EARTHQUAKE RESEARCH INSTITUTE UNIVERSITY OF TOKYO

スロー地震学
Science of Slow Earthquakes

東京大学
THE UNIVERSITY OF TOKYO

Fracture of random media: mechanical instability and enhanced susceptibility

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Fracture of random media involves power law

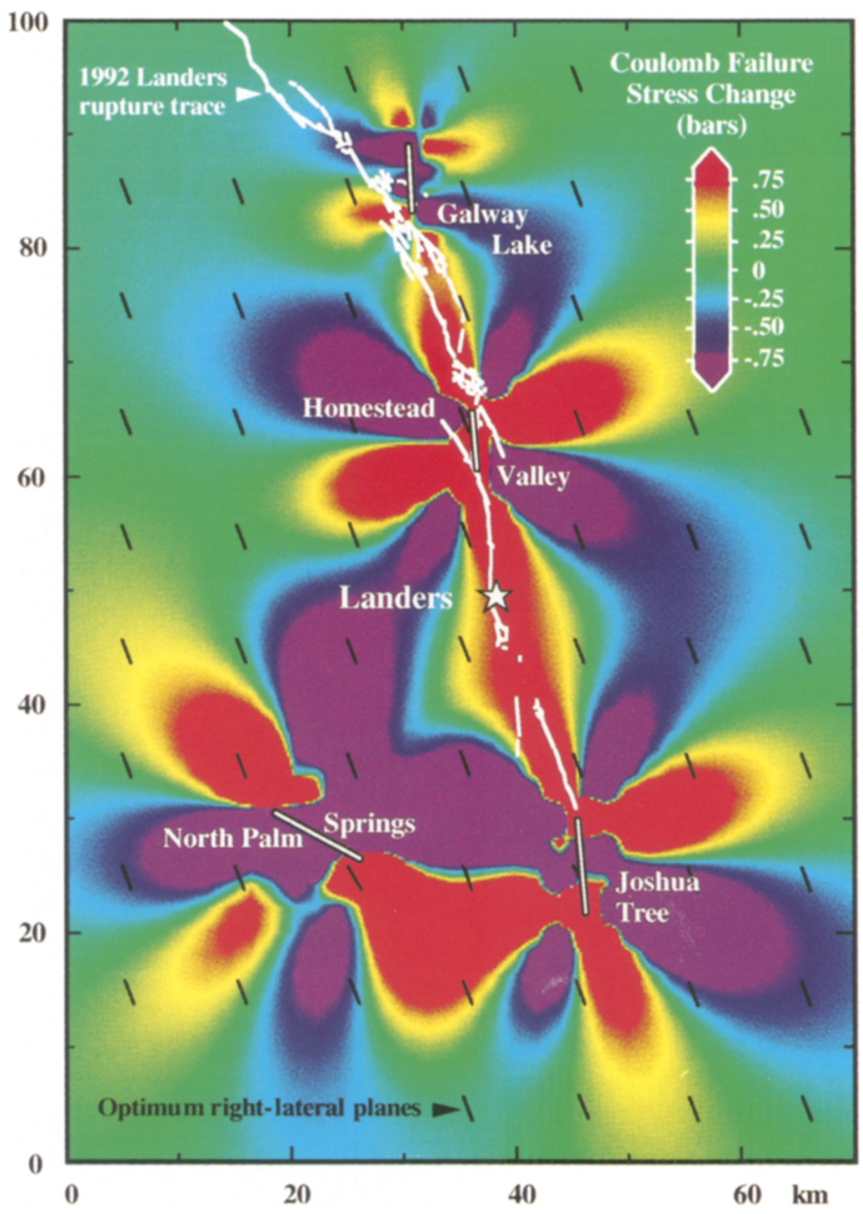
e.g. event-size distribution

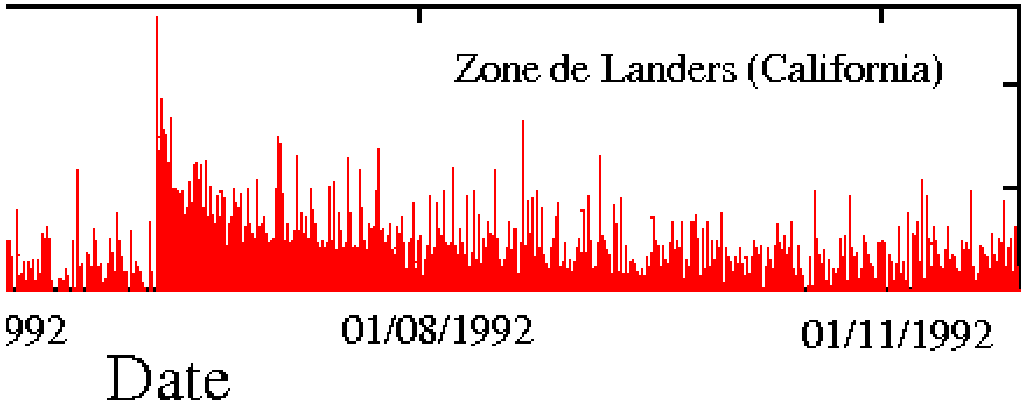
this talk = power law in the time domain

two example: earthquakes and creep

earthquakes: aftershocks

an earthquake triggers more earthquakes





due to abrupt stress change
caused by the “mainshock”

inferred stress change by Landers earthquake

(King et al. 1994)

Omori law for aftershocks

number of aftershocks per unit time at time t

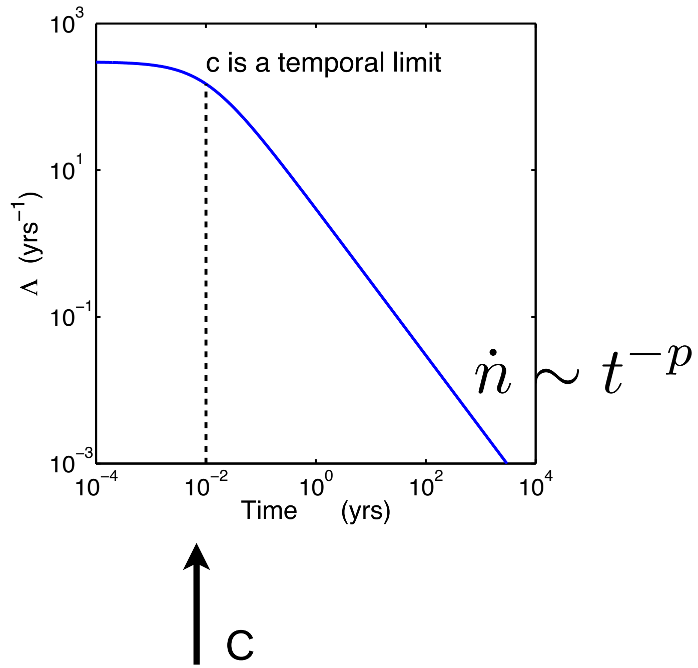
(time elapsed from mainshock)

$$\dot{n} \propto (t+c)^{-p}$$

p = 0.8 to 1.5

c ~ O(1) to O(100) min.

some relaxation process after the abrupt change of stresses(?)



time constant c may depend on stress?

