Analysis of B wave function contributions in $B \to \pi$ form factor in perturbative QCD

°H.Hayakawa, T.Kurimoto
Department of Physics, Toyama University

At Yukawa Institute for Theoretical Physics

1 Introduction

B meson decays $\Rightarrow$ CP violation, Nature of Hadron

Experimentally: Belle and Babar B factories and other facilities
$\Rightarrow$ High statistics of B decay data

Theoretically: Light-cone sum rules, Lattice QCD, pertubative QCD(PQCD)

$B \to \pi$ transition matrix elements

$$\langle \pi(P_2)|\overline{q}(z)\gamma_\mu b(0)|\overline{B}(P_1)\rangle$$
$$= F_+(q^2)\left[(P_1+P_2)_\mu - \frac{M_B^2 - M_\pi^2}{q^2} q_\mu\right] + F_0(q^2)\frac{M_B^2 - M_\pi^2}{q^2} q_\mu$$

$P_1(P_2)$: B meson (Pion) momentum, $M_B(M_\pi)$: B meson (Pion) mass
q : Momentum transfer

$\star$ $F_{+,0}$: $B \to \pi$ Form Factor

$B \to \pi$ Form Factor :

$\star$ Light-cone sum rules : (P.Ball, JHEP**09**,005(1998))

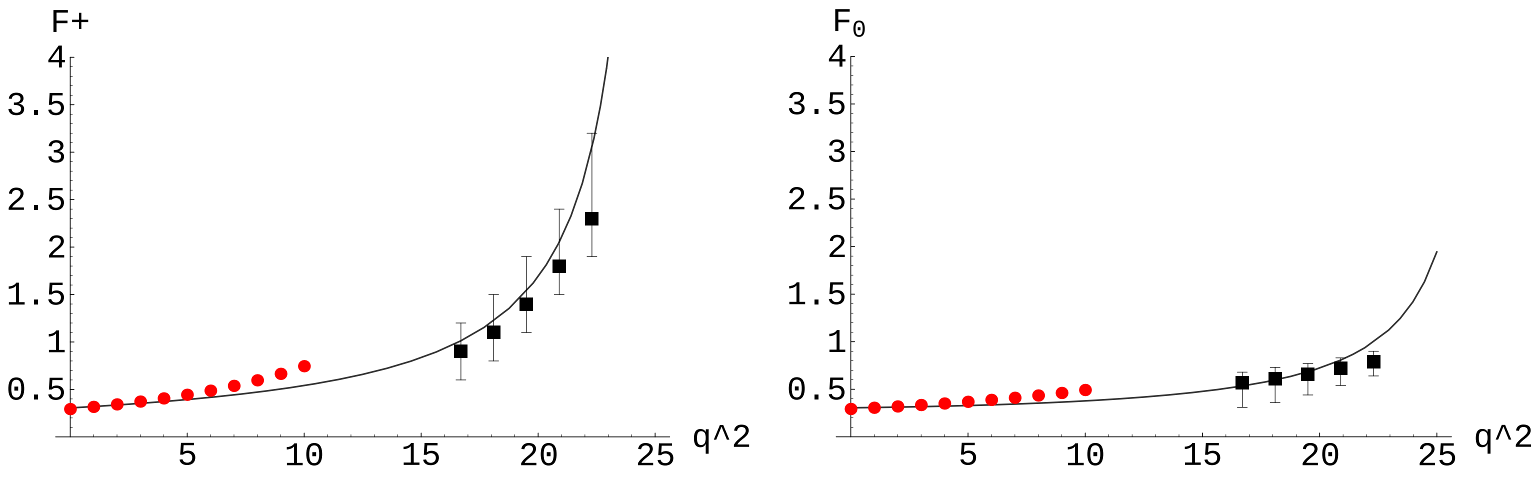
$$F_{+,0}(q^2) = \frac{F_{+,0}}{1 - a_F(q^2/M_B^2) + b_F(q^2/M_B^2)^2} \; ; \qquad \begin{aligned} F_{+,0}(0) &= 0.305 \\ a_F &= 0.266 \\ b_F &= -0.752 \end{aligned}$$

$\star$ Lattice QCD : (UKQCD Collaboration (K.C.Bowler *et al.*), Phys.Lett. **B486**, 111(2000))

$$F_{+,0}(q^2) = \frac{F_{+,0}}{1 - c_F(q^2/M_B^2)} \; ; \qquad \begin{aligned} F_{+,0}(0) &= 0.310 \\ c_F &= 0.760 \end{aligned}$$

