Physics of Jammed Matter--
Yukawa Institute for Theoretical Physics, Kyoto University

Reversible-irreversible transitions in particle trajectories near the jamming transition

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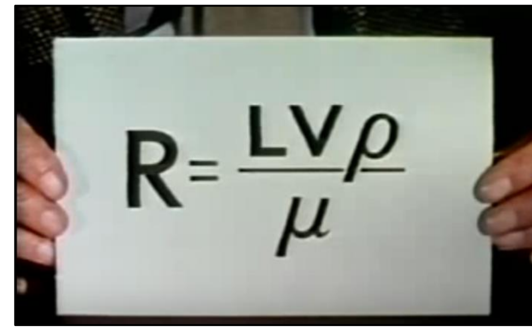
K. Nagasawa, K. Miyazaki, and T. Kawasaki , to be submitted.
T. Kawasaki and L. Berthier, Phys. Rev. E **94**, 022615 (2016).

Taylor's experiment

Low Reynolds number flow: a particle trajectory can be reversible!



$$(\nabla . \mathbf{v}) = 0,$$

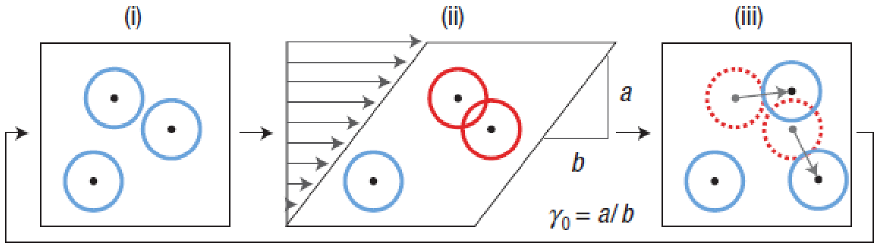
$$0 = -\nabla P + (\nabla . \boldsymbol{\tau})$$

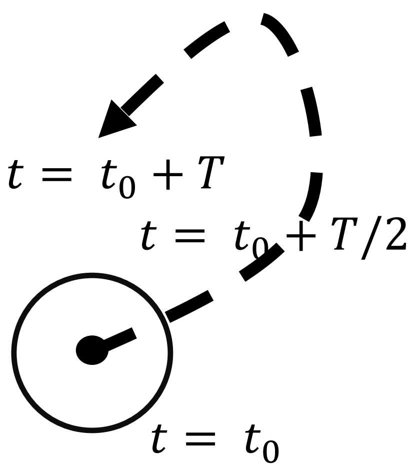
$$\boldsymbol{\tau} = \mu[\nabla \mathbf{v} + (\nabla \mathbf{v})^t].$$

Taylor & Friedman (1966), Low Reynolds Number Flows
(National Committee on Fluid Mechanics Films, Encyclopedia Britannica Education Corp., United States)

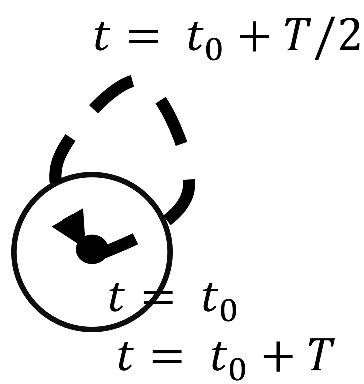
Reversible-irreversible (R-I) transition is a non-equilibrium phase transition!

Oscillatory shear experiment of low density colloidal suspensions ($\varphi = 0.20$)





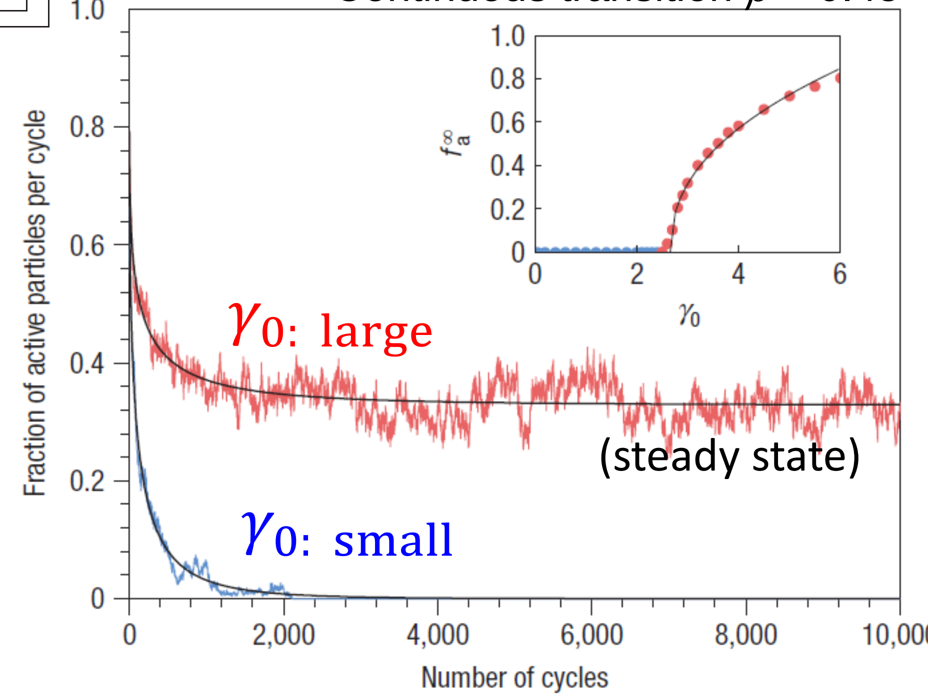
Irreversible



Reversible

(Absorbing state)

Continuous transition $\beta = 0.45$



This absorbing state transition shows Directed Percolation (DP) ($\beta_{DP} = 0.58$) like critical behavior.

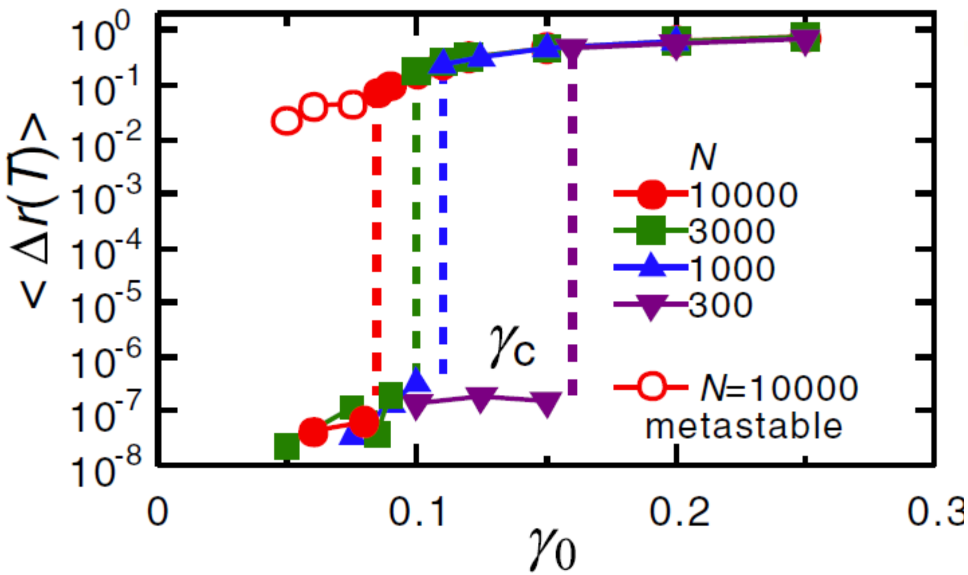
L. Corté, P. M. Chaikin, J. P. Gollub & D. J. Pine, Nat Phys **4**, 420 - 424 (2008).

Even in very high density system, R-I transition takes place -- behaving differently from low density suspensions

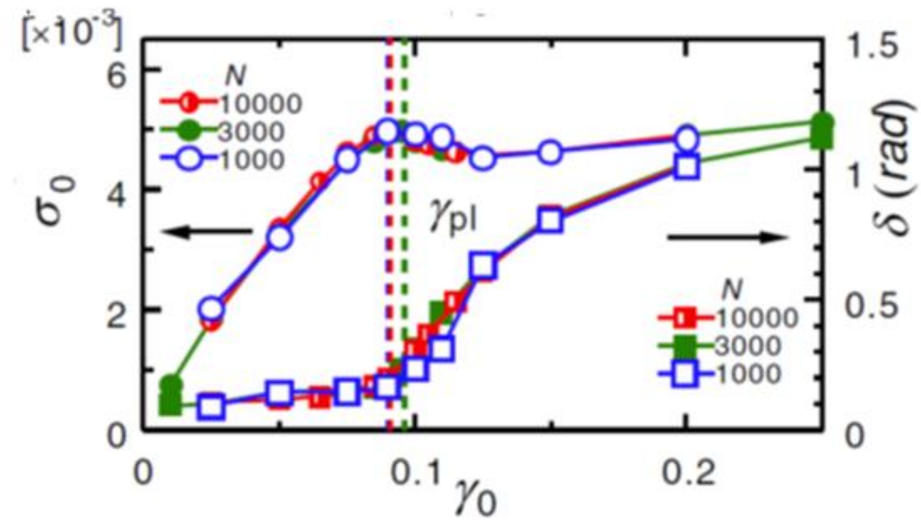
Oscillatory shear numerical experiment in highly dense suspensions in 3D

($\varphi = 0.80$ cf. $\varphi_J = 0.647$)

Time averaged one cycle displacement : $\langle \Delta r(T) \rangle$



Stress-strain curve:



"Discontinuous" RI transition

$$\gamma_c \approx 0.0885$$

Yielding transition

$$\gamma_{pl} \approx 0.09$$

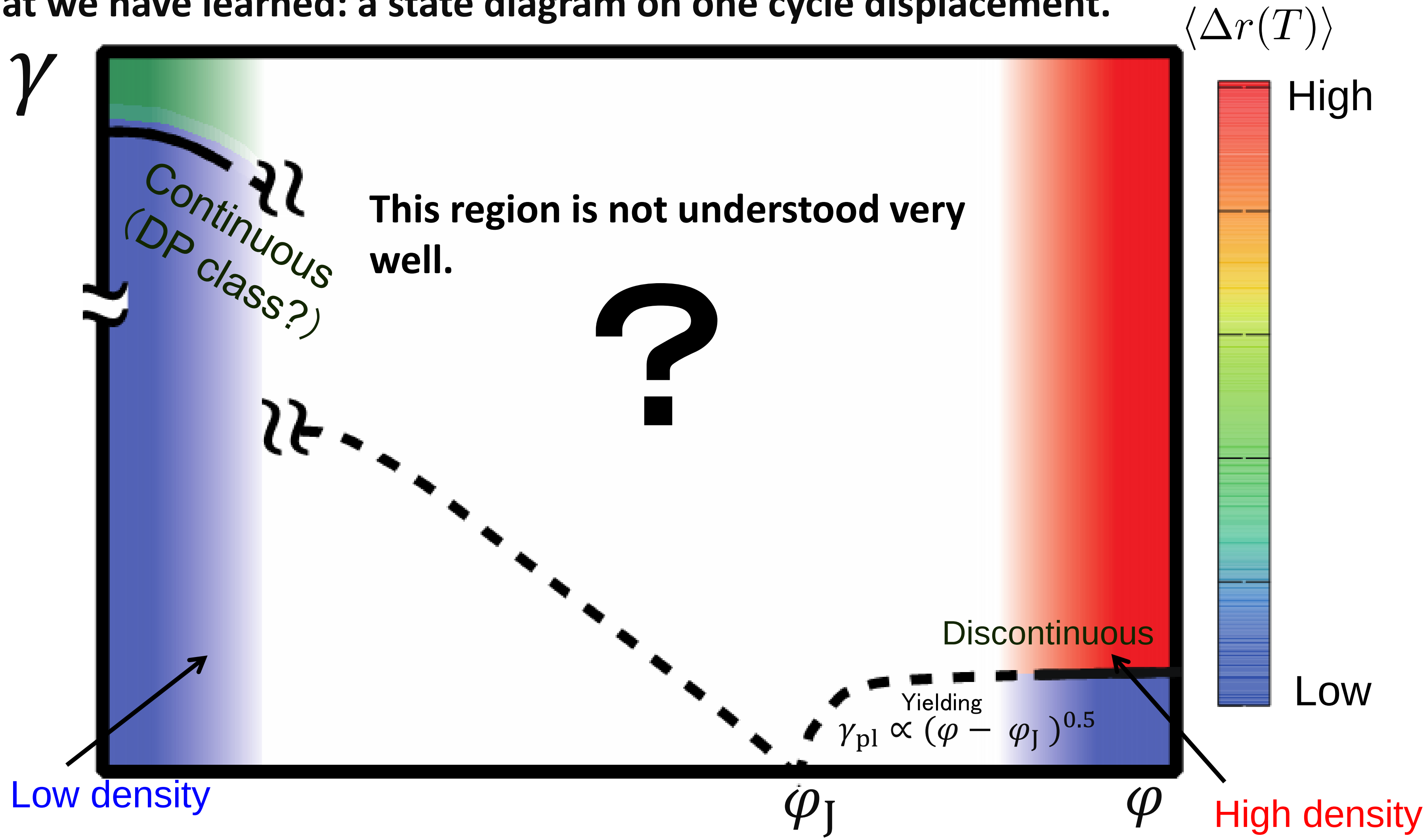
close relation

Yielding transition is an origin of discontinuous RI transition.

T. Kawasaki and L. Berthier, Phys. Rev. E **94**, 022615 (2016).

Purpose

What we have learned: a state diagram on one cycle displacement.



L. Corté, et al., Nat Physics **4**, 420 (2008)

T. Kawasaki and L. Berthier, Phys. Rev. E **94**, 022615 (2016).
E. Tjhung and L. Berthier, Phys. Rev. E **96**, 050601 (2017).

Purpose : To understand RI transition in wide range of density across jamming transition.

Method : Numerical simulations of athermal particles with oscillatory shear.

Numerical methods

- **2D overdamped athermal particles with oscillatory shear, (Lees- Edwards boundary condition)**

$$\xi_{\mathrm{s}}\left[\frac{\partial \mathbf{r}_i}{\partial t}-\dot{\gamma}(t)y_i\mathbf{e}_x\right]+\sum_j\frac{\partial U(r_{ij})}{\partial \mathbf{r}_i}=\mathbf{0}$$

- **Interaction potential (Harmonic sphere)**

$$U(r_{ij})=\frac{\epsilon}{2}\left(1-r_{ij}/a_{ij}\right)^2\Theta(a_{ij}-r_{ij}),$$

- **Strain evolution with oscillation**

$$\dot{\gamma}(t)=\gamma_0\omega\sin(\omega t)\qquad \omega=2\pi/T$$

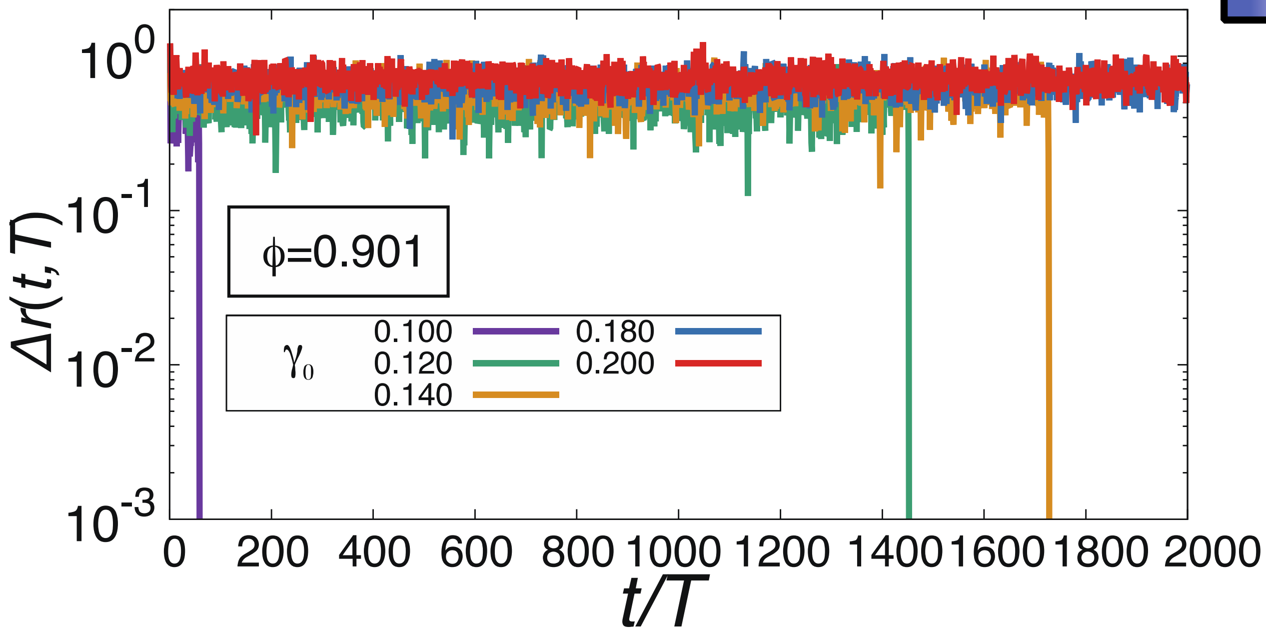
$$\gamma(t)=\gamma_0[1-\cos(\omega t)],$$

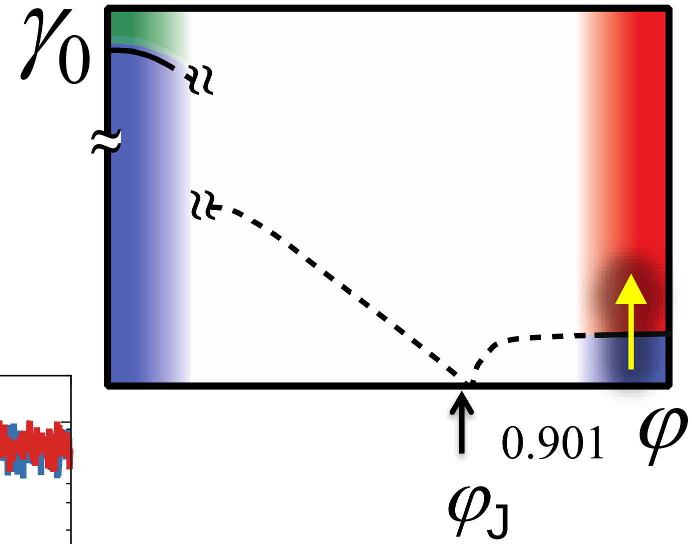
- **Oscillation period:** $T=10^4\tau$ $(\tau=\xi_s\sigma^2/\varepsilon)$
- **Simulation time:** $4000T$
- **System size:** $L=20\sigma$ ($L=40\sigma, 80\sigma, 160\sigma$: check of finite size effect)
- **Packing fraction:** $\varphi=0.70-1.0$ cf: $\varphi_J(2D)=0.843$

