2018/10/26,27
Physics of Jammed Matter
@ YITP, Kyoto Univ.

Particles flows behind an intruder: from Stokesian flow to turbulent flow

Satoshi Takada (ERI, Univ. Tokyo)
Hisao Hayakawa (YITP, Kyoto Univ.)

Fluid flows around an intruder

Drag of an intruder characterizes the fluid flow.

- Slow-speed region (Stokes' law)

$$F = \begin{cases} 6\pi\eta a V \text{ (sphere, rough surface)} \\ 4\pi\eta a V \text{ (sphere, smooth surface)} \end{cases}$$

  (a: radius of intruder, η: viscosity)

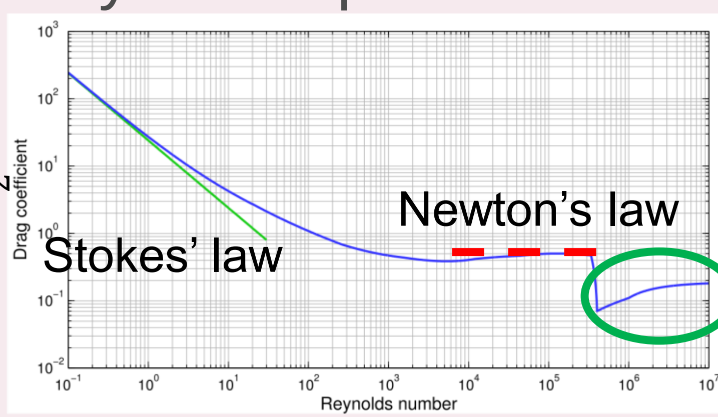
  ☛ M. Itami & S. Sasa, J. Stat. Phys. **161**, 532 (2015)

- High-speed region (Newton's law)

$$F \propto a^2 V^2$$

  for fully developed turbulence

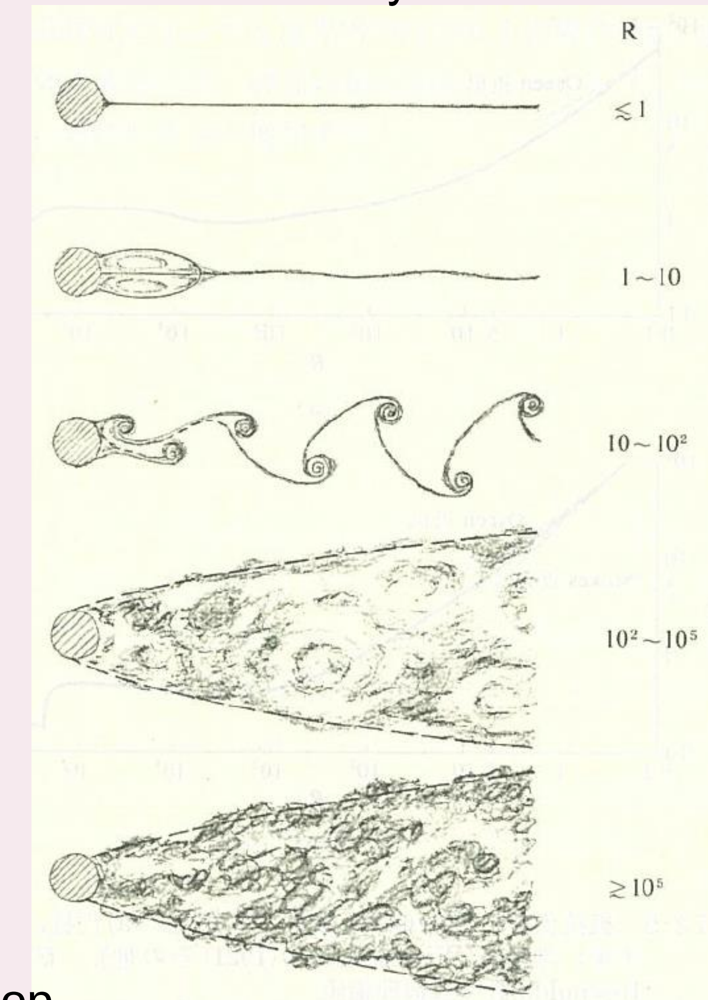
$$C_D = \frac{F}{\frac{1}{2}\rho V^2 S}$$



drag drop

ρ: density
S: collision cross section

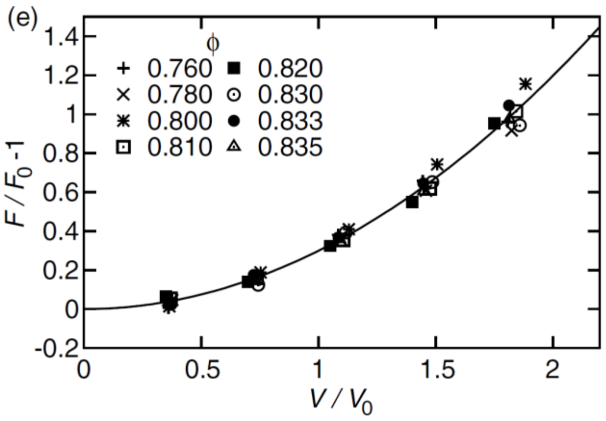
Reynold's number



Tomomasa Tatsumi
"Hydrodynamics" (Baifukan)

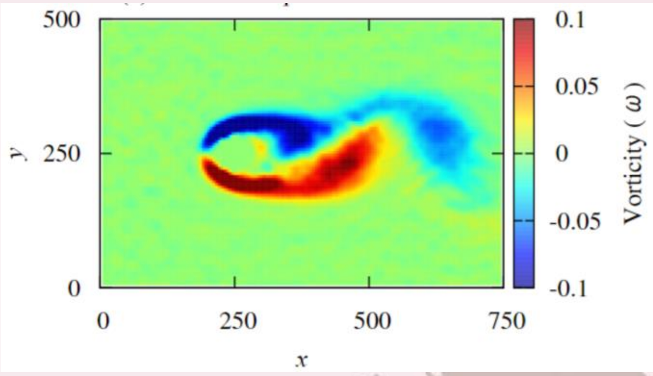
Drag law for particle systems

Y. Takehara & K. Okumura, PRL, **112**, 148001 (2014).



How does the drag law change for particle systems.