$\star$ PQCD : (T.Kurimoto, H-n.Li and A.I.Sanda, Phis.Rev D**65**, 014007 (2002))

$$F_{+,0}(0) = 0.297$$



$B \to \pi$ form factor
as the function of q^2

●: PQCD
■: Lattice
—: Sum Rules

In PQCD approach :

$$F_{+,0} = \int_0^1 dx_1 dx_2 \int_0^{1/\Lambda} db_1 db_2 \; \mathrm{Tr}\Big[\Psi_\pi(x_2, b_2) \times T_H \times \Psi_B(x_1, b_1)\Big] \times E(t)$$

T_H : Hard amplitude
$\Psi_{B,\pi}$: B meson and Pion wave function
$E(t)$: Evolution factor
(α_s and Sudakov factor; t: hard scale)

$x_{1,2}$: momentum fraction
$b_{1,2}$: Impact parameter
(separation between valence quarks of meson)

For the light mesons (π, K, etc.)
Light-cone DAs exists for both leading and higher twists.
(V.M.Braun, I.E.Filyanov, Z.Phys. C**48**, 239(1999); P.Ball, JHEP**9901**, 010(1999))

For the heavy mesons (B, D)
We have only the analytic solutions with two-parton approximate.
(H.Kawamura, J.Kodaira, C-F.Qiao and K.Tanaka, Phys.Lett.B523,111(2001), hep-ph/0112174)
In PQCD, A single model DA is used.

⇒ Reconsider the contribution from B meson wave function.

2 B meson wave function

♠ B meson wave function

(A.G.Grozin and M.Neubert, Phys.Rev.D**55**, 272 (1997))

$$
\begin{aligned}
&\int \frac{d^4z}{(2\pi)^2} \mathrm{e}^{ik_1 z} \langle 0|\overline{q}_\alpha(z) b_\beta(0)|\overline{B}(P_1)\rangle \\
&\quad = -\frac{i}{\sqrt{2N_c}} \left[(\not{P}_1 + M_B)\gamma_5 \left(\tilde{\phi}_B^+(k_1) - v^+\gamma^- \left(\tilde{\phi}_B^-(k_1) - \tilde{\phi}_B^+(k_1) \right) \right) \right]_{\beta\alpha} \\
&\quad = -\frac{i}{\sqrt{2N_c}} \left[(\not{P}_1 + M_B)\gamma_5 \left(-v^-\gamma^+ \tilde{\phi}_B^+(k_1) - v^+\gamma^- \tilde{\phi}_B^-(k_1) \right) \right]_{\beta\alpha}
\end{aligned}
$$

⇒ <u>Two distribution amplitudes $\tilde{\phi}_B^\pm(k_1)$</u>

In the PQCD analysis

→ To keep k_T → Fourier transformation for k_1

$$
\phi_B^\pm(x_1, b) = \int dk_1^+ d^2k_{1T}\ \mathrm{e}^{i\overrightarrow{k}_{1T}\cdot\overrightarrow{b}}\, \tilde{\phi}_B^\pm(k_1)
$$

♠ Distribution Amplitudes (DA)

We $\left\{ \begin{array}{l} \text{treat only the two-parton distribution amplitudes.} \\ \text{neglect the contributions from higher Fock states.} \end{array} \right.$

⋆ KKQT DA (H.Kawamura, J.Kodaira, C-F.Qiao and K.Tanaka)

$$\phi_B^{\pm}(x,b) = \psi_B^{\pm}(x)\xi(x,b) \quad \big(\text{in the Wandzura-Wilczek approximation}\big)$$

- $\psi_B^{\pm}(x)$: Light-cone distribution amplitudes ($\theta(x)$: step function, $\overline{\Lambda} = M_B - m_b$)

$$\psi_B^{+}(x) = \frac{x}{2\overline{\Lambda}/M_B}\theta(x)\theta(2\overline{\Lambda}/M_B - x), \quad \psi_B^{-}(x) = \frac{2\overline{\Lambda}/M_B - x}{2(\overline{\Lambda}/M_B)^2}\theta(x)\theta(2\overline{\Lambda}/M_B - x)$$

- $\xi(x,b)$: Transverse distribution amplitude (Bessel function $J_0(y)$)

$$\xi(x,b) = J_0\left(M_B b\sqrt{x(2\overline{\Lambda}/M_B - x)}\right)$$

Numerical parameter is $\overline{\Lambda}/M_B$ only.

