Physics of Jammed Matter,
Yukawa Institute for Theoretical Physics, Kyoto, Japan

Rheology laws in the pressure controlled dense granular system under an oscillatory shear

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Introduction

Characteristic properties of granular matters

- They have a liquid and solid region.

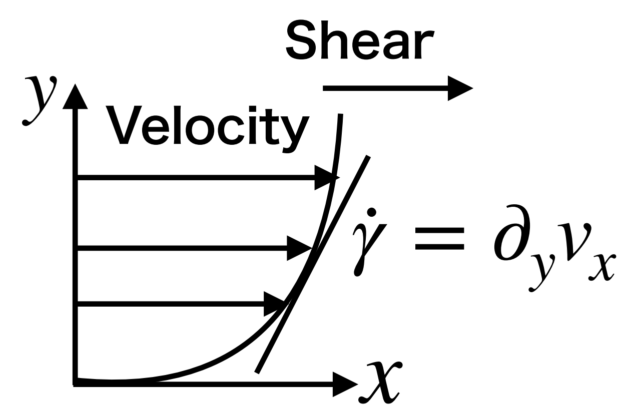
"https://www.youtube.com/watch?v=50_-zqsgDA4"

In the liquid region we use the viscosity η.

$$\eta := \frac{\sigma_{xy}}{\dot{\gamma}}$$

σ_{xy}: shear stress

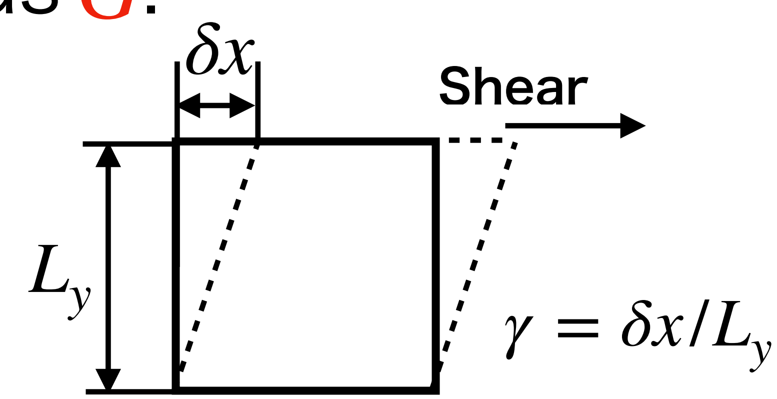
$\dot{\gamma}$: shear rate



In the solid region we use the shear modulus G.

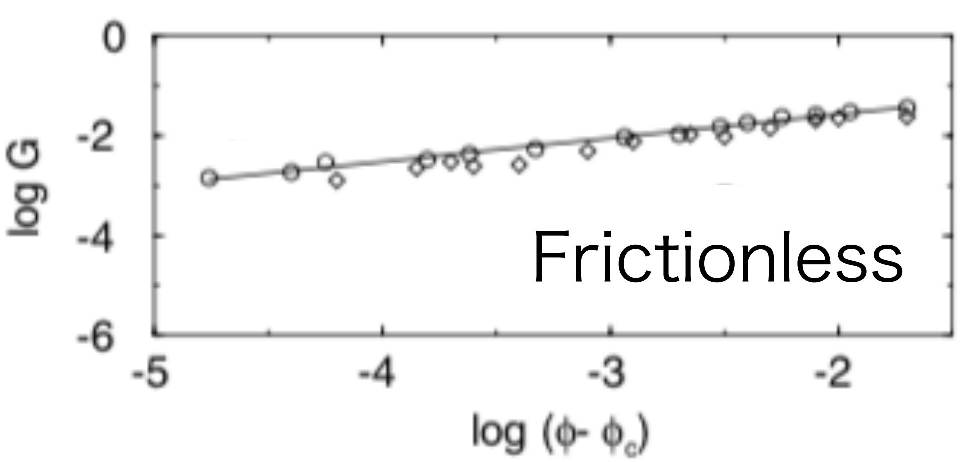
$$G := \frac{\sigma_{xy}}{\gamma}$$

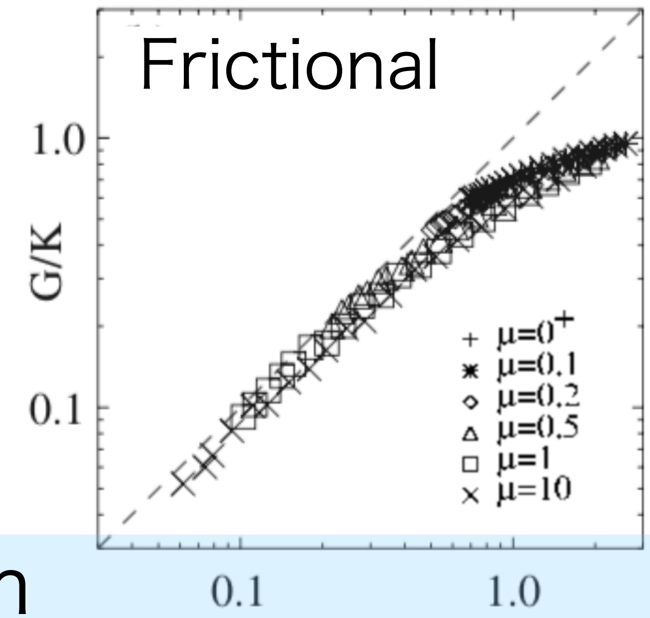
γ: shear strain

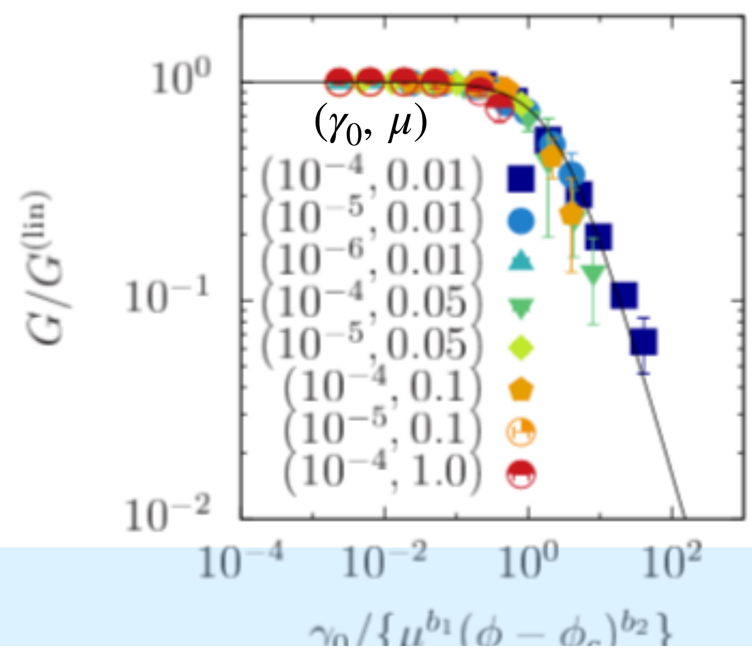


Oscillatory shear

Volume controlled system







$G \sim (\phi - \phi_J)^{1/2}$ $(\gamma_0 \to 0)$

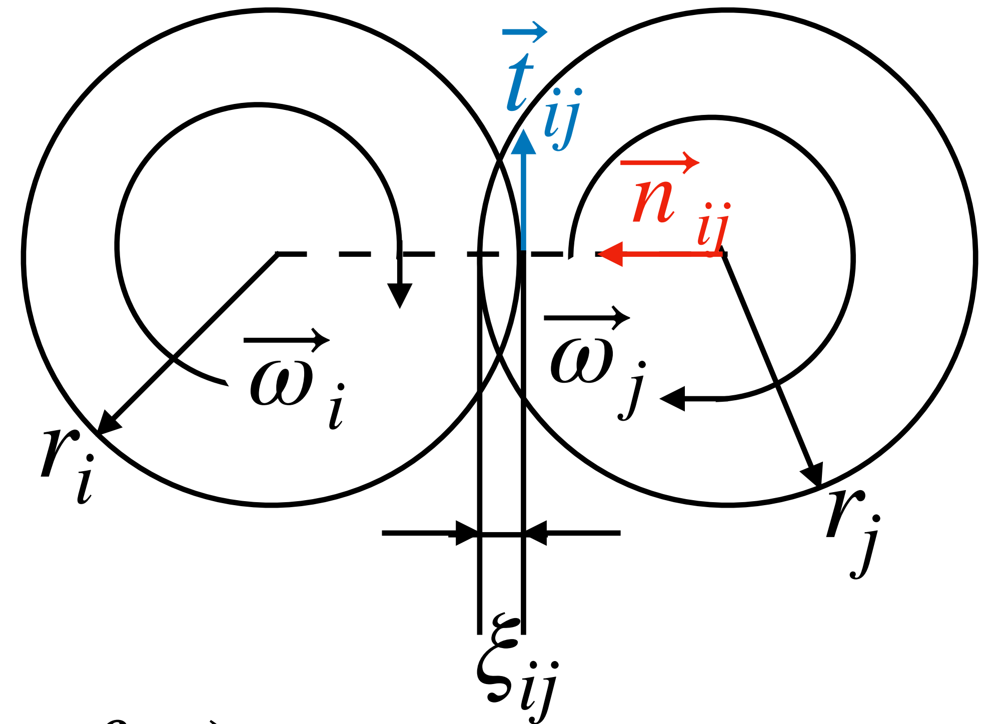
Pressure controlled system

Dilatancy exists.

What is an appropriate rheology law?

Contact force: linear spring and dash pod

$$\vec{f}_{ij} = \left(\vec{f}_{ij,n} + \vec{f}_{ij,t}\right)\Theta(r_{ij} - x_{ij})$$



Normal part

$$\vec{f}_{ij,n} = k_n \xi_{ij} \vec{n}_{ij} - \eta_n \vec{v}_{ij,n}$$

Tangential part

$$\vec{f}_{ij,t} = \begin{cases} k_t \zeta_{ij} \vec{t}_{ij} - \eta_t \vec{v}_{ij,t} & (f_{ij,t} < \mu_c f_{ij,n}) \\ \mu_c f_{ij,n} \vec{t}_{ij} & (\text{otherwise}) \end{cases}$$

S. Luding, Granular Matter ***10***, *235 (2008).*

Normal damping constant

$$\eta_n / \sqrt{m k_n} = 1 \quad (\text{restitution coefficient} \simeq 0.043)$$