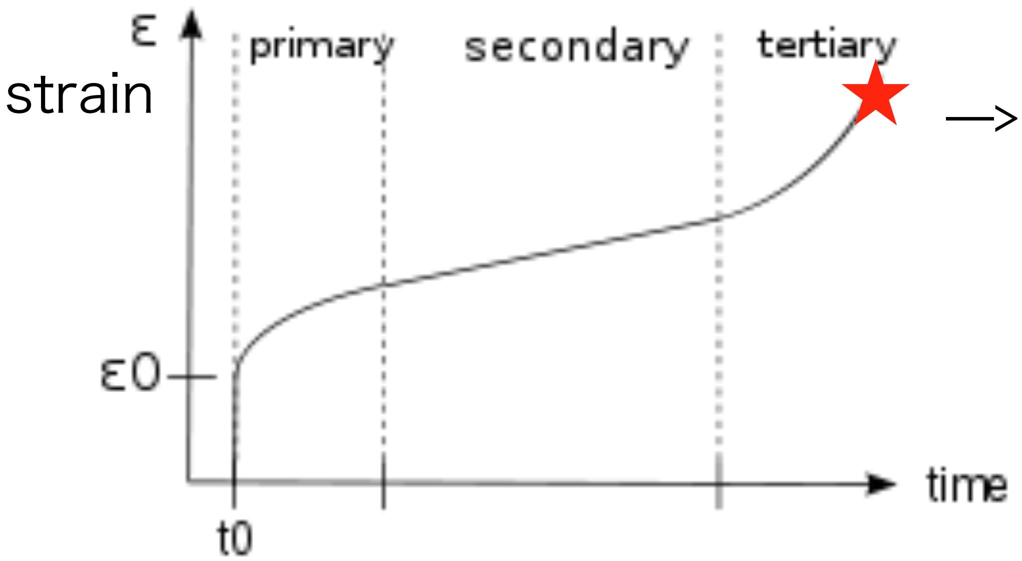
stress increases —> c decreases

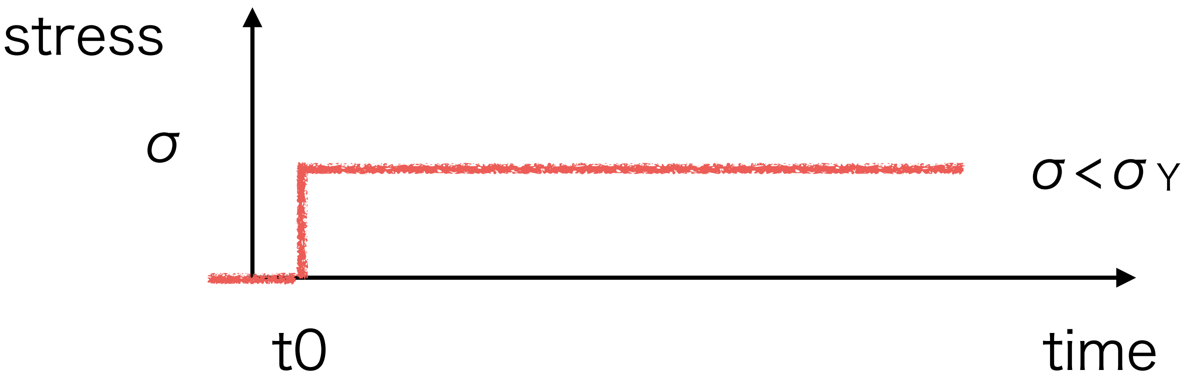
(Narteau et al. 2009; TH 2015)

If so, one can infer the stress level in the crust from the c-value!

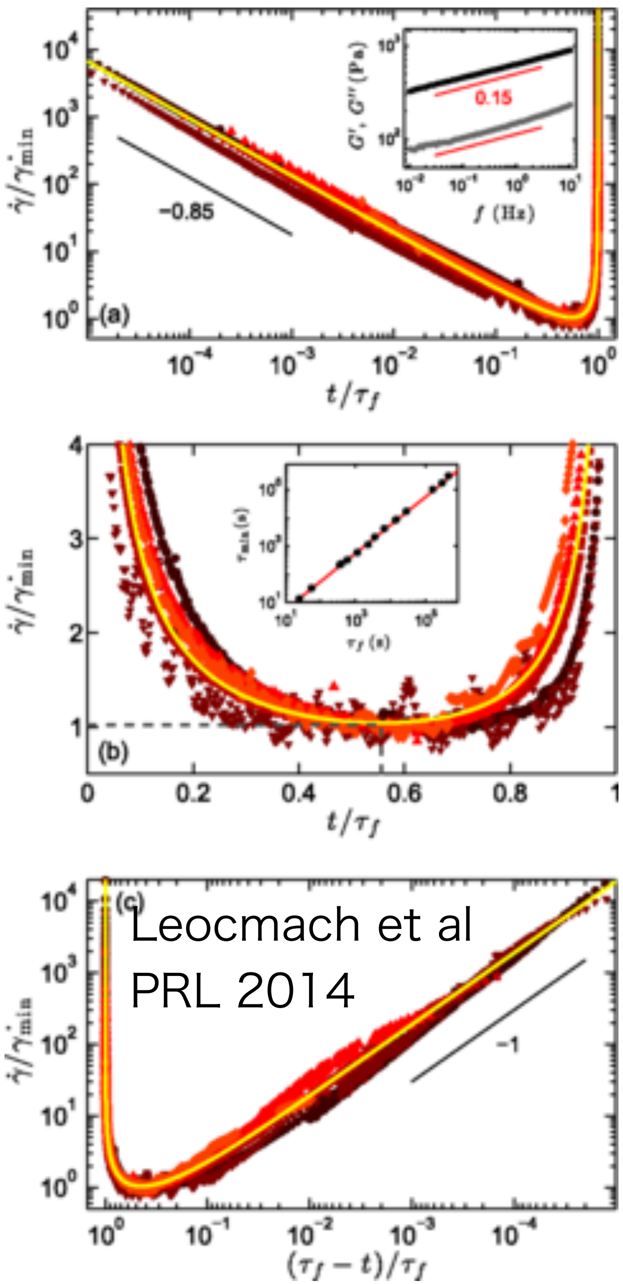
creep is the tendency of a solid material to move slowly or deform permanently under the influence of mechanical stresses. It can occur as a result of <u>long-term exposure to high levels of stress that are still below the yield strength of the material</u>.

(Wikipedia)





strain rate vs time: power laws



Leocmach et al PRL 2014

primary

$$\dot{\epsilon} \propto t^{-\alpha}$$

"Andrade creep"

α=0.6 to 1.0

secondary

almost constant strain rate
(but very slow)

tertiary

$$\dot{\epsilon} \propto (t_f - t)^{-\alpha'}$$

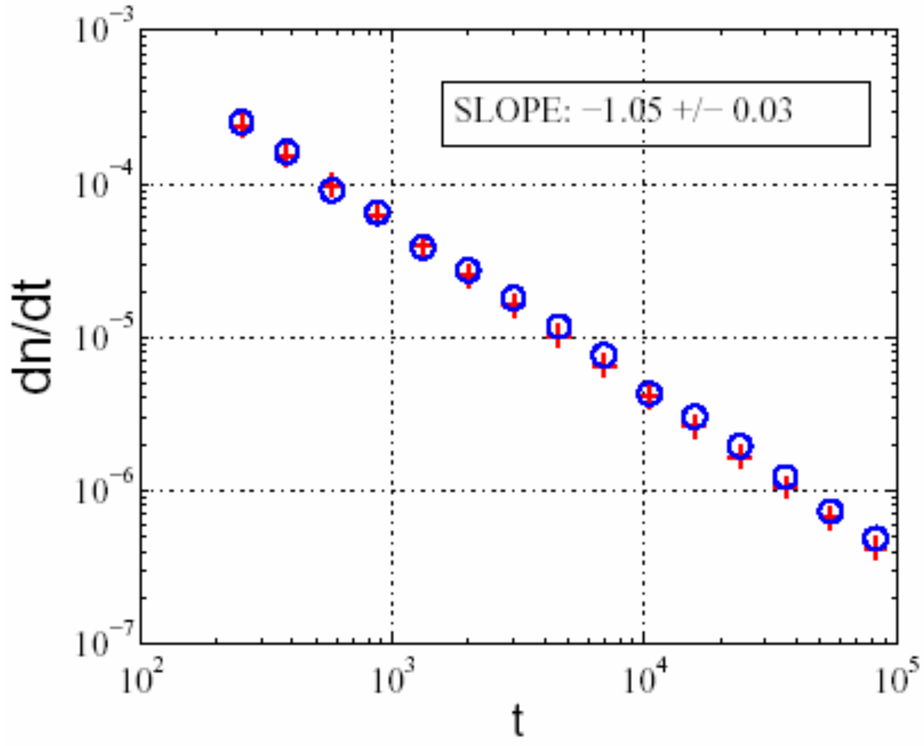
t_f = time of breakdown

α' is not necessarily identical to α

α'=0.6 to 1.0

micro-fracture event rate vs time in creep

creep test of a glass cylinder (Mallick et al. 2009)



