Dense Hadronic Matter and Finite Nuclei in an RMF Model with a Chiral Potential

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In constructing the dense matter equation of state (EOS), it is desired to respect both chiral symmetry and hypernuclear physics. In dense matter, strangeness is expected to play a decisive role and the partial restoration of chiral symmetry would modify the hadron properties. For chiral symmetry side, We have recently developed a chiral SU(2) symmetric RMF model with logarithmic sigma potential derived in the strong coupling limit (SCL) of the lattice QCD. In this SCL model, we can describe not only symmetric nuclear matter but also bulk properties of finite nuclei. We find that the EOS of symmetric matter is softened by the scalar meson with hidden strangeness, which couples with sigma meson through the determinant interaction. In this paper, we present this RMF model including chiral SU(3) potential and its results. We discuss the importance of hyperon in neutron star in this chiral SU(3) RMF model and show an effect to nuclear star maximum mass by introducing this potential. We also inquire the effect of meson-meson-baryon coupling which is corresponding to density dependent couplings.

Keywords: Neutron star, Hypernuclei

1. Introduction

The equation of state (EOS) of dense hadronic matter is one of the key ingredients in nuclear physics. In addition, astrophysical compact events, such as neutron stars and/or supernovae, are strongly affected by the properties of the dense matter EOS. It is, however, essentially difficult to observe the character of dense matter EOS directly. Thus, it is preferable to examine the properties of dense matter EOS through the theoretical models which can give a reasonable description about the experimental data of finite nuclei. In particular, hyperons are expected to appear at relatively small densities, $\sim (2-3)\rho_0$ and to make the hadronic star matter EOS softened. But YN and YY interactions can not be measured directly because of the short life times of hyperons for weak decay. From this point of view, it is also desired to investigate the property of hyperons through hypernuclei, hyperon bound states in nuclei.

Besides, the chiral symmetry is known as another important ingredient in dense matter. It is one of fundamental symmetries of QCD with massless quarks, and its spontaneous symmetry breaking generates constituent quark and thus hadron masses.[1] Therefore, it is preferable for hadronic many-body theories to include the chiral symmetry by considering its spontaneous breaking and partial restoration at finite baryon densities. We regard a relativistic mean field (RMF) model [2–7] as a candidate which satisfy such a theoretical need.

Recently, we have constructed a chiral SU(2) RMF model, the SCL2 RMF model,[6] with a logarithmic σ potential and its extension to the hypernuclear systems, the SCL3 RMF model .[7] They have succeeded in reproducing not only the properties of "normal" nuclear systems, for example, the saturation property of the symmetric nuclear matter and the BEs and charge rms radii of finite nuclei, but also the ones of Λ and Σ hypernuclear states. However the SCL3 RMF model failed to explain the observed maximum mass of neutron star if we adopt the parameter sets based on normal nuclear and hypernuclear experimental data. In this work, we examine the extension of the SCL logarithmic chiral potential where the finite temperature effect is taken into account. In addition, we also study $\bar{\Psi}_B\sigma\omega\Psi_B$ type NLO coupling regarded as density dependent type coupling since naively the σ meson field is proportional to the baryon density ρ_B.

This paper is organized as follows. In the Sec. 2, we present brief chiral RMF model description including a new chiral potential derived from the finite temperature calculation in the strong coupling limit of lattice QCD. We also introduce the density dependent type coupling for the meson-baryon coupling part of RMF Lagrangian. Next, we reveal the present results of these extended SCL3 RMF model abbreviated as SCL3ASH and SCL3DDC in Sec. 3. Finally, we summarize in Sec. 4

2. Model description

In this section, we introduce the formalization of the extended SCL3 RMF model.

Lagrangian to be solved in an RMF model can be written as

$$\begin{aligned}\mathcal{L} = &\sum_i \bar{\psi}_i \left[i\partial\!\!\!/ - M_i^* - \gamma_\mu U_i^\mu \right] \psi_i \\ &+ \frac{1}{2}\,\partial_\mu \varphi_\sigma \partial^\mu \varphi_\sigma - \frac{1}{2}\,m_\sigma^2 \varphi_\sigma^2 + \frac{1}{2}\,\partial_\mu \varphi_\zeta \partial^\mu \varphi_\zeta - \frac{1}{2}\,m_\zeta^2 \varphi_\zeta^2 \\ &- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{m_\omega^2}{2}\omega_\mu \omega^\mu - \frac{1}{4}R_{\mu\nu}R^{\mu\nu} + \frac{m_\rho^2}{2}R_\mu R^\mu \\ &- \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{m_\phi^2}{2}\phi_\mu \phi^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &+ \frac{c_\omega}{4}\left(\omega_\nu \omega^\nu\right)^2 - V_{\sigma\zeta}\left(\varphi_\sigma, \varphi_\zeta\right) \ , \end{aligned} \tag{1}$$

