Stochastic Approach to Flat Direction during Inflation

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based on arXiv:1207.1165

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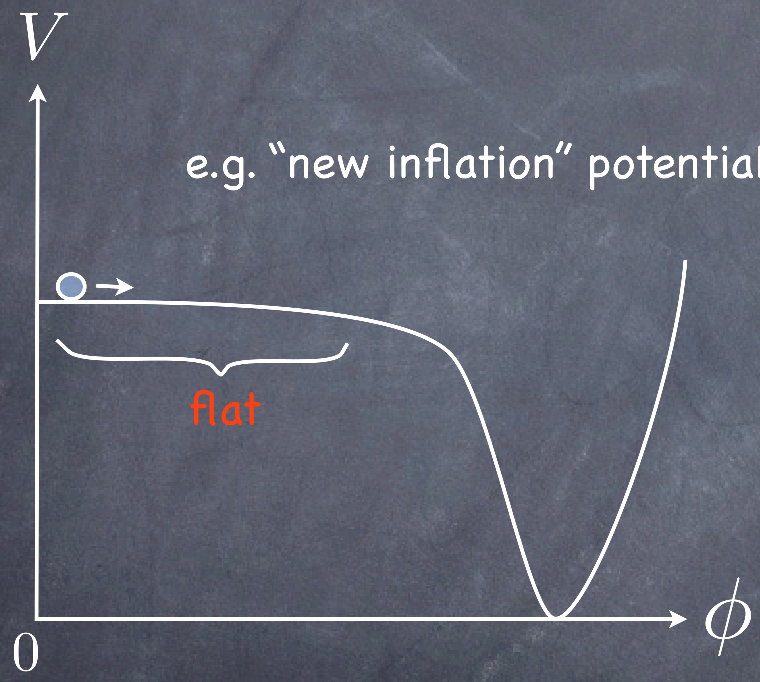
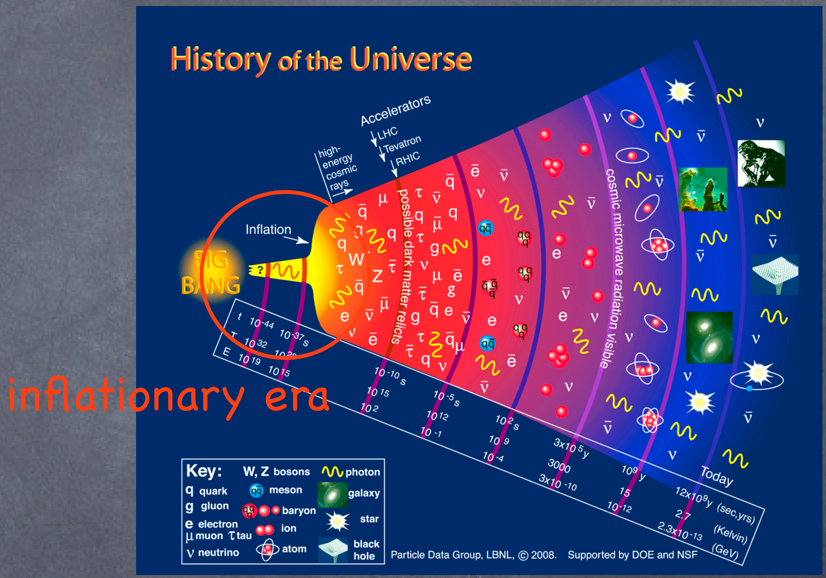
Introduction

Inflation is a key ingredient to solve the problems in the standard cosmology. To obtain a successful inflation, we need a scalar field which has a very flat potential.





