Fluctuations in Mechanical Response of Dense Granular System to External Shear Flow

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Introduction

Jamming Transition

- Emergence of rigidity in macroscopic disordered materials
  ex.) Foams, emulsions, colloids, pastes and granular media

- Second-order phase transition-like features have been pointed out [1]
  *one example will be explained in detail later

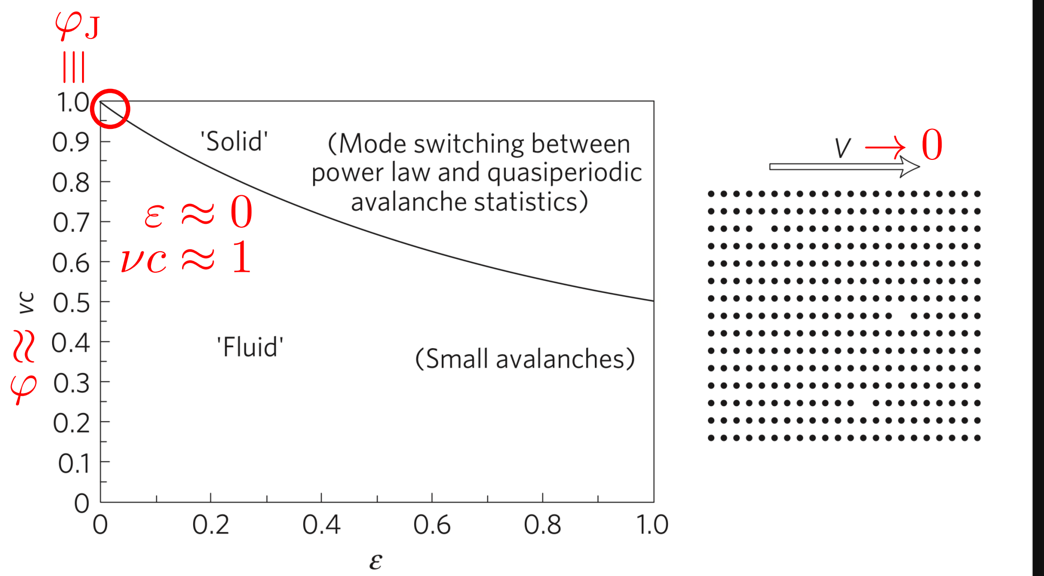
[2] Foams

[3] Paste

[4] Granular Matter

[1] Dinkgreve, Paredes, Michels and Bonn, *Phys. Rev. E* **92**, 012305 (2015)
[2] https://atgirl.com/
[3] https://ws-plan.com/
[4] https://ja.wikipedia.org/wiki/

Avalanche Dynamics in Granular Flow



$$\tau_i(t) = J o_i \left[\left(\sum_{m=1}^{N} o_m u_m(t)/N \right) - u_i(t) \right] + K_L \left(Vt - u_i(t) \right)$$

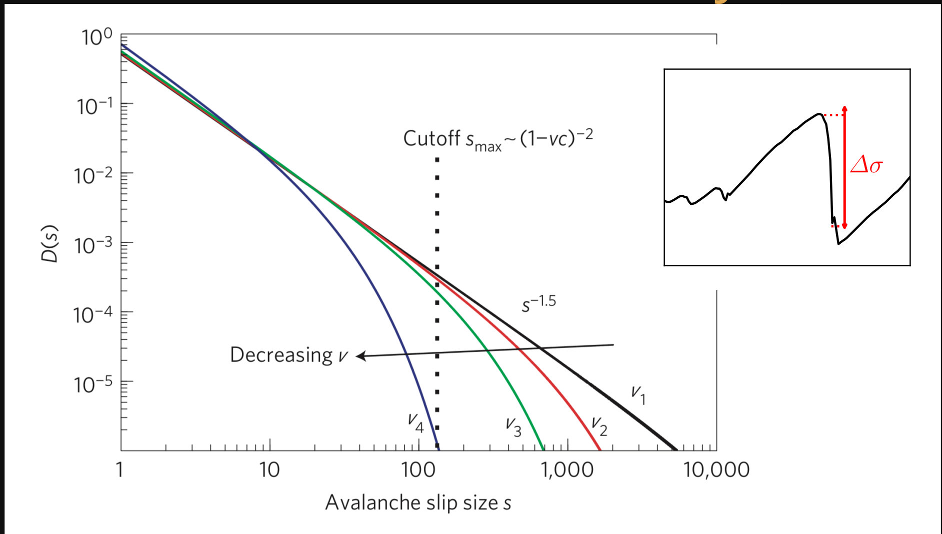
- $c \equiv J/(J + K_L)$: a constant of order unity
- ν: the rescaled packing fraction $\Phi/\Phi_{\max} (\simeq \varphi/\varphi_J)$
- ϵ: difference between effective static and dynamic friction

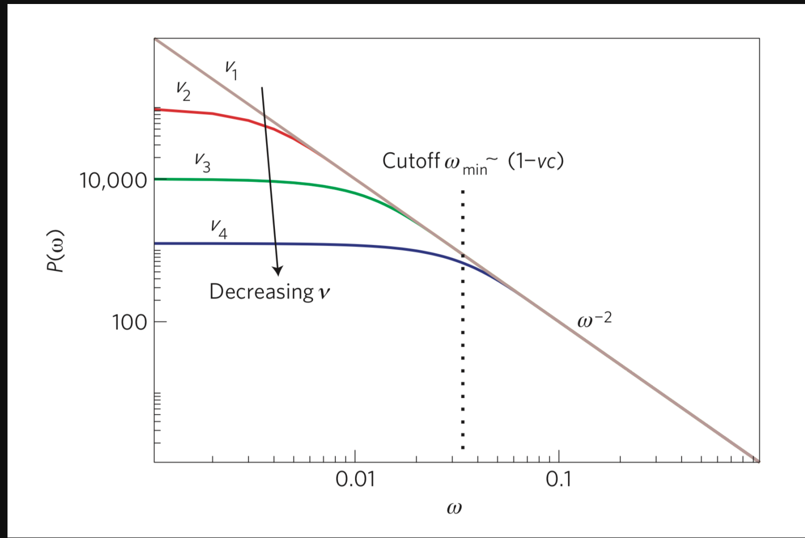
Dahmen, Ben-Zion and Uhl, *Nature Physics* **7**, 554 (2011)

Fluctuations around Jamming Transition

- Mean field and renormalization group theory approach -

Avalanche Dynamics in Granular Flow





- Power-law decay of the avalanche size distribution
- Power-law of the power spectrum of the total slip rate
- Cutoff values are also prediccted

Dahmen, Ben-Zion and Uhl, *Nature Physics* **7**, 554 (2011)

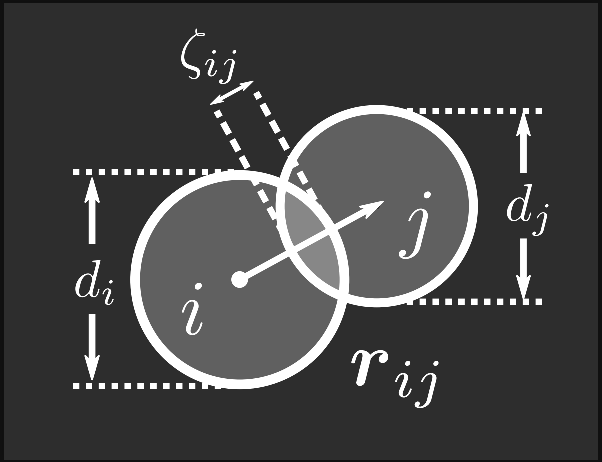
System Setups

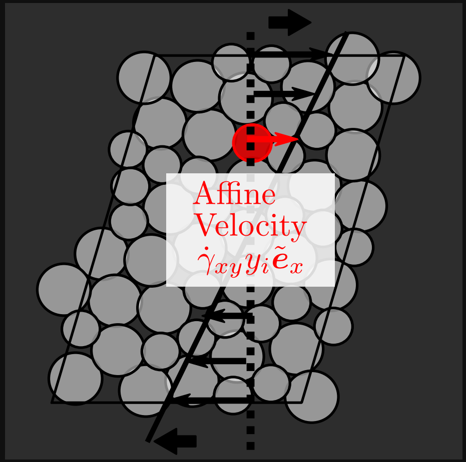
Repulsive Interaction with Dissipation - Linear spring-dashpod model -

$$\boldsymbol{f}_{ij} = \begin{cases} -\left(k_{\mathrm{s}}\zeta_{ij} - \eta\dot{\zeta}_{ij}\right)\tilde{\boldsymbol{r}}_{ij} & (\zeta_{ij} > 0) \\ 0 & (\text{otherwise}) \end{cases}$$

$$\zeta_{ij} = \frac{1}{2}\left(d_i + d_j\right) - r_{ij}$$

(k_{s} :spring constant, η : damping coefficient)





