

# **Proximity effect of pair correlation in the inner crust of neutron stars with Hartree-Fock-Bogoliubov theory**

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OMEG15, YITP, JAPAN

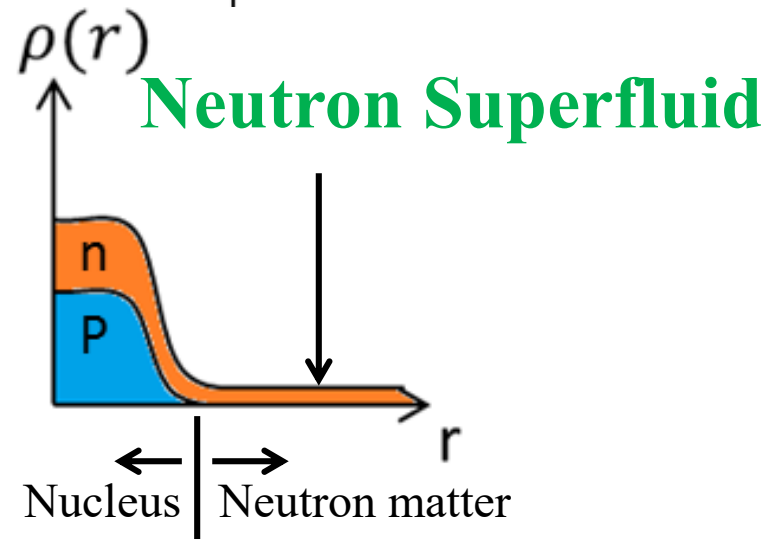
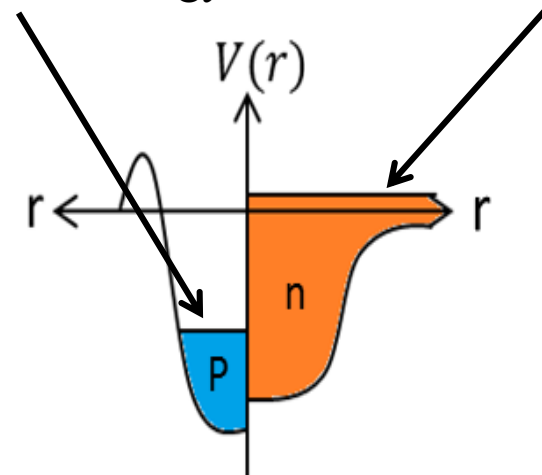
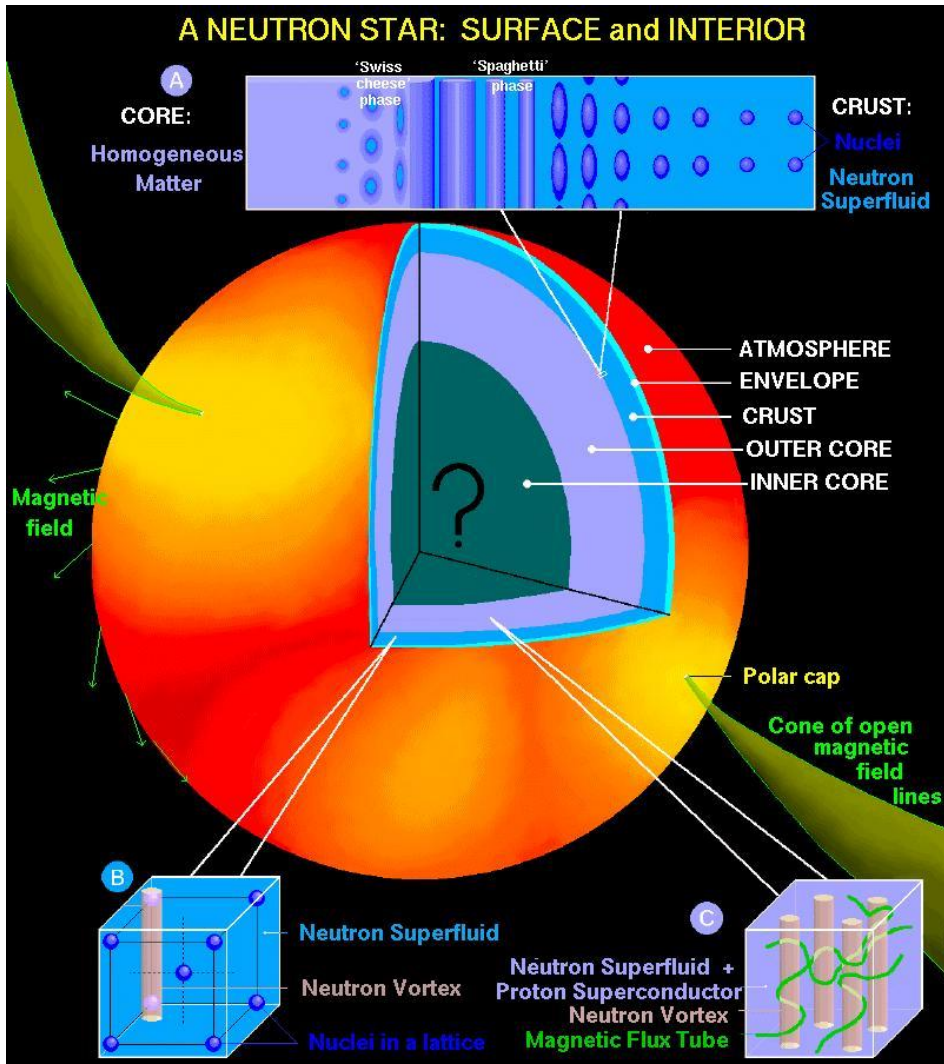
Program No. 8-4: Nuclear matter and Neutron stars

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# Inner crust

negative fermi energy

positive fermi energy



Neutron-rich nuclei  $N \gg Z$   
 +  
 Neutron Superfluid

# Interplay between nuclei and neutron superfluid

↔ Coupling between nuclei and neutron superfluid.

↔ Thermal conductivity of the inner crust.

