linear base fields from non-linear ones

$$E_7 \xrightarrow[\phi_{27}]{} E_6$$

$$56 = \begin{pmatrix} 1_3 \\ 27_1 \\ 27^*_{-1} \\ 1_{-3} \end{pmatrix} \qquad \xi_{56}(\phi_{27}) \begin{pmatrix} \psi_{56}(1) \\ 0 \\ 0 \\ 0 \end{pmatrix} \equiv \Psi_{56}(\phi_{27})$$

$$E_6 \xrightarrow[\phi_{16}]{} E_5$$

$$27 = \begin{pmatrix} 1_4 \\ 16_1 \\ 10_{-2} \end{pmatrix} \qquad \xi_{27}(\phi_{16}) \begin{pmatrix} \psi_{27}(1) \\ 0 \\ 0 \end{pmatrix} \equiv \Psi_{27}(\phi_{16})$$

$$E_5 \xrightarrow[\phi_{10}]{} E_4$$

$$16 = \begin{pmatrix} 1_5 \\ 10_1 \\ 5^*_{-3} \end{pmatrix} \qquad \xi_{16}(\phi_{10}) \begin{pmatrix} \psi_{16}(1) \\ 0 \\ 0 \end{pmatrix} \equiv \Psi_{16}(\phi_{10})$$

56's

$$\xi_{56}(\phi_{27}) \begin{pmatrix} \psi'_{56}(1) \\ \Psi_{27}(\phi_{16}) \\ 0 \\ 0 \end{pmatrix} \qquad = \xi_{27}(\phi_{16}) \begin{pmatrix} \psi'_{27}(1) \\ 0 \\ 0 \end{pmatrix}$$

$$\xi_{56}(\phi_{27}) \begin{pmatrix} \psi''_{56}(1) \\ \xi_{27}(\phi_{16}) \begin{pmatrix} \psi'_{27}(1) \\ \Psi_{16}(\phi_{10}) \\ 0 \end{pmatrix} \\ 0 \\ 0 \end{pmatrix} \qquad \xi_{16}(\phi_{10}) \begin{pmatrix} \psi_{16}(1) \\ 0 \\ 0 \end{pmatrix}$$

$$\Psi_3 \equiv \xi_{56}(\phi_{27}) \begin{pmatrix} \langle \psi_{56}(1) \rangle \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{array}{c} \begin{pmatrix} 1 \\ \phi_{27} \\ \phi_{27} \times \phi_{27} \\ (\phi_{27} \times \phi_{27}) \cdot \phi_{27} \end{pmatrix} \end{array} \begin{array}{c} 1 \\ 27 \\ \overline{27} \\ 1 \end{array} \times \langle \psi_{56}(1) \rangle$$

$$\Psi_2 \equiv \xi_{56}(\phi_{27}) \begin{pmatrix} \langle \psi'_{56}(1) \rangle \overset{\text{と採る}}{=} 0 \\ \xi_{27}(\phi_{16}) \begin{pmatrix} \langle \psi_{27}(1) \rangle \\ 0 \\ 0 \end{pmatrix} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \Phi_{27}(\phi_{16}) \\ \phi_{27} \times \Phi_{27}(\phi_{16}) \\ \phi_{27} \cdot (\phi_{27} \times \Phi_{27}(\phi_{16})) \end{pmatrix} \langle \psi_{27}(1) \rangle$$

$$\Psi_1 \equiv \xi_{56}(\phi_{27}) \begin{pmatrix} \langle \psi''_{56}(1) \rangle = 0 \\ \xi_{27}(\phi_{16}) \begin{pmatrix} \langle \psi'_{27}(1) \rangle = 0 \\ \xi_{16}(\phi_{10}) \begin{pmatrix} \langle \psi_{16}(1) \rangle \\ 0 \\ 0 \end{pmatrix} \\ 0 \end{pmatrix} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \Phi_{27}(\phi_{16}, \phi_{10}) \\ \phi_{27} \times \Phi_{27}(\phi_{16}, \phi_{10}) \\ \phi_{27} \cdot (\phi_{27} \times \Phi_{27}(\phi_{16}, \phi_{10})) \end{pmatrix} \langle \psi_{16}(1) \rangle$$

where

$$\Phi_{27}(\phi_{16}) \equiv \begin{pmatrix} 1 \\ \phi_{16} \\ \phi_{16} \sigma_A \phi_{16} \end{pmatrix} \begin{array}{l} \\ \cdots 16 \\ \cdots 10 \end{array} = \xi_{27}(\phi_{16}) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\Phi_{16}(\phi_{10}) \equiv \xi_{16}(\phi_{10}) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ \phi_{10} \\ \phi_{10} \times \phi_{10} \end{pmatrix} \begin{array}{l} \\ \\ \cdots 5^* \end{array}$$

$$\Phi_{27}(\phi_{16}, \phi_{10}) = \xi_{27}(\phi_{16}) \begin{pmatrix} 0 \\ \Phi_{16}(\phi_{10}) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \Phi_{16}(\phi_{10}) \\ \phi_{16} \sigma_A \Phi_{16}(\phi_{10}) \end{pmatrix}$$

