THE TROJAN HORSE METHOD: BASICS AND RECENT RESULTS

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SUMMARY

1) Indirect methods: the Trojan Horse case

2) Trojan Horse Method: ingredients and checks

3) THM, RIBs and n-induced reactions

GAMOW WINDOW → 10-100 keV (non-explosive scenarios)

↓

Nano- Picobarn (even less!)

↓

Miserable S/N ratio

↓

Extrapolation

↓

Dedicated Experiments / Lab (LUNA)

↓

Electron Screening

↓

Extrapolation BACK AGAIN

Indirect Methods (CD, ANC, **THM**)

Caveat: NO free lunches in Nature

Trojan Horse Method

Main application: measurements of charged particle cross sections at astrophysical energies

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Phys. Lett. B, Vol 176 (1986)

BREAKUP REACTIONS AS AN INDIRECT METHOD TO INVESTIGATE LOW-ENERGY CHARGED-PARTICLE REACTIONS RELEVANT FOR NUCLEAR ASTROPHYSICS

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Received 18 April 1986; revised manuscript received 10 July 1986

It is proposed to use breakup reactions as a means to extract information on charged-particle induced reactions at low relative energies. The Coulomb penetration factor, which diminishes tremendously the two-particle cross section, is overcome in the three-body scattering approach. The assumptions and possibilities of such a method are discussed and applications to astrophysically relevant nuclear reactions are indicated.

The study of charged-particle reactions at low relative energies is of special interest for the synthesis of the elements in the universe [1]. A great problem in the direct experimental study of such reactions at the relevant astrophysical energies is the very low cross section due to the Coulomb barrier of the incident particles. Usually a mixture of experimental information at higher energies and theoretical arguments and calculations is used in order to extrapolate the astrophysical S-factor down to the relevant energies.

In this letter it is proposed to obtain information about the low-energy charged-particle induced reaction

$$A+x \to c+C \qquad (1)$$

by means of the three-body type of reaction

$$A+(b+x) \to b+c+C. \qquad (2)$$

A "spectator" particle b is attached to particle x, to form a projectile a = (b+x). The bombarding energy E_a is chosen to overcome the Coulomb barrier in the incident channel of reaction (2). In this way, particle x can be brought into the nuclear reaction zone to induce the reaction (1) of particle x with A. If the Fermi motion of particle x inside a compensates for the initial projectile velocity v_a, this reaction (1) is induced at very low (even vanishing) relative energy between A and x. This "trojan horse method" is illustrated schematically in fig. 1. It is now suggested to study reaction (2) experimentally under conditions which correspond to astrophysically relevant energies between x and A. The problem is then to obtain, from the experimentally determined coincidence cross section $d^3\sigma/d\Omega_c d\Omega_b dE_b$, information about the astrophysically interesting cross section

$$\sigma_{Ax\to cC} = \frac{\pi}{q_x^2}\sum_l (2l+1)\,|S_{lc}|^2. \qquad (3)$$

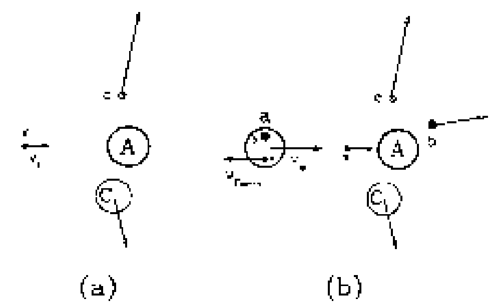

Fig. 1. At astrophysically relevant energies the two-particle reaction A+x→c+C is strongly hindered by the Coulomb potential (part (a)). In the three-body approach (b), particle x is brought into the nuclear reaction zone of the target nucleus A inside the projectile a = (b+x) with velocity v_a and it induces the reaction at the low relative energies corresponding to $v_x = v_a - v_{Fermi}$ in which one is interested.

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THM: a primer

Idea: get the 2-body cross-section of the process

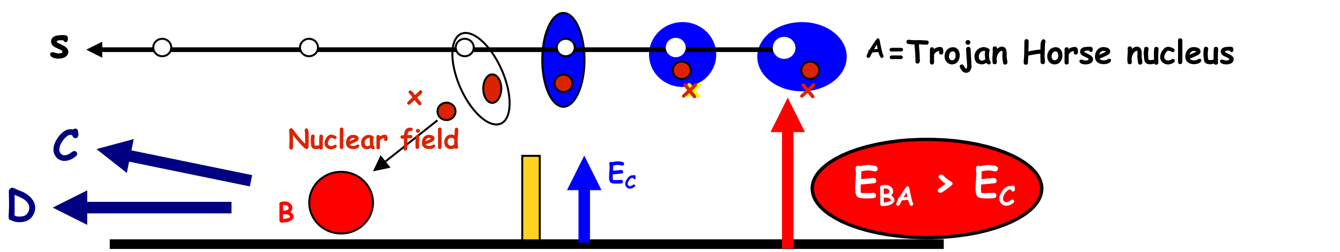
$B + x \rightarrow C + D$

At astrophysical energies from the QUASI-FREE contribution

of a 3-body reaction *(C. Spitaleri, Folgaria 1990)*

$B + A \rightarrow C + D + S$

$A = x \otimes S$



$E_{Bx} = E_{CD} - Q_2$

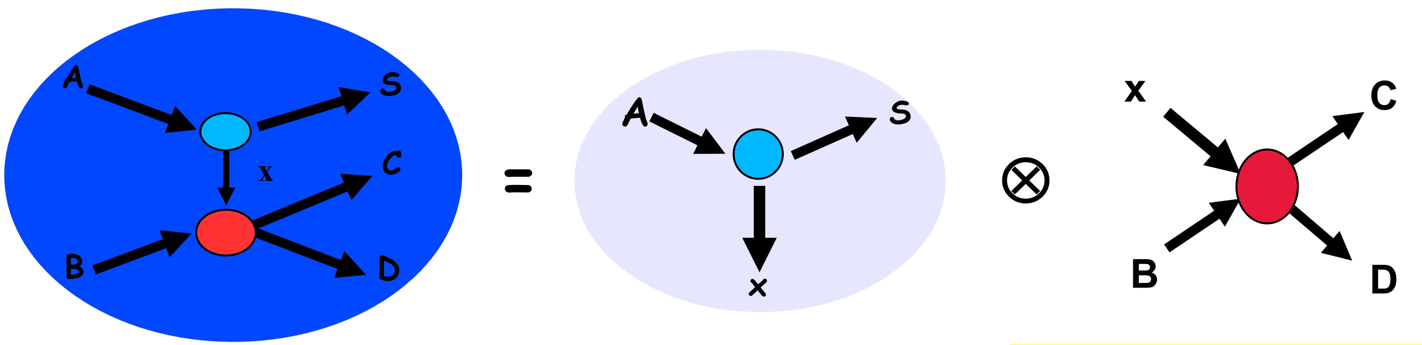
E_{Bx} = interaction energy B-x

E_C = Coulomb barrier between A and B

E_{BA} = relative energy between A and B

Electron screening removed by construction

Assuming that a Quasi-free mechanism is dominant one can use PWIA:



3-body Reaction

Virtual Decay

Virtual reaction (astrophysical process)

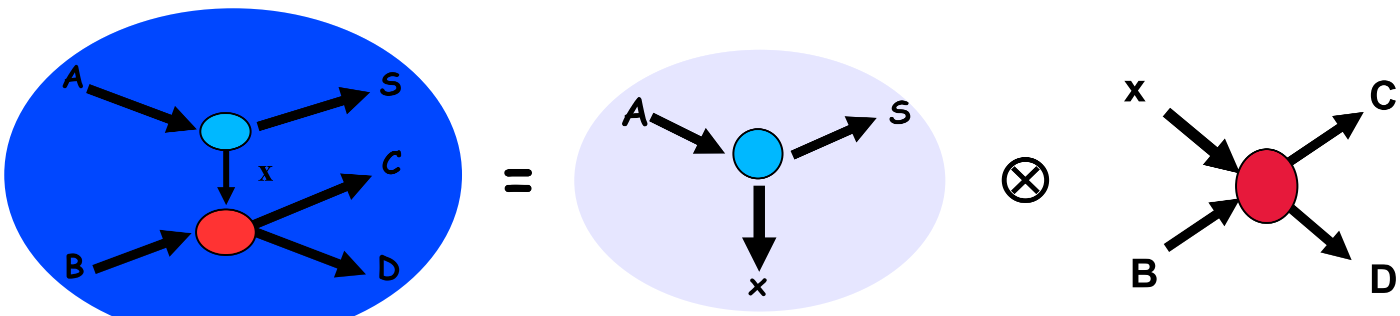
Nuclear!

$$\frac{d^3\sigma}{d\Omega_C\, d\Omega_D\, dE_{cm}} \propto KF \cdot |\Phi\ (P_s)|^2 \bullet \frac{d\sigma^N}{d\Omega}$$

$E_{Bx} = E_{CD} - Q_{2b}$

And by inverting this…

Assuming that a Quasi-free mechanism is dominant one can use PWIA:



3-body Reaction

Virtual Decay

Virtual reaction
(astrophysical process)

$$\frac{d^3\sigma}{d\Omega_C\, d\Omega_D\, dE_{cm}} \;\backslash\; KF \cdot |\Phi(P_s)|^2 \propto \frac{d\sigma^N}{d\Omega}$$

Nuclear!

