k_T factorization

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Mini-workshop on
"QCD for B decays"
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Kyoto University
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B - physics   Big and serious problem

TREATMENT of the hadronic matrix element

$$\langle X_1 X_2 | H | B \rangle$$

"FACTORIZATION" is a powerful strategy

Ex.   Two body decay

$$M(B \to X_1 + X_2) = c_1 f_1 P_{1\mu} \langle X_2 | V^\mu | B \rangle + c_2 f_2 P_{2\mu} \langle X_1 | V^\mu | B \rangle + c_B f_B P_{B\mu} \langle X_1 X_2 | V^\mu | 0 \rangle$$

where $\langle X_i | A_\mu | 0 \rangle = P_{i\mu} f_i$

USING the techniques proposed by Brodsky and Lepage, PRD 22 (1980) 2157

$$\langle X_i | V^\mu | B \rangle = \mathrm{Tr}\left(\overline{\Psi_i}\, T_h^\mu\, \Psi_B \right)$$

- quark-antiquark DA Ψ_i , Ψ_B
- hard scattering amplitude T_h

THE idea of factorization
has been developed and improved

tomorrow session

- ☆ H. Hayakawa PQCD
- ☆ T. Kurimoto PQCD
- ☆ Guohuai Zhu QCD factorization
- ☆ Y. Y. Keum PQCD
- ☆ S. Mishima PQCD

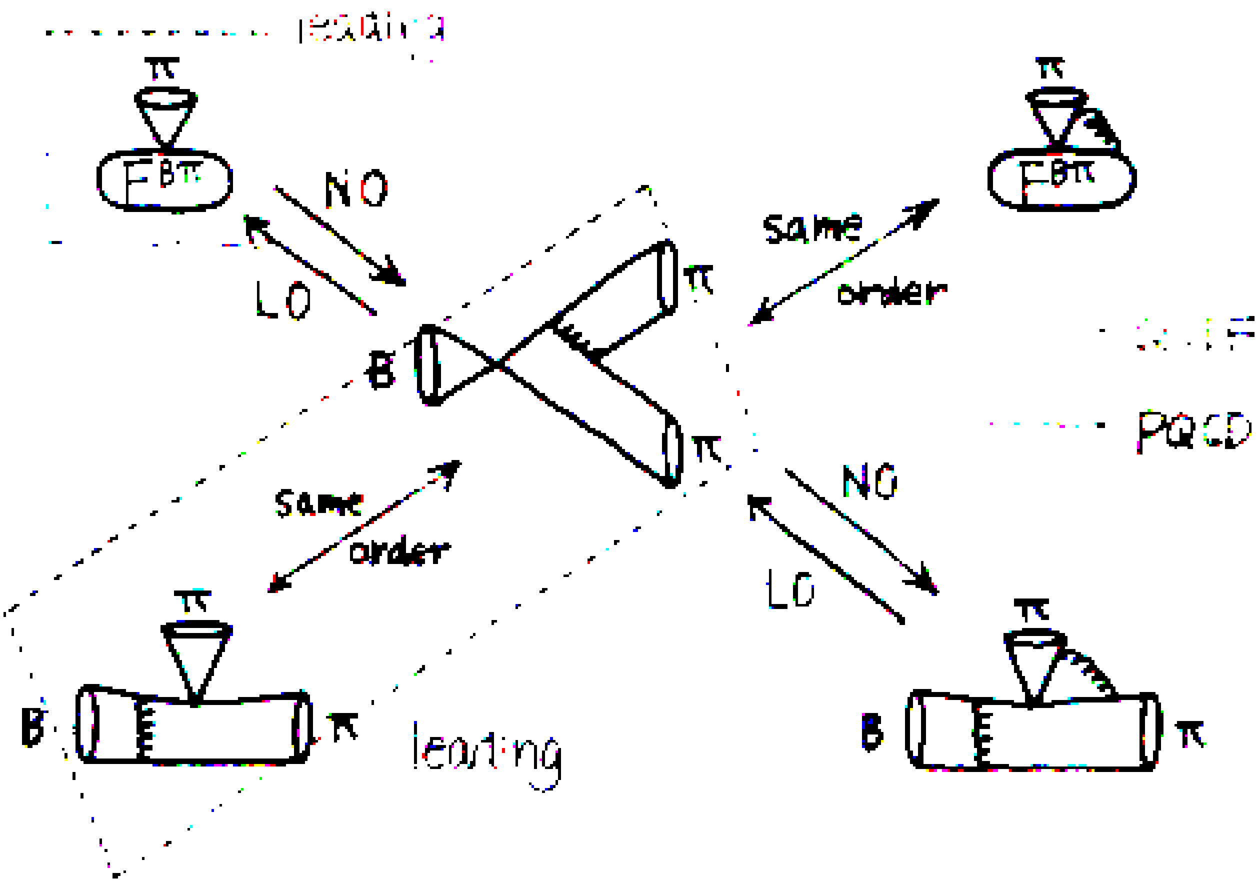
THERE are plural factorization approaches