$\dot{n} \propto t^{-\alpha}$ for fracture event rate

—> essentially the same with Omori law!

our problem here

creep: thermal activation process is essential

earthquake: athermal (?? always some time delay)

very different details but same empirical law $\dot{n} \propto (t+c)^{-p}$

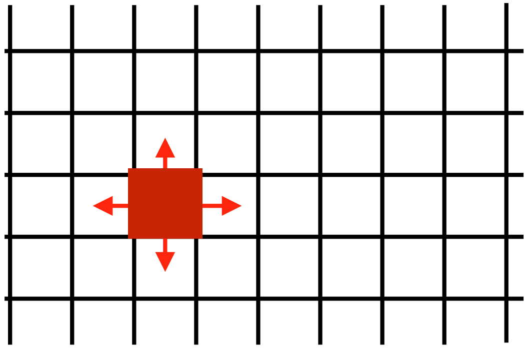
Time constant c is important in earthquakes but not investigated in creep tests

1. what determines the c-value?

(may give some hint to earthquakes)

2. A range of exponent is obtained for p. What causes this variation?
(0.6 - 1.0)

Our approach: toy models (Self-Organized Criticality)



internal variable = f_{ij} (interpreted as stress)

each site has its own fracture strength $f_{ij}^{(c)}$

(randomness)

if $f_{ij} > f_{ij}^{(c)}$ —> rupture —> redistribution of internal variable f_{ij}

$f_{ij} = 0$ (no longer support load)

Redistribution rule:

e.g. short range (nearest neighbors), long range (power-law), etc…

$$F = \sum_{i,j} f_{ij}$$ external load

kept constant —> more unstable if some elements are killed

time evolution:

1. Scan each element: if stress is above its threshold, it is broken.

2. Broken fibers redistribute their load

Fracture of a fiber and redistribution of stress take some time
—> time scale —> a single time step

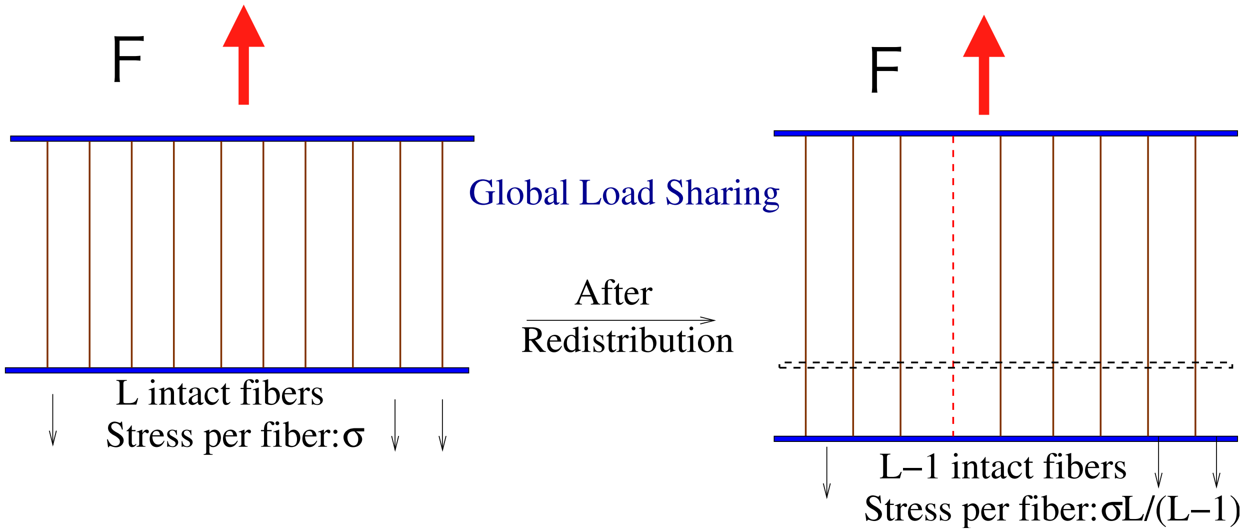
1. Mean-field model

2. Nearest-neighbor model

3. effect of thermal activation process

mean-field model = stretching bundle of brittle fibers

Fibers are coupled in parallel



mean field model: force is equally redistributed to all the sites

f_{ij} is uniform : $f_{ij} = f = F/L$

#fibers	L fibers —> (L-1) fibers
force per fiber	F / L —> F / (L-1)

time evolution equation

(remaining fibers) = (initial) - (broken ones)

$$L_i = L_0 - \int_0^{f_{i-1}} L_0 p(y) dy$$

p(x) = probability distribution of failure threshold

$$n_i = 1 - \int_0^{f_{i-1}} p(y) dy.$$

$$n_i = L_i / L_0 = f_0 / f_i$$

$$\longrightarrow \qquad f_i = \frac{f_0}{1 - \int_0^{f_{i-1}} p(y) dy} = \frac{f_0}{\int_{f_{i-1}}^{\infty} p(y) dy}$$

recursive relation

τ : time needed for load redistribution and failure

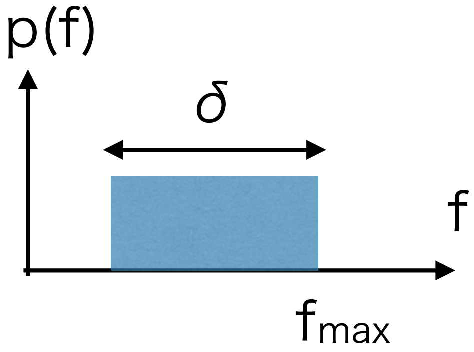
$$(f_{i+1} - f_i)/\tau \simeq \dot{f}$$

$$f_i = \frac{f_0}{1 - \int_0^{f_{i-1}} p(y)dy} \longrightarrow \tau \dot{f} = \frac{f_0}{1 - \int_0^{f} p(x)dx} - f$$

obtain mean-field time evolution equation for f(t)

(force per fiber)

the case of uniform threshold distribution



$$p(f) = 1/\delta \qquad \text{if} \quad f \in [f_{\max} - \delta, f_{\max}]$$

$$= 0 \qquad \text{otherwise}$$

$$x \equiv f/f_{\max}$$

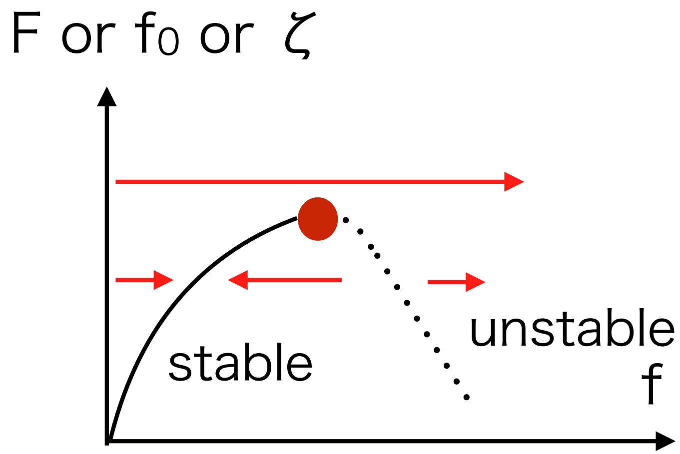
$$(2) \longrightarrow \quad \tau \dot{x} = \frac{\left(x - \frac{1}{2}\right)^2 + \zeta}{1 - x}$$