Tangential damping constant

$$\eta_t / \eta_n = 0.5$$

Tangential spring constant

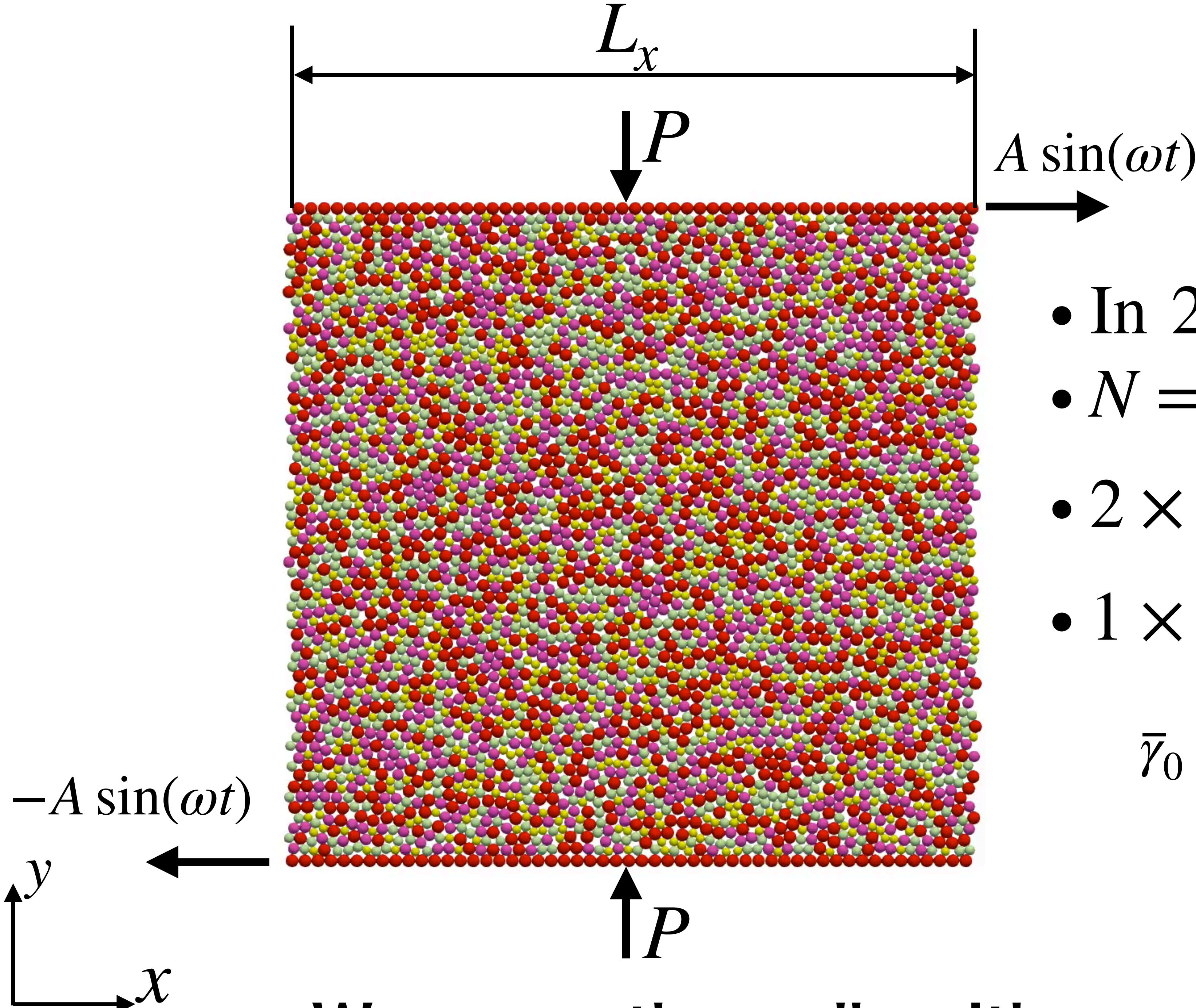
$$k_t / k_n = 0.25$$

Friction coefficient

$$\mu_c = 0 \sim 1$$

Oscillatory shear system

We explain how to apply oscillatory shear.



- In 2 dimensional
- $N = 4000$ (tetradisperse)
- $2 \times 10^{-5} \leq P/k_n \leq 6 \times 10^{-2}$
- $1 \times 10^{-6} \leq \bar{\gamma}_0 \lesssim 1$

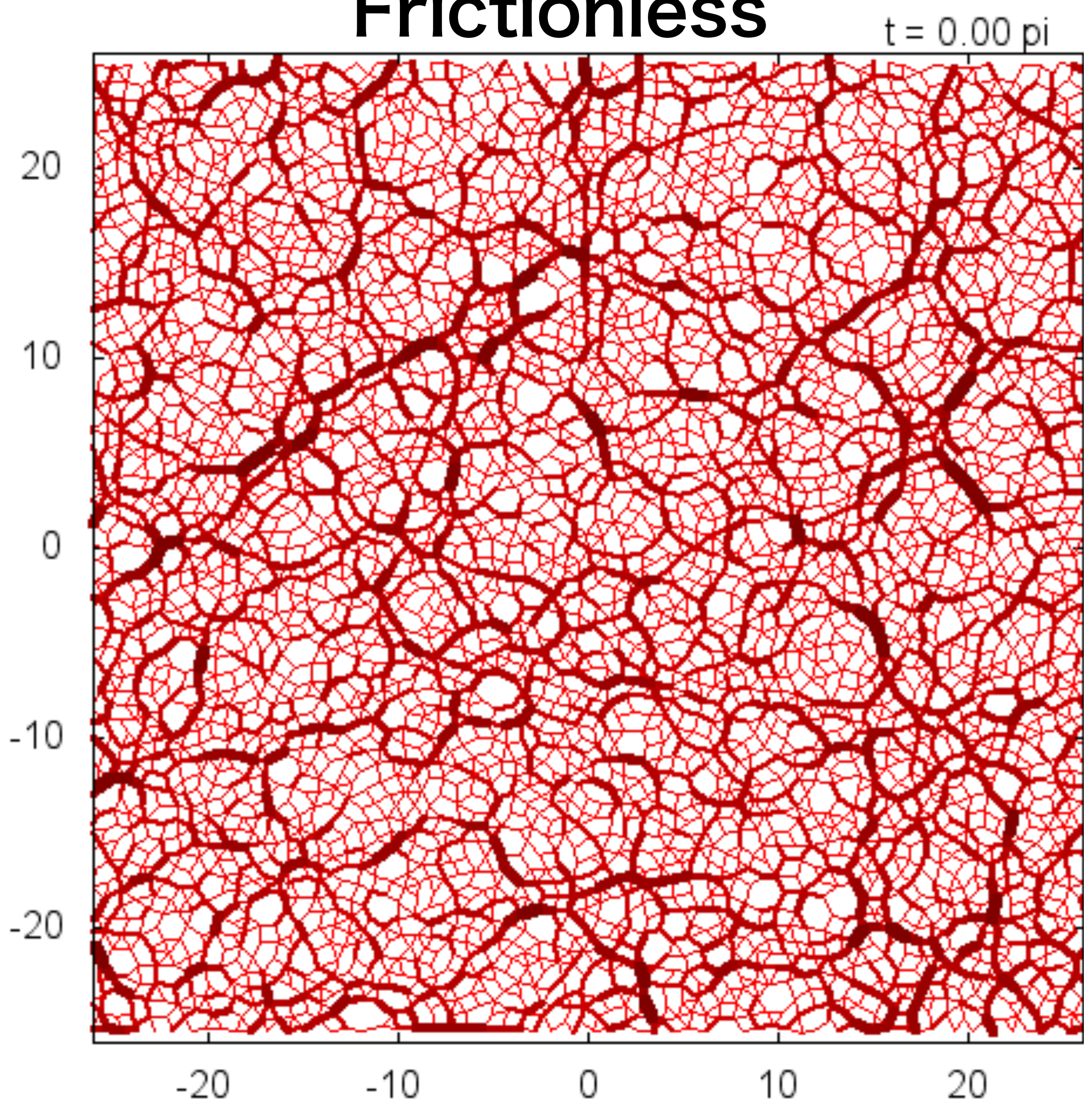
$$\bar{\gamma}_0 := \frac{A}{L_x/2} \quad (A : \text{amplitude})$$

We press the walls with a pressure P and they move according to $\pm A \sin(\omega t)$

Set up of oscillatory shear

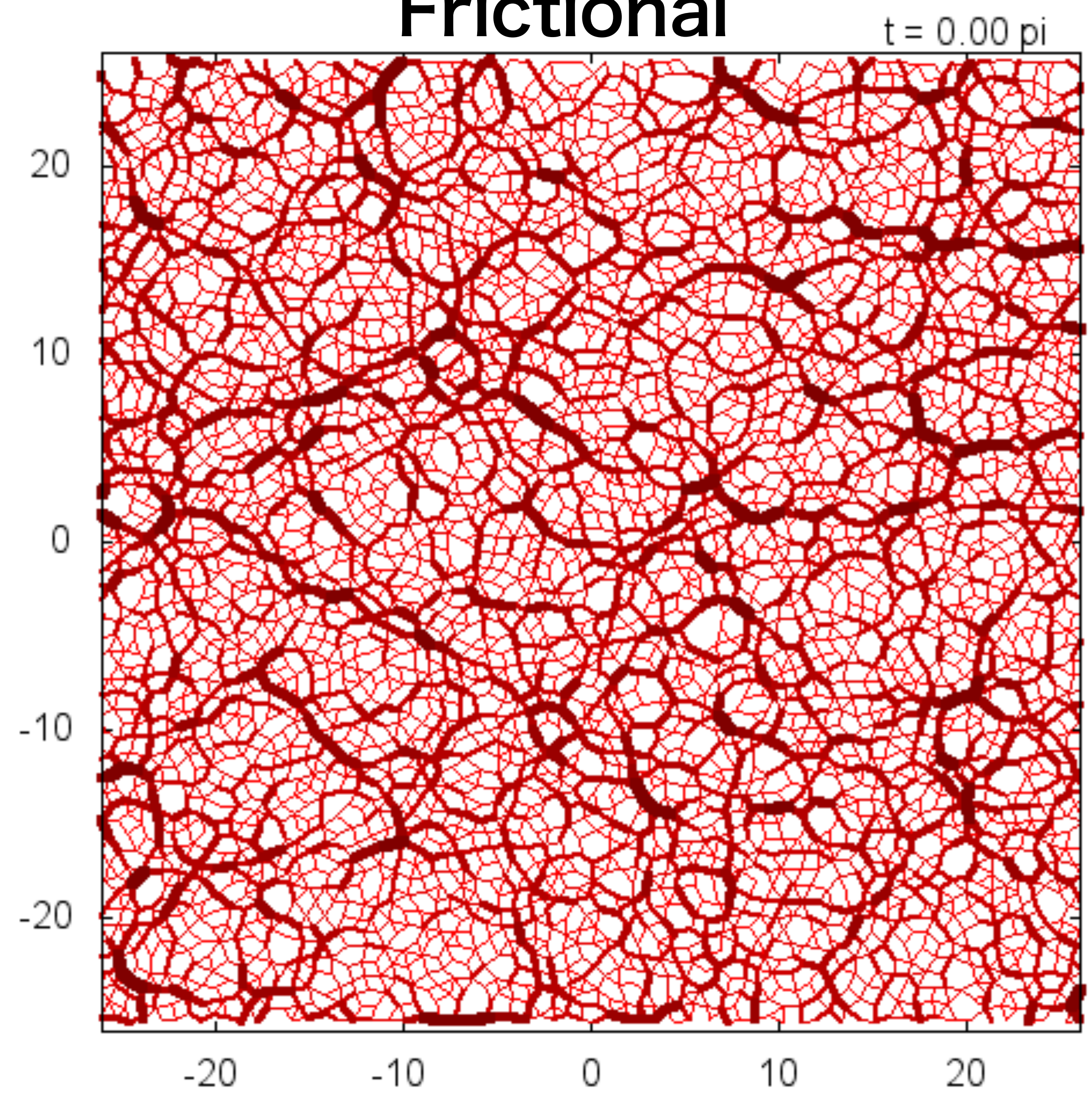
In our protocol of oscillatory state, each physical quantity is averaged over 10 cycles after initial 10 cycles.

Frictionless



$\overline{\gamma_0} = 6 \times 10^{-1}$, $P/k_n = 2 \times 10^{-3}$.