D. N. Aguilera, *et al.*, Phys. Rev. Lett. 102, 091101(2009).

↔ Microscopic QRPA analysis

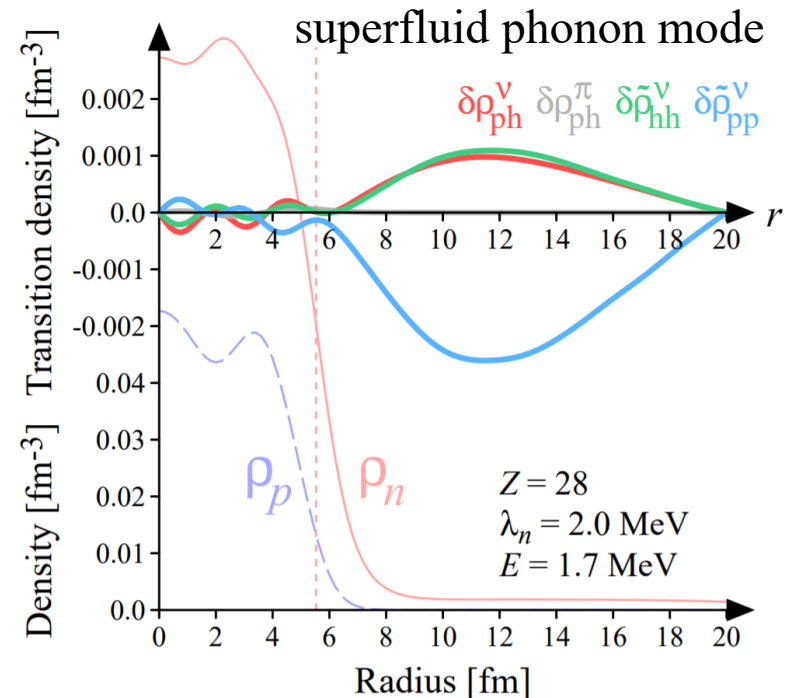
T. Inakura and M. Matsuo, Phys. Rev. C 96, 025806(2017).

Phonon excitation of neutron superfluid.

The couple is weak.

## In the present work

- ◆ we want to investigate pairing property of the ground state.
- ◆ We study interplay between nuclei and neutron superfluid.



# Purpose of present study

- ◆ The nuclei and neutron superfluid are very different matters.

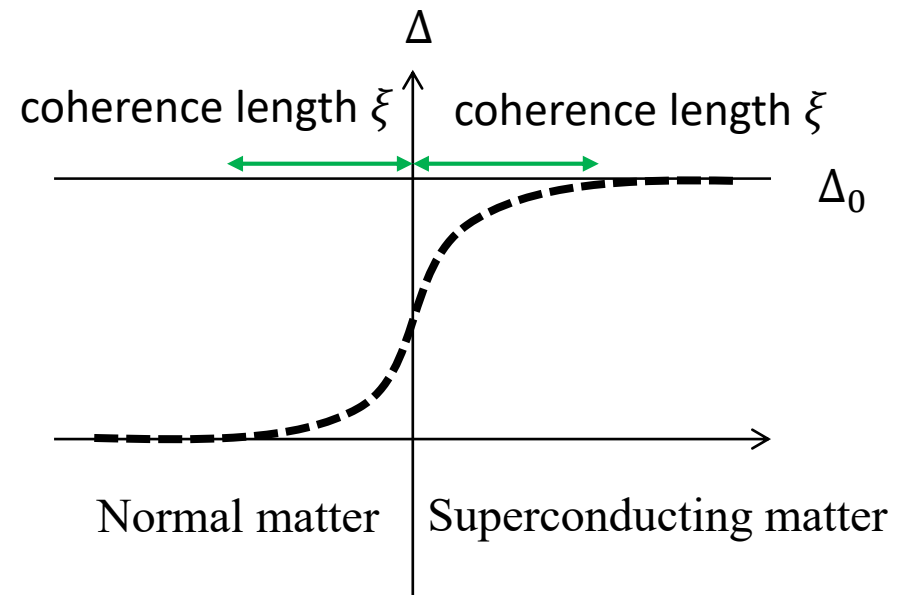
Focus: proximity effect of pair correlation.

- ◆ In superconductivity theory, if superconducting matter and normal matter are contact, the cooper pair leaks into the normal metal.

P. D. De Gennes, Rev. Mod. Phys. 36. 225 (1964).

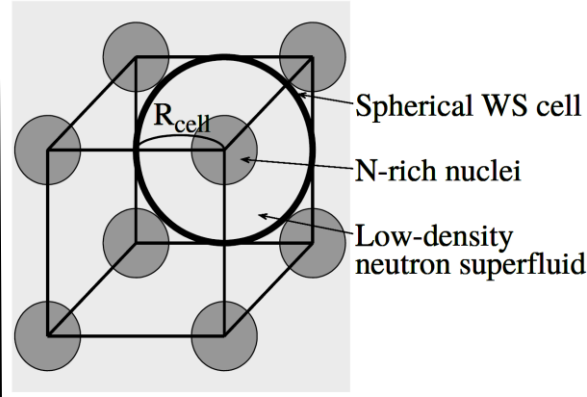
Proximity effect: If different superfluid matters contact,  
the pair correlation around the border is affected.

- ✓ We study how neutron-rich nuclei affect pairing of neutron superfluid.
- ✓ We focus the proximity effect of pair correlation that appears in pair density (pair condensation).



# Model

## 1. Spherical Wigner-Seitz cell



### 2. Hartree-Fock-Bogoliubov equation

$$\begin{bmatrix} h(r) - \lambda & \Delta(r) \\ \Delta(r) & -h(r) + \lambda \end{bmatrix} \begin{bmatrix} \phi_1(r) \\ \phi_2(r) \end{bmatrix} = E \begin{bmatrix} \phi_1(r) \\ \phi_2(r) \end{bmatrix}$$

**3. Method**  
diagonalization  
(symmetry)

### 4. Fermi energy $\lambda$

We put positive  $\lambda_n$  to control external neutron density.  
We set  $\lambda_p$  to express the proton number Z.

### 5. HF potential

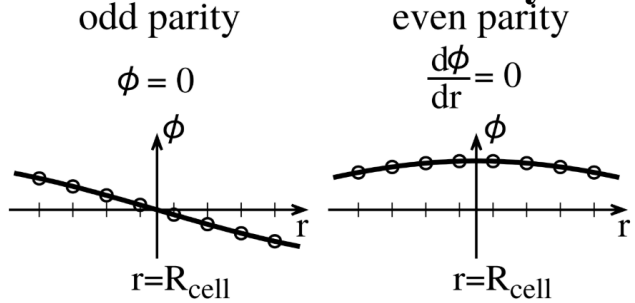
Skyrme interaction: **SLy4**  
SLy4 reproduce EOS of neutron matter.  
E. Chabanat, *et al.*, Nucl. Phys. A635, 213 (1998).

### 6. Pairing correlation: DDDI

$$V_{pair,q}(r_1, r_2) = V_0 \left( 1 - \eta \left( \frac{\rho_q}{\rho_0} \right)^\alpha \right) \left( \frac{1 - P_\sigma}{2} \right) \delta(r_1 - r_2)$$

We use two parameter set.