Matter 5^* の導入 ← anomally matching

$$\xi_{10}(\phi_{10})\begin{array}{c}\begin{pmatrix}5^*\\0\end{pmatrix}\end{array}\begin{array}{l}\\ \cdots 5\end{array} = \mathbf{10}(5^*) = \begin{pmatrix}5^*\\ \phi_{10}\times 5^*\end{pmatrix}$$

$$\Phi_{\overline{27}}(5^*) \equiv \xi_{\overline{27}}(\phi_{16})\begin{pmatrix}\mathbf{10}(5^*)\\ 0\\ 0\end{pmatrix}\begin{array}{l}\\ \cdots \overline{16}\\ \cdots 1\end{array}$$

$$= \begin{pmatrix}\mathbf{10}(5^*)\\ \phi_{16}\times\mathbf{10}(5^*)\\ \phi_{16}\cdot\mathbf{10}(5^*)\,\phi_{16}\end{pmatrix}$$

almost

.. $\overline{27}$ "linear" basis

$h_{E_6}(g,\phi_{27})$

invariants

by contracting with $\Phi_{27}(\phi_{16})\langle\psi_{27}(1)\rangle \supset \phi_{16}\,\mathbf{10}(5^*)\,\phi_{16}$

$\Phi_{\overline{27}}(5^*)$ × $\Phi_{27}(\phi_{16},\phi_{10})\langle\psi_{16}(1)\rangle \supset \phi_{16}\,\mathbf{10}(5^*)\,\Phi_{16}(\phi_{10})$

しかし、$\underline{\phi_{27}}$ と contract して invariant は作れない！

と couple させることは難しい！

$\phi_{27} = 16 + (5+5^*) + 1$

↑これと組んで massive にはできない。

Φ_{27} 中の 5
anomaly matching 5^*) を Higgs にできるか?

⇒ 無理.

(∵) Φ_{27} と couple できない (だろう)

の中の $5+5^*$ は 対等に出てきて
一方を matter, 一方を Higgs にはできない
(R-parity を assign できない)

$56 \cdot 56$

$$\begin{pmatrix}1\\27\\\overline{27}\\1\end{pmatrix}\begin{pmatrix}1\\27\\\overline{27}\\1\end{pmatrix} \quad \overline{27} = 27\times 27$$

$$27^3 = 1\cdot\cancel{10}\cdot\cancel{10} + 16\,\cancel{10}\,16$$

$$\downarrow \qquad\qquad \downarrow$$

$$1\cdot 5\cdot 5^* \qquad \left(\begin{array}{l}10\cdot 5\cdot 10\\ 10\cdot 5^*\cdot 5^*\end{array}\right.$$

$56 \cdot \Phi_{133}\, 56$

$$\Phi_{133} = \begin{pmatrix} \Phi_{\overline{27}}, 2 \\ \Phi_{1+78}, 0 \\ \Phi_{27}, -2 \end{pmatrix} \qquad \begin{pmatrix} 1_3 \\ 27_1 \\ \overline{27}_{-1} \\ 1_{-3} \end{pmatrix} \begin{pmatrix} \Phi_{\overline{27}} \cdot 27_1 + \Phi_1 \cdot 1_3 \\ \Phi_{\overline{27}} \times \overline{27}_{-1} + \Phi_{1+78} \cdot 27_1 \\ \Phi_{27} \times 27_1 + \Phi_{1+78}\, \overline{27}_{-1} \\ \Phi_{27} \cdot \overline{27}_{-1} + \Phi_1 \cdot 1_{-3} \end{pmatrix}$$

$\overline{27} \sim 27^2$

$\sim \times \sim$

$\bigcirc \times \bigcirc$

$27 \cdot \Phi_{27} \cdot 27$

Yukawa term

$\Phi_3 \propto \langle \varphi_{56}(1) \rangle$

$\Phi_2 \propto \langle \varphi_{27}(1) \rangle$

$\Phi_1 \propto \langle \varphi_{16}(1) \rangle$

Higgs
Φ_{133} must be linear

If Φ_{133} made from non-linear one

$$\Phi_{\overline{27}}(5^*) = \begin{pmatrix} \cancel{10}(5^*) \\ \phi_{16} \times \cancel{10}(5^*) \\ \phi_{16}\, \cancel{10}(5^*)\, \phi_{16} \end{pmatrix}$$

$$\Phi_{133}(5^*) = \xi_{133}(\phi_{27}) \begin{pmatrix} \Phi_{\overline{27}}(5^*) \\ 0 \\ 0 \end{pmatrix} \begin{matrix} \overline{27} \\ 1+78 \\ 27 \end{matrix}$$

$$= \begin{pmatrix} \Phi_{\overline{27}} \\ \phi_{27} \times \Phi_{\overline{27}} \quad (1+78) \\ (\phi_{27} \times \phi_{27}) \times \Phi_{\overline{27}} \end{pmatrix} \rightarrow \Phi_{27}$$

the above interaction is <u>NOT Yukawa interaction</u>;

$$27 \cdot \underset{\substack{| \\ \phi_{27}^2 \cdot \Phi_{\overline{27}}}}{\Phi_{27}} \cdot 27 = 27^4 \cdot \overline{27}$$