▶ Frictionless system

Macroscopic Mechanical Response: Shear Stress

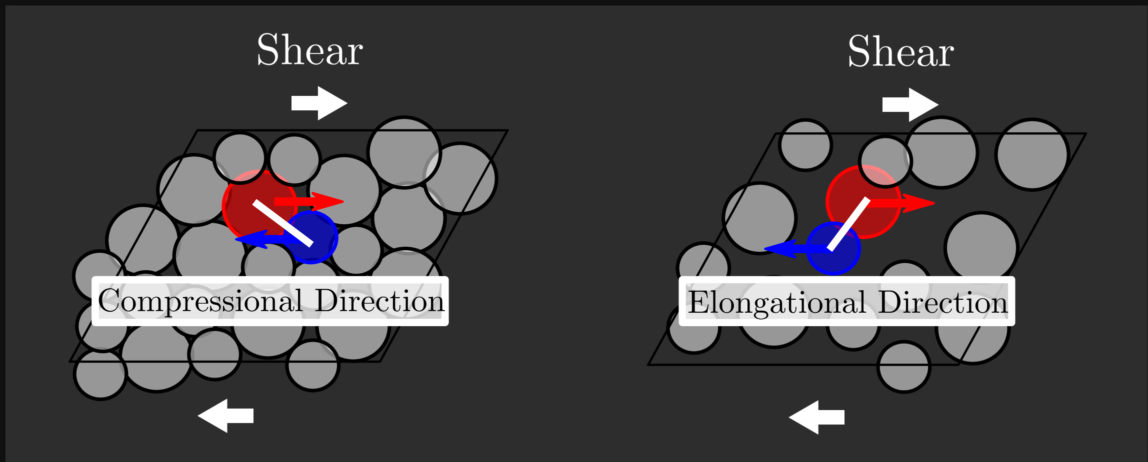
Contact term only

$$\sigma = \left\langle \frac{1}{2}\left(\sigma_{xy} + \sigma_{yx}\right)\right\rangle$$

$$\sigma_{\alpha\beta} = \frac{1}{L^3} \sum_i \sum_{j \neq i} f_{ij\alpha} r_{ij\beta}$$

$$(\alpha, \beta \in \{x, y, z\} \text{ and } i, j \in \{1, 2, \cdots N\})$$

Dense system | Dilute system



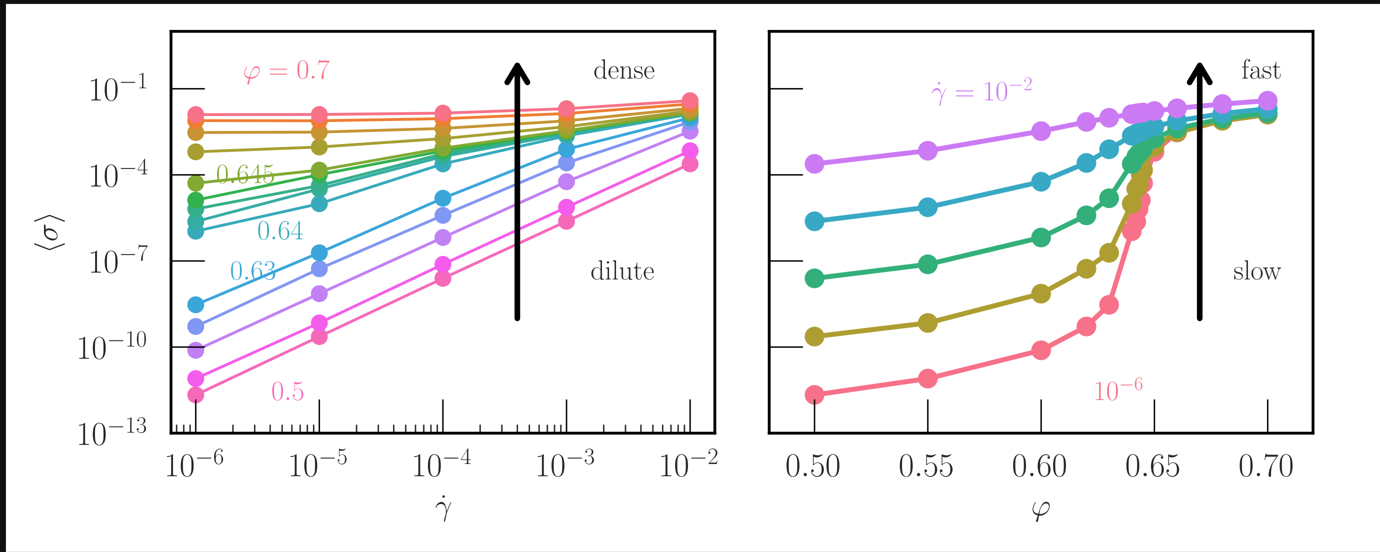
Positive value | (can be) Negative value

- Systems are three dimensional

Results

Flow Curves

Mechanical response to the external shear



Analogy to critical phenomena ...

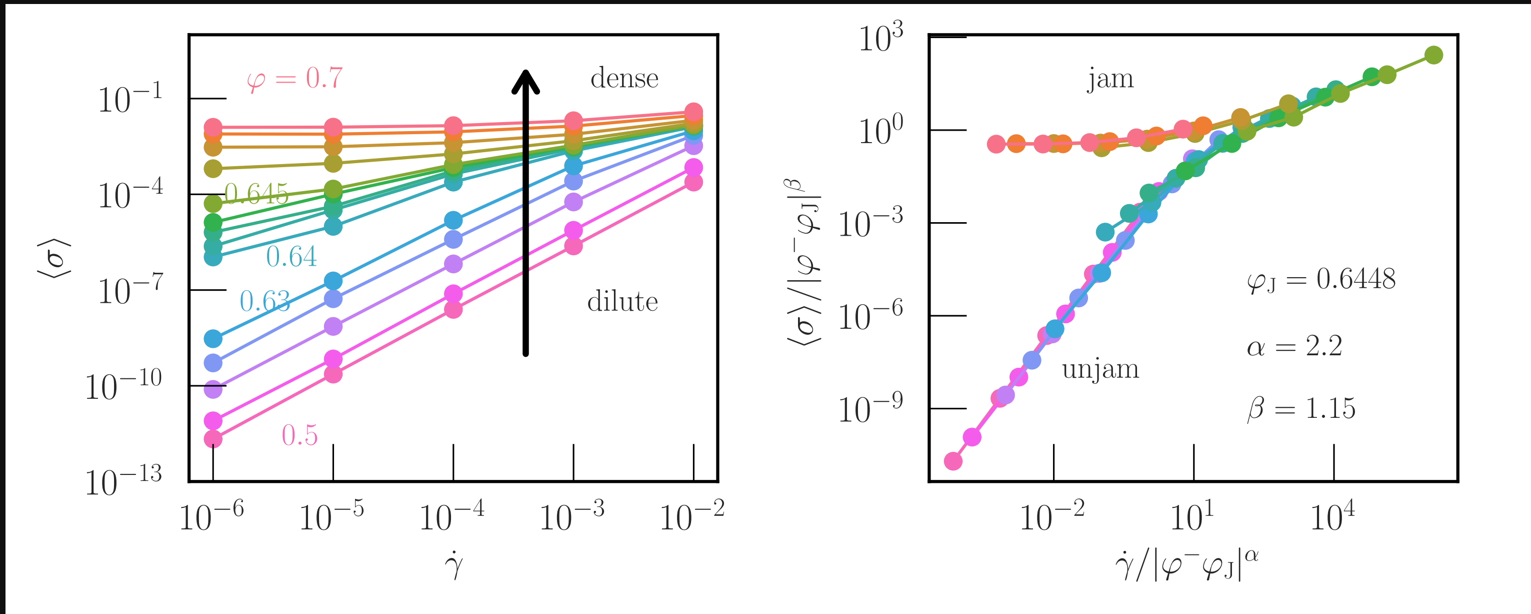
- The control parameter: volume fraction φ