- For $T = 0$, the drag law is Newtonian $F_{\mathrm{drag}} \propto V^2$.
- When the system is thermalized, the drag law changes to $F_{\mathrm{drag}} \propto V$.
- Kármán's vortex is also observed from MD simulation.



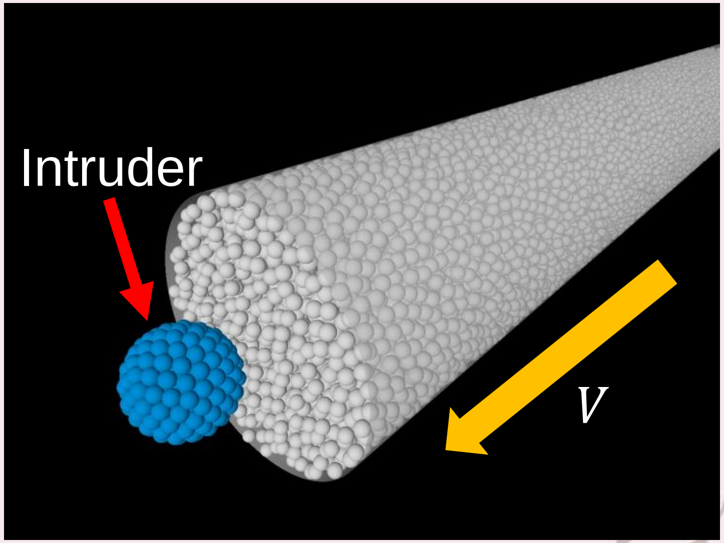
Asano et al., JCP, **148**, 144901 (2018)

<u>Question</u>:
What is the fluid flow behind an intruder in 3-D particle flows?
How does the fluid flow depend on the temperature?

Setup and model

We perform MD simulation (monodisperse particles)

- Beam particles
  - $N = 30{,}000, 101{,}250,$ and $810{,}000$ particles
  - mass m, diameter d
  - elastic collision ($e = 1$)
  - initial packing fraction: $\phi = 0.40$
  - thermalized at T
  - the translational speed V is added at $t = 0$.



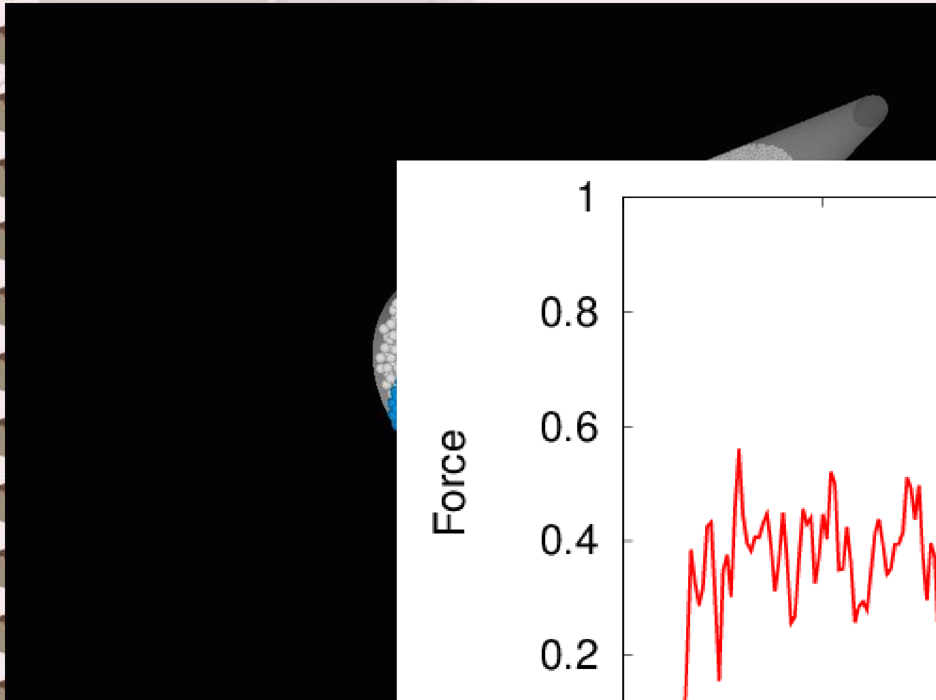
- Intruder
  - Core particle (diameter D_{core}) + N_{surface} small particles on the surface $(D_{\mathrm{core}}/d, N_{\mathrm{surface}}) = (5{,}144), (13, 744), (28{,}1600)$
  - mass $M \to \infty$, diameter $D \simeq 7d, 15d, 30d$
  - set at the origin

- Boundary condition (intruder & particle)
  - Particles reflect at random with the temperature T
- Boundary condition (in front of the intruder)
  - cylinder with the radius $R = 10d$ for ($N = 30{,}000, 101{,}250$) and $R = 20d$ for ($N = 810{,}250$): a simple reflection
- Two cases behind the intruders are examined
  - Free scattering behind the intruder
  - Scattered particles are confined in a tube
- Introduction of dimensionless parameters
  - dimensionless velocity: $R = V/v_{\mathrm{T}} = V\sqrt{m/(2T)}$
    ($R \propto$ "Reynold's number" because the viscosity $\sim\sqrt{T}$.)
  - dimensionless drag force: $C_{\mathrm{D}} = F_{\mathrm{drag}}/\{(1/2)\rho V^2 S\}$
    (ρ: density of the beam particles, S: the collision cross section)

All quantities are nondimensionalized
in terms of m, d, κ (linear spring constant).

