B Meson Light-Cone Wavefunctions and Structure Functions in QCD in the Heavy Quark Limit

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Exclusive B meson decays $\rightsquigarrow$ CKM matrix

$B \to \pi \ell \bar{\nu}$

$B \to \rho \gamma$

$B \to K^* \ell^+ \ell^-$

$B \to \pi \pi$

$$A(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i \, \underline{C_i(\mu)} \, \underline{\langle M_1 M_2 | O_i | B \rangle_{(\mu)}}$$

λ_i : CKM factor

Fundamental scales $\quad M_W \gg m_b \gg \Lambda_{QCD}$

How to compute $\langle M_1 M_2 | O_i | B \rangle$?

(How to separate short and long distance contributions?) $\sim 1/m_b \qquad \sim 1/\Lambda_{QCD}$

Rigorous approaches

QCD factorization

…

pQCD

$m_b \gg \Lambda_{QCD}$ (in the leading order in $1/m_b$)

$$\langle M_1 M_2 | O_i | B \rangle = \sum_j F_j^{B\to M_1}(m_2^2) \int_0^1 du\, T_{ij}^{I}(u)\, \phi_{M_2}(u) + (M_1 \leftrightarrow M_2)$$

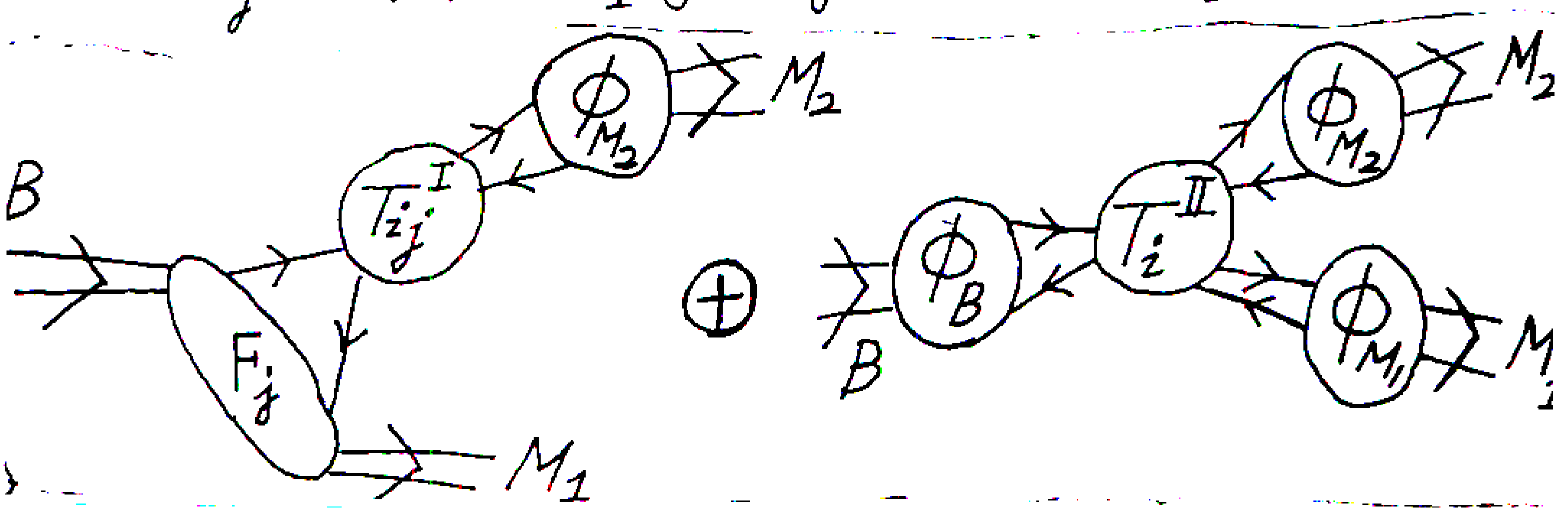
$$+ \int_0^1 d\xi\, du\, dv\, T_i^{II}(\xi, u, v)\, \phi_B(\xi)\, \phi_{M_1}(u)\, \phi_{M_2}(v)$$

T_{ij}^{I}, T_i^{II} : hard scattering amp. $(\sim 1/m_b)$

ϕ_B, ϕ_{M_1}, ϕ_{M_2} : light-cone DAs

$F_j^{B\to M_1}$: $B \to M_1$ form factor

} $(\sim 1/\Lambda_{QCD})$



T^{I}_{ij} , T^{II}_{i} : calculable by PT

$F^{B\to M_1}_{j}$ — nonperturbative,

ϕ_B , ϕ_{M_1} , ϕ_{M_2} — but universal!

ϕ_π, ϕ_K, ϕ_ρ, ϕ_{K^*} : light-meson DAs

well-known

systematic model-independent formalism exists!

ϕ_B , ϕ_{B^*} : B-meson DAs

NOT well-known

major source of uncertainty in the calculation of decay ratios

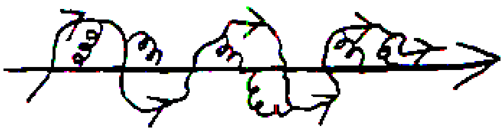
$$\phi_B(\xi) \sim \int d^2k_T \; \Phi_B(\xi, k_T)$$

DAs (distribution amplitudes) — WFs (wavefunctions)

Heavy quark symmetry

Isgur, Wise ('89)
Georgi ('90)
Eichten, Hill ('90)
Grinstein ('90)

$$\bar{B} = \bar{q} b$$



$$P_b^\mu = m_b v^\mu + k^\mu$$

$$v^2 = 1, \quad k^\mu = O(\Lambda_{QCD})$$

$$\mathcal{L}_b = \bar{b}\,(i\not{D} - m_b)\,b$$

$$b(x) \approx e^{-i m_b v\cdot x} h_v(x)$$

$$\not{v}\, h_v(x) = h_v(x)$$

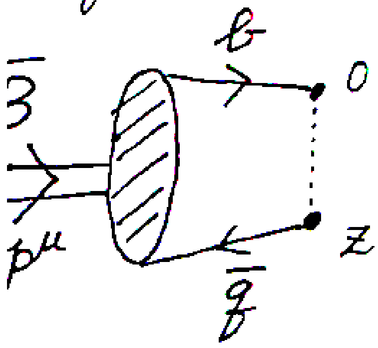
$$\mathcal{L}_b = \bar{h}_v\, i v\cdot D\, h_v + O(1/m_b)$$