$E_{Bx} = E_{CD} - Q_{2b}$

... one obtains this!

Interlude

Up to now no specific calculation procedure has been applied.

Only the possibility of factorizing the 3 body cross-section is really important.

Technical (i.e. theoretical) Approaches used so far BY OUR GROUP

- **PWIA (Kondratiev) 1994**
- **MPWBA (Typel-Wolter) (roughly) 2000**
- **PWIA+DWBA+many others (Mukhamedzanov+Bertulani) 2003→ to date**

First THM theoretical analysis: V. Kondratiev

$^6Li+d \rightarrow \alpha+\alpha$ from $^6Li+^6Li \rightarrow \alpha+\alpha+\alpha_s$

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM}^{\text{HOES}} = \Sigma_k a_k P_k\left(\cos\vartheta_{CM}\right)$$

$$a_k = \left(\hat{J}_A \hat{J}_x\right)^{-2} \sum_{\langle f|}\sum_{|i\rangle} \langle s_f l_f | T_j | s_i l_i \rangle \langle s_f l'_f | T_{j'} | s_i l'_i \rangle \bullet$$

$$(-1)^{s_i - s_f}\, \hat{l}_i \hat{l}_{i'} \hat{J}\, \hat{J}'\, \left(l_i 0 l'_i 0 | K 0\right) \bullet Wig\left(l_i J l'_i J'_i; S_i K\right)$$

(see Cherubini et al, ApJ 457 (1996) 855 for details)

In the T matrix expression we considered RESONANT (sub-threshold ^8Be state), NON-RESONANT and INTERFERENCE terms.

a_k fixed by fitting data for 2-body NUCLEAR cross-section

First THM paper
ApJ 457 (1996)

…urnal, 457:855–858, 1996 February 1

INDIRECT INVESTIGATION OF THE $d + {}^6$Li REACTION AT …
FOR NUCLEAR ASTROPHY…

S. Cherubini, V. N. Kondratyev,[1] M. L…
Università di Catania and INFN–Laborator…
Đ. Miljanić …
Ruđer Bošković …
Forschungszentrum Jül…

BUT …

NO FREE LUNCHES IN NATURE!

Only 1 parameter but no absolute cross sections

The indirect investig…
sidered, employing…
reaction at proj…
tion of the ^{6}…
factor ex… …al
Subj…

… …atical conditions. At these … …ess is considered to be well … …ynman diagram of Figure 1. The … …med to disintegrate into the clusters α_s … …sidered to be a spectator to the ^{6}Li(d, α)α … …at proceeds within the region of the nuclear … …his description allows direct comparison of the … …erential cross section $d^3\sigma/(dE_1\, d\Omega_1\, d\Omega_2)$ of the … …Li, $2\alpha)\alpha_s$ reaction with the cross section for the ^{6}Li(d, α)α nuclear process. Using the distorted wave impulse approximation, we express the triple-differential cross section, measured in an α_1-α_2 coincidence experiment, through the nuclear part of the two-body reaction cross section as (see Chant & Roos 1977 and references therein):

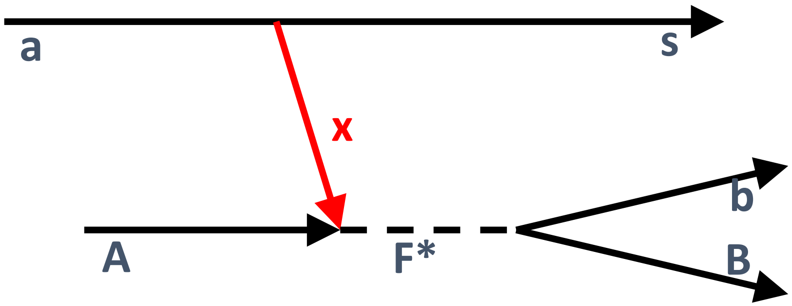
$$\frac{d^3\sigma}{dE_1\, d\Omega_1\, d\Omega_2} = \mathrm{KF}|\Phi_{p_{12}}(p_s)|^2 \frac{d\sigma^N}{d\Omega}, \quad (1)$$

where KF is a kinematical factor. The distorted, spectator

r…
nu…
well … …ent
comes … …hich is
due to … …o the inci-
dent ener… …action of the
astrophysic… …s is performed by
extrapolation… …ion taken at higher
energies. This p… …ssumption of a theoretical

Slide by Marco la Cognata

The full THM: the resonant case (A. Mukhamedzhanov)



R. Tribble et al. ... Prog. Phys. **77** (2014) 10...

Amplitude of the TH ...

Same R-matrix term as in OES cross section but for the appearence of the inverse penetration factor making it possible to obser... suppressed resonances at low energies

negligible at usual experimental precisions

$M^{\text{PWA(prior)}}(P, k_{aA}) \ldots$

$\times \ldots$

$\ldots_{lm_l}(-\hat{k}_{bB})$

$\ldots(\hat{p}_{xA})$

$\ldots P_{l'}^{-1/2}(k_{xA}, R_{xA})(j_{l'}(p_{xA}R_{xA})$

$\ldots_{xA}) - 1) - D_{xAl'}(p_{xA}, R_{xA})]$

$$+ 2 \ldots_{A} e^2 \mu_{xA} \int_{R_{xA}}^{\infty} \mathrm{d}r_{xA} \frac{O_{l'}(k_{xA}, r_{xA})}{O_{l'}(k_{xA}, R_{xA})} j_{l'}(p_{xA}r_{xA})).$$

...rier ...form of ...ative motion wave function

solid sphere scattering phase shift

Inverse penetration

R-matrix-like boundary condition

BUT ...

NO FREE LUNCHES IN NATURE!

Absolute cross sections but many parameters

Optimization of the «ingredients» of the method

PHYSICAL REVIEW C **71**, 058801 (2005)

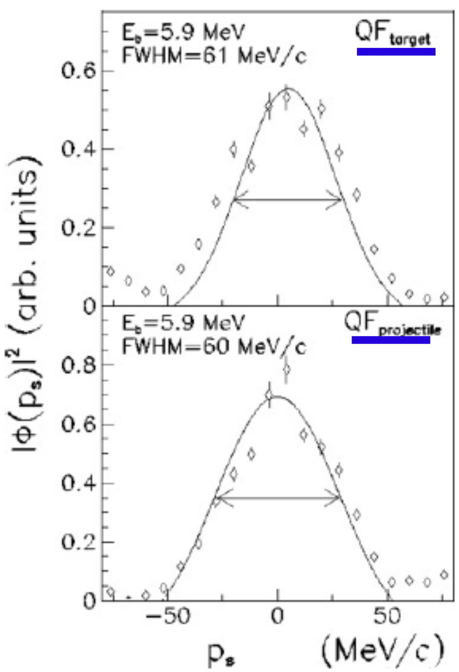


FIG. 1. Experimental momentum distribution for the α particle inside ^{6}Li derived according to the guidelines given in the text for the ^{6}Li(^{6}Li, $\alpha\alpha$)^{4}He reaction. The upper and lower parts refer to the target and projectile breakup cases, respectively.