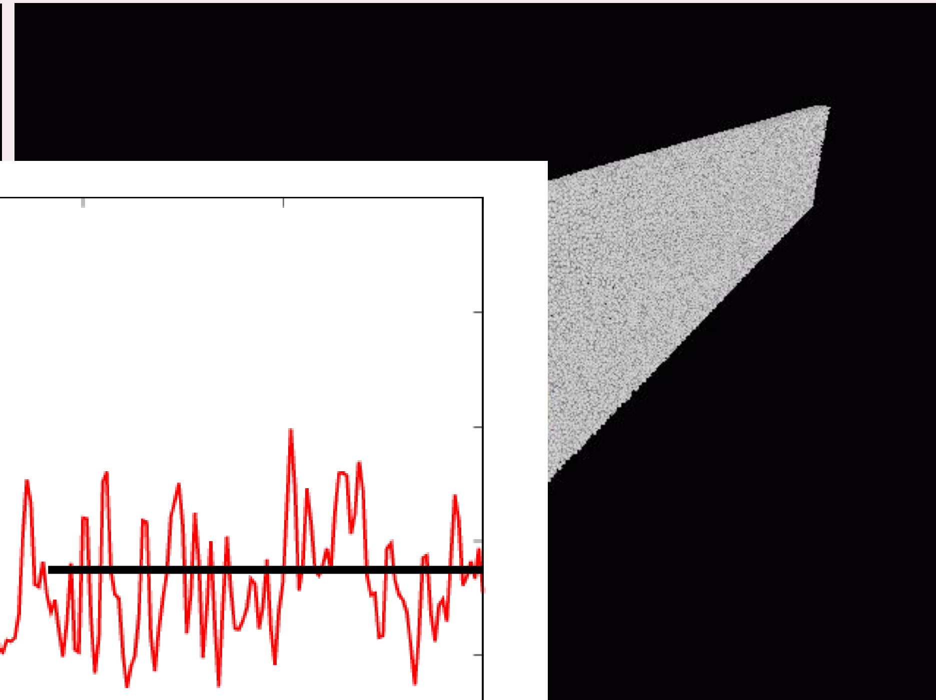
Free scattering

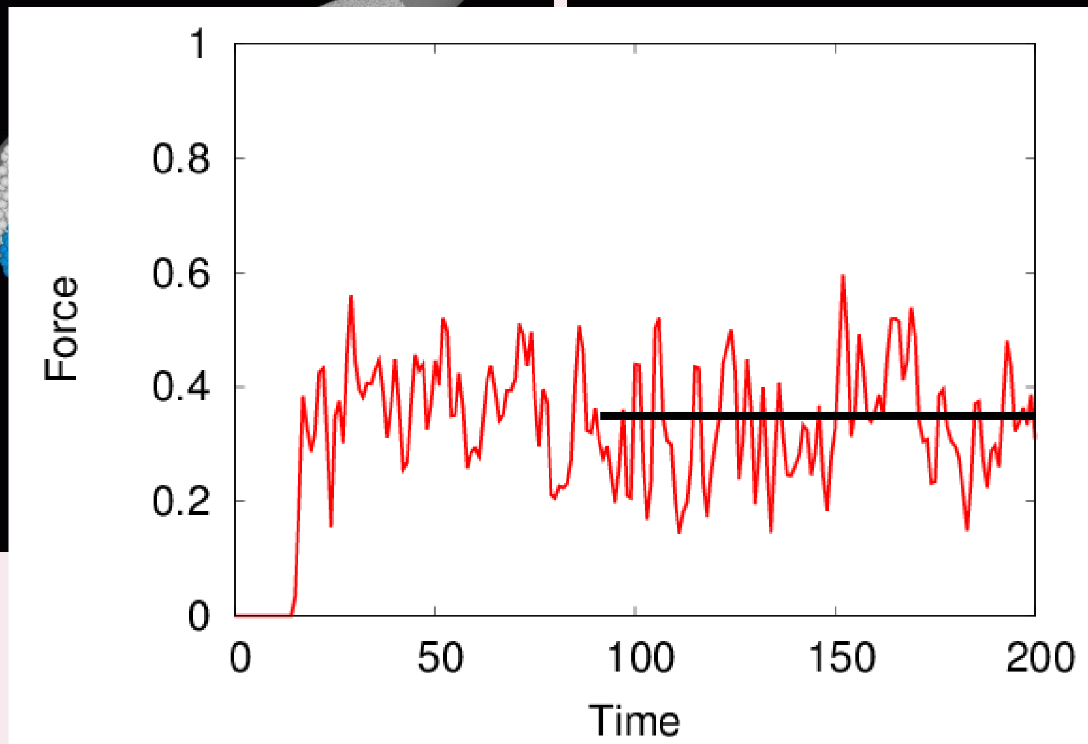
$(N, D/d) = (30{,}000, 7), R = 10^3$



Confined