  (inverse temperature β in the Ising model)
- The external field: shear rate $\dot{\gamma}$

  (magnetic field h in the Ising model)
- The order parameter: average shear stress $\langle\sigma\rangle$

Flow Curves

Data collapse by critical scaling[1]

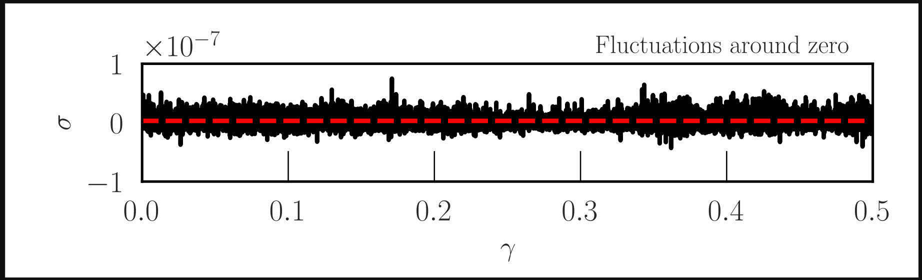


- Scaling by the distance from the critical point $|\varphi - \varphi_J|$ gives distinct two branches: below and above φ_J
- The critical point is determined as $\varphi \simeq 0.6448$

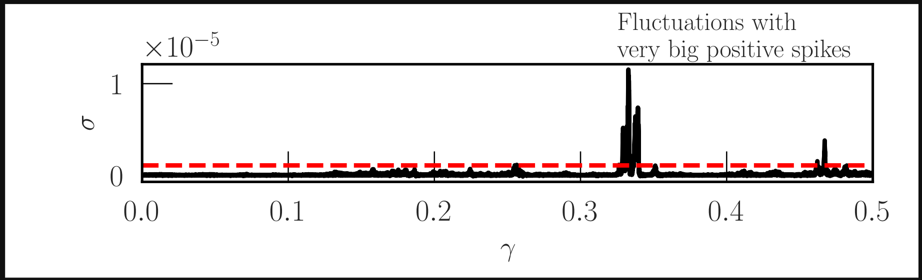
[1] Dinkgreve, Paredes, Michels and Bonn, *Phys. Rev. E* **92**, 012305 (2015)
Bonn, Denn, Berthier, Divoux and Manneville, *Rev. Mod. Phys.* **89**, 035005 (2017)
etc.

Stress-Strain Curves

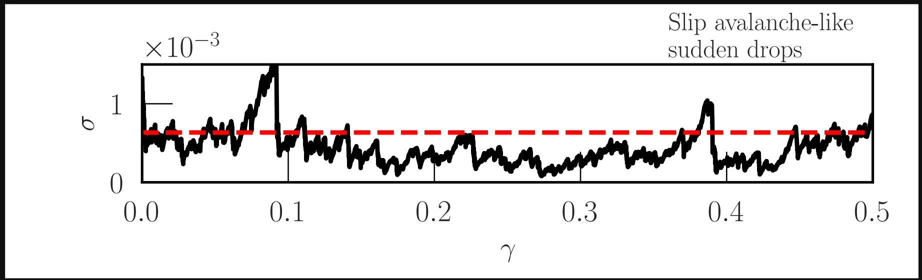
Blow the jamming point $\varphi = 0.63 < \varphi_J$



