Equation of State from Neutron Star Mass and Radius Measurements

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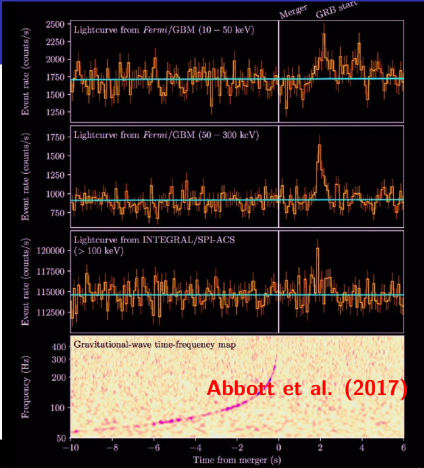
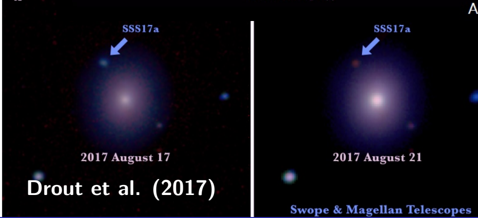
Main Topics

- Constraints on Radii and Masses from GW170817
- The Neutron Star Maximum Mass
- Estimating Neutron Star-Black Hole Merger Properties from LVC Announcements
  - Application to S190426c

GW170817

- LIGO-Virgo (LVC) detected a signal consistent with a BNS merger, followed 1.7 s later by a weak sGRB.
- 16600 orbits observed over 165 s.
- $\mathcal{M} = 1.187 \pm 0.001\ M_\odot$
- $M_{\rm T,min} = 2^{6/5}\mathcal{M} = 2.726 M_\odot$
- $E_{\rm GW} > 0.025 M_\odot c^2$
- $D_L = 40 \pm 10$ Mpc
- $75 < \tilde{\Lambda} < 560$ (90%)
- $M_{\rm ejecta} \sim 0.06 \pm 0.02\ M_\odot$
- Blue ejecta: $\sim 0.01 M_\odot$
- Red ejecta: $\sim 0.05 M_\odot$
- Possible r-process production
- Ejecta + GRB: $M_{max} \lesssim 2.2 M_\odot$





Properties of Known Double Neutron Star Binaries

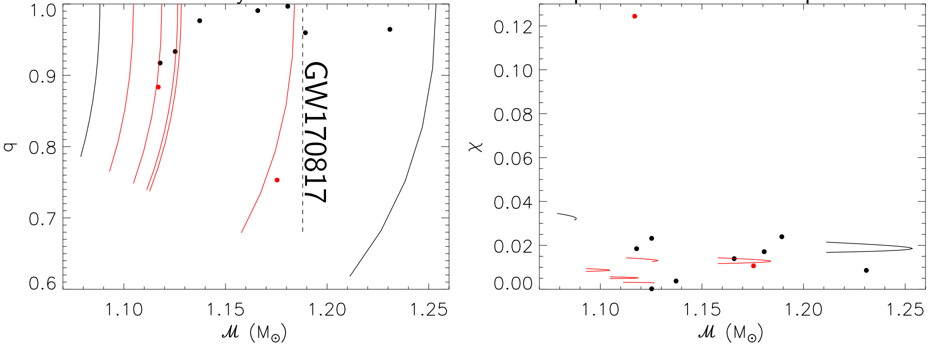
- Both component masses are accurately measured (9)

— Only the total binary mass is accurately measured (7)

Binaries with $\tau_{GW} > t_{\rm universe}$ (7)

$q = M_2/M_1$ is the binary mass ratio for a system

$\chi = cJ/(GM^2)$ is the dimensionless spin parameter for individual pulsars



$\mathcal{M} = (M_1 M_2)^{3/5}(M_1 + M_2)^{-1/5}$ is the chirp mass

Limits to $\mathcal{M}$ and q

Neutron stars have masses in the range $M_{min} \leq M \leq M_{max}$.

- $M_{min} \gtrsim 1.1 M_\odot$ from models of CCSNe models and ν-trapped remnants. Smallest well-measured mass is PSR J0453+1559 companion [Martinez et al. 2015] with $1.174 \pm 0.004 M_\odot$.
- $M_{max} \gtrsim 2 M_\odot$ from PSR J0348 + 0432 [Antoniadis et al. 2013] with $2.01 \pm 0.04 M_\odot$, PSR J0740 + 6620 [Cromartie et al. 2019] with $2.17^{+0.11}_{-0.10} M_\odot$, and PSR J2215-5135 [Linares et al. 2018] with $2.27^{+0.17}_{-0.15} M_\odot$. The first has smaller uncertainty, but involves white dwarf evolutionary assumptions; the second is a Shapiro-delay measurement; the third is a black widow like system with large companion modeling uncertainties.

$$2^{-1/5} M_{min} = 1.02 M_\odot < \mathcal{M} < 2^{-1/5} M_{max} = 1.89 M_\odot$$

GW170817: $\mathcal{M} = 1.187 M_\odot$; $q \geq 0.735$; $1.365 \leq M_1/M_\odot \leq 1.600$

Waveform Model Parameters

There are 13 wave-form free parameters including finite-size effects at third PN order $(v/c)^6$. LVC17 used a 13-parameter model; De et al. (2018) used a 9-10 parameter model.