QCDF (factorization)

P (perturbative) QCD

CRUSIAL difference between QCDF and PQCD
can be shown by a following picture

CORRELATION in $\overline{B}^0 \to \pi^+ \pi^-$



LO : LOWEST ORDER

NO : NEXT ORDER

THE different aspects between the two approaches lead to the different phenomenological predictions.

Ex. CP asymmetry in $\overline{B^0} \to \pi^+\pi^-$

- M. Beneke hep-ph/0207228

  QCDF approach

- C.D. Lü, K. Ukai and M.Z. Yang, PRD 63, 074009 (2001)

  Y.Y. Keum and A.I. Sanda hep-ph/0209014

  Ukai's doctoral thesis

  PQCD approach

  $$\downarrow$$

  $C_{\pi\pi} \sim -0.4$ for $\phi_2 \sim 80^\circ$

exp. $C_{\pi\pi} \sim -0.57 \pm 0.19$ (average) J. Rosner hep-ph/0208243

PQCD approach is based on k_T factorization.

THE PQCD statement

If there is no end-point singularity developed in a hard amplitude,

collinear factorization works.

If such a singularity occurs, indicating the breakdown of collinear factorization,

k_T factorization should be employed

Exclusive B meson decays belong to the latter category.

M. N. and H-n. Li
PRD 67, 034001 (2003)

nonleptonic $\longleftrightarrow$ $\langle X_1 X_2 | H | B \rangle$

THE problem has not been resolved yet.

PREFERABLY

we start considering the simplest case
&
we get the feel of the factorization

k_T factorization for the process $\pi \gamma^* \to \gamma$

$O(\alpha_s^0)$ factorization (the lowest order)

$O(\alpha_s^1)$ factorization

CHECK that the $O(\alpha_s)$ wave-functions can be reproduced by the $O(\alpha_s)$ terms of the nonlocal matrix element in the b space.

k_T : transverse degrees of freedom

b : Fourier conjugation to k_T

THE key points of the k_T factorization

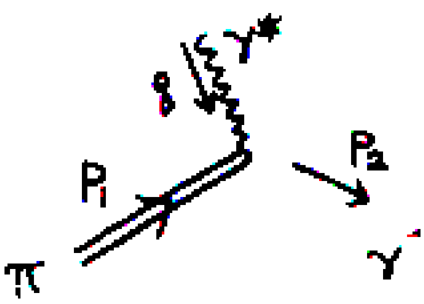
the valence partons

- are initially on-shell,
- carry only longitudinal momenta,
- acquire nonvanishing k_T through collinear gluon exchanges.

IN that case,

a procedure of the proof for the k_T factorization is similar to that for the collinear factorization.

(details of collinear factorization, M.N. and H-n. Li hep-ph/0202127)



momentum for initial state π

$$P_1 = (P_1^+,\ 0,\ \vec{0}_T)$$

momentum for final real photon γ

$$P_2 = (0,\ P_2^-,\ \vec{0}_T)$$

$$P_1^+ = P_2^- \equiv \frac{Q}{\sqrt{2}}$$

WE concentrate on the kinematic region with large Q.

$$q^2 = (P_2 - P_1)^2 = -Q^2$$

HERE we use the coordinate

momentum components are defined as

$$p = (p_0, p_1, p_2, p_3) \longleftarrow \text{ordinary}$$

$$\equiv (p^+, p^-, \vec{p}_T)$$

where

$p^\pm$ [illegible]

[illegible]

