Shear jamming, discontinuous shear thickening and fragile state in frictional grains under oscillatory shear

arXiv:1810.03846

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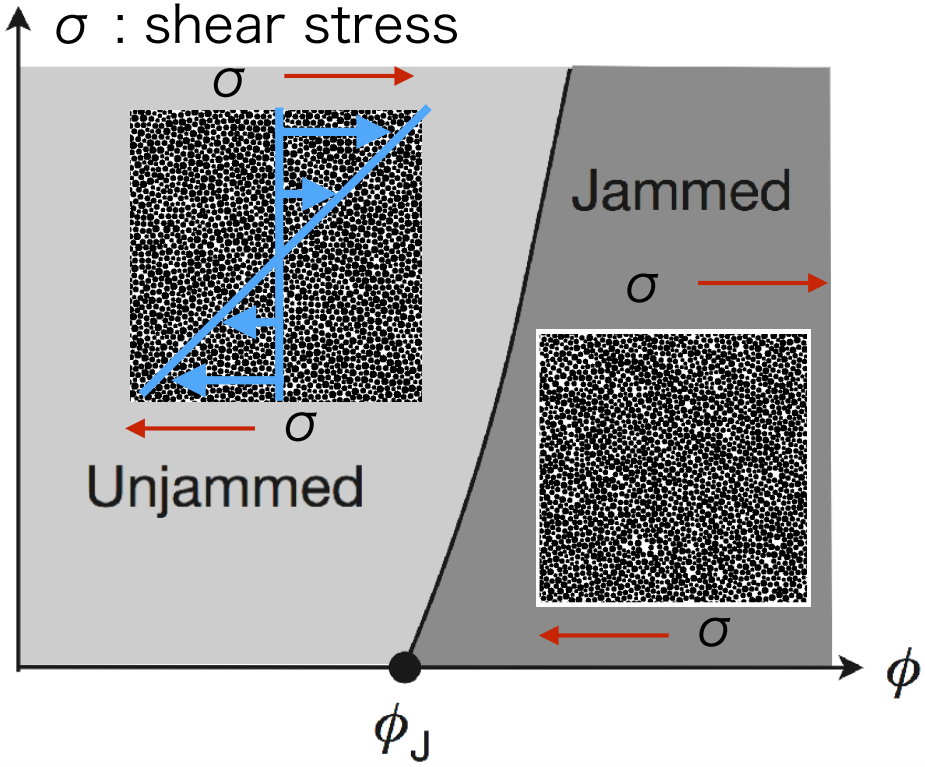
Jamming phase diagram **(Frictionless)**



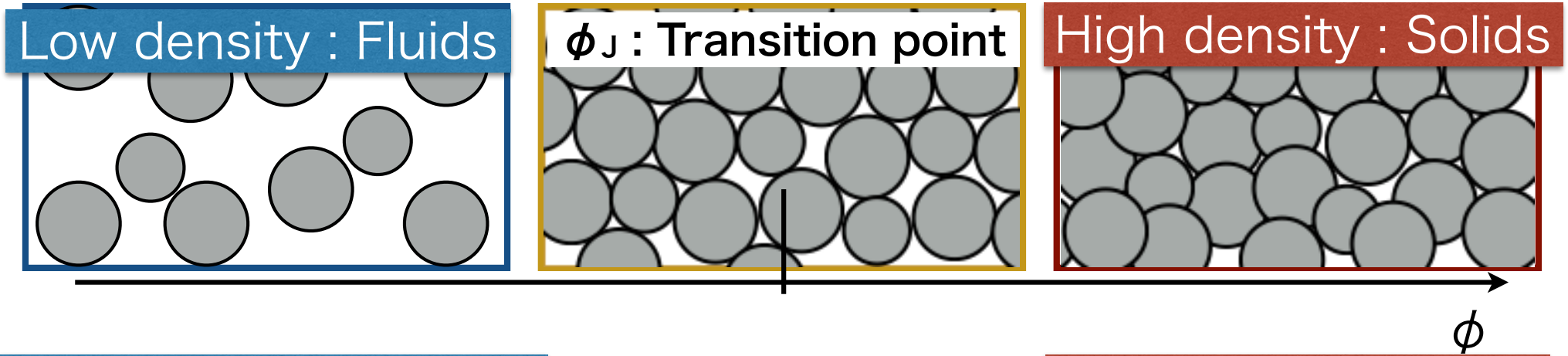
$\phi < \phi_J$

- Granular materials behave like fluids.
- There is no contact network.

$\phi > \phi_J$

- They behave as solids with ever-lasting contacts under small σ.
- They can flow if σ is larger than the yield stress.