Results: One cycle trajectory above jamming transition density

Displacement per cycle

$$\Delta r(t,T) = \frac{1}{N}\sum_j |\vec{r}_j(t+T) - \vec{r}_j(t)|$$



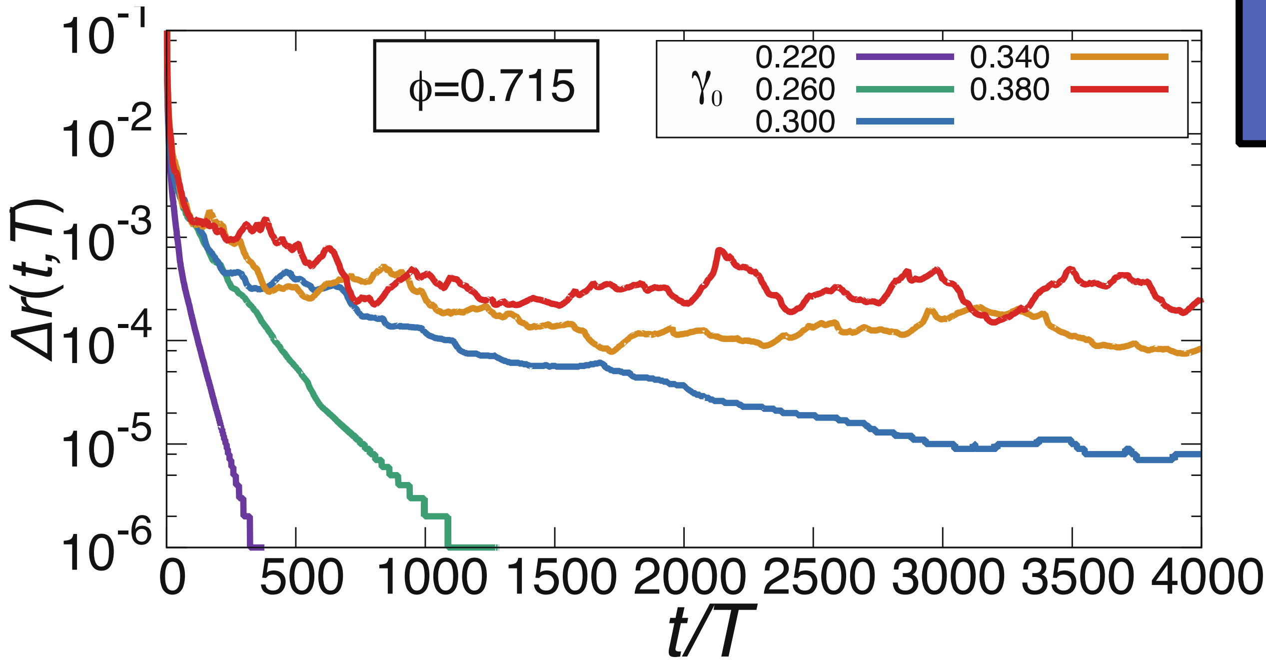


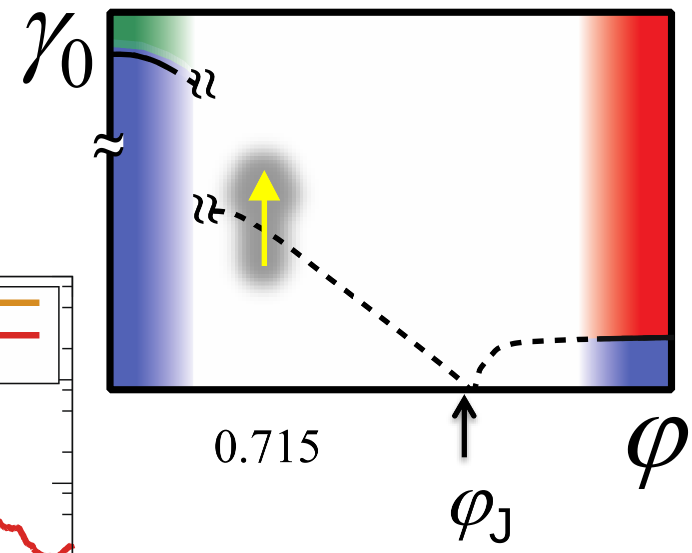
Discontinuous transition

Results: One cycle trajectory in low density

Displacement per cycle

$$\Delta r(t,T) = \frac{1}{N}\sum_j |\vec{r}_j(t+T) - \vec{r}_j(t)|$$



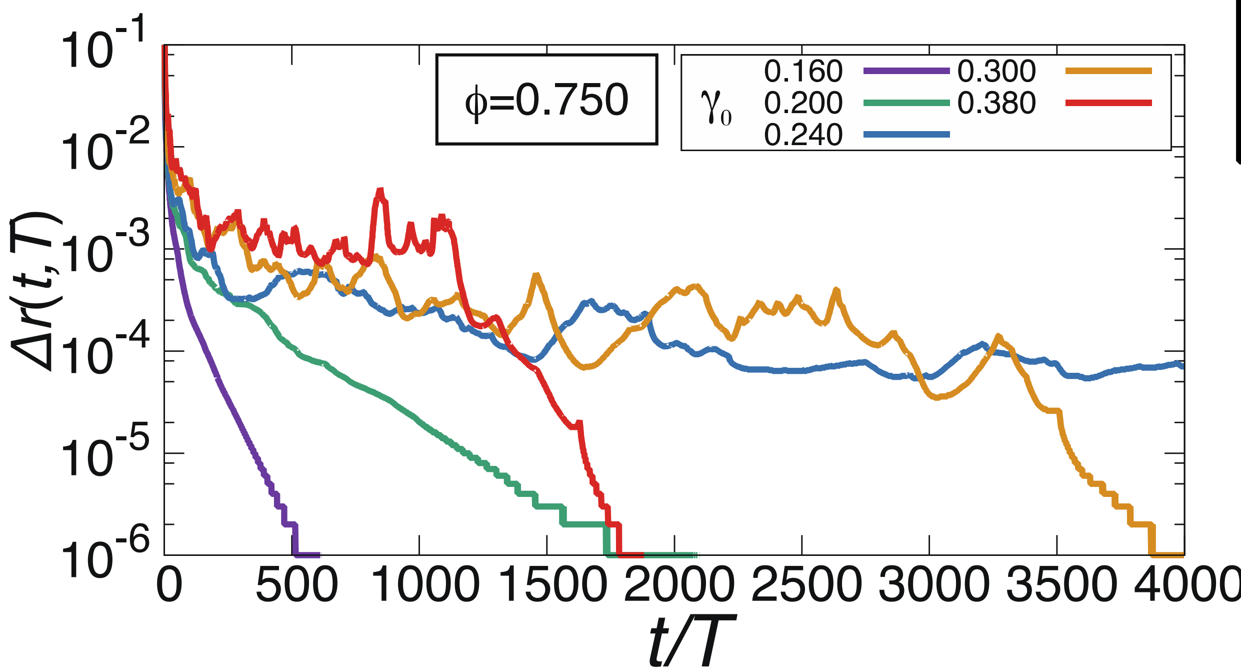


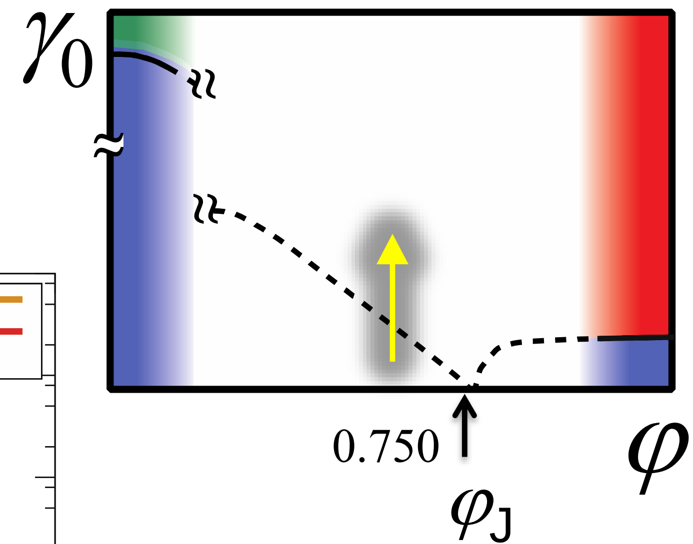
Continuous transition

Results: One cycle trajectory below jamming transition density

Displacement per cycle

$$\Delta r(t,T) = \frac{1}{N}\sum_j |\vec{r}_j(t+T) - \vec{r}_j(t)|$$

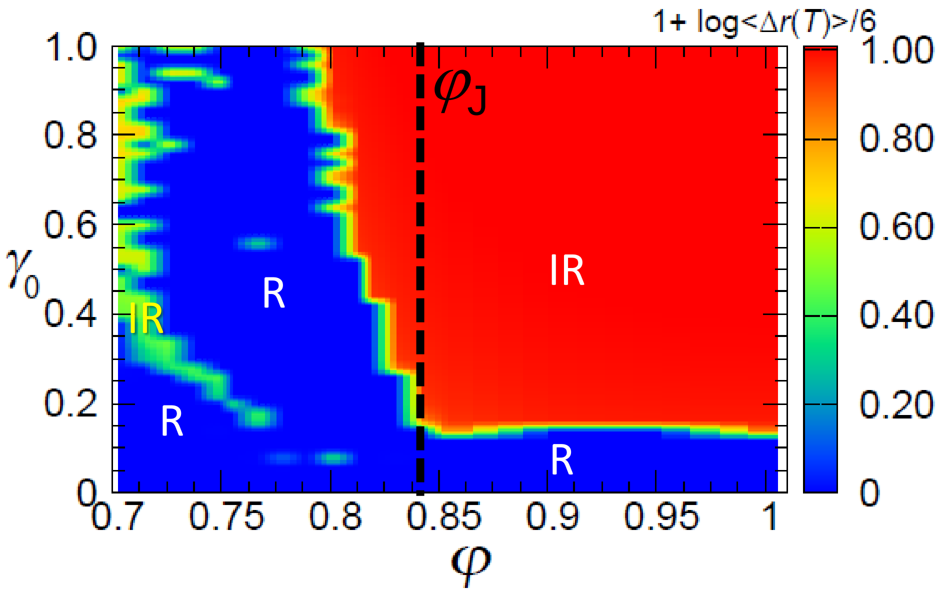




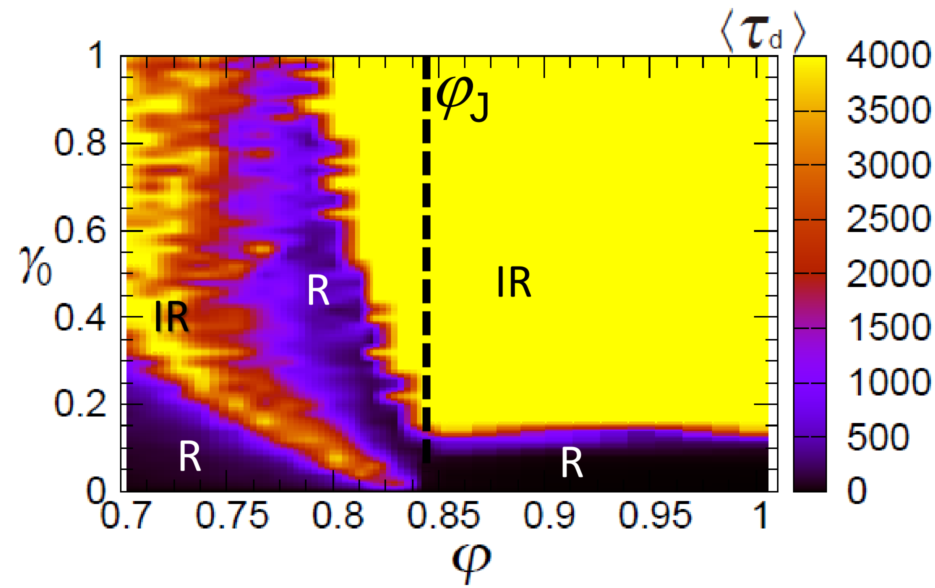
Reentrant transition

Results: State diagrams for R-I transitions