Extrinsic:

- Sky location (2) EM data
- Distance (1) EM data
- Inclination (1)
- Coalescence time (1)
- Coalescence phase (1)
- Polarization (1)

Intrinsic:

- Component masses (2)
- Spin parameters (2)
- Tidal deformabilities (2) correlated with masses

Tidal Deformability

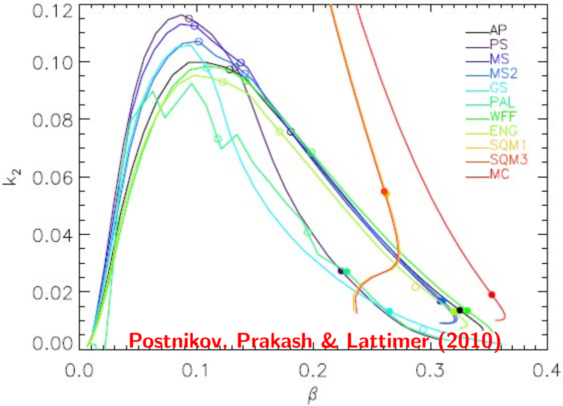
The tidal deformability λ is the ratio of the induced dipole moment Q_{ij} to the external tidal field E_{ij}, $Q_{ij} \equiv -\lambda E_{ij}$.

Work with the dimensionless quantity

$$\Lambda = \frac{\lambda c^{10}}{G^4 M^5} \equiv \frac{2}{3} k_2 \left(\frac{Rc^2}{GM} \right)^5$$

k_2 is the dimensionless Love number.

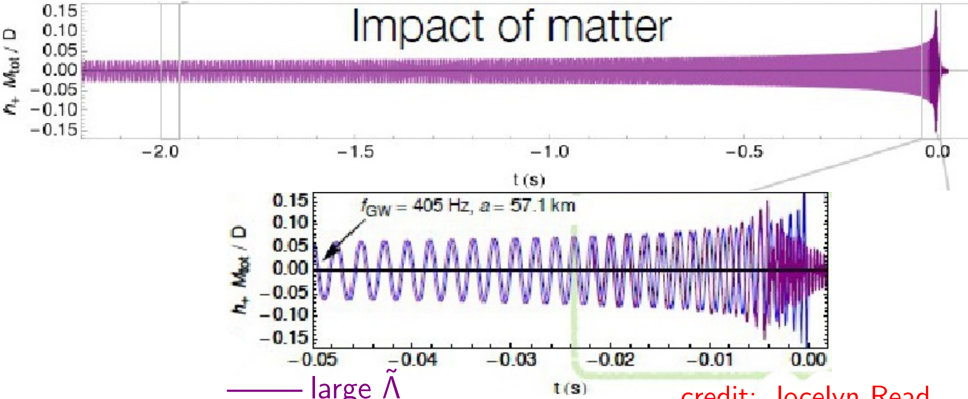
For a neutron star binary, $\tilde{\Lambda}$ is the relevant quantity:



$$\tilde{\Lambda} = \frac{16}{13} \frac{(1+12q)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5}, \qquad q = M_2/M_1 \leq 1$$

The Effect of Tides

Tides accelerate the inspiral and produce a phase shift compared to the case of two point masses.



large $\tilde{\Lambda}$

small $\tilde{\Lambda}$

credit: Jocelyn Read

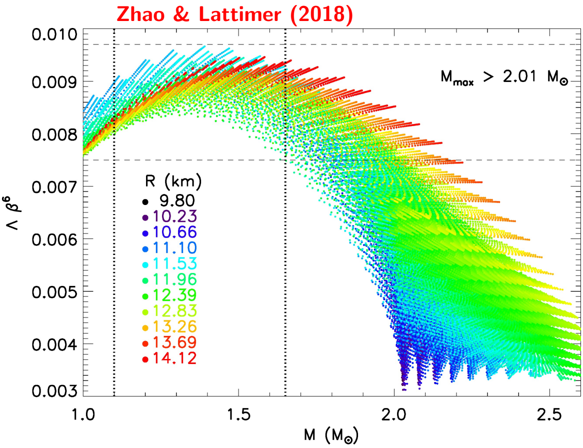
$$\delta\Phi_t = -\frac{117}{256}\frac{(1+q)^4}{q^2}\left(\frac{\pi f_{GW} G\mathcal{M}}{c^3}\right)^{5/3}\tilde{\Lambda} + \cdots .$$