### A. Dirichlet-Neumann boundly condition



J. W. Negele and D. Vauthrin (1973).

### I. Strong paring

Reproduce:

- ◆ BCS gap
- ◆ Scattering length  
 $a = -18.5 \text{ fm}$

$$\begin{aligned} V_0 &= -458.4 \text{ MeV} \\ \eta &= 0.845 \\ \alpha &= 0.59 \\ \rho_0 &= 0.08 \text{ fm}^{-3} \end{aligned}$$

M. Matsuo, PRC(2006).  
M. Matsuo, *et al.*, NPA(2007).

### II. Weak paring

Reproduce:

- ◆ Ab-initio gap

$$\begin{aligned} V_0 &= -400 \text{ MeV} \\ \eta &= \text{same} \\ \alpha &= \text{same} \\ \rho_0 &= \text{same} \end{aligned}$$

### B. Numerical detail

9-point difference method  
 $\Delta r = 0.2 \text{ fm}$

### C. Cut off

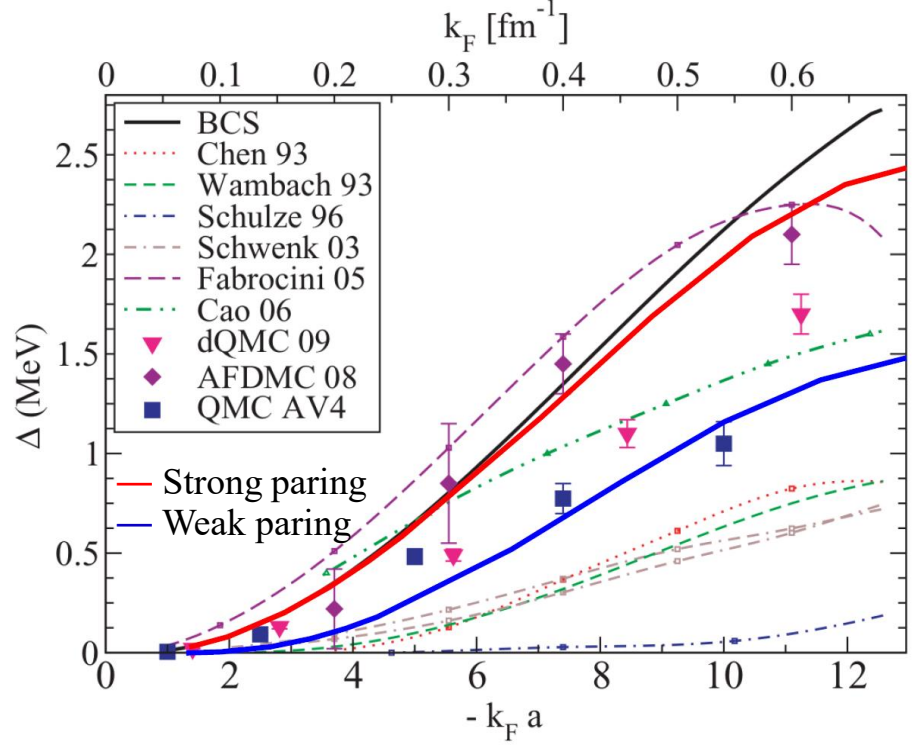
$E_{\text{max}} = 60 \text{ MeV}$   
 $l_{\text{max}} = 2 \times R_{\text{cell}}$

# Pair gap in neutron matter

BCS gap is stronger than ab-initio gap.

Strong pairing :  $V_0 = -458.4\text{MeV}$

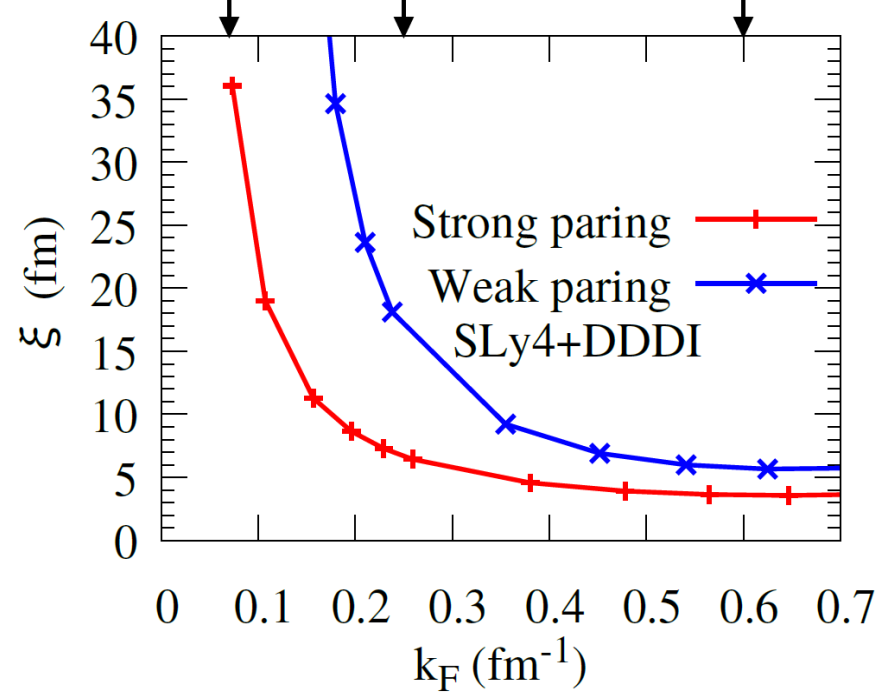
Weak pairing :  $V_0 = -400\text{MeV}$



A. Gezerlis and J. Carlson, PRC (2010).

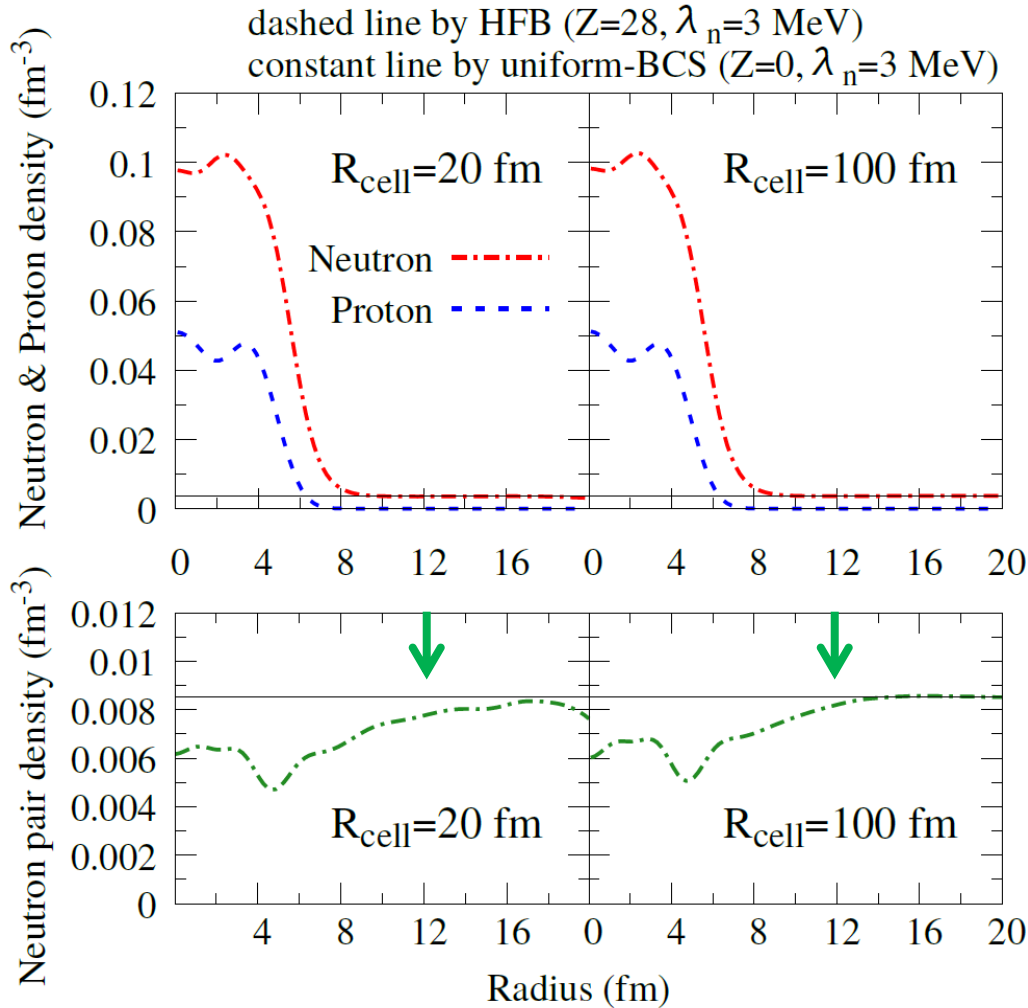
The length of proximity effect  $\sim \xi$