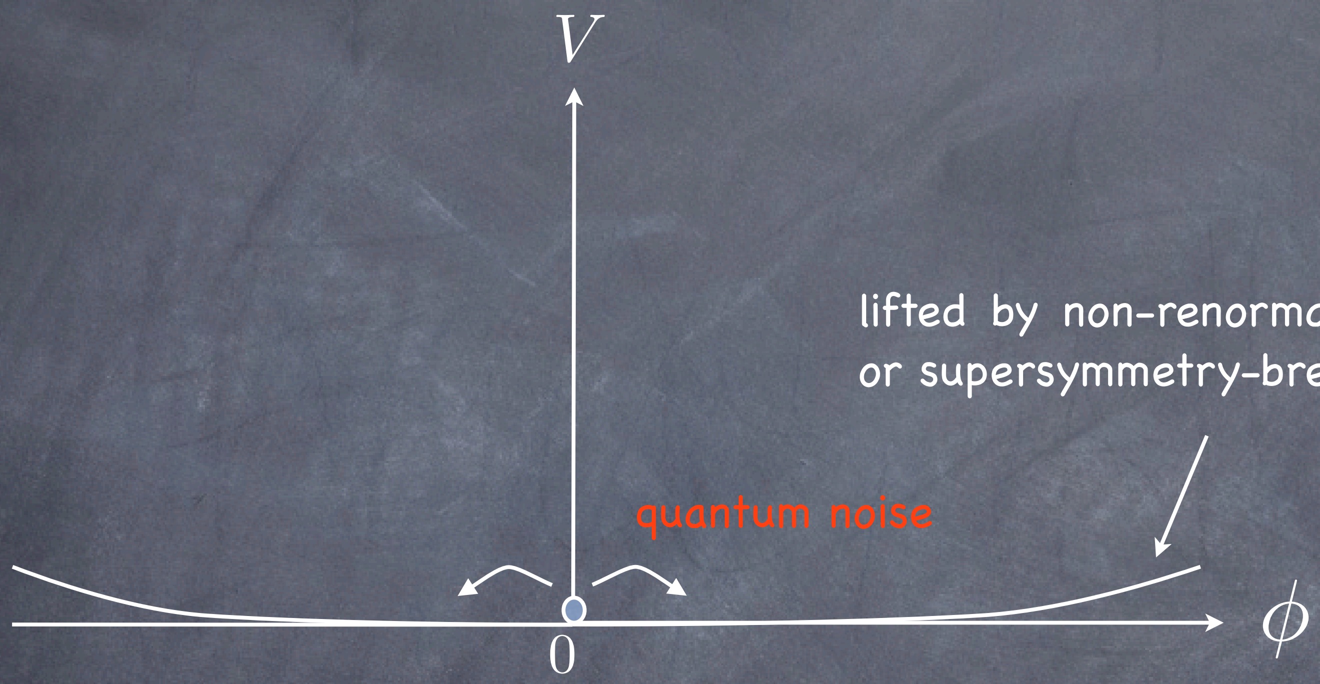
http://www.particleadventure.org/images/history-universe-08.jpg

In supersymmetric models of particle physics, there are many flat directions. In the following, we consider a flat direction in the framework of supersymmetry.

Flat direction ϕ

A flat direction has a very flat potential.

A.Linde (1982)
A.Vilenkin, L.H.Ford (1982)
A.A.Starobinsky (1982)



During inflation, quantum noise drives the flat direction to dilute.
The variance of the flat direction is given by the following formulae:

$$\langle \phi^2 \rangle = \frac{H^2}{4\pi^2} N, \quad (\tilde{m} = 0)$$

$$\langle \phi^2 \rangle = \frac{3H^4}{8\pi^2 \tilde{m}^2}, \quad (\tilde{m} \ll H)$$

Here, $N = Ht$

$\tilde{m}$: effective mass

Stochastic approach (single field)

Starobinsky (1984)

M.Sasaki, Y.Nambu, K.Nakao (1987)

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Definition of IR modes

$$\phi(x) = \Phi(x) + \phi_{\rm UV}(x) \quad , \dot{\phi}(x) = \Pi(x) + \Pi_{\rm UV}(x)$$

$$\Phi(x) \equiv \int \frac{d^3k}{(2\pi)^3}\theta(\epsilon aH - k)\left(a_{\mathbf{k}}\varphi_k(t) + a^\dagger_{-\mathbf{k}}\varphi^*_k(t)\right)e^{i\mathbf{k}\cdot\mathbf{x}}$$

(IR mode for conjugate momentum is defined in the same way)

$$\varphi_k(t) = \sqrt{\frac{\pi}{4k^3}} H e^{i\left(\frac{\pi\nu}{2}+\frac{\pi}{4}\right)}\left(\frac{k}{aH}\right)^{3/2} H^{(1)}_\nu\left(\frac{k}{aH}\right),$$

$$\nu^2 = \frac{9}{4} - \frac{\tilde{m}^2}{H^2}$$

$$\tilde{m}^2 = \left\langle \frac{\partial^2 V}{\partial\phi^2}\right\rangle$$

Equations of motion for IR modes (Langevin eqs.)

$$\dot{\Phi}(x) = \Pi(x) + s^{(\phi)}(x),$$

$$\dot{\Pi}(x) = -3H\Pi(x) - V'(\Phi) + s^{(\pi)}(x).$$

where the noise term is given by

$$s^{(\phi)}(x) = \epsilon aH^2 \int \frac{d^3k}{(2\pi)^3}\delta(k - \epsilon aH)\left(a_{\mathbf{k}}\varphi_k(t) + a^\dagger_{-\mathbf{k}}\varphi^*_k(t)\right)e^{i\mathbf{k}\cdot\mathbf{x}}$$

variance of the noise term:

$$\langle s^{(\phi)}(x)s^{(\phi)}(x')\rangle = \left(\frac{H}{2\pi}\right)^2 H\delta(t-t')j_0(\epsilon aH|\mathbf{x}-\mathbf{x}'|)$$

when the effective mass is exactly zero!!