In the PQCD approach to the B meson exclusive decay processes

- Leading DA: ϕ_B and Sub-leading DA: $\overline{\phi}_B$

  Linear combination of two DAs $\phi_B^{\pm}$ e.g. $\phi_B = \frac{1}{2}\left(\phi_B^+ + \phi_B^-\right)$, $\overline{\phi}_B = \frac{1}{2}\left(\phi_B^+ - \phi_B^-\right)$

- Only Leading contribution has been taken into account in the analyses.

  Sub-leading Contribution : Next-to-leading power of $\overline{\Lambda}/M_B$
  $\Rightarrow$ It has been neglected.

$\star$ Question: $\left\{ \begin{array}{l} \text{contribution from } \phi_B \text{ : Leading} \\ \text{contribution from } \overline{\phi}_B \text{ : Sub-leading} \end{array} \right. \Rightarrow$ OK?

(S.Descotes-Genon, C.T.Sachrajda, Nucl.Phys.B625, 239 (2002))

Numerically
Contribution from $\overline{\phi}_B$ becomes important in the $B \to \pi$ form factor.
(Z-T.Wei, M-Z.Yang, Nucl.Phys.B642, 263 (2002))

Reconsider B meson wave function:

★ There is a freedom to put an arbitrary function $f(x)$.

$$\Psi_B = -\frac{i}{\sqrt{2N_c}}\left[(\not{P}_1 + M_B)\gamma_5\left\{\phi_B^+ - v^+\gamma^-(\phi_B^- - \phi_B^+)\right\}\right]_{\beta\alpha}$$

$(\not{v}+1)\gamma_5$
$= (\not{v}+1)\gamma_5(-\not{v})$
$(\not{v}\not{v}=1)$

$$= -\frac{i}{\sqrt{2N_c}}\left[(\not{P}_1 + M_B)\gamma_5\left\{\phi_B^+ - v^+\gamma^-(\phi_B^- - \phi_B^+) + f + \not{v}f\right\}\right]_{\beta\alpha} \quad \Leftarrow \text{Add Zero}$$

$$= -\frac{i}{\sqrt{2N_c}}\left[(\not{P}_1 + M_B)\gamma_5\left\{(\phi_B^+ + \phi_B^- + f) + v^+\gamma^-(\phi_B^+ + f) + v^-\gamma^+(\phi_B^- + f)\right\}\right]_{\beta\alpha}$$

$$\equiv -\frac{i}{\sqrt{2N_c}}\left[(\not{P}_1 + M_B)\gamma_5\left\{\phi_B^L + v^+\gamma^-\phi_B^{N1} + v^-\gamma^+\phi_B^{N2}\right\}\right]_{\beta\alpha}$$

We define as

$$\phi_B^L \equiv \phi_B^+ + \phi_B^- + f, \quad \phi_B^{N1} \equiv \phi_B^+ + f, \quad \phi_B^{N2} \equiv \phi_B^- + f \ .$$

⇒ B meson WF generally is divided into three parts $(L, N1, N2)$ which have an arbitrariness of $f(x)$.

3 $B \to \pi$ form factor

We evaluate the $B \to \pi$ form factor with above B meson WF and 2 parton pion WF (P.Ball) according to T.Kurimoto, H-n.Li and A.I.Sanda.

$B \to \pi$ form factor:

- Three parts.

$$\begin{aligned}\Psi_B \quad &\propto \quad (\not{P}_1 + M_B)\gamma_5 \{\underbrace{(\phi_B^+ + \phi_B^- + f)}_{} + \underbrace{v^+\gamma^-(\phi_B^+ + f)}_{} + \underbrace{v^-\gamma^+(\phi_B^- + f)}_{}\} \\ \Rightarrow \quad F_{+,0} \quad &= \quad F_{+,0}^L[\phi_B^L] \quad + \quad F_{+,0}^{N1}[\phi_B^{N1}] \quad + \quad F_{+,0}^{N2}[\phi_B^{N2}]\end{aligned}$$

- $F_{+,0}^{L,N1,N2}$ are a functional of $f(x)$.

e.g. $(i = +, 0)$

▷ $f = 0$ or the original independent of f;

$$F_i = F_i^L[\phi_B^+ + \phi_B^-] + F_i^{N1}[\phi_B^+] + F_i^{N2}[\phi_B^-] = -F_i^{N1}[\phi_B^-] - F_i^{N2}[\phi_B^+]$$