Impulse distribution of a+d in ^{6}Li

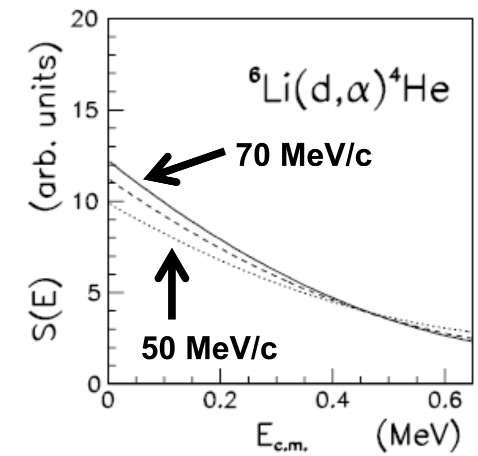
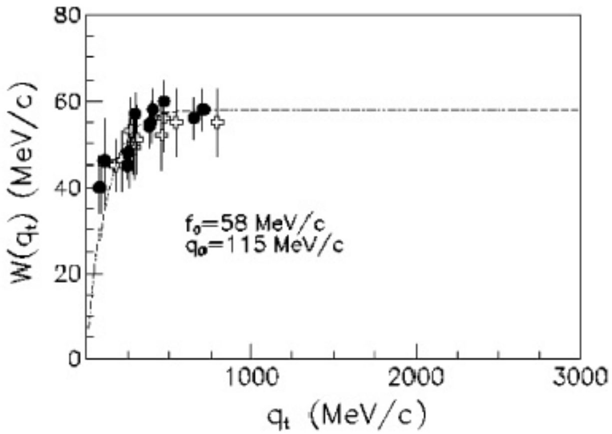


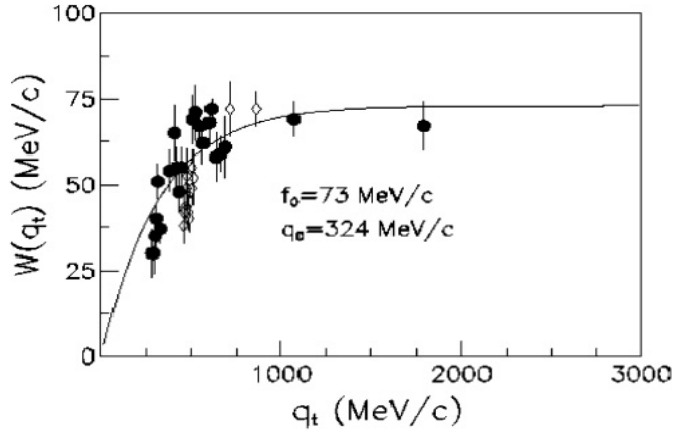
FIG. 3. Experimental ^{6}Li$(d,\alpha)^4$He $S(E)$ factor, extracted via the THM, for different choices of the $w(q_t)$ for the α momentum distribution inside ^{6}Li. The solid line represents the case of $w(q_t) =$ 70 MeV/c, the dashed line is for $w(q_t)$ = 61 MeV/c, and dotted line is for $w(q_t) = 50$ MeV/c.

Dependence of the final result from the impulse distribution width

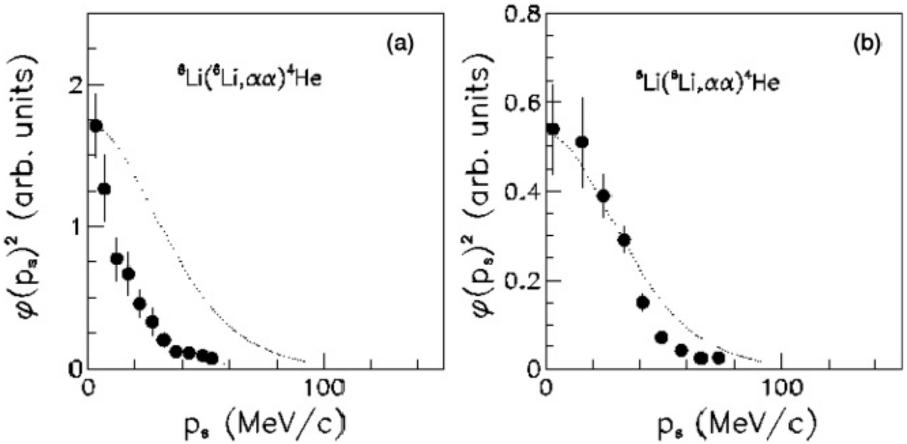
(IN)dependence from the Trojan Horse nucleus also verified

Optimization of the «ingredients» of the method





$$\mathbf{q}_t^{QF} = m_x \sqrt{\frac{1}{m_A m_B}}\, \mathbf{p}_A$$



RG Pizzone et al
Phys Rev C 80,
025807 (2009)

Treiman-Yang Criterion: a bit of history

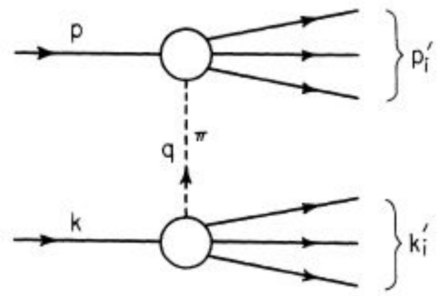


FIG. 1. Diagram for single-pion exchange.

TESTS OF THE SINGLE-PION EXCHANGE MODEL

S. B. Treiman
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

and

C. N. Yang
Institute for Advanced Study, Princeton, New Jersey

(Received December 14, 1961)

The differential reaction cross section $d\sigma$ is given by

$$J d\sigma = f \prod_i dp_i' \delta(p_i'^2 + m_i^2) \times \prod_j dk_j' \delta(k_j'^2 + \mu_j^2)\delta(p + k - \sum p_i' - \sum k_j'). \quad (1)$$

where J is the relative current of the incident particles, f is the square of the invariant transition amplitude, and all energies are positive-definite. The crucial remark is that, on the peripheral collision picture, f has the structure

$$f = G(p, p_i')H(k, k_i'). \quad (2)$$

The implications of this restriction on the structure of f are best brought out in the reference frames in which one or another of the initial particles is at rest. Thus:

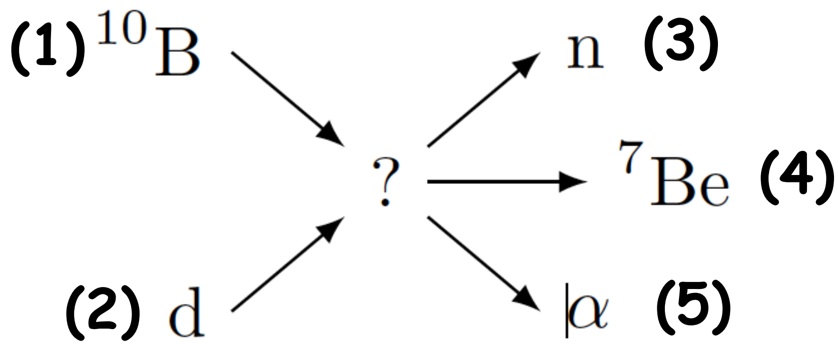
1. In the system where p is at rest (the laboratory system, if p is in fact the target particle), the differential cross section should be invariant under the simultaneous rotation of all three-vectors $\vec{p}_i'$ about the momentum vector $\vec{q}$ of the virtual meson: $\vec{q} = \vec{k} - \sum_i \vec{k}_i' = \sum_i \vec{p}_i'$. This result follows from inspection of Eqs. (1) and (2).

2. Similarly, in the system where k is at rest the differential cross section should be invariant under simultaneous rotation of all three-vectors $\vec{k}_i'$ about $\vec{q} = -\sum_i \vec{k}_i' = \sum_i \vec{p}_i' - \vec{p}$.