Λ is Highly Correlated With M and R

- $\Lambda = a\beta^{-6}$
  $\beta = GM/Rc^2$

  $a = 0.0086 \pm 0.0011$
  for
  $M = 1.35 \pm 0.25\ M_\odot$

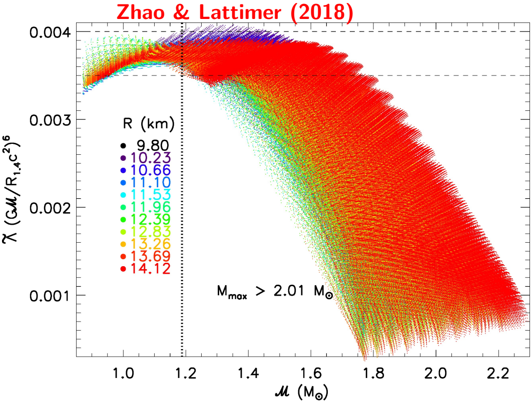
- If $R_1 \simeq R_2 \simeq R_{1.4}$
  it follows that
  $\Lambda_2 \simeq q^{-6}\Lambda_1$.



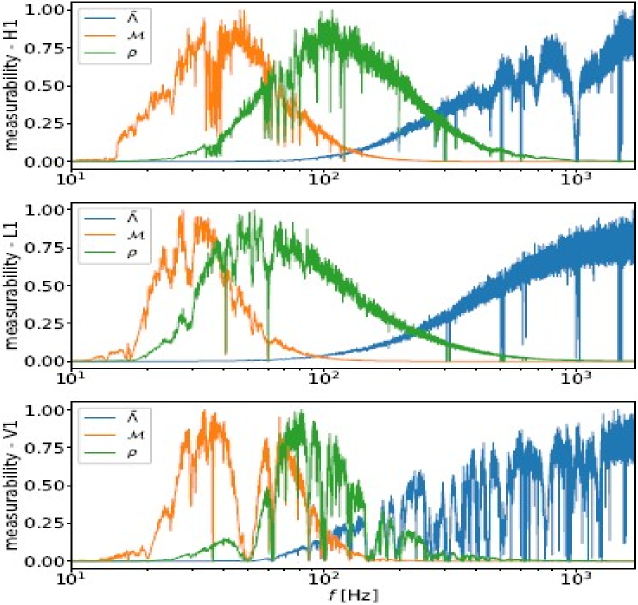
Binary Deformability and the Radius

$$\tilde{\Lambda} = \frac{16}{13}\frac{(1+12q)\Lambda_1 + q^4(12+q)\Lambda_2}{(1+q)^5} \simeq \frac{16a}{13}\left(\frac{R_{1.4}c^2}{G\mathcal{M}}\right)^6 \frac{q^{8/5}(12-11q+12q^2)}{(1+q)^{26/5}}$$

- $\tilde{\Lambda} = a'(R_{1.4}c^2/G\mathcal{M})^6$
  $a' = 0.0035 \pm 0.0006$
  for
  $\mathcal{M} = 1.2 \pm 0.2\ M_\odot$
- GW10817:
  $a' = 0.00375 \pm 0.00025$
- $R_{1.4} =$
  $11.5 \pm 0.3\ \frac{\mathcal{M}}{M_\odot}\left(\frac{\tilde{\Lambda}}{800}\right)^{1/6}$ km
- GW10817:
  $R_{1.4} = 13.4 \pm 0.1\ \left(\frac{\tilde{\Lambda}}{800}\right)^{1/6}$ km



Measurability of Tidal Deformability



De et al. (2018)

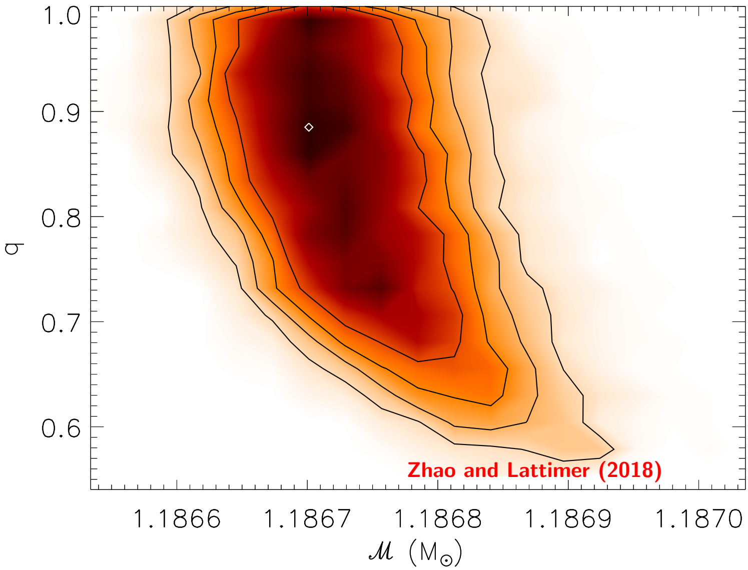
Re-Analysis of GW170817 (De et al. 2018)