$\rho_n = 0.00004 \text{ fm}^{-3}$     $\rho_n = 0.0006 \text{ fm}^{-3}$     $\rho_n = 0.009 \text{ fm}^{-3}$   
 $\lambda_n = 0.2 \text{ MeV}$     $\lambda_n = 1 \text{ MeV}$     $\lambda_n = 5 \text{ MeV}$



# Finite size effect and proximity effect

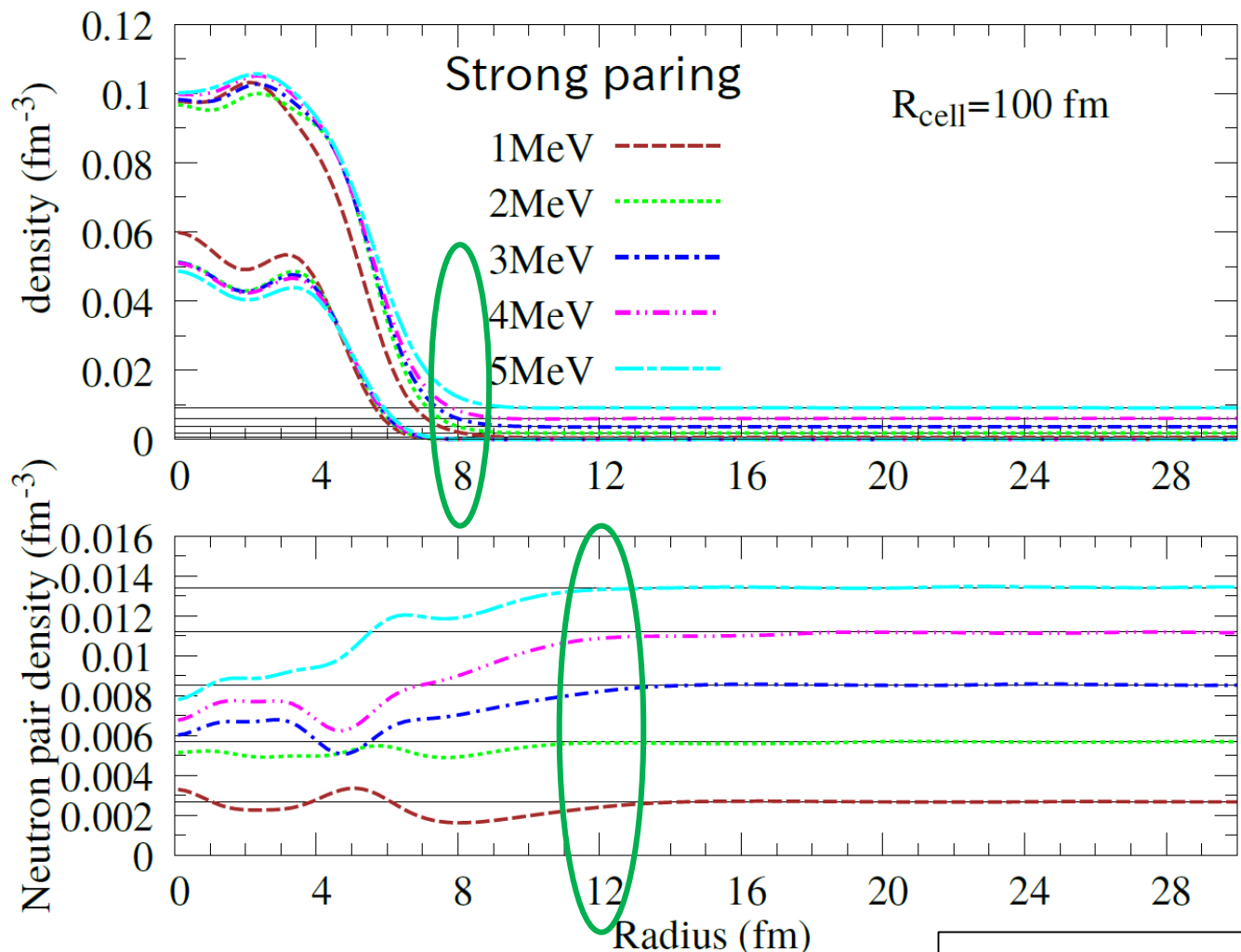
- ◆ In order to discuss proximity effect, we want to remove the finite size effect.
- ◆ Uniform-BCS and HFB gives same result in uniform neutron matter.



- ◆ The neutron & proton density is same in 20fm and 100fm.
- ◆ In the 20fm case, the neutron pair density does not converge.  
← Finite size effect
- ◆ We use  $R_{\text{cell}} = 100$ fm and discuss proximity effect.

# Typical result for the strong pairing $Z=28(\text{Ni})$

- ◆ Neutron density converge to that of uniform-BCS at  $\sim 8\text{fm}$ .
- ◆ Neutron pair density converge to that of uniform-BCS at  $\sim 12\text{fm}$ .
- ◆ The length of proximity effect is short,  $\sim 4\text{fm}$ .



The constant line by uniform-BCS.



