar system constraints. (Vainshtein mechanism)

4) Two graviton eigen modes are superposition of two metric perturbations, which are mass eigen states at low frequencies and δg and $\delta\tilde{g}$ themselves at high frequencies.

5) Graviton oscillations occur only at around the crossover frequency, but there is some chance for observation.

Ghost free bi-gravity

$$\frac{L}{M_G^2} = \frac{\sqrt{-g}R}{2} + \frac{\sqrt{-\tilde{g}}\tilde{R}}{2\kappa} + \frac{\sqrt{-g}}{2}\sum_{n=0}^{4} \underline{c_n V_n} + \frac{L_{matter}}{M_G^2}$$

$$V_0 = 1,\ V_1 = \tau_1, V_2 = \tau_1^2 - \tau_2,\ \cdots$$

$$\tau_n \equiv Tr\left[\gamma^n\right] \quad \gamma^i_j \equiv \sqrt{g^{ik}\tilde{g}_{kj}}$$

only 5 possible terms including 2 cosmological constants.

When $\tilde{g}$ is fixed, de Rham-Gabadadze-Tolley massive gravity.

Even if $\tilde{g}$ is promoted to a dynamical field, the model remains to be free from ghost.

(Hassan, Rosen (2012))

FLRW background

(Comelli, Crisostomi, Nesti, Pilo (2012))

Generic homogeneous isotropic metrics

$$ds^2 = \underline{a^2(t)}\left(-dt^2 + dx^2\right)$$

$$d\tilde{s}^2 = \underline{b^2(t)}\left(-\underline{c^2(t)}dt^2 + dx^2\right)$$

$$\xi \equiv b/a$$

$$\Rightarrow \quad \underbrace{\left(6c_3\xi^2 + 4c_2\xi + c_1\right)}_{\text{branch 1}}\underbrace{\left(cba' - ab'\right)}_{\text{branch 2}} = 0$$

branch 1 : Pathological:

Strong coupling

Unstable for the homogeneous anisotropic mode.