Effective mass effects in noise terms

Recently, Enqvist et al (2012) have analyzed the flat direction using stochastic approach. In the their works, the effective mass effects are not included in the noise terms. In this study, we improve this point.

The noise correlation functions integrated for a small intervals is given by:

$$S^{(\phi)}(r,t;\mathrm{d}t) \equiv \int_t^{t+\mathrm{d}t} \mathrm{d}t_1 \int_t^{t+\mathrm{d}t} \mathrm{d}t_2 \, \langle \mathrm{vac}| \, s^{(\phi)}(\mathbf{x_1},t_1) s^{(\phi)}(\mathbf{x_2},t_2) \, |\mathrm{vac}\rangle$$

$$= \left(\frac{H}{2\pi}\right)^2 H\mathrm{d}t \; j_0(\epsilon a H r) \; \frac{\pi}{2}\epsilon^3 |H_\nu^{(1)}(\epsilon)|^2 \times \begin{cases} 1 & (\nu = \mathrm{real}), \\ \mathrm{e}^{-\pi\mu} & (\nu = i\mu), \end{cases}$$

$$S^{(\pi)}(r,t;\mathrm{d}t)$$

$$\equiv \int_t^{t+\mathrm{d}t} \mathrm{d}t_1 \int_t^{t+\mathrm{d}t} \mathrm{d}t_2 \, \langle \mathrm{vac}| \, s^{(\pi)}(\mathbf{x_1},t_1) s^{(\pi)}(\mathbf{x_2},t_2) \, |\mathrm{vac}\rangle$$

$$= \left(\frac{H^2}{2\pi}\right)^2 H\mathrm{d}t \; j_0(\epsilon a H r) \; \frac{\pi}{2}\epsilon^3 \left| \left(\frac{3}{2} - \nu\right) H_\nu^{(1)}(\epsilon) + \epsilon H_{\nu-1}^{(1)}(\epsilon) \right|^2 \times \begin{cases} 1 & (\nu = \mathrm{real}), \\ \mathrm{e}^{-\pi\mu} & (\nu = i\mu), \end{cases}$$

the effective mass is embedded in $\nu^2 = \frac{9}{4} - \frac{\tilde{m}^2}{H^2}$

The zero-point fluctuation in the noise terms

Mode function of the zero-point fluctuation is given by

$$\varphi_k^{\rm zero}(t) = \frac{1}{\sqrt{2\omega_k(t)a(t)}} {\rm e}^{-i\int^t {\rm d}t' \sqrt{k^2/a^2+\tilde{m}^2}}$$

$\omega_k(t) = \sqrt{k^2/a^2+\tilde{m}^2}$

$a(t)$:scale factor

Inserting this mode function into the formulae for the noise correlation functions, we identify the zero-point contribution as:

for $\tilde{m}=0$

$$\bar{S}^{(\phi){\rm zero}}(r,N;{\rm d}N) = \frac{{\rm d}N}{(2\pi)^2} j_0(\epsilon a H r)\epsilon^2,$$

$$\bar{S}^{(\pi){\rm zero}}(r,N;{\rm d}N) = \frac{{\rm d}N}{(2\pi)^2} j_0(\epsilon a H r)\epsilon^2\left(1+\epsilon^2\right).$$

for $\tilde{m}\gg H$

$$\bar{S}^{(\phi){\rm zero}}(r,N;{\rm d}N) \simeq \frac{{\rm d}N}{(2\pi)^2} j_0(\epsilon a H r)\epsilon^3 \frac{H}{\tilde{m}},$$

$$\bar{S}^{(\pi){\rm zero}}(r,N;{\rm d}N) \simeq \frac{{\rm d}N}{(2\pi)^2} j_0(\epsilon a H r)\epsilon^3 \frac{\tilde{m}}{H}\left(1+\frac{9}{4\tilde{m}^2/H^2}\right)$$

$\propto \tilde{m}$

Should we remove the zero-point fluctuation from the noise terms ???