USING the above coordinate,

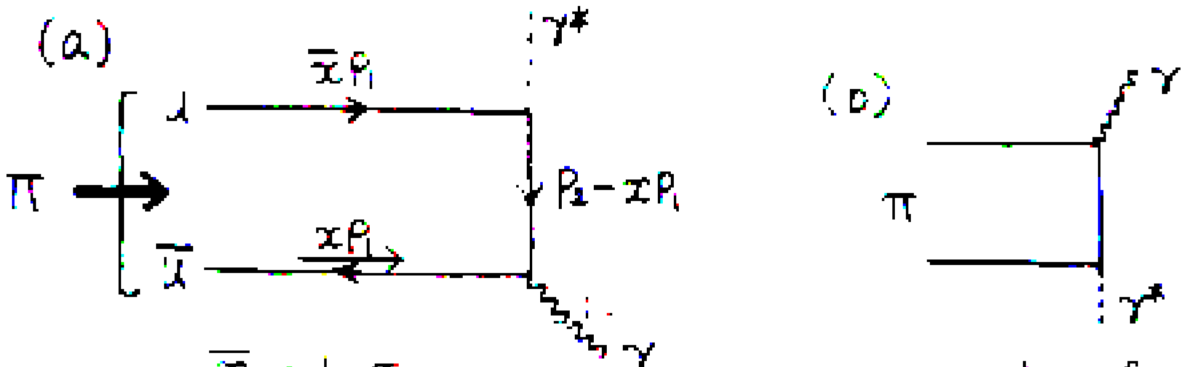
$p \cdot q$ and $\not{p}$ are expressed as follows

$$p \cdot q = p^+ q^- + p^- q^+ - \vec{p}_T \cdot \vec{q}_T$$

$$\not{p} = p^+ \gamma^- + p^- \gamma^+ - \vec{\gamma}_T \cdot \vec{\gamma}_T$$

THE lowest order

(Because of no any additional gluon exchanges, this case is explicitly same as collinear factorization)



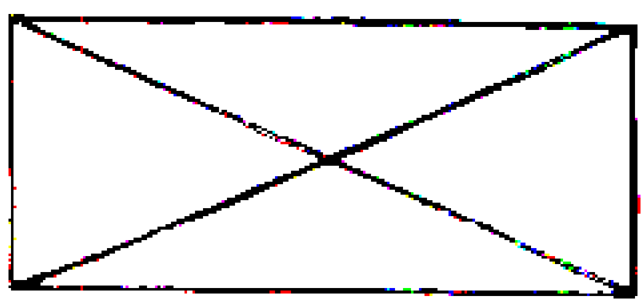
$\bar{x} = 1 - x$ $\qquad$ x : momentum fraction

WE pick out the left side diagram (a)

$$\tilde{G}^{(0)}(x) = -ie^2\, \overline{q_u(xP_1)}\, \not{\epsilon}\, \frac{\not{P}_2 - x\not{P}_1}{(P_2 - xP_1)^2}\, \gamma_\mu\, q_u(\bar{x}P_1)$$

ϵ : polarization vector of out going photon

THE factorization in the fermion flow is achieved by inserting Fierz transformation



In this case, only the structure contributes to $\widetilde{G}^{(*)}(x)$

$$\tilde{G}^{(0)}(x) = \psi^{(0)}(x)\, \tilde{H}^{(0)}(x)$$

$$\psi^{(0)}(x) = \frac{1}{4P_1^+}\, \overline{q_u(xP_1)}\, \gamma_5 \not{n}_-\, q_u(xP_1) \qquad n_- = (0, 1, \vec{0}_T)$$

$$\tilde{H}^{(0)}(x) = i e^2 \frac{tr(\not{\epsilon} \not{P}_2 \gamma_\mu \not{P}_1 \gamma_5)}{2x P_1 \cdot P_2}$$

For the discussion at the next order $O(\alpha_s)$,

Perform the $O(\alpha_s^0)$ factorization in b space.

