Naïve dimensional analysis in holography

2012 07/18 @ YITP
素粒子物理学の進展2012
Ryoichi Nishio from KIPMU & U.Tokyo

in collaboration with Taizan Watari, Tsutomoto T. Yanagida, and Kazuya Yonekura

based on arXiv: 1205.2949 [hep-ph]
accepted in PRD
Motivation

- Naïve Dimensional Analysis (NDA)
  - an ansatz for coupling constants among composite states (hadrons) in (arbitrary) strongly coupled theories
  - widely used not only in QCD, but also in many models beyond standard models
Motivation

• Naïve Dimensional Analysis (NDA)
  – an ansatz for coupling constants among composite states (hadrons) in (arbitrary) strongly coupled theories

  – widely used not only in QCD, but also in many models beyond standard models

However, there is no clear justification for NDA.

We examine the NDA ansatz from gauge/gravity duality, by estimating glueball couplings
Statement of NDA ansatz

NDA ansatz claims that the effective action of an SU(Nc) gauge theory is

\[ S = \int d^4x \frac{N_c^2}{(4\pi\beta)^2} \mathcal{L}(\phi(x), \partial_\mu, \Lambda_{\text{NDA}}), \]

\[ \phi : \text{fields of composite states (glueball)} \]

\[ \beta \text{ is order 1.} \]

\[ \text{Dimensionless coefficients of every terms in } L \text{ are order 1.} \]

Essential point:

\[ \text{overall } 4\pi (\sim 13) \text{ factor which is sizable compared with 1.} \]
Coupling constants from NDA ansatz

NDA ansatz claims that the effective action of an SU(Nc) gauge theory is

\[ S = \int d^4x \frac{N_c^2}{(4\pi\beta)^2} \mathcal{L}(\phi(x), \partial_\mu, \Lambda_{\text{NDA}}), \]

\( \phi : \text{fields of composite states (glueball)} \)

\( \Lambda_{\text{NDA}} \) is mass scale of hadrons.

After rescaling so that phi’s are canonical normalized,

(I+2)-point coupling terms are given by

\[ \left( \frac{4\pi\beta}{N_c} \right)^I \frac{\Lambda_{\text{NDA}}^{-I+2}}{(I+2)!} \phi^{I+2} \]

\[ \left( \frac{4\pi\beta}{N_c} \right)^I \frac{\Lambda_{\text{NDA}}^{-I}}{2!} \phi^I (\partial_\mu \phi \partial^\mu \phi) \]

(I + 2)-point coupling = \( \left( \frac{4\pi\beta}{N_c} \right)^I \) in units of \( \Lambda_{\text{NDA}} \)
QCD seems to be consistent with NDA

From chiral Lagrangian, \[ \mathcal{L}_{\pi\pi\pi} \simeq f_{\pi}^{-1}(\partial\pi)^2\pi \]
\[ g_{\pi\pi\pi} = f_{\pi}^{-1} \simeq (93 \text{ MeV})^{-1} \]

On the other hand, according to NDA, 
\[ g_{\rho\pi\pi} \simeq \frac{4\pi}{\sqrt{N_c}} \frac{1}{M_\rho} \simeq (110 \text{ MeV})^{-1} \]

From decay of vector mesons, 
\[ g_{\rho\pi\pi} \simeq \frac{4\pi}{\sqrt{N_c}} \times (0.83) \]  
\[ g_{\phi K^+K^-} \simeq \frac{4\pi}{\sqrt{N_c}} \times (0.87) \]
Argument in favor of NDA ansatz

ansatz of “loop saturation”

Tree level diagram ~ loop diagram @ energy scale ~ hadron mass

\[
1 \sim \frac{1}{q^2 + M^2} g^2 \int \frac{d^4 p}{(2\pi)^4} \frac{g^2 \text{Vol}(S^3)}{2(2\pi)^4} \left( \frac{g}{4\pi M} \right)^2
\]

4\pi : loop factor
Argument in favor of NDA ansatz

ansatz of “loop saturation”

Tree level diagram $\sim$ loop diagram @ energy scale $\sim$ hadron mass

Loop saturation is also an ansatz. Loop saturation seems to be inconsistent with large $N_c$. 
Hadron coupling in holography

• Today, it is well-known and straightforward to calculate the mass spectra and coupling constants of hadrons in a given gravity background.

  – ex)
    • Csaki, Ooguri, Oz, Terning ‘98 glueball mass spectra
    • Hong, Yoon, Strassler ‘04, ’05 three point coupling of vector meson pions, decay of meson
    • Sakai, Sugimoto ’04, ‘05

• More than 3-point couplings were little studied, because it looked merely tired calculation.