ϕ : Packing fraction



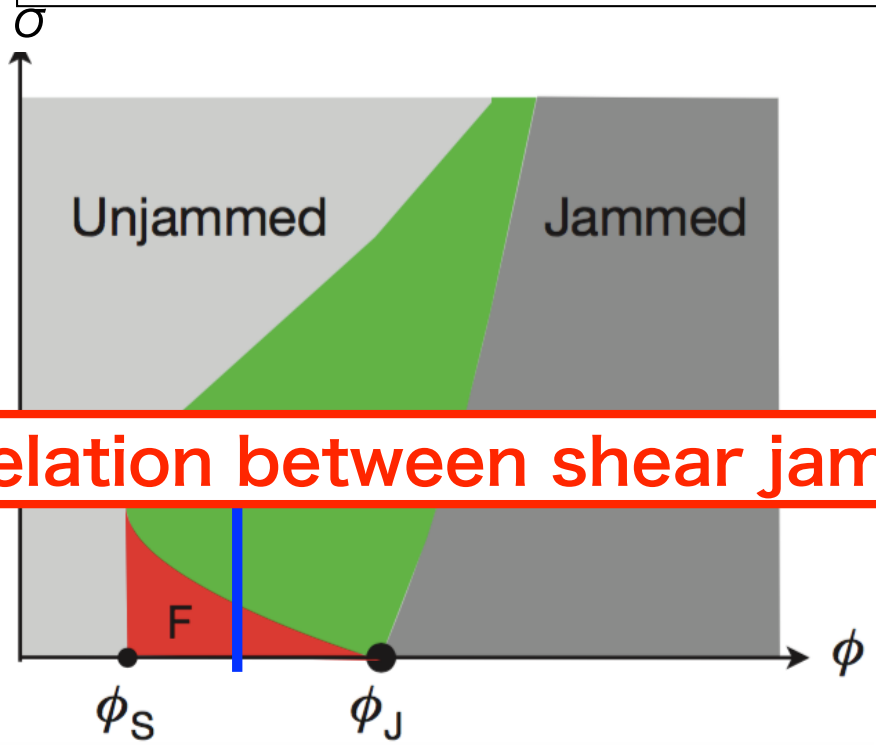
Unnamed : Liquid-like phase

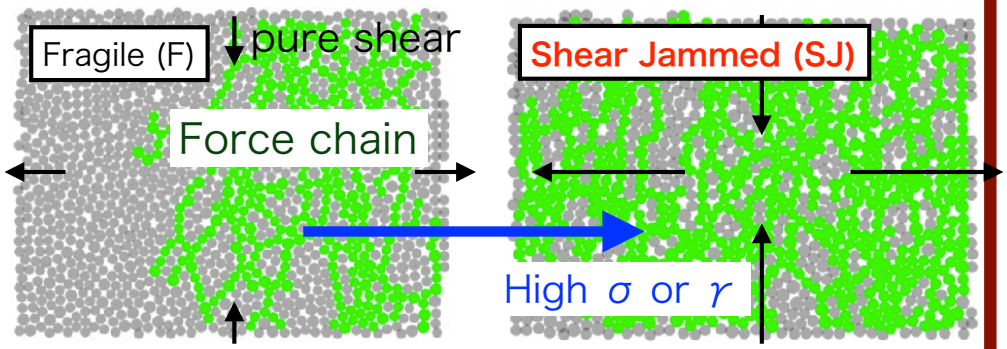
Jammed : Solid-like phase

Effect of friction

Phase diagram : Shear Jamming

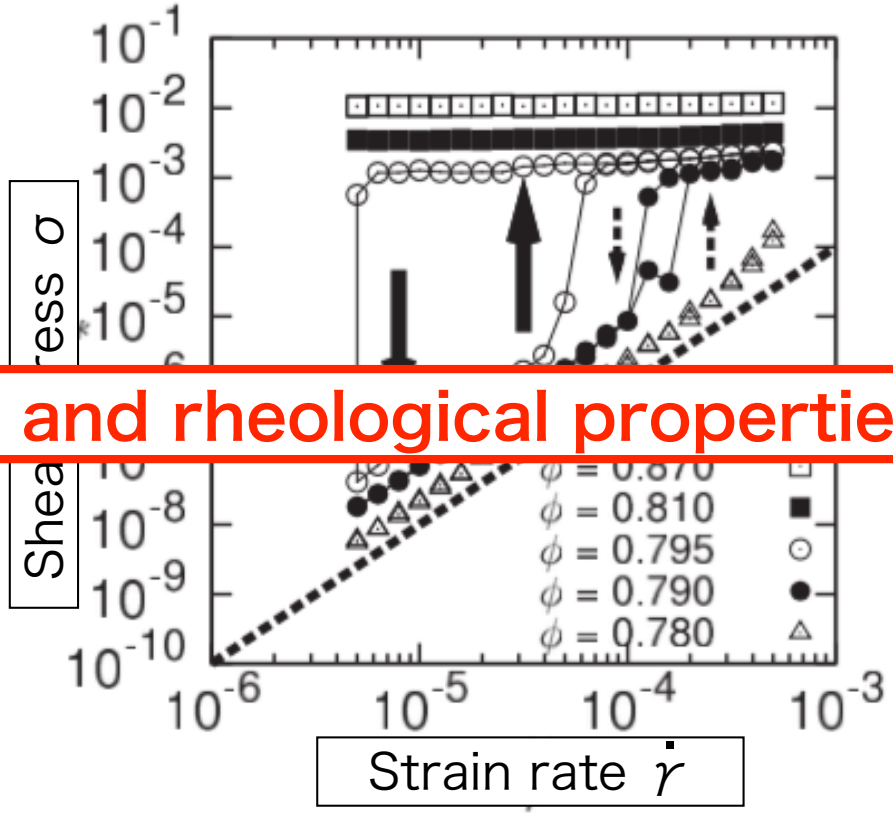
D. Bi, J. Zhang, B. Chakraborty, and R. P. Behringer., (2011).





Rheology : DST

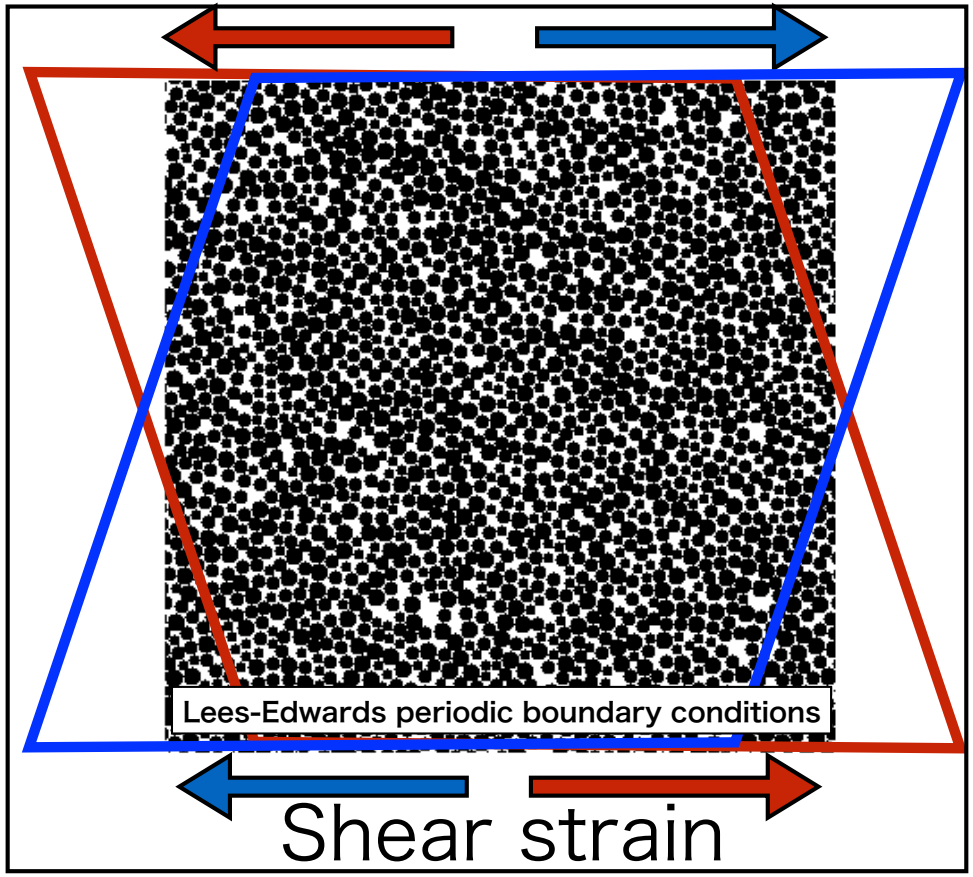
Otsuki, Hayakawa, PRE (2011)



Relation between shear jamming and rheological properties

Discontinuous shear thickening (DST)

Model : Dry granular particles(DEM)



- Oscillatory shear strain :
$$\gamma(t) = \gamma_0(1 - \cos\omega t)$$
- Frequency : ω, Strain amplitude : γ_0