The model

According to Enqvist et al (2012), we consider the following potential.

$$V = \frac{1}{2}\lambda_e^2\left(\phi^2 + h^2\right)\bar{e}^2 + \frac{1}{8}g_2^2 h^4 + \frac{1}{8}g_1^2\left(h^4 + 4\bar{e}^4 - 4h^2\bar{e}^2\right) + \frac{\phi^6}{M_{\rm P}^2}$$

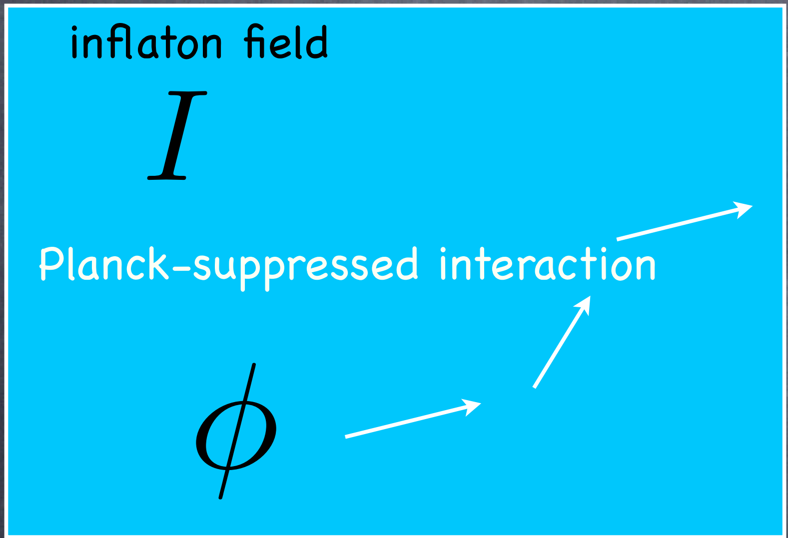
Then, the discretized coupled Langevin eq. to be solved numerically is:

$$\begin{aligned}
&\bar{\Phi}(N+{\rm d}N) = \bar{\Phi}(N) + \bar{\Pi}(N){\rm d}N + \bar{S}^{(\phi)},\\
&\bar{\Pi}(N+{\rm d}N) = \bar{\Pi}(N) - 3\bar{\Pi}(N){\rm d}N - \left(\lambda_e^2\bar{E}(N)^2\bar{\Phi}(N) + 6\frac{H^2}{M_{\rm P}^2}\bar{\Phi}(N)^5\right){\rm d}N + \bar{S}^{(\pi_\phi)},\\
&\bar{E}(N+{\rm d}N) = \bar{E}(N) + \bar{\Pi}_e(N){\rm d}N + \bar{S}^{(e)},\\
&\bar{\Pi}_e(N+{\rm d}N) = \bar{\Pi}_e(N) - 3\bar{\Pi}_e(N){\rm d}N\\
&\qquad\qquad - \left(\left(\lambda_e^2\bar{\Phi}(N)^2 + \left(\lambda_e^2 - g_1^2\right)\bar{h}(N)^2\right)\bar{E}(N) + 2g_1^2\bar{E}(N)^3\right){\rm d}N + \bar{S}^{(\pi_e)},\\
&\bar{h}(N+{\rm d}N) = \bar{h}(N) + \bar{\Pi}_h(N){\rm d}N + \bar{S}^{(h)},\\
&\bar{\Pi}_h(N+{\rm d}N) = \bar{\Pi}_h(N) - 3\bar{\Pi}_h(N){\rm d}N\\
&\qquad\qquad - \left(\left(\lambda_e^2 - g_1^2\right)\bar{E}(N)^2\bar{h}(N) + \frac{1}{2}\left(g_1^2 + g_2^2\right)\bar{h}(N)^3\right){\rm d}N + \bar{S}^{(\pi_h)}.
\end{aligned}$$

Hubble induced mass

E.D.Stewart (1995)
M.Dine, L.Randall, S.D.Thomas (1995)

It has been pointed out that a scalar field in Inflationary era generically obtain an effective mass of the order of Hubble scale in the framework of (local) supersymmetry.



$$V \supset \frac{|\phi|^2}{M_{\rm P}^2}\rho_I = 3H_I^2|\phi|^2.$$

ρ_I : inflaton energy density
$M_{\rm P}$: reduced Planck mass
H_I : Hubble parameter during inflation

Such a effect lift flat directions drastically.