▷ $f = \phi_B^{\text{any}} - \phi_B^+ - \phi_B^-$;

$$F_i = F_i^L[\phi_B^{\text{any}}] + F_i^{N1}[\phi_B^{\text{any}} - \phi_B^-] + F_i^{N2}[\phi_B^{\text{any}} - \phi_B^+]$$

In Numerical calculation, $f_B = 0.19$, $f_\pi = 0.13$, $\Lambda_{\rm QCD} = 0.25$ GeV

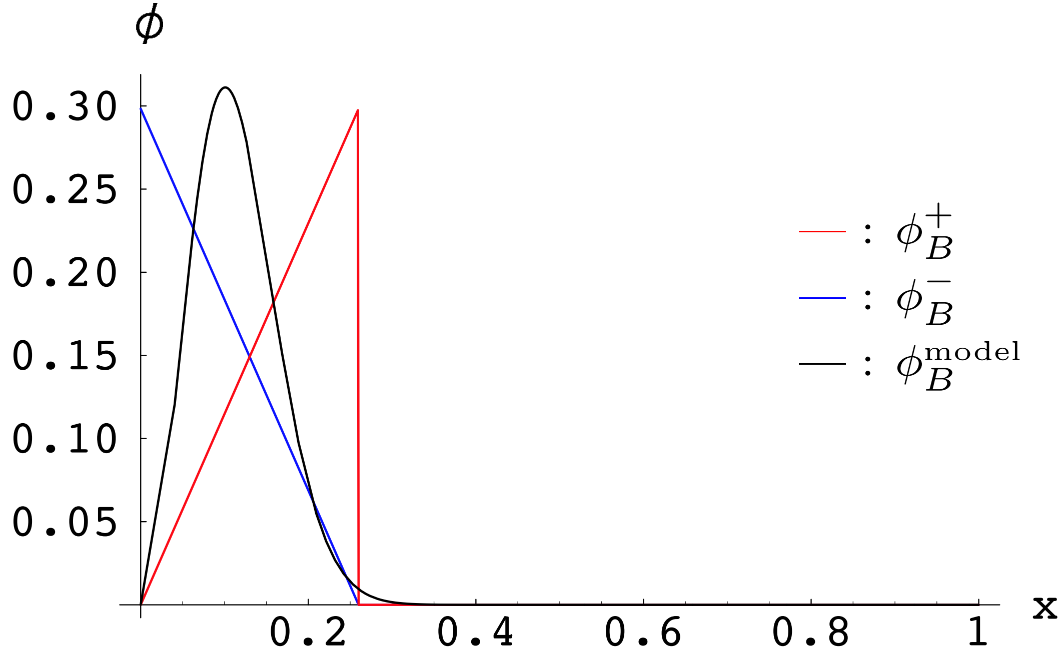
We compute $F^L_{+,0}[\phi^L_B]$, $F^{N1}_{+,0}[\phi^{N1}_B]$, $F^{N2}_{+,0}[\phi^{N2}_B]$ with DAs $\phi^+_B, \phi^-_B, \phi^{\rm model}_B$ for $\phi^{L,N1,N2}_B$.

$\phi^{\rm model}_B$ is the model adopted in PQCD method:

$$\phi^{\rm model}_B(x,b) = N_B x^2 (1-x)^2 \exp\left[\frac{1}{2}\left(\frac{xM_B}{\omega_B}\right)^2 - \frac{\omega_B^2 b^2}{2}\right];$$

$\omega_B = 0.4$: a shape parameter
N_B: a normalization constant

Shapes of
$\phi^\pm_B(x,0)$; $(\overline{\Lambda}/M_B = 0.13)$
and $\phi^{\rm model}_B(x,0)$



♠ $F^{L,N1,N2}[DA]$ and $F_{+,0}$ in maximum recoil region $q^2 = 0$:

legend :

DA	F^L	F^{N1}	F^{N2}
ϕ^+	$F^L[\phi^+] = 0.293$	$\cdots$	$F^{N2}[\phi^+] -0.192$
ϕ^-	$\cdots$	$F^{N1}[\phi^-] = -0.211$	$\cdots$

$$F_i = F^L[\phi^+] + F^L[\phi^-] + F^{N1}[\phi^+] + F^{N2}[\phi^-]$$
$$= -F^{N1}[\phi^-] - F^{N2}[\phi^+]$$

⋆ Model DA

ω_B	DA	F^L	F^{N1}	F^{N2}
0.4	$\phi_B^{\rm model}$	0.295	-0.103	-0.192

We have $F_{+,0}(q^2 = 0) \sim 0.3$ as $\overline{\Lambda}/M_B = 0.125 \sim 0.130$. ($\overline{\Lambda} \sim 0.67$ GeV)

⇒ KKQT WF is good.