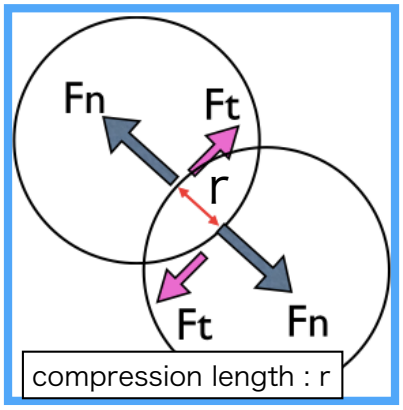
  ω is sufficiently small
- Shear stress : σ

Storage modulus :

$$G' = -\frac{\omega}{\pi}\int_0^{2\pi/\omega} dt \frac{\sigma(t)\cos(\omega t)}{\gamma_0}$$

Loss modulus :

$$G'' = \frac{\omega}{\pi}\int_0^{2\pi/\omega} dt \frac{\sigma(t)\sin(\omega t)}{\gamma_0}$$



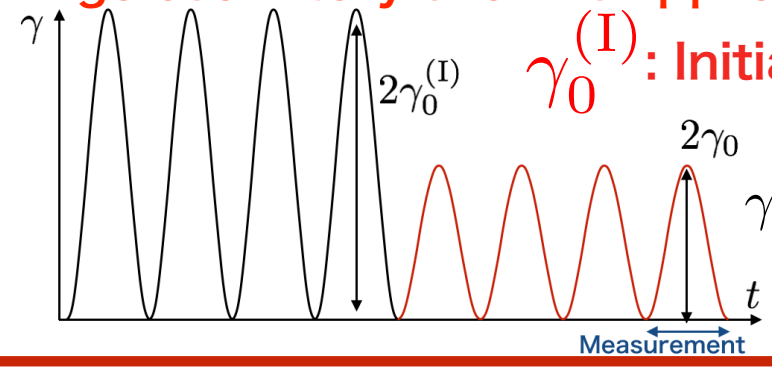
F_n : Normal force

F_t : Tangential force

μ : Friction coefficient

Coulomb's law : $F_t \leq \mu F_n$

Large oscillatory shear is applied first.



$\gamma_0^{(I)}$: Initial amplitude

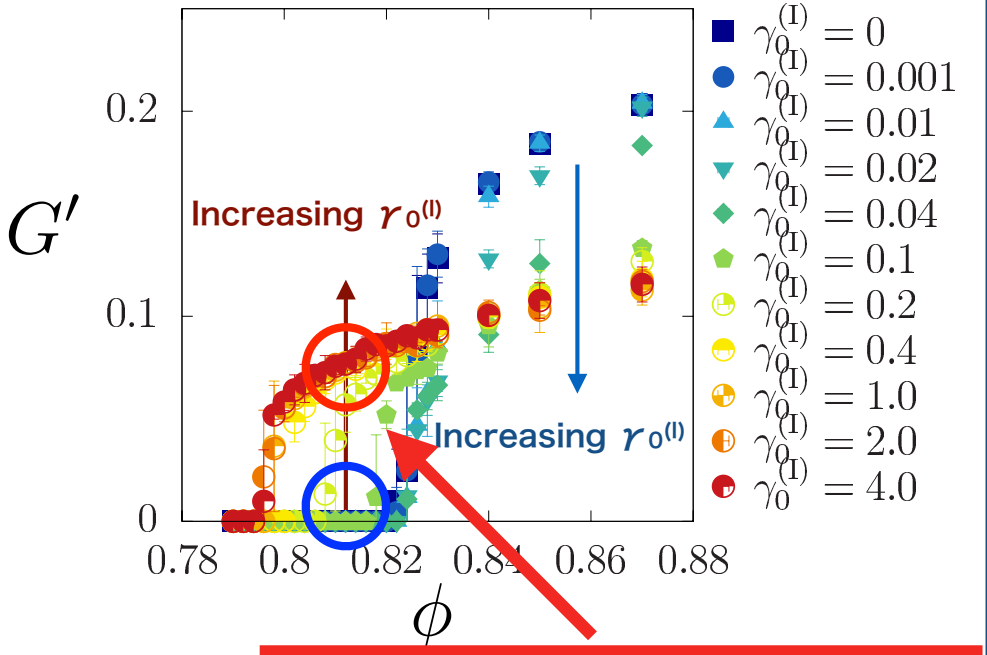
Control parameter

$\gamma_0^{(I)} = 0 \sim 2.0$

$\gamma_0 = 10^{-4}$

Effect of initial shear on shear modulus

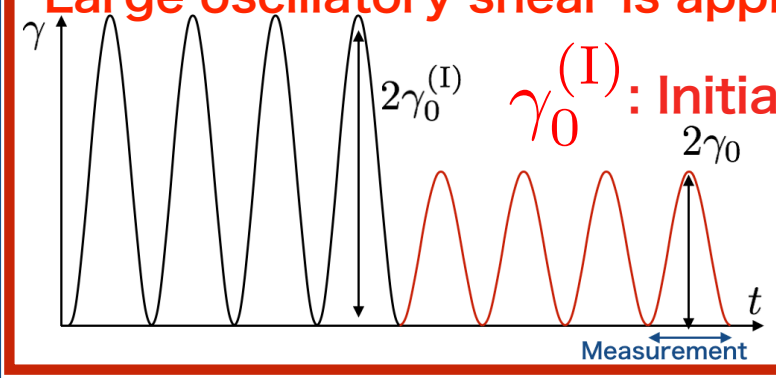
$\mu = 1.0, \gamma_0 = 10^{-4}$



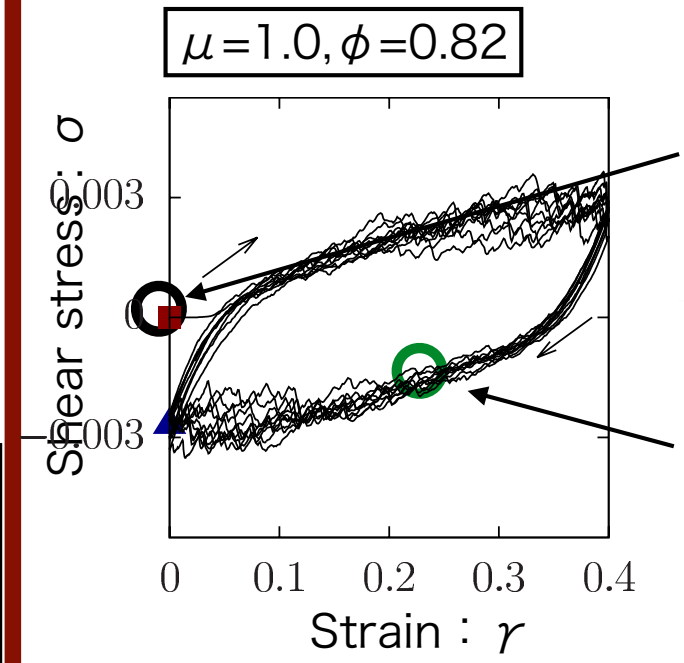
Shear induced jamming

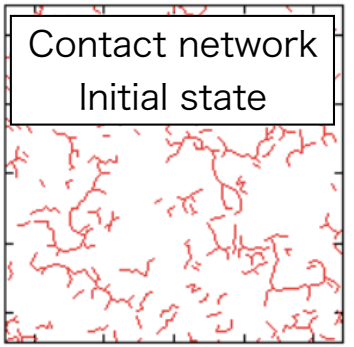
- ϕ_J decreases.
- Large ϕ : G' decreases.
- Small ϕ : G' becomes finite.