Preserving flat direction during inflation

To avoid the Hubble-induced tree-level effective mass, we can assume the following:

- D-term inflation scenario — E.D.Stewart (1995)

or

- a Heisenberg symmetry — MK.Gaillard, H.Murayama, KA.Olive (1995)

Even if we assume the above, a radiative correction is exist.

$$|\tilde{m}^2_{\phi,\mathrm{rad.}}| \simeq 10^{-2}\lambda_e^2 H^2$$

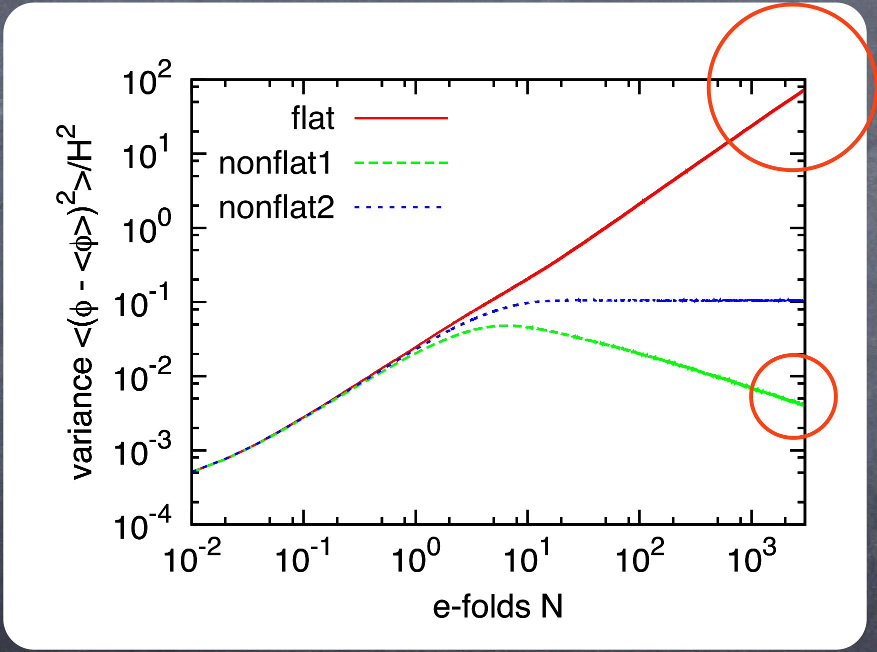
MK.Gaillard, H.Murayama, KA.Olive (1995)
Garbrecht (2007)

If the tree-level effective mass is highly suppressed, this radiative correction dominates the potential until the non-renormalizable interactions become important.

without the zero-point contribution to noise terms

(the time evolution of IR modes)

$$\epsilon = 10^{-2}$$
$$\lambda_e = g_1 = g_2 = 1$$



evolves linearly

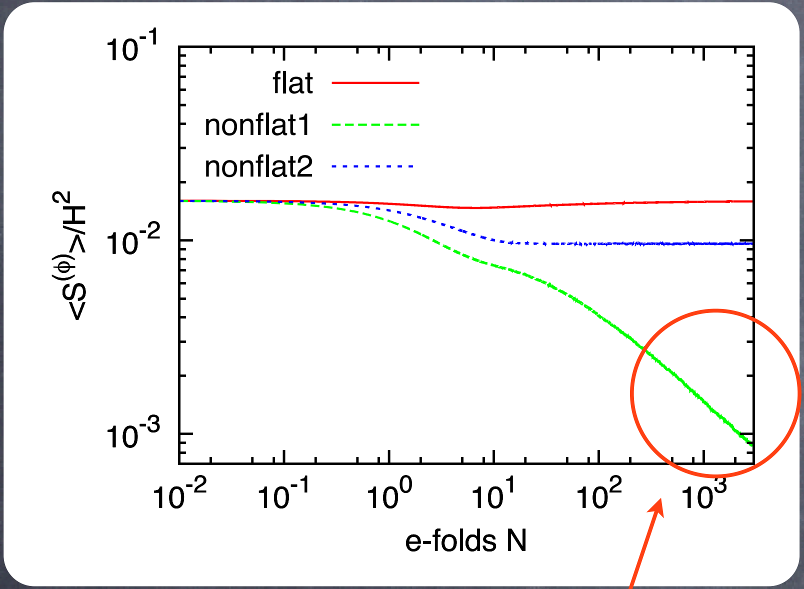
decreases infinitely

- without the zero-point contribution to noise terms

(the time evolution of noise terms)

