場の理論的シミュレーションに基づく アクシオン宇宙ひもの解析

関口豊和 (ICRR, Univ. of Tokyo)

arXiv:1012.5502 with 平松尚志(YITP), 川崎雅裕(ICRR), 山口昌英(TITech), 横山順一(RESCEU)

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Outline

- Introduction
- Field theoretic simulation of axionic strings
- Scaling property of the string network
- Energy spectrum of radiated axions
- Constraint on the axion decay constant
- Summary

I. Introduction

Axion:

- Pseudo-NG boson of the spontaneously broken anomalous Peccei-Quinn U(I) symmetry.
- Rich implications in cosmology:
 - A candidate of CDM
 - Isocurvature perturbations $H_{
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⇒Another possibility: the PQ symmetry restores in the early Universe.

Axionic string

 Since the PQ symmetry is a global U(I) symmetry, when U(I)PQ breaks spontaneously, I-dim topological defect (axionic string) can form.



 If the SSB of U(I)PQ occurs in the early Universe, a cosmological network of axionic strings is generated.

Fate of cosmological axionic strings

Scaling solution

• Number of strings in a horizon stays constant.



- At QCD phase transition, axionic domain walls (DWs) bounded by axionic strings are generated, and both of them quickly disappear if $N_{\rm DW} = 1$.
- Strings lose their energy by emitting massless axions.
 - Emitted axions finally become CDM.

 $\bar{\rho}_{\mathrm{axion}}(t_0) = m_{\mathrm{axion}}\bar{n}_{\mathrm{axion}}(t_0)$

• Number density of radiated axions:

$$\bar{n}_{\rm axion}(t) = \int \frac{dk}{2\pi^2} \frac{R(t)}{k} P(k, t).$$

- Energy spectrum of radiated axions: P(k)
 - At small momenta, P(k) peaks at the horizon scale ~1/H.
 - At higher momenta...

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 - Scales as power-law? (← strings lose energy during several oscillations) [Harari & Sikivie (87), Hagmann & Sikivie (91), ...]
 - \Rightarrow There has been a controversy!

A previous study

- A solution to the controversy was given by Yamaguchi, Kawasaki & Yokoyama (99).
 Spectrum of axions emitted between t=65t, an 0.01 Example of axions emitted between t=65t, and and a sector of axions emitted between t=65t, and a
 - A field theoretic lattice simulation of the PQ scalar is performed.
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/⁴⁰ δk)∼

Main purpose:

To improve the accuracy of the analysis, using a sophisticated statistical method.

2. Field theoretic simulation of axionic strings

• Field theoretic simulation

- Axionic strings are not well-localized. (cf. string-based Nambu-Goto action used for local strings)
- First-principles calculation, free from theoretical uncertainties.

$$\ddot{\Phi}(\vec{x},t) + 3H(t)\Phi(\vec{x},t) - \frac{1}{R(t)^2}\nabla^2\Phi(\vec{x},t) = -\frac{\partial V[\Phi,T]}{\partial\Phi^*}$$

• Dynamical range is limited.

$$\frac{R(t)}{R(t_{\rm crit})} \lesssim \sqrt{N_{\rm grid}^{1/3} \frac{\eta}{M_{\rm Pl}}}.$$

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two requirements I. Resolve inner structure of strings \Rightarrow comoving size $\propto 1/R(t)$.

2. Simulation box > a horizon volume

 \Rightarrow comoving size $1/R(t)H(t) \propto R(t)$

Details of simulation

• Parameters

- PQ scale: $\eta = 10^{16} \mathrm{GeV}$
- coupling constant: $\lambda = 1$
- number of relativistic dof: $g_* = 1000$
- comoving size of simulation box at final time: $L = 1.6/H(\tau_{end})$
- number of grids: $N_{\rm grid} = 512^3$
- time range: $t_{\text{ini}} = 0.25t_{\text{crit}}, t_{\text{end}} = 25t_{\text{crit}}$
- spacial resolution: $R(t_{
 m end})\Delta x = 0.7 d_{
 m string}$, with $d_{
 m string} = 1/\sqrt{2\eta}$
- Initial condition is randomly drawn from the thermal distribution.
- Equation of motion is integrated using the leap-frog method.