Heavy Quark Effective Theory (HQET)

spin-flavor symmetry

Eq. of motion: $i v\cdot D h_v = 0$

Light-cone WFs for B meson



B-S WFs at equal light-cone time

[illegible]

$p^\mu = M v^\mu \qquad (v^2 = 1)$

$$\langle 0 | \bar{q}(z) \ldots h_v(0) | \bar{B}(p) \rangle \sim \tilde{\Phi}(v \cdot z, z^2)$$

$P\exp[ig\int_0^z dx \cdot A(x)]$

Γ : [illegible]

$\tilde{\Phi}_+(v \cdot z, z^2), \quad \tilde{\Phi}_-(v \cdot z, z^2)$

$\tilde{\Phi}_+(v \cdot z, z^2), \quad \tilde{\Phi}_-(v \cdot z, z^2)$

$\tilde{\Phi}_+(v \cdot z, z^2), \quad \tilde{\Phi}_-(v \cdot z, z^2)$

1, [illegible] $\quad 0$

heavy quark spin sym. $\Rightarrow$ only 2 WFs $\tilde{\Phi}_\pm$

$\not{v} h_v = h_v$

v_μ [illegible] $= \ldots \not{v} h_v(0) \ldots - \ldots h_v(0)$

(cf.) Light-cone WFs for π

$$\langle 0 | \bar{q}(z) \Gamma q(0) | \pi(p) \rangle \sim \tilde{\varphi}_\pi(v \cdot z, z^2)$$

4 independent WFs !

$$\langle 0|\bar q(z)\gamma^\mu\gamma_5 h_v(0)|\bar B(p)\rangle = i f_B M \left\{ v^\mu \tilde\Phi_+(v\cdot z, z^2) - \frac{z^\mu}{2v\cdot z}\left[\tilde\Phi_+(v\cdot z, z^2) - \tilde\Phi_-(v\cdot z, z^2)\right]\right\}$$

$$\langle 0|\bar q(z)\gamma_5 h_v(0)|\bar B(p)\rangle = -\frac{i}{2} f_B M \left[\tilde\Phi_+(v\cdot z, z^2) + \tilde\Phi_-(v\cdot z, z^2)\right]$$

$$\langle 0|\bar q(z)\sigma^{\mu\nu}\gamma_5 h_v(0)|\bar B(p)\rangle = -\frac{i}{2} f_B M \varepsilon^{\mu\nu\alpha\beta} v_\alpha z_\beta \frac{1}{v\cdot z}\left[\tilde\Phi_+(v\cdot z, z^2) - \tilde\Phi_-(v\cdot z, z^2)\right]$$

$$\langle 0|\bar q(0)\gamma^\mu\gamma_5 h_v(0)|\bar B(p)\rangle = i f_B M v^\mu:\ \tilde\Phi_\pm(0,0) = 1$$

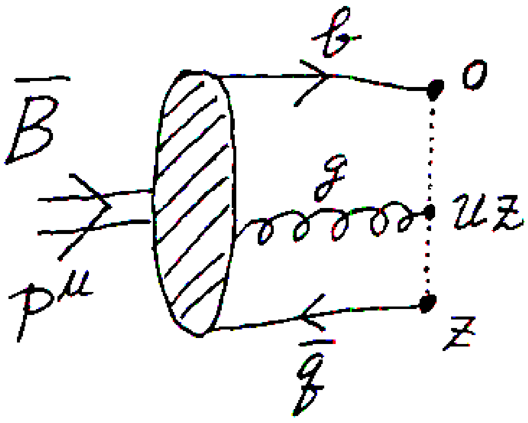
$$\langle 0|\bar q(z)\Gamma h_v(0)|\bar B(p)\rangle = -\frac{i f_B M}{2}\mathrm{Tr}\left[\gamma_5 \Gamma \frac{1+\not v}{2}\left\{\tilde\Phi_+(v\cdot z, z^2) - \not z\,\frac{\tilde\Phi_+(v\cdot z, z^2) - \tilde\Phi_-(v\cdot z, z^2)}{2v\cdot z}\right\}\right]$$

Relation with DAs:

$$\tilde\phi_\pm(v\cdot z) = \tilde\Phi_\pm(v\cdot z, z^2)\Big|_{z^2\to 0}$$

$$\left(\phi_\pm(\xi) = \int d^2k_T\, \Phi_\pm(\xi, k_T)\right)$$

Three-particle WFs for B meson



$$\langle 0|\bar{q}(z)\, gG_{\mu\nu}(uz)h_v(0)|\bar{B}(p)\rangle \sim \tilde{\Psi}(v\cdot z, u, z^2)$$

[illegible]	$\tilde{\Psi}_V$			
[illegible]		$\tilde{\Psi}_A$	$\tilde{X}_A$	$\tilde{Y}_A$
[illegible]	$\tilde{\Psi}_V$	$\tilde{\Psi}_A$		$\tilde{Y}_A$
[illegible]			$\tilde{X}_A$	$\tilde{Y}_A$
[illegible]	0			

$$\langle 0|\bar{q}(z)\, gG_{\mu\nu}(uz)\, z^\nu\, h_v(0)|\bar{B}(p)\rangle$$
$$= \frac{1}{2} f_B M\, \mathrm{Tr}\Big[\gamma_5\, \frac{1+\not{v}}{2}$$
$$\times \Big\{(v_\mu \not{z} - v\cdot z\, \gamma_\mu)\, \big(\tilde{\Psi}_A(v\cdot z,\, u) - \tilde{\Psi}_V(v\cdot z,\, u)\big)$$
$$-i\,\sigma_{\mu\nu} z^\nu\, \tilde{\Psi}_V(v\cdot z,\, u) - z_\mu\, \tilde{X}_A(v\cdot z,\, u)$$
$$+\frac{z_\mu}{v\cdot z}\, \not{z}\tilde{Y}_A(v\cdot z,\, u)\Big\}\Big] + \cdots$$

heavy quark spin symmetry

$\Longrightarrow$ 4 independent WFs $\tilde{\Psi}_V, \tilde{\Psi}_A, \tilde{X}_A, \tilde{Y}_A$

Constraints from QCD equations of motion

Exact identities:

$$\frac{\partial}{\partial z^\mu}\bar{q}(z)\gamma^\mu\Gamma h_v(0) = \bar{q}(z)\overleftarrow{\not{D}}\Gamma h_v(0) + i\int_0^1 du\, u\, \bar{q}(z)\, g\, \boxed{G_{\mu\nu}(uz)}\, z^\nu \gamma^\mu \Gamma h_v(0)$$

$$v^\mu \frac{\partial}{\partial z^\mu}\bar{q}(z)\Gamma h_v(0) = -\bar{q}(z)\Gamma\, v\cdot D\, h_v(0) + i\int_0^1 du\,(u-1)\,\bar{q}(z)\, g\, \boxed{G_{\mu\nu}(uz)}\, v^\mu z^\nu \Gamma h_v(0) + v_\mu \partial^\mu\{\bar{q}(z)\Gamma h_v(0)\}$$