⋆ KKQT DA

$\overline{\Lambda}/M_B$	DA	F^L	F^{N1}	F^{N2}	$F_{+,0}$
0.100	ϕ^+	0.293	-0.101	-0.192	0.403
	ϕ^-	1.089	-0.211	-0.877	
0.105	ϕ^+	0.274	-0.096	-0.179	0.381
	ϕ^-	1.039	-0.202	-0.836	
0.110	ϕ^+	0.258	-0.091	-0.167	0.361
	ϕ^-	0.991	-0.195	-0.797	
0.115	ϕ^+	0.241	-0.086	-0.155	0.342
	ϕ^-	0.947	-0.188	-0.761	
0.120	ϕ^+	0.226	-0.082	-0.145	0.325
	ϕ^-	0.907	-0.180	-0.726	
0.125	ϕ^+	0.212	-0.077	-0.135	0.308
	ϕ^-	0.869	-0.173	-0.697	
0.130	ϕ^+	0.200	-0.073	-0.126	0.293
	ϕ^-	0.833	-0.166	-0.666	
0.135	ϕ^+	0.188	-0.070	-0.118	0.279
	ϕ^-	0.799	-0.161	-0.639	
0.140	ϕ^+	0.177	-0.066	-0.111	0.265
	ϕ^-	0.768	-0.154	-0.613	
0.145	ϕ^+	0.167	-0.063	-0.104	0.253
	ϕ^-	0.736	-0.149	-0.589	

♠ Effect of the function $f(x)$ in B meson WF

$$F_i = F_i^L[\phi_B^+ + \phi_B^- + f] + F_i^{N1}[\phi_B^+ + f] + F_i^{N2}[\phi_B^- + f]$$

⋆ Some linear combination of two DAs $\phi_B^\pm$ as $f(x)$

⇒ $F^L[\phi_B^L]$ and $F^{N1,N2}[\phi_B^{N1,N2}]$ from combining the values of above Table

e.g. • $f = -\phi_B^-$ with $\overline{\Lambda}/M_B = 0.100$ • $f = \frac{1}{2}(\phi_B^+ - \phi_B^-)$ with $\overline{\Lambda}/M_B = 0.130$

$$\left.\begin{array}{l} F^L[\phi_B^+] = 0.29 \\ F^{N1}[\phi_B^+ - \phi_B^-] = 0.11 \\ F^{N2}[0] = 0.00 \end{array}\right\} F_{+,0} = 0.40$$

$$\left.\begin{array}{l} F^L[\frac{1}{2}(\phi_B^+ + \phi_B^-)] = 0.52 \\ F^{N1}[\frac{1}{2}(\phi_B^+ - \phi_B^-)] = 0.05 \\ F^{N2}[\frac{1}{2}(\phi_B^- - \phi_B^+)] = -0.27 \end{array}\right\} F_{+,0} = 0.30$$

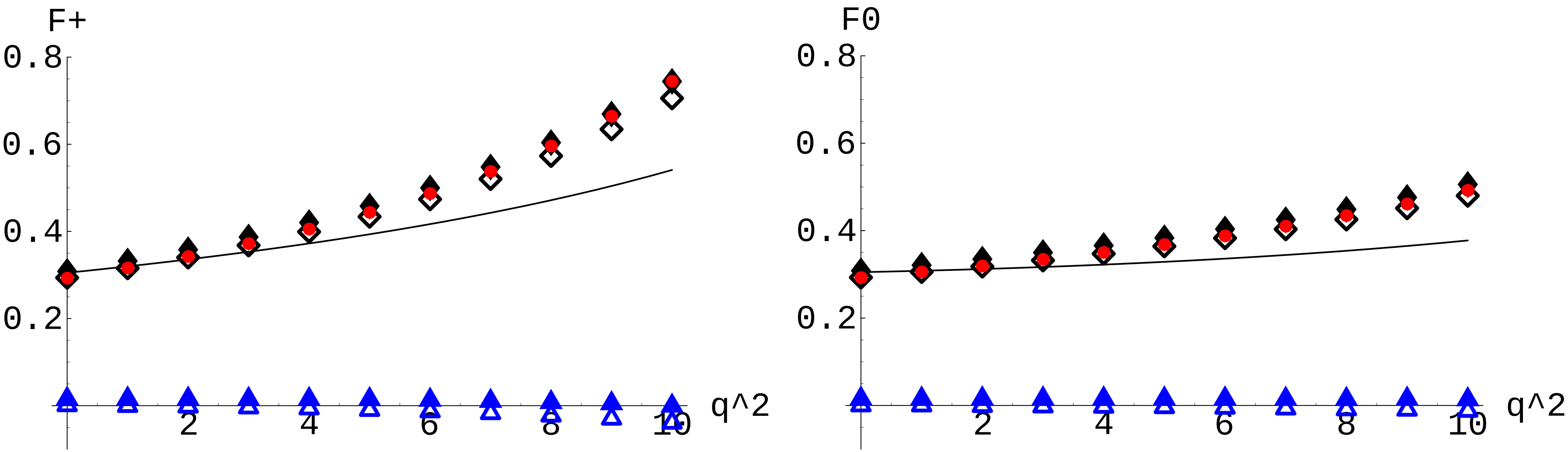
⋆ $f = \phi_B^{\rm model} - \phi_B^+ - \phi_B^-$