Frictional



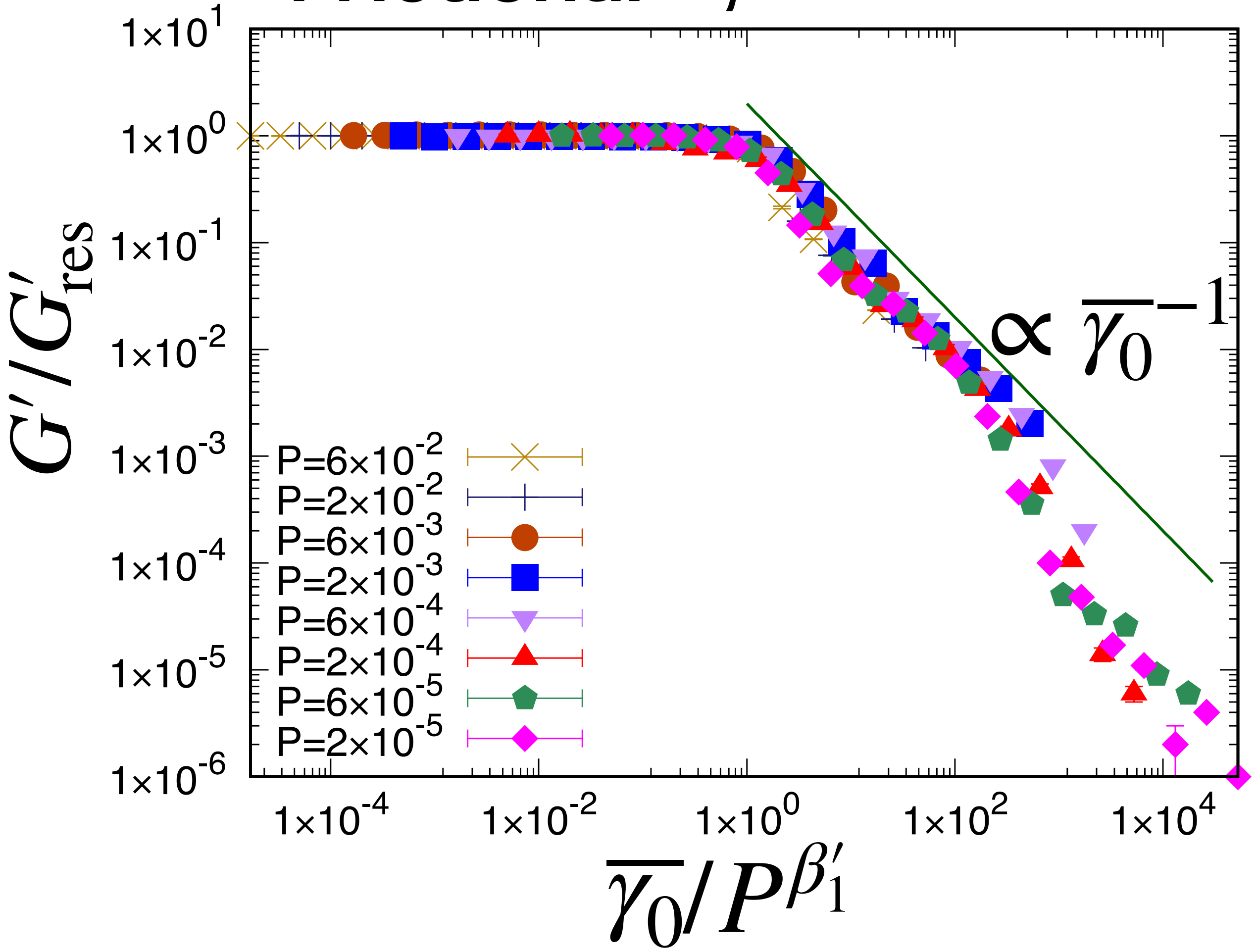
$\overline{\gamma_0} = 6 \times 10^{-1}$, $P/k_n = 2 \times 10^{-3}$.

Scaling laws for storage and loss modulus

$$G' = \lim_{\gamma \to \overline{\gamma}_0} \frac{\tilde{\sigma}}{\gamma}. \quad \left(\tilde{\sigma} = \sigma - \langle \sigma \rangle\right)$$

$$G'' = \omega \lim_{\gamma \to 0 (\tilde{\sigma} \geq 0)} \frac{\tilde{\sigma}}{\dot{\gamma}}. \quad \left(\tilde{\sigma} = \sigma - \langle \sigma \rangle\right)$$

Frictional $\mu = 1$



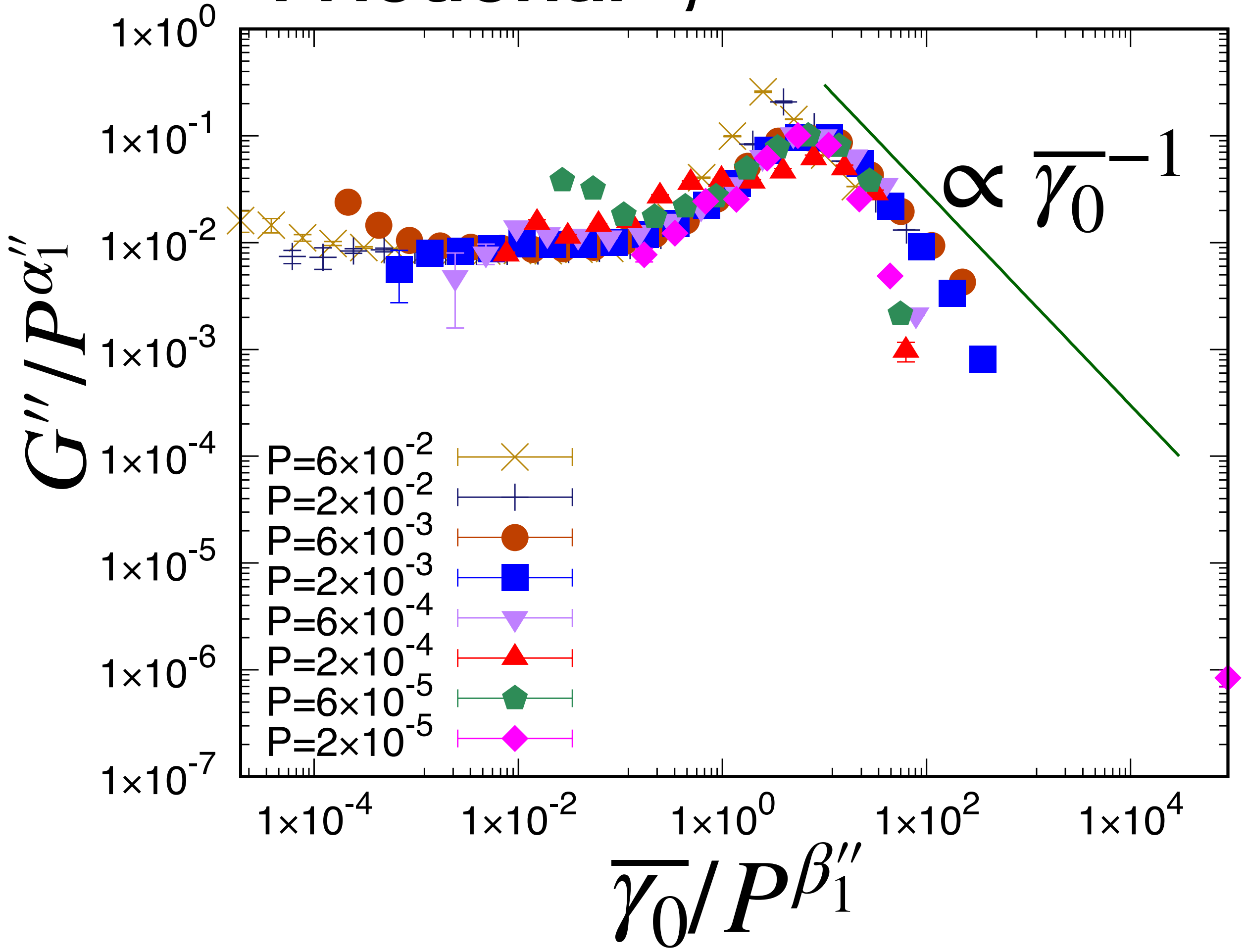
$$G'_{\rm res} = \lim_{\overline{\gamma_0} \searrow 0} G' > 0, \ \beta'_1 = 1.$$

M. Otsuki & H. Hayakawa, Phys. Rev. E 95, 062902 (2017).

c.f. G' **might be scaled by** P **using** $P \propto (\phi - \phi_J)$?