One cycle displacement



Life time of the irreversible state



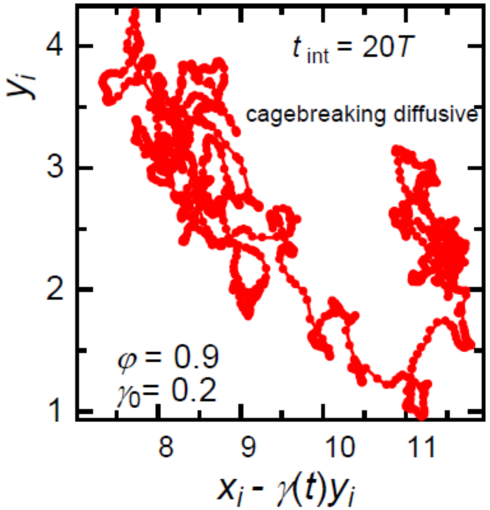
R: reversible, IR: irreversible

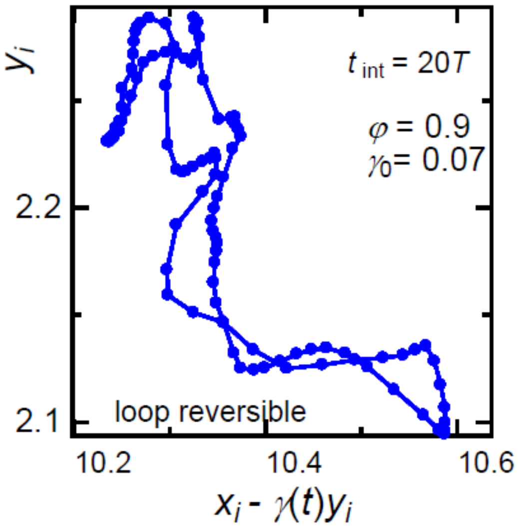
➡ To clarify the details of R-I states, we investigate single-particle trajectory, geometry of the particles, and mechanical property.

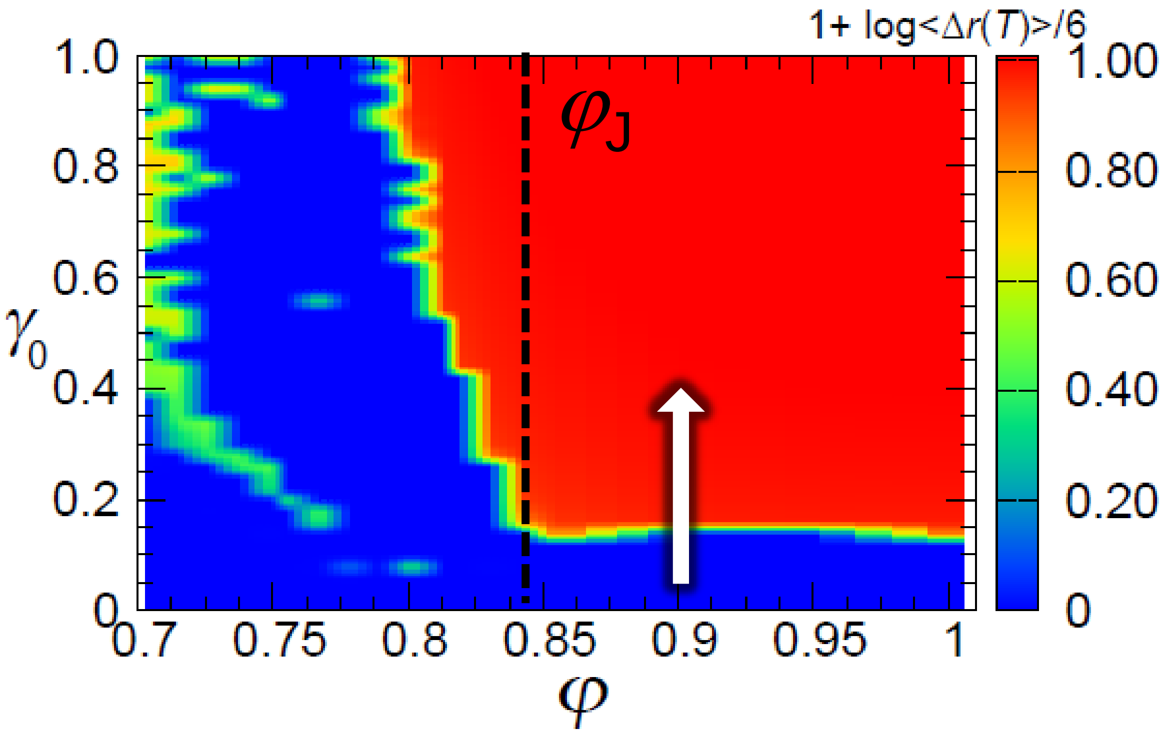
Results: Classifications of the particle trajectories above jamming

Non-affine single particle trajectory

i.e., a trajectory whose affine deformation is removed (**φ** =0.90, **20 cycles)**

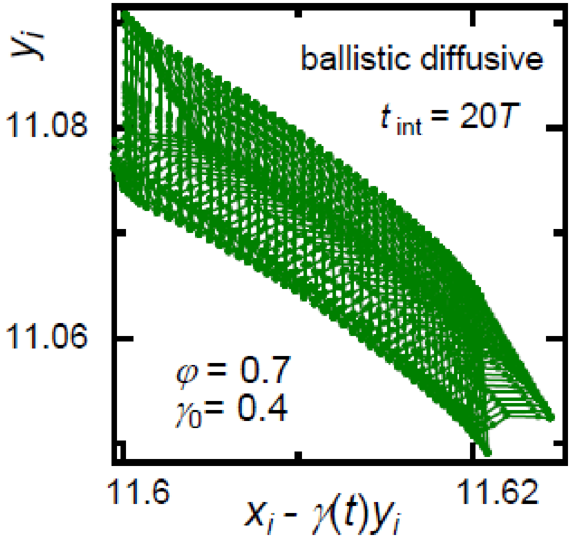


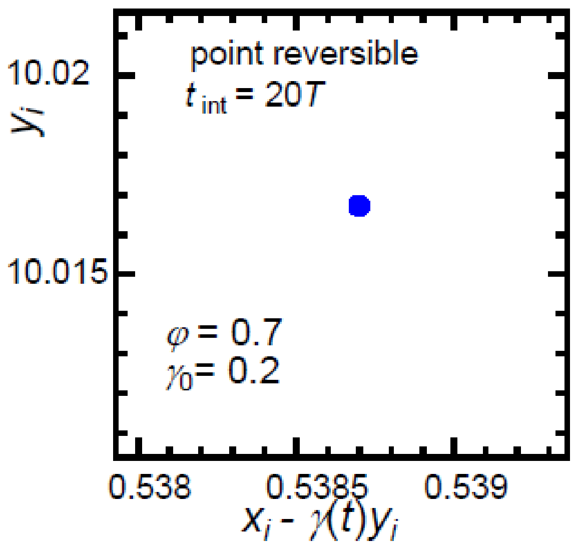


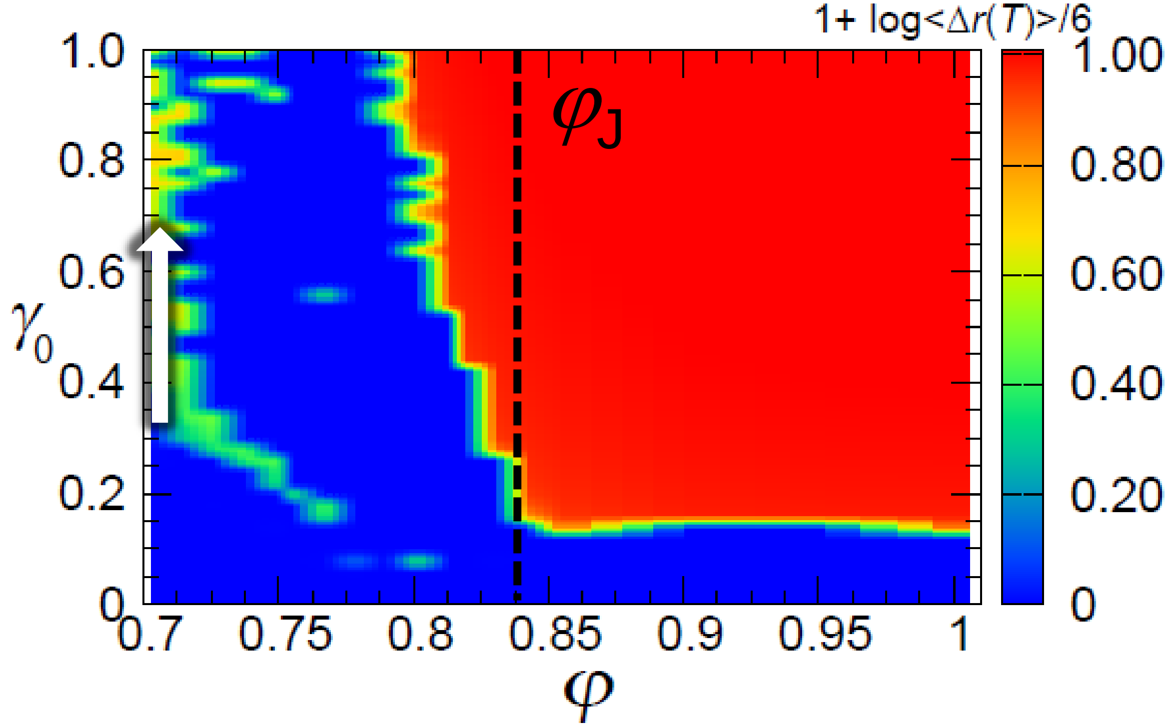


Classifications of the particle trajectories below jamming

Non-affine single particle trajectory (φ =0.70, 20 cycles)