$$\zeta := \frac{f_0 \delta}{f_{\max}^2} - \frac{1}{4}$$

$\zeta < 0$ —> two fixed points

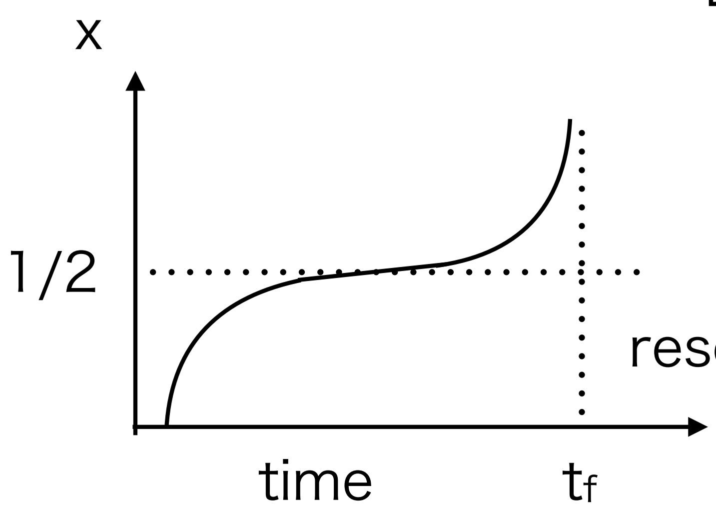
$\zeta = 0$ —> saddle-node bifurcation

$\zeta > 0$ —> runaway (breakdown)



can obtain exact solution for $\zeta > 0$

$$t_m - t = \frac{1}{2}\log\left[\left(\frac{1}{2}-x\right)^2+\zeta\right]+\frac{1}{2\sqrt{\zeta}}\tan^{-1}\left(\frac{1/2-x}{\sqrt{\zeta}}\right)$$



tm : constant

resembles strain-time relation in creep experiments

ζ<<1 —> $x \simeq \frac{1}{2}+\sqrt{\zeta}\tan[2\sqrt{\zeta}(t-t_m)]$

derivation of creep laws

$$x \simeq \tfrac{1}{2} + \sqrt{\zeta}\tan[2\sqrt{\zeta}(t - t_m)]$$

primary creep

$$t \simeq 0 \quad \longrightarrow \quad x \simeq \frac{1}{2} - \frac{1}{2t + c(x_0)}$$

$$\longrightarrow \quad \dot{x} \simeq [t + c(x_0)]^{-2}$$

$$c(x_0) \simeq \frac{1}{1/2 - x_0}$$

tertiary creep

$$\sqrt{\epsilon}(2t_f - a) \simeq \pi/2$$

expansion around t_f

$$\longrightarrow \quad \dot{x} \simeq (t_f - t)^{-2}$$

case of general threshold distribution

$$f_{i+1} = \Phi(f_i) \qquad \Phi(f) \equiv \frac{f_0}{1 - \int_0^f p(x)dx} = \frac{f_0}{\int_f^\infty p(x)dx}$$

$\Phi(0) = f_0$

$\Phi(f)$ is a monotonically increasing function

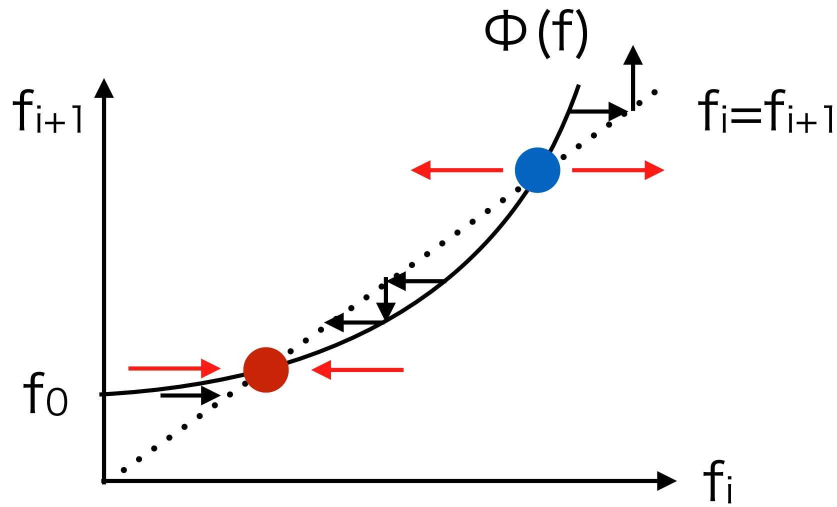
—> the sign of $\partial^2\Phi/\partial f^2$ is crucial.

<u>if $\partial^2\Phi/\partial f^2 > 0$ (concave case)</u>

1. For sufficiently small f_0

—> two fixed points

(stable & unstable)

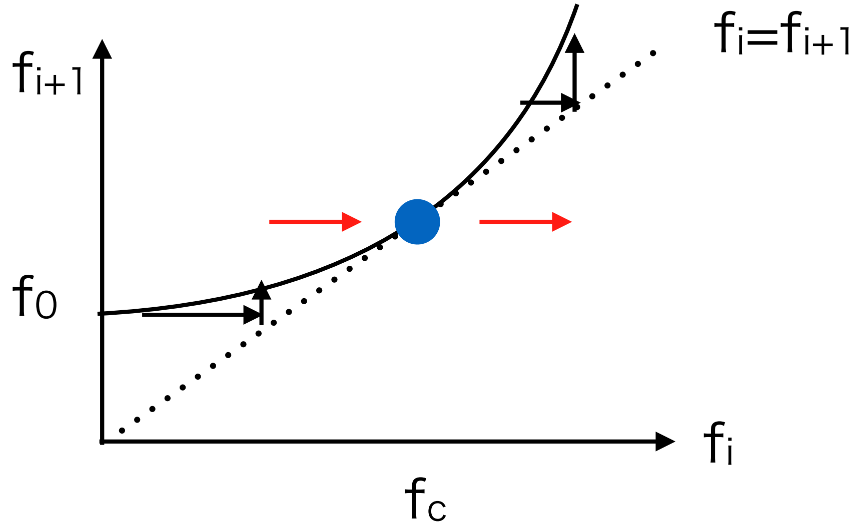


concave case ($\partial^2\Phi/\partial f^2 > 0$)

2. At critical f_0

two fixed points merge at f_c

—> saddle-node bifurcation



3. If f_0 only slightly larger than the critical value

$$\Phi(f) \simeq \epsilon + f_c + (f - f_c) + a(f - f_c)^2 + \cdots$$

$$\tau \dot{f} = \Phi(f) - f \simeq \epsilon + a(f - f_c)^2.$$

condition to neglect third order term?

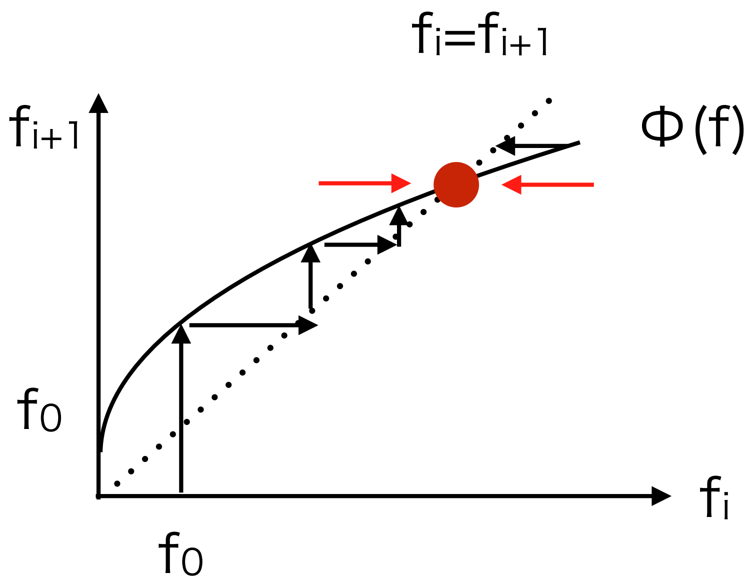
$$\longrightarrow \quad \dot{f} = \frac{\tau/a}{[t + \tau/a(f_c - f_0)]^2}$$

obtain Omori law

case of $\partial^2\Phi/\partial f^2 < 0$

only one fixed point

(irrespective of f_0)



No breakdown —> desirable materials?

what kind of p(x) satisfies this condition?