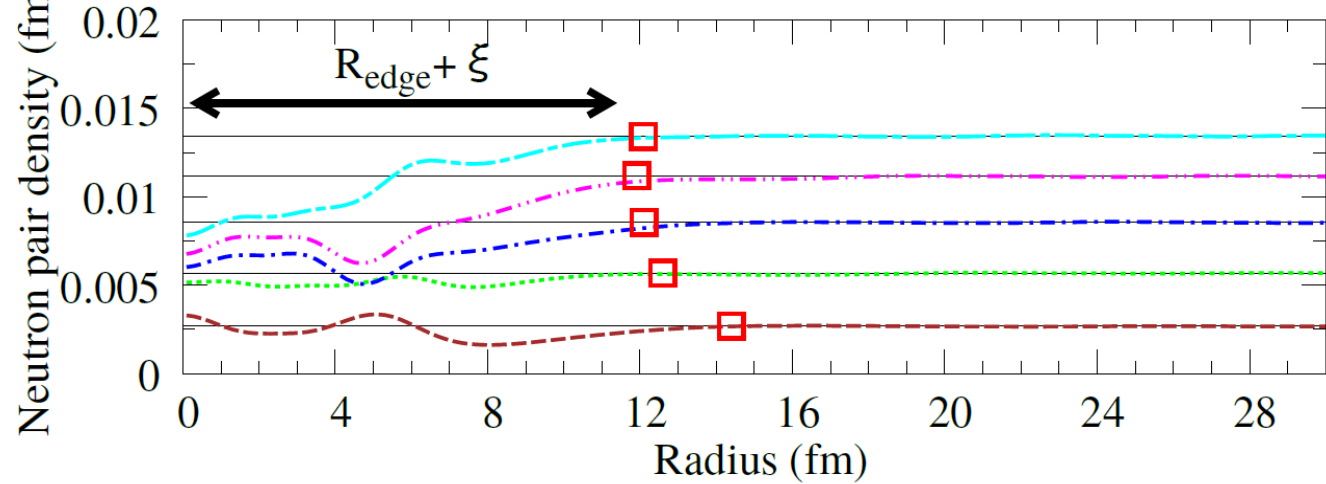
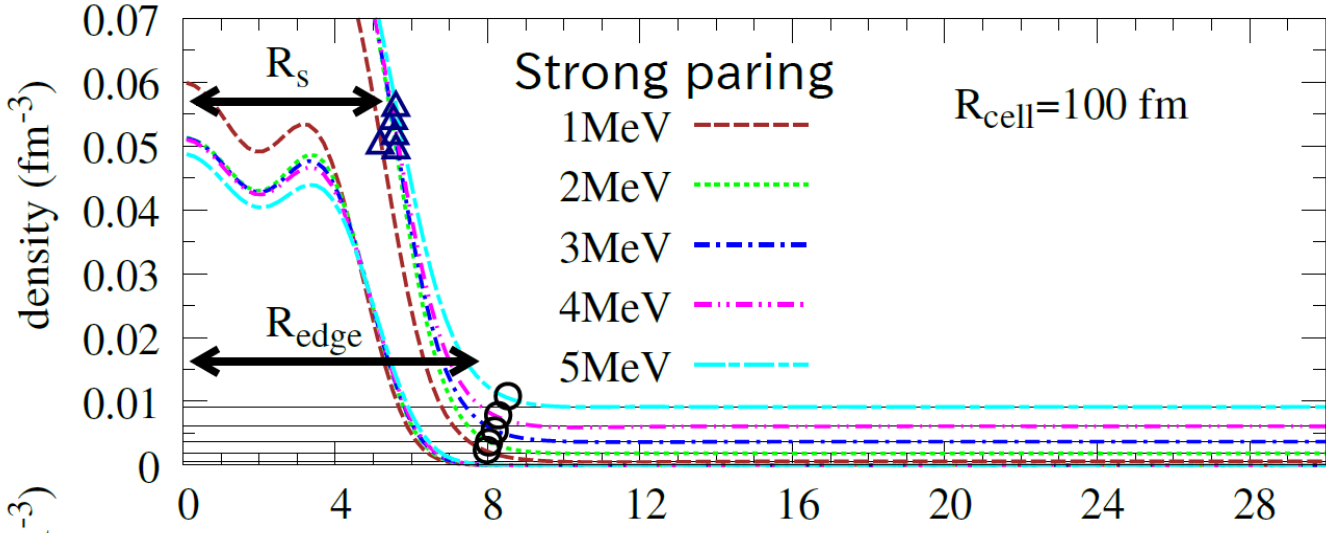
# The length of the proximity effect

- ◆ Pair density converges to uniform-BCS around red point.
- ◆ The coherence length describes the proximity effect very well.
- ◆ The length of proximity effect is short  $\sim 4 - 6$  fm.

Woods-Saxon function

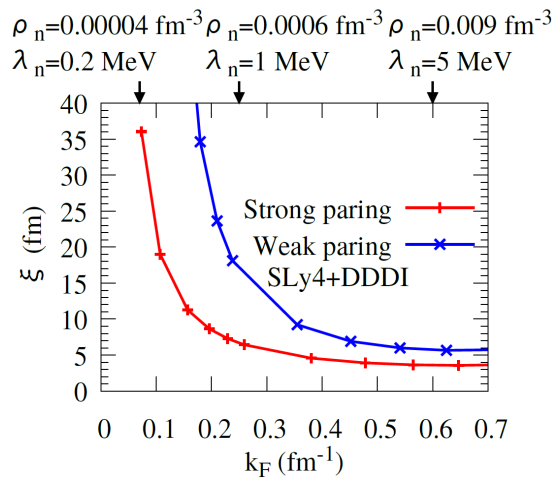
$$\left( 1 + \exp\left(\frac{r - R_s}{a}\right) \right)^{-1}$$