Take $\langle 0 | \cdots | \bar{B}(p) \rangle$

$$\frac{\partial}{\partial z^\mu} \bar{q}(z) \gamma^\mu \Gamma h_v(0) = \bar{q}(z) \overleftarrow{\not{D}} \Gamma h_v(0) \quad (\to 0)$$

$$+ i \int_0^1 du\, u\, \bar{q}(z)\, g\, G_{\mu\nu}(uz)\, z^\nu \gamma^\mu \Gamma h_v(0)$$

⇑ $\tilde{\Phi}_\pm, \ \dfrac{\partial \tilde{\Phi}_\pm}{\partial z^\mu}$ ⇓

⇑ $\tilde{\Psi}_V, \ \tilde{\Psi}_A, \ \tilde{X}_A, \ \tilde{Y}_A$ ⇓

$$v^\mu \frac{\partial}{\partial z^\mu} \bar{q}(z) \Gamma h_v(0)$$

$$= - \bar{q}(z) \Gamma\, v \cdot \overleftarrow{D}\, h_v(0) \quad (\to 0)$$

$$+ i \int_0^1 du\, (u-1)\, \bar{q}(z)\, g\, G_{\mu\nu}(uz)\, v^\mu z^\nu \Gamma h_v(0)$$

$$+ v_\mu \partial^\mu \left\{ \bar{q}(z) \Gamma h_v(0) \right\}$$

← $\bar{\Lambda} \times \tilde{\Phi}_\pm$

$$\bar{\Lambda} = \frac{i v_\mu \partial^\mu \langle 0 | \bar{q} \Gamma h_v | \bar{B}(p) \rangle}{\langle 0 | \bar{q} \Gamma h_v | \bar{B}(p) \rangle} = M - m_b$$

"effective mass" of meson states in the HQET

$\Gamma = 1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}$

4 independent constraint equations between $\tilde{\Phi}_+, \tilde{\Phi}_-$ and $(\tilde{\Psi}_V, \tilde{\Psi}_A, \tilde{X}_A, \tilde{Y}_A)$

- [I] $z^2 \to 0$ with $(\tilde{\Psi}_V, \tilde{\Psi}_A, \tilde{X}_A, \tilde{Y}_A)$
  exact differential equations to determine DAs $\tilde{\phi}_\pm(v \cdot z) = \tilde{\Phi}_\pm(v \cdot z, z^2)\ _{z^2 \to 0}$

  Kawamura, Kodaira, Qiao, Tanaka, Phys. Lett. B523 (2001) 111 [E: B536 (2002) 344]

- [II] $z^2 \neq 0$, neglecting $(\tilde{\Psi}_V, \tilde{\Psi}_A, \tilde{X}_A, \tilde{Y}_A)$
  differential equations for $\tilde{\Phi}_\pm(v \cdot z, z^2)$
  in the "Wandzura-Wilczek approximation"

  Kawamura, Kodaira, Qiao, Tanaka, hep-ph/0112174 (to be published in Mod. Phys. Lett. A)

[I] $z^2 \to 0$

$\tilde{\Phi}_\pm(v\cdot z, z^2) \to \tilde{\phi}_\pm(v\cdot z)$, $\qquad \dfrac{\partial\tilde{\phi}_\pm(v\cdot z)}{\partial z^2} \equiv \dfrac{\partial\tilde{\Phi}_\pm(v\cdot z, z^2)}{\partial z^2}\Big|_{z^2\to 0}$

1st identity: $\bar{q}\overleftarrow{\not{D}} = 0$

$t \equiv v\cdot z$

$$\frac{d\tilde{\phi}_-(t)}{dt} - \frac{1}{t}\left(\tilde{\phi}_+(t) - \tilde{\phi}_-(t)\right) = 2t\int_0^1 du\, u\left(\tilde{\Psi}_A(t,u) - \tilde{\Psi}_V(t,u)\right) \quad (1)$$

$$\frac{d\tilde{\phi}_+(t)}{dt} - \frac{d\tilde{\phi}_-(t)}{dt} - \frac{1}{t}\left(\tilde{\phi}_+(t) - \tilde{\phi}_-(t)\right) + 4t\frac{\partial\tilde{\phi}_-(t)}{\partial z^2} = 2t\int_0^1 du\, u\left(\tilde{\Psi}_A(t,u) + 2\,\tilde{\Psi}_V(t,u) + \tilde{X}_A(t,u)\right) \quad (2)$$

2nd identity: $v\cdot D h_v = 0$

$$\frac{d\tilde{\phi}_+(t)}{dt} - \frac{1}{2t}\left(\tilde{\phi}_+(t) - \tilde{\phi}_-(t)\right) + i\bar{\Lambda}\tilde{\phi}_+(t) + 2t\frac{\partial\tilde{\phi}_-(t)}{\partial z^2} = t\int_0^1 du\,(u-1)\left(\tilde{\Psi}_A(t,u) + \tilde{X}_A(t,u)\right) \quad (3)$$

$$\frac{d\tilde{\phi}_+(t)}{dt} - \frac{d\tilde{\phi}_-(t)}{dt} + \left(i\bar{\Lambda} - \frac{1}{t}\right)\left(\tilde{\phi}_+(t) - \tilde{\phi}_-(t)\right) + 2t\left(\frac{\partial\tilde{\phi}_+(t)}{\partial z^2} - \frac{\partial\tilde{\phi}_-(t)}{\partial z^2}\right) = 2t\int_0^1 du\,(u-1)\left(\tilde{\Psi}_A(t,u) + \tilde{Y}_A(t,u)\right) \quad (4)$$

- Exact in QCD in the heavy-quark limit
- By combining eqs.(2) and (3), we can eliminate $\partial\tilde{\phi}_-(t)/\partial z^2$