where the couplings between scalar mesons and baryons correspond to attractive parts of baryon interaction contrary to the ones between vector mesons and baryons corresponding to repulsive parts. For meson fields, we include effective potential of mesons as shown in Eq. (1) with $V_{\sigma\zeta}$ and ω^4 potential as same as Ref. 4 which succeeded in reproducing normal nuclear propertyV_ω. Then, it should be investigated what form the meson potential should have. In previous work,[6,7] we regard chiral symmetry between σ and π as a constraint and thus, we apply the chiral potential derived from the strong coupling limit of lattice QCD(SCL-LQCD). In those works, the $\mathrm{SU}_f(2)$ and SU_3 chiral potentials are examined about normal nuclear and hypernuclear properties.

The $\mathrm{SU}_f(2)$ and $\mathrm{SU}_f(3)$ chiral potential has a logarithmic form derived from analytic calculation on SCL-LQCD and written as

$$\begin{aligned} V_\chi &= -\frac{a_\sigma}{2}\,\log\left(\det MM^\dagger\right) + \frac{b_\sigma}{2}\,\mathrm{tr}(MM^\dagger) - c_\sigma \sigma \\ &= -a_\sigma\,\log(\sigma^2 + \boldsymbol{\pi}^2) + \frac{b_\sigma}{2}\,(\sigma^2 + \boldsymbol{\pi}^2) - c_\sigma \sigma \\ &\simeq -2a_\sigma\, f_{\mathrm{SCL}}(\frac{\varphi_\sigma}{f_\pi}) + \frac{1}{2}m_\sigma^2 \varphi_\sigma^2 + \frac{1}{2}m_\pi^2 \boldsymbol{\pi}^2 \ , \end{aligned} \tag{2}$$

$$f_{\mathrm{SCL}}(x) = \log(1-x) + x + \frac{x^2}{2} \ , \tag{3}$$

where the explicit chiral symmetry breaking term $-c_\sigma\sigma$ is introduced. SCL3 RMF model which include $\mathrm{SU}_f(3)$ logarithmic chiral potential, however, has failed in explaining the maximum mass of neutron star if we apply the parameter sets determined hypernuclear data such as S_Λ and/or $\Delta B_{\Lambda\Lambda}$. In Fig. 5 of Ref. 7, we show the scalar and vector potential of nucleon in symmetric nuclear matter as a function of ρ_B. Both scalar and vector potentials seem to be in good agreement with RBHF results[8] around rho_0 Vector potential shows certain deviation from the RBHF result on the contrary to that scalar potential in the wide range of ρ_B. The strength of introduced ω^4 effective potential c_ω in SCL3 model($c_\omega \sim 300$) is 2.5 times larger than the one introduced in TM model and this potential is regarded as a suppressor of vector potential. Thus, we need some mechanism which stabilize nuclear matter and do not need too strong c_ω. In this work, we examine arcsinh type chiral potential and density dependent type coupling between meson and baryon.

First, we introduce arcsinh type chiral potential extended from the logarithmic chiral potential. If we calculate Lattice action in SCL-LQCD on finite temperature and then we take zero temperature limit, logarithmic chiral potential should be corrected as

$$\begin{aligned}
&-\frac{a}{2}\log(y) + by^2 + c_\sigma y \quad y = \sqrt{\frac{\sigma^2}{+}\pi^2 f_\pi^2} \\
\to &-\frac{a'}{2}\mathrm{arcsinh}(\alpha y) + b'y^2 + c_\sigma y \\
= &-a'\left\{\mathrm{arcsinh}\left(\alpha\frac{f_\pi+\varphi}{f_\pi}\right) - \beta\frac{\varphi}{f_\pi} + \frac{\beta^3}{2}\left(\frac{\varphi}{f_\pi}\right)^2\right\} + \frac{m_\sigma^2}{2}\varphi + \frac{m_\pi^2}{2}\pi^2 \qquad (4)
\end{aligned}$$

where $\beta = \alpha/\sqrt{1+\alpha^2}$. In the case of the arcsinh chiral potential, we have an additional parameter α compared to the logarithmic one as shown in Eq. (4). We adjust it so as to reproduce TM1 non-linear σ potential since TM1 does not need so much c_ω to reproduce normal nuclear property. This result is shown in Fig. 1. Logarithmic chiral potential has an infinite barrier at $\sigma = 0$ and thus, full chiral restoration does not occur with this potential in all range of temperature. On the other hand, in arcsinh chiral potential has only finite (negative) derivative but no divergent behavior at $\sigma = 0$. Therefore, in finite temperature, chiral symmetry may restore and this point may be important when we calculate the equation of state in finite temperature for examining supernovae process.