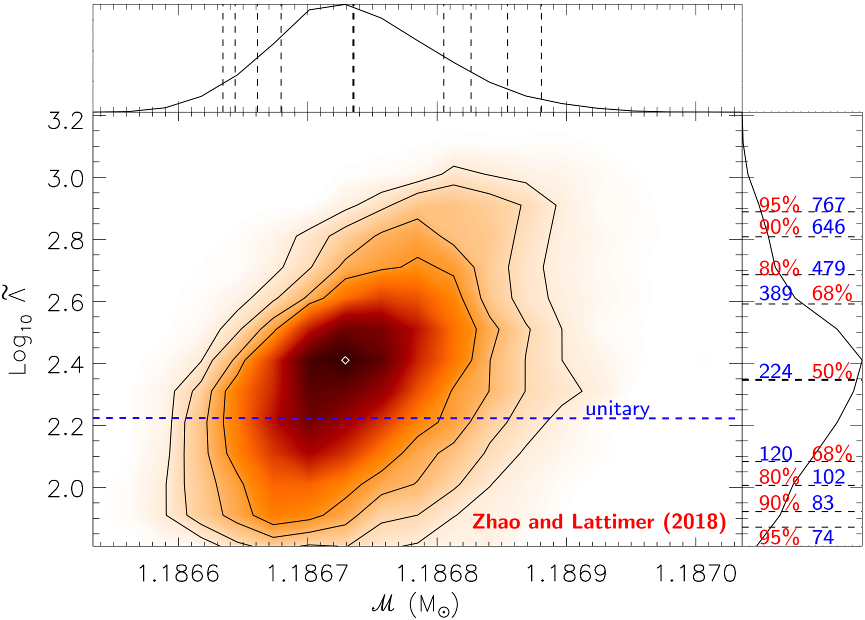
- De18 takes advantage of the precisely-known electro-magnetic source position (Soares-Santos et al. 2017).
- Uses existing knowledge of H_0 and the redshift of NGC 4993 to fix the distance (Cantiello et al. 2017).
- Assumes both neutron stars have the same equation of state, which implies $\Lambda_1 \simeq q^6 \Lambda_2$.
- Baseline model effectively has 9 instead of 13 parameters.
- Explores variations of mass, spin and deformability priors.
- Low-frequency cutoff taken to be 20 Hz, not 30 Hz as in LVC17, doubling the number of analyzed orbits.

De18 find that including $\Lambda - M$ correlations

- establishes a lower 90% confidence bound to $\tilde{\Lambda}$ (which is above the causal minimum value), and
- reduces the upper 90% confidence bound to $\tilde{\Lambda}$ by 30%.

68%, 80%, 90% and 95% Confidence Bounds



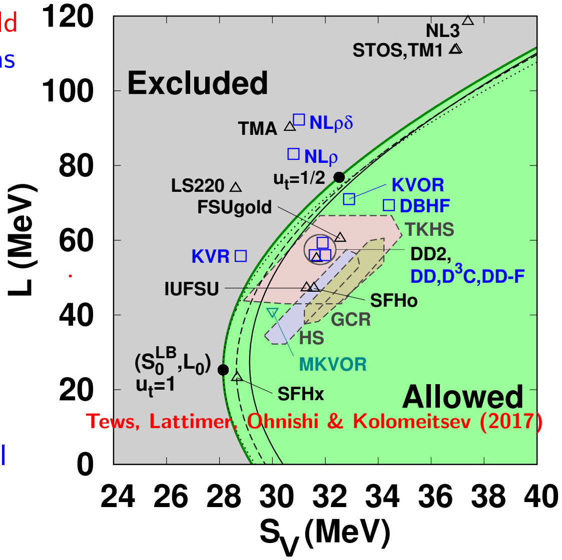

3.2
3.0
2.8
2.6
2.4
2.2
2.0
$\mathrm{Log}_{10}\,\tilde{\Lambda}$
1.1866
1.1867
1.1868
1.1869
1.1870
$\mathcal{M}\ (M_\odot)$
unitary
Zhao and Lattimer (2018)
95% 767
90% 646
80% 479
389 68%
224 50%
120 68%
80% 102
90% 83
95% 74