It is easy to prove that the above two tests are exhaustive for fixed incoming energy. There are

VOLUME 8, NUMBER 3 PHYSICAL REVIEW LETTERS FEBRUARY 1, 1962

First things first…



Spin avaraged $|M|^2$ for a reaction with n-2 bodies in the final state

1 + 2 → 3 + 4 + 5 (n=5 in this case)

depends on 3n-10 indipendent variables:

→5 variables for 3 bodies in the final state

→2 variables for 2 bodies in the final state

→0 variables for a two body decay

Mandelstam variables

A good and well known choice for kinematical variables are the Mandelstam invariants

Four momentum $\underline{P} = (p_x, p_y, p_z, E)$ with metrics (-1,-1,-1,1)

$$|\underline{P}|^2 = E^2 - p^2 = m^2 \qquad (c=1, \quad E = K+m)$$

For any pair of particles 1 and 2 the s and t Mandelstam variables are defined as

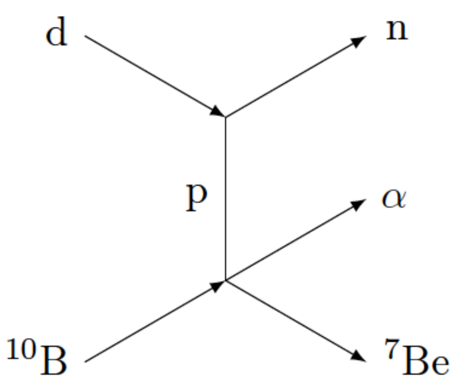
$$s_{12} = (\underline{P}_1 + \underline{P}_2)^2 \qquad t_{12} = (\underline{P}_1 - \underline{P}_2)^2$$

So, for a reaction with 3 bodies in the final state, a choice of 5 independent variables is

$$s_{12}, \quad t_{13}, \quad t_{24}, \quad s_{35}, \quad s_{45}$$

Mechanism-specific invariants

For a given mechanism LESS than 5 variables can contribute. E.g.:

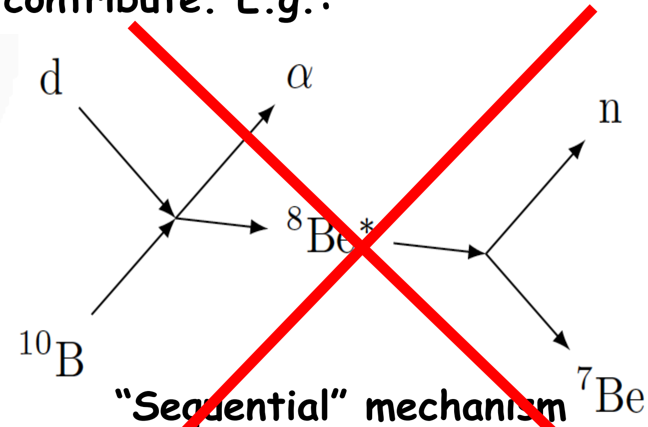


Quasi-free mechanism

Virtual decay d→p+n: 0 variables

Virtual reaction: 2 variables among $s_{\alpha-Be}$, t_{B-Be}, t_{B-a} (i.e. u)

$|\underline{P}_p|^2 = t_{d-n}$



"Sequential" mechanism

2-body d(^{10}B,a)^{8}Be* reaction: 2 var. among s_{d-B}, $t_{d-\alpha}$, t_{B-a} (i.e. u)

Decay ^{8}Be*→^{7}Be+n: 0 variables

$|\underline{P}_p|^2 = s_{Be-n}$

If $|M|^2$ does NOT depend on s_{d-B} sequential is ruled out

Treiman Yang idea in simple words

In a 1+2 → 3+ 4 +5 reaction

Keep

$s_{4\text{-}5}$, $t_{1\text{-}4}$, $t_{1\text{-}5}$ (i.e. u)

i.e.: $s_{\alpha-Be}$, $t_{B\text{-}Be}$, $t_{B\text{-}a}$

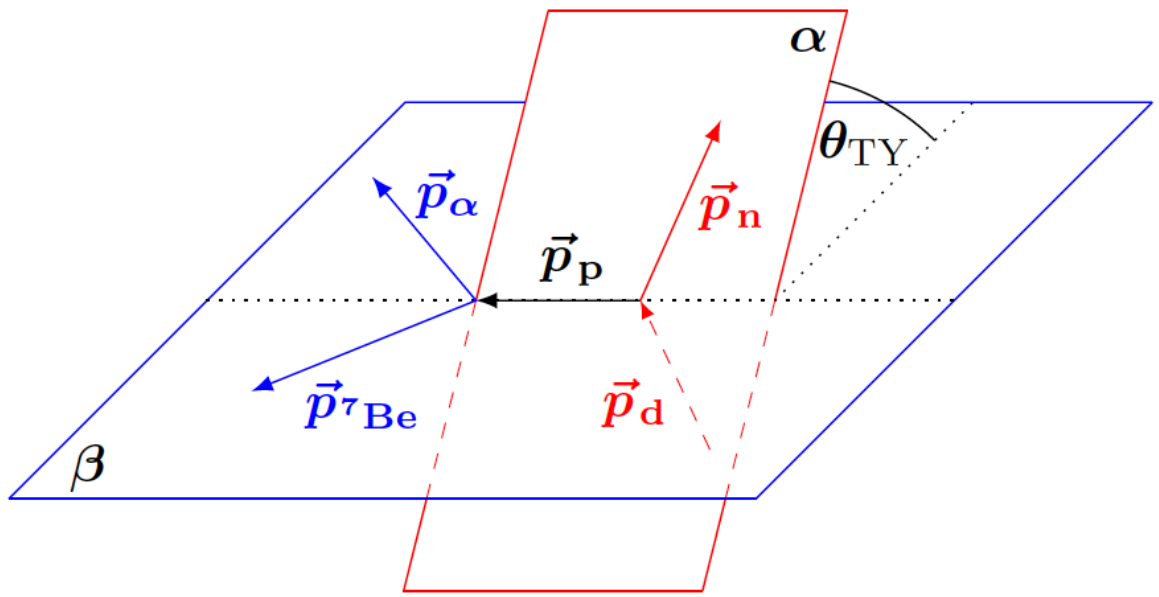
constant

and change the others. If QF dominates, then

$$|M|^2 = \text{const}$$

Note. This is still NOT a sufficient condition for QF dominance, but it is a very very strong one. It becomes even stronger if the invariance keep true by changing the beam energy (i.e. $s_{d\text{-}B}$)

Treiman Yang rotation



The previous statement is equivalent to have an invariance of $|M|^2$ under rotations of plane α with respect to plane β (in the reference frame where the particle that does not breakup is at rest, ^{10}B in this case)

$\theta_{\rm TY}$ is the Treiman-Yang rotation angle

Treiman Yang Creterion Summary

We have applied TY criterion roughly 35 year after first attempts by the Catania group

Results were reasonably good

TY is a powerful tool in connection with THM for Nuclear Astrophysical studies, as the signature of the QF mechanism provided by TY invariance is very strong

Paper in preparation

Future: make TY routinely used in THM studies (requires use of bidim detectors).

$^{18}F+p \rightarrow {}^{15}O + \alpha$ VIA THM at CRIB

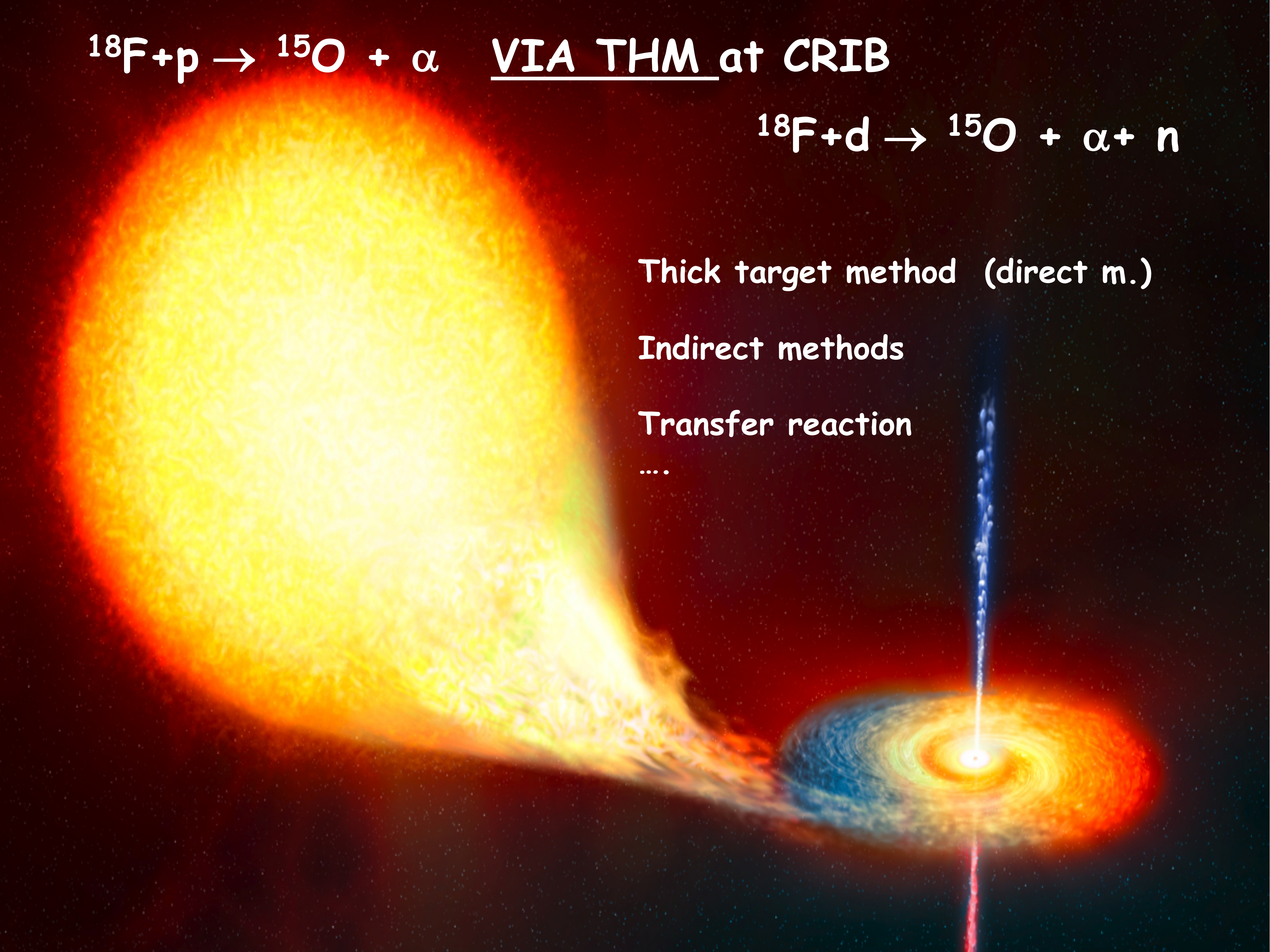
$^{18}F+d \rightarrow {}^{15}O + \alpha + n$