Reason

in our paper on collinear factorization (hep-ph/0202127)

$O(\alpha_s)$ radiative correction is written as

$$\psi^{(1)}(x) \Big[\underbrace{\tilde{H}^{(0)}(x)}_{\text{without}} - \underbrace{\tilde{H}^{(0)}(\xi)}_{\text{with}} \Big] \qquad \xi = x - \frac{\lambda}{P}$$

λ : dependence

in the momentum space

momentum of an additional gluon

$\tilde{H}^{(0)} \longrightarrow$ calculable hard part

Keeping two terms, calculation is complicated.

$$\left[\phi^{(1)}(x, b) - \phi^{(1)}(x, \xi, b) \right] \underline{H^{(0)}(\xi, b)}$$

- Fourier transformation
- in the b space

$\Longrightarrow$ can be done !

$$\tilde{G}^{(0)}(x) = \int d^2b \; \Phi^{(0)}(x,\xi,b)\, H^{(0)}(\xi,b)$$

$$\Phi^{(0)}(x,\xi,b) = \int d^2 l_T \; \psi^{(0)}(x)\, \delta(\xi-x)\, \delta^2(l_T)\, e^{-i\vec{l}_T\cdot\vec{b}_T}$$

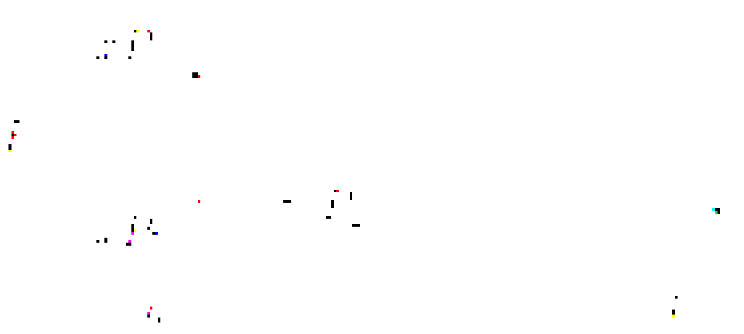
$$H^{(0)}(\xi,b) = \int d^2 k_T \; \tilde{H}^{(0)}(\xi,k_T)\, e^{i\vec{k}_T\cdot\vec{b}}$$

NEXT we check the $O(\alpha_s)$ factorization by considering radiative corrections.

Diagrams for Proof of k_T Factorization

$$\pi\gamma^* \to \gamma,\ B \to \gamma l \bar{\nu}$$

Lowest Order Diagram

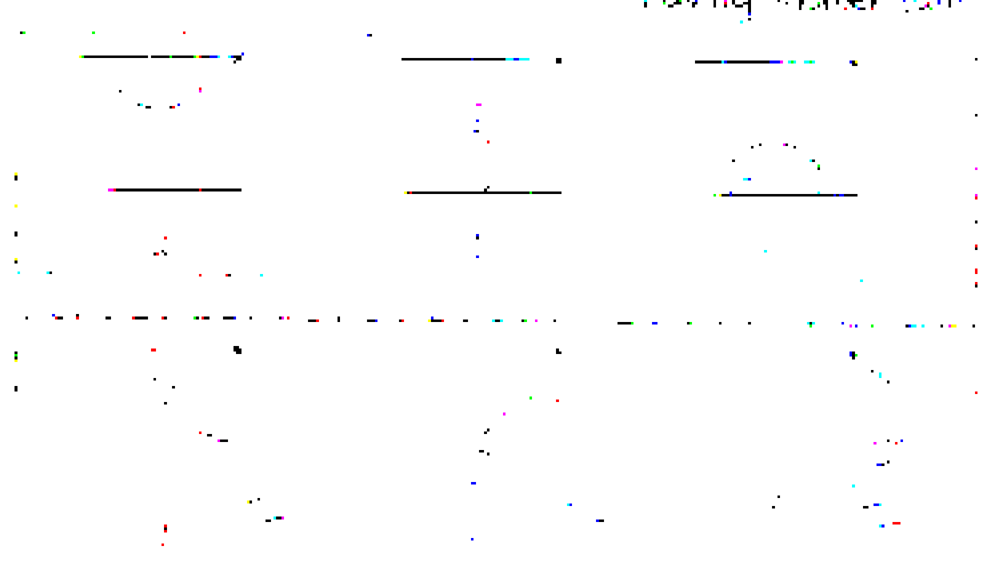


$O(\alpha_s)$ Corrections to ...

same ... an additional gluon having momentum l

reducible

irreducible



C(αs) factorization

focusing on the diagram (a) and (d)