Second, we study density NLO meson-bayron coupling written as

$$\mathcal{L}_{\mathrm{DDC}} = \bar{\Psi}_i g_{\sigma\omega i}\frac{\varphi_\sigma}{f_\pi}\not{\omega}\Psi_i \qquad (5)$$

where this coupling regarded as density dependent type coupling (DDC) since σ field is naively proportional to scalar density ρ_s, $\varphi_\sigma \sim \sum_i \frac{g_{\sigma i}}{m_\sigma^2}\rho_{si}$ in RMF models. As seen in Fig. 5 of Ref. 7, σ field seems to saturate in high density phase. Thus, this coupling should be more effective around normal density than in high density. From this point, this coupling can be regarded

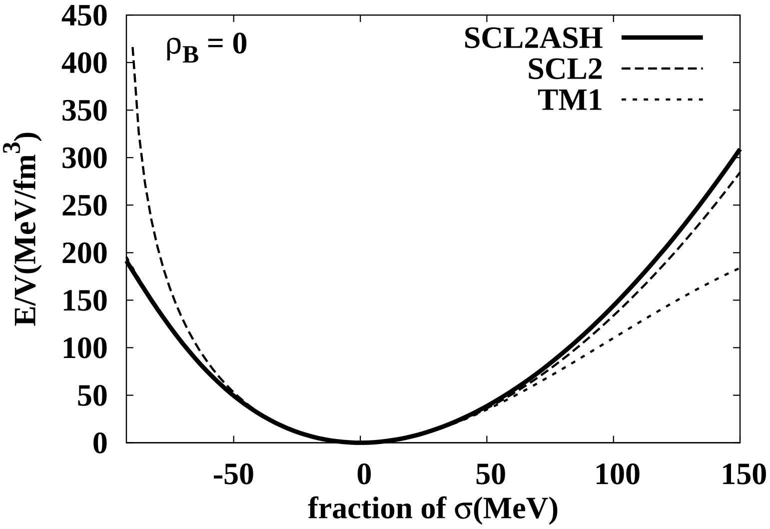

Fig. 1. Chiral potential of SCL3 and SCL2ASH RMF model. We also present effective non-linear potential of TM1.[4]

as a substitute of ω^4 potential around ρ_0 but should not suppress vector potential compared to ω^4 potential case.

There are several model parameters in two RMF models, SCLfASH RMF model and SCLfDDC RMF model which include arcsinh chiral potential and/or NLO density dependent type coupling, respectively. First, the parameters of chiral potential are constrained by the masses of mesons. As we mentioned, additional parameter α is determined by reproducing TM1 effective σ potential in the case of SCLfASH RMF model. The remained parameter m_σ and the strength of ω^4 potential, c_σ, and the coupling constants between mesons and baryons are determined so as to explaining the saturation property of symmetric nuclear matter, BE and charge rms radii of normal stable nuclei, S_Λ of single Λ hypernuclei, and $\Delta B_\Lambda\Lambda$ of ${}^{6}_{\Lambda\Lambda}$He. For the coupling constants between vector mesons and hyperons, we adopt SU$_f$(3) relation briefly introduced in Ref. 7. In SCLfDDC case, DDC parameters should be determined so as to reduce c_ω compared to previous SCL3 RMF model.

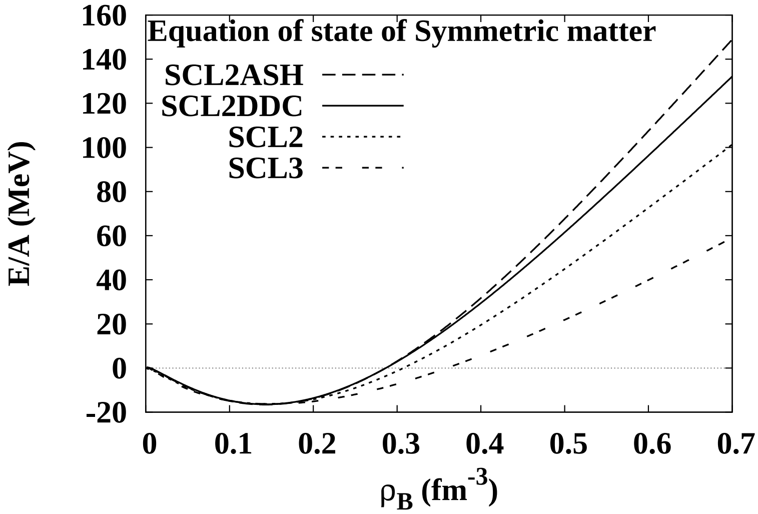

Fig. 2. Symmetric nuclear matter EOS of SCL2ASH and SCL2DDC RMF model.