Unitary Gas Bounds

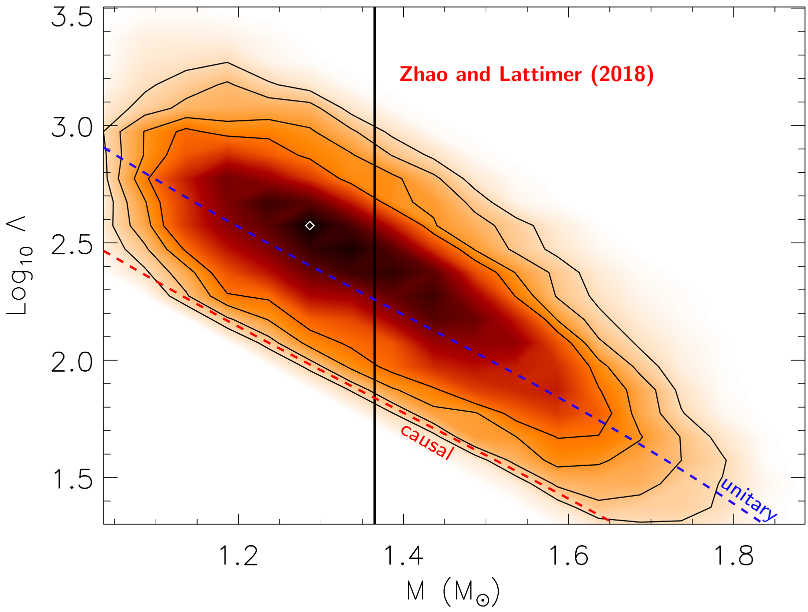
Neutron matter energy should be larger than the unitary gas energy $E_{UG} = \xi_0 (3/5) E_F$

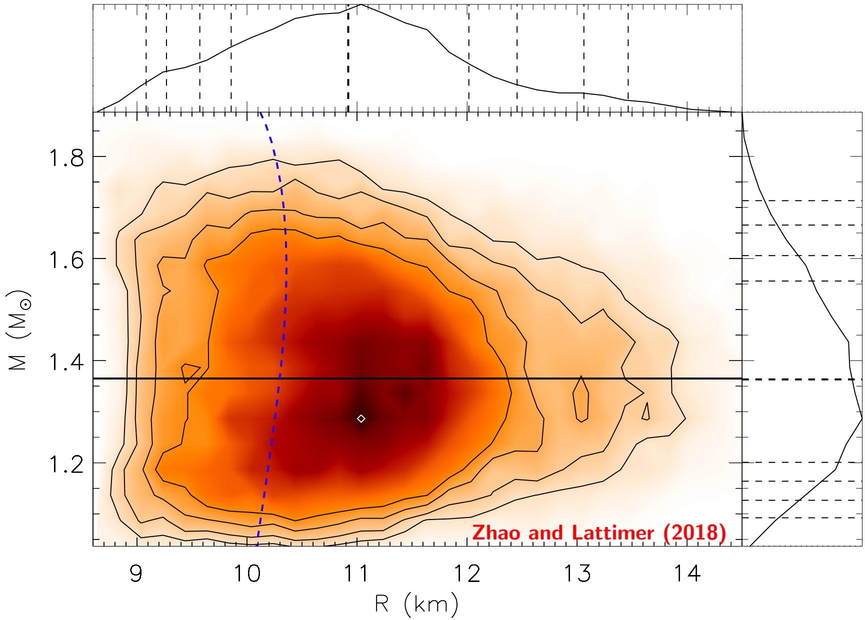
$$E_{UG} = 12.6 \left(\frac{n}{n_s}\right)^{2/3} \mathrm{MeV}$$

The unitary gas refers to fermions interacting via a pairwise short-range s-wave interaction with an infinite scattering length and zero range. Cold atom experiments show a universal behavior with the Bertsch parameter $\xi_0 \simeq 0.37$.