TRYING to separate the infrared divergences from the amplitude,

ℓ has collinear-like components

$$\ell = (\ell^+, \ell^-, \ell_T)$$

$$\ell^+ \sim Q$$

$$\ell^- \sim 0$$

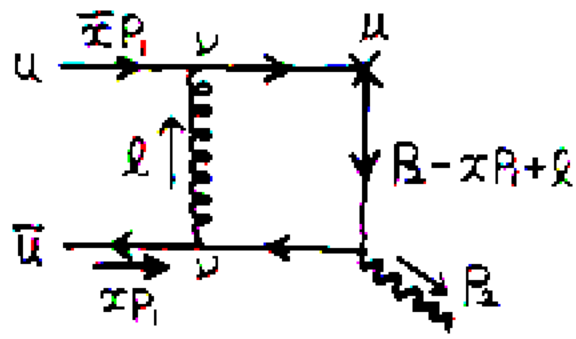
$$\ell_T \sim \Lambda$$

(Λ represents a small scale)

IN this region

$$\ell^2 = -\ell_T^2$$

(b)



$$l = (l^+,\ 0,\ \vec{l}_T)$$

$$I_b^{(1)} = e^2 g^2 C_F\, \overline{q_u}(\bar{x}P)\, \gamma_\nu \frac{x\not P_1 - \not l}{(xP_1 - l)^2}\, \not\epsilon\, \frac{\not P_2 - x\not P_1 + \not l}{(P_2 - xP_1 + l)^2}\, \gamma_\mu$$
$$\times \frac{\bar{x}\not P_1 + \not l}{(\bar{x}P_1 + l)^2}\, \gamma^\nu\, q_u(\bar{x}P)\, \frac{1}{l^2}$$

Fierz transformation

$$= \Psi_b^{(1)}(x, \xi, l_T)\ \tilde{H}^{(0)}(\xi, l_T)$$

$$\Psi_b^{(1)}(x, \xi, l_T) = \frac{ig^2}{4P_1^+} C_F\, \overline{q_u}\, \gamma_\nu \frac{x\not P_1 - \not l}{(xP_1 - l)^2}\, \gamma_5 \not P_-\, \frac{\bar{x}\not P_1 + \not l}{(\bar{x}P_1 + l)^2}\, \gamma^\nu q_u\, \frac{1}{l^2}$$

$$\tilde{H}^{(0)}(\xi, l_T) = i e^2 \frac{\mathrm{tr}\left(\not\epsilon\,(\not P_2 - \xi\not P_1)\gamma_\mu \not P_1 \gamma_5\right)}{2\xi P_1 \cdot P_2 + l_T^2}$$

$$(P_2 - xP_1 + l)^2 = -2P_2 \cdot (xP_1 - l) - l_T^2$$
$$= -\left(l_T^2 + 2\xi P_1 \cdot P_2\right) \qquad \xi = x - \frac{l^+}{P_1^+}$$

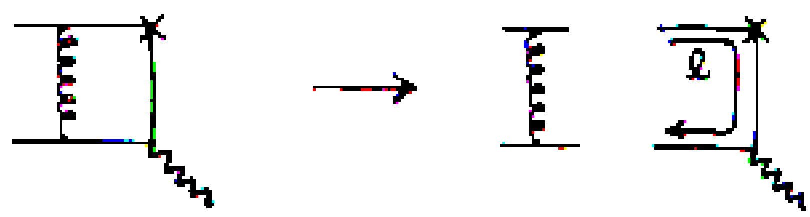
THESE functions are in the momentum space.

USING the Fourier transformation,

$$\psi_b^{(1)}(x,\xi,l_T)\quad \tilde{H}^{(0)}(\xi,l_T)$$

$$\Downarrow$$

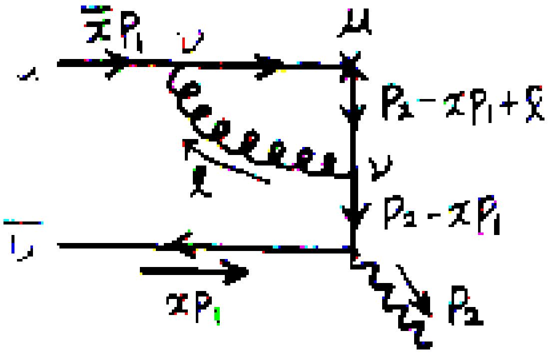
$$\Phi_b^{(1)}(x,\xi,b)\quad H^{(0)}(\xi,b)$$



$\psi_b^{(1)}(x,\xi,l_T)$

$$\Phi_b^{(1)}(x,\xi,l_T) = \int d^2 l_T\ \psi_b^{(1)}(x,\xi,l_T)\ e^{-i\vec{l}_T\cdot\vec{b}}\ \delta\left(\xi-x+\frac{l^+}{P_1^+}\right)$$