• However, it is interesting and physically important to examine whether the NDA rule exist, which governs a lot of coupling constants.
Setup of gravitational dual

The gravity dual of conformal gauge theory: $\text{AdS}_5 \times W$

$$ds^2 = R^2/z^2(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) + R^2 ds^2_W, \quad 0 < z < \infty,$$

$$W = S^5, T^{1,1}, Y^{p,q}$$

The gravity dual of confining gauge theory:

$$ds^2 = (f(z))^{-2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) + R^2 ds^2_{Wz}, \quad 0 < z < z_{\text{max}}$$

We consider a confining gauge theory which is conformal in UV limit

$$(f(z))^{-2} \to R^2/z^2, \quad W_z \to W, \quad (z \to 0)$$
Effective 4d action from gravity description

Gauge/gravity duality implies that we can obtain 4d hadron action by dimensional reduction of 10d sugra action on the corresponding background.

\[ S_{\text{sugra}} = \int d^{10}X \mathcal{L}[\Phi(X), \ldots] \]

Mode decomposition of sugra field

\[ \Phi(X) = \sum_i \chi_i(x) \psi_i(z, \theta) \]

Integrate out W and z directions

\[ S_{\text{SUGRA}} \to \int d^4x \mathcal{L}[\chi_i(x)] = S_{\text{hadron}} \]
Intermediate 5d description

Integrating out the compact W direction
for simplicity, consider only two scalars, dilaton and RR scalar, and
moreover only consider the constant mode on W

In the case of conformal gauge theory,

\[
S = \frac{(R^5 \text{Vol}(W)) \times R^3}{2 \kappa_{10}^2 g_s^2} \int d^4x \int_0^\infty \frac{dz}{z^3} \left( -\frac{1}{2} \left( (\partial \phi)^2 + (\partial_z \phi)^2 \right) - \frac{1}{2} e^{2\phi} \left( (\partial \alpha')^2 + (\partial_z \alpha')^2 \right) \right) + \ldots,
\]

backgrounds

\[
ds^2 = R^2 / z^2 (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2) + R^2 ds_W^2, \quad 0 < z < \infty,
\]

\[
e^\Phi = g_s, \quad F_5 = \frac{(2\pi \sqrt{\alpha'})^4 N_c}{\text{Vol}(W)} (1 + *) \text{vol}(W) \quad R^4 = 4\pi g_s N_c \alpha'^2 \frac{\text{Vol}(S^5)}{\text{Vol}(W)}
\]
The overall factor (coupling const.) in 5d action

\[
S = \frac{(R^5 \text{Vol}(W)) \times R^3}{2\kappa_{10}^2 g_s^2} \int d^4 x \int_0^\infty \frac{dz}{z^3} \left( -\frac{1}{2} ((\partial \phi)^2 + (\partial_z \phi)^2) - \frac{1}{2} e^{2\phi} ((\partial c)^2 + (\partial_z c)^2) \right) + \ldots,
\]

\[
\frac{1}{2\kappa_{10}^2 g_s^2} \times (R^5 \text{Vol}(W)) \times R^3 = \frac{4a}{8\pi^2}, \quad 4a \equiv p' N_c^2, \quad p' \equiv \frac{\text{Vol}(S^5)}{\text{Vol}(W)}.
\]

by rescaling 5d fields,

\[
S = \int d^4 x \int_0^\infty \frac{dz}{z^3} \left[ -\frac{1}{2} ((\partial \phi')^2 + (\partial_z \phi')^2) - \frac{1}{2} ((\partial c')^2 + (\partial_z c')^2) \right.
\]

\[
- \sum_{I=1}^\infty \left( \frac{4\pi}{\sqrt{2a}} \right)^I \frac{1}{I!2!} (\phi')^I ((\partial c')^2 + (\partial_z c')^2) \right].
\]
5d action in confining geometry

Simply insert extra factor $Y(z)$. $Y(z)$ represents the deviation from CFT.