$(N, D/d) = (810{,}000, 30), R = 10^1$



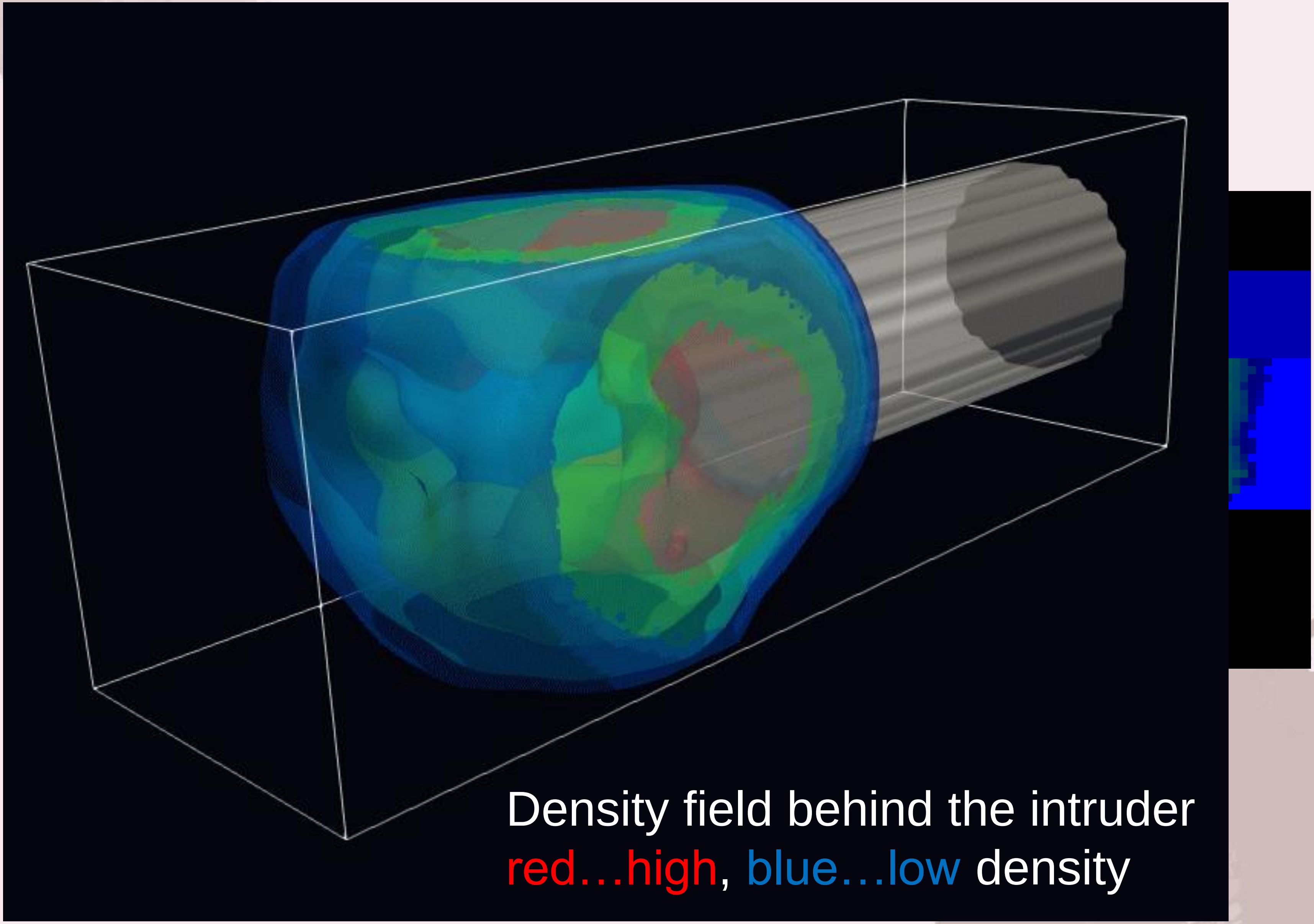


rticles for
isualized.

We measure the mean force acting on the intruder.