3. Numerical results

In this section, we present brief results of the SCL3ASH RMF model and the SCL2DDC RMF model, respectively.

First, we show the *BE* of normal stable nuclei and symmetric nuclear matter in SCL2ASH RMF model and SCL2DDC RMF model with NL, TM, SCL2, and SCL3 RMF model. Both calculated symmetric nuclear matters of SCL2ASH and SCL2DDC succeed in explaining the saturation property of symmetric nuclear matter as shown in Fig. 2. Their incompressibilities are also in the empirically acceptable range. The parameter $g_{\rho N}$ is determined by reproducing *BE* of normal nuclei and both two models can explain with reasonable $g_{\rho N}$.

In addition, both two models succeed in reducing c_ω values in each their model, which are equal to 71.3 (66.5) in SCL2ASH (SCL2DDC) RMF model, respectively. In Fig. 3 and 4, we show calculated neutron star matter EOS and neutron star mass. Here, the results of SCL2ASH RMF model are almost similar to the ones of SCL2DDC RMF model. Therefore, we omit to plotting the results of SCL2ASH in these figures. As easily seen, calculated NS maximum masses of both two model show a enhancement compared to SCL2 RMF model result. From this result, these two extension is important

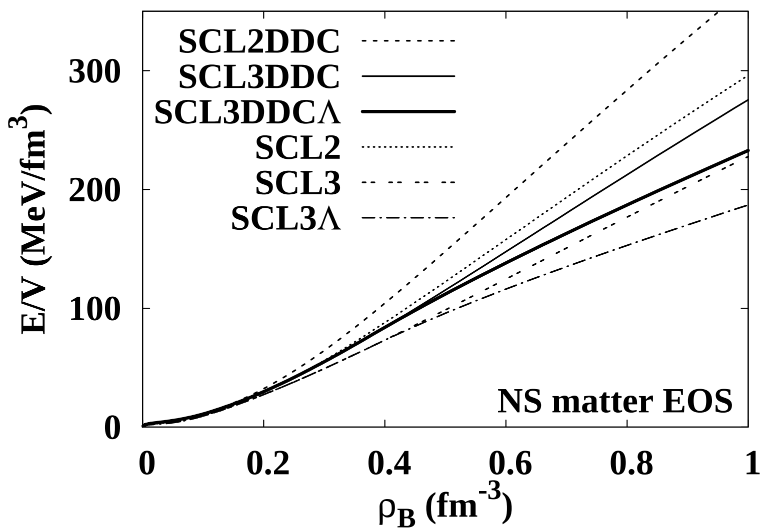

Fig. 3. Calculated neutron star matter EOS and neutron star mass.

for explaining the observed property of neutron star matter EOS.

Second, we introduce hyperon degree of freedom. Unfortunately, it is not well established to extend $\mathrm{SU}_f(2)$ arcsinh chiral potential to $\mathrm{SU}_f(3)$ one. Therefore, we concentrate on adopting SCL3DDC RMF model for hypernuclear physics. Calculated S_Λ in SCL3DDC RMF model are presented in Fig. 5. DDC parameter of Λ are determined by naive quark counting, $2g_{\sigma\omega N}/3$. Combining S_Λ and $\Delta B_{\Lambda\Lambda}$, we can fix two parameter for Λ, $g_{\sigma\Lambda}$ and $g_{\zeta\Lambda}$.

With these parameters, we can calculate neutron star matter EOS based on hypernuclear data. Back to Fig. 3 and 4, we show the neutron star matter EOS and neutron star mass on SCL3DDC RMF model where Λ hyperon is taken into account. From these results, hyperons make EOS softened certainly but calculated maximum mass of neutron star exceeds observed one.