Identification of strings

- Non-trivial task!! Only $\Phi(\vec{x}, t)$ at discrete lattice points are known.
- We developed a completely new method for identification of strings:



 We can determine >99% of string positions in quadrates penetrated by strings.





Scaling parameter

 $\xi \equiv \frac{\rho_{\rm string} t^3}{\mu_{\rm string} t}$

= (# of strings in a horizon volume)





• Completely deferent identification methods give similar results: $\xi = 1.0 \pm 0.08$ [YKY99], $\xi \simeq 0.8$ [Yamaguchi & Yokoyama (03)].



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• Cf. For local string, $\xi\simeq 13$. [Bennett & Bouchet (90), Allen & Shellard (90), ...]

• Axion is the phase of the PQ scalar

$$\Phi(\vec{x},t) = \left(\eta + \frac{\sigma(\vec{x},t)}{2}\right) \exp\left[i\frac{a(\vec{x},t)}{\sqrt{2}\eta}\right]$$



• Mean kinetic energy of axion is given by

$$\bar{\rho}_{\rm axion}(t) = \frac{1}{V} \int d^3x \frac{1}{2} \left[\frac{da}{dt}(\vec{x}, t) \right]^2 = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2V} \left| \frac{da}{dt}(\vec{k}, t) \right|^2.$$

• Energy spectrum:

$$P(k,t) = \int \frac{d\hat{k}}{4\pi} \frac{k^2}{2V} \left| \frac{da}{dt}(\vec{k},t) \right|^2$$

• Number density:

$$\bar{n}_{\text{axion}}(t) = \int \frac{dk}{2\pi^2} \frac{R(t)}{k} P(k, t).$$

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- Removal of string contamination is crucial!! In YKY99, this is done by using only selected sub-volumes found without strings.
- We adopted the pseudo-power spectrum estimator (PPSE), which is often used in data analysis of CMB.



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4. Spectrum of masked fluctuation

$$\tilde{P}(k) \approx \int \frac{d\hat{k}}{4\pi} \left| \tilde{\dot{a}}(\vec{k}) \right|^2$$

• Biased due to masking $\langle \tilde{P}(k) \rangle \neq P_{\rm free}(k)$



• Energy spectra are estimated in eight sub-boxes with a same size.



• Total box size $L = 1/H(t_{end})$

Grid points away from strings by $< 3d_{string}$ are masked.

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• PPSE successfully removes the string contaminations!

Result: Differential spectrum

• Differential spectrum (= energy spectrum of net radiated axions) $\Delta \hat{P}(k; t_1, t_2) = R(t_2)^4 \hat{P}(k, t_2) - R(t_1)^4 \hat{P}(k, t_1)$



comoving wavenumber k

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comoving wavenumber k

Energy dependence of radiated axions

- Energy spectrum of radiated axion P(k):
 - Sharply peaks around the horizon scale.
 - Suppressed exponentially toward higher momenta k.
 - Consistent with the previous YKY99 and supports the claim of Davis & Shellard.
- Mean momentum of radiated axions:

$$\overline{k(t)} = \frac{\int dk \Delta P_{\text{free}}(k,t)}{\int dk \frac{1}{k} \Delta P_{\text{free}}(k,t)} = \frac{\Delta[\bar{\rho}_{\text{axion}} R^4]}{\Delta[\bar{n}_{\text{axion}} R^3]}$$

• $\overline{k(t)}$ ~ Hubble scale, $\overline{k(t)}^{-1} = 0.23 \pm 0.02 \frac{t}{R(t)2\pi}.$

 \Rightarrow consistent with 0.25±0.18 in YKY99.