—> it is concave most plausible distributions

the c-value in Omori law

we obtained $\dot{x} \simeq [t + c(x_0)]^{-2}$ for primary creep

so dx/dt does not diverge at t=0

$c(x_0)$ defines a characteristic time for creep

$$c(f_0)/\tau \simeq \frac{1}{1/2 - f_0/f_{\max}}$$

increasing function of initial force

cf. opposite to granular avalanche / earthquakes

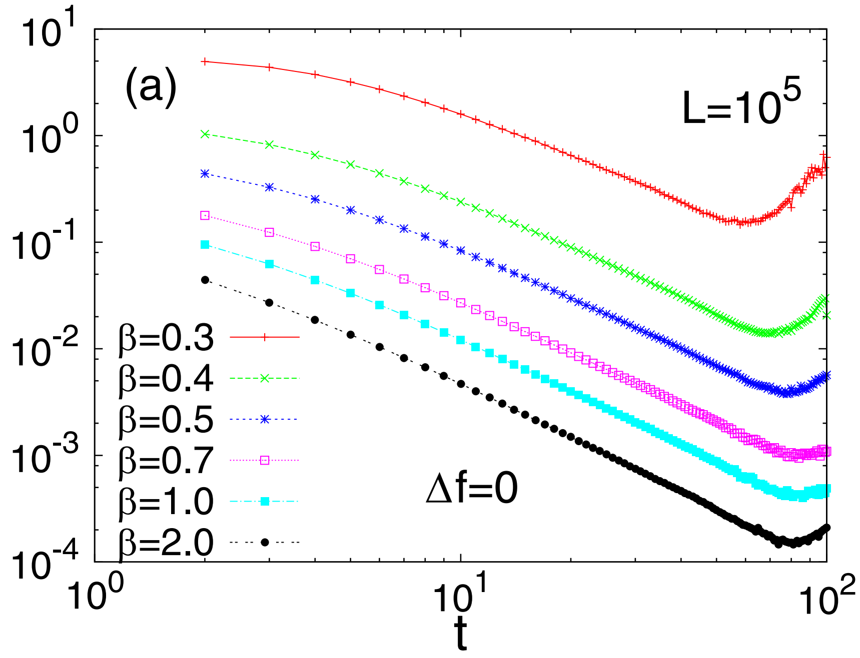
decreasing function of fmax

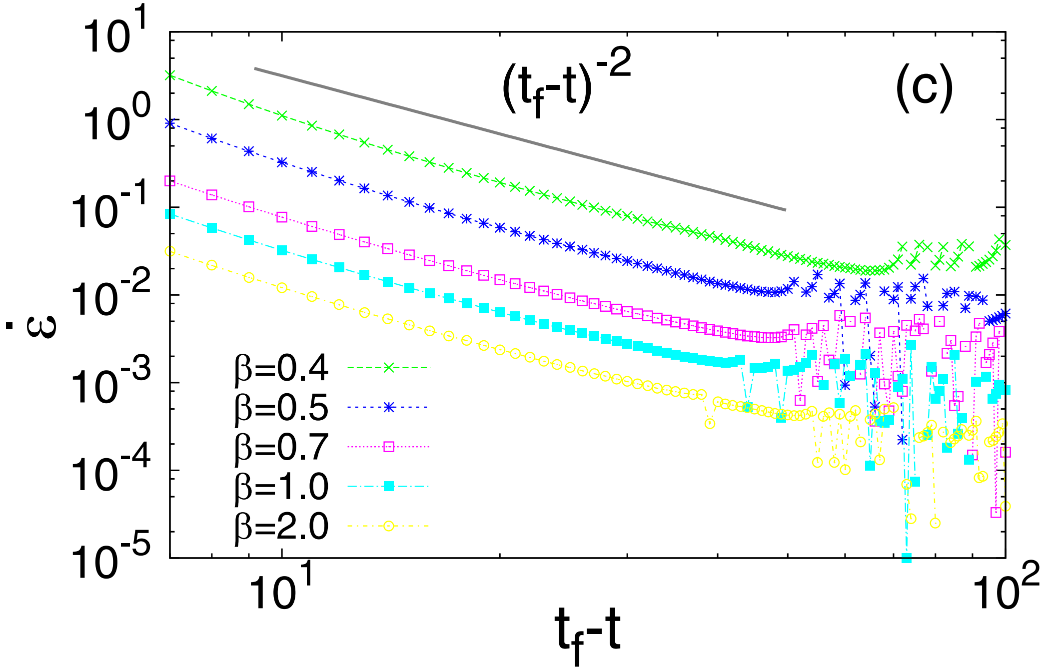
more disorder —> smaller c-value

check other types of distribution

Weibull distribution

$$p(x) \propto x^{\beta-1} \exp(-x^{\beta}) \longrightarrow \partial^2\Phi/\partial f^2 > 0$$



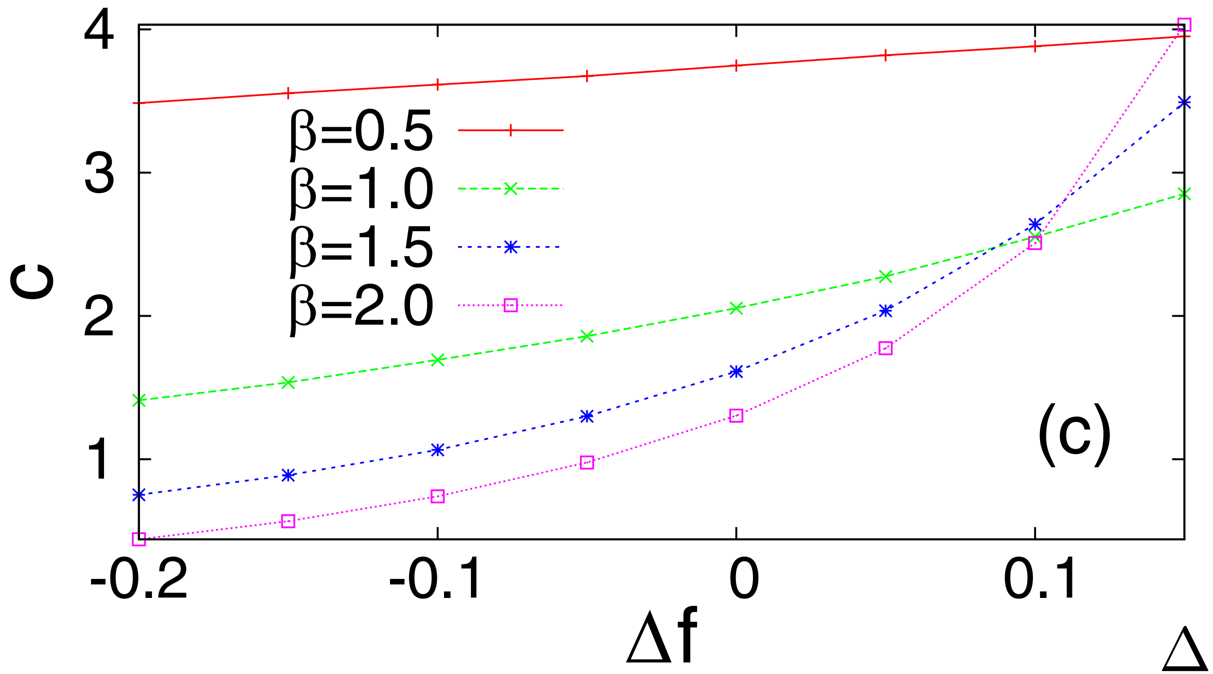


$$\dot{x} \simeq [t + c(x_0)]^{-2}$$

$$\dot{x} \simeq (t_f - t)^{-2}$$

should be universal as long as $\partial^2\Phi/\partial f^2 > 0$

stress dependence of c-value (Weibull distribution case)



$$\Delta f = f_0 - f_c$$

total load increases —> c-value increases

β increases —> c-value decreases

(more variance in strength distribution)