4 independent equations

$\Longrightarrow$ 2 "good" equations without $\partial\tilde{\phi}_\pm(t)/\partial z^2$

$$\frac{d\tilde{\phi}_-(t)}{dt} - \frac{1}{t}\left[\tilde{\phi}_+(t) - \tilde{\phi}_-(t)\right]$$
$$= 2t \int_0^1 du\, u\, (\tilde{\Psi}_A(t,u) - \tilde{\Psi}_V(t,u))$$

$$\frac{d\tilde{\phi}_+(t)}{dt} + \frac{d\tilde{\phi}_-(t)}{dt} + 2i\bar{\Lambda}\tilde{\phi}_+(t)$$
$$= -2t \int_0^1 du \left[\tilde{\Psi}_A(t,u) + \tilde{X}_A(t,u) + 2u\,\tilde{\Psi}_V(t,u)\right]$$

For going over to the momentum-space:

$$\tilde{\phi}_\pm(v\cdot z) = \int d\omega\; e^{-i\omega v\cdot z}\phi_\pm(\omega)$$

- ωv^+ : "k^+" of $\bar{q}$

3-particle WFs in the momentum space

$\Psi_V(\rho,\xi), \Psi_A(\rho,\xi), X_A(\rho,\xi), Y_A(\rho,\xi)$

$$\tilde{F}(v\cdot z, u) = \int d\rho d\xi\; e^{-i(\rho+\xi u)v\cdot z}F(\rho,\xi),$$
$$(F = \{\Psi_V, \Psi_A, X_A, Y_A\})$$

- ρv^+ : "k^+" of $\bar{q}$
- ξv^+ : "k^+" of g

$$\omega \frac{d\phi_-(\omega)}{d\omega} + \phi_+(\omega) = I(\omega)$$
$$(\omega - 2\bar{\Lambda})\, \phi_+(\omega) + \omega\phi_-(\omega) = J(\omega)$$

$I(\omega)$, $J(\omega)$: "source" terms due to 3-particle WFs

$$I(\omega) = 2\frac{d}{d\omega} \int_0^\omega d\rho \int_{\omega-\rho}^\infty \frac{d\xi}{\xi} \frac{\partial}{\partial \xi} \left[\Psi_A(\rho, \xi) - \Psi_V(\rho, \xi)\right]$$
$$J(\omega) = -2\frac{d}{d\omega} \int_0^\omega d\rho \int_{\omega-\rho}^\infty \frac{d\xi}{\xi} \left[\Psi_A(\rho, \xi) + X_A(\rho, \xi)\right] -4 \int_0^\omega d\rho \int_{\omega-\rho}^\infty \frac{d\xi}{\xi} \frac{\partial \Psi_V(\rho, \xi)}{\partial \xi}$$

Solution:

$$\phi_\pm(\omega) = \phi_\pm^{(WW)}(\omega) + \phi_\pm^{(g)}(\omega)$$

- $\phi_\pm^{(WW)}(\omega)$: solution with $I(\omega) = J(\omega) = 0$ ("Wandzura-Wilczek part")
- $\phi_\pm^{(g)}(\omega)$: induced by the source terms due to three-particle WFs

WW-part:

Explicit analytic solution:

$$\phi_+^{(WW)}(\omega) = \frac{\omega}{2\bar{\Lambda}^2}\theta(2\bar{\Lambda} - \omega)$$

$$\phi_-^{(WW)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\bar{\Lambda}^2}\theta(2\bar{\Lambda} - \omega)$$

three-particle part:

$$\phi_+^{(g)}(\omega) = \frac{\omega}{2\bar{\Lambda}}\Phi(\omega)$$

$$\phi_-^{(g)}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\bar{\Lambda}}\Phi(\omega) + \frac{J(\omega)}{\omega}$$

$$\begin{aligned}\Phi(\omega) &= \theta(2\bar{\Lambda} - \omega)\left\{\int_0^\omega d\rho \frac{K(\rho)}{2\bar{\Lambda} - \rho} - \frac{J(0)}{2\bar{\Lambda}}\right\} \\ &- \theta(\omega - 2\bar{\Lambda})\int_\omega^\infty d\rho \frac{K(\rho)}{2\bar{\Lambda} - \rho} \\ &- \int_\omega^\infty d\rho \left(\frac{K(\rho)}{\rho} + \frac{J(\rho)}{\rho^2}\right)\end{aligned}$$

$$K(\rho) = I(\rho) - \frac{1}{2\bar{\Lambda}}\frac{d}{d\rho}J(\rho)$$

$$\begin{aligned}I(\omega) &= 2\frac{d}{d\omega}\int_0^\omega d\rho \int_{\omega-\rho}^\infty \frac{d\xi}{\xi}\frac{\partial}{\partial\xi}[\Psi_A(\rho,\xi) - \Psi_V(\rho,\xi)] \\ J(\omega) &= -2\frac{d}{d\omega}\int_0^\omega d\rho \int_{\omega-\rho}^\infty \frac{d\xi}{\xi}[\Psi_A(\rho,\xi) + X_A(\rho,\xi)] \\ &- 4\int_0^\omega d\rho \int_{\omega-\rho}^\infty \frac{d\xi}{\xi}\frac{\partial\Psi_V(\rho,\xi)}{\partial\xi}\end{aligned}$$

[[illegible] Λ "distributions"]

Mellin moments $(n = 0, 1, 2, \cdots)$:

$$
\begin{aligned}
\langle \omega^n \rangle_\pm &= \int_0^\infty d\omega\ \omega^n \phi_\pm(\omega) \\
&= \int_0^\infty d\omega\ \omega^n \phi_\pm^{(WW)}(\omega) + \int_0^\infty d\omega\ \omega^n \phi_\pm^{(g)}(\omega) \\
&\equiv \langle \omega^n \rangle_\pm^{(WW)} + \langle \omega^n \rangle_\pm^{(g)}
\end{aligned}
$$

$$
\begin{aligned}
\langle \omega^n \rangle_+^{(WW)} &= \frac{2}{n+2} (2\bar{\Lambda})^n \\
\langle \omega^n \rangle_-^{(WW)} &= \frac{2}{(n+1)(n+2)} (2\bar{\Lambda})^n
\end{aligned}
$$

$$
\begin{aligned}
\langle \omega^n \rangle_+^{(g)} &= \frac{2}{n+2} \sum_{i=1}^{n-1} (2\bar{\Lambda})^{i-1} \sum_{j=1}^{n-i} \binom{n-i}{j} \\
&\times \left(\left\{ (n+1-i) \frac{2j+1}{j+1} + 1 \right\} [\Psi_A]_j^{n-i} \right. \\
&+ (n+2-i)\, [X_A]_j^{n-i} \\
&\left. + (n+3-i) \frac{j}{j+1} [\Psi_V]_j^{n-i} \right) \\
\langle \omega^n \rangle_-^{(g)} &= \frac{1}{n+1} \langle \omega^n \rangle_+^{(g)} \\
&- \frac{2n}{n+1} \sum_{j=1}^{n-1} \binom{n-1}{j} \frac{j}{j+1} \left([\Psi_A]_j^{n-1} - [\Psi_V]_j^{n-1} \right)
\end{aligned}
$$