M. Otsuki & H. Hayakawa, Phys. Rev. E 80, 011308 (2009).

Frictional $\mu = 1$



$$\alpha''_1 = 0.2, \ \beta''_1 = 1.05.$$

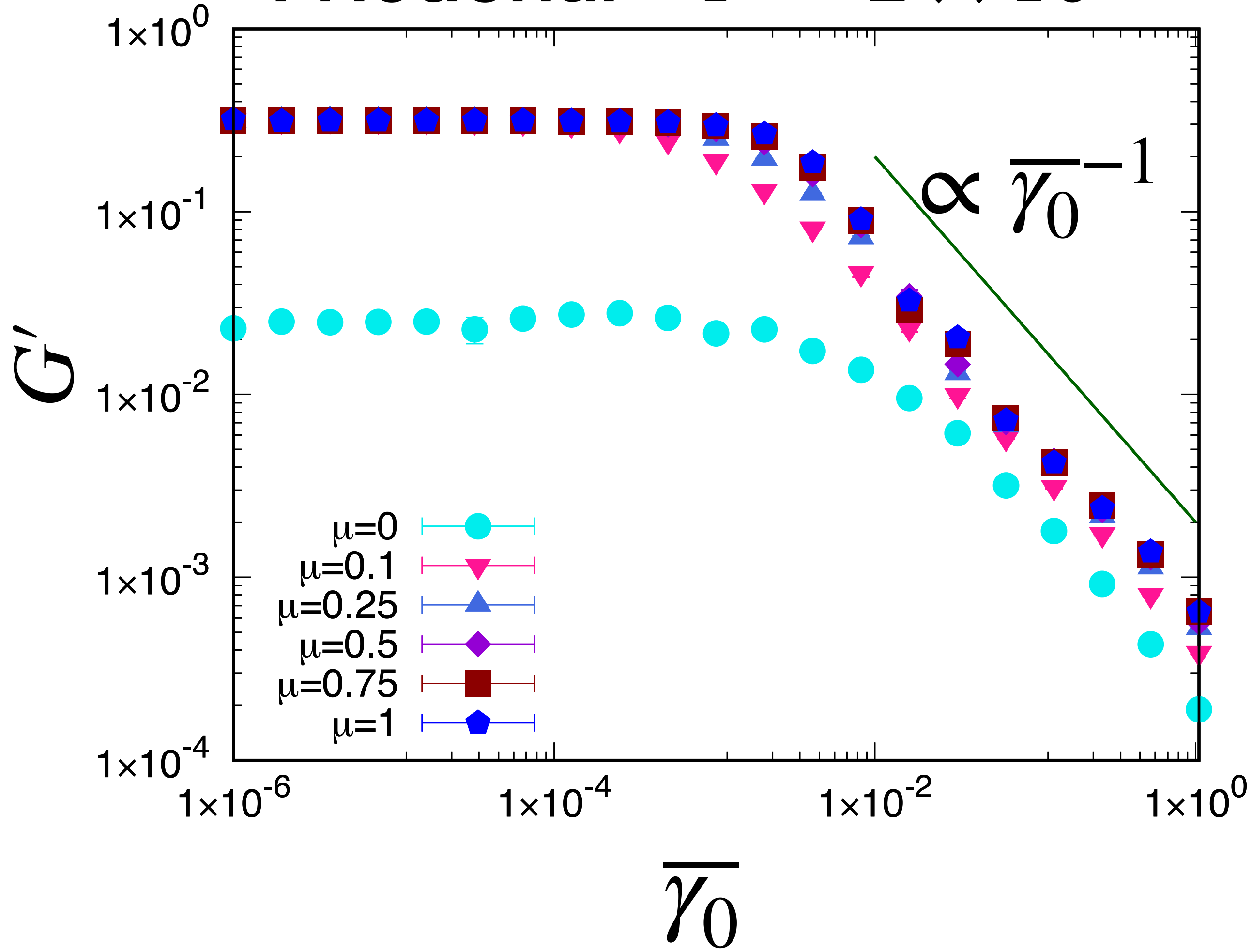
G'' **has a peak.**

Influence of friction coefficient

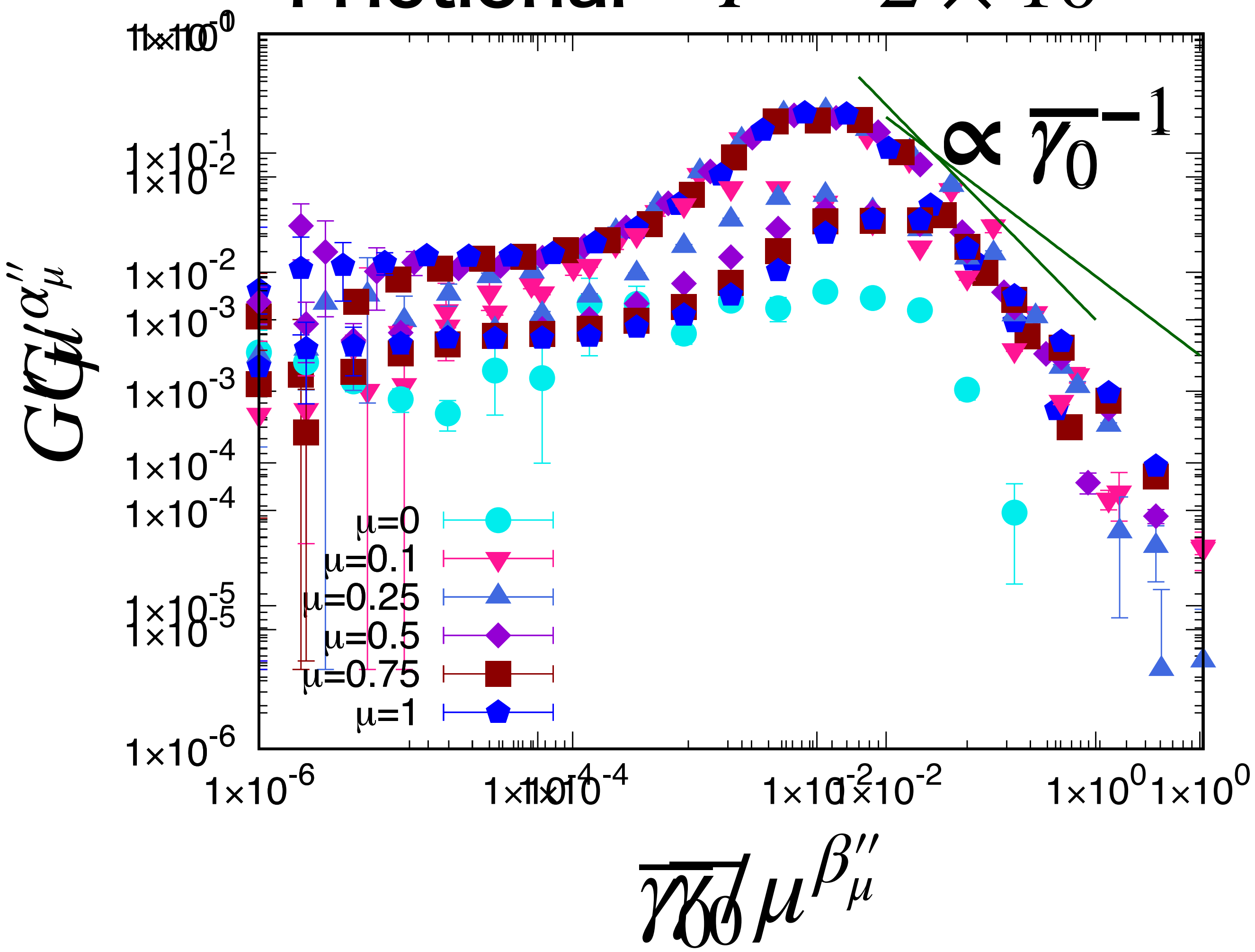
$$G' = \lim_{\gamma \to \overline{\gamma}_0} \frac{\tilde{\sigma}}{\gamma}. \quad \left(\tilde{\sigma} = \sigma - \langle \sigma \rangle\right)$$

$$G'' = \omega \lim_{\gamma \to 0 (\tilde{\sigma} \geq 0)} \frac{\tilde{\sigma}}{\dot{\gamma}}. \quad \left(\tilde{\sigma} = \sigma - \langle \sigma \rangle\right)$$

Frictional $P = 2 \times 10^{-3}$



Frictional $P = 2 \times 10^{-3}$

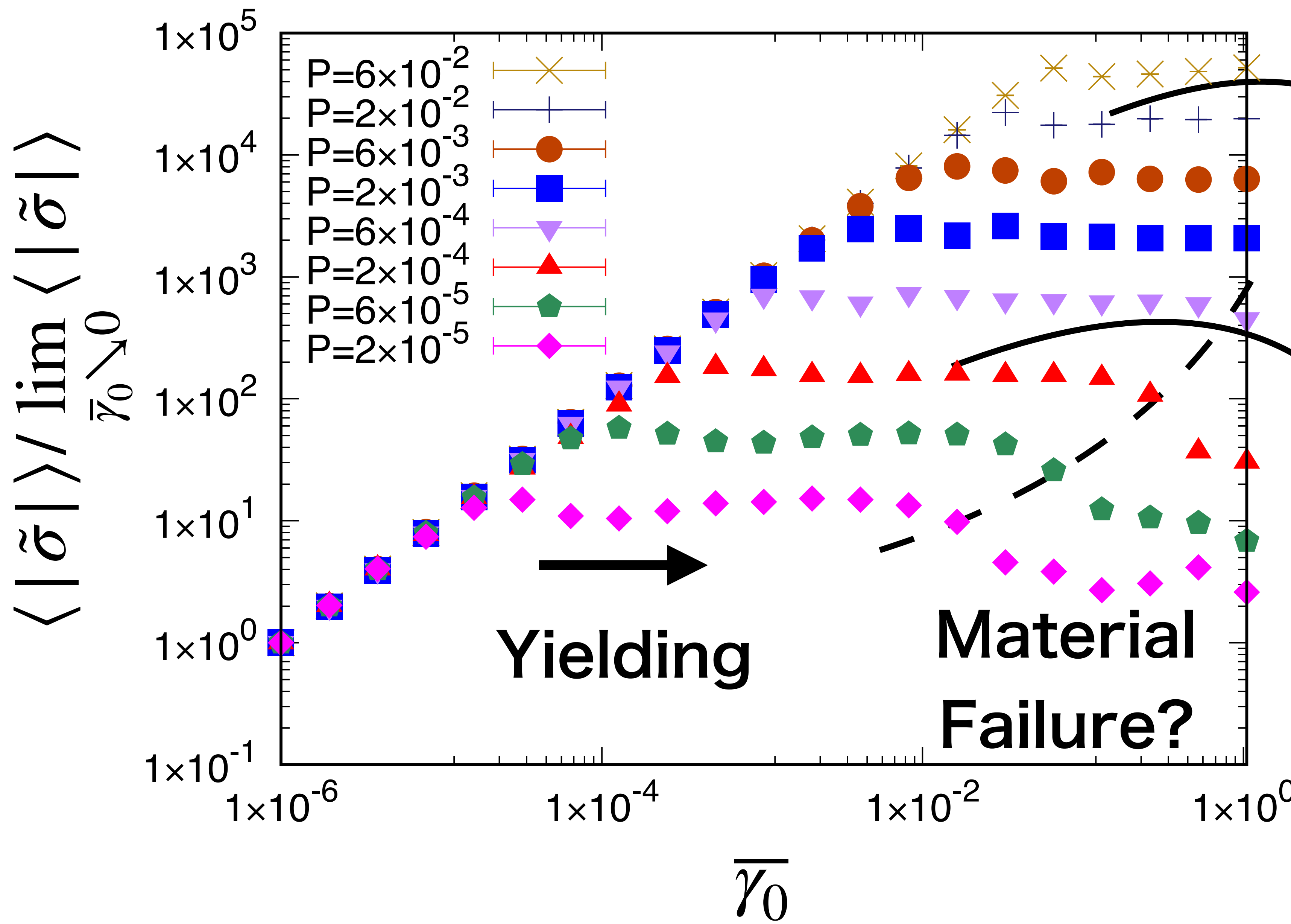


$\alpha''_\mu = 0.3, \; \beta''_\mu = 0.75.$

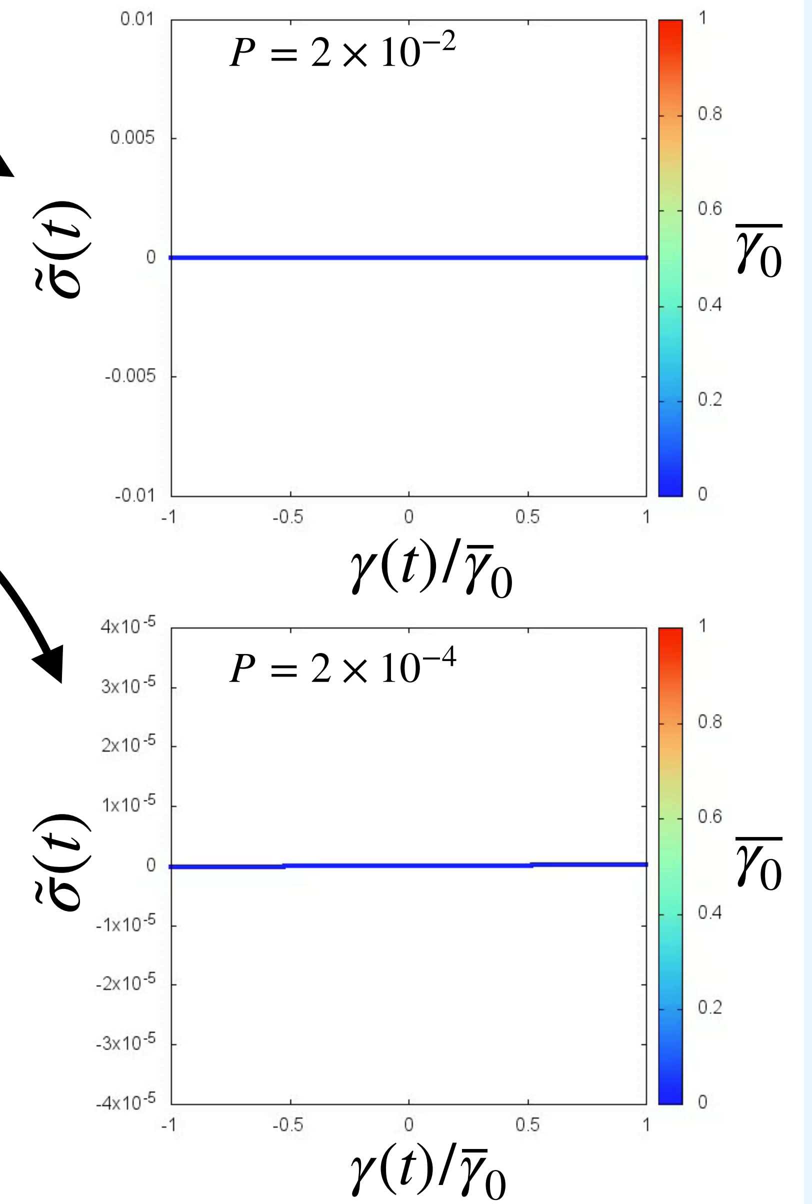
$G'|_{\mu=0}$ **and** $G''|_{\mu=0}$ **are significantly different from** $G'|_{\mu \geq 0.1}$ **and** $G''|_{\mu \geq 0.1}$ **respectively.**