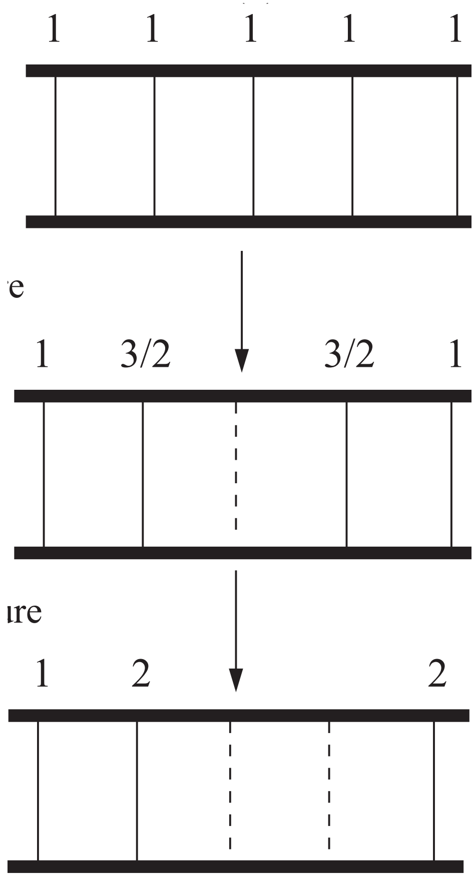
more disorder —> small c-value

(same as uniform distribution case)

case of nearest-neighbor interaction

the load is redistributed within +- R nearest-neighbors

R=1 case



R > Rc —> mean-field

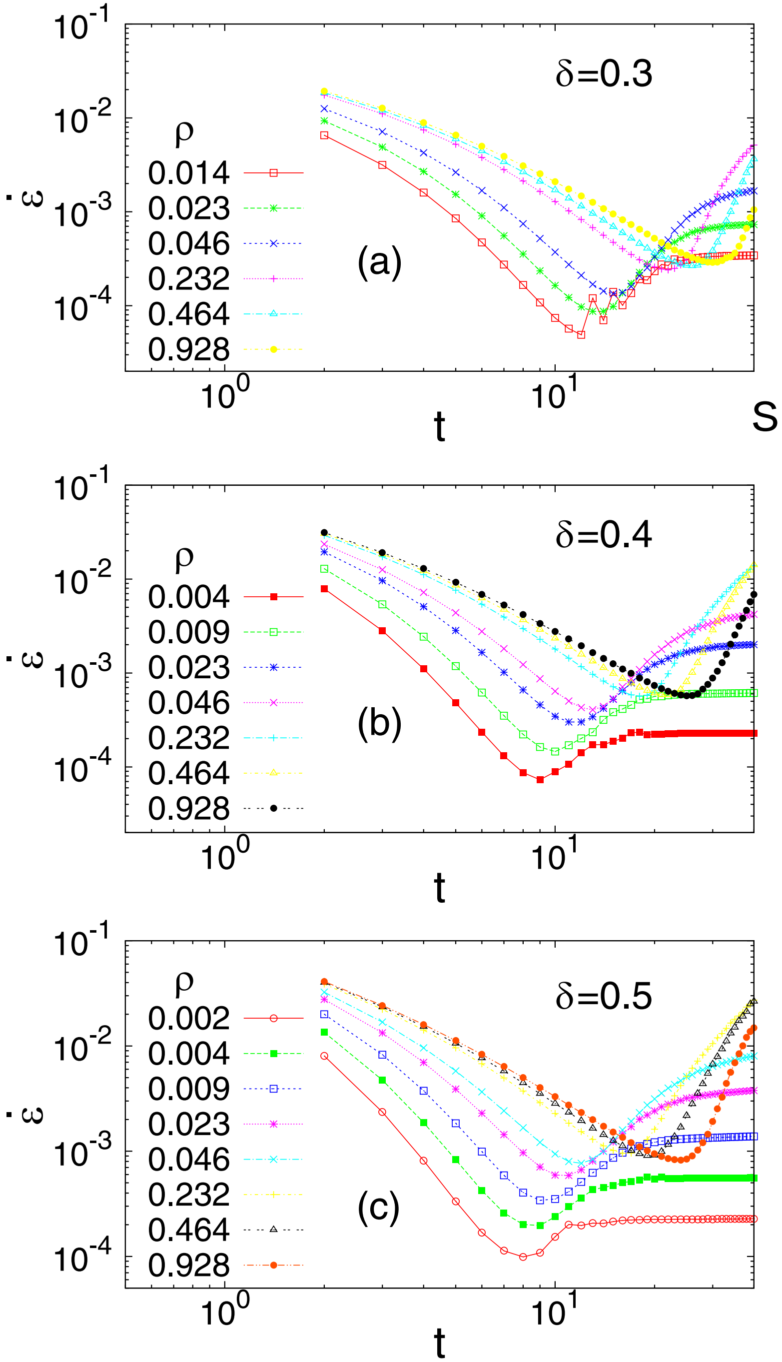
Rc ~ $L^{3/2}$ (Biswas et al. PRE 2015)

Here R < Rc is investigated

parameter for interaction range

$$\rho = R/R_c$$

primary creep for nearest-neighbor model

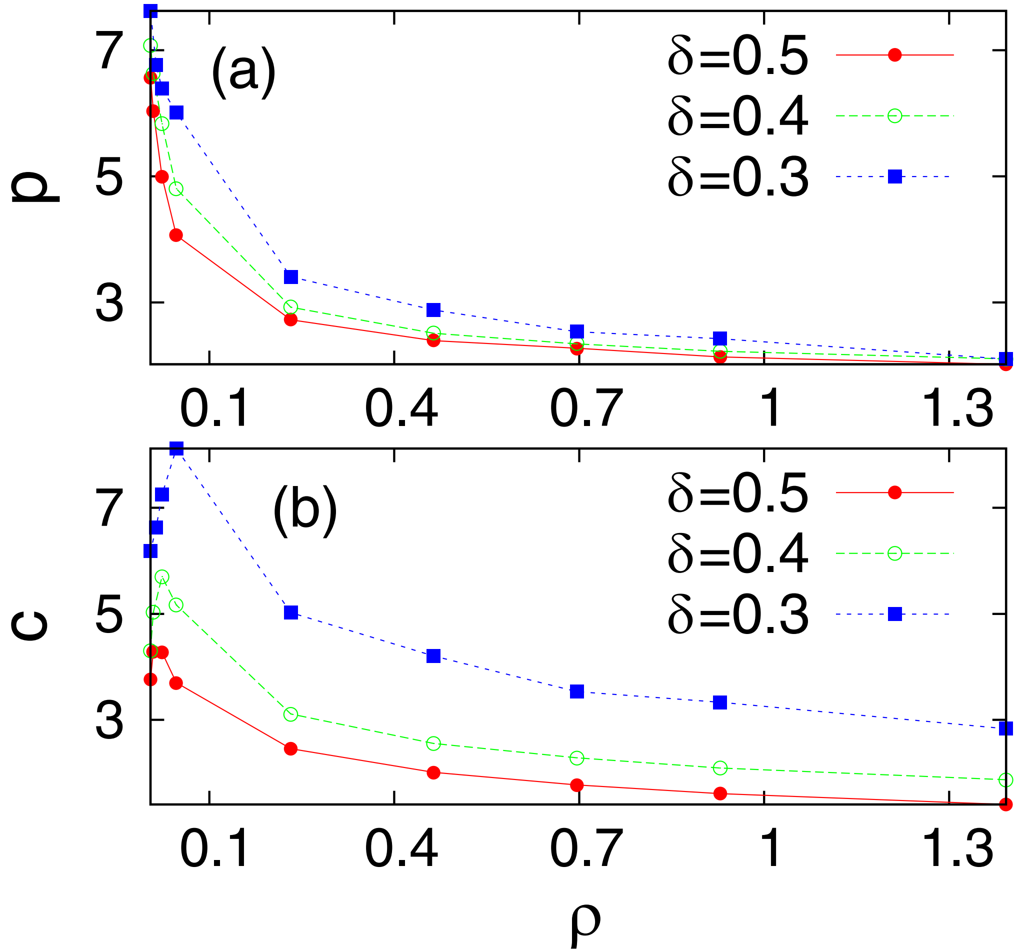


even larger exponent! (p>2)