Close to the jamming point $\varphi = 0.64 \approx \varphi_J$

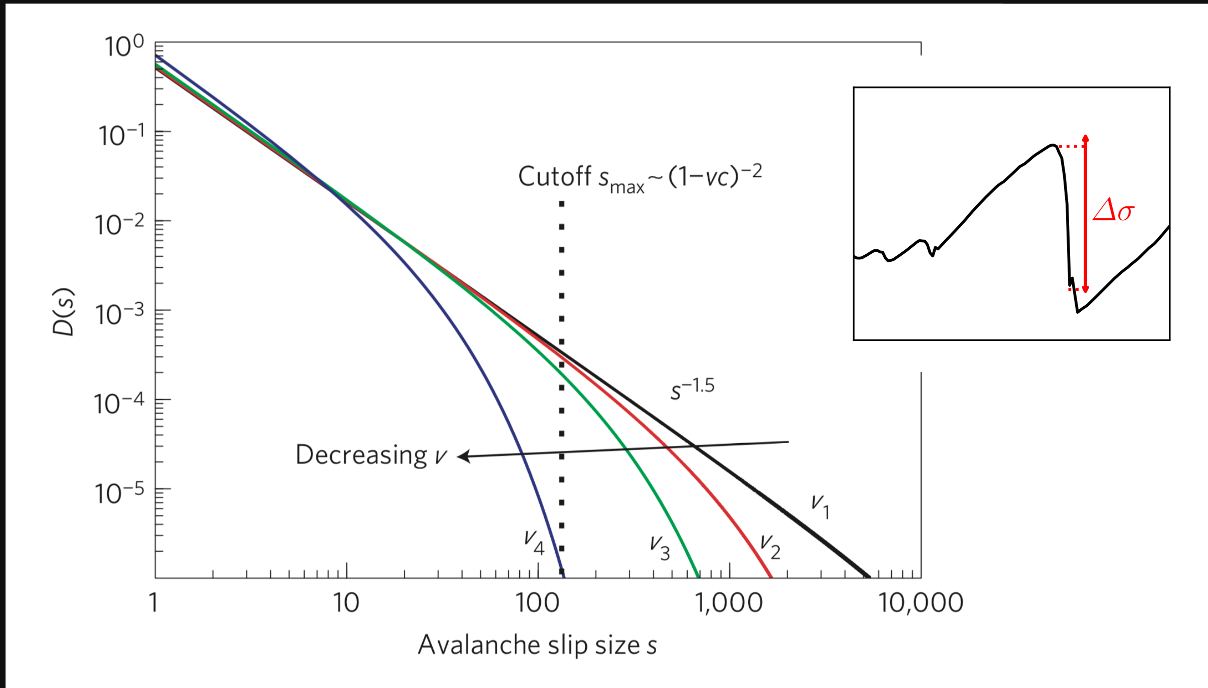


Above the jamming point $\varphi = 0.65 > \varphi_J$



Avalanche Theory for Granular Flows 1

Avalanche size distribution

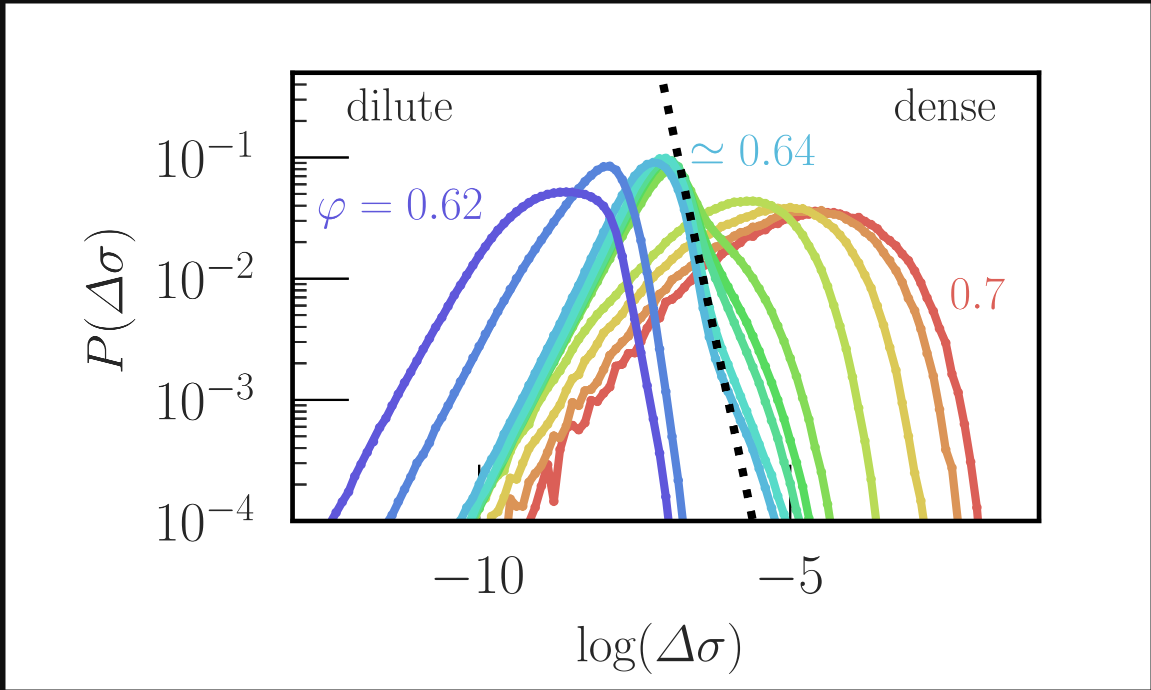


- Power law decay ($\sim s^{-1.5}$) with an exponential cutoff
- The cutoff avalanche size scales as $(1 - \nu c)^{-2}$

Dahmen, Ben-Zion and Uhl, *Nature Physics* **7**, 554 (2011)

Stress Drop Distribution

Avalanche size distribution in the simulation ($\dot{\gamma} = 10^{-6}$)



- Qualitative behavior is different from the theory
- Power-law behavior is observed only around φ_J [1]

[1] Hatano, Narteau and Shebalin, *Sci. Rep.* **5**, 12280 (2015)

Why Only at the Critical Point?

Assumptions made by the theory [1]

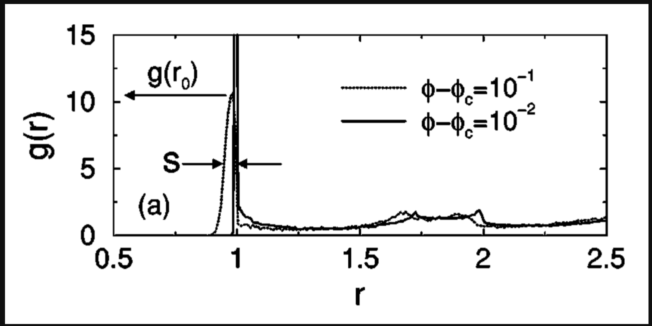
1. Steady State
2. High Density
   (force chain percolation)
3. Isotropy and Homogeneity
4. Statistical Properties

[1] Dahmen, Ben-Zion and Uhl, *Nature Physics* **7**, 554 (2011)

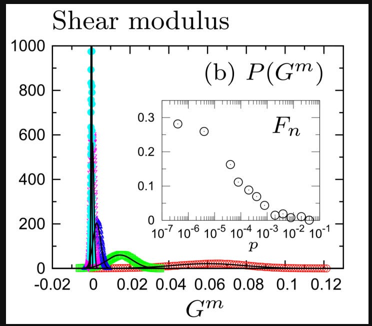
Anomalies at the Critical Point

Everything becomes homogeneous

- Delta function-like first peak in $g(r)$ [1]
- Vanishing elastic heterogeneities [2]
- Hyperuniformity [3]



[1]

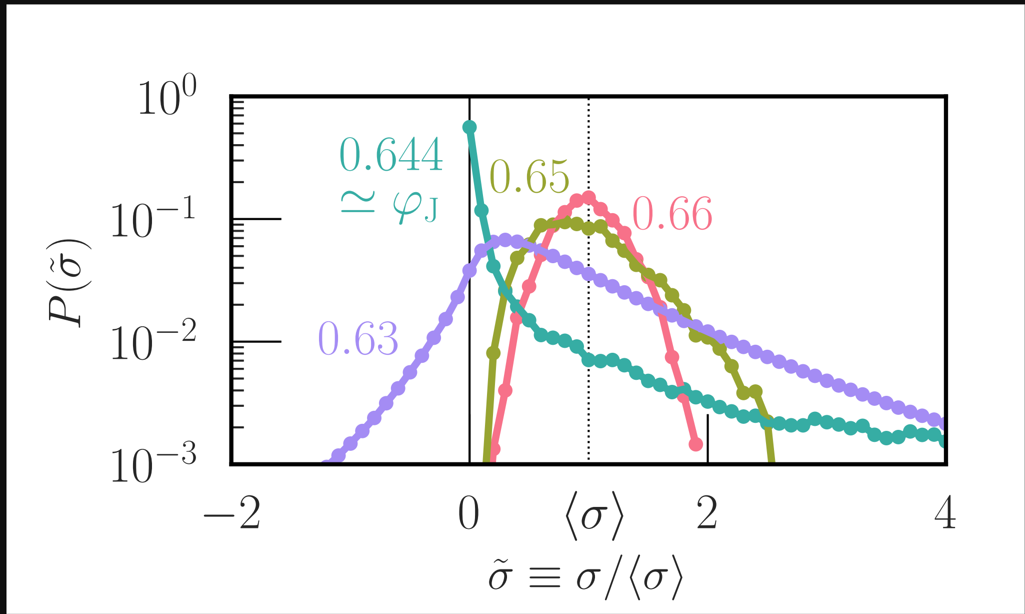


[2]

[1] O'Hern, *et al., Phys. Rev. E* **68**, 011306 (2003)
[2] Mizuno, Silbert and Sperl, *Phys. Rev. Lett.* **116**, 068302 (2016)
[3] Berthier, *et al., Phys. Rev. Lett.* **106**, 120601 (2011)

For Reference ...

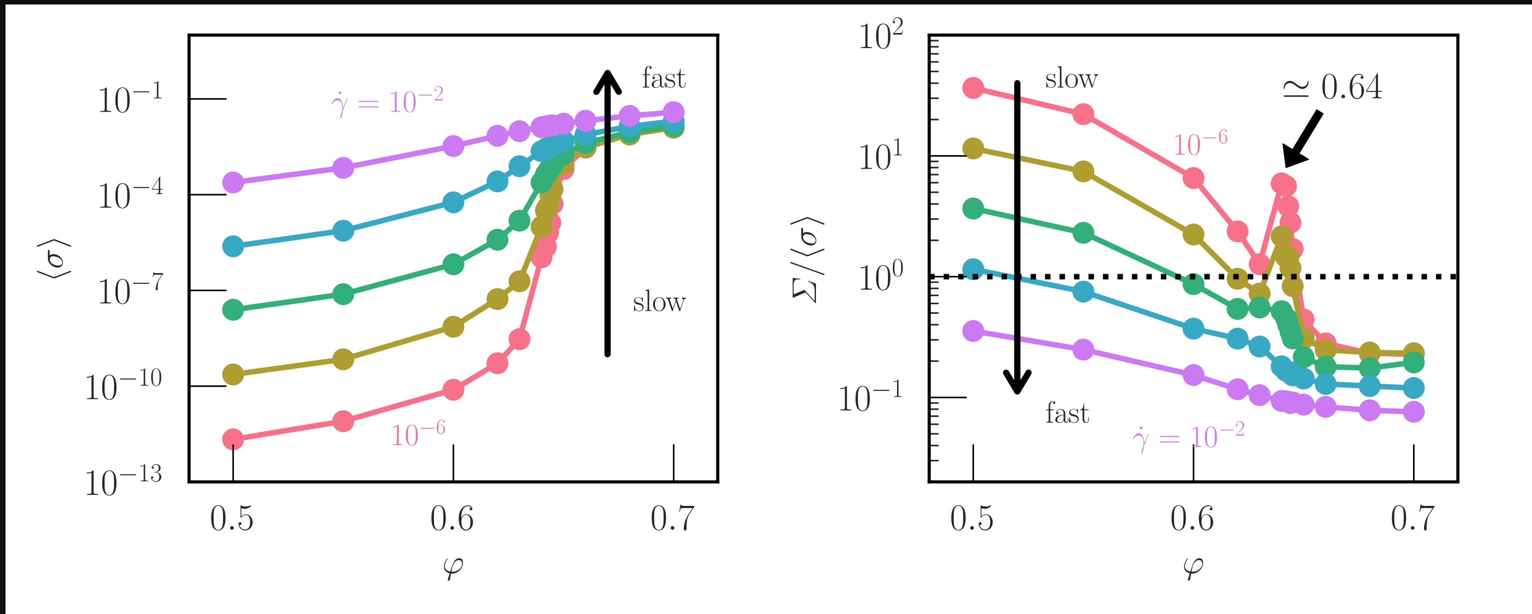
The stress probability distribution



- Below: Skewed and broad with negative values
- Around critical: Skewed and broad starting from zero
- Above: Close to Gaussian around the average