Discussion - $G' \propto \overline{\gamma_0}^{-1}$

Averaged shear stress curve



$\overline{\gamma_0}$ dependence

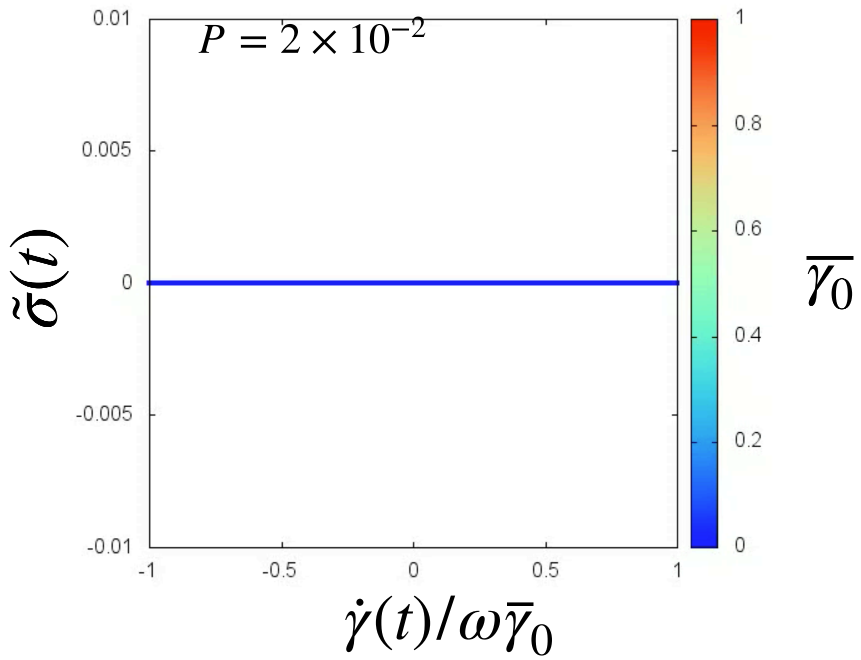


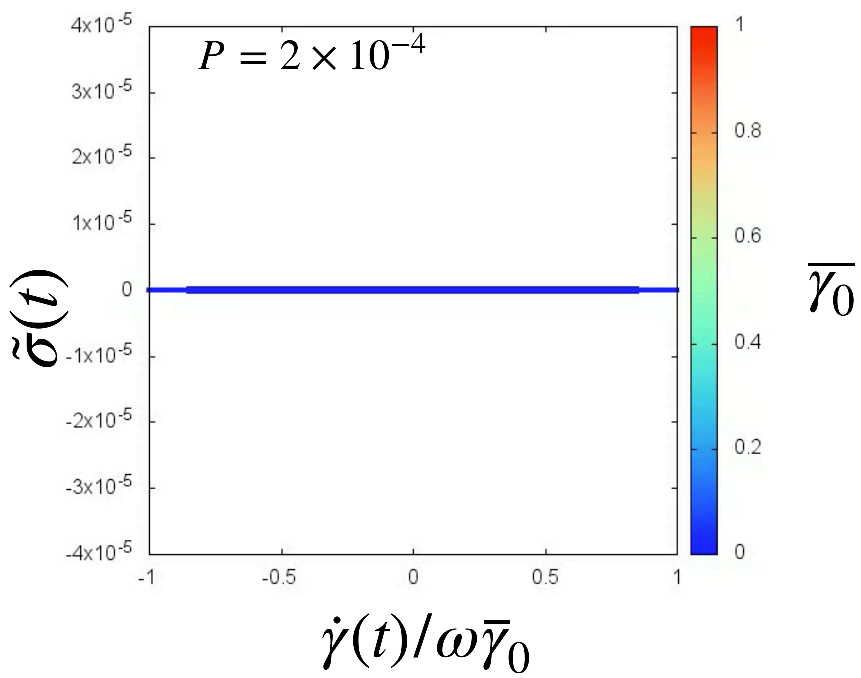
$$\max \tilde{\sigma}(t) \simeq \text{const} \quad (\overline{\gamma}_0 > \overline{\gamma}_{0,c}) .$$

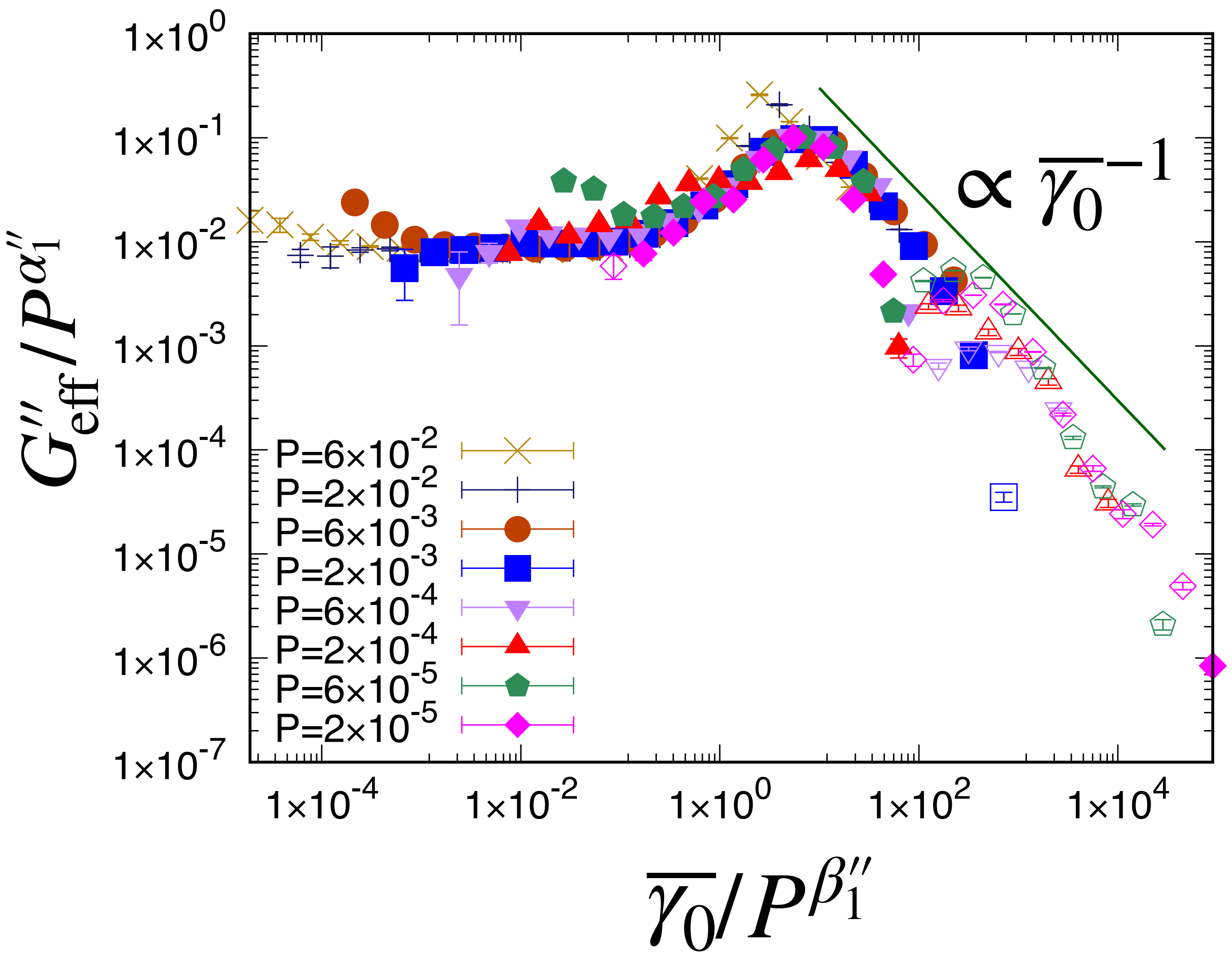
The shear stress yields.

$$\Rightarrow \quad G' = \lim_{\gamma \to \overline{\gamma}_0} \frac{\tilde{\sigma}}{\gamma} \propto \overline{\gamma_0}^{-1}$$

Discussion - $G'' < 0$







$$\alpha''_1 = 0.2,\ \beta''_1 = 1.05.$$

$$G''_{\rm eff} = \omega \lim_{\gamma\to 0(\tilde{\sigma}\geq 0)} \frac{\tilde{\sigma}}{|\dot{\gamma}|}. \quad (\tilde{\sigma} = \sigma - \langle\sigma\rangle)$$

$G'' < 0$ due to the phase delay,

but $G''_{\rm eff}$ satisfies the scaling law.

Discussion - Peak of the loss modulus

0~1 G'' is constant.

Coordination number(Z) is constant.

1~2 G'' increases.

Z decreases.

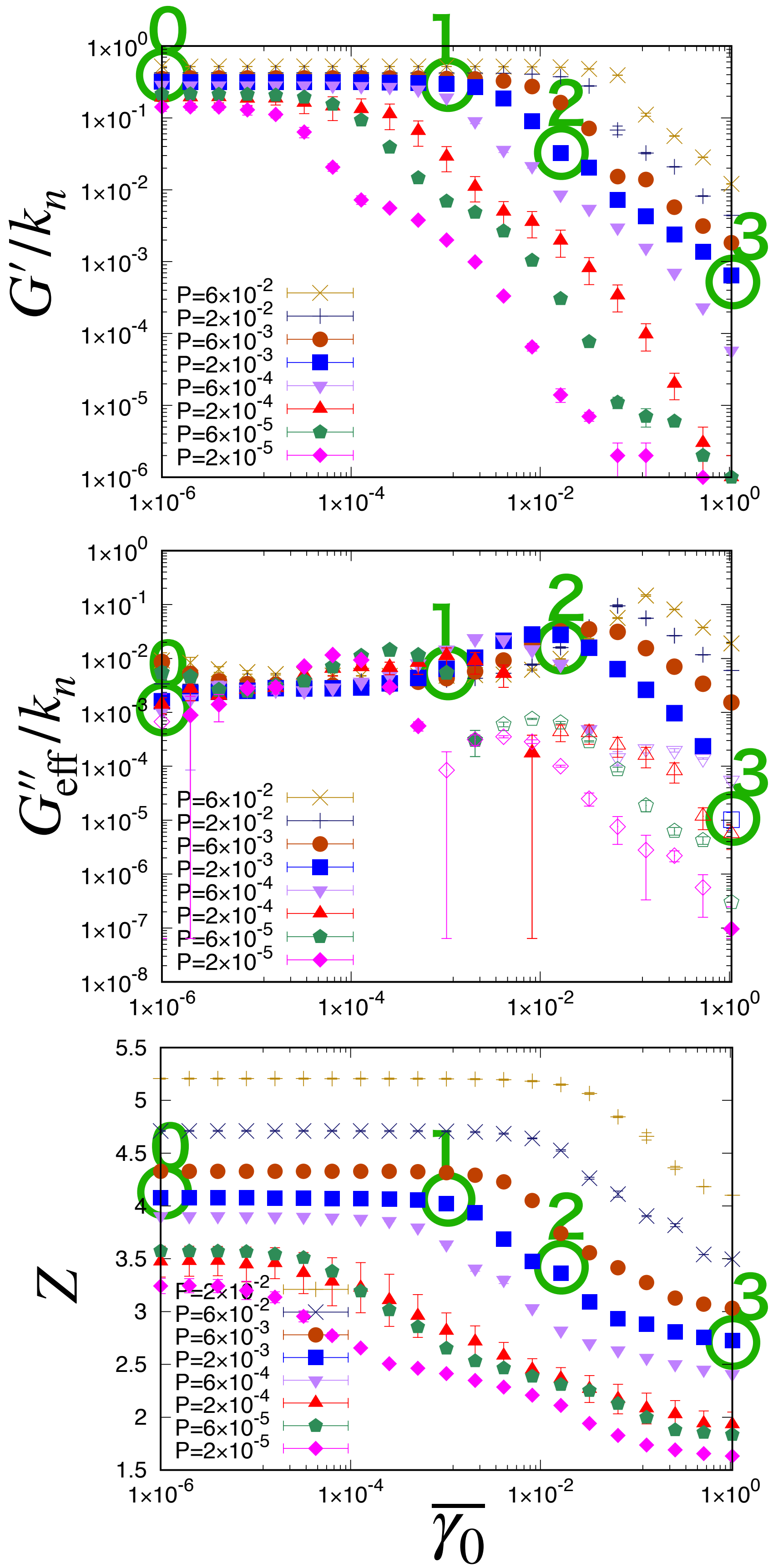
The system begins to flow.