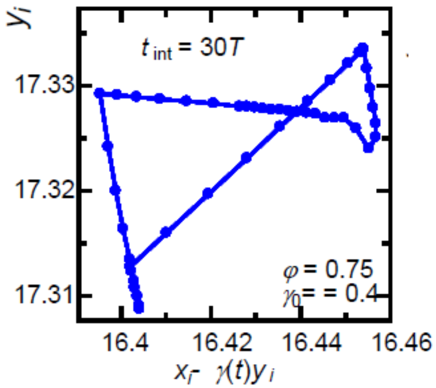


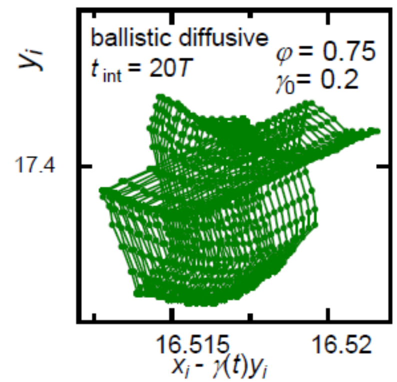


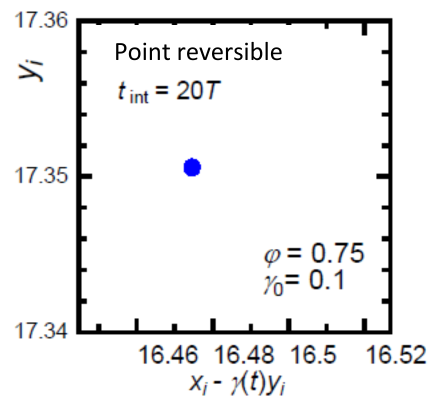


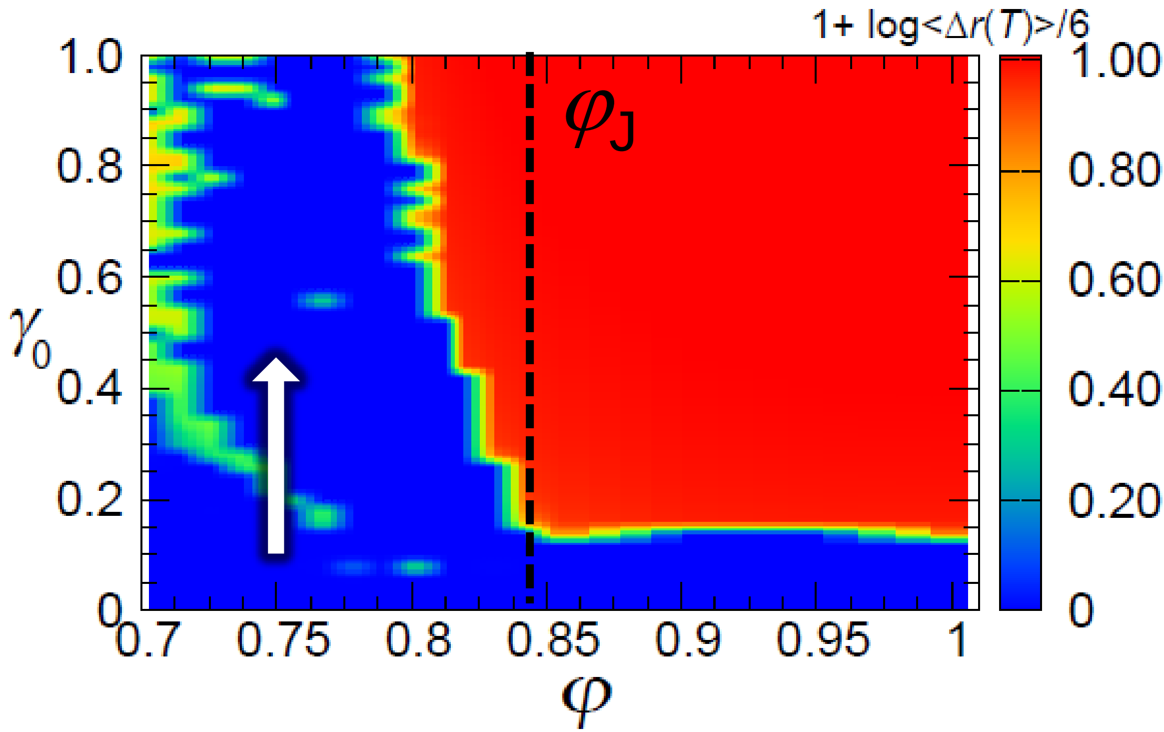
Classifications of the particle trajectories at reentrant regime

Non-affine single particle trajectory (φ =0.75, 20 cycles)