$$\Rightarrow \quad F_{+,0} = F_{+,0}^L[\phi_B^{\rm model}] + F_{+,0}^{N1}[\phi_B^{\rm model} - \phi_B^-] + F_{+,0}^{N2}[\phi_B^{\rm model} - \phi_B^+]$$

$F_{+,0}^L[\phi_B^{\rm model}]$ is Leading contribution in T.Kurimoto, H-n.Li and A.I.Sanda.

If $F_{+,0}^{N1,N2}$ are small or vanish or cancel in that case,
⇒ The use of $\phi_B^{\rm model}$ only is granted in the PQCD approach to B meson decays.

$B \to \pi$ form factors F_+ and F_0 as function of q^2 in $f = \phi_B^{\rm model} - \phi_B^+ - \phi_B^-$.



— : Light-cone sum rules (P.Ball)

♦ (◊): $F_{+,0}$ with $\overline{\Lambda}/M_B = 0.125(0.130)$, ● : $F^L_{+,0}[\phi_B^{\rm model}]$

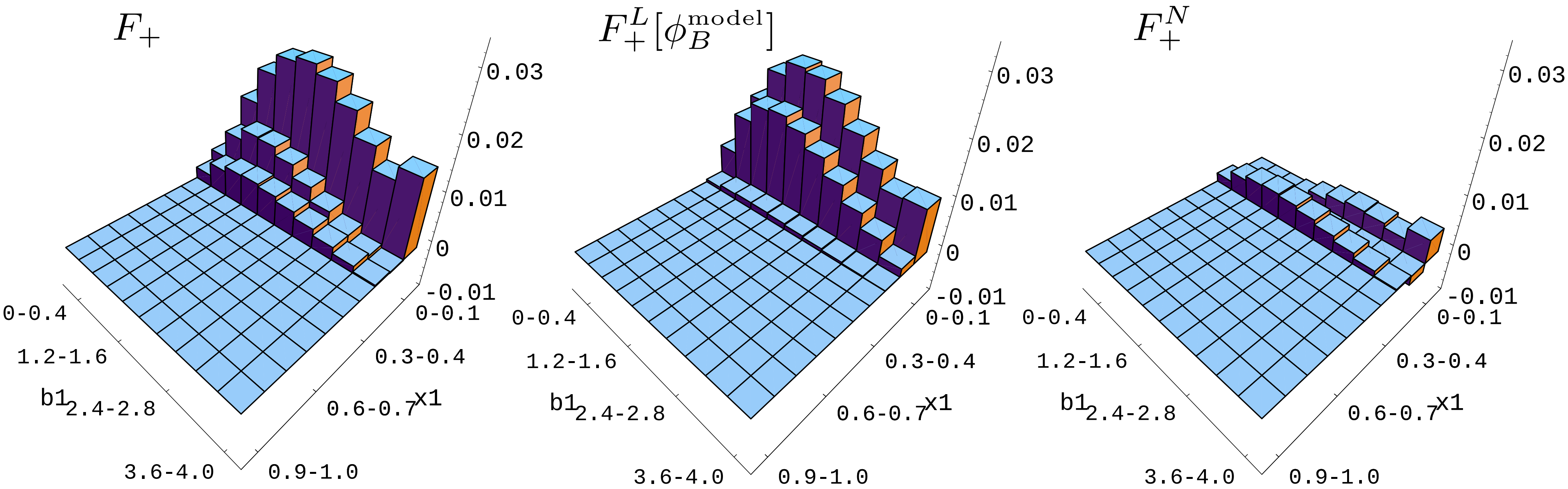
▲ (△): $F^N_{+,0}$ with $\overline{\Lambda}/M_B = 0.125(0.130)$; $F^N_{+,0} \equiv F^{N1}_{+,0}[\phi_B^{\rm model} - \phi_B^-] + F^{N2}_{+,0}[\phi_B^{\rm model} - \phi_B^+]$

- $F_{+,0}$ and $F^L_{+,0}[\phi_B^{\rm model}]$ agree with each other for $q^2 = 0 \sim 10$.