(d)



$$\ell = (\ell^+, 0, \vec{\ell}_T)$$

$$I_d^{(1)} = -e^2 g^2 C_F \, \overline{\xi_u(xP_1)} \, \not{\epsilon} \, \frac{\not{P}_2 - x\not{P}_1}{(P_2 - xP_1)^2} \gamma_\nu \frac{\not{P}_2 - x\not{P}_1 + \not{\ell}}{(P_2 - xP_1 + \ell)^2} \gamma_\mu$$
$$\times \frac{\bar{x}\not{P}_1 + \not{\ell}}{(\bar{x}P_1 + \ell)^2} \gamma^\nu \xi_u(\bar{x}P_1) \frac{1}{\ell^2}$$

dominant contribution

$$\gamma^\nu = \gamma^+ , \quad \gamma_\nu = \gamma_+ = \gamma^-$$

$$\underset{\gamma^+}{(\not{P}_2 - x\not{P}_1)} \underset{\gamma^-}{\gamma_\nu} \underset{\gamma^+}{(\not{P}_2 - x\not{P}_1 + \not{\ell})} \approx 2 P_{2\nu} (\not{P}_2 - \xi \not{P}_1)$$

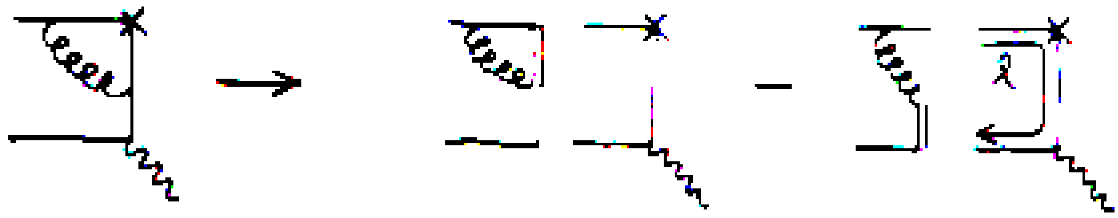
$\longleftarrow$ dominant contribution

$$\frac{2P_{2\nu}}{(P_2 - xP_1)^2 (P_2 - xP_1 + \ell)^2} \approx \frac{n_{-\nu}}{n_- \cdot \ell} \left[\frac{1}{(P_2 - xP_1)^2} - \frac{1}{(P_2 - \xi P_1)^2} \right]$$

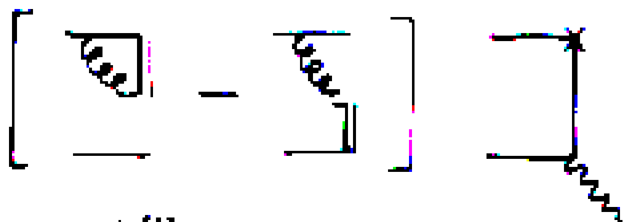
$$= \Psi_d^{(1)}(x, \xi, \ell_T) \left[\widetilde{H}^{(0)}(x) - \widetilde{H}^{(0)}(\xi, \ell_T) \right]$$

$$\Psi_d^{(1)}(x, \xi, \ell_T) = \frac{-i g^2}{4P_1^+} C_F \, \bar{\xi}_u \, \gamma_5 \not{n}_- \frac{\bar{x}\not{P}_1 + \not{\ell}}{(\bar{x}P_1 + \ell)^2} \gamma^\nu \xi_u \frac{1}{\ell^2} \frac{n_{-\nu}}{n_- \cdot \ell} .$$

In the momentum space



$\Downarrow$ Fourier transformation



$\Phi_d^{(1)}(x,\xi,b)$ $\quad$ $H^{(0)}(\xi,b)$

$\Psi_d^{(1)}(x,\xi,l_T)$

$\downarrow$

$$\Phi_d^{(1)}(x,\xi,b) = \int d^2 l_T \, \Psi_d^{(1)}(x,\xi,l_T)\left[\delta(\xi-x) - \delta\left(\xi-x+\frac{l^+}{P_1^+}\right)e^{-i\vec{l}_T\cdot\vec{b}}\right]$$