$a$ : diffuseness  
 $R_s$ : nuclear surface



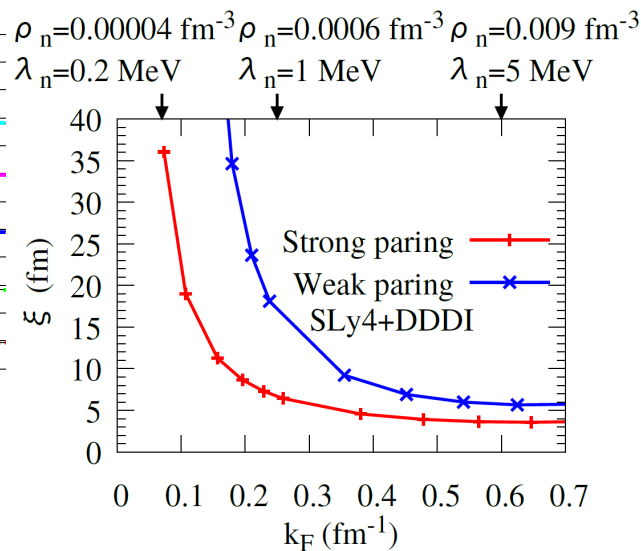
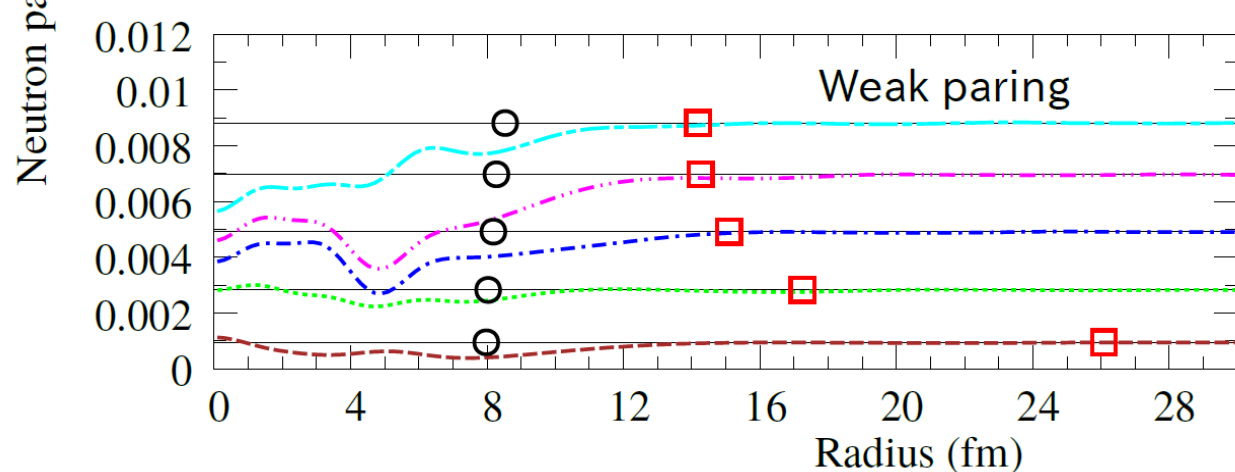
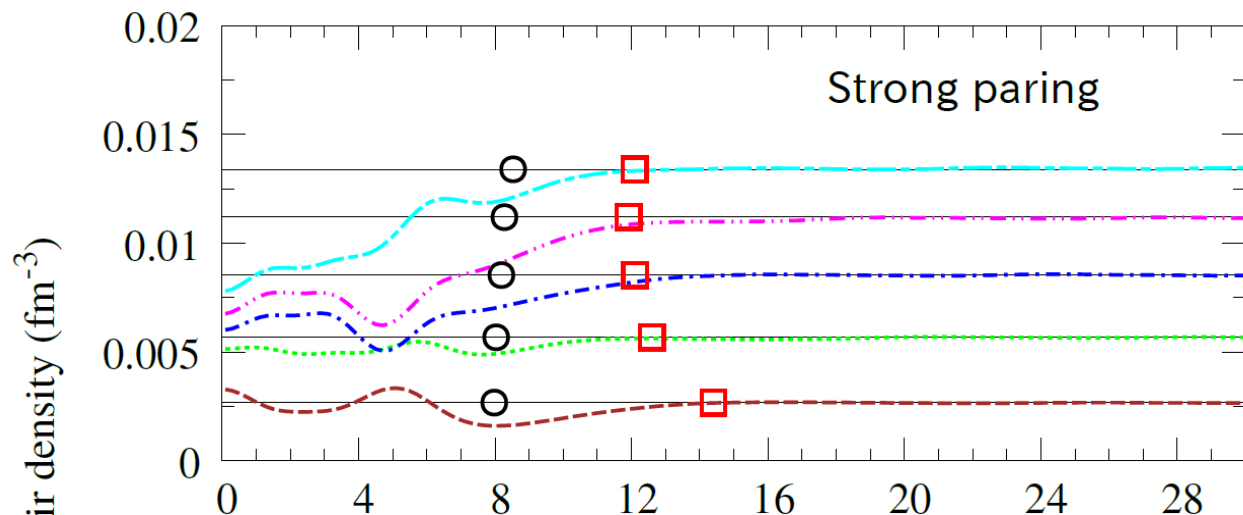
The nucleus edge:

$$R_{\text{edge}} = R_s + 4a$$



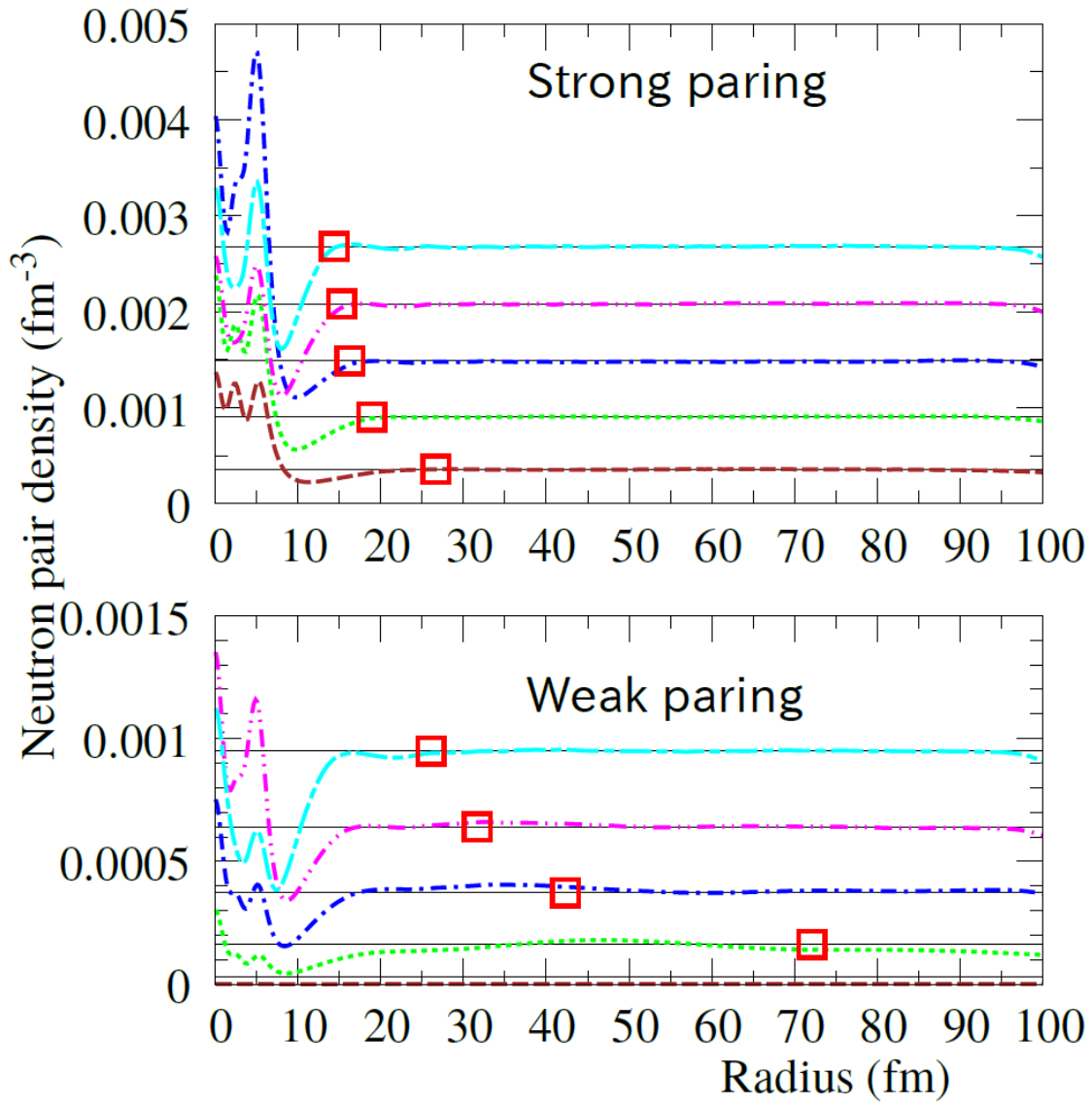
# Strong paring vs weak paring

$$\lambda_n = 1 - 5 \text{ MeV}$$



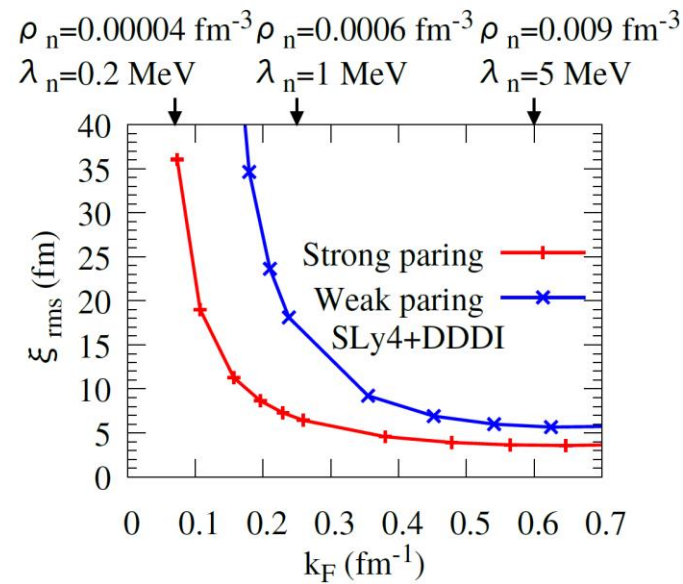
# Strong paring vs weak paring

The case of low density:  $\lambda_n = 0.2 - 1.0 \text{ MeV}$



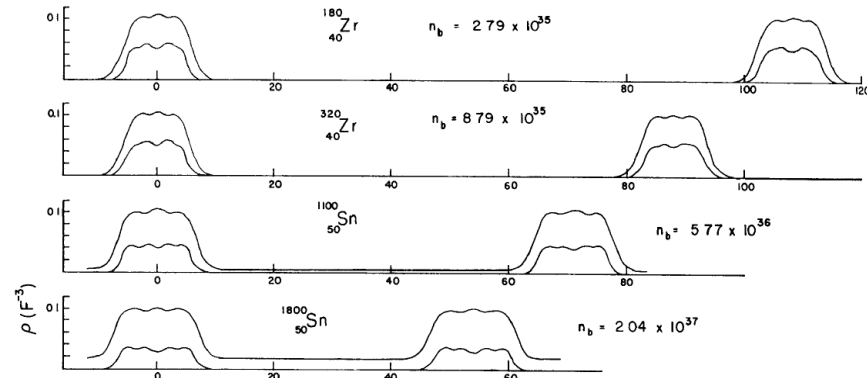
$R_{\text{cell}}=100 \text{ fm}$

0.2MeV ---  
 0.4MeV ...  
 0.6MeV -.-  
 0.8MeV -.-  
 1.0MeV ---

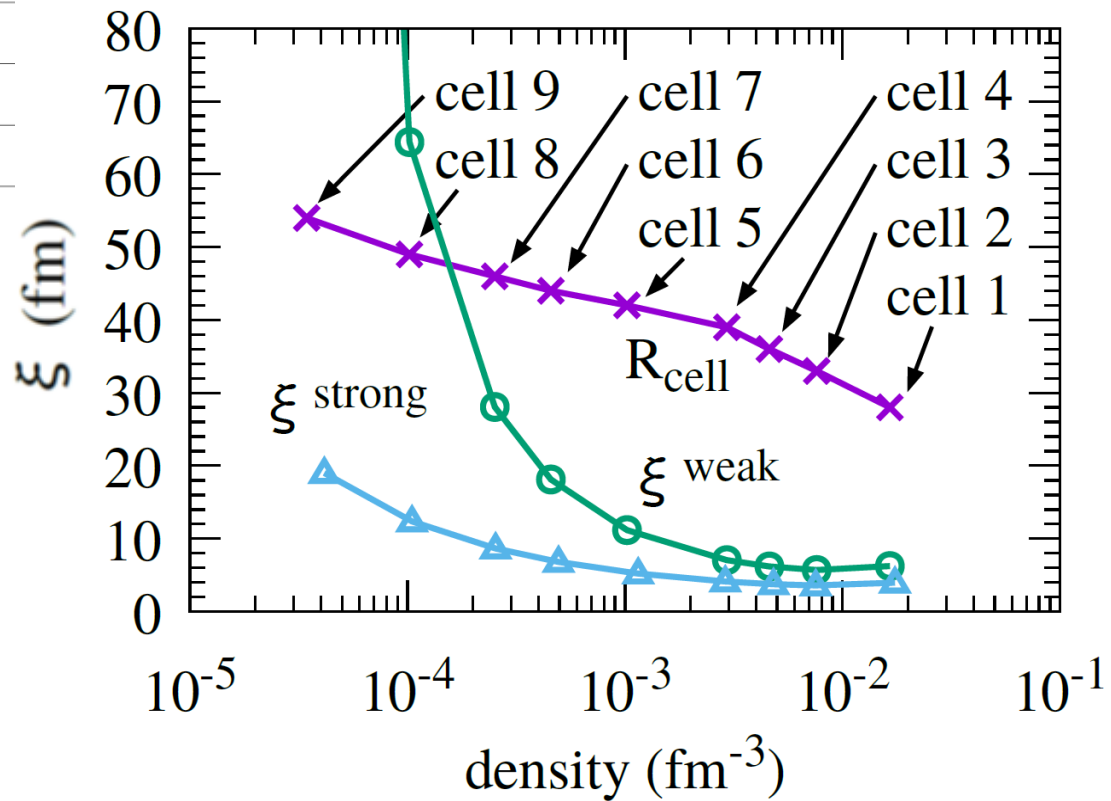


# Realistic Wigner-Seitz cell: $R_{\text{cell}}$ vs $\xi$

	nuclei	$\rho_{\text{ext}}(\text{fm}^{-3})$	$R_{\text{cell}}(\text{fm})$
1	$^{180}\text{Sn}$	$1.74 \times 10^{-2}$	28
2	$^{1350}\text{Sn}$	$7.53 \times 10^{-3}$	33
3	$^{1100}\text{Sn}$	$4.82 \times 10^{-3}$	36
4	$^{950}\text{Sn}$	$2.89 \times 10^{-3}$	39
5	$^{500}\text{Zr}$	$1.15 \times 10^{-3}$	42
6	$^{320}\text{Zr}$	$4.95 \times 10^{-4}$	44
7	$^{250}\text{Zr}$	$2.54 \times 10^{-4}$	46
8	$^{200}\text{Zr}$	$1.05 \times 10^{-4}$	49
9	$^{180}\text{Zr}$	$4.15 \times 10^{-5}$	54