Thick target method (direct m.)

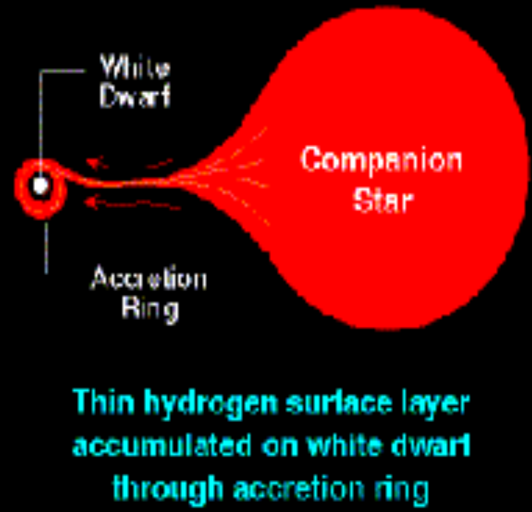
Indirect methods

Transfer reaction

….



$^{18}F(p,\alpha)^{15}O$

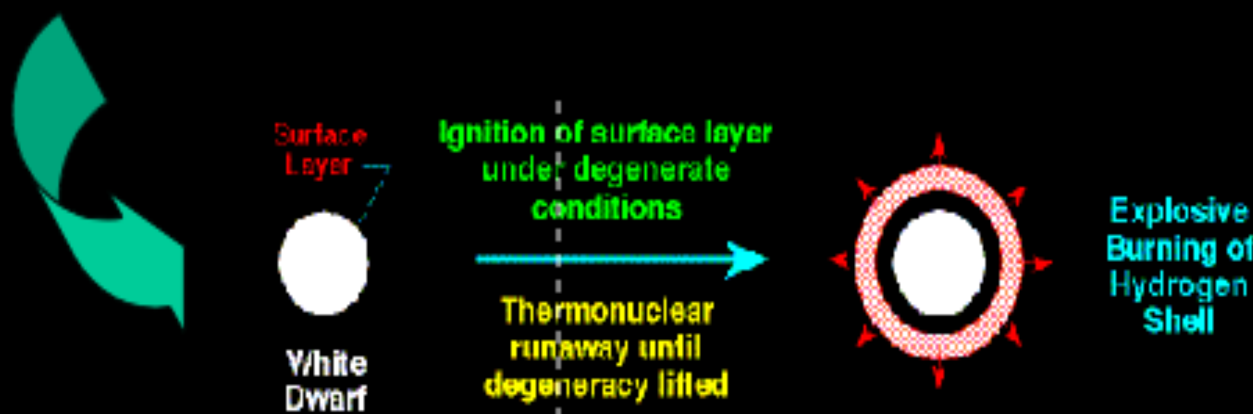


Observed γ- rays come from e^+e^-

e^+ come from ^{18}F decay mostly

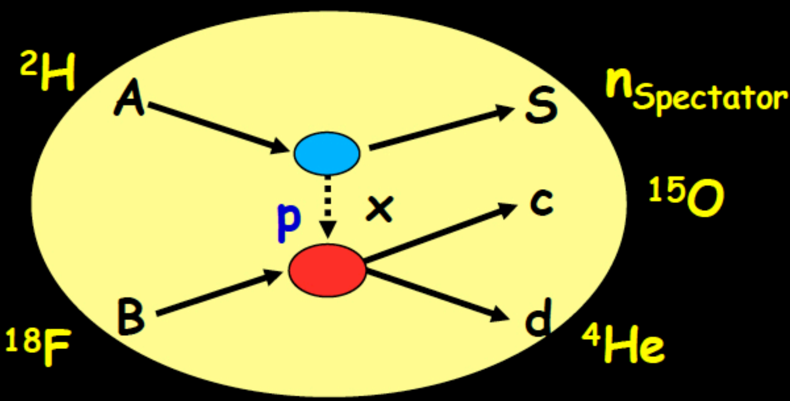
At novae temperatures (100 - 500 keV) ^{18}F can be mainly destroyed by