4. Summary and discussion

In this paper, we examine two RMF models, SCLfASH RMF model and SCLfDDC RMF model which include arcsinh chiral potential and/or NLO density dependent type coupling, respectively. They are extended from

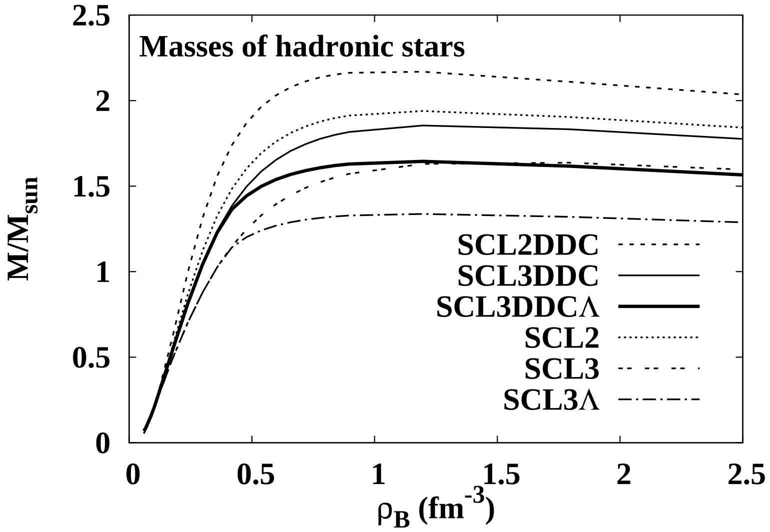

Fig. 4. Calculated neutron star matter EOS and neutron star mass.

SCL3 RMF model so as to explain observed maximum mass of neutron star with a chiral potential derived from SCL-LQCD.

In SCLfASH RMF model, the arcsinh chiral potential derived from finite temperature calculation is adopted in place of logarithmic chiral potential of SCL3 RMF model. Besides, we examine NLO density dependent type coupling between mesons and baryons in SCLfDDC RMF model. With these two RMF models, we can explain the saturation point of symmetric nuclear matter and *BE* of normal nuclei. Unfortunately, it is not well established to extend SCL-LQCD to $SU_f(3)$ case in finite temperature. Thus, we study hypernuclear systems with only SCL3DDC RMF model so far. Combining the $SU_f(3)$ symmetric relation for the coupling between vector mesons and baryons, the parameters of Λ hyperon, $g_{\sigma\Lambda}$ and $g_{\zeta\Lambda}$ are determined by reproducing S_Λ of single Λ hypernuclei and $\Delta B_{\Lambda\Lambda}$ of ${}^{6}_{\Lambda\Lambda}$He. With these parameters, SCL3DDC RMF model succeeds in explaining the maximum mass of neutron star even if it includes Λ hyperon. Thus, these results indicate that the chiral RMF model derived from SCL-LQCD can reproduce not only finite nuclear properties including hypernuclear data but also explain the observed neutron star maximum mass. We should proceed to further investigation about Σ and Ξ hyperons in order to discuss

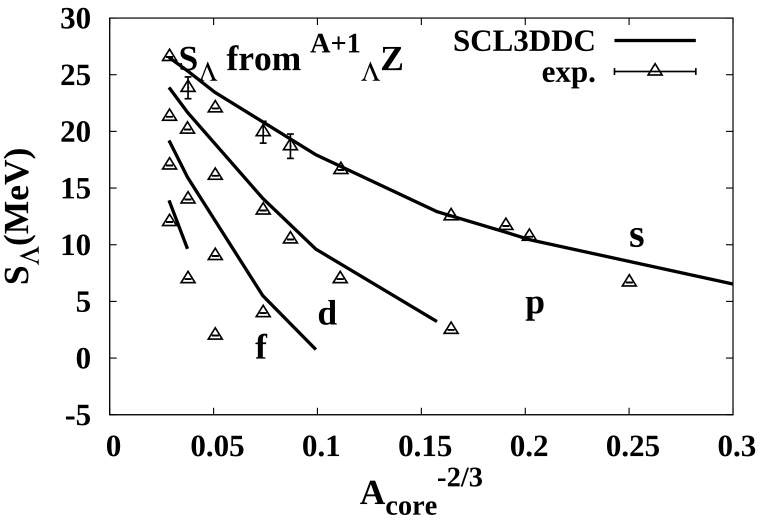

Fig. 5. Separation energy of Λ from single Λ hypernuclei.

neutron star matter more properly.

5. acknowledgments

This work was supported in part by KAKENHI from MEXT and JSPS under the grant numbers, 17070002, 19540252 and 20·4326, Global COE Program "The Next Generation of Physics, Spun from Universality and Emergence", and the Yukawa International Program for Quark-hadron Sciences (YIPQS).

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