- $F^N_{+,0}$ is very small.

$$F^N_{+,0} = F^{N1}_{+,0}[\phi_B^{\rm model}] - F^{N1}_{+,0}[\phi_B^-] + F^{N2}_{+,0}[\phi_B^{\rm model}] - F^{N2}_{+,0}[\phi_B^+] \cong 0$$

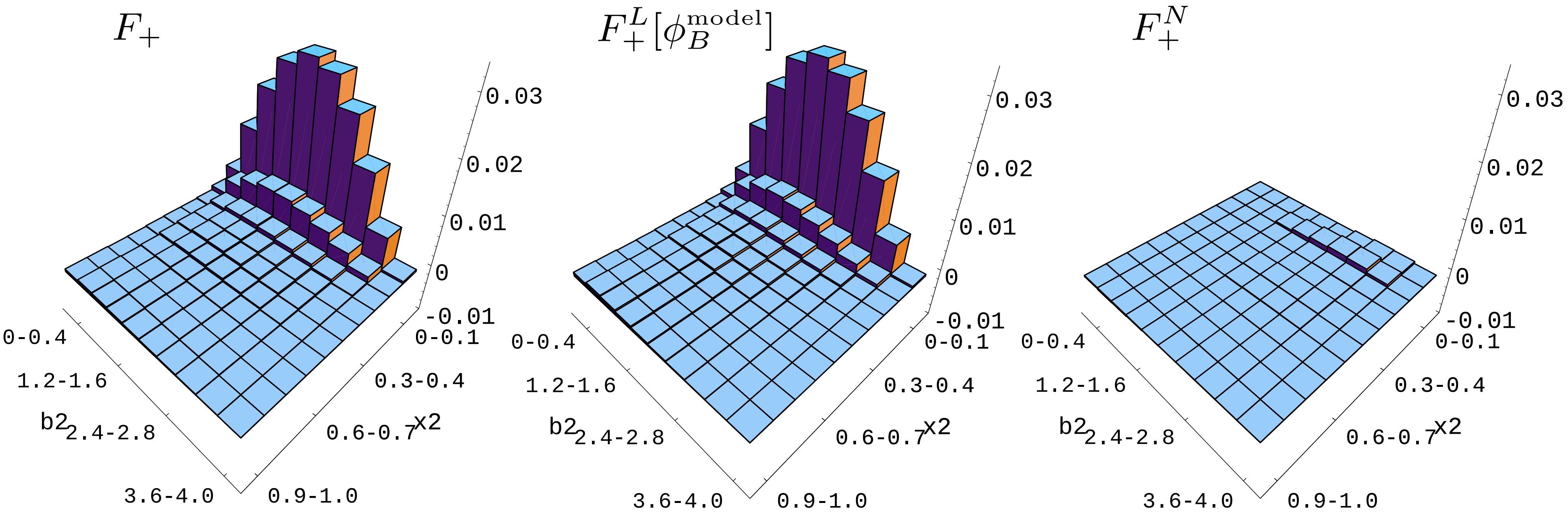
Dependence of F_+, $F_+^L[\phi_B^{\rm model}]$ and F_+^N on the parameter space of x_1-b_1 $(\overline{\Lambda}/M_B = 0.130,\ q^2 = 0)$



- F_+ and $F_+^L[\phi_B^{\rm model}]$ are occupied by small x_1 region. $\leftarrow$ Shapes of DAs
- F_+^N is small.

$\Rightarrow$ $F_+^L[\phi_B^{\rm model}]$ might be Leading contribution.