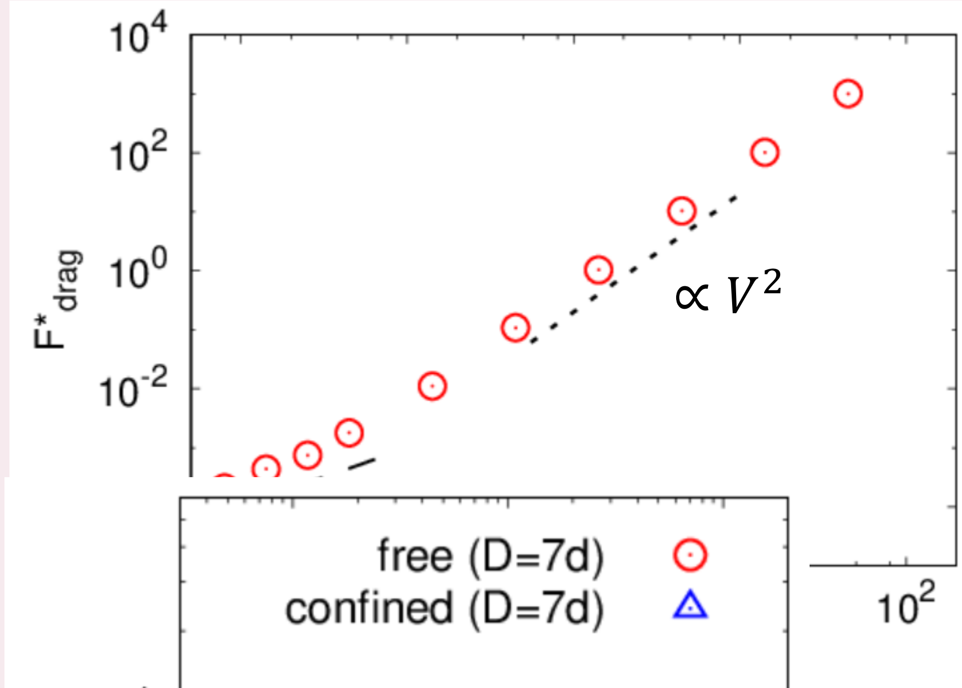
Density and temperature fields

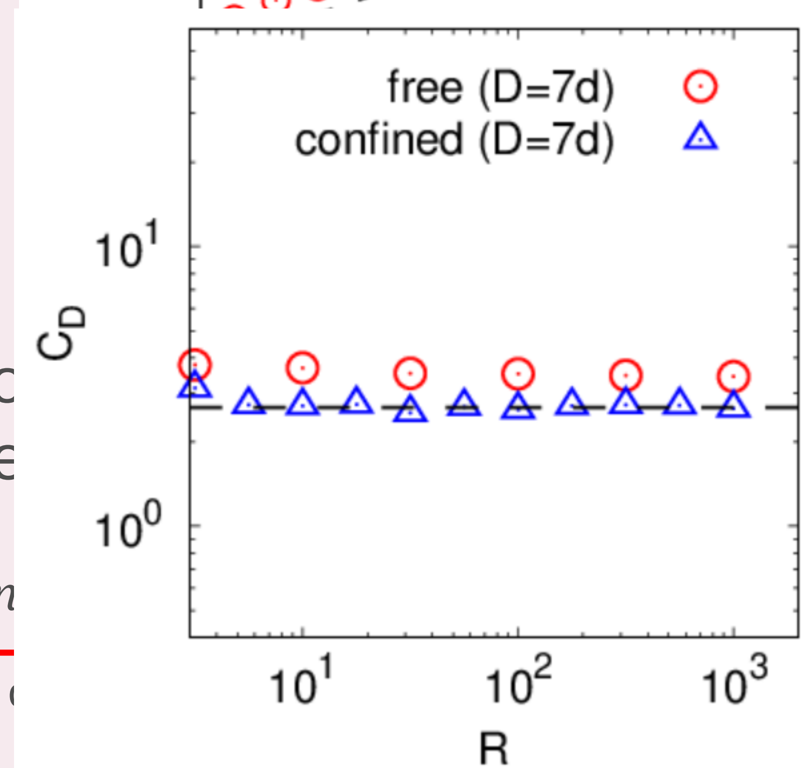
$R = 10^3$



F_{drag} vs. V

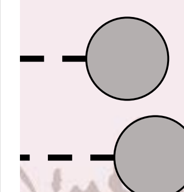
- High V regime
  $F_{\text{drag}} \propto V^2$
- Low V regime
  $F_{\text{drag}} \sim V$





High V regime

The drag force is understo
by a simple collision mode



$$F_{\text{coll}} = \int_{0\le\theta<\pi} d\hat{S}\, 2mV\cos^2\theta \cdot n$$

Surface integral Number

er density

$$\Rightarrow C_{\text{D}} = \frac{F_{\text{coll}}}{\frac{1}{2}nmV^2\frac{\pi}{4}D^2} = \frac{8}{3}$$

$\Rightarrow$ consistent with the simulation.

Low V regime

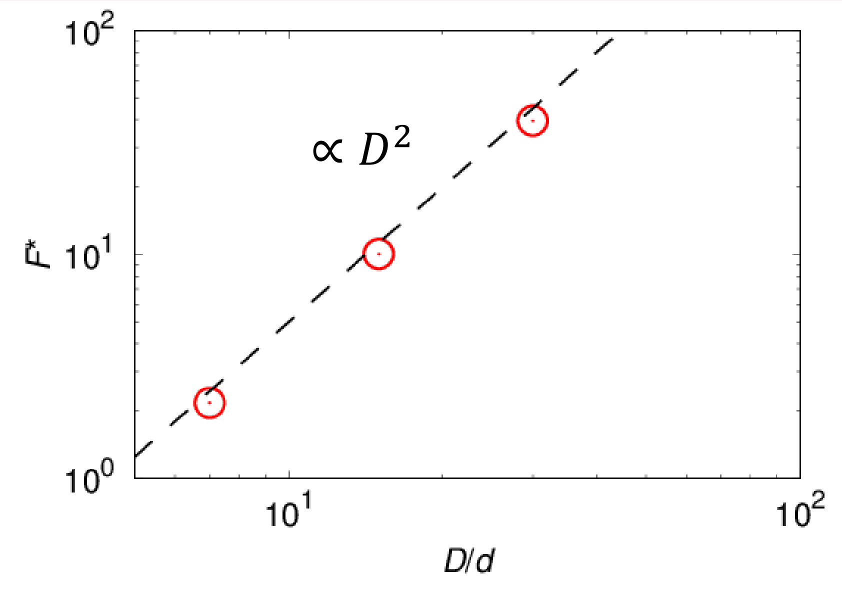
$F_{\mathrm{drag}} \sim V$ ⇒ Stokes' law?