(cannot regarded as power law)

shorter interaction range —> large exponent

effect of disorder



larger interaction range

more disorder

—> smaller exponent

& smaller c-value

partial summary: athermal case

1. Mean-field fiber bundle model reproduces three stages of creep

power-law slow dynamics is due to saddle-node bifurcation

large exponent: 2

2 Nearest-neighbor models exhibit even larger exponent, $p>2$.

3 The c-value is increasing function of the total load

but decreases with the degree of disorder in strength

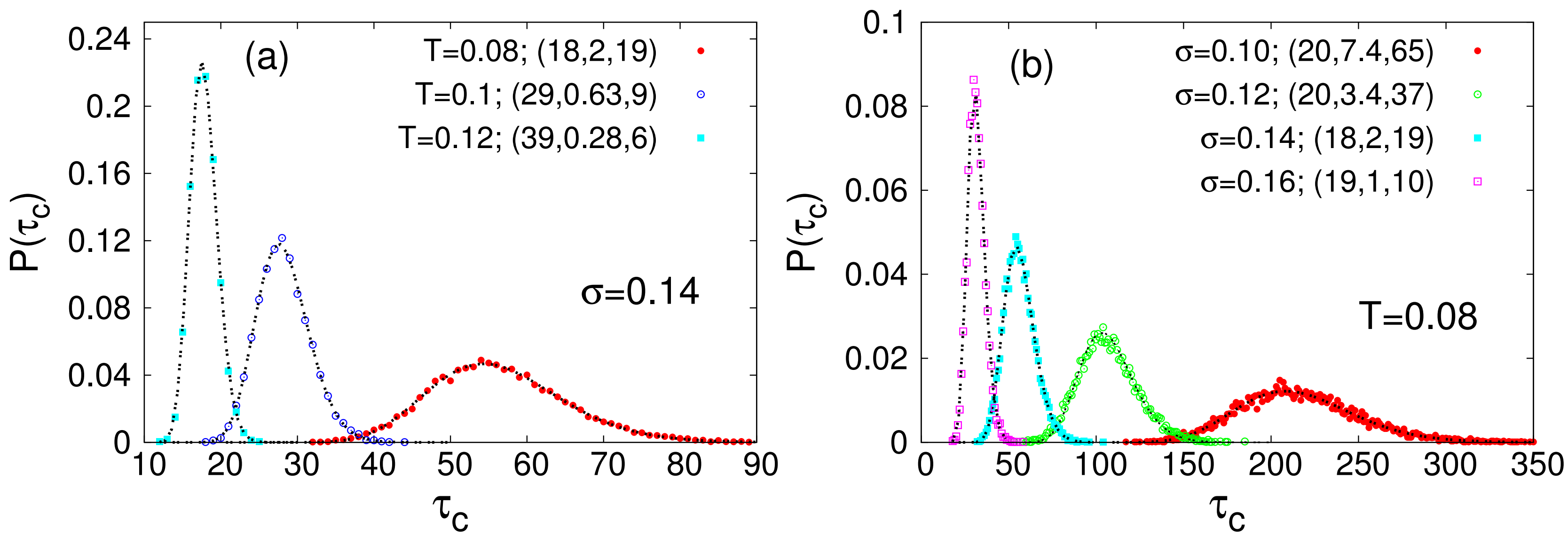
ongoing work: thermal case (mean-field)

rupture probability (rate) of fiber i:

$$P_r(t,i) = P' exp\left[-\frac{\sigma_{th}(i) - \sigma(i)}{T}\right]$$

—> time-evolution is stochastic

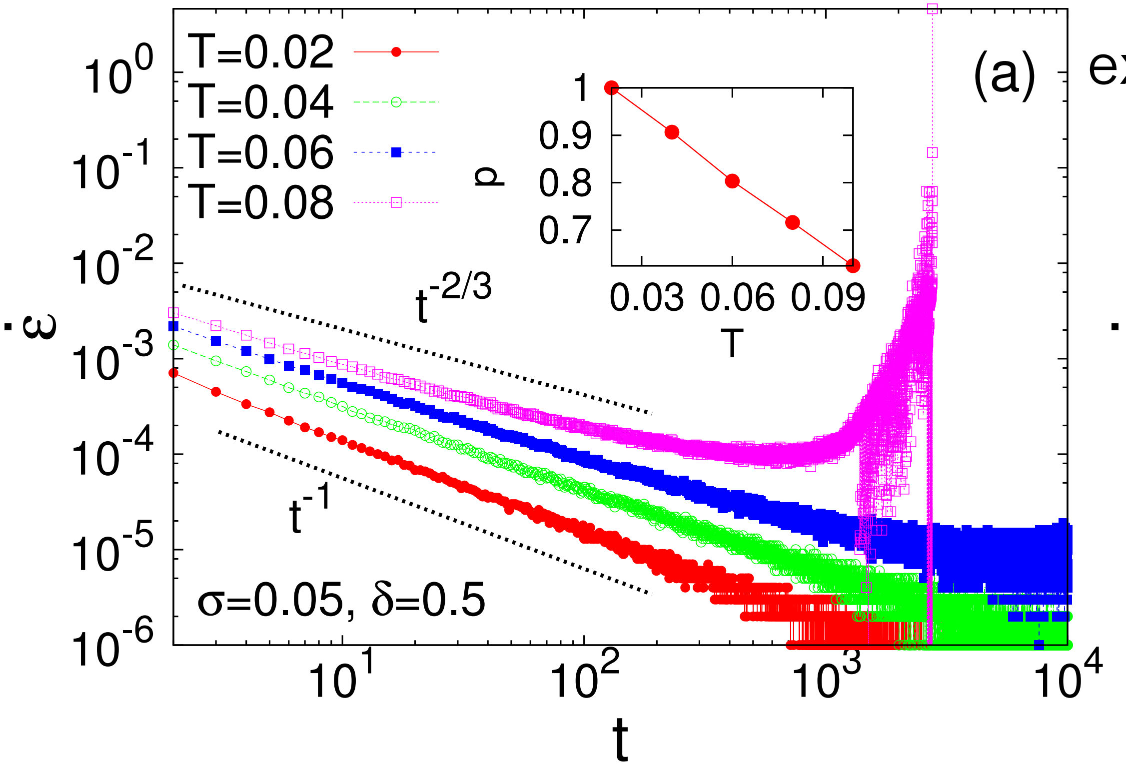
probability distribution for time of failure



—> fitted with Gamma distribution (theory?)