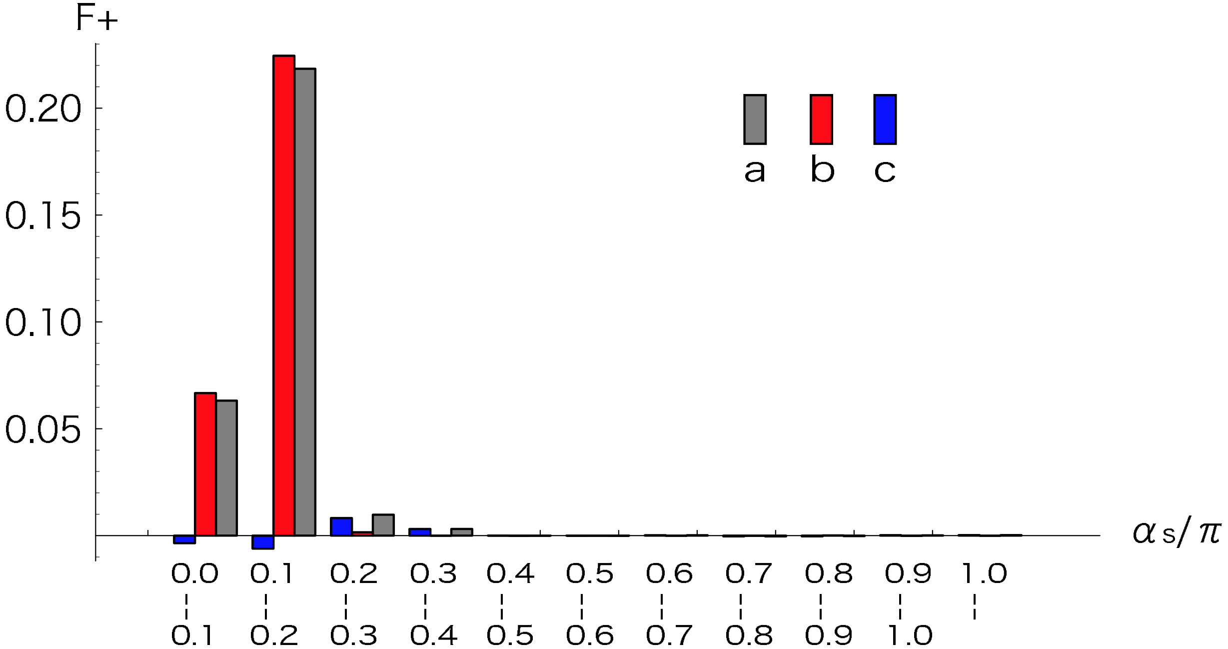
Dependence of F_+, $F_+^L[\phi_B^{\rm model}]$ and F_+^N on the parameter space of x_2-b_2 $(\overline{\Lambda}/M_B = 0.130,\ q^2 = 0)$



- $F_+ \cong F_+^L[\phi_B^{\rm model}]$
- F_+^N is small.

$\Rightarrow$ $F_+^L[\phi_B^{\rm model}]$ might be Leading contribution also.

F_+, $F_+^L[\phi_B^{\rm model}]$, F_+^N from different ranges of α_s/π at $q^2 = 0$ with $\overline{\Lambda}/M_B = 0.130$



a) F_+
b) $F_+^L[\phi_B^{\rm model}]$
c) F_+^N

- Almost all contributions come from the region where $\alpha_s/\pi < 0.2$.

- $F_+ \cong F_+^L[\phi_B^{\rm model}]$ and F_+^N is small compared to $F_+^L[\phi_B^{\rm model}]$.

$\Rightarrow$ Reliability of perturbative method is assured.

4 Summary

We have calculated $B \to \pi$ form factor with the general form of B meson wave function in the PQCD approach.

⋆ KKQT WF ($\overline{\Lambda}/M_B \sim 0.130$) $\Rightarrow$ OK. ($F_{+,0}(0) \sim 0.3$)

⋆ B meson WF has a freedom to put an arbitrary function $f(x)$.

⋆ A specific form of $f(x)$ ($f = \phi_B^{\rm model} - \phi_B^+ - \phi_B^-$)

$\Rightarrow$ $F_{+,0} \cong F_{+,0}^L[\phi_B^{\rm model}]$ and $F_{+,0}^N$ is small.

$F_{+,0}^L[\phi_B^{\rm model}]$ (Simple calculation using a single model DA) can be regarded as Leading contribution in $B \to \pi$ form factor.

⋆ Other decays ($B \to \pi\pi$, $B \to D_s\pi$ etc.)

These are dominated by the contribution from $B \to \pi$ transition matrix elements, that is, $B \to \pi$ form factor.

$\Rightarrow$ Uncertainly from sub-leading B meson DA is small.