- $\binom{n}{j} = n!/[j!(n-j)!]$
- the double moments of the three-particle WFs:

$$
[F]_j^i = \int_0^\infty d\rho \int_0^\infty d\xi\ \rho^{i-j} \xi^{j-1} F(\rho, \xi)
$$

$$
(F = \{\Psi_V, \Psi_A, X_A\})
$$

For low moments $(n = 1, 2)$,

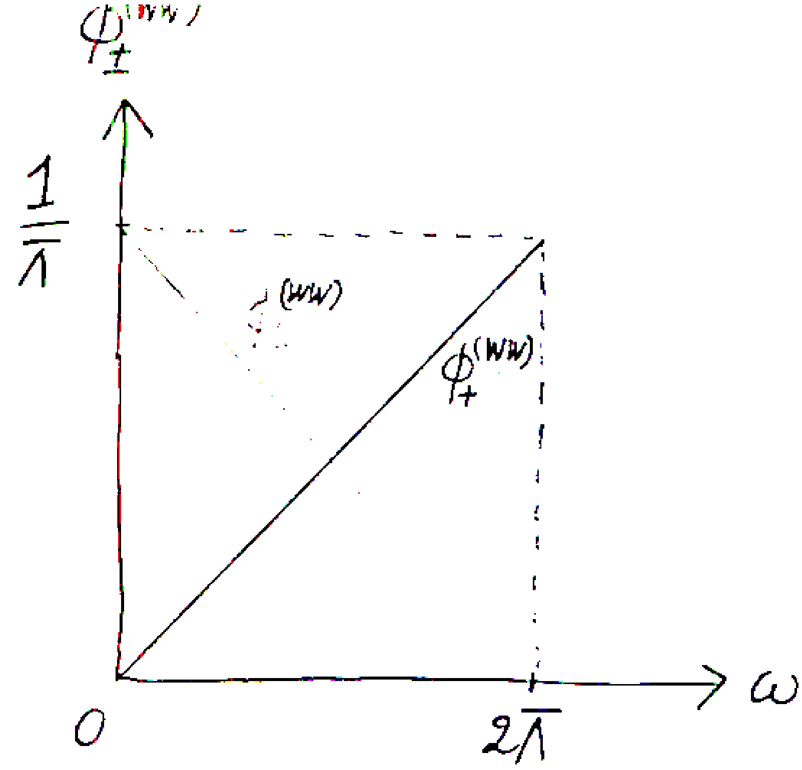
$$\langle \omega \rangle_+ = \frac{4}{3}\bar{\Lambda}$$
$$\langle \omega \rangle_- = \frac{2}{3}\bar{\Lambda}$$

$$\langle \omega^2 \rangle_+ = 2\bar{\Lambda}^2 + \frac{2}{3}\lambda_E^2 + \frac{1}{3}\lambda_H^2$$
$$\langle \omega^2 \rangle_- = \frac{2}{3}\bar{\Lambda}^2 + \frac{1}{3}\lambda_H^2$$

$$[\Psi_A]_1^1 = \frac{1}{3}\lambda_E^2, \quad [\Psi_V]_1^1 = \frac{1}{3}\lambda_H^2, \quad [X_A]_1^1 = 0$$

$$\langle 0|\bar{q} g E \cdot \alpha \gamma_5 h_v|\bar{B}(p=0)\rangle = f_B M \lambda_E^2$$
$$\langle 0|\bar{q} g H \cdot \sigma \gamma_5 h_v|\bar{B}(p=0)\rangle = i f_B M \lambda_H^2$$

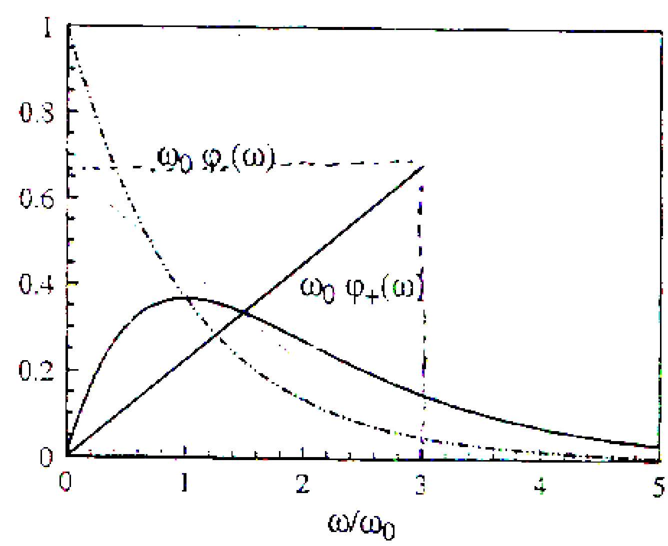
- $n = 1, 2$ results exactly coincide with the relations obtained by Grozin, Neubert ('97)
- $\lambda_E^2/\bar{\Lambda}^2 = 0.36 \pm 0.20$, $\lambda_H^2/\bar{\Lambda}^2 = 0.60 \pm 0.23$ $(\mu \simeq 1\text{GeV})$ by QCD sum rules



◉ consistent with "perturbative wave fn."

$\phi_+ \sim \omega \ , \quad \phi_- \sim 1$

$(\omega \to 0)$



$\omega_c = \frac{2}{3}\bar{\Lambda}$

FIG. 7. Model wave functions $\varphi_\pm(\omega)$ defined in Eq. (4.18).