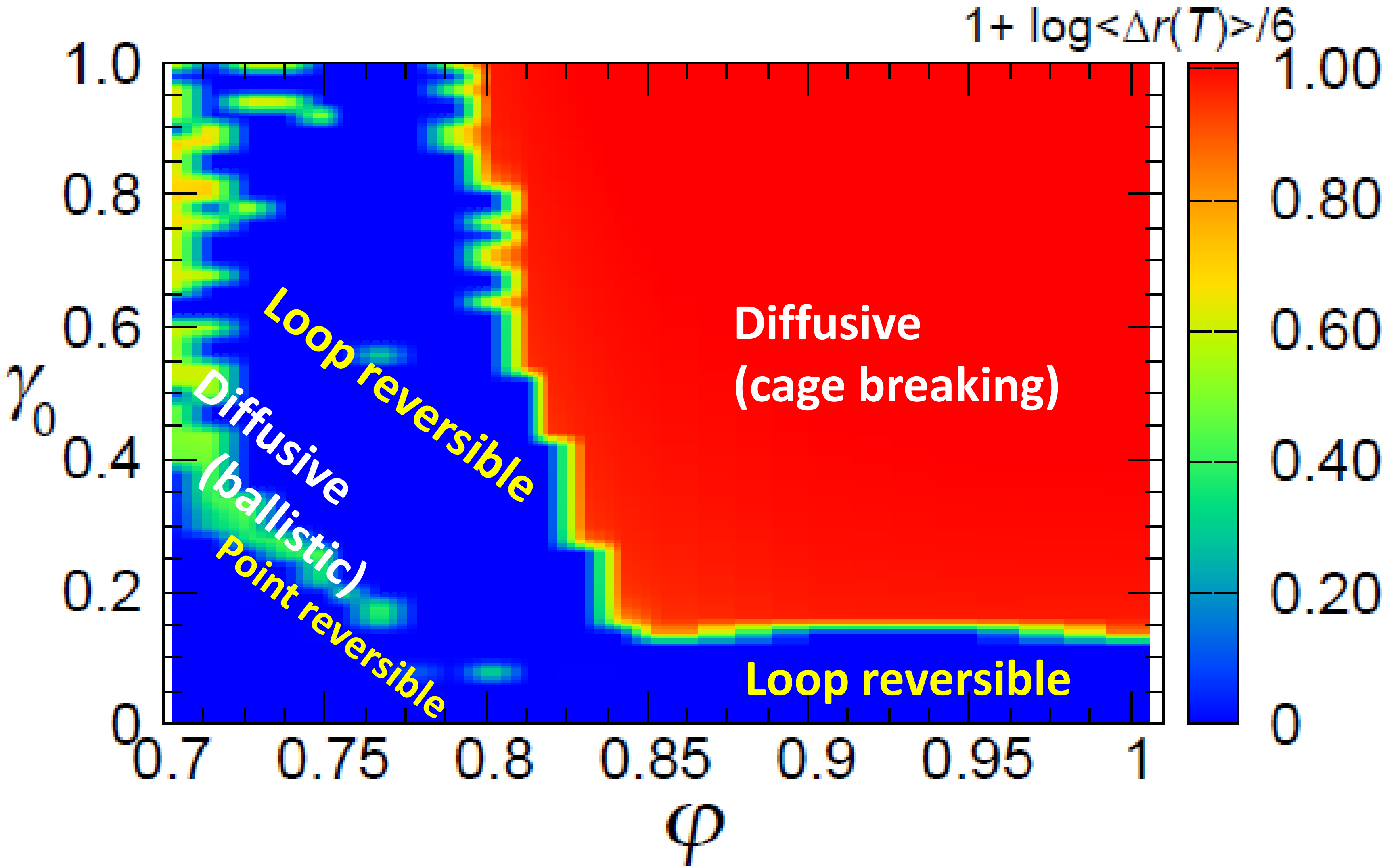






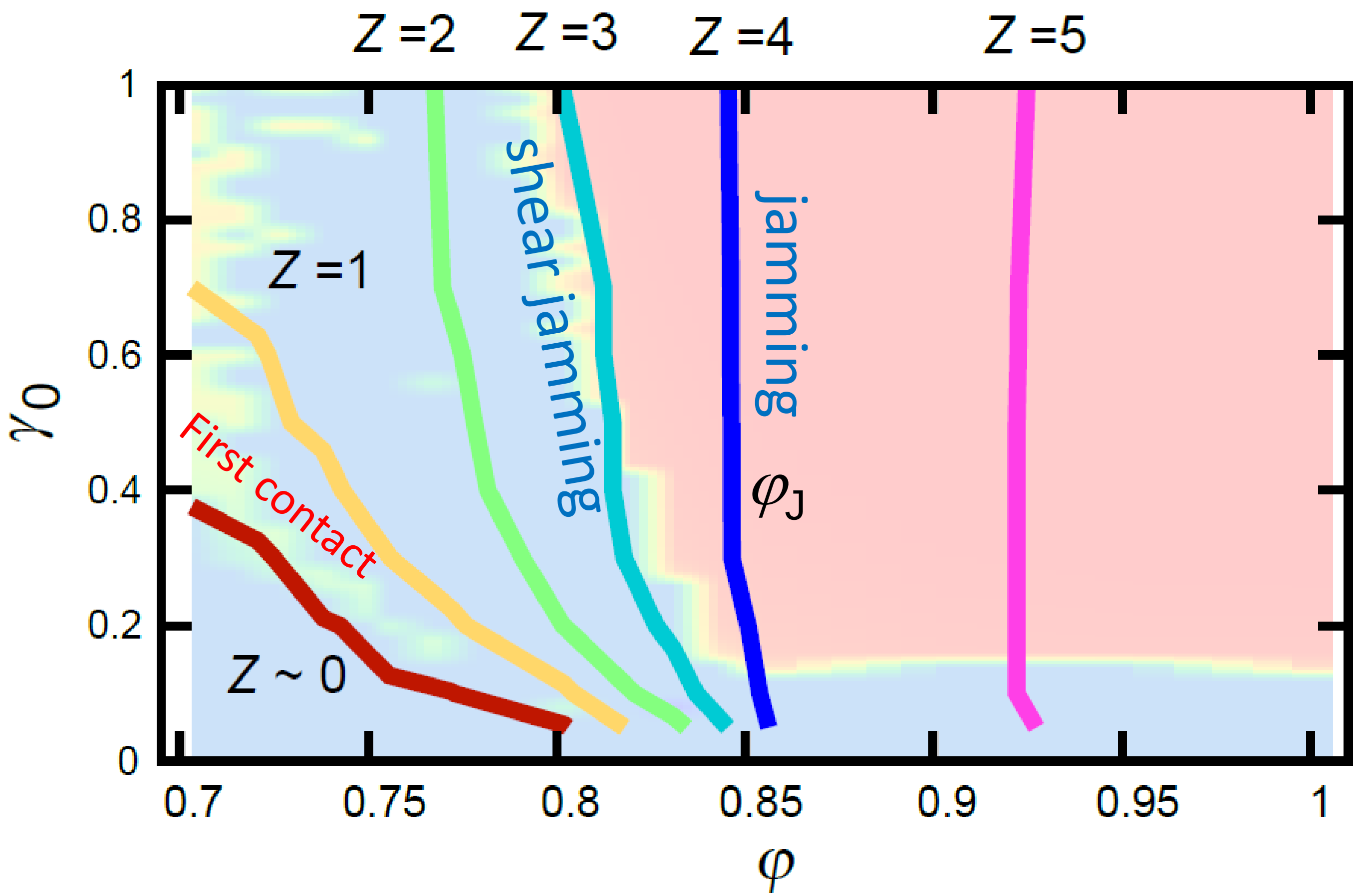


Classifications of the particle trajectories: summary



Relation between R-I transition and coordination number

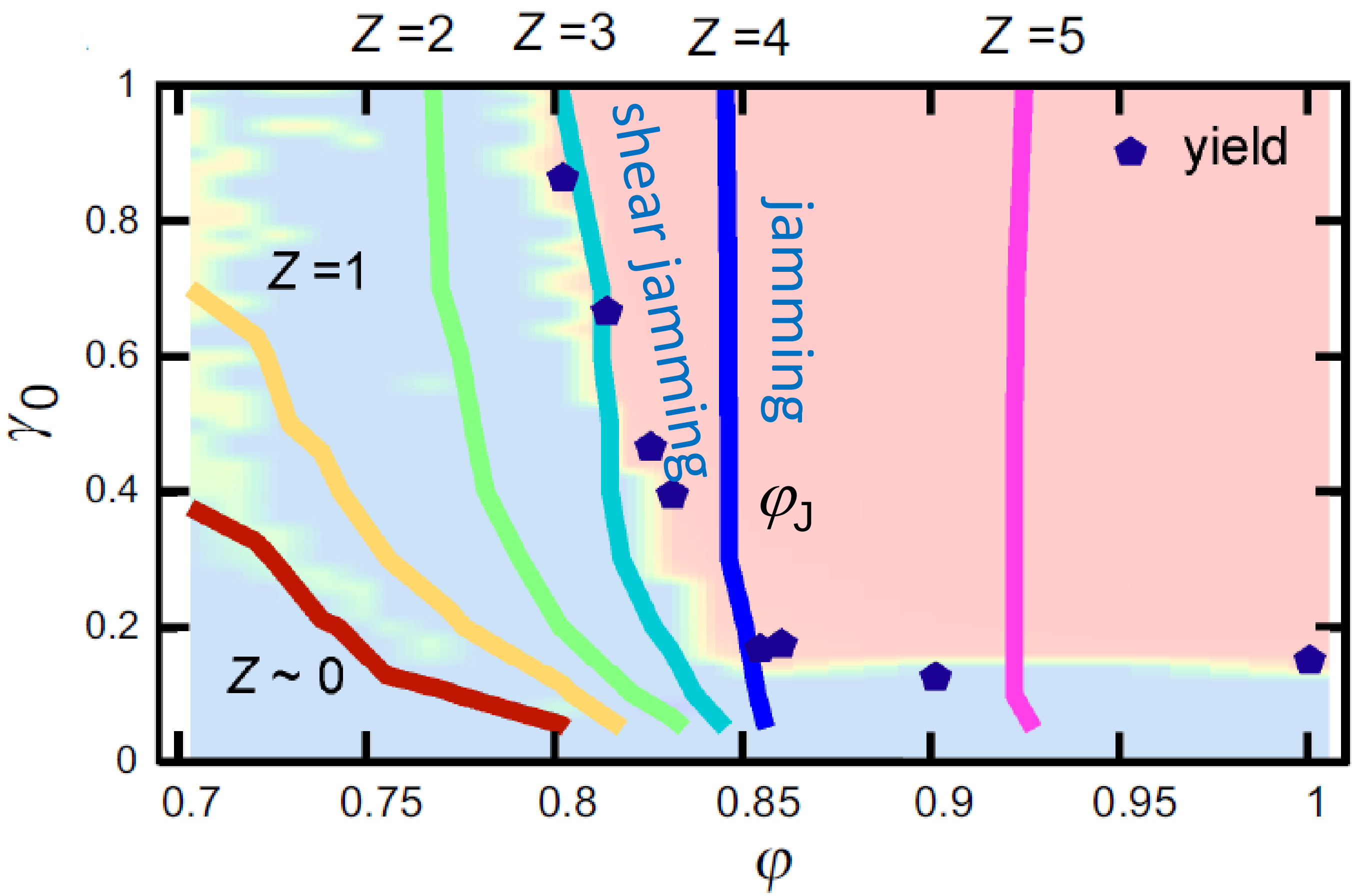
Condition for a stable packing: $Z \geq 4\ (= 2d\ \ isostatic)$



(1) $Z = 3$ in unjam regime: discontinuous R-I transition ➡ Related to shear jamming. [H. A. Vinutha and S. Sastry, Nat Phys **12**, 578 (2016).]

(2) $Z \sim \delta (> 0)$: Continuous transition (3) $Z = 1$: Reentrant transition?

Relationship among the geometry of the particles, mechanical property and RI transition



Yielding is related to the RI transition even below jamming transition density which should be related to a shear jamming.

➡ Finding the correlation among the RI transitions, coordination numbers, and mechanical properties.

Summary

We investigated RI transitions for wide-range of density across the jamming transition.

(1) Found various types of RI transitions around jamming density.

(2) Classified RI transitions on the single particle trajectories.

(3) Found correlations among RI transitions, particle geometry, and mechanical property.

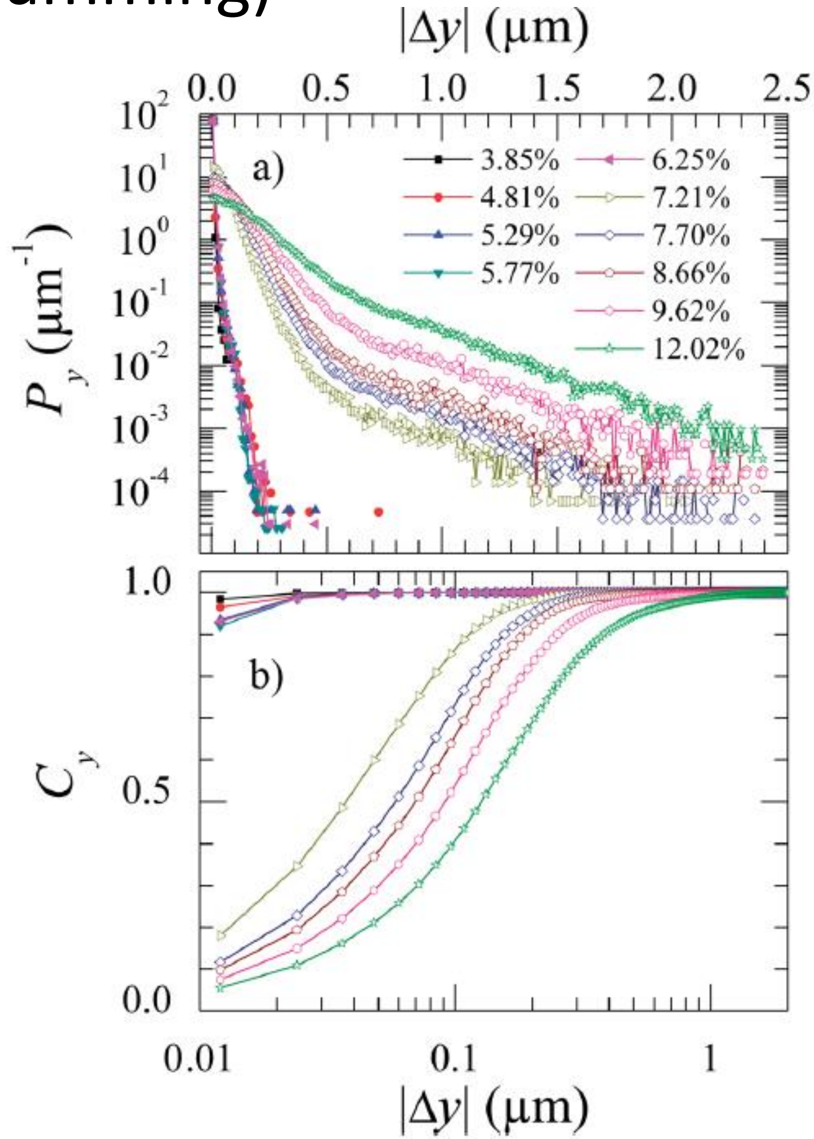
Future work:

Relation between RI transition and yielding transition very close to jamming transition.

K. Nagasawa, K. Miyazaki, and T. Kawasaki , to be submitted.
T. Kawasaki and L. Berthier, Phys. Rev. E **94**, 022615 (2016).