$\epsilon = 10^{-2}$

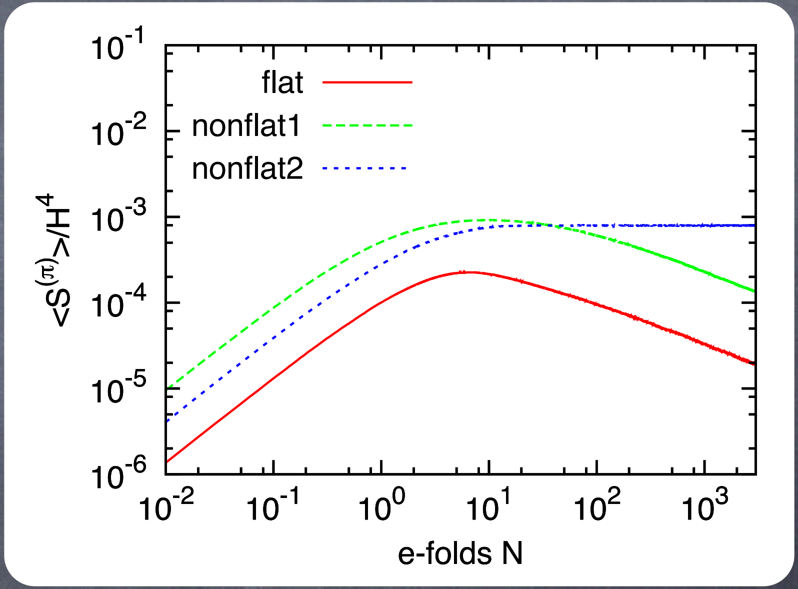
$\lambda_e = g_1 = g_2 = 1$



highly suppressed

$\epsilon = 10^{-2}$

$\lambda_e = g_1 = g_2 = 1$

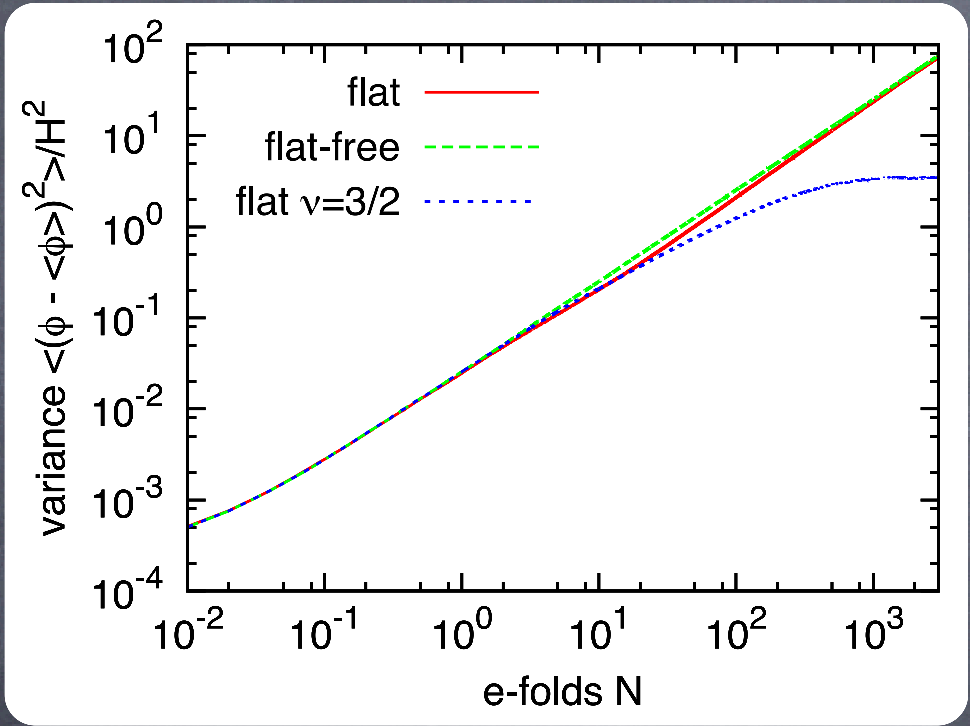


very small compared to $\langle S^{(\phi)} \rangle$

- without the zero-point contribution to noise terms

(comparison)

$\epsilon = 10^{-2}$



flat
$\lambda_e = g_1 = g_2 = 1$

flat-free
$\lambda_e = 0, g_1 = g_2 = 1$

flat ν=3/2
$\lambda_e = g_1 = g_2 = 1$
using the noise for $\tilde{m} = 0$

The flat direction eventually behaves as an exactly flat direction.