→ Grozin, Neubert ('97)

(based on QCD SR estimate of $n=1,2$ moments)

[II] Constraints from QCD equations of motion for $z^2 \neq 0$

$$\tilde{\Phi}_\pm(v \cdot z, \; -z_T^2) = \int d\omega d^2k_T \; e^{-i\omega v\cdot z + i k_T \cdot z_T} \Phi_\pm(\omega, k_T)$$

$$z^\mu = (0, z^-, z_T), \qquad z^2 = -z_T^2 \neq 0$$

$$\phi_\pm(\omega) = \int d^2k_T \, \Phi_\pm(\omega, k_T)$$

$\Longrightarrow$ Transverse momentum distribution

- also unknown
- necessary for computing power corrections and for estimating the transition form factors for $B \to D$, $B \to \pi$, etc

$$\omega \frac{\partial \Phi_-}{\partial \omega} + z_T^2 \left(\frac{\partial \Phi_+}{\partial z_T^2} - \frac{\partial \Phi_-}{\partial z_T^2} \right) + \Phi_+ = 0$$

$$\omega \left(\frac{\partial \Phi_+}{\partial \omega} - \frac{\partial \Phi_-}{\partial \omega} \right) - 4 \frac{\partial^2}{\partial \omega^2} \frac{\partial \Phi_+}{\partial z_T^2} + 2 \left(\Phi_+ - \Phi_- \right) = 0$$

$$(\omega - \bar{\Lambda}) \frac{\partial \Phi_+}{\partial \omega} - 2 \frac{\partial^2}{\partial \omega^2} \frac{\partial \Phi_+}{\partial z_T^2} + \frac{1}{2} \left(3\Phi_+ - \Phi_- \right) = 0$$

$$(\omega - \bar{\Lambda}) \left(\frac{\partial \Phi_+}{\partial \omega} - \frac{\partial \Phi_-}{\partial \omega} \right) - 2 \frac{\partial^2}{\partial \omega^2} \left(\frac{\partial \Phi_+}{\partial z_T^2} - \frac{\partial \Phi_-}{\partial z_T^2} \right) + 2 \left(\Phi_+ - \Phi_- \right) = 0$$

- "(ω, z_T)-representation"

$$\tilde{\Phi}_\pm(v \cdot z, -z_T^2) = \int d\omega \; e^{-i\omega v\cdot z} \Phi_\pm(\omega, -z_T^2)$$

- "Wandzura-Wilczek approximation", neglecting three-particle WFs Ψ_V, Ψ_A, X_A, Y_A

Analytic solution

$$\Phi_{\pm}^{(WW)}(\omega, -z_T^2) = \phi_{\pm}^{(WW)}(\omega) J_0\left(|z_T|\sqrt{\omega(2\bar{\Lambda} - \omega)}\right)$$

light-cone WFs for transverse separation z_T

k_T -representation:

$$\Phi_{\pm}^{(WW)}(\omega, -z_T^2) = \int d^2k_T\, e^{ik_T \cdot z_T} \Phi_{\pm}^{(WW)}(\omega, k_T)$$

$$\Phi_{+}^{(WW)}(\omega, k_T) = \frac{\omega}{2\pi\bar{\Lambda}^2}\theta(\omega)\theta(2\bar{\Lambda} - \omega)\delta\left(k_T^2 - \omega(2\bar{\Lambda} - \omega)\right)$$

$$\Phi_{-}^{(WW)}(\omega, k_T) = \frac{2\bar{\Lambda} - \omega}{2\pi\bar{\Lambda}^2}\theta(\omega)\theta(2\bar{\Lambda} - \omega)\delta\left(k_T^2 - \omega(2\bar{\Lambda} - \omega)\right)$$

Transverse momentum distribution for WW-part

- k_T-dependence is completely different from conventional models

  transverse and longitudinal momenta are strongly coupled through the combination $k_T^2/[\omega(2\bar{\Lambda} - \omega)]$

  $\Longleftrightarrow$

  $$\Phi_{mo}(\omega, k_T) = N\omega^2(1-\omega)^2 \exp\left(-\frac{\omega^2}{2\omega_0^2}\right) \times \exp\left(-\frac{k_T^2}{2K^2}\right)$$

  $(\omega_0 = 0.3\text{GeV},\ K = 0.4\text{GeV})$

- slow-damping for large transverse separation z_T

Effect of three-particle contributions

$$\Phi_\pm(\omega, k_T) = \Phi_\pm^{(WW)}(\omega, k_T) + \Phi_\pm^{(g)}(\omega, k_T)$$

1st moment of k_T^2

$$\langle k_T^2 \rangle_\pm \equiv \int d\omega d^2 k_T \; k_T^2 \Phi_\pm(\omega, k_T) = 4 \left. \frac{\partial \tilde{\Phi}_\pm(v\cdot z, z^2)}{\partial z^2} \right|_{z^2\to 0,\; v\cdot z\to 0}$$

From results for $z^2 \to 0$ (eqs. (3) and (4))

$t \equiv v \cdot z$

$$\frac{d\tilde{\phi}_+(t)}{dt} - \frac{1}{2t}\left[\tilde{\phi}_+(t) - \tilde{\phi}_-(t)\right] + i\bar{\Lambda}\tilde{\phi}_+(t) + 2t\frac{\partial\tilde{\phi}_+(t)}{\partial z^2} = t\int_0^1 du\,(u-1)\left(\tilde{\Psi}_A(t,u) + \tilde{X}_A(t,u)\right)$$

$$\frac{d\tilde{\phi}_+(t)}{dt} - \frac{d\tilde{\phi}_-(t)}{dt} + \left(i\bar{\Lambda} - \frac{1}{t}\right)\left[\tilde{\phi}_+(t) - \tilde{\phi}_-(t)\right] + 2t\left(\frac{\partial\tilde{\phi}_+(t)}{\partial z^2} - \frac{\partial\tilde{\phi}_-(t)}{\partial z^2}\right) = 2t\int_0^1 du\,(u-1)\left(\tilde{\Psi}_A(t,u) + \tilde{Y}_A(t,u)\right)$$

$$\langle k_T^2 \rangle_\pm = \frac{2}{3}\left(\bar{\Lambda}^2 + \lambda_E^2 + \lambda_H^2\right)$$

$\lambda_E^2/\bar{\Lambda}^2 = 0.36 \pm 0.20$, $\lambda_H^2/\bar{\Lambda}^2 = 0.60 \pm 0.23$ (QCDSR)

- three-particle contributions might considerably broaden the transverse momentum distribution

Summary for the light-cone WFs

QCD factoization approach for exclusive B meson decays

B meson light-cone WFs for $m_b \to \infty$

- heavy quark symmetry
- QCD (HQET) equations of motion

[I] exact analytic solution for $z^2 \to 0$ (DAs)

- explicit analytic solution for WW-part in terms of $\bar{\Lambda}$
- integral representations for three-particle contributions

Some remarks:

- Both leading twist ϕ_+ and higher-twist ϕ_- receive three-particle contributions. $\phi_\pm = \phi_\pm^{(WW)} + \phi_\pm^{(g)}$
- $\langle \omega^n \rangle_\pm$ by dim$= n+3$ local two- and three-particle operators
- $\phi_+^{(WW)}(\omega) \sim \omega$, $\phi_-^{(WW)}(\omega) \sim$ const. as $\omega \to 0$.
- sharp behavior (discontinuity) at $\omega = 2\bar{\Lambda}$: "distributions"

[II] exact analytic solution for $z^2 \neq 0$ for WW-part

- k_T-dependence of valence Fock WFs
- explicit solution in terms of $\bar{\Lambda}$

Some remarks:

- "non-factorization" of longitudinal and transverse directions
- "slow-damping" for transverse directions
- three-particle contributions for $\langle k_T^2 \rangle_\pm$ broadening

Our solution:

All relevant QCD constraints are satisfied!