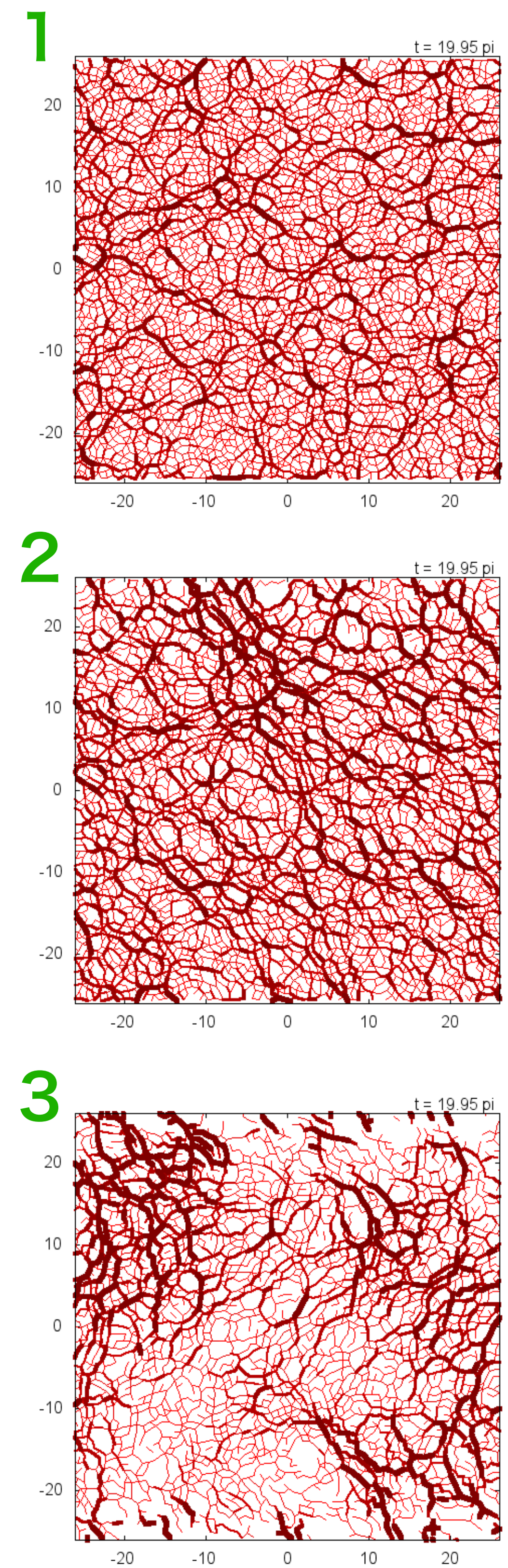
2~3 G'' decreases.

The contact network is completely broken.

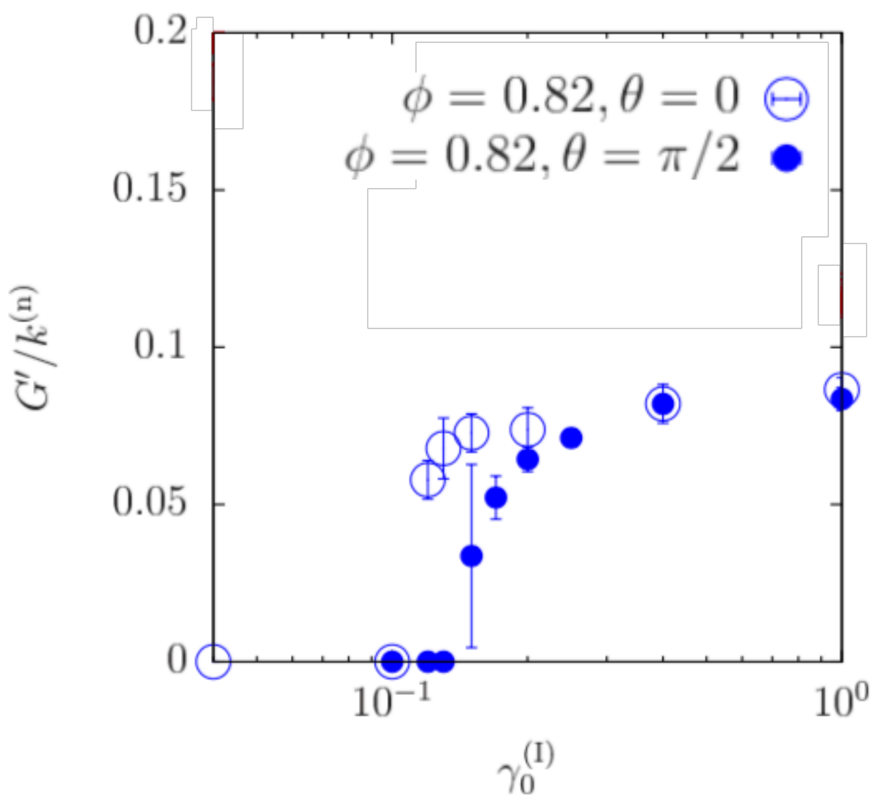
It's easy to flow.