$$S = \int d^4 x \int_0^{z_{\text{max}}} \frac{dz}{z^3} Y(z) \left[ -\frac{1}{2} ((\partial \phi')^2 + (\partial_z \phi')^2) - \frac{1}{2} ((\partial c')^2 + (\partial_z c')^2) \right]$$

$$- \frac{1}{2!I!} \sum_{I=1}^{\infty} \left( \frac{4\pi}{\sqrt{2a}} \right)^I (\phi')^I ((\partial c')^2 + (\partial_z c')^2) \right] + \ldots,$$

$$Y(z) = \frac{(f(z))^{-3}}{(R/z)^3} \frac{\text{Vol}(W_z)}{\text{Vol}(W_{z=0})},$$

$$Y(z) \to 1, \quad (z \to 0)$$
Effective action in 4d

\[ S = \int d^4x \int_0^{z_{\text{max}}} \frac{dz}{z^3} Y(z) \left[ -\frac{1}{2}((\partial \phi')^2 + (\partial_z \phi')^2) - \frac{1}{2}((\partial c')^2 + (\partial_z c')^2) \right. \\
- \frac{1}{2!} \sum_{I=1}^{\infty} \left( \frac{4\pi}{\sqrt{2a}} \right)^I (\phi')^I ((\partial c')^2 + (\partial_z c')^2) \right] + \ldots , \]

Substitute mode decompositions,

\[ \phi'(x, z) = \sum_{m=1}^{\infty} \psi_m(z) \tilde{\phi}_m(x), \quad c'(x, z) = \sum_{n=1}^{\infty} \psi_n(z) \tilde{c}_n(x) . \]

\( \tilde{\phi}_m(x) \): m-th excited glueball corresponding to dilaton

\( \tilde{c}_m(x) \): m-th excited glueball corresponding to RR scalar

Def. of mode functions

\[ z^3 Y^{-1}(z) \partial_z (z^{-3} Y(z) \partial_z \psi_n(z)) = m_n^2 \psi_n(z), \quad \int_0^{z_{\text{max}}} \frac{dz}{z^3} Y(z) \psi_n(z) \psi_m(z) = \delta_{nm} , \]
4d coupling constants among glueballs

\[ \mathcal{L}_{\text{int}} = - \sum_{I=1}^{\infty} \sum_{n_1n_2m_1 \ldots m_I} \left[ a_{n_1n_2,m_1 \ldots m_I}^{(I)} \frac{\Lambda_{\text{NDA}}^{-I}}{2! I!} \tilde{\phi}_m \cdots \tilde{\phi}_m (\partial_\mu \tilde{c}_n) (\partial_\mu \tilde{c}_n) + b_{n_1n_2,m_1 \ldots m_I}^{(I)} \frac{\Lambda_{\text{NDA}}^{2-I}}{2! I!} \tilde{\phi}_m \cdots \tilde{\phi}_m \tilde{c}_n \tilde{c}_n \right]. \]

Dimensionless (I+2)-point coupling constant

\[ a_{n_1n_2m_1 \ldots m_I}^{(I)} = \left( \frac{4\pi}{\sqrt{2a}} \right)^I \times \Lambda_{\text{NDA}}^I \int_0^{z_{\text{max}}} dz Y(z) \psi_{n_1} \psi_{n_2} \psi_m \cdots \psi_m, \]

\[ b_{n_1n_2m_1 \ldots m_I}^{(I)} = \left( \frac{4\pi}{\sqrt{2a}} \right)^I \times \Lambda_{\text{NDA}}^{I-2} \int_0^{z_{\text{max}}} dz Y(z) (\partial_z \psi_{n_1}) (\partial_z \psi_{n_2}) \psi_m \cdots \psi_m. \]
Rough estimation of overlap integral

( if $n$ and $l$ are not large)

\[ \psi_n \sim z_{\text{max}}, \quad \partial_z \psi_n \sim m z_{\text{max}} \]

$m$: typical glueball mass

\[
|a_{n_1n_2m_1...m_l}^{(I)}| \sim \left( \frac{4\pi}{\sqrt{2a}} \Lambda_{\text{NDA}} z_{\text{max}} \right)^I
\]

\[
|b_{n_1n_2m_1...m_l}^{(I)}| \sim \left( \frac{m}{\Lambda_{\text{NDA}}} \right)^2 \left( \frac{4\pi}{\sqrt{2a}} \Lambda_{\text{NDA}} z_{\text{max}} \right)^I
\]

If we take $\Lambda_{\text{NDA}} \sim m$

\[
|a_{n_1n_2m_1...m_l}^{(I)}|, |b_{n_1n_2m_1...m_l}^{(I)}| \sim \left[ \left( \frac{4\pi}{\sqrt{2a}} \right)^I (m z_{\text{max}}) \right]
\]
Rough estimation of overlap integral

(if n and I are not large)

$$\psi_n \sim z_{\max}, \quad \partial_z \psi_n \sim m z_{\max}$$

m: typical glueball mass

Remember, according to NDA,

$$|a_{n_1 n_2 m_1 \ldots m_I}^{(I)}, |b_{n_1 n_2 m_1 \ldots m_I}^{(I)}| \sim \left( \frac{4\pi \beta}{N_c} \right)^I$$