If so, the drag force $F \propto D$

HOWEVER, $F_{\mathrm{drag}} \propto D^2$!

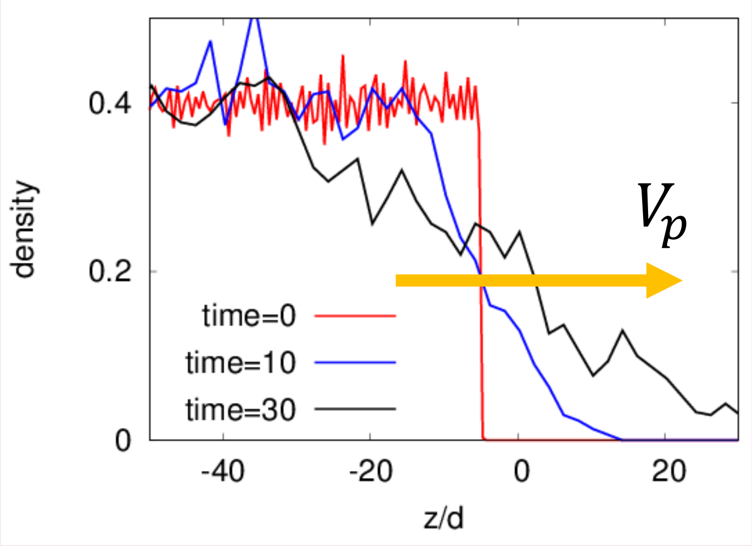
⇒ ❌ Stokes' law ❌

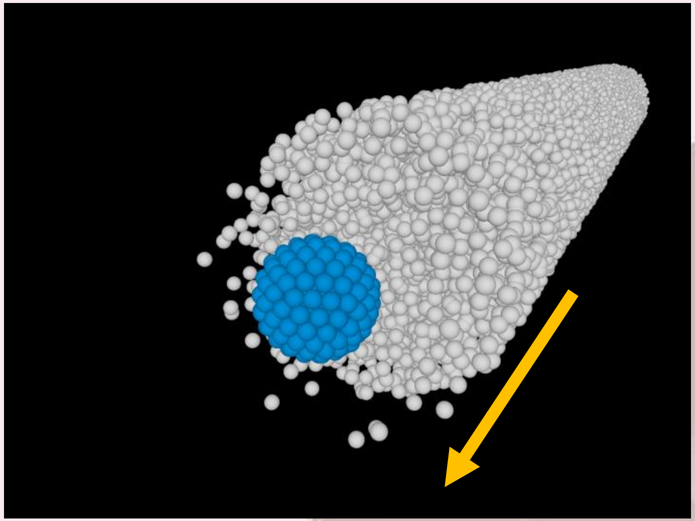
What happens?



⇒ "A shock wave" expands into vacuum.

The propagation speed V_p is higher than the input speed V.



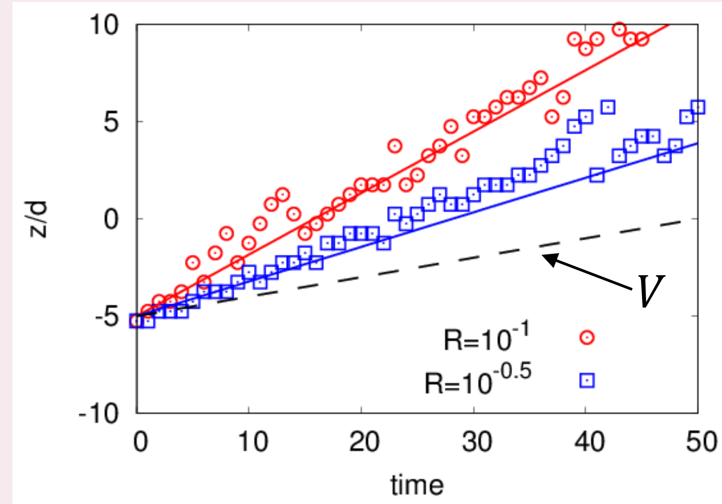


This propagation speed V_p is approximately given by

$$V_p = \sqrt{V v_{\mathrm{T}}} = \frac{V}{\sqrt{R}}$$

⇒ A simple collision model with V_p

$$F_{\mathrm{coll}} = \frac{\pi}{3} n m D^2 V_p^2, \quad C_{\mathrm{D}} = \frac{8}{3R}$$