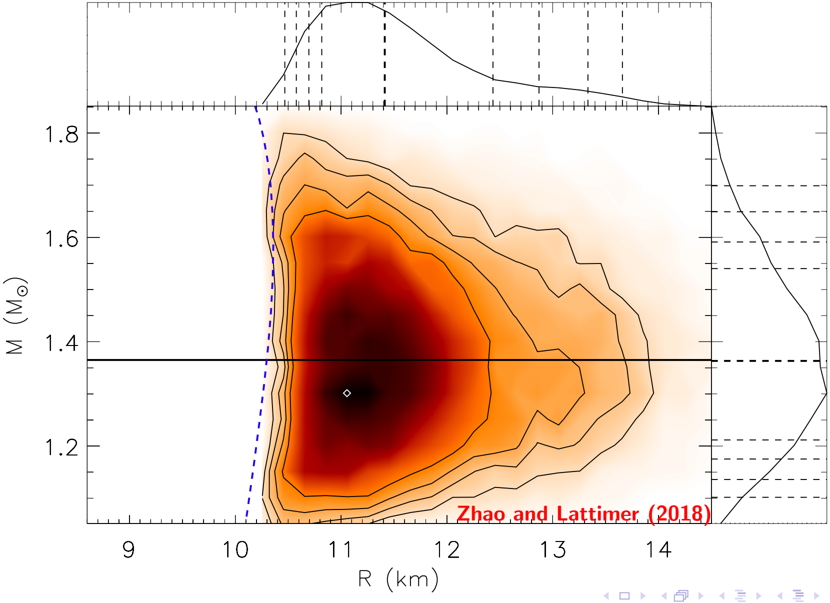
$S_v \geq 28.6$ MeV; $L \geq 25.3$ MeV; $p_0(n_s) \geq 1.35$ MeV fm^{-3}; $R_{1.4} \geq 9.7$ km


Zhao and Lattimer (2018)
3.5
3.0
2.5
2.0
1.5
$\mathrm{Log}_{10}\,\Lambda$
1.2
1.4
1.6
1.8
$M\ (M_\odot)$
causal
unitary

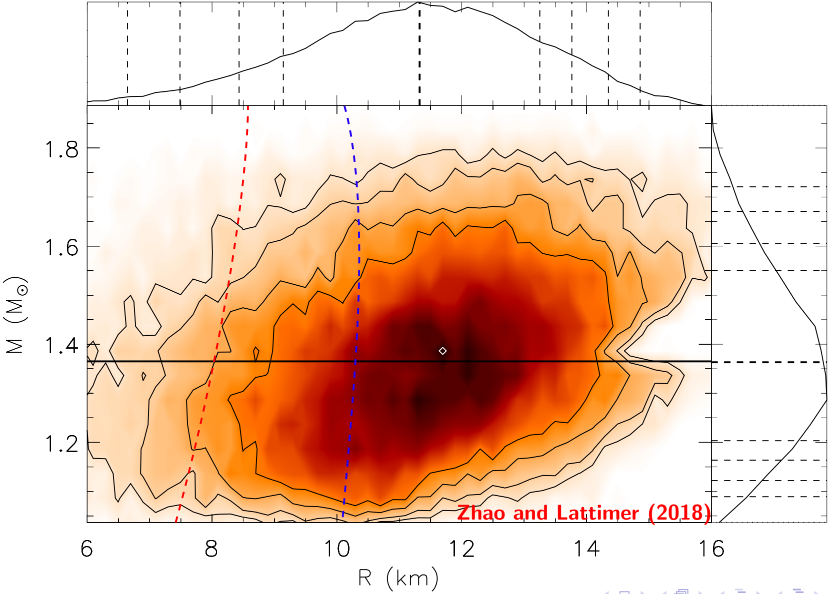


1.8
1.6
1.4
1.2
M (M⊙)
9
10
11
12
13
14
R (km)
Zhao and Lattimer (2018)

$M - R$ With Unitary Gas Limit Imposed



$M - R$ With No $\Lambda - M$ Correlations



Maximum Mass Constraint From GW170817

- Pulsar observations imply that slowly rotating neutron stars have a maximum mass $M_{max} \gtrsim 2M_\odot$.
- Initially, the remnant is differentially rotating, but quickly (~ 0.1s) uniformizes its rotation.
- Differentially-rotating stars likely have $M_{max,d} \gtrsim 1.5M_{max}$.
- Maximally uniformly rotating stars have $M_{max,u} = \xi M_{max}$ with $1.17 \lesssim \xi \lesssim 1.21$.
- *Hypermassive* stars, with $M > M_{max,u}$, promptly collapse to a BH.
- *Supermassive* stars, with $M_{max} \leq M \leq M_{max,u}$, are metastable but have much longer lifetimes.
- Inspiralling mass $M_T = M_1 + M_2 = \mathcal{M} q^{-3/5}(1+q)^{6/5}$ is between $2.73M_\odot$ ($q = 1$) and $2.78M_\odot$ ($q = 0.7$).
- Whether or not a supermassive star promptly collapses depends on M_T and binding energy; use baryon masses.