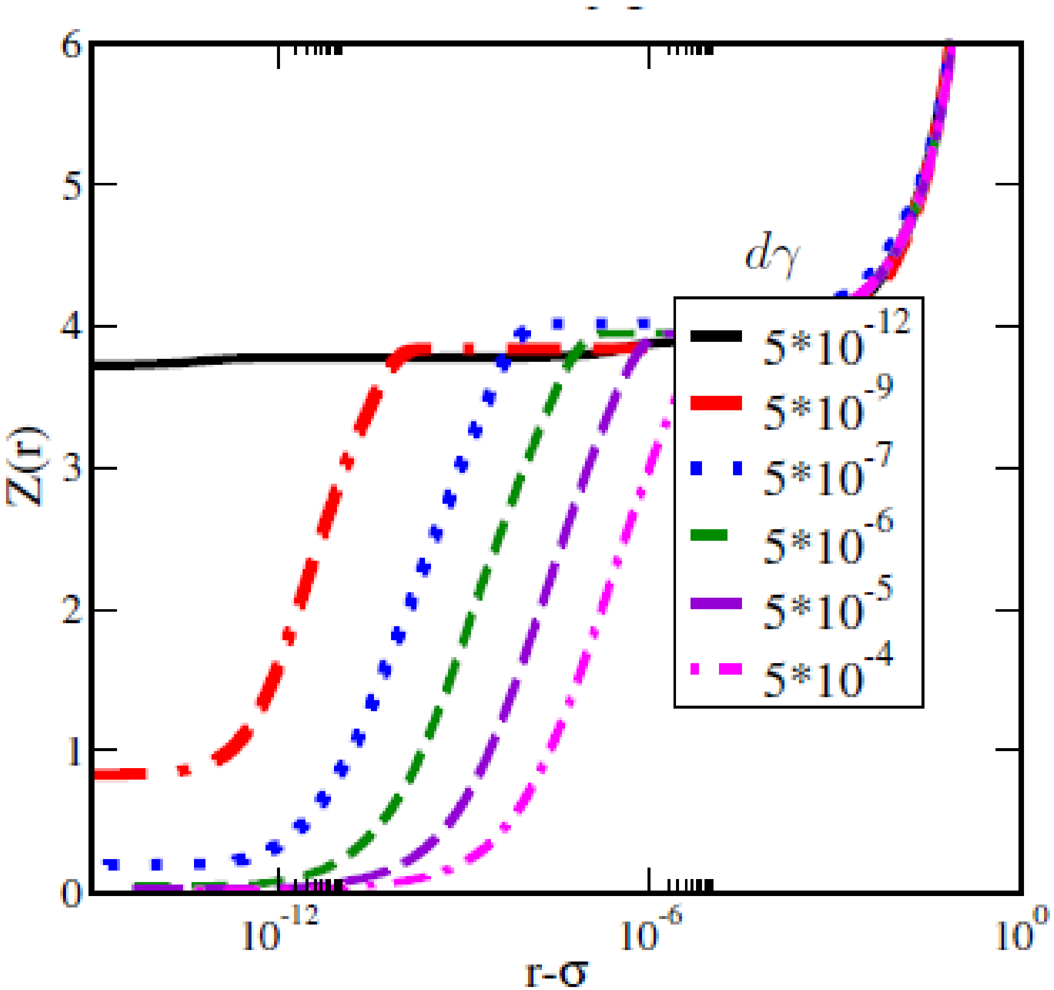
Experiments of emulsions

φ =0.74 (above jamming)



E. D. Knowlton, D. J. Pine, and L. Cipelletti, Soft Matter **10**, 6931 (2014).

Contact number below jamming



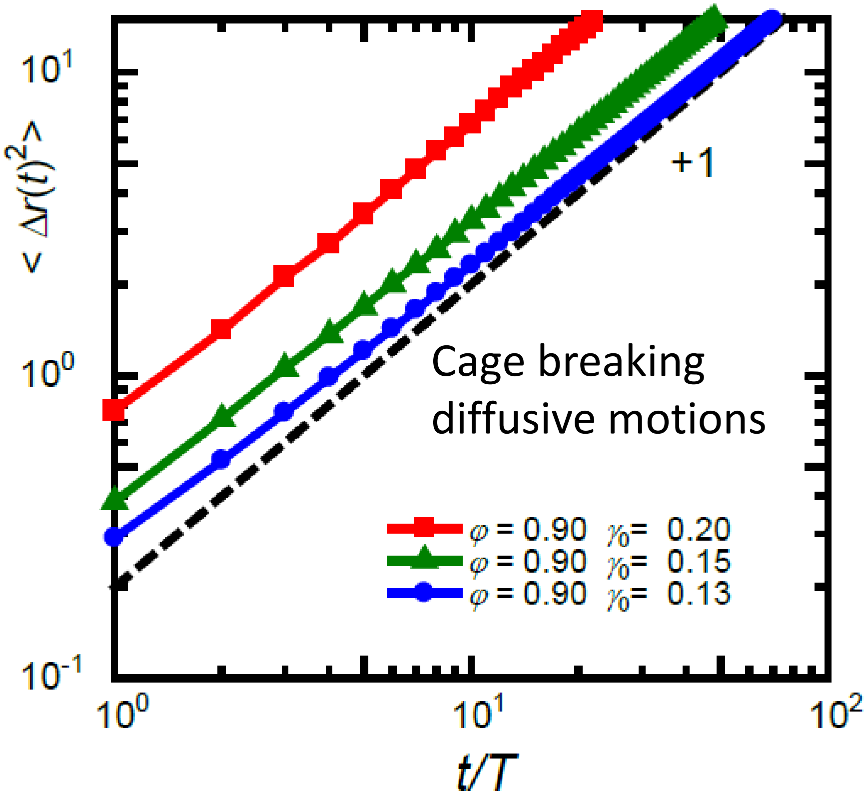
r : Threshold distance for determining the nearest neighbors.

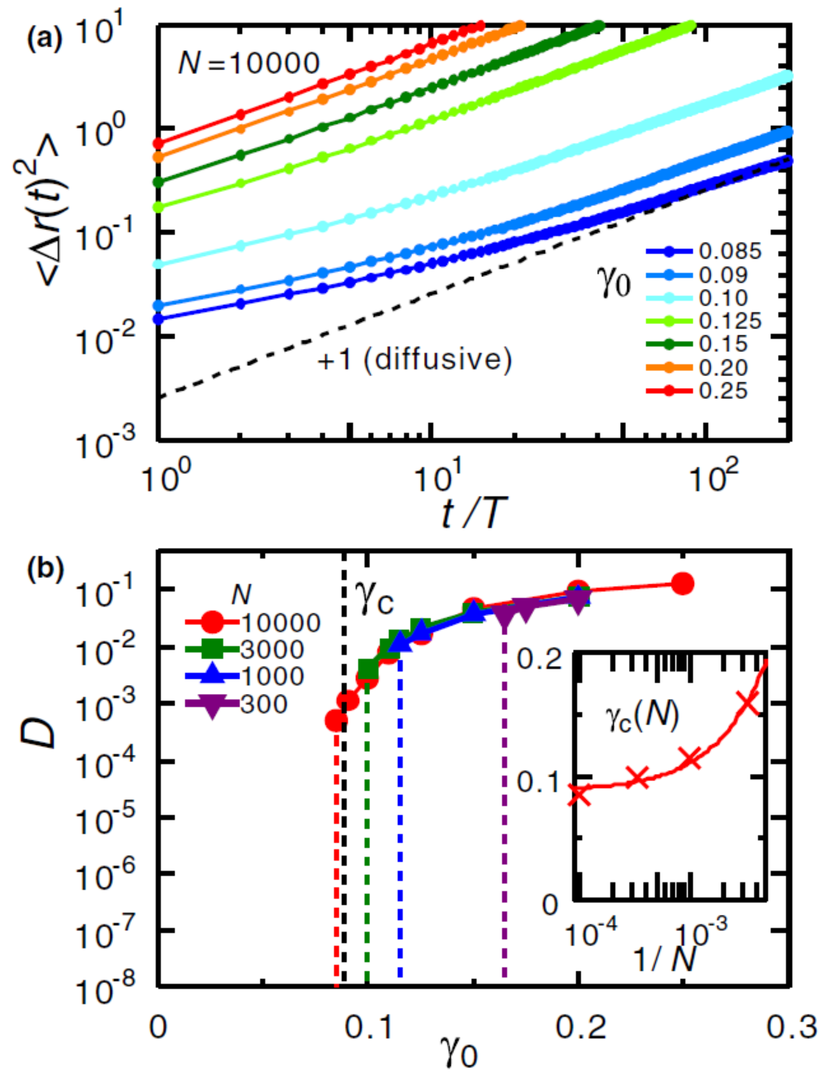
H. A. Vinutha and S. Sastry, Nat Phys **12**, 578 (2016).

Results: Mean squared displacements

(Previous study) 3 dimensional system : $\varphi > \varphi_J$

Mean squared displacements (φ =0.90)