ongoing work: thermal case (mean-field)

primary creep



exponent depends on temperature

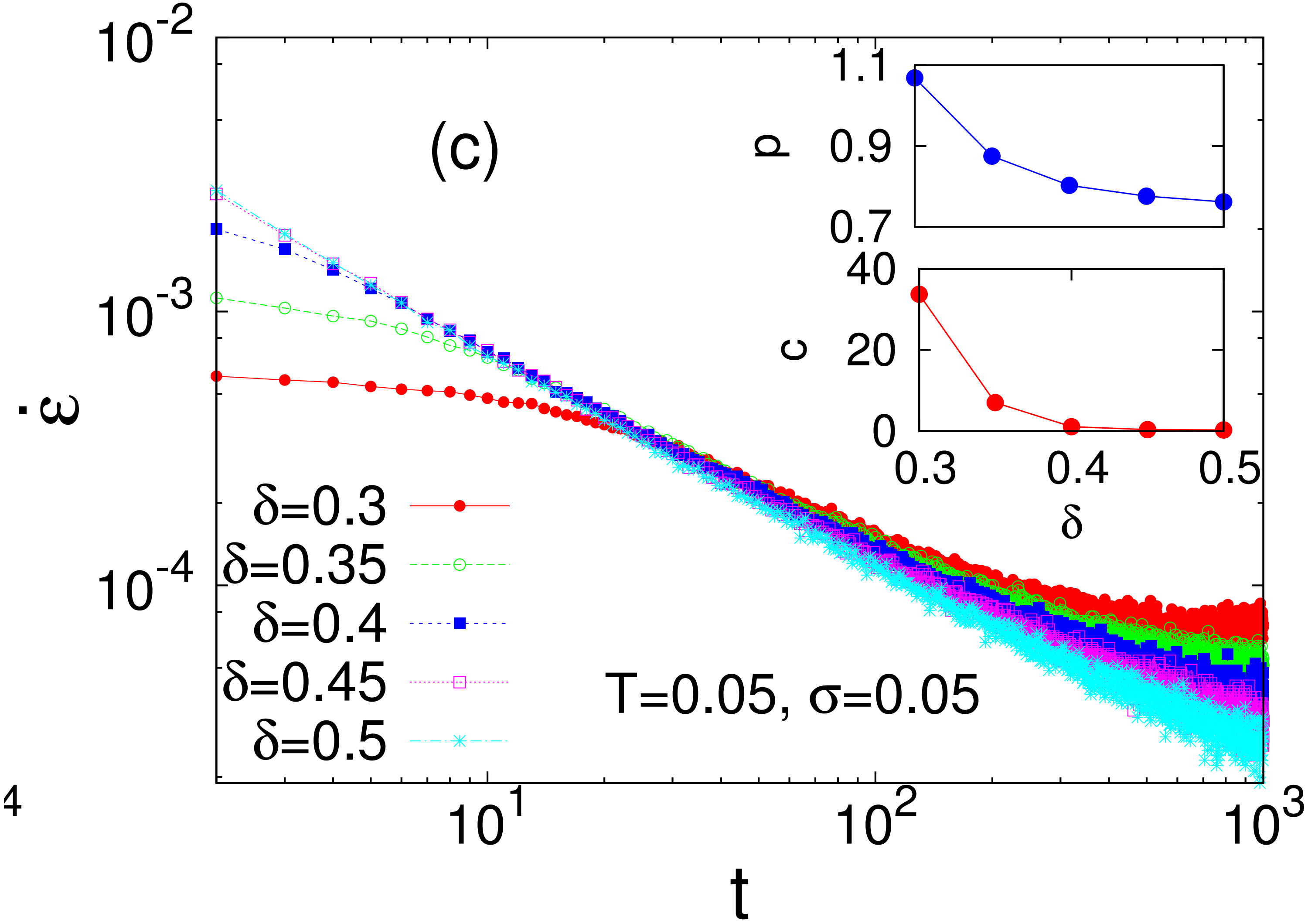
lower T —> larger exponent

(why?)

however, we cannot find inverse Omori law for tertiary creep

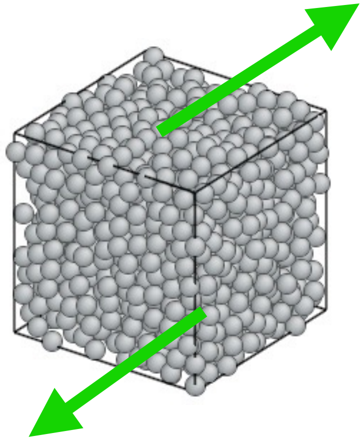
ongoing work: thermal case (mean-field)

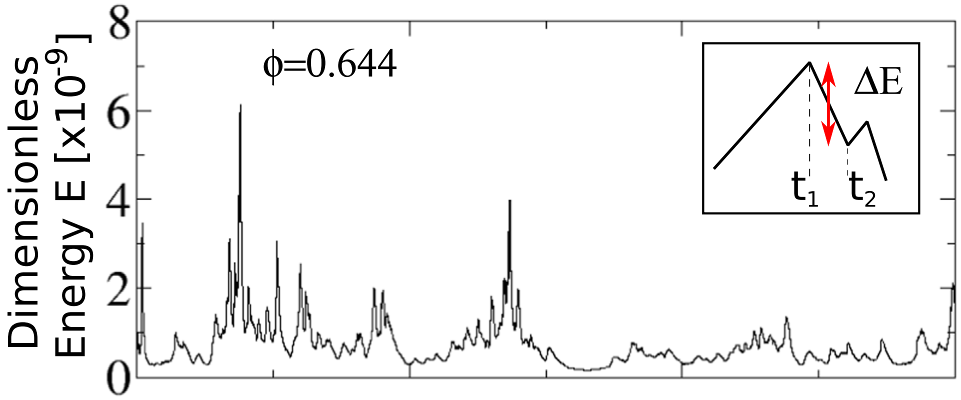
c-value decreases with disorder (again)

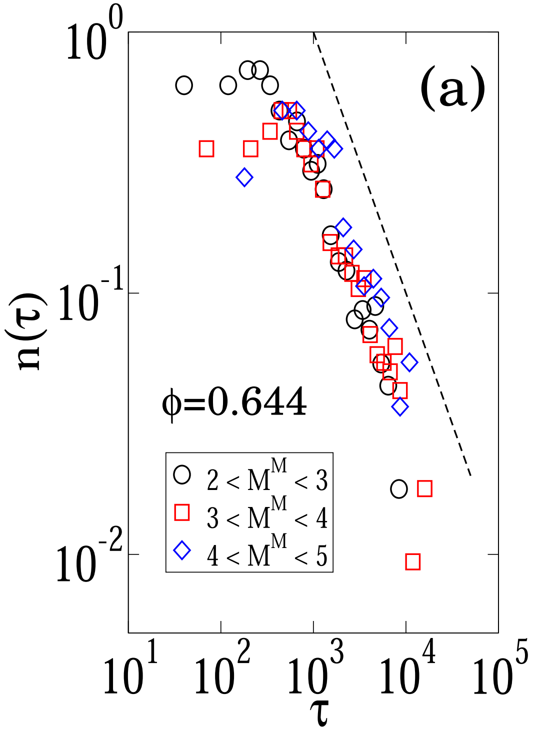


the c-value in Omori law: granular avalanche

TH, Narteau, & Shebalin, Scientific Reports 5, 12280 (2015)

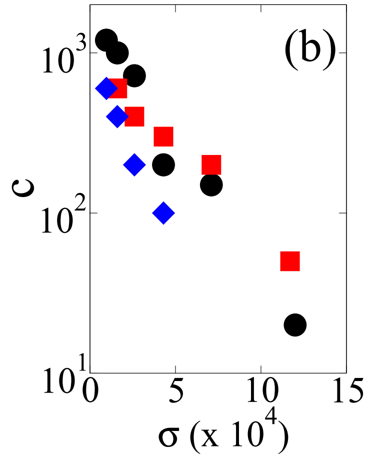






$$\dot{n} \propto (t+c)^{-1}$$

$$c \propto \exp(-f/f_*)$$



c is a decreasing function of stress, f

(same tendency with earthquakes)

summary

1. An SOC-like mean-field model resembles creep behaviors

2. Omori law and inverse Omori law are reproduced.

3. Exponent is -2 irrespective of the threshold distribution

   due to saddle-node bifurcation for mean-field model

   Nearest-Neighbor models exhibit even larger exponent

4. Nonzero c-value is obtained. Larger disorder leads to smaller c-value

5. Thermally-activated rupture reduces the exponent for Omori law

   (ongoing work)

S. Roy and TH, PHYSICAL REVIEW E **97**, 062149 (2018)