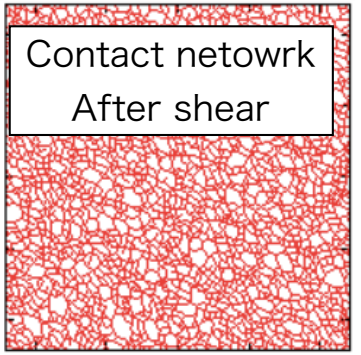
Large oscillatory shear is applied first.



Change in the initial shear







Phase diagram on $\gamma_0^{(\mathrm{I})}$-ϕ plane

Storage modulus : $G'\left(\phi, \gamma_0^{(\mathrm{I})}\right)$

G' without shear:

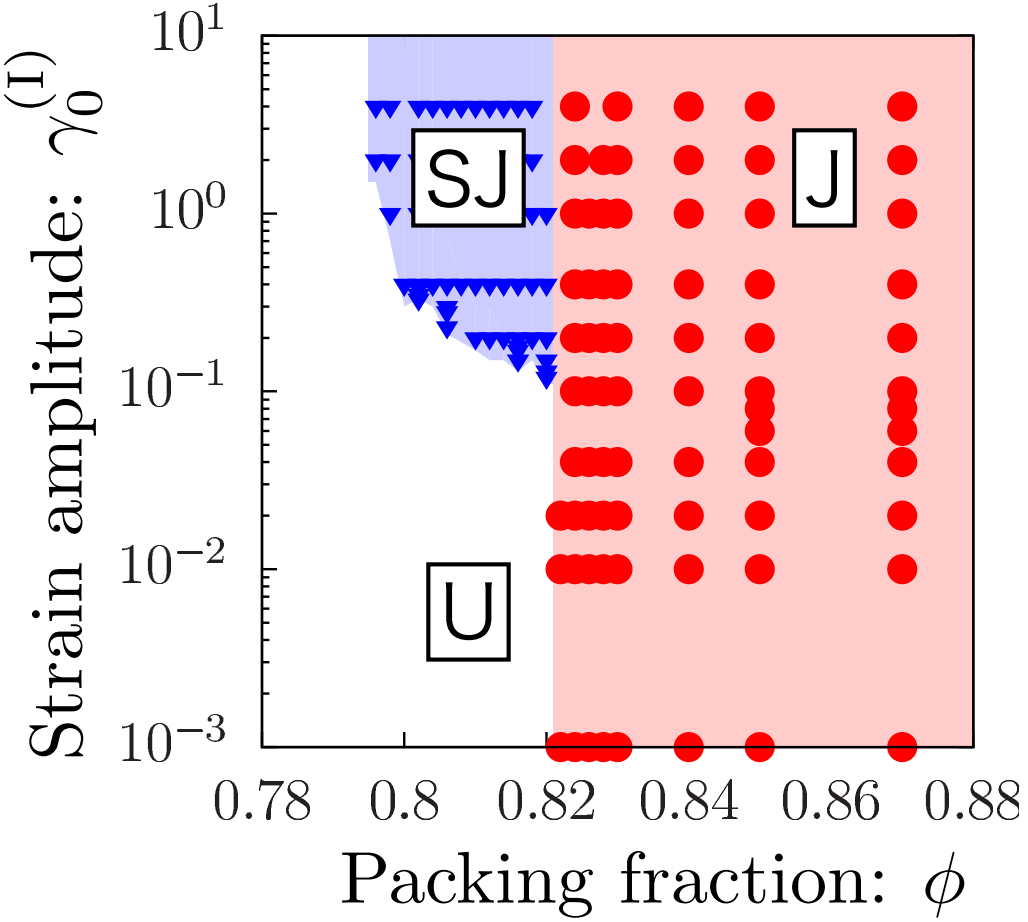
$$G'_0(\phi) \equiv \lim_{\gamma_0^{(\mathrm{I})} \to 0} G'\left(\phi, \gamma_0^{(\mathrm{I})}\right)$$

Unjammed (U) $G'\left(\phi, \gamma_0^{(\mathrm{I})}\right) = 0$

Jammed (J) $G'_0(\phi) > 0$

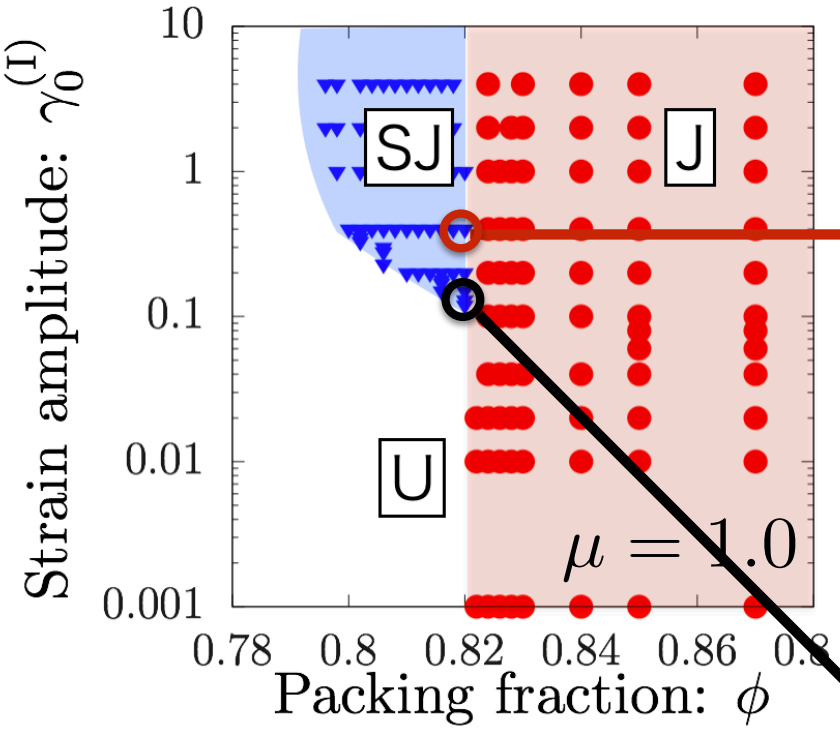
Shear Jammed (SJ)