＊Phase delay exists

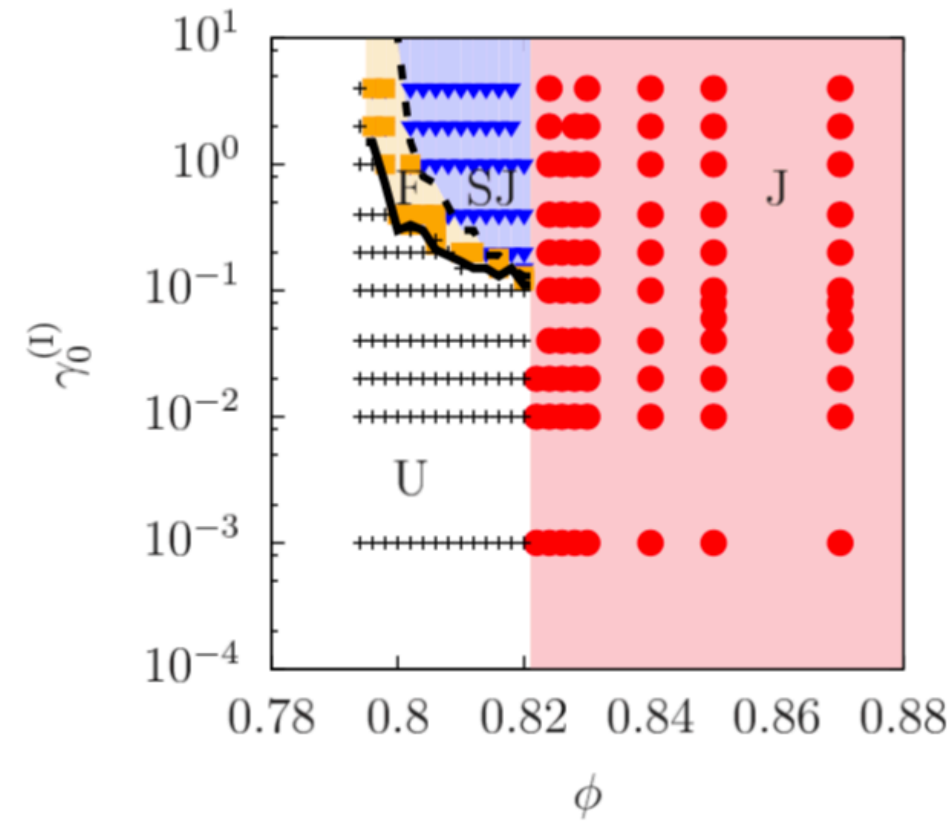




Discussion - Shear jamming



Shear jamming

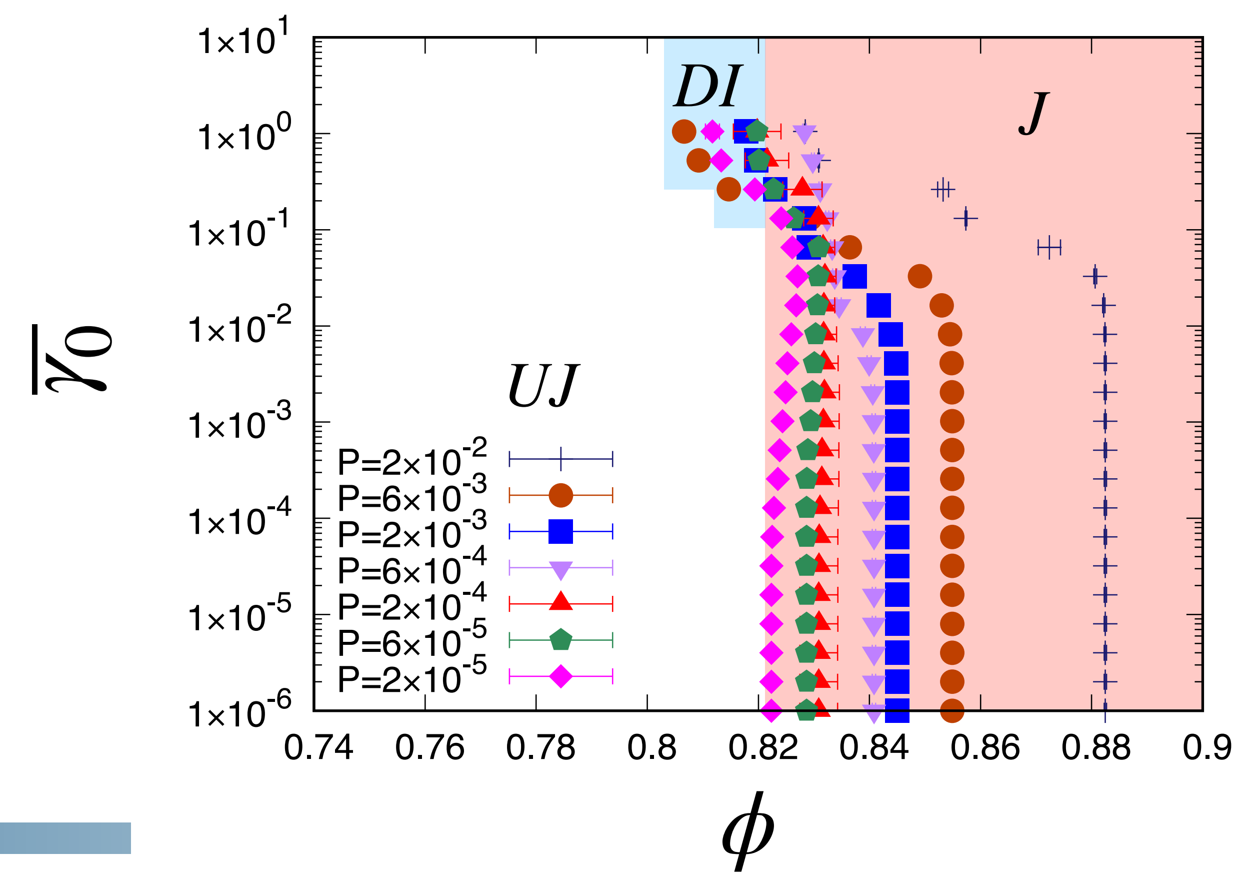


Otsuki & Hayakawa, arXiv:1810.03846

In our cases

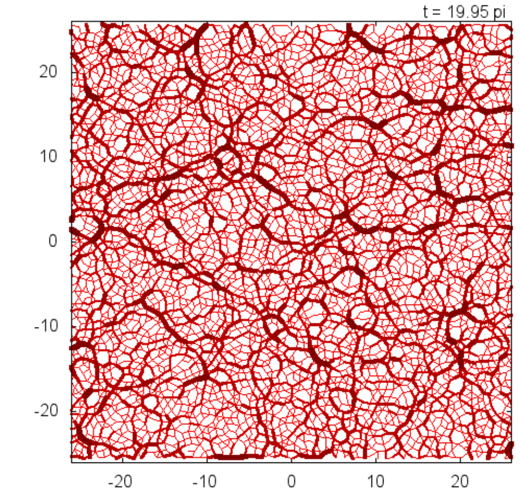
$$G'|_{\phi<\phi_J} > 0.$$

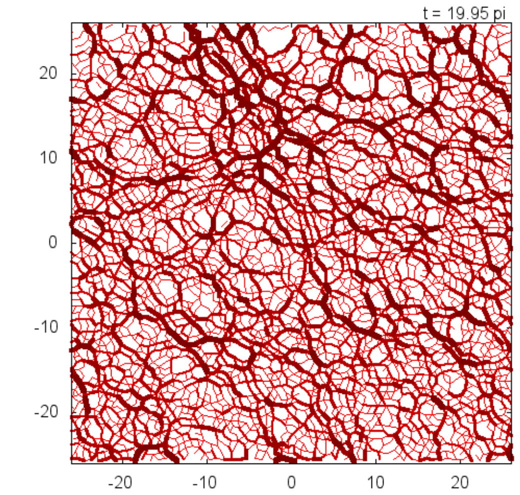
∴Shear jamming appears.

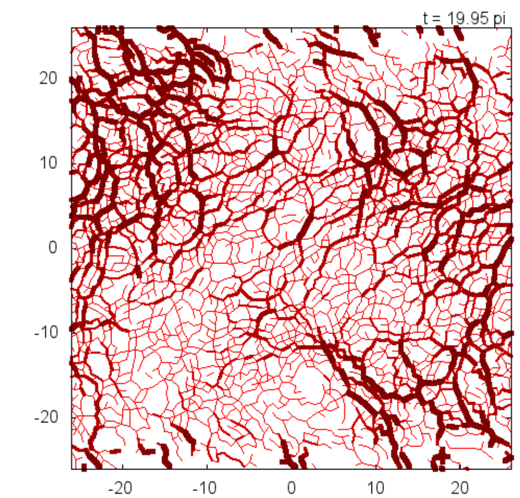


Conclusion

We investigate the rheology in the oscillatory shear.







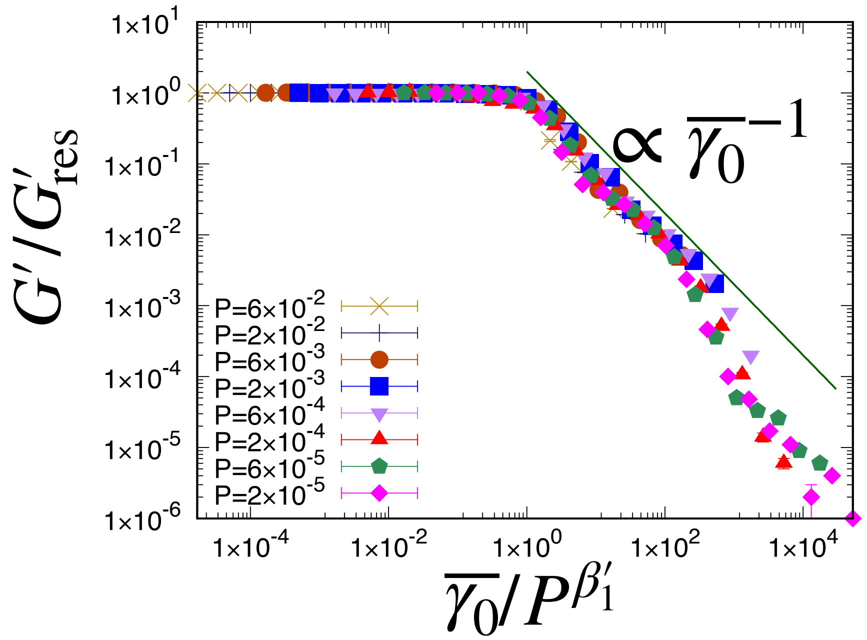
Rheology law

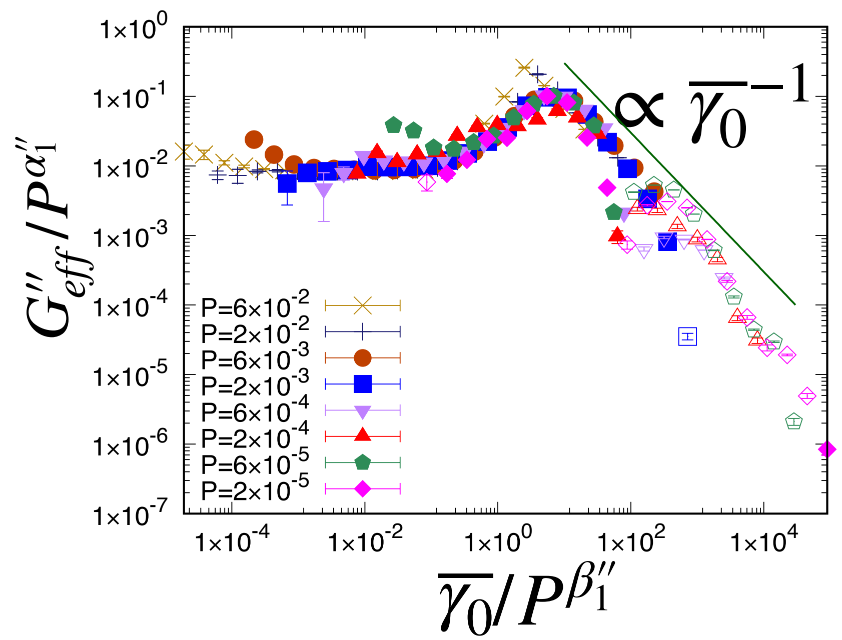
- **Storage modulus ··· The scaling laws exist.**
- **Loss modulus ··· G'' has a peak.**