$^{18}F(p,\alpha)^{15}O$

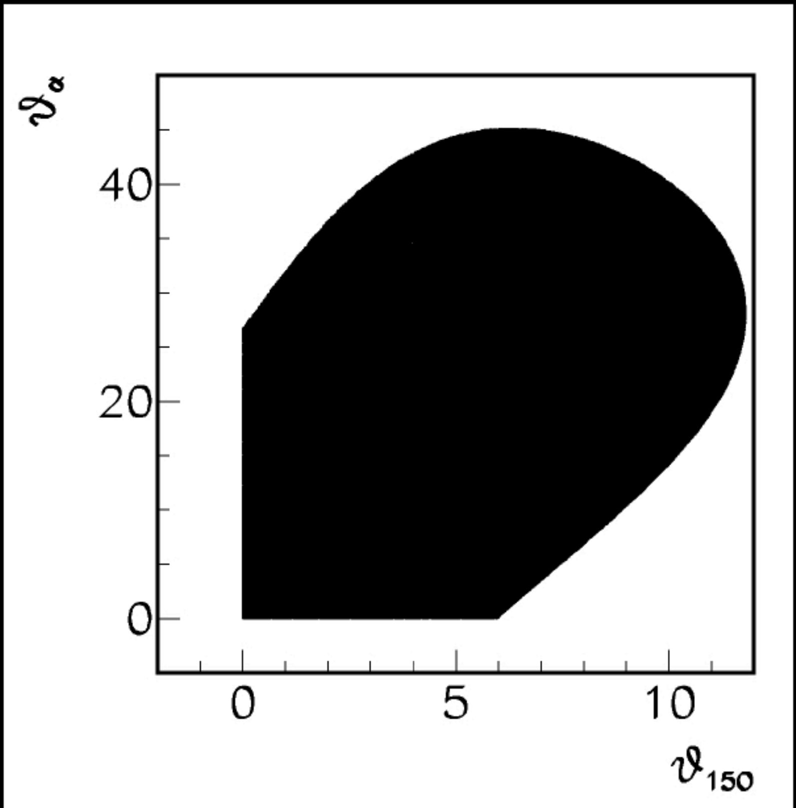


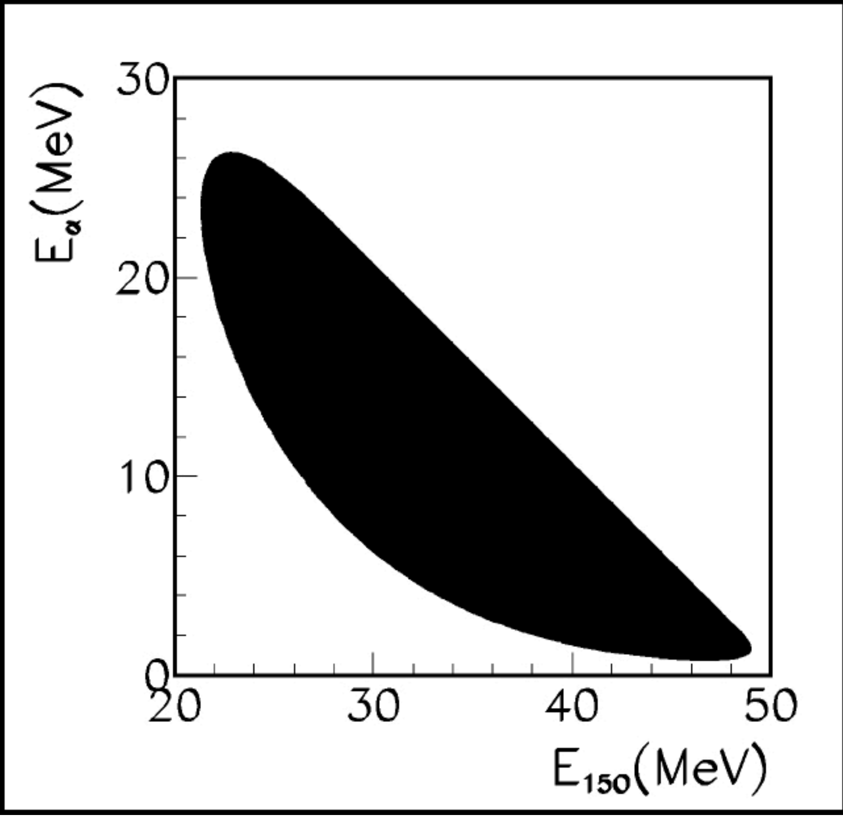
For the star energetics this are peanuts!

THM Experiment kinematics... needs all!



E(^{18}F) = 50 MeV

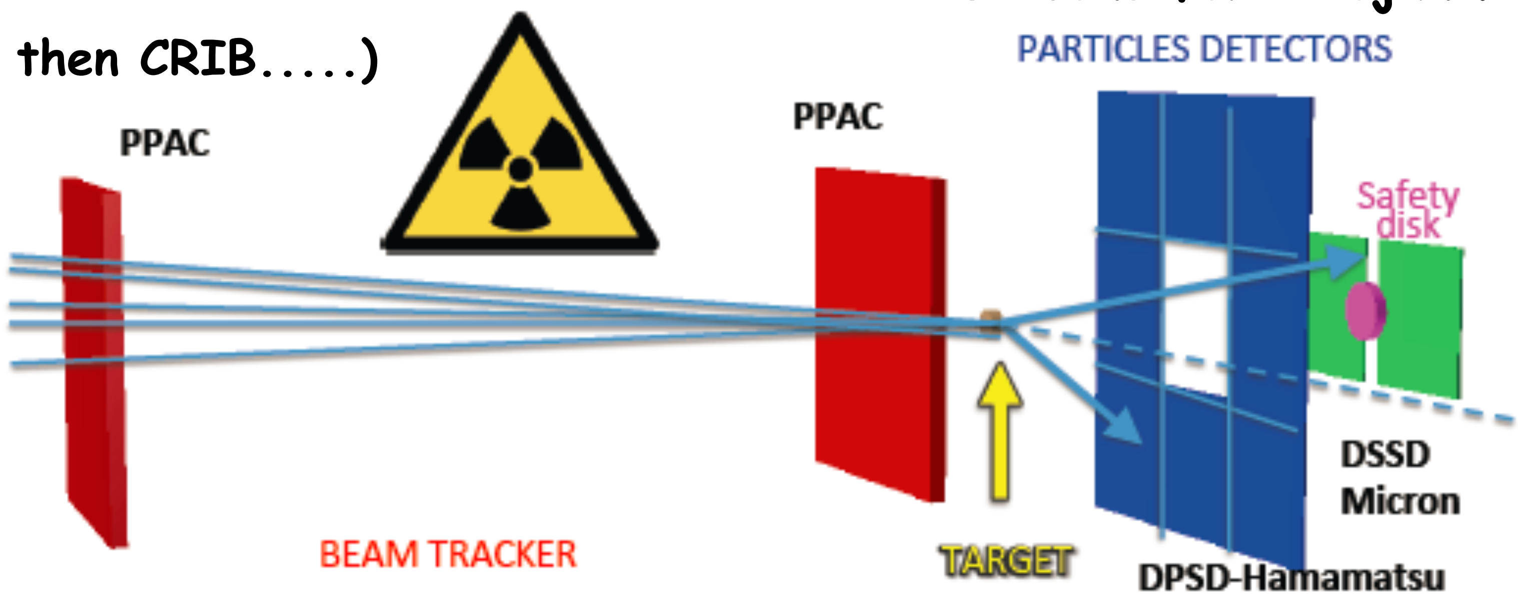


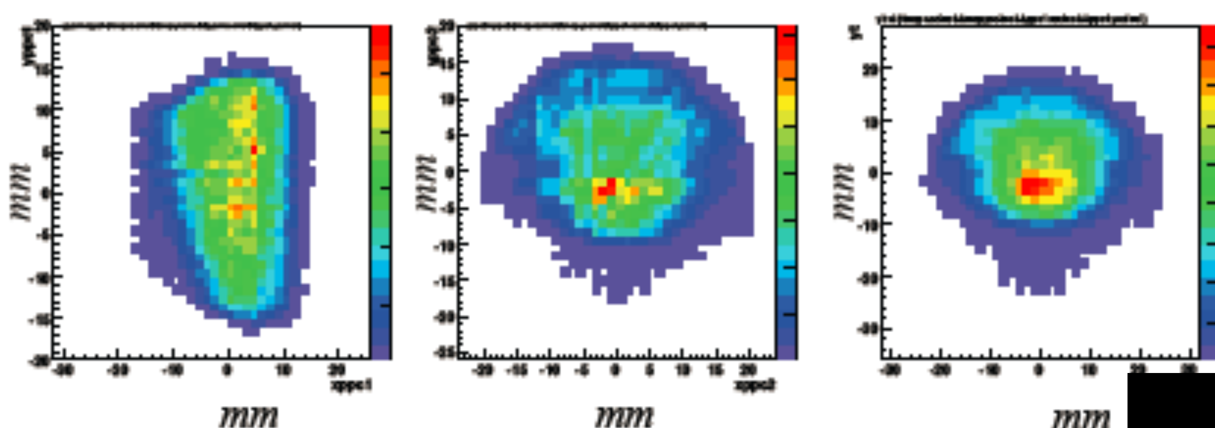


EXPERIMENTAL SETUP

ASTRHO: Array of Silicons for TRojan HOrse

(other then CRIB.....)