For Reference ... 2

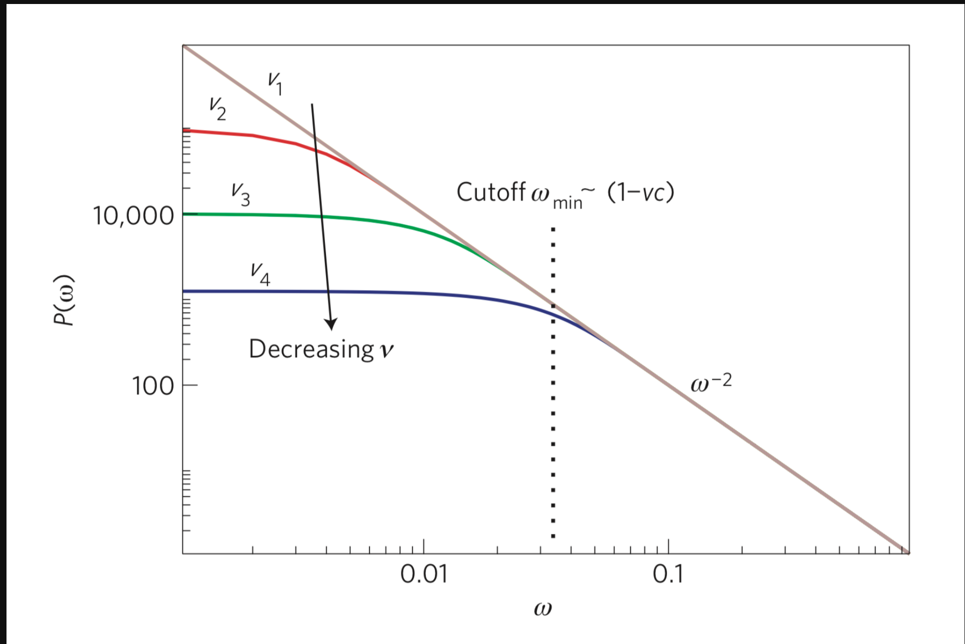
The standard deviation of stress $\Sigma \equiv \left(\langle\sigma^2\rangle - \langle\sigma\rangle^2\right)^{0.5}$



- Distinct peak around the critical point φ_{J}

Avalanche Theory for Granular Flows 2

Power spectrum of the total slip rate ($\simeq$ the stress fluctuation)



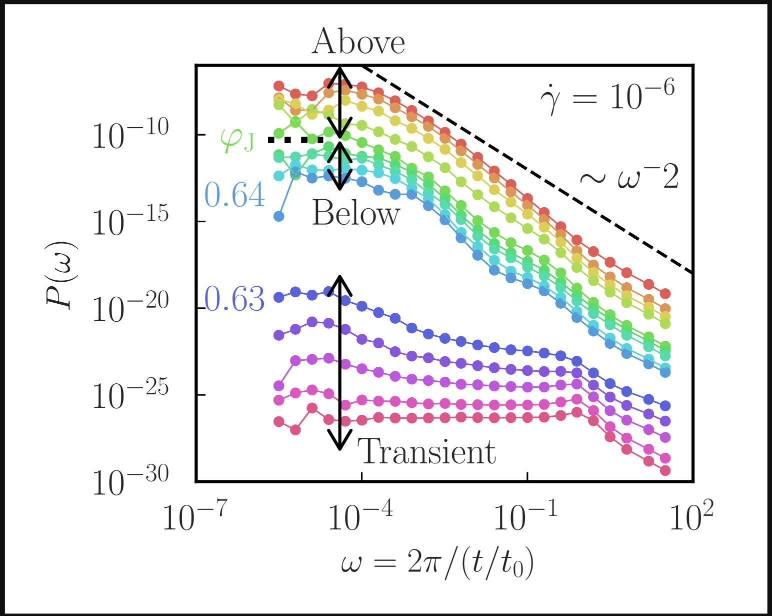
- Crossover from plateau to power-law decay ($\simeq \omega^{-2}$)
- Cutoff frequency scales as $\sim (1 - \nu c) \simeq |\varphi_J - \varphi|$
- Height of plateau scales as $\sim (1 - \nu c)^{-2} \simeq |\varphi_J - \varphi|^{-2}$

Dahmen, Ben-Zion and Uhl, *Nature Physics* **7**, 554 (2011)

Power Spectrum of Stress Fluctuations

Another theoretical prediction

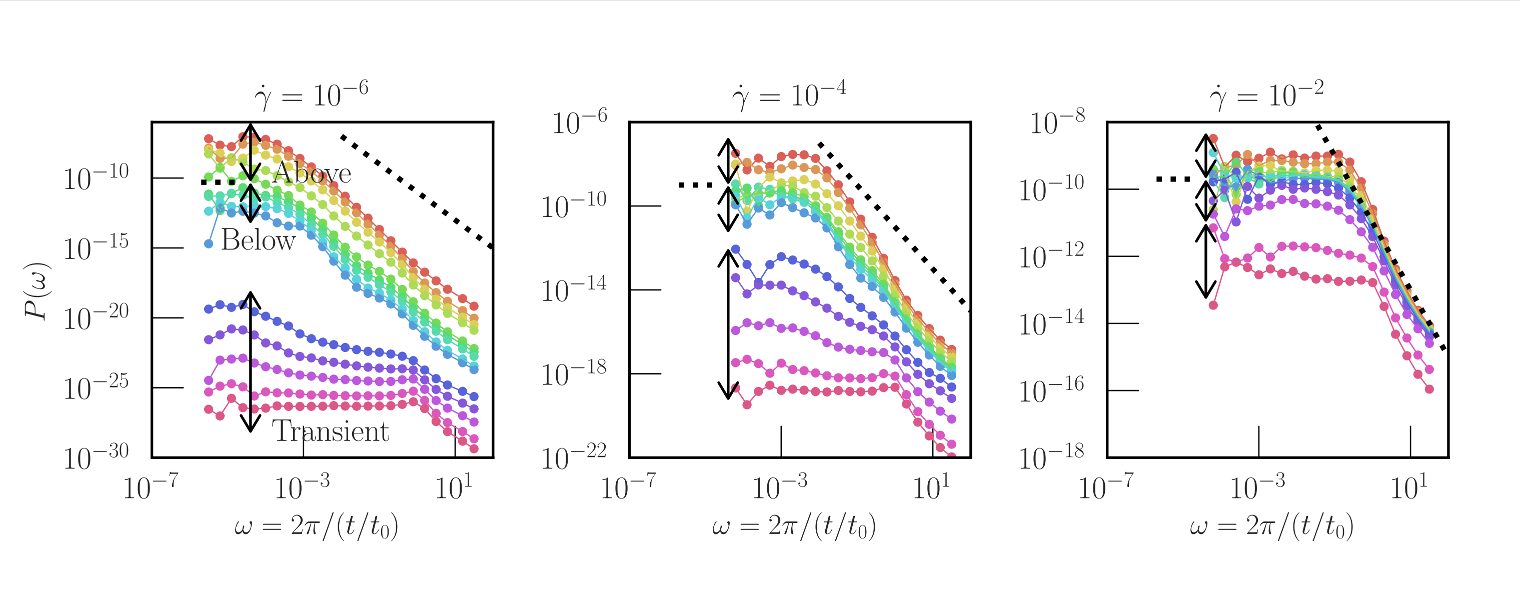
$$P(\omega) = \delta\sigma^2(\omega), \quad \delta\sigma(\omega) = \int \exp i\omega t(\sigma(t) - \langle\sigma\rangle)$$