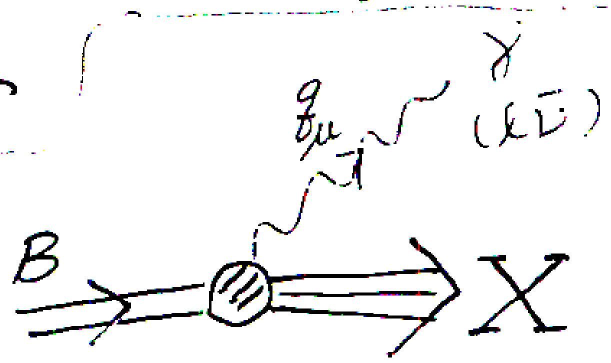
- building up the B meson light-cone WFs

  heavy quark symmetry $\Longrightarrow$ WFs for B, [illegible], D mesons
- phenomenological applications (exclusive B decays, ...)

B meson light-cone structure functions for inclusive decays

$B \to X_s \gamma$

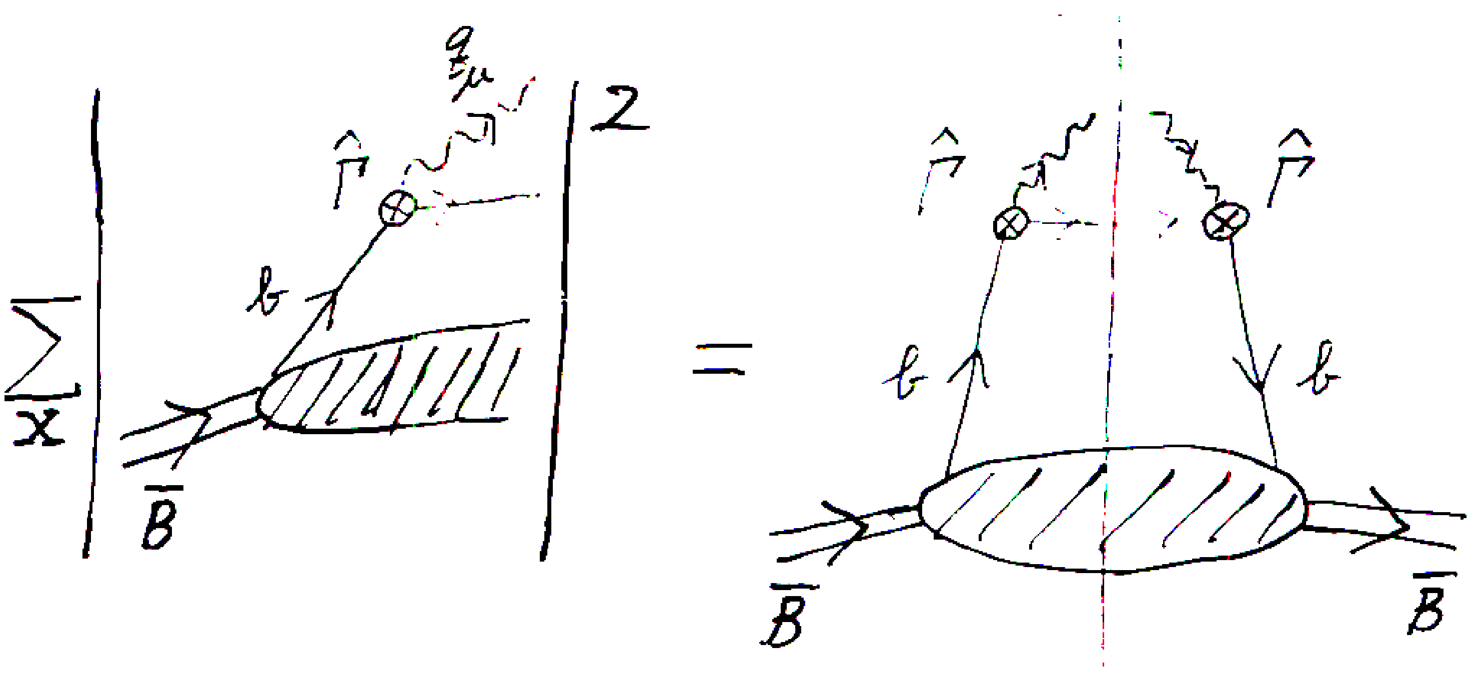
$B \to X_u l \bar{\nu}$



$$d\Gamma \sim \sum_X \left| \langle X \gamma_{(l\bar{\nu})} | H_{eff} | \bar{B} \rangle \right|^2 \delta^{(4)}(p - p_X - q)$$

$$\sim \text{Im}\, T(q^2, p\cdot q)$$

$$T(q^2, p\cdot q) = -i \int d^4x\, e^{-iq\cdot x} \langle \bar{B}(p) | T[\bar{b}(x) \hat{\Gamma} q(x)\ \bar{q}(0) \hat{\Gamma} b(0)] | \bar{B}(p) \rangle$$