Shear Jamming

- **The dilatancy is related to the Shear Jamming.**

Scaling laws in frictional systems

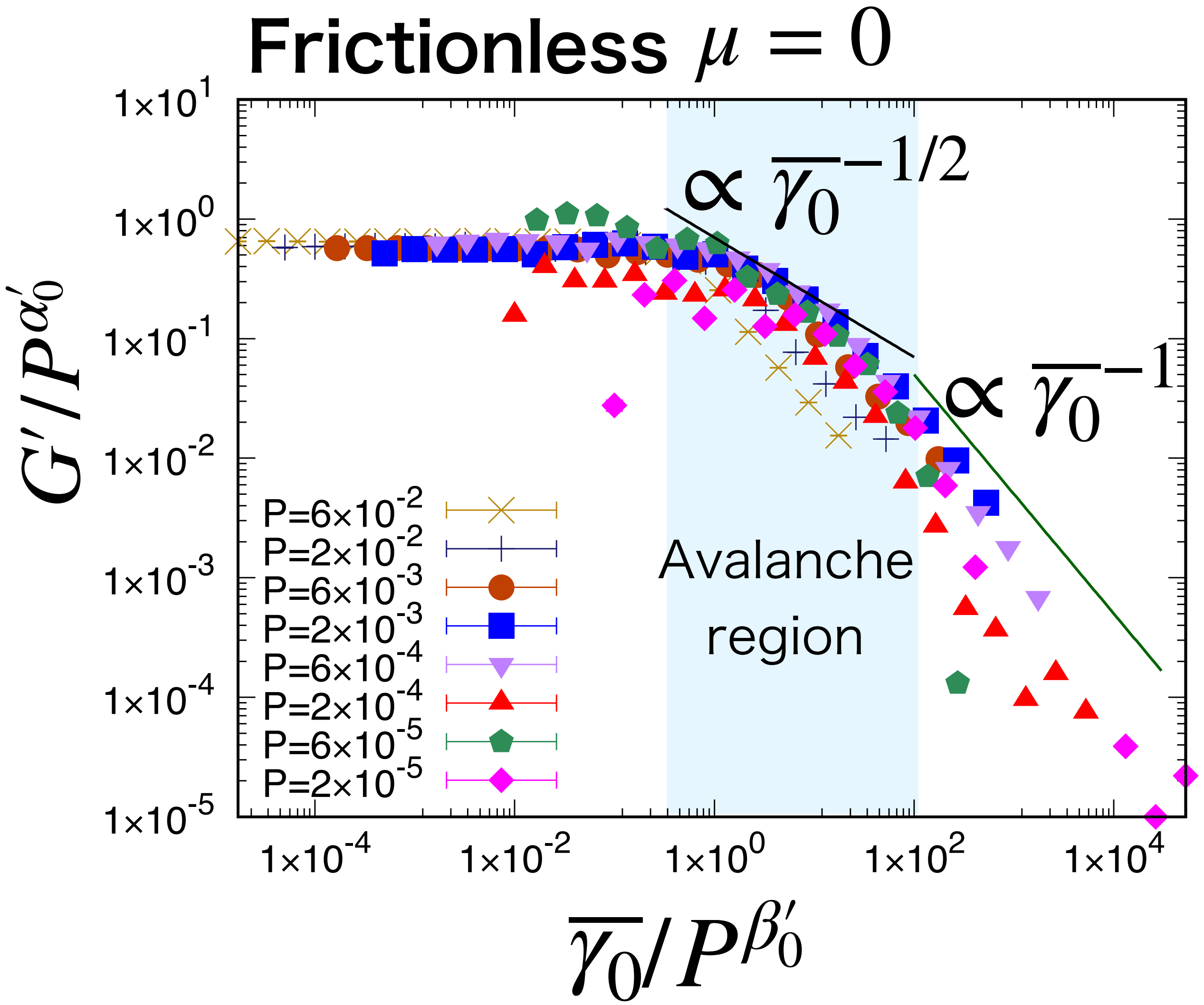




Appendixes

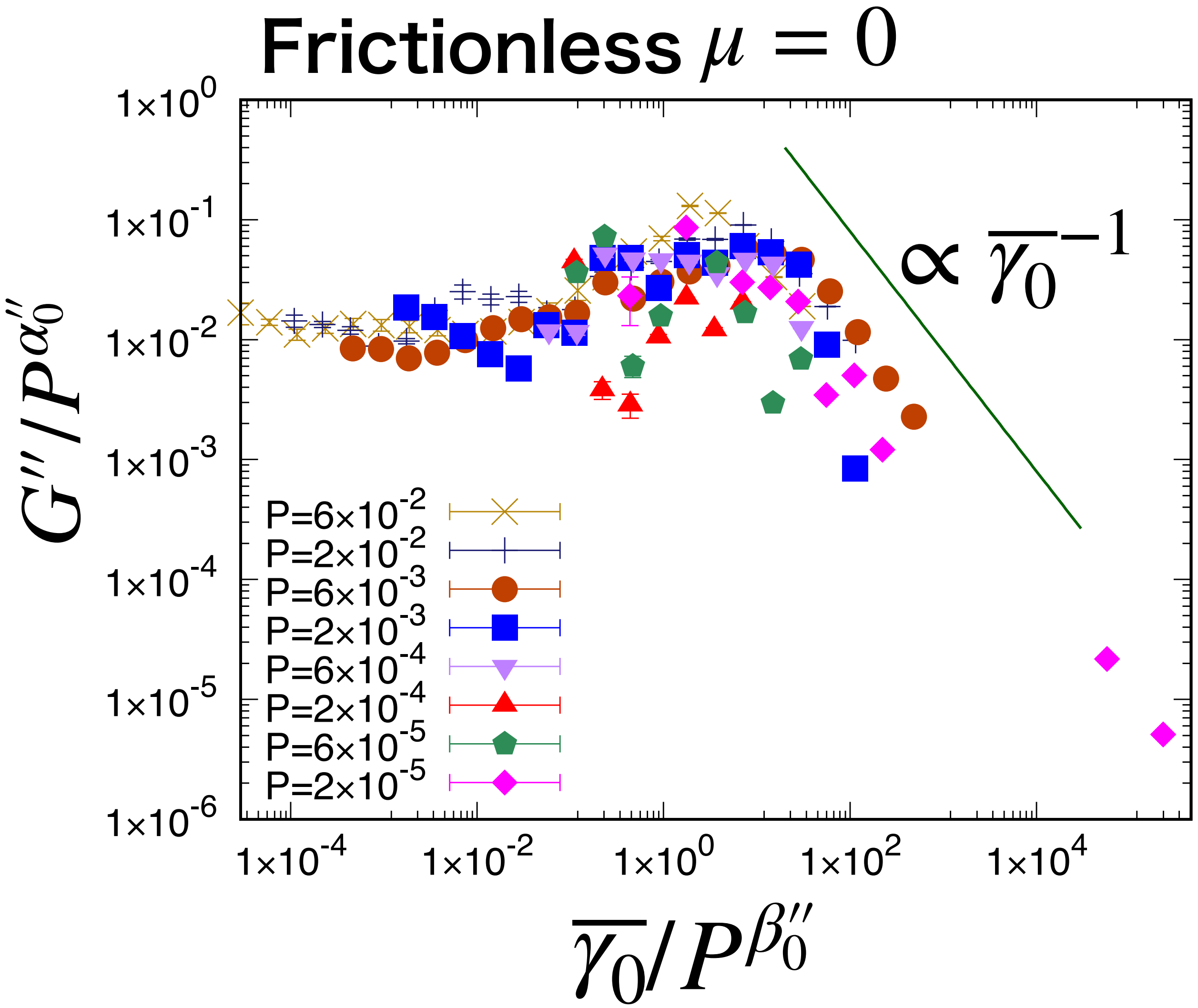
Scaling laws in the frictionless systems

$$G' = \lim_{\gamma \to \overline{\gamma}_0} \frac{\tilde{\sigma}}{\gamma}. \quad \left(\tilde{\sigma} = \sigma - \langle \sigma \rangle\right)$$



$\alpha_0' = 0.5,\ \beta_0' = 1.$

*M. Otsuki, H. Hayakawa, Phys. Rev. E 9**0**, 042202 (2014).*



$\alpha_0'' = 0.35,\ \beta_0'' = 1.2.$

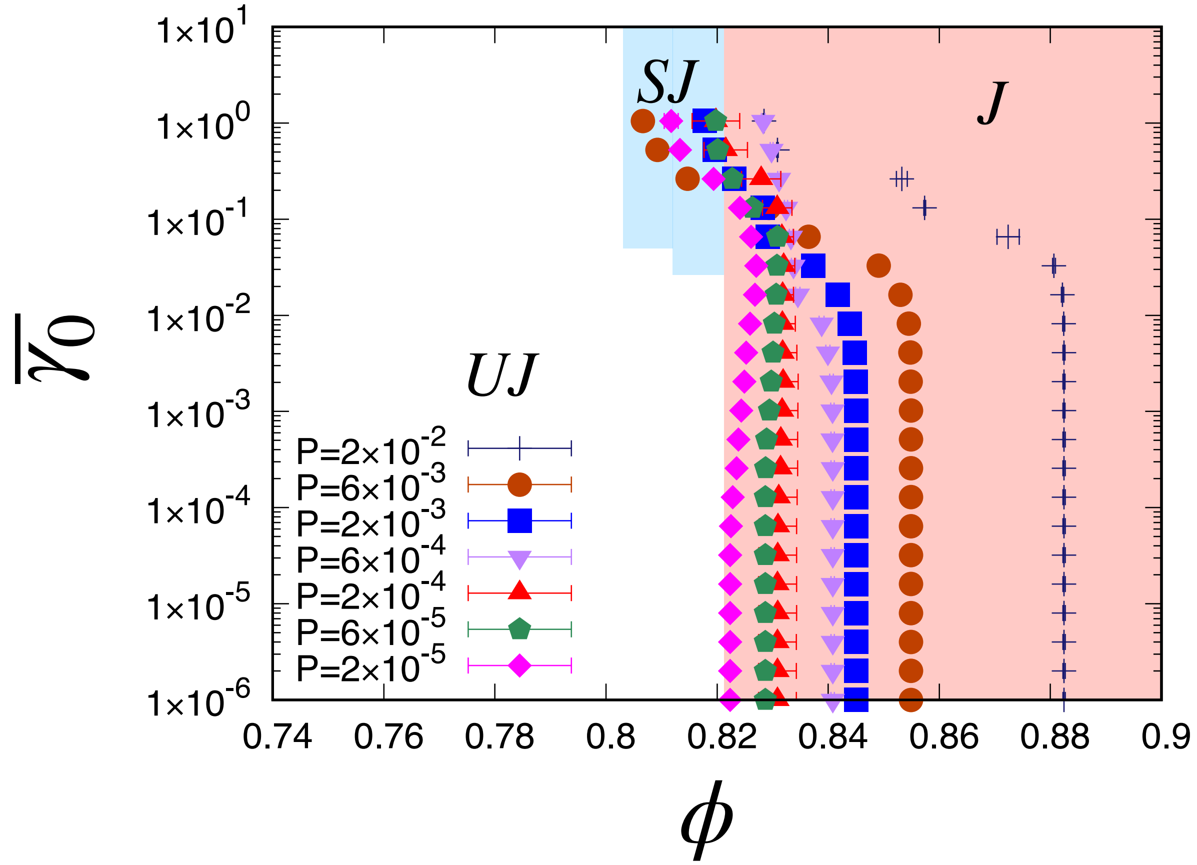
*M. Otsuki, H. Hayakawa, Phys. Rev. E 9**5**, 062902 (2017).*

c.f. G' **might be scaled by** P **using** $P \propto (\phi - \phi_J)$?

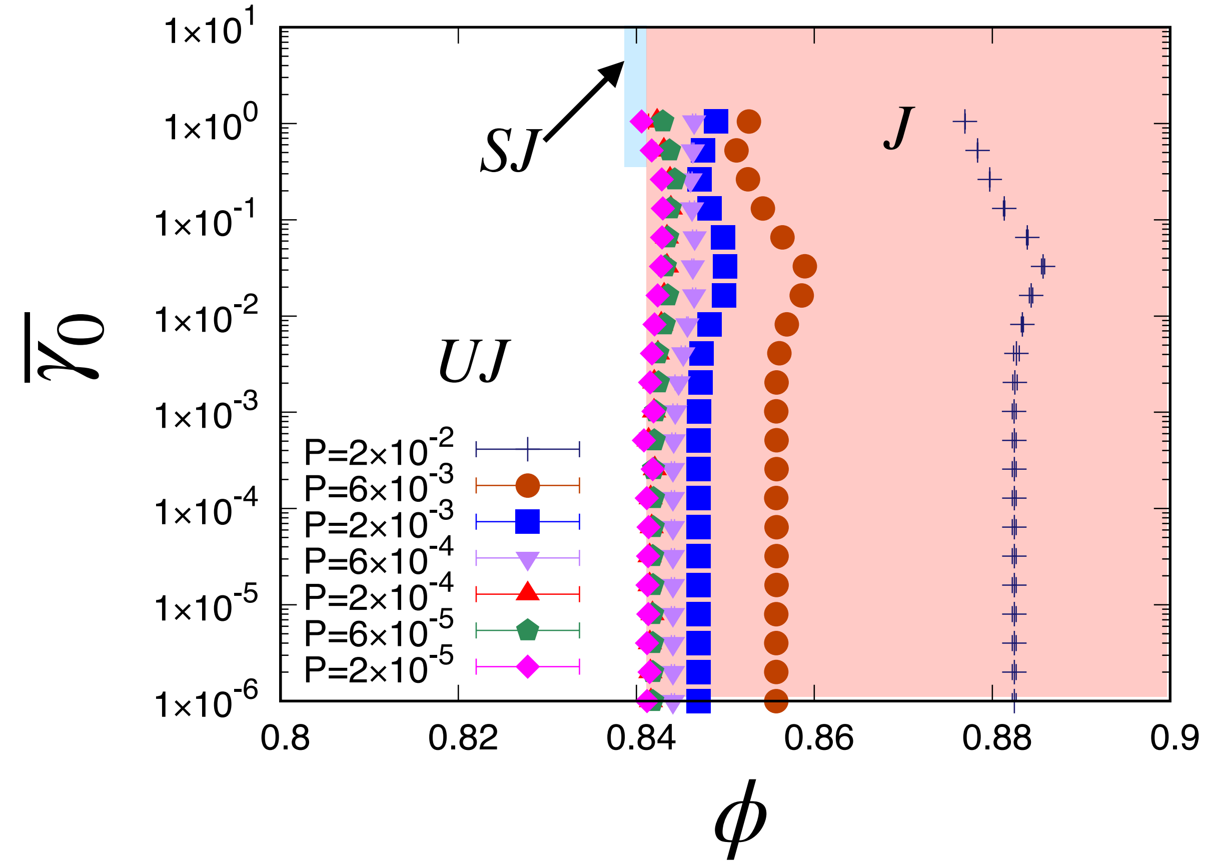
M. Otsuki, H. Hayakawa, Phys. Rev. E 80, 011308 (2009).

Discussion - Shear jamming in the frictionless system

Frictional



Frictionless



In both cases

$$G'|_{\phi<\phi_J} > 0.$$

∴Shear jamming appears.

Pressure dependence of $G'_{\rm res}$

$$G'_{\rm res} := \lim_{\overline{\gamma_0}\searrow 0} G'.$$

$$G'_{\rm res} \simeq a \log P + b,$$
$$a = 0.04 \pm 0.01,$$
$$b = 0.62 \pm 0.03.$$