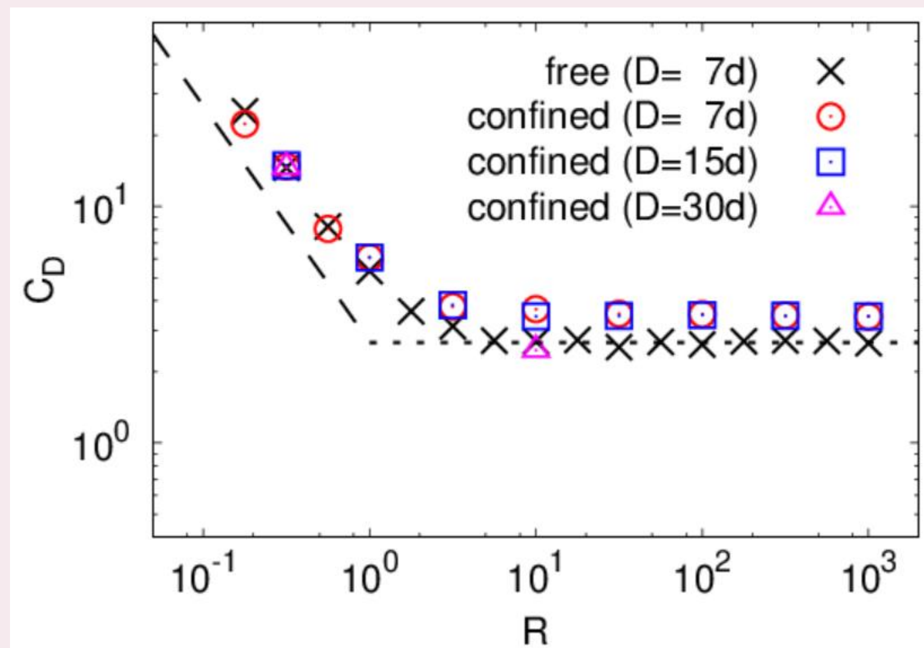


C_{D} vs R

$$C_{\mathrm{D}} = \frac{8}{3} \ (R \gtrsim 1)$$

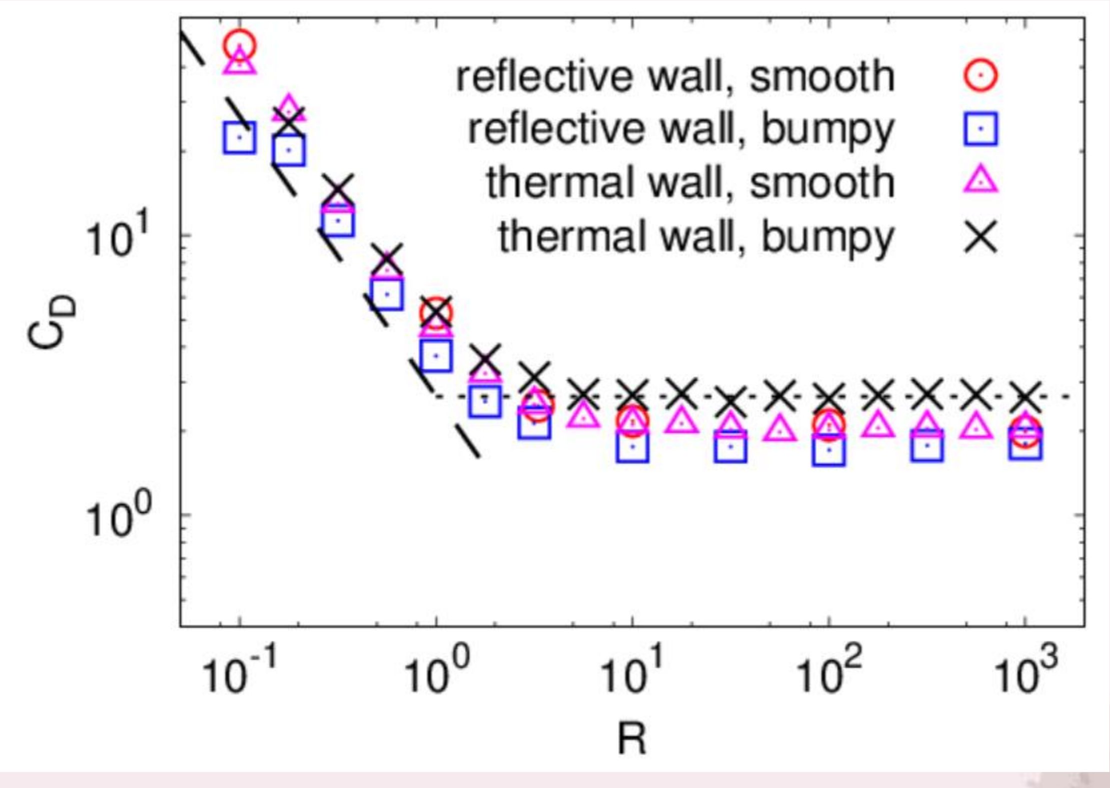
$$C_{\mathrm{D}} = \frac{8}{3R} \ (R \lesssim 1)$$

No drag drop for larger R!



Effect of boundary condition

- Effect of the boundary condition between the intruder and particles



The results are independent of the boundary condition.