$$G'_0(\phi) = 0,\ G'\left(\phi, \gamma_0^{(\mathrm{I})}\right) > 0$$

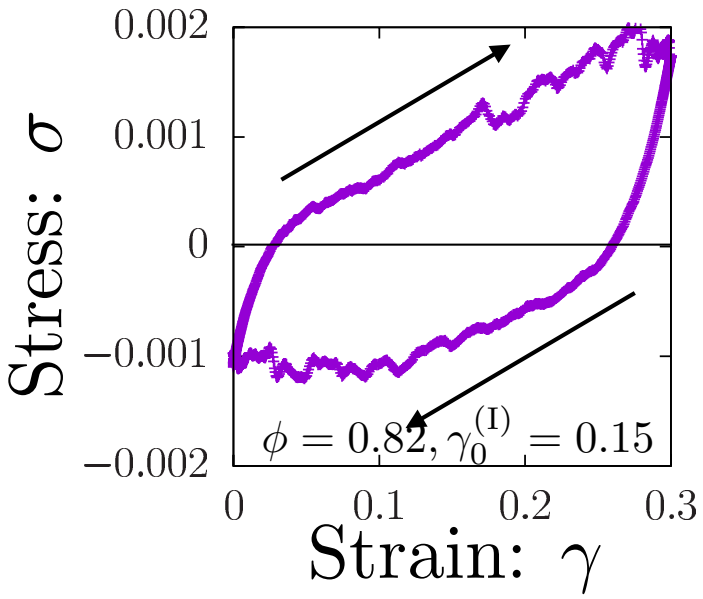


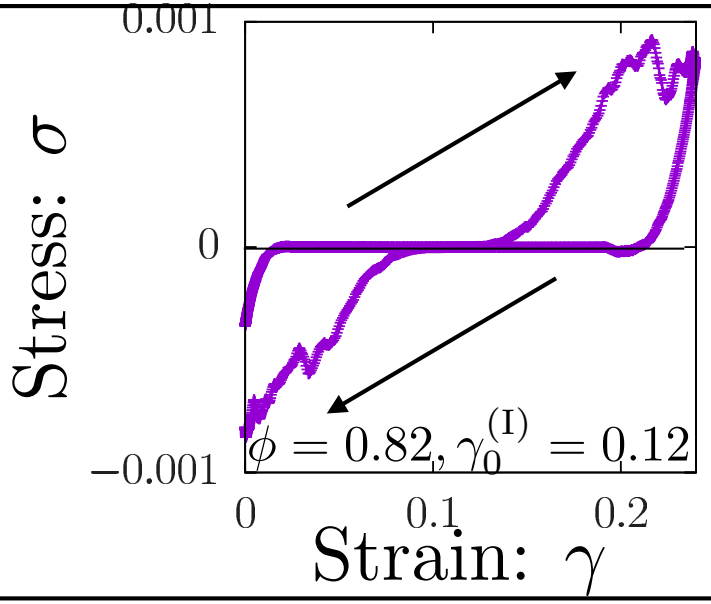
- SJ is above U.
- The critical strain for SJ is finite.

“Fragile”?



Stress-Strain curve in the initial shear





- There is a **“Fragile”** phase **in SJ**.
- “Fragile” : There are solid-like and liquid like phase in a cycle.

Definition of Fragile state: Dependence on phase

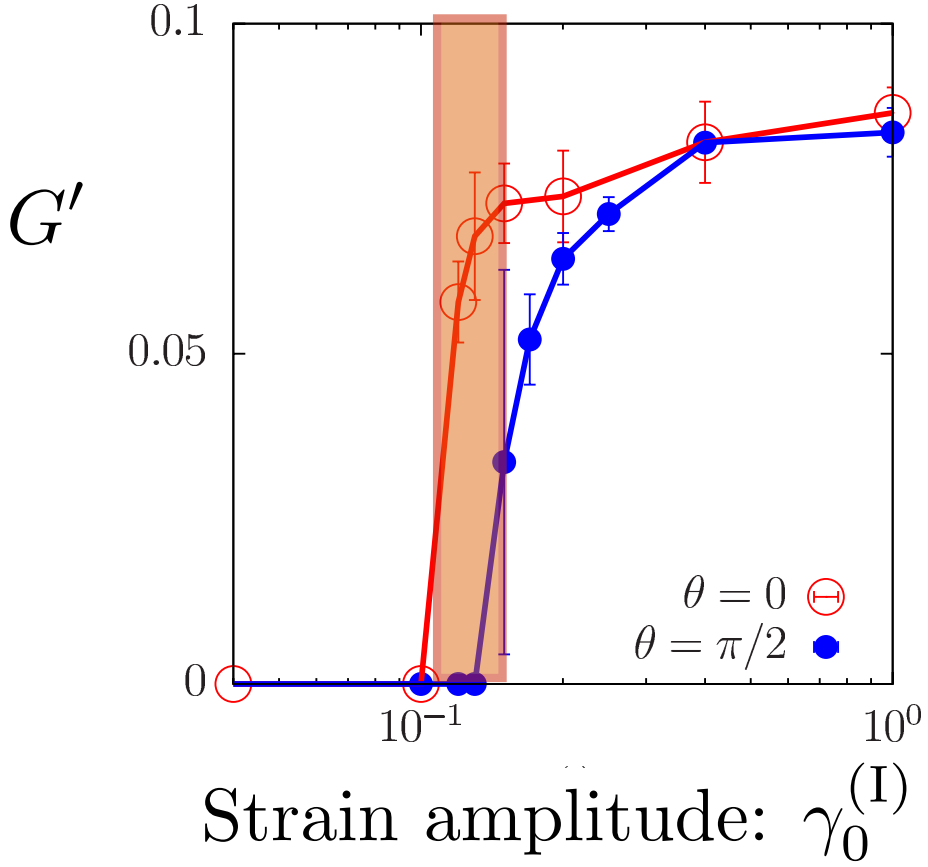
Generalized oscillatory shear:

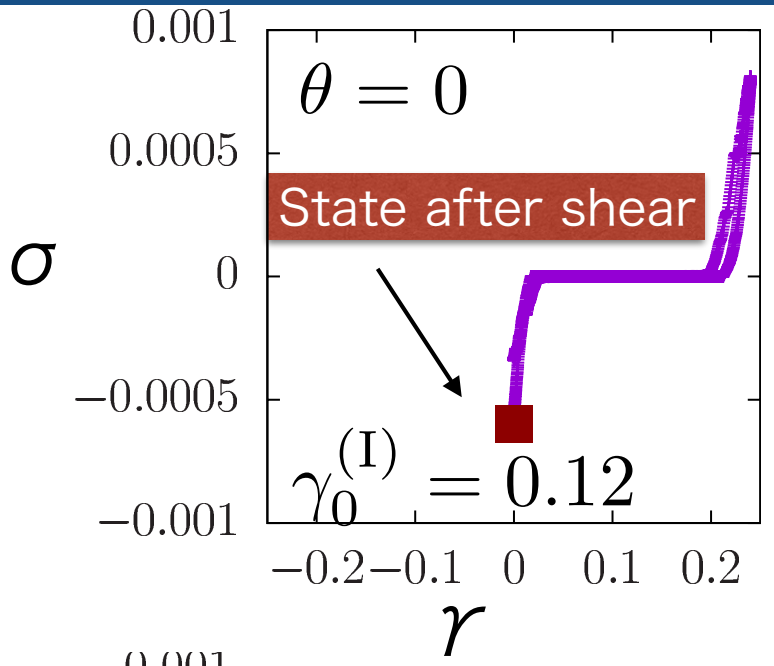
$$\gamma(t) = \gamma_0^{(\mathrm{I})}\{\cos\theta - \cos(\omega t + \theta)\}$$