Maximum Mass Constraint

- Define BE$= M^b - M$,

$$\frac{\mathrm{BE}}{M} \simeq (0.058 \pm 0.006M) + (0.013 \pm 0.001)M^2 = aM + bM^2.$$

- Define $M^b_{max,u}/M^b_{max} = \xi_b$.
- If $M^b_T > M^b_{max,d}$, remnant promptly collapses to a BH before a gamma-ray burst or radio jets can form or disc ejecta occurs (which were observed).
- If $M^b_T < M^b_{max,u}$, remant will be indefinitely stable, but disc ejecta likely poisioned by neutrinos which de-neutronize it and destroy the r-process.
- $M^b_{max,u} < M^b_T < M^b_{max,d}$, modulo ejecta $\Delta \gtrsim 0.05 M_\odot$.
- $\xi_b M^b_{max} < M^b_T - \Delta$ is a cubic equation for M_{max}, with approximate solution

$$M_{max}/M_\odot < 2.18 + 0.52(1-q)^2 \simeq 2.18 - 2.25.$$

S190426c: First Black Hole-Neutron Star Merger?

Information from LVC indicates a marginal case, with 14% chance of being 'terrestrial'.

Assuming it is cosmic, GCN circular 24411 stated $p_{\rm BHNS} = 0.60, p_{\rm gap} = 0.35, p_{\rm BNS} = 0.15, p_{\rm BBH} < 0.01$, $p_{\rm HasNS} > 0.99$ and $p_{\rm rem} = 0.72$.

LVC defines NS if $M \leq 3M_\odot$, BH if $M \geq 5M_\odot$ and gap if either mass satisfies $3M_\odot < M < 5M_\odot$.

LVC will not release the chirp mass $\mathcal{M}$ (even though it is known precisely), the mass ratio $q = M_1/M_2 > 1$ (and therefore M_1 or M_2, known much less precisely), or the spin parameter χ if one component is a BH.

But it is possible to recover $\mathcal{M}, M_1, M_2$ and χ in cases where $p_{\rm BHNS}, p_{\rm gap}, p_{BNS}$ and $p_{\rm rem}$ are nonzero.

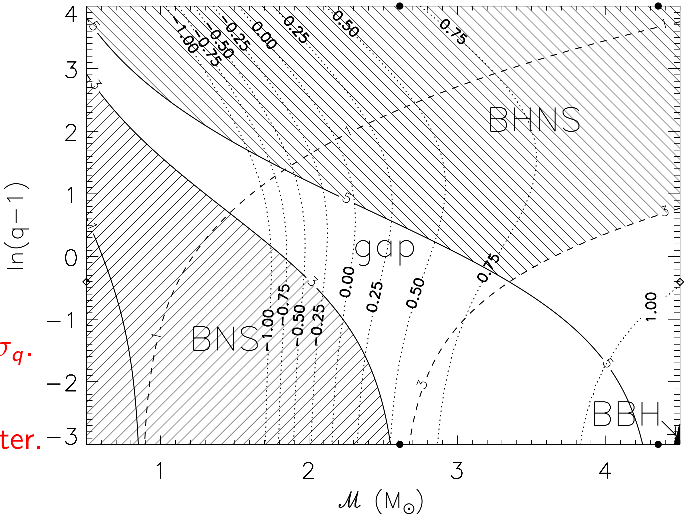
Suitable Variables

$\mathcal{M}$ has small uncertainty $\sigma_{\mathcal{M}}$.

q has large uncertainty, but $q \in [1, \infty]$.

$\bar{q} = \ln(q-1)$ has $\bar{q} \in [-\infty, \infty]$ and large uncertainty σ_q.

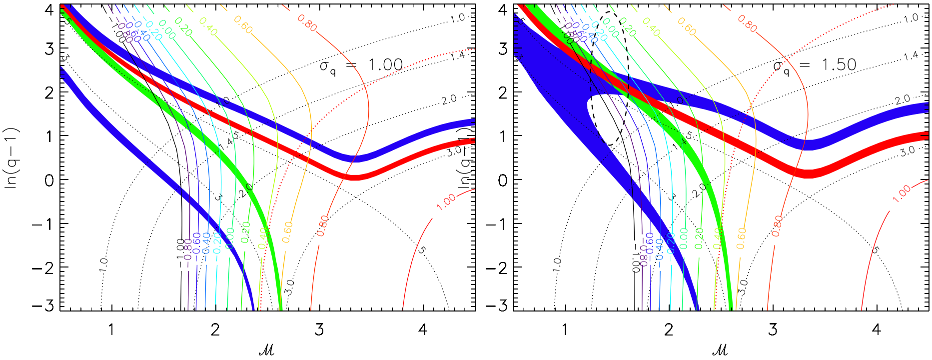
σ_q is the most important parameter.