light-cone dominance

Universal

$p^\mu = M v^\mu$

$z^2 = 0$

$$\tilde{f}(v\cdot z) = \int d\omega\, e^{i\omega v\cdot z} f(\omega)$$

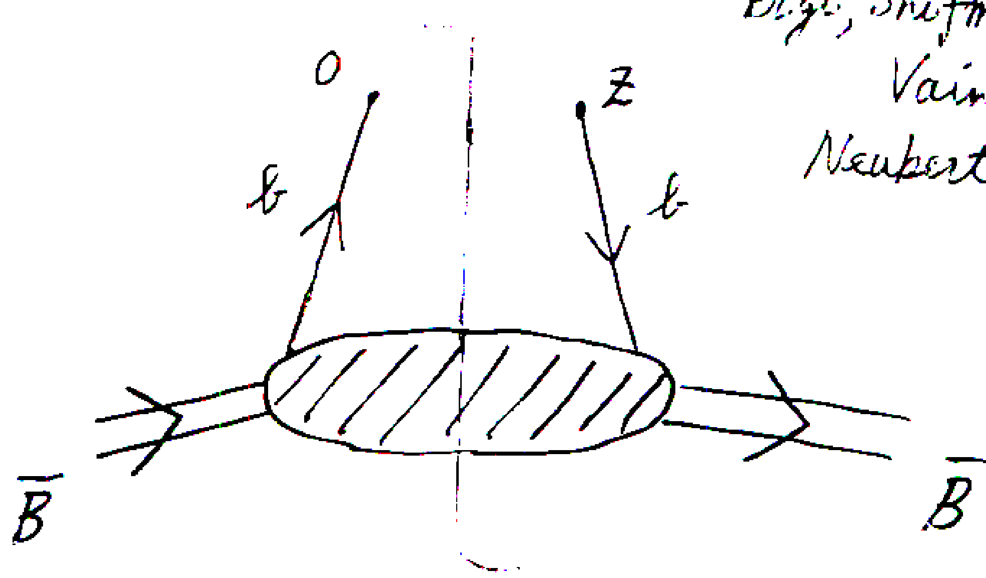
$m_b \to \infty$

$$\tilde{f}(v\cdot z) = \frac{1}{2}\langle \bar{B}(v)|\bar{h}_v(z)\, h_v(0)|\bar{B}(v)\rangle$$

(between $\bar{h}_v(z)$ and $h_v(0)$:) $P\exp\left[ig\int_0^z dx\cdot A(x)\right]$

Bigi, Shifman, Uraltsev, Vainshtein ('94)

Neubert ('94)



$$y \equiv \frac{2p\cdot q}{m_b^2} \sim \frac{2q^0}{m_b}$$

$$d\Gamma_{B\to X_s\gamma} \propto S(y)$$

spectral function

$f(\omega)$

$$d\Gamma_{B\to X_u\ell\bar{\nu}} \propto [F(y)\theta(1-y) + F(1)S(y)]$$

shape function

$f(\omega)$

$f(\omega)$
- universal structure fn.
- also called "shape fn."
- light-cone residual momentum distribution of the heavy quark inside B-meson (ωv^+: k^+ of HQ)

[illegible]

$z \cdot A = z^- A^+ = 0$ gauge

$$\bar{h}_v(z) = \bar{h}_v(0) \sum_{j=0}^{\infty} \frac{1}{j!} (z^- \overleftarrow{D}^+)^j$$

$$f(\omega) = \frac{v^+}{2} \int \frac{dz^-}{2\pi} e^{-i\omega v^+ z^-} \langle \bar{B}(v) | \bar{h}_v(0) \underbrace{\sum_{j=0}^{\infty} \frac{1}{j!} (-z^- \overrightarrow{D}^+)^j}_{= e^{i(iz^- D^+)}} h_v(0) | \bar{B}(v) \rangle$$

$$= \frac{v^+}{2} \langle \bar{B}(v) | \bar{h}_v(0) \delta(iD^+ - \omega v^+) h_v(0) | \bar{B}(v) \rangle$$

We need full "functional form" of $f(\omega)$ to describe decay spectra for $y \sim 1$

Support property of $f(\omega)$

structure fn. in full QCD $\qquad p^\mu = M v^\mu$

$$\frac{1}{2}\langle \bar{B}(p)|\bar{b}(z)\gamma^\mu b(0)|\bar{B}(p)\rangle = p^\mu \tilde{f}_0(p\cdot z) + O(\text{twist}4)$$

$$\Downarrow \; m_b \to \infty$$

$$\frac{1}{2} M e^{i m_b v\cdot z}\langle \bar{B}(v)|\bar{h}_v(z)\gamma^\mu h_v(0)|\bar{B}(v)\rangle = M v^\mu e^{i m_b v\cdot z}\tilde{f}(v\cdot z)$$

$$\begin{cases} b(z) = e^{-i m_b v\cdot z} h_v(z) + O(1/m_b) \\ |\bar{B}(p)\rangle = \sqrt{M}\,|\bar{B}(v)\rangle + O(1/m_b) \end{cases}$$

$$\langle \bar{B}(p)|\bar{B}(p')\rangle = 2p^0(2\pi)^3\delta^{(3)}(\vec{p}-\vec{p}')$$

$m_b \to \infty$:

$$\tilde{f}_0(p\cdot z) = e^{i m_b v\cdot z}\tilde{f}(v\cdot z)$$

$$\tilde{f}_0(p\cdot z) = \int dx\, e^{i x p\cdot z} f_0(x), \qquad \tilde{f}(v\cdot z) = \int d\omega\, e^{i\omega v\cdot z} f(\omega)$$

$$f(\omega) = \int dx\, \delta(\omega - [Mx - m_b])\, f_0(x)$$

$f_0(x) \geq 0 \qquad 0 \leq x \leq 1$

$$\Rightarrow \boxed{f(\omega) \geq 0 \qquad -\infty < \omega \leq \bar{\Lambda}}$$

$(-m_b \leq \omega \leq M - m_b)$

$$f_0(x) = \delta(x-1)$$

$$f(\omega) = \int dx\, \delta(\omega - [Mx - m_b])\, \delta(x-1)$$

$$= \delta(\omega - \bar{\Lambda})$$

$$= \delta(\omega)$$

A "realistic" model

one parameter model by Mannel-Neubert:

$$f^{MN}(\xi) = \frac{32}{\pi^2 \bar\Lambda}\left(1-\frac{\xi}{\bar\Lambda}\right)^2 \exp\left[-\frac{4}{\pi}\left(1-\frac{\xi}{\bar\Lambda}\right)^2\right]\theta\left(1-\frac{\xi}{\bar\Lambda}\right)$$

$$\int d\xi\, f^{MN}(\xi) = 1\,, \qquad \int d\xi\, \xi\, f^{MN}(\xi) = 0$$

$$\int d\xi\, \xi^2 f^{MN}(\xi) = \left(\frac{3\pi}{8}-1\right)\bar\Lambda^2 \simeq (0.42\bar\Lambda)^2$$

$$\int d\xi\, \xi^3 f^{MN}(\xi) = -\left(2-\frac{5\pi}{8}\right)\bar\Lambda^3 \simeq -(0.33\bar\Lambda)^3$$

$\Rightarrow \bar\Lambda = 0.57$ GeV ($m_b \simeq 4.71$ GeV)

	M-N model	SR	lattice
λ_1	-0.17 GeV^2	$(-0.25$ GeV$^2)$ -0.6 GeV^2	-0.09 GeV^2 -0.45 GeV^2
$A_3 = -\frac{a_3}{12}$	$-(190$ MeV$)^3$	$-(270$ MeV$)^3$	