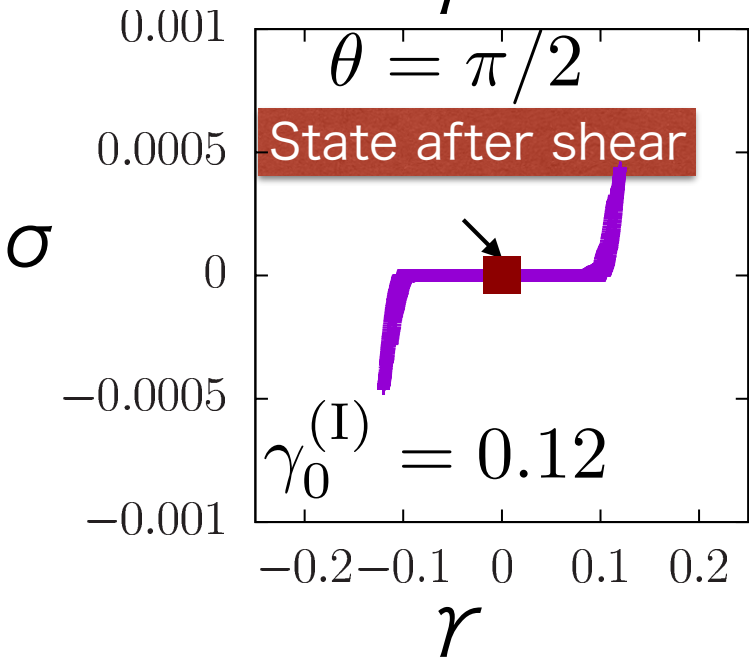
$$\dot{\gamma}(t) = \gamma_0^{(\mathrm{I})}\omega\sin(\omega t + \theta)$$

θ : (Initial or Final) phase of shear

Previous results: $\theta = 0$







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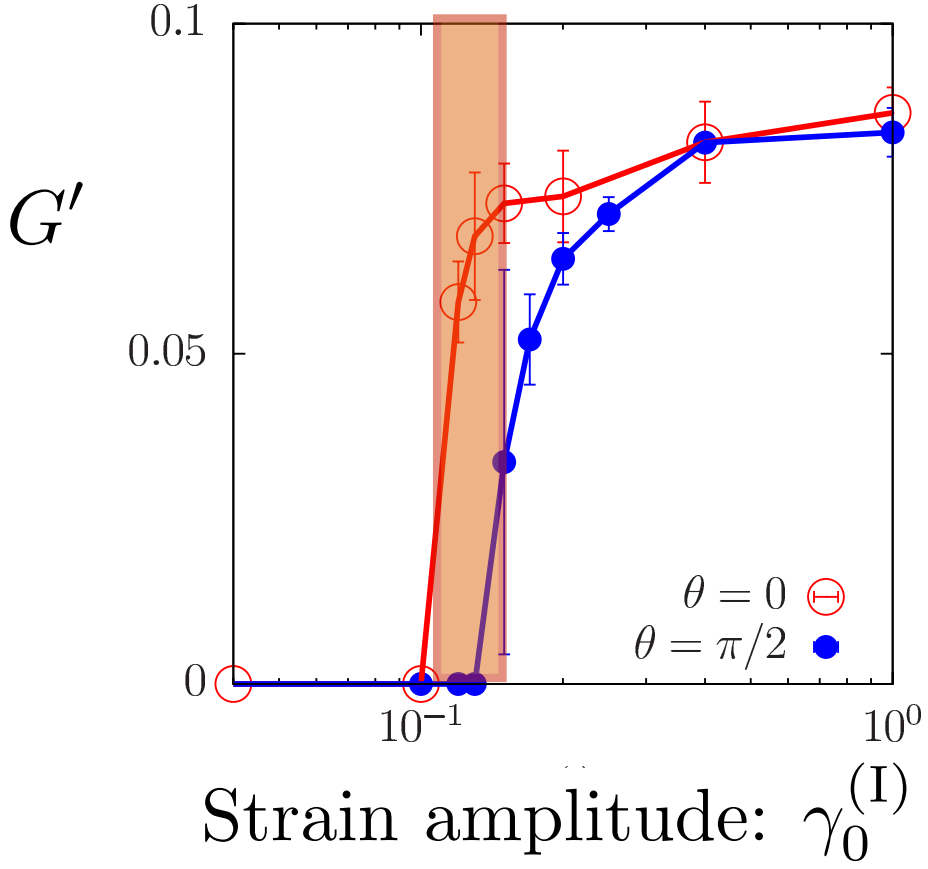
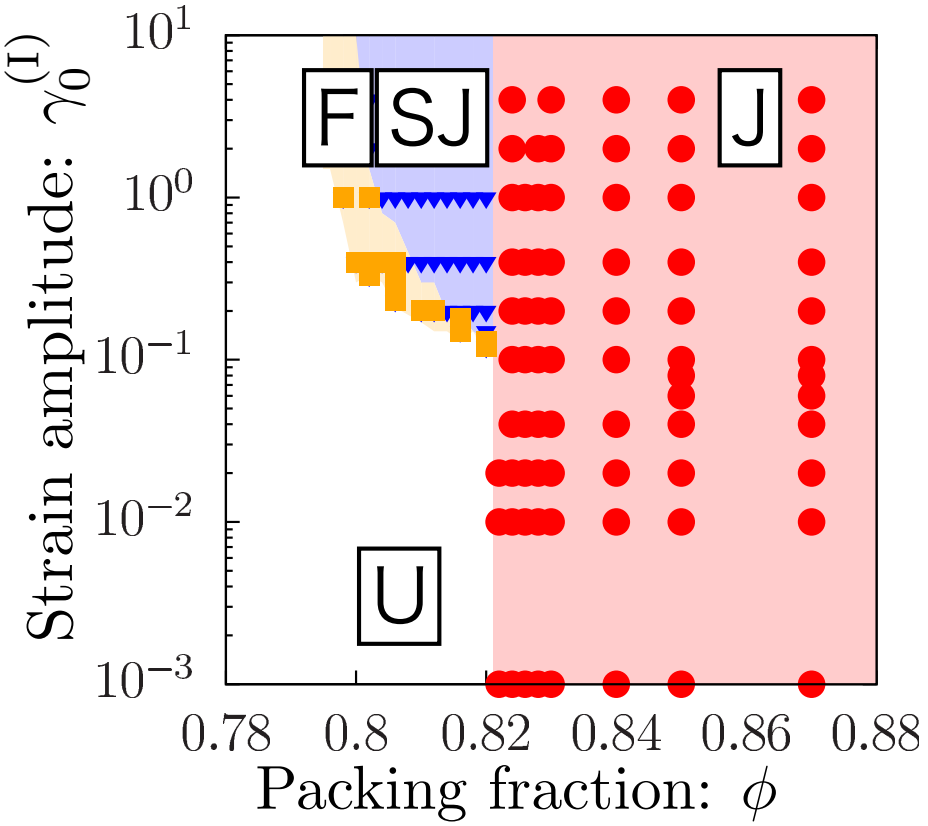