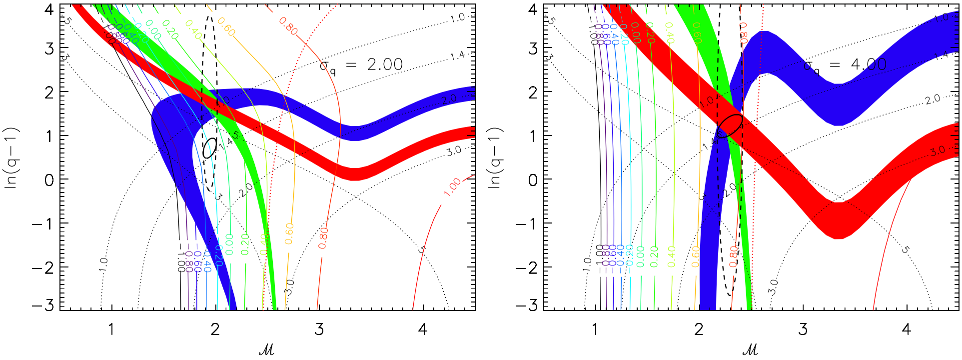


Probabilities

Assume

$$\frac{d^2p}{d\mathcal{M}d\bar{q}} = \frac{1}{2\pi\sqrt{\sigma_\mathcal{M}\sigma_q}}\exp\left[-\frac{(\mathcal{M}-\mathcal{M}_0)^2}{2\sigma_\mathcal{M}^2}-\frac{(\bar{q}-\bar{q}_0)^2}{2\sigma_q^2}\right].$$





σ_q = 2.00
σ_q = 4.00
ln(q−1)
$\mathcal{M}$

Spin

LVC uses model of Foucart et al. (2012, 2018) to determine mass M_d remaining outside the remnant more than a few ms after a BHNS merger:

$$M_d/M_{\rm NS}^b \simeq \alpha' \eta^{-1/3}(1-2\beta) - \hat{R}_{\rm ISCO}\beta\beta'\eta^{-1} + \gamma',$$

$\beta = GM_{\rm NS}/R_{\rm NS}c^2$, $\eta = q(1+q)^{-2}$ and
$\hat{R}_{\rm ISCO} = R_{\rm ISCO}c^2/GM_{\rm BH}$. $\alpha' \simeq 0.406$, $\beta' \simeq 0.139$, $\gamma' = 0.255$.

For the Kerr metric

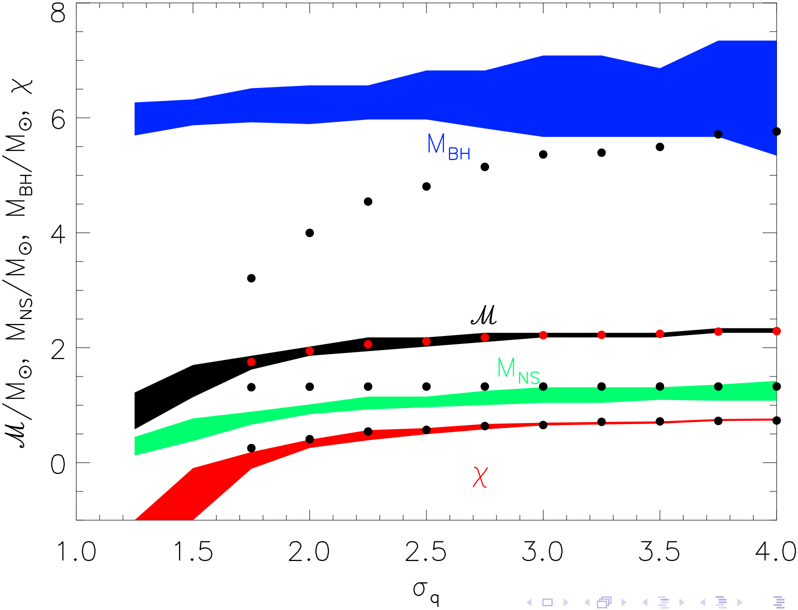
$$\chi = \sqrt{\hat{R}_{\rm ISCO}}\left(4/3 - \sqrt{\hat{R}_{\rm ISCO}/3 - 2/9}\right).$$

$M_d = 0$ implies

$$\hat{R}_{\rm ISCO} = (\beta'\beta)^{-1}(\alpha'\eta^{2/3}(1-2\beta) + \gamma'\eta).$$

χ is found from $p_d = \int\int_{M_d \geq 0} \frac{d^2 p}{d\mathcal{M} d\bar{q}} d\mathcal{M} d\bar{q}$.

Convergence For Large σ_q



Summary

- GW170817 provided R and EOS information compatible with expectations from nuclear theory, experiment and other astrophysical obserations.
- GW170817 also hints that M_{max} is not far above the minimum provided by pulsar timing.
- NICER should soon provide complementary radius information from X-ray sources.
- Future GW measurements should be additive if sources are similar.
- A promising BHNS candidate seems to have $M_{\rm BH} = 6M_\odot$, $M_{\rm NS} = 1.4M_\odot$ and $\chi = 0.3$.