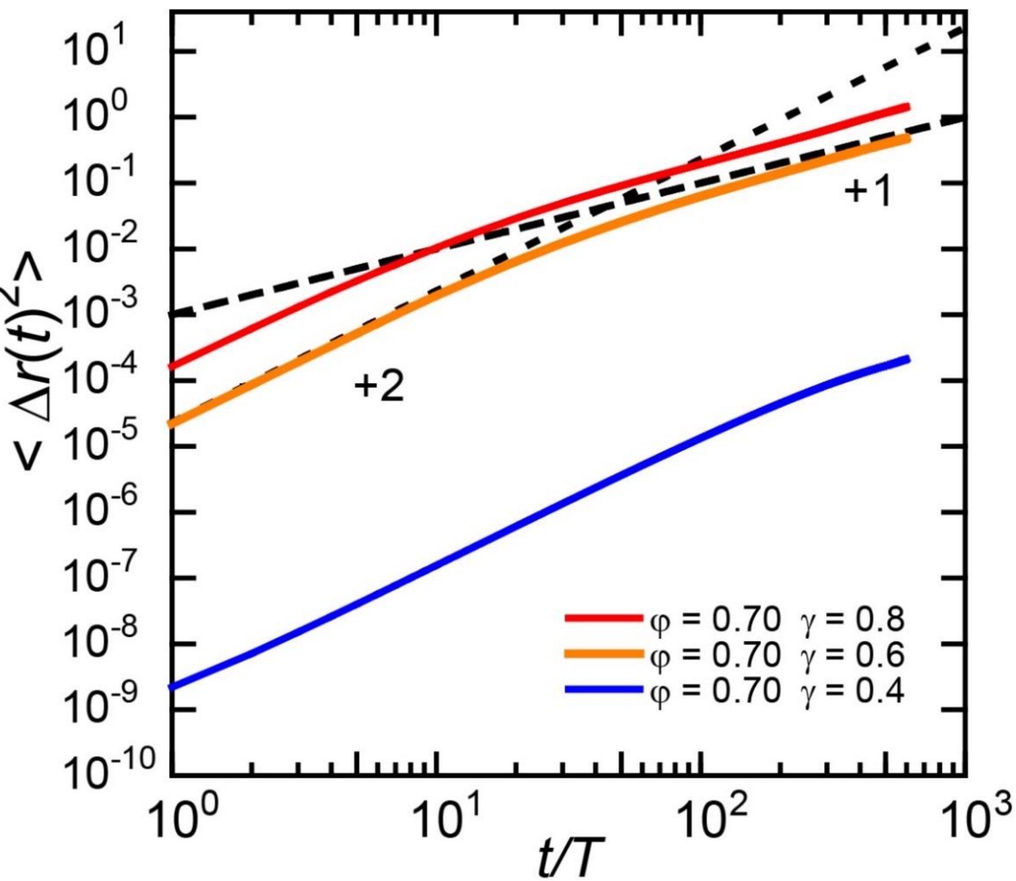


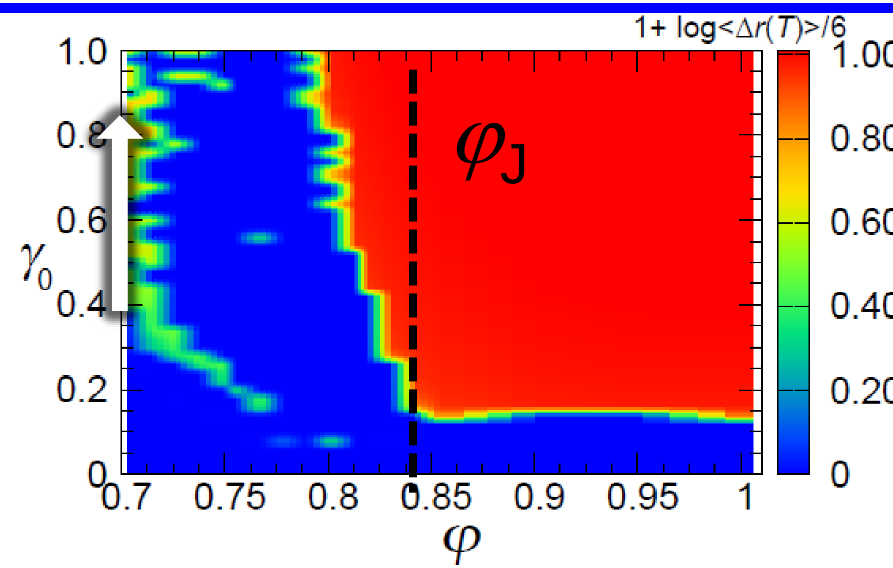


T. Kawasaki and L. Berthier, Phys. Rev. E **94**, 022615 (2016).

Results: Mean squared displacements below jamming

Mean squared displacements (φ =0.70)



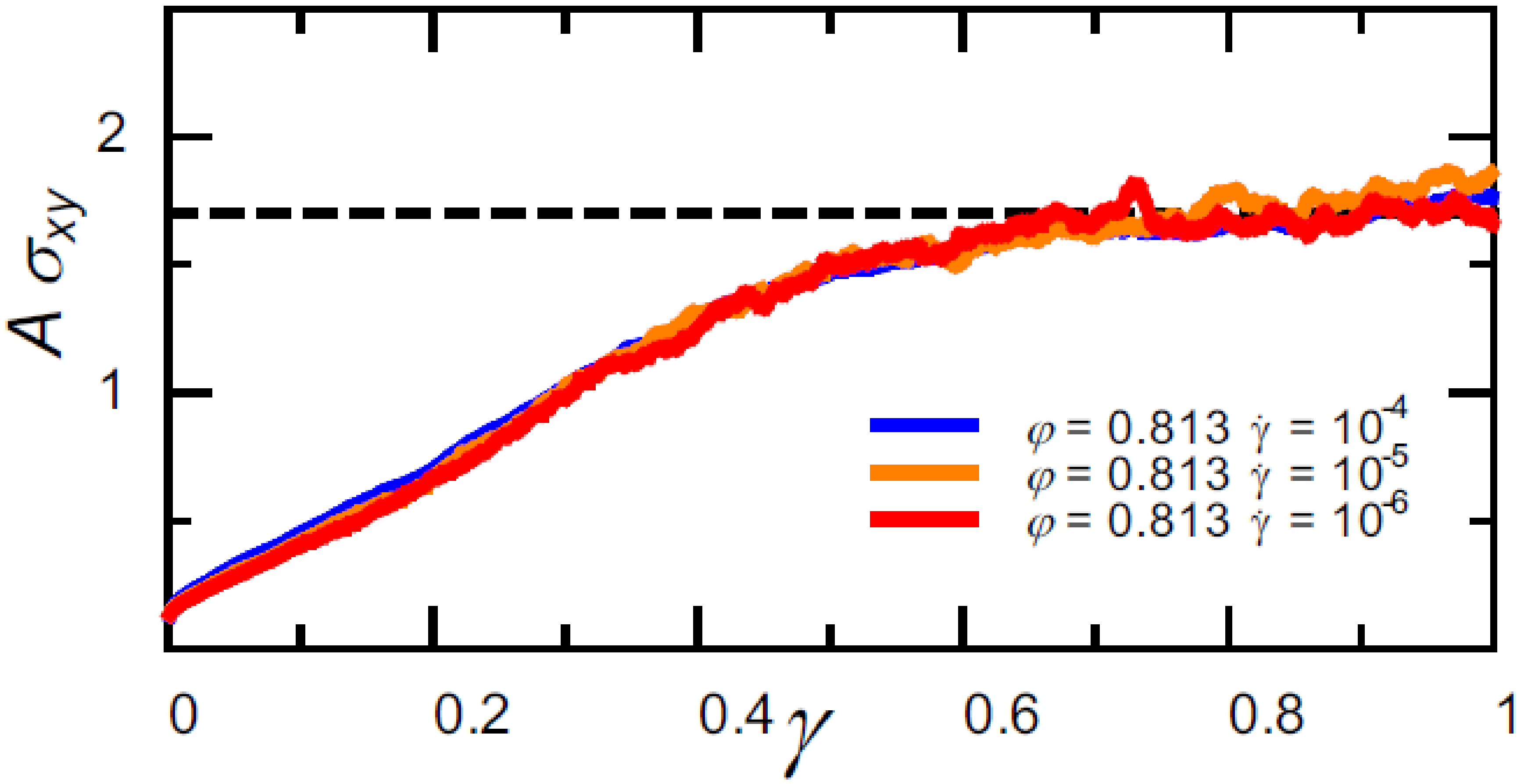


・In shot time regime, particles move ballistically.

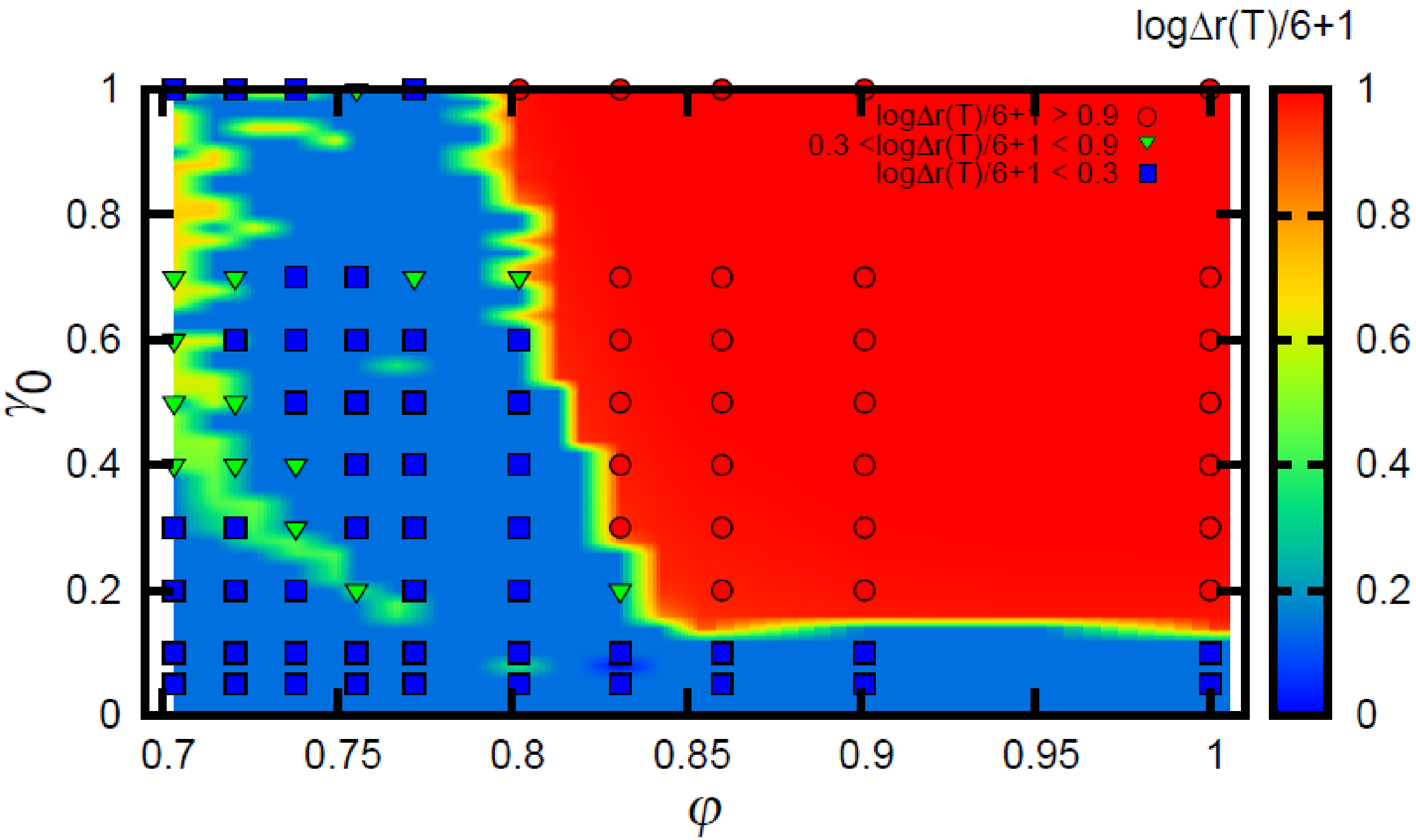
・The lasting time of the ballistic motion becomes critically long close to RI transition point.

Stress strain curve below jamming

Shear rate dependence



AQS



Color map: Finite frequency, $T = 10^4$

Dots: AQS