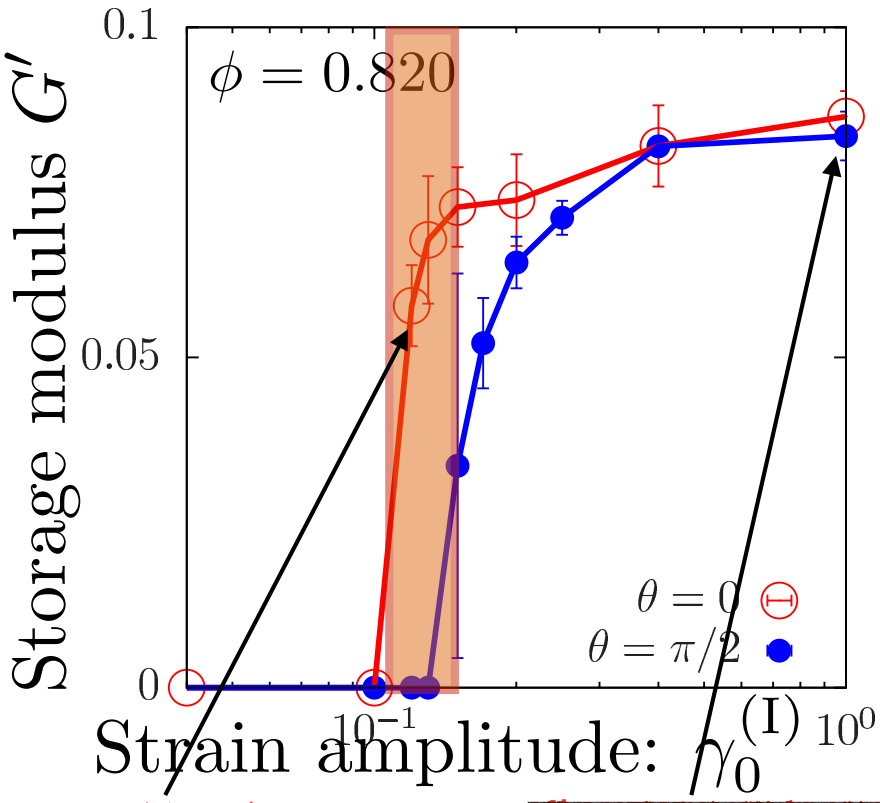


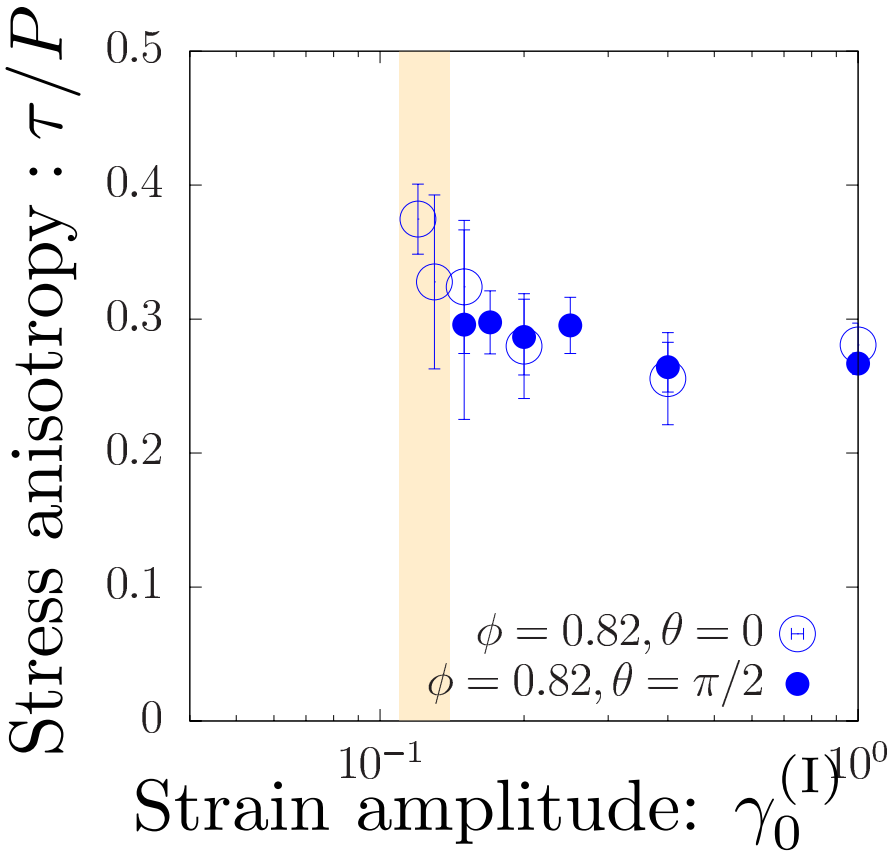
Fragile

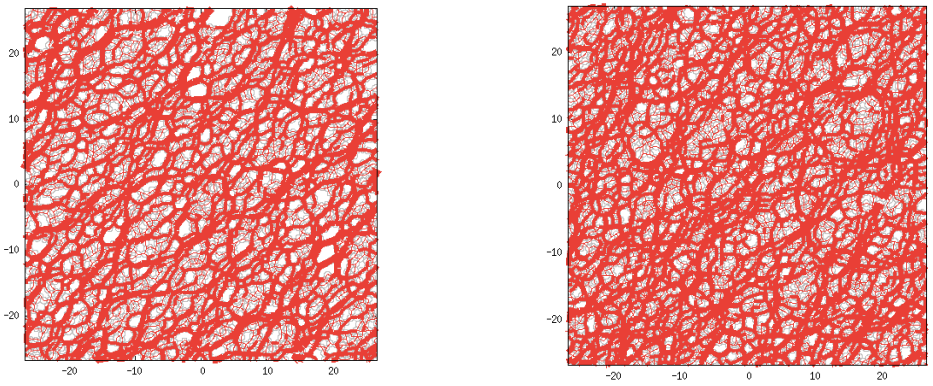
$$G' > 0 \text{ for } \theta = 0$$

$$G' = 0 \text{ for } \theta = \pi/2$$

Anisotropy in fragile phase







Principal stresses: σ_1, σ_2

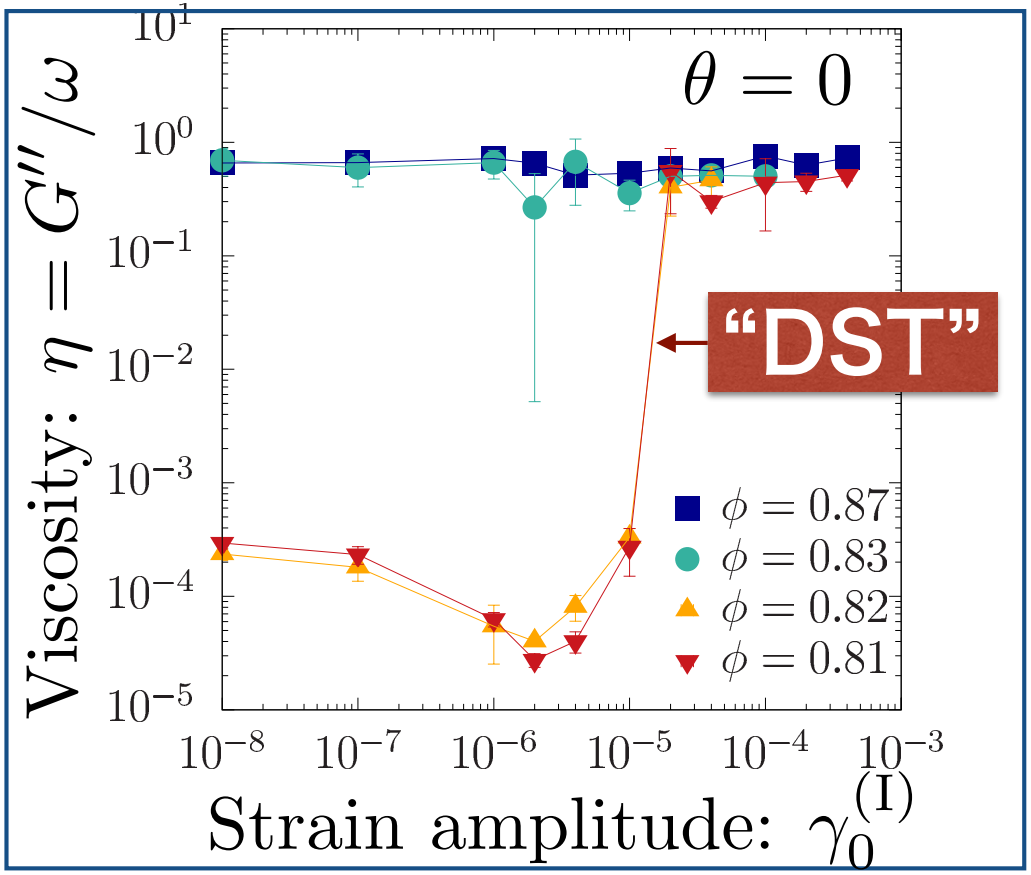
$P = (\sigma_1 + \sigma_2)/2$

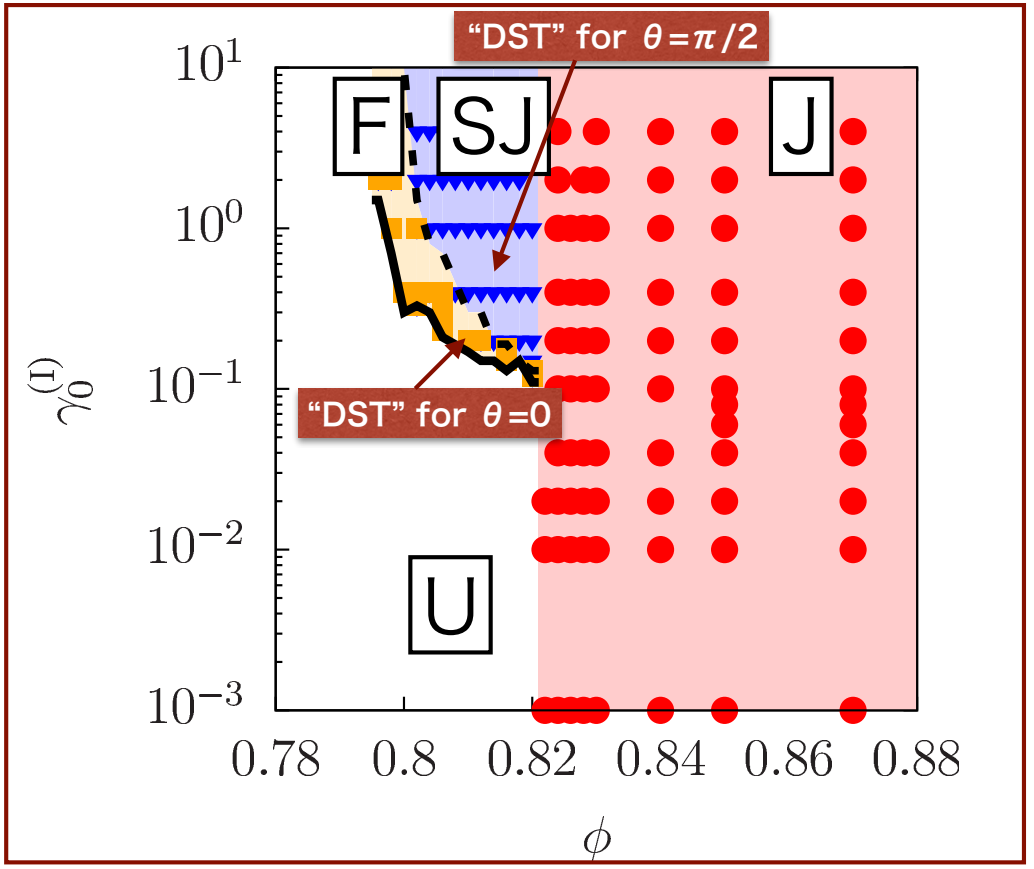
$\tau = (\sigma_2 - \sigma_1)/2$

Stress anisotropy has a peak in the fragile phase.

DST-like behavior in loss modulus

Loss modulus : $G'' = \frac{\omega}{\pi}\int_0^{2\pi/\omega} dt \frac{\sigma(t)\sin(\omega t)}{\gamma_0}$





- η exhibits a rapid increase, which is similar to the discontinuous shear thickening (DST).
- "DST" depending on θ is observed in the fragile phase.

Summary

- Topic : Shear modulus of frictional grains. arXiv:1810.03846
- Due to large initial shear, **the shear jamming** occurs and **the shear modulus drastically changes.**
- **The jamming phase diagram** is obtained from mechanical properties.
- There is **"Fragile" phase,** where G' depends on the phase of the shear.
- **DST-like behavior** is observed in the Fragile phase.