5. Constraint on the axion decay constant

- Extrapolate our result down to $\eta = f_a \simeq 10^{12} {
 m GeV}.$
- In the scaling regime, the energy density of strings are given by

$$\bar{
ho}_{
m string}(t) = rac{\xi}{t^2} 2\pi f_a^2 \ln\left(rac{t}{\sqrt{\xi}d_{
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ight), \quad ext{ with } d_{
m string} = f_a/\sqrt{2}.$$

• Axionic strings lose their energy via emitting axions:

$$\left[\frac{d\bar{\rho}_{\rm axion}}{dt}\right] = -\left[\frac{d\bar{\rho}_{\rm string}}{dt}\right]$$

 \Rightarrow Net energy density of radiated axions:

$$\frac{1}{R(t)^4} \frac{d[R(t)^4 \bar{\rho}_{axion}(t)]}{dt} \simeq \frac{\xi}{t^3} 2\pi f_a^2 \ln\left(\frac{f_a t}{\sqrt{2\xi}}\right)$$

Number of radiated axions in a unit comoving volume:

$$R(t)^{3}\bar{n}_{\mathrm{axion}}(t) = \int^{t} dt \overline{k(t)}^{-1} \frac{d[R(t)^{4}\bar{\rho}_{\mathrm{axion}}(t)]}{dt}.$$

Constraint on the axion decay constant(cont'd)

• Strings continue emitting axions till the wall domination occurring at

$$T_w \simeq 0.67 \text{ GeV} \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{-0.12}$$
. $\longleftarrow m_{axion} \propto f_a^{-1} T^{-3.39}$
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- After the wall domination, the number of axions is conserved.
- The energy density of CDM axions from strings $\Omega_{\rm axion}h^2 \simeq 8.7 \left(\frac{\xi}{\epsilon}\right) \left(\frac{f_a}{10^{12} {\rm GeV}}\right)^{1.19} \text{ with } \begin{cases} \xi = 0.87 \pm 0.14\\ 1/\epsilon = 0.23 \pm 0.02 \end{cases}$

• Constraint on the decay constant of axion:

$$f_a \lesssim 1.3 \times 10^{11} \text{GeV} \qquad \longleftarrow \begin{array}{c} \Omega_{\text{CDM}} h^2 = 0.11 \\ \text{[Komatsu+(10)]} \end{array}$$

Other sources of axion CDM

- Emission from DWs [Hagmann & Sikivie (91), Lyth (92), Nagasawa & Kawasaki (94), ...]
 - DWs quickly disappear after formation.

 $\Omega_{\rm axion} h^2 \simeq \frac{1.8}{\gamma} \left(\frac{f_a}{10^{12} {\rm GeV}} \right)^{1.19} \frac{\gamma: \text{Lorentz factor}}{\text{of radiated axions}}$

• Numerical simulation gives $\gamma \simeq 7.$ [Chang, Hagmann & Sikivie (98)]

 $f_a \lesssim 4.9 \times 10^{11} \text{GeV}.$

• Coherent oscillation

$$\Omega_{\rm axion} h^2 \simeq 0.10 \ \theta_i^2 \left(\frac{f_a}{10^{12} {\rm GeV}}\right)^{1.19}$$

 θ_i : initial misalignment

• The canonical value is $\langle \theta_i^2 \rangle = \pi^2/3$. $f_a \lesssim 4.0 \times 10^{11} \text{GeV}.$

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• Constraints from other sources are comparable. $f_a \lesssim 10^{11} \text{ GeV}$.

Summary

- We performed a field theoretic simulation of cosmological axionic strings of the largest scales ($N_{\rm grid} = 512^3$) so far.
- We developed a new method for identification of strings, which allows determination of string positions with >99% efficiency.
- We estimated the energy spectrum of radiated axions from strings, using the pseudo-power spectrum estimator. We successfully removed contributions from string cores and achieved precise estimation of the spectrum.
- The spectrum is consistent with YKY99, showing exponential damping at large momenta. Our result supports the claim of Davis & Shellard.
- We obtained a constraint on the axion decay constant,

 $f_a \lesssim 10^{11} \text{ GeV}.$

Result: Energy spectrum

Energy spectrum



Note!! Not all of axions are emitted within the scaling regime.