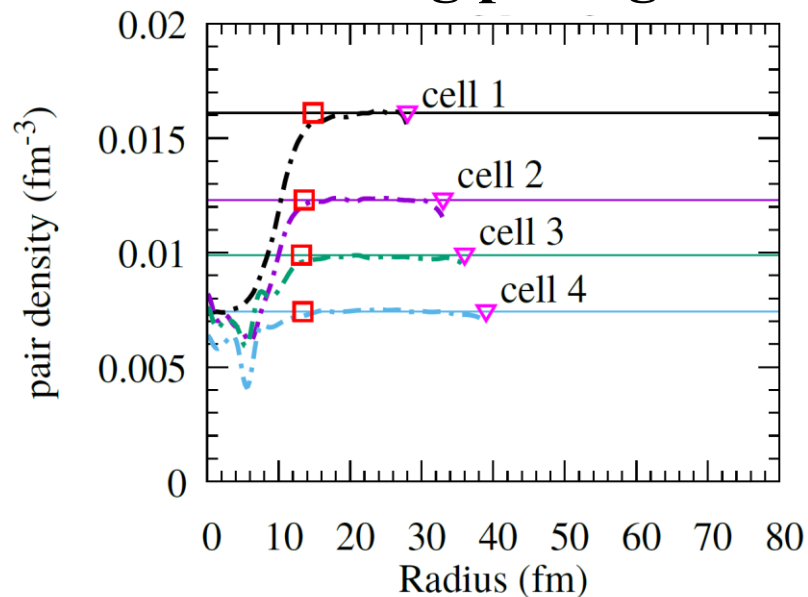


J. W. Negele and D. Vauthrin Nucl. Phys. A 207, 298 (1973).

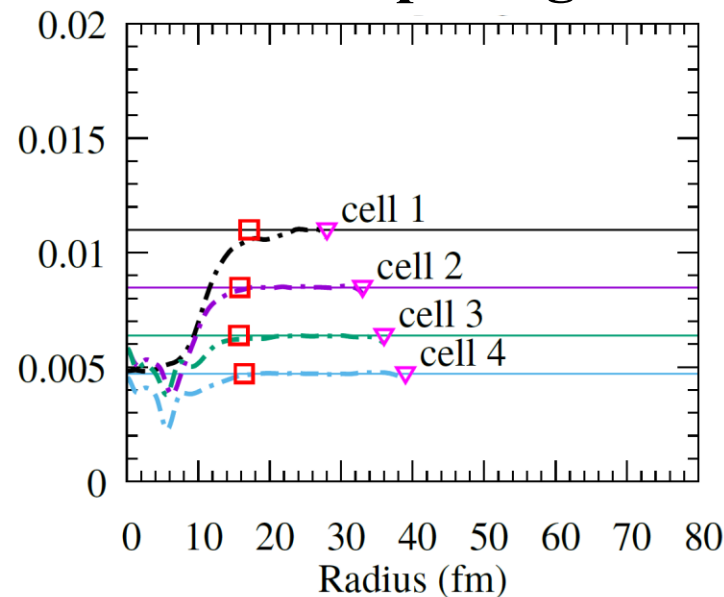


# Realistic Wigner-Seitz cell $R_{\text{cell}}$ vs $R_{\text{edge}} + \xi$

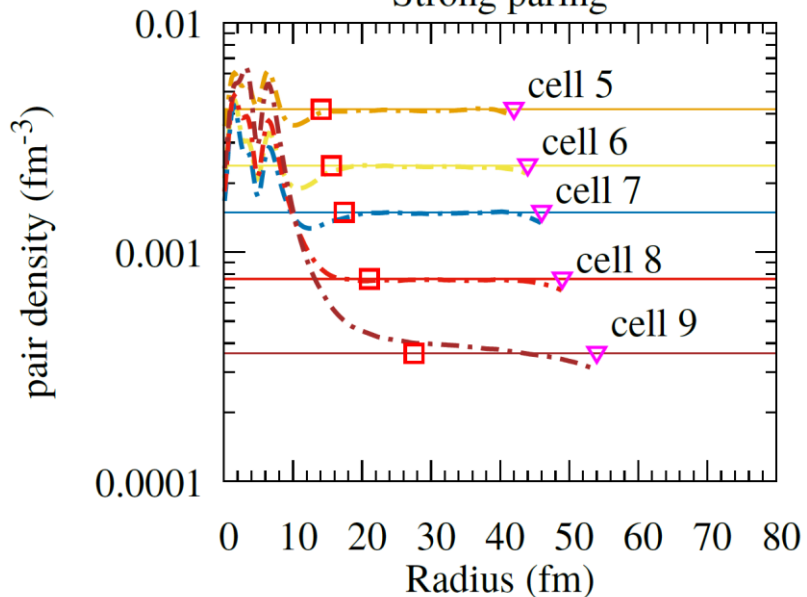
## Strong paring



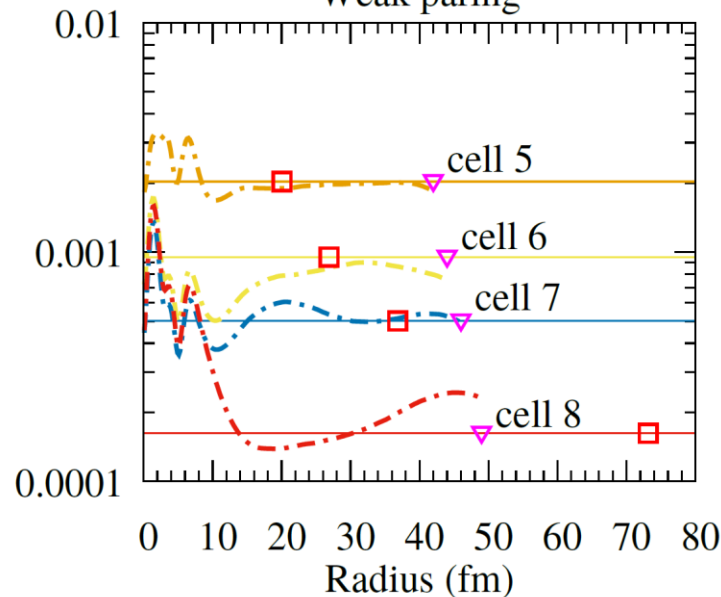
## Weak paring



## Strong paring



## Weak paring



# Conclusion

We have studied proximity effect of pair correlation in the inner crust of neutron stars.

We use strong pairing (BCS gap) and weak pairing (ab-initio gap).

- ◆ In the case of strong pairing, proximity effect is short in the all cells.
- ◆ In the case of weak pairing and low-density, proximity effect is long. In this case, pair density does not converge to uniform-BCS.
- ◆ The coupling between nuclei and neutron superfluid is weak at high density.
- ◆ But, in the case of low-density, influence of nuclei is strong. In this case, neutron superfluid in the inner crust may be different from the uniform neutron superfluid.