branch 2 : Healthy

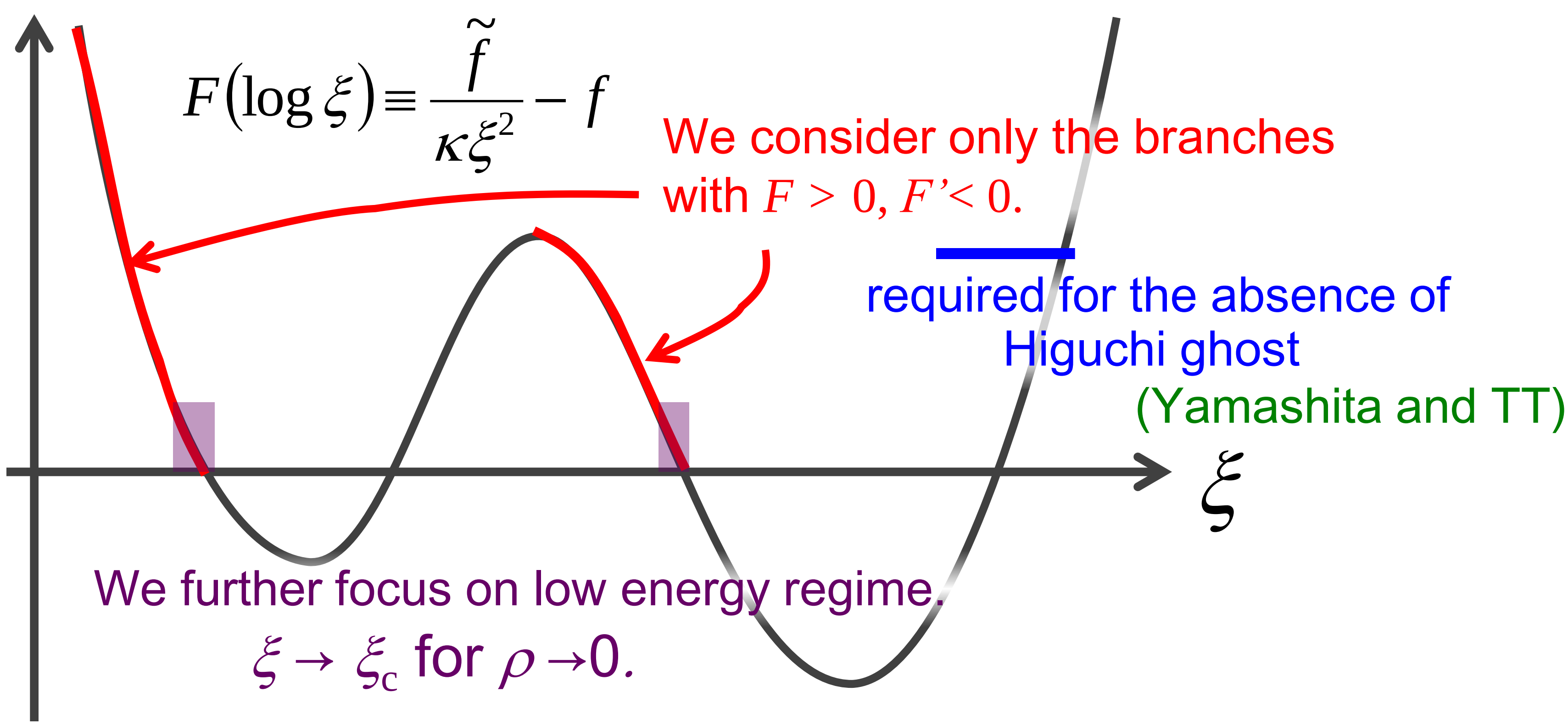
Branch 2 background

A very simple relation holds:

$$\frac{\rho}{M_G^2} + f - \tilde{f}/\kappa\xi^2 = 0 \qquad f(\log\xi) := c_0 + 3c_1\xi + 6c_2\xi^2 + 6c_3\xi^3$$

$$\tilde{f}(\log\xi) := c_1\xi + 6c_2\xi^2 + 18c_3\xi^3 + 24c_4\xi^4$$

$\xi \equiv b/a$ is algebraically determined as a function of ρ.



Branch 2 background

We expand with respect to $\delta\xi = \xi - \xi_c$.

$$H^2 = \frac{\rho}{3M_G^2} + \frac{f}{3} \quad \Rightarrow \quad H^2 = \frac{\rho}{3\left(1+\kappa\xi_c^2\right)M_G^2}$$

effective energy density due to mass term

Effective gravitational coupling is weaker because of the dilution to the hidden sector.

$$\frac{1}{c-1}\frac{\xi'}{\xi} = \frac{a'}{a} \quad \Rightarrow \quad c-1 = \frac{3(\rho+P)}{\mu^2 M_G^2}$$

Effective graviton mass

$$\mu^2 = \left(1 + \frac{1}{\kappa\xi_c^2}\right) f_c'$$

natural tuning to coincident light cones (c=1) at low energies ($\rho \to 0$)!

Solar system constraint: basics

- vDVZ discontinuity

In GR, this coefficient is 1/2

current bound $<10^{-5}$

$$\delta g_{\mu\nu} \propto \Box^{-1}\left(T_{\mu\nu} - \frac{1}{3} g_{\mu\nu} T\right)$$

To cure this discontinuity
we go beyond the linear perturbation (Vainshtein)

Schematically

Correction to the Newton potential Φ

$$\Delta\delta\Phi + \mu^{-2}(\partial\partial\delta\Phi)^2 = G_N\rho$$

$$\Rightarrow \frac{\delta\Phi}{\Phi} \approx \frac{\mu r^2\sqrt{G_N\rho}}{r^2 G_N\rho} \approx \mu\sqrt{\frac{r^3}{r_g}}$$

$$10^{-10} \geq \mu\sqrt{\left(10^{13}cm\right)^3/\left(10^5 cm\right)} \Rightarrow \mu^{-1} \geq 300Mpc$$

Gravitational potential around a star in the limit $c \to 1$

Spherically symmetric static configuration:

$$ds^2 = -e^{u-v}dt^2 + e^{u+v}\left(dr^2 + r^2 d\Omega^2\right)$$

$$d\tilde{s}^2 = \xi_c^2\left[-e^{\tilde{u}-\tilde{v}}dt^2 + e^{\tilde{u}+\tilde{v}}\left(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2\right)\right] \qquad \tilde{r} = e^R r$$

Erasing $\tilde{u}, \tilde{v}$ and R ,

$$\Rightarrow \quad \left(\Delta - \mu^2\right)u - \frac{C}{\mu^2}\left(\left(\Delta u\right)^2 - \left(\partial_i \partial_j u\right)^2\right) \approx \frac{\rho_m}{M_G^2}$$

$C \propto f_c''$, which can be tuned to be extremely large.

Then, the Vainshtein radius $\quad r_V \approx \left(\dfrac{C r_g}{\mu^2}\right)^{1/3}$

can be made very large, even if $\mu^{-1} \ll 300\mathrm{Mpc}$.

Solar system constraint: $\quad \sqrt{C}\mu^{-1} \geq 300\mathrm{Mpc}$

$\Delta v \approx \dfrac{\rho_m}{\tilde{M}_G^2}$ $\quad v$ is excited as in GR. $\qquad H^2 = \dfrac{\rho}{3\tilde{M}_G^2}$

Excitation of the metric perturbation on the hidden sector:

Erasing u, v and R

$$\Rightarrow \left(\Delta-\mu^2\right)\tilde{u}-\frac{\tilde{C}}{\mu^2}\left(\left(\Delta\tilde{u}\right)^2-\left(\partial_i\partial_j\tilde{u}\right)^2\right)\approx\frac{\rho_m}{M_G^2}$$

$$\Delta\tilde{v}\approx\frac{\rho_m}{\tilde{M}_G^2}$$

$\tilde{u}$ is also suppressed like u.