- Qualitative behaviors partly obey theory robustly
  (from plateau to power-law with ω^{-2})
  (cutoff frequency does not depend on φ)
- Below critical point, there are two regimes

Power Spectrum of Stress Fluctuations

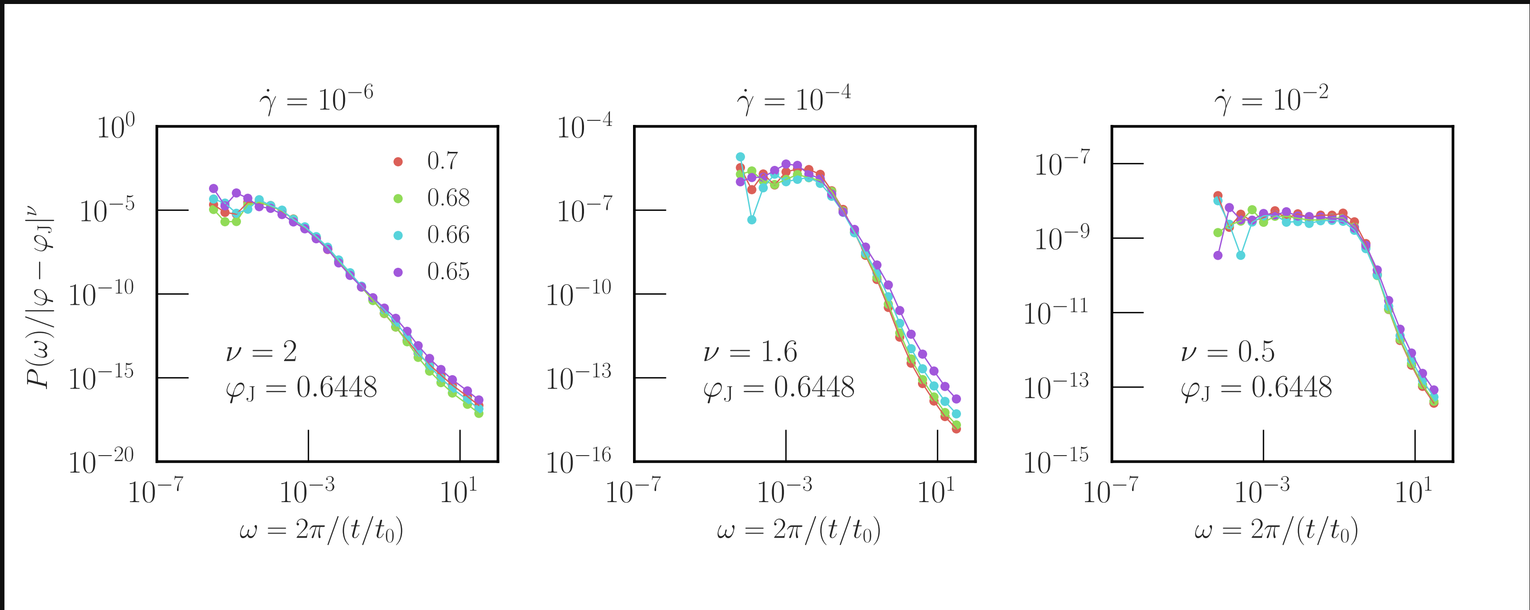
External field dependence



- Qualitative behaviors are still very robust
- The threshold between the below and transient regime becomes small

Scaling by the Volume Fraction (above φ_{J})

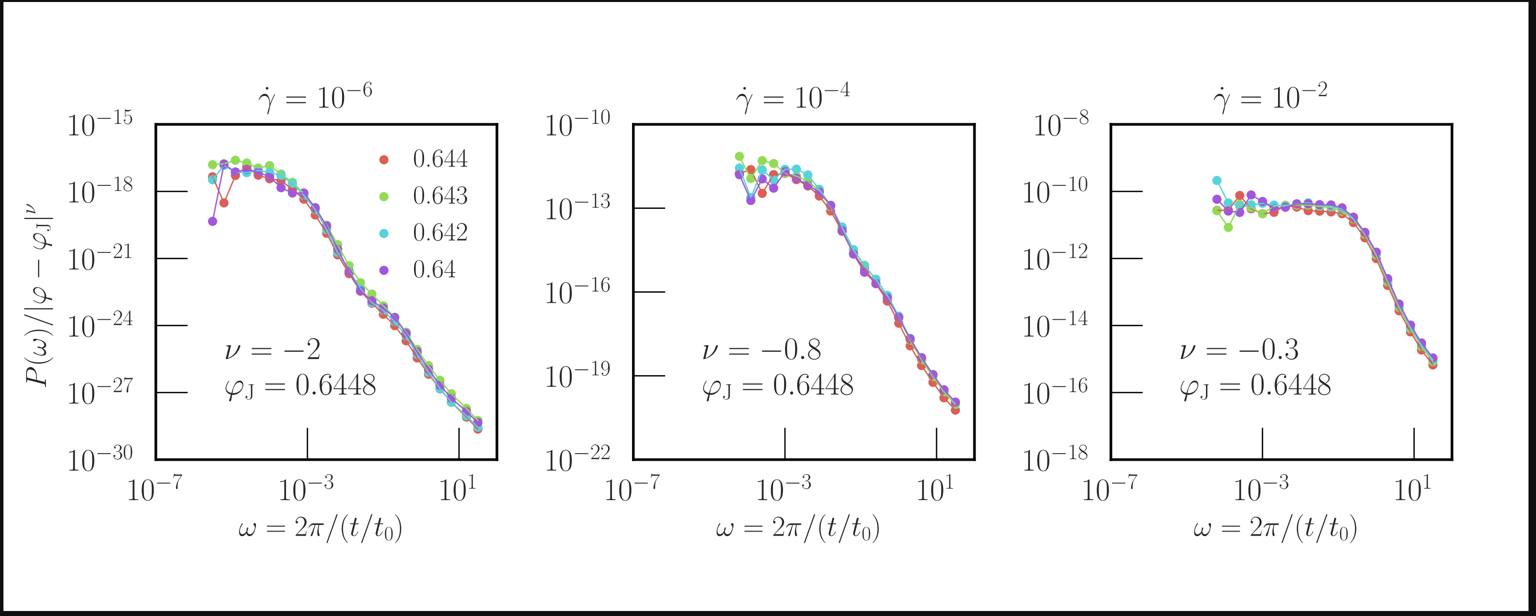
The data above jamming point $\varphi < \varphi_{\mathrm{J}}$



- The critical value is constant
- The exponents are positive

Scaling by the Volume Fraction (below φ_J)

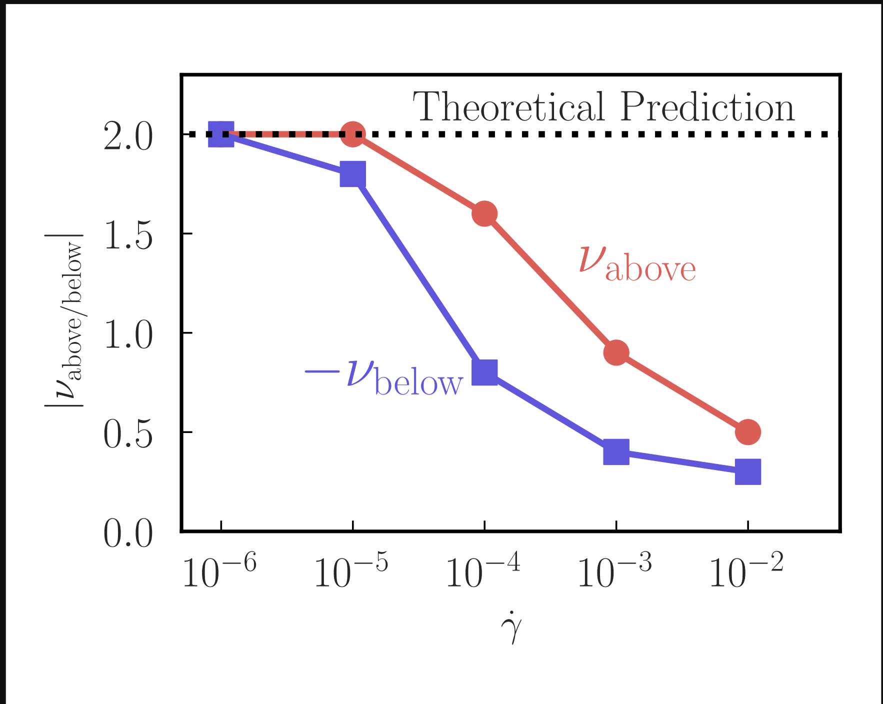
The data below jamming point $\varphi < \varphi_\mathrm{J}$