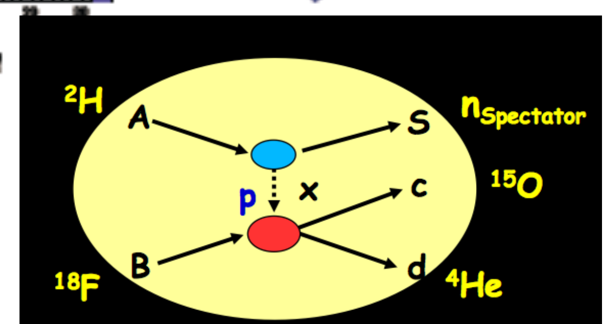


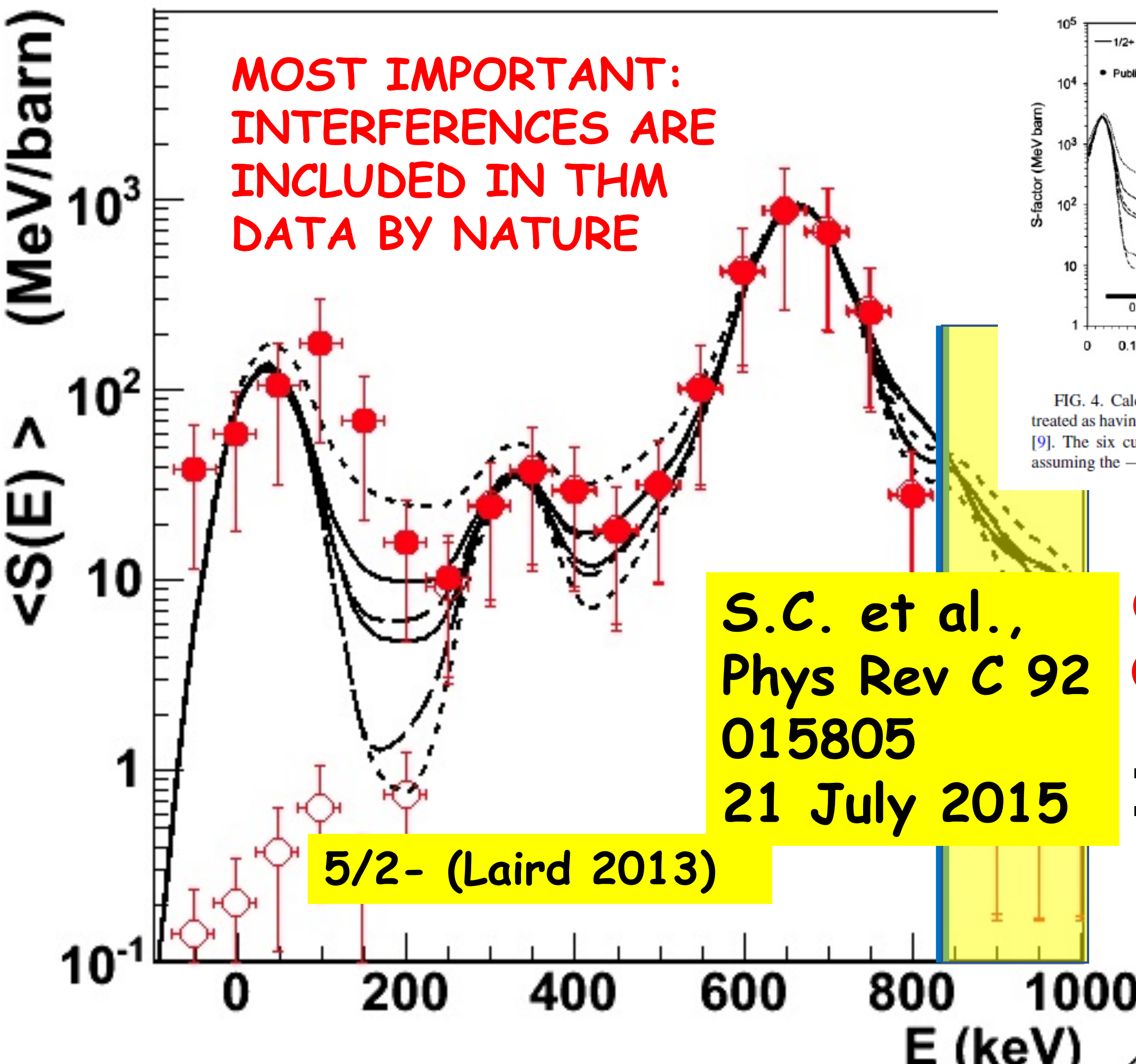


BEAM TRACKED EVENT- BY - EVENT

Ebeam (^{18}F) = 50 MeV

Geometric efficiency simulated using GEANT3





THM data

C.E. Beer, Phys. Rev. C 83, 042801(R) (2011)

Smeared to THM resolution

C. E. BEER *et al.*

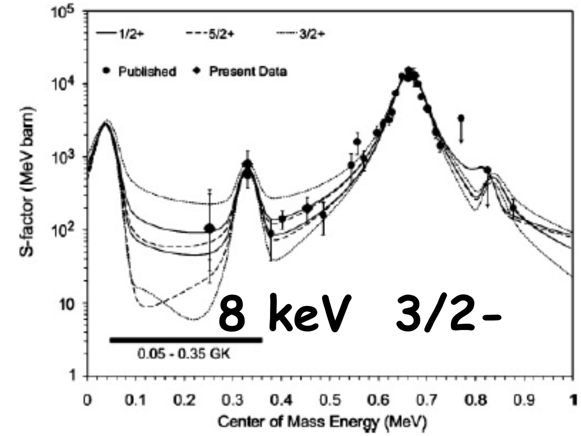
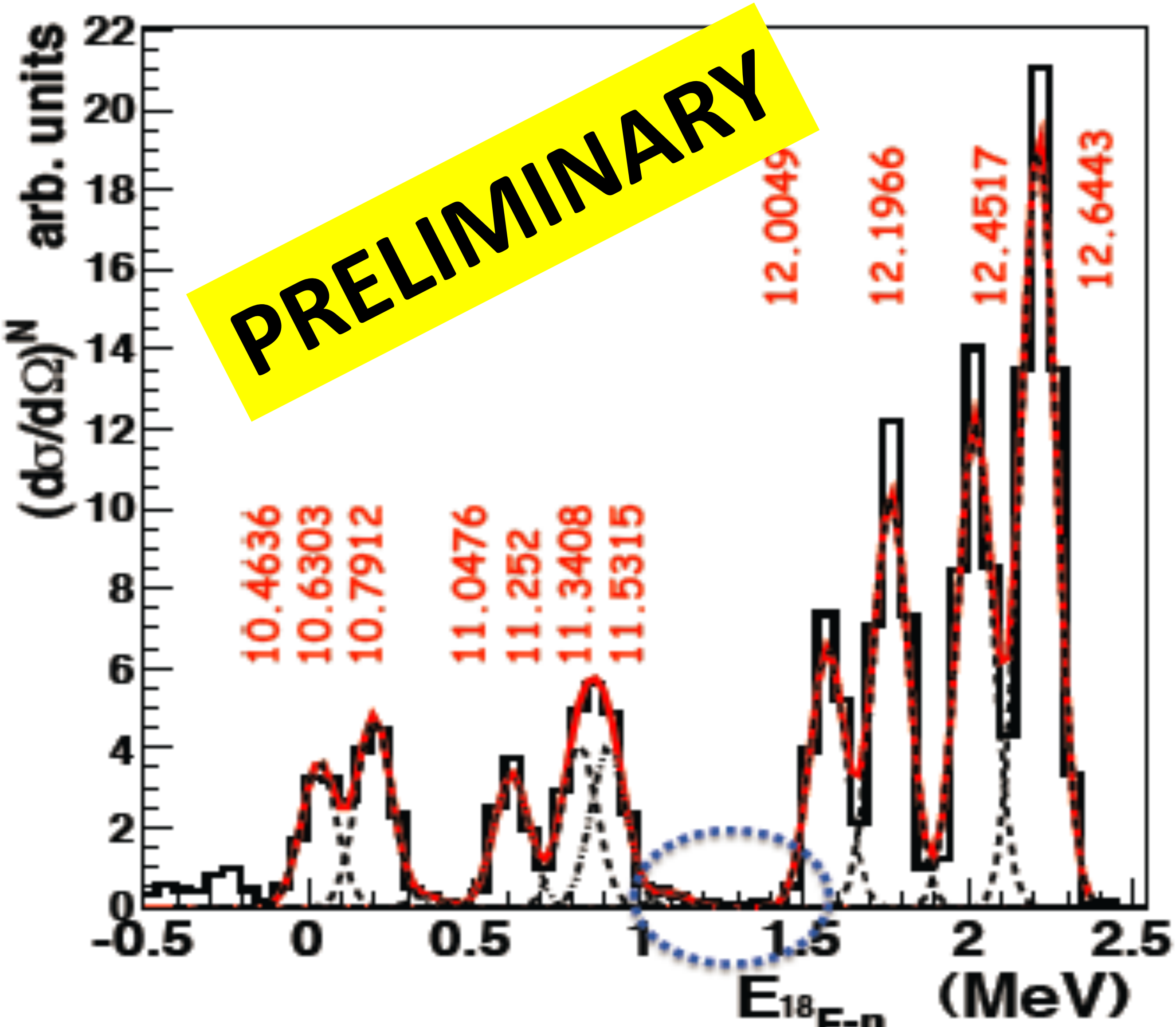


FIG. 4. Calculated $^{18}F(p,\alpha)^{15}O$ S factors with the 8 keV state treated as having a spin-parity of $3/2^-$ using the Adekola parameters [9]. The six curves correspond to the upper and lower S factors, assuming the -121 keV resonance to be $1/2^+$, $5/2^+$, or $3/2^+$.