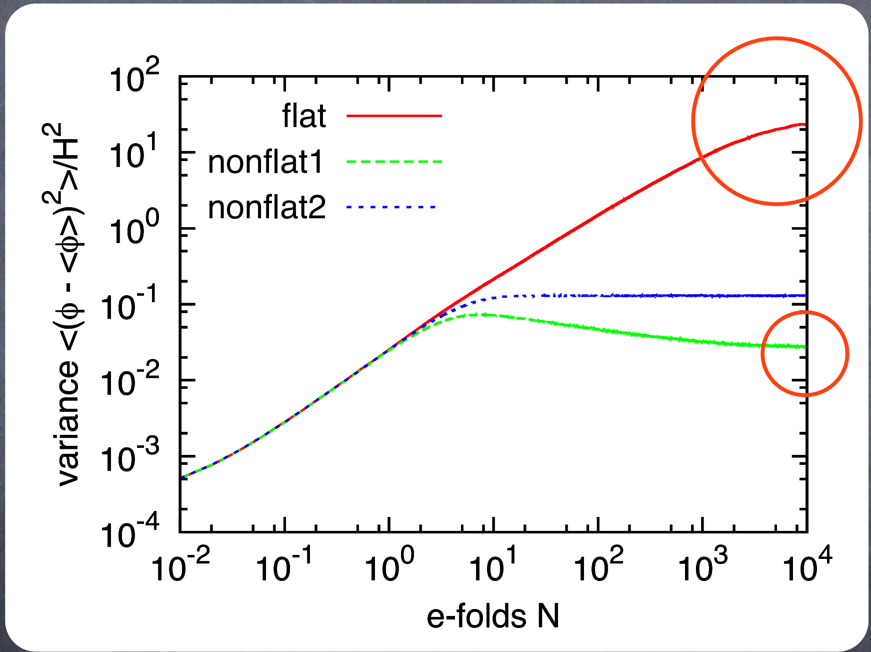
→ The flat direction reaches an exactly flat direction in the tree-level argument.

- with the zero-point contribution to noise terms

  (the time evolution of IR modes)

$\epsilon = 1$

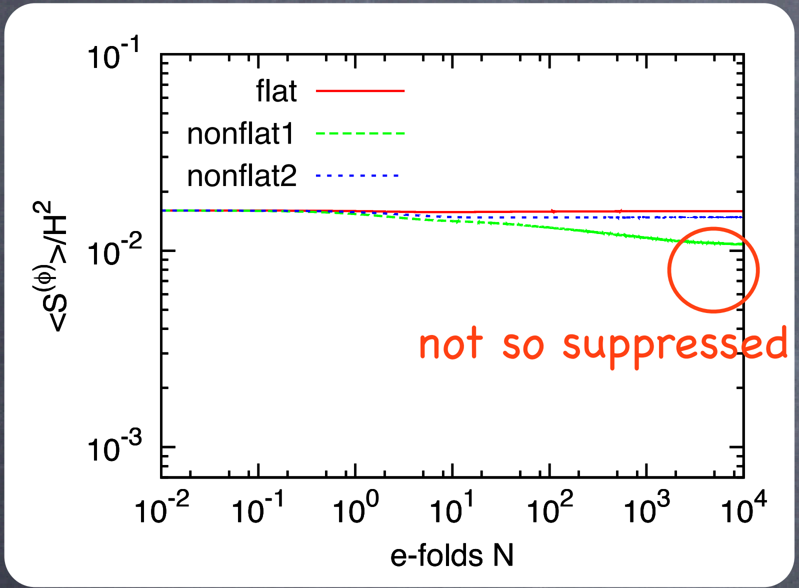
$\lambda_e = g_1 = g_2 = 1$

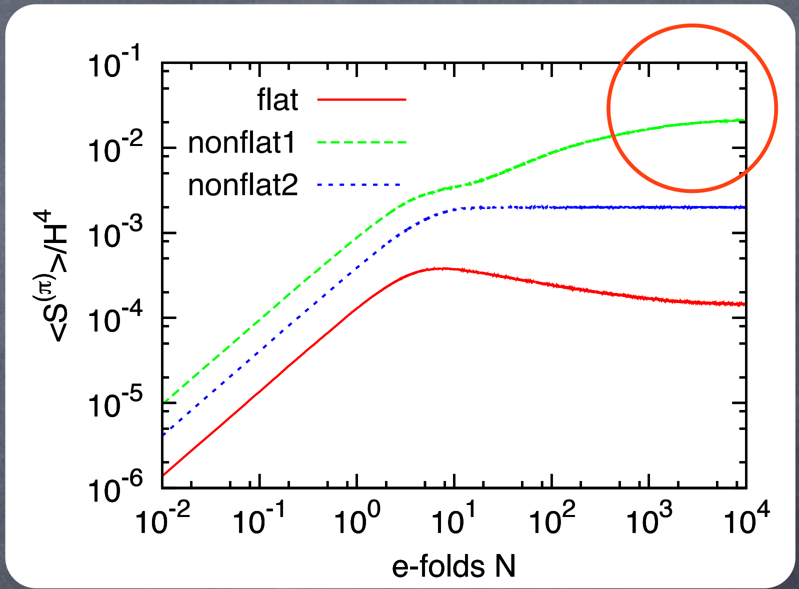


with the zero-point contribution to noise terms

(the time evolution of noise terms)