SINCE we consider the factorization in b space,

$$\Phi^{(1)}(x,\xi,b) \propto \langle 0|\,\overline{q_u(y)}\,\gamma_5 \not{n}_- \left(-ig\int_0^y ds\cdot A(s)\right) q_u(0)\,|\pi\rangle$$

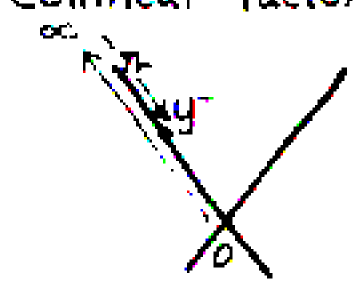
where

$$y = (0,\, y^-,\, \vec{b}\,)$$

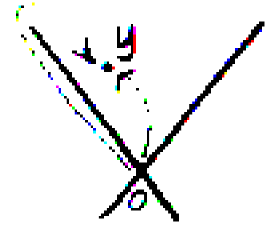
THE coordinate is shifted from y^- to y

the path for the Wilson link is modified

collinear factorization



k_T factorization



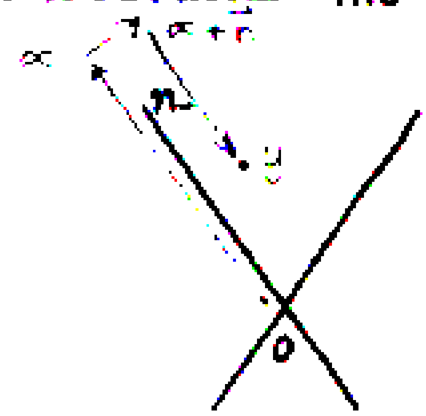
$0 \to \infty$
$\infty \to y^-$
move on the n_- axis

How does it go to y ?

CONSIDERING the path

check whether the above $\Phi^{(1)}(x,\xi,b)$ reproduce the factor $\frac{n_-\cdot v}{n_-\cdot \xi}$ and $\frac{n_-\cdot v}{n_-\cdot \xi}\, e^{-i\vec{k}_T\cdot\vec{b}}$

Assuming the path is composed of three pieces,



(i) $0 \to \infty$ along the direction of n_-

(ii) $\infty \to \infty + \vec{b}$

(iii) $\infty + \vec{b} \to y$ along n_-

we can check it easily.

(i)

$$-ig\int_0^\infty dz\, n_- \cdot A(zn) \longrightarrow -ig\int_0^\infty dz\, e^{iz(n\cdot l + i\epsilon)}\, n_- \cdot \tilde{A}(l)$$

$$= g \boxed{\frac{n_{-\alpha}}{n_- \cdot l}}\, \tilde{A}^\alpha(l)$$

ϵ : infinitesimal constant

$$A^\alpha(zn) = \int \frac{d^4 l}{(2\pi)^4}\, e^{il\cdot nz}\, \tilde{A}^\alpha(l)$$

(ii)

$$-ig\int_\infty^{\infty+\vec{b}} ds \cdot A(s) \propto e^{-s\epsilon}\Big|_{s=\infty}^{s=\infty+\vec{b}} = 0 \qquad (e^{-\infty} = 0)$$

(iii)

$$-ig\int_\infty^{y^-} dz\, n_- \cdot A(zn+\vec{b}) \longrightarrow -ig\int_\infty^{y^-} dz\, e^{i(zn\cdot l - \vec{l}_T\cdot\vec{b})}\, n_- \cdot \tilde{A}(l)$$

$$= -g \boxed{\frac{n_{-\alpha}}{n_- \cdot l}\, e^{-i\vec{l}_-\cdot\vec{b}}}\, \tilde{A}^\alpha(l)$$

The factor $\frac{n_{-\alpha}}{n_- \cdot l}$, $\frac{n_{-\alpha}}{n_- \cdot l}\, e^{-i\vec{l}_T\cdot\vec{b}}$ are reproduced

In this talk

(1)

I explained

the difference between QCDF and PQCD approaches

PQCD is based on collinear / k_T factorization

For exclusive B decays, k_T factorization is employed

(2)

I showed

the procedure of k_T factorization

at the lowest order

at $O(\alpha_s)$

process we chose $\pi\gamma^* \to \gamma$

(3)

We checked

$O(\alpha_s)$ WF can be reproduced

by $O(\alpha_s)$ terms of the nonlocal matrix element

in the b space.