$\tilde{v}$ is also excited like v.

The metric perturbations are almost conformally related with each other: $d\tilde{s}^2\approx\xi_c^2 ds^2$

Non-linear terms of $\tilde{u}$ (or equivalently u) play the role of the source of gravity.

Gravitational wave propagation

Short wavelength approximation :

$$k >> m_g >> H$$

$$h'' - \Delta h + m_g^2\left(h - \tilde{h}\right) = 0$$

$$\tilde{h}'' - c^2 \Delta \tilde{h} - \frac{c m_g^2}{\kappa \xi_c^2}\left(h - \tilde{h}\right) = 0$$

$$m_g^2 = \frac{f'}{3} + \frac{(c-1)}{6}\left(f'' - f'\right)$$

(Comelli, Crisostomi, Pilo (2012))

$$\mu^2 := m_g^2 \frac{1+\kappa\xi^2}{\kappa\xi^2}$$

$$k_c := \frac{\mu}{\sqrt{2(c-1)}}$$

k

mass term is important.

Eigenmodes are

$$h + \tilde{h}, \quad \kappa\xi_c^2 h - \tilde{h}$$

modified dispersion relation due to the effect of mass

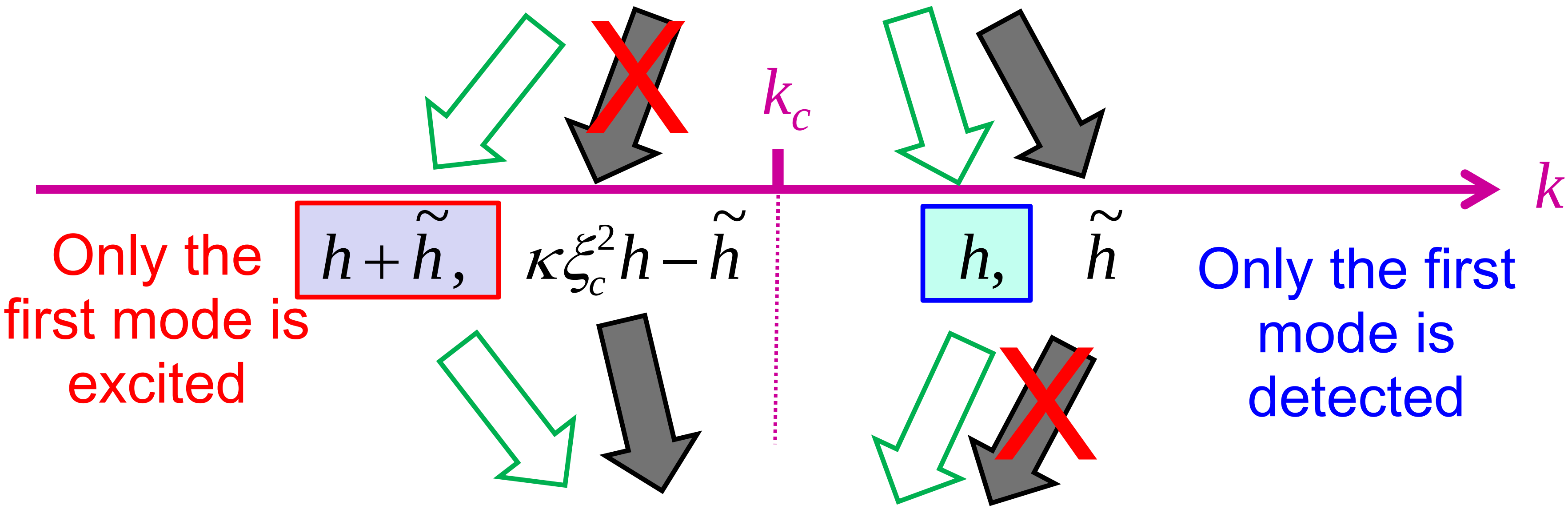
$C \neq 1$ is important.

Eigenmodes are

$$h, \quad \tilde{h}$$

modified dispersion relation due to different light cone

At the GW generation, both h and $\tilde{h}$ are equally excited.



We can detect only h.

Only modes with $k \sim k_c$ picks up the non-trivial dispersion relation of the second mode.

Interference between two modes. ⇒ Graviton oscillations

If the effect appears ubiquitously, such models would be already ruled out by other observations.

Summary

Gravitational wave observations open up a new window for modified gravity.

Even the radical idea of graviton oscillations is not immediately denied. We may find something similar to the case of solar neutrino experiment in near future.

Although space GW antenna is advantageous for the gravity test in many respects, more that can be tested by KAGRA will be remaining to be uncovered.