- The critical value is constant
- The exponents are negative

Crossover of the Critical Exponent

$\dot{\gamma}$ dependent critical exponent



- Consistent with theory[1] for small $\dot{\gamma}$

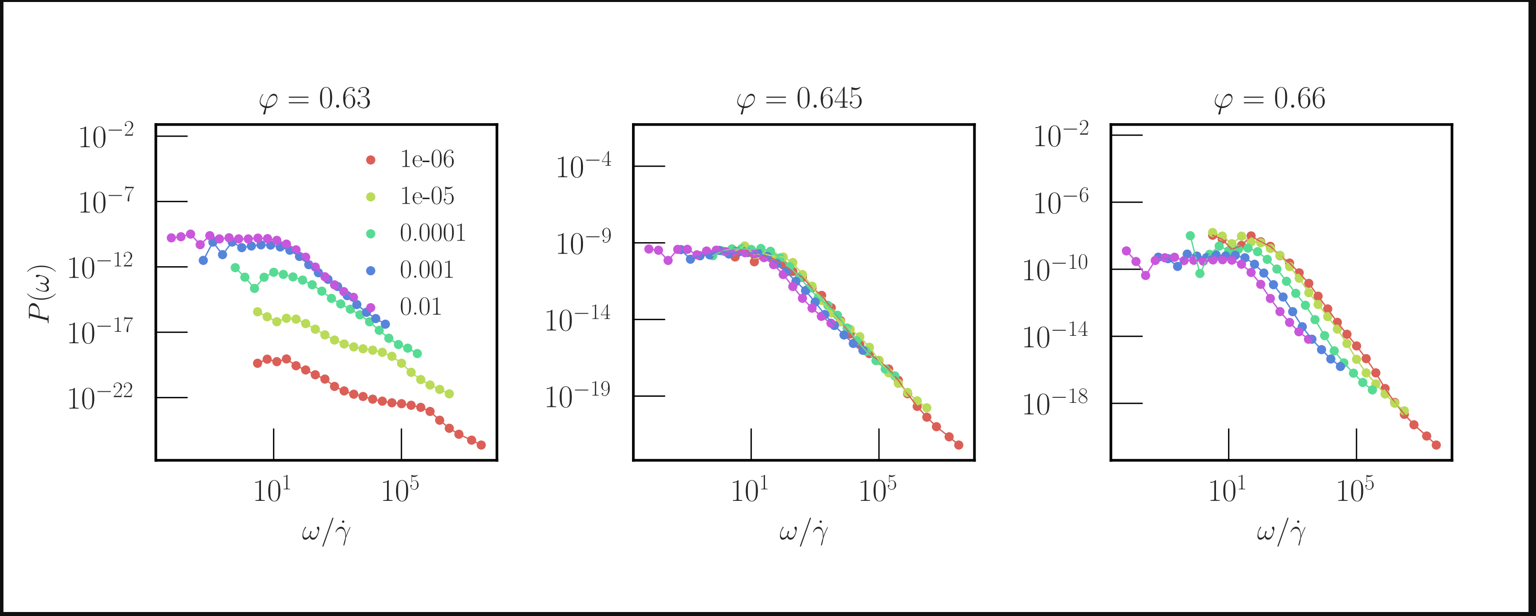
  (if the densest possible packing Φ_{max} in the theory is φ_{J})

- Cross over between different universality class?

Dahmen, Ben-Zion and Uhl, *Nature Physics* **7**, 554 (2011)

Scaling by the shear rate

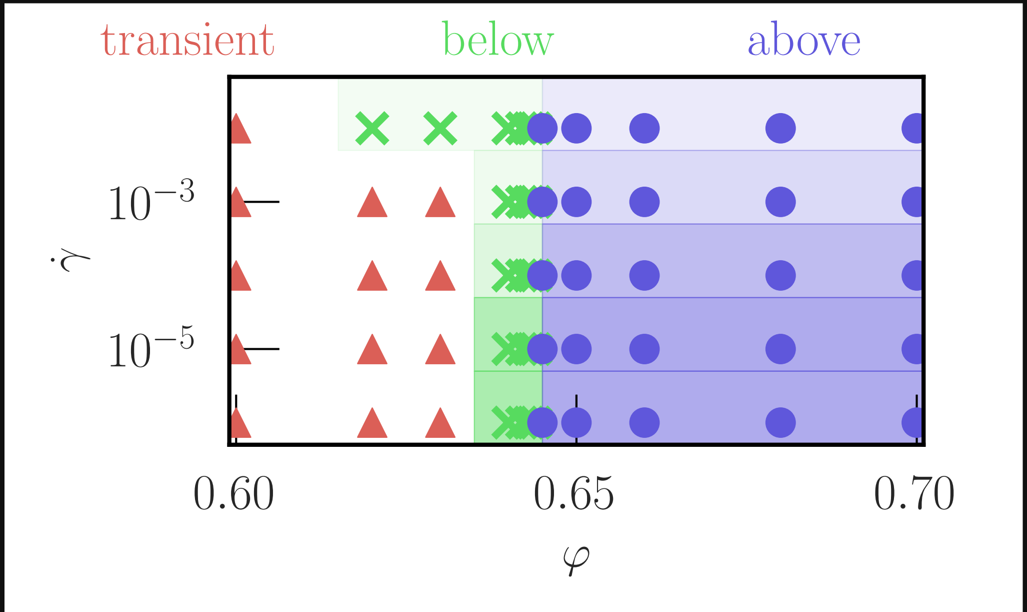
Rescale of the frequency alone



- Data collapse only in the vicinity of the critical point
- The scaling exponent is unity

"Phase" Diagram

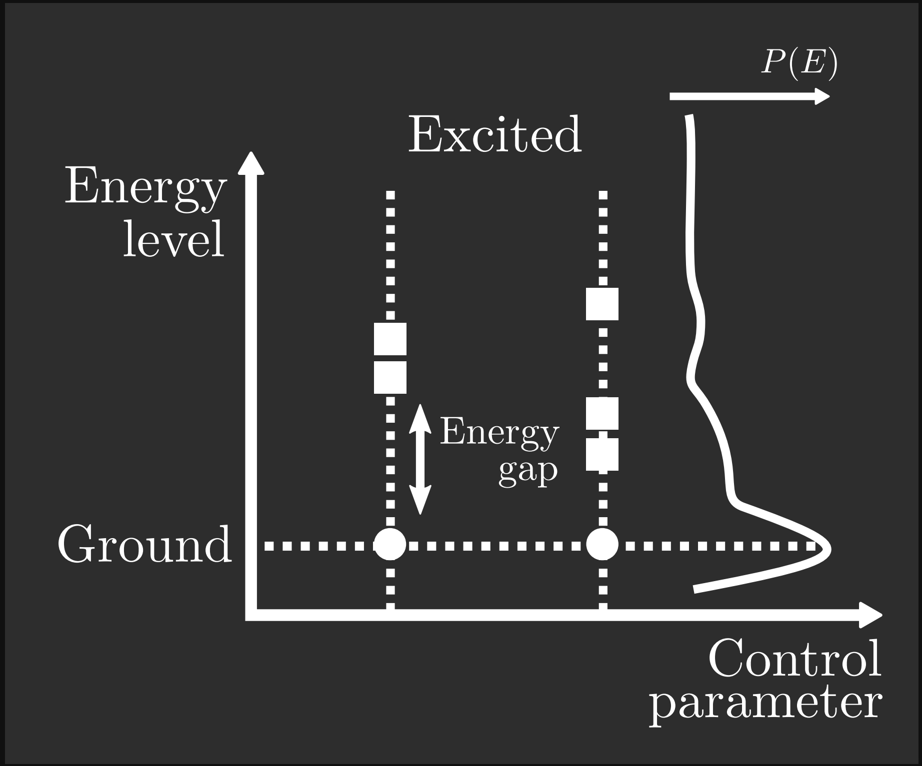
At least three different regimes



- MFT corresponds to the below regime

Meaning of Regimes in Fluctuations?