large compared to $\langle S^{(\phi)} \rangle$





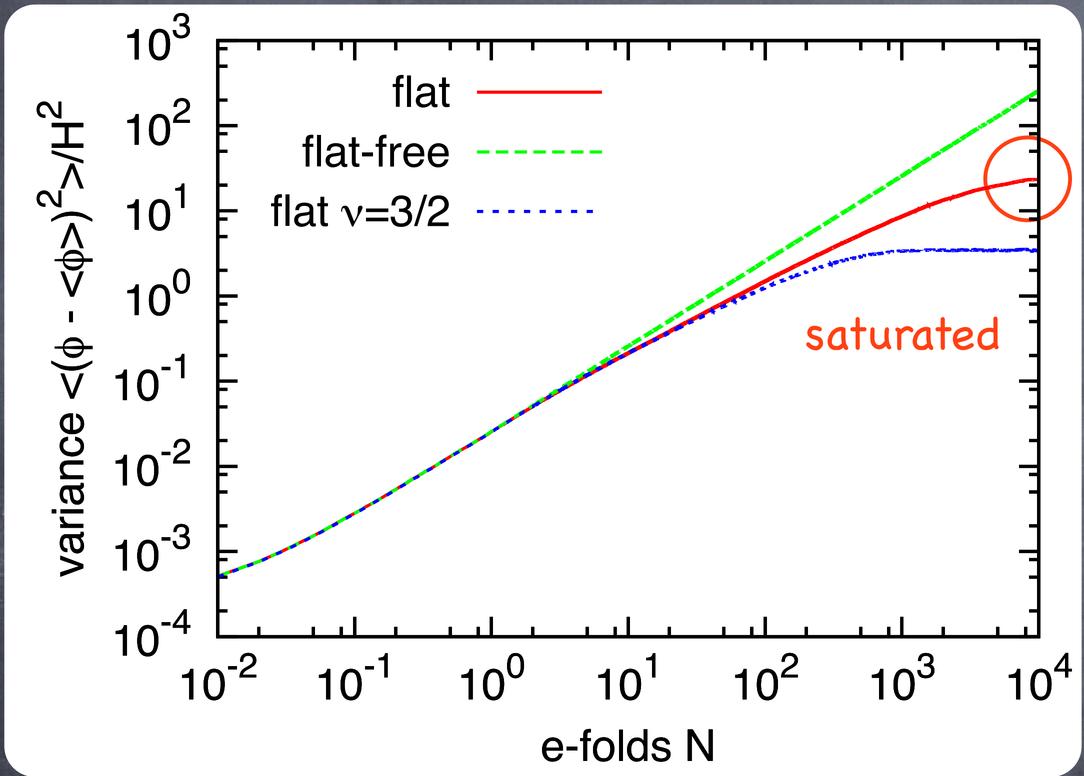
$\epsilon = 1$
$\lambda_e = g_1 = g_2 = 1$

$\epsilon = 1$
$\lambda_e = g_1 = g_2 = 1$

with the zero-point contribution to noise terms

(comparison)

$\epsilon = 1$



flat
$\lambda_e = g_1 = g_2 = 1$

flat-free
$\lambda_e = 0, g_1 = g_2 = 1$

flat ν=3/2
$\lambda_e = g_1 = g_2 = 1$
using the noise for $\tilde{m} = 0$

⇒ The flat direction saturates at last in the tree-level argument.

Summary

- We have analyzed the time evolution of a flat direction coupled to non-flat directions by using stochastic formalism. We include the Effective mass effects in the noise terms.

- Since non-flat directions have large effective masses and small noise terms, non-flat directions have only tiny effects on the growth of the flat direction's variance.

[without zero-point fluctuation in the noise term]

- The flat direction reaches an exactly flat direction in the tree-level argument.

[with zero-point fluctuation in the noise term]

- The flat direction saturates at last in the tree-level argument.

However, little is known how to treat the zero-point fluctuation in the noise terms.