18F+n→15N+α



PRELIMINARY
(dσ/dΩ)^N arb. units
22
20
18
16
14
12
10
8
6
4
2
0
-0.5
0
0.5
1
1.5
2
2.5
E18F-n (MeV)
10.4636
10.6303
10.7912
11.0476
11.252
11.3408
11.5315
12.0049
12.1966
12.4517
12.6443

NEW 18F+d experiment @ CRIB

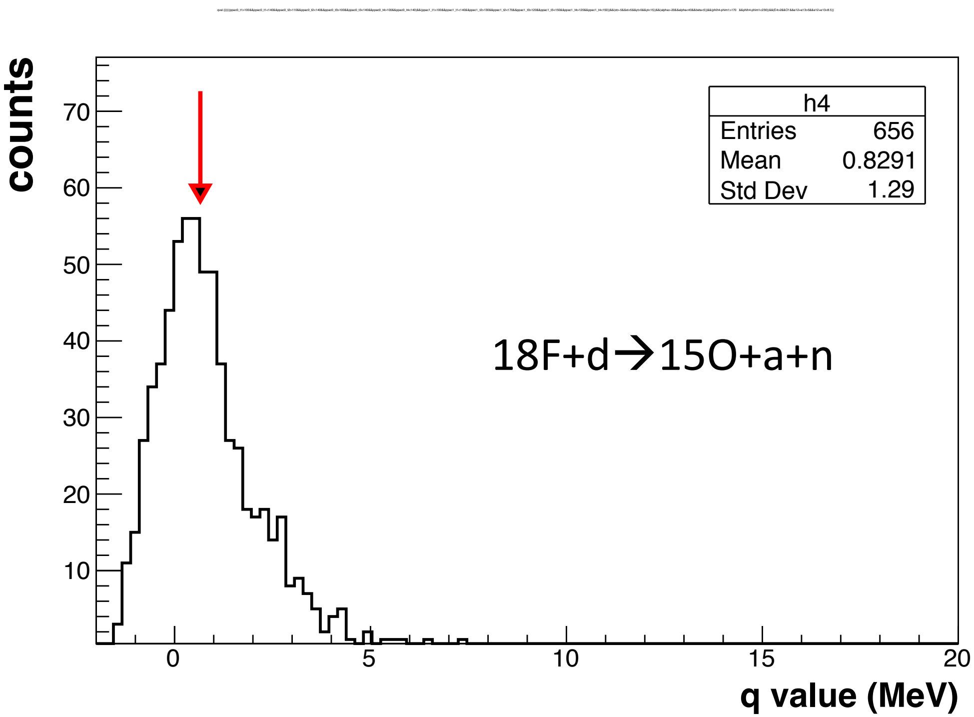
Performed October-November 2015

+ Setup upgrade: DE stage for DSSD added

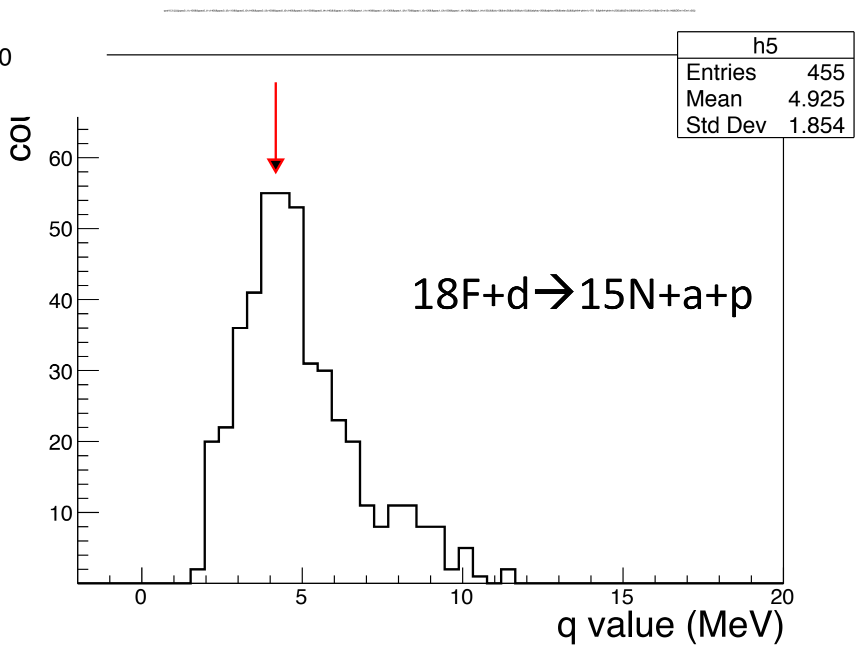
+ 15 days of (relatively) smooth data taking

- we got more beam intensity than previous experiment (on average), we expected even more…
- The beam quality was unstable
- Data Analysis under way

Q-value for the 3-body reactions



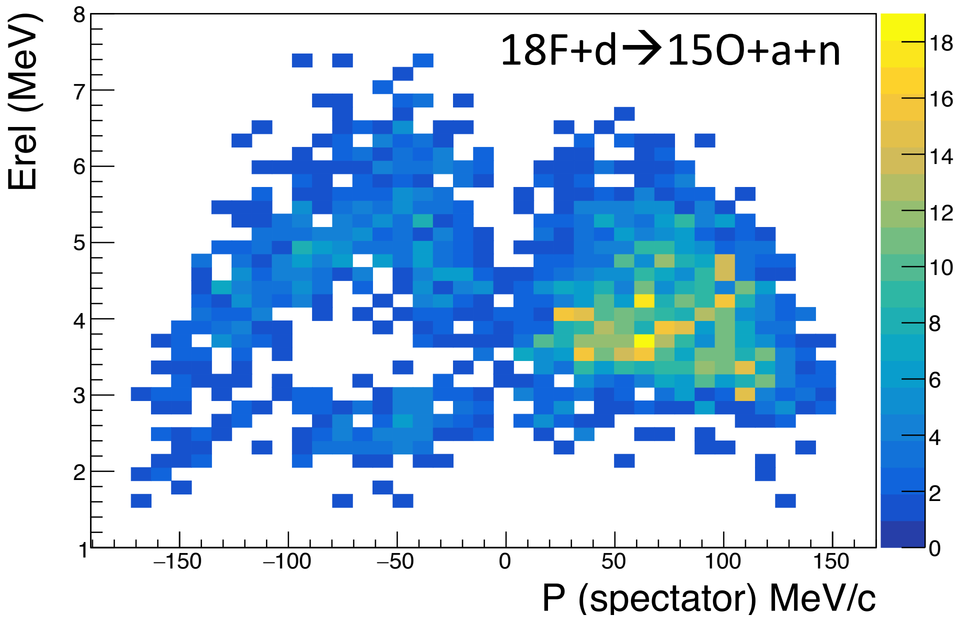
.... after some cuts on beam



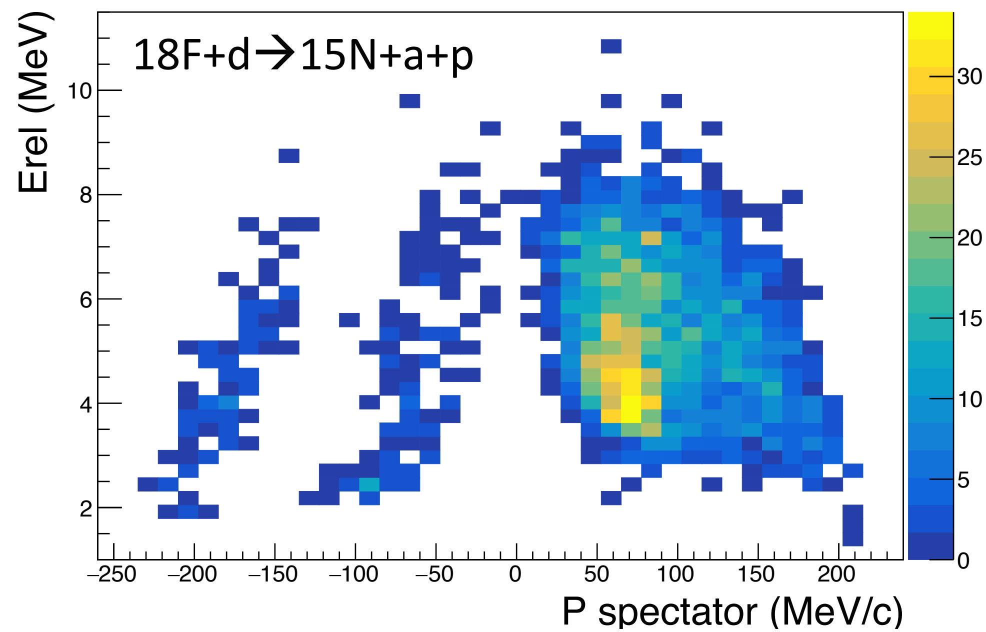
runs 114-132

Only 1 hamamatsu detector in coincidence with one micro-strips telescope

Selection of O and N on DE%E spectra



Erel vs momentum of spectator



THANKS FOR YOUR ATTENTION

THM was developped by the ASFIN Collaboration since 1990.

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