Differences only in excited states



- Can be reflected in dynamics (fluctuations) under external field

Summary

- what are done in this work -

- ☑ Studied the fluctuations in stress response
- ☑ Many features reveal there are three regimes
- ☑ MFT captures the properties below φ_{J} very well

Future Problems

- Issues to be tackled -

- ☐ Dimensionality dependence (studies in 2D)
- ☐ Finite size effects
- ☐ Effects of tangential friction between particles

THE END

Thank You for Your Kind Attention!!

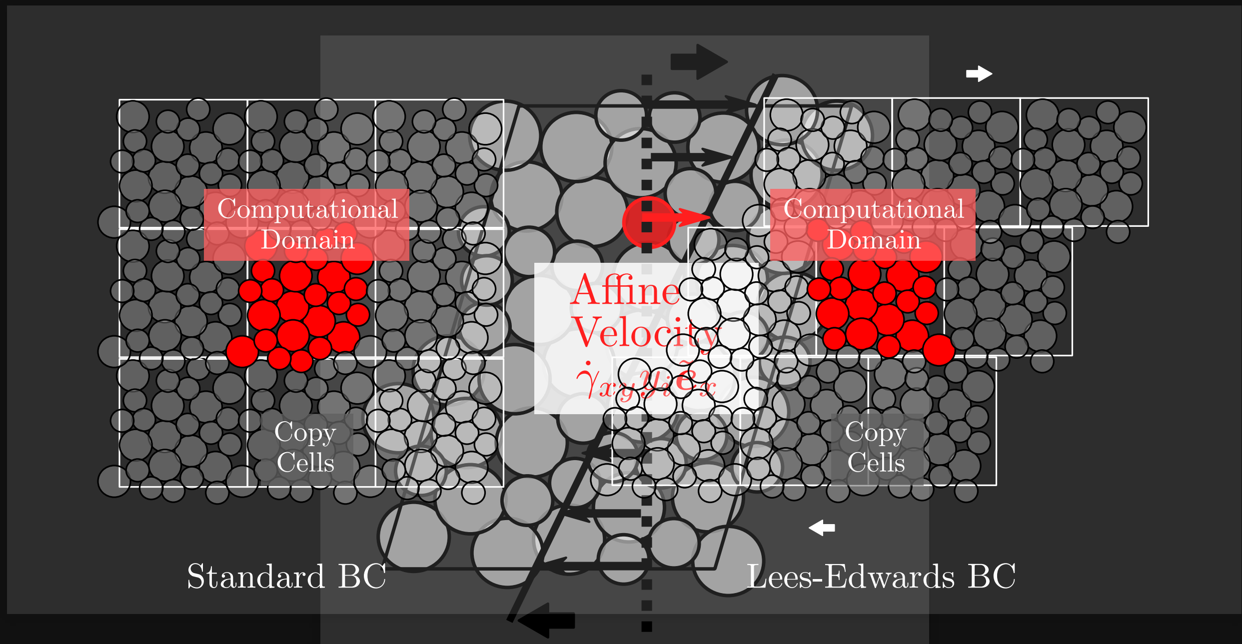
Appendix

Steady simple shear

- with Lees-Edwards boundary condition -

$$\dot{\boldsymbol{r}}_i = \boldsymbol{u}_i = \delta \boldsymbol{u}_i + \dot{\gamma}_{xy} y_i \tilde{\boldsymbol{e}}_x$$

$$\delta \dot{\boldsymbol{u}}_i = \frac{1}{m} \sum_{j \neq i} \boldsymbol{f}_{ij}$$



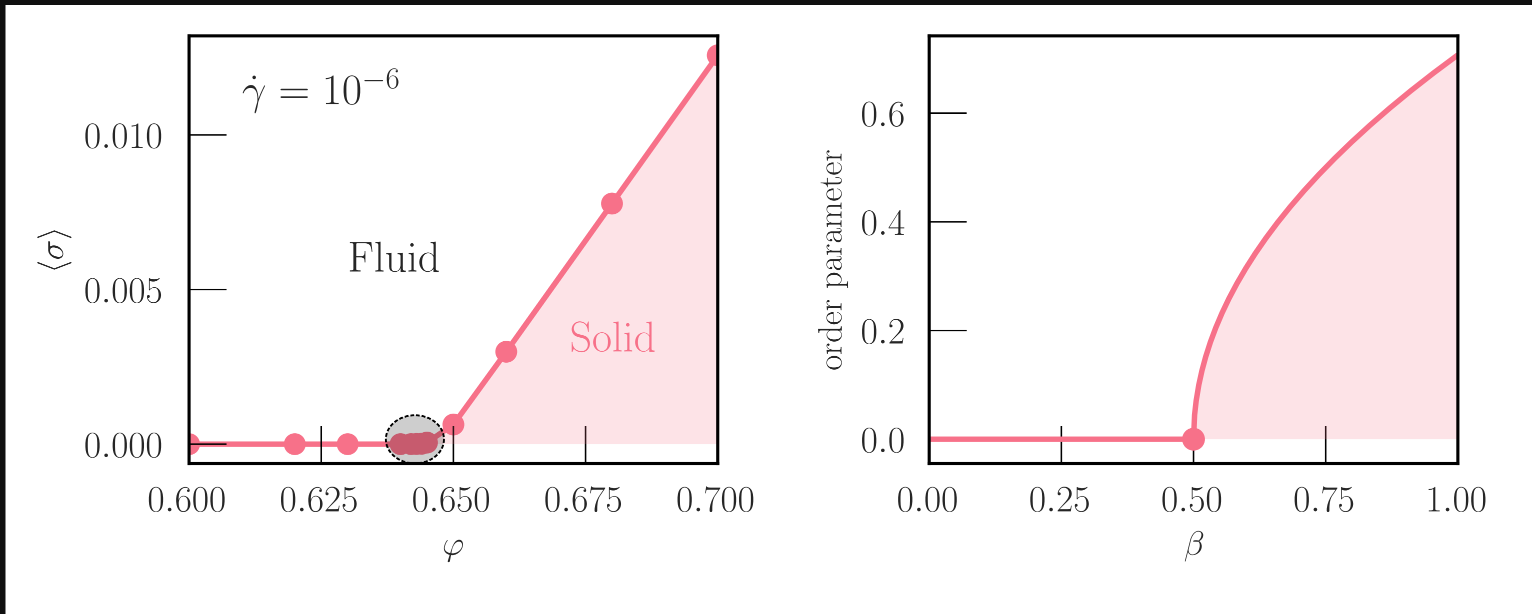
- Macroscopic variables will be studied

Parameters

50% : 50% binary mixture	$d_{\mathrm{A}} = 1$ and $d_{\mathrm{B}} = 1.4$
Number of particles	N = 1024
Shear rate	$\dot{\gamma} t_{\mathrm{m}} = 10^{-7} - 10^{-2}$
Mean volume fraction	$\varphi = 0.5 - 0.7$ $(\varphi_{\mathrm{c}} \sim 0.644)$
Spring constant	$k_{\mathrm{s}} = 2$
Viscosity	$\eta = 2$
Restitution Coefficient	$e = 0$ (contact overdamp)

Theory vs. Numerical Experiments

Consistency lies below the critical point



- The theory is based on the solid state assumption [1]
- Consistent with the simulation results under φ_J

[1] Dahmen, Ben-Zion and Uhl, *Nature Physics* **7**, 554 (2011)