Visualization of vortex tube for large R

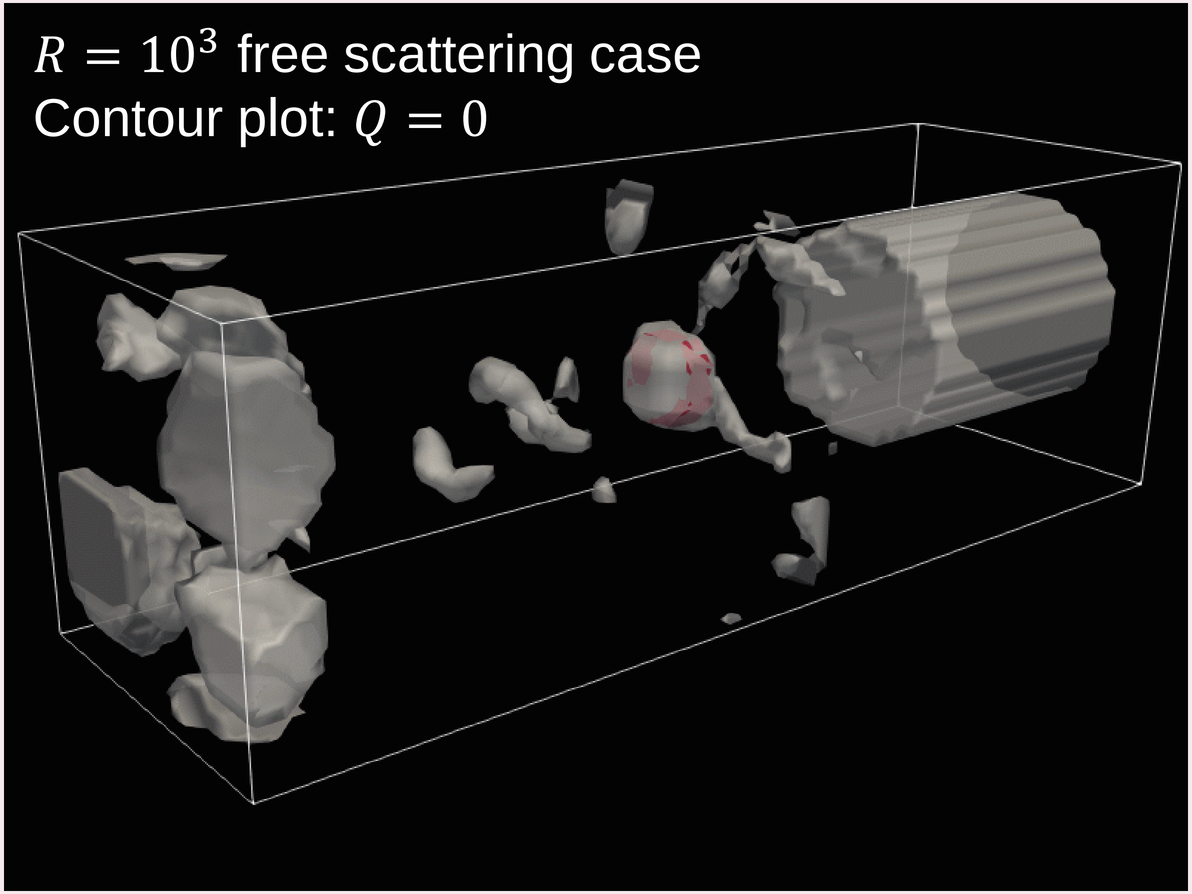
Can we observe vortex structures? – Yes.

Introduction of second invariant:

$$Q = \frac{1}{2}\left(-S_{ij}S_{ij} + W_{ij}W_{ij}\right)$$

$$S_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

$$W_{ij} = \frac{1}{2}\left(u_{i,j} - u_{j,i}\right)$$



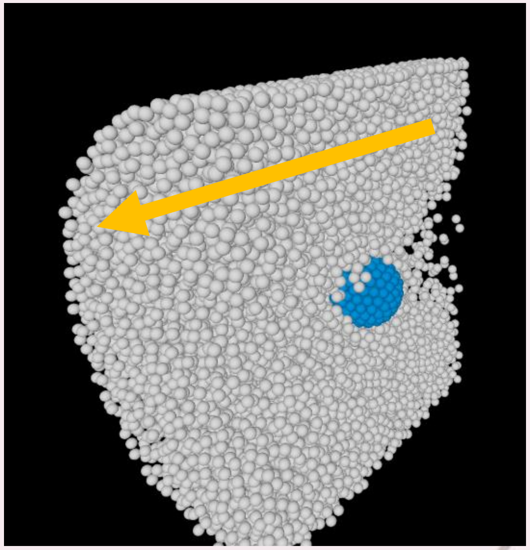
$Q < 0$:
vorticity < viscous diss.

$Q > 0$:
vorticity > viscous diss.

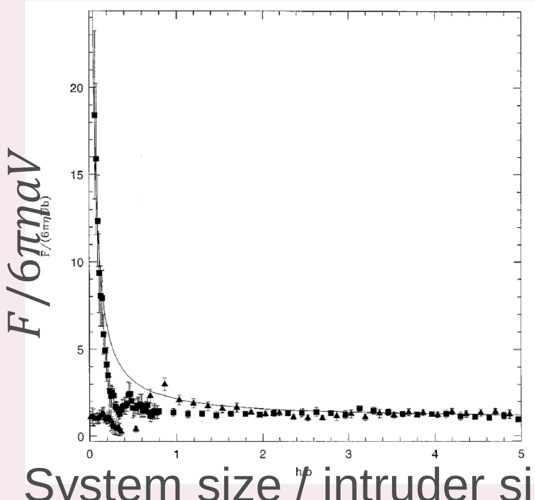
$\Rightarrow$ Appearance of vortices

Discussion

- Why no Stokes' law?
  At initial time, the space around the intruder is vacuum.
  ⇒ "A shock wave" propagates.
  This determines the drag law.



- How can we realize Stokes' law?
  Setup is very important.
  We should put the intruder
  into the beam particles.



M. Vergeles, et al., PRE 53, 4852 (1996)
reported that the Stokes' law is satisfied
if the system is much larger than the intruder.

Summary

We have performed MD simulations and investigated the drag law.

- The drag coefficient is proportional to
  - $\mathrm{const.}$ ...Newtonian based on the simple **collision model.**
  - $1/R$ ... "Shock wave" with expansion speed $V_p = \sqrt{V v_T}$ simple collision model with speed V_p.

  No drag drop for large R.

  The results are independent of the boundary condition.
- No separation vortices appear, but Kármán-like vortices are observed for large R.

Future work

- Stokes' flow / Analysis of turbulence