If we take $$\Lambda_{NDA} \sim m$$

$$|a_{n_1 n_2 m_1 \ldots m_I}^{(I)}, |b_{n_1 n_2 m_1 \ldots m_I}^{(I)}| \sim \left[ \left( \frac{4\pi}{\sqrt{2a}} \right) (m z_{\max}) \right]^I$$
Numerical result in hard wall model \((Y(z)=1)\)

<table>
<thead>
<tr>
<th>(a_{\text{typ}}^{(I)})</th>
<th>(\sigma_{\ln a}^{(I)})</th>
<th>(b_{\text{typ}}^{(I)})</th>
<th>(\sigma_{\ln b}^{(I)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>((4\pi/\sqrt{2a}\Lambda_{\text{NDA}} z_{\text{max}} \cdot 0.8)^{-I})</td>
<td>((4\pi/\sqrt{2a}\Lambda_{\text{NDA}} z_{\text{max}} \cdot 0.8)^{-I})</td>
<td>((4\pi/\sqrt{2a}\Lambda_{\text{NDA}} z_{\text{max}} \cdot 0.8)^{-I})</td>
<td>((4\pi/\sqrt{2a}\Lambda_{\text{NDA}} z_{\text{max}} \cdot 0.8)^{-I})</td>
</tr>
<tr>
<td>(I = 1)</td>
<td>0.3</td>
<td>(\ln[2.])</td>
<td>((\frac{m_1}{\Lambda_{\text{NDA}}}^2 \times 0.4)</td>
</tr>
<tr>
<td>(I = 2)</td>
<td>0.3</td>
<td>(\ln[2.])</td>
<td>((\frac{m_1}{\Lambda_{\text{NDA}}}^2 \times 0.3)</td>
</tr>
<tr>
<td>(I = 3)</td>
<td>0.4</td>
<td>(\ln[2.])</td>
<td>((\frac{m_1}{\Lambda_{\text{NDA}}}^2 \times 0.2)</td>
</tr>
<tr>
<td>(I = 4)</td>
<td>0.4</td>
<td>(\ln[2.])</td>
<td>((\frac{m_1}{\Lambda_{\text{NDA}}}^2 \times 0.4)</td>
</tr>
</tbody>
</table>

\(a_{\text{typ}}, b_{\text{typ}}^{(I)}\) : geometric means of \((I + 2)\)-point couplings up to 3\(^{rd}\) excited mode

\[a_{\text{typ}}^{(I)}, b_{\text{typ}}^{(I)} \sim (0.2 \sim 0.4) \left[\left(\frac{4\pi}{\sqrt{2a}}\right) m_1 z_{\text{max}} 0.8\right]^{I}\]

\(m_1 z_{\text{max}} \sim 5.1\)
Table 2: Geometric means

<table>
<thead>
<tr>
<th>$a^{(I)}<em>{\text{typ}} \left( \frac{4\pi}{\sqrt{2}a</em>{\text{eff}}} \Lambda_{\text{NDA}} z_{\text{max}} \cdot 1.5 \right)^{-I}$</th>
<th>$\sigma_{\ln a}^{(I)}$</th>
<th>$b^{(I)}<em>{\text{typ}} \left( \frac{4\pi}{\sqrt{2}a</em>{\text{eff}}} \Lambda_{\text{NDA}} z_{\text{max}} \cdot 1.5 \right)^{-I}$</th>
<th>$\sigma_{\ln b}^{(I)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I = 1$</td>
<td>0.3</td>
<td>$\ln[2.]$</td>
<td>$\ln[3.]$</td>
</tr>
<tr>
<td>$I = 2$</td>
<td>0.3</td>
<td>$\ln[1.]$</td>
<td>$\ln[4.]$</td>
</tr>
<tr>
<td>$I = 3$</td>
<td>0.4</td>
<td>$\ln[2.]$</td>
<td>$\ln[4.]$</td>
</tr>
<tr>
<td>$I = 4$</td>
<td>0.5</td>
<td>$\ln[2.]$</td>
<td>$\ln[3.]$</td>
</tr>
</tbody>
</table>

\(a(z)\) can be defined in non-conformal geometry. \(a_{\text{eff}}\) is its typical value.
Discussion

Result from holography

\[ |a^{(I)}_{n_1n_2m_1...m_I}|, |b^{(I)}_{n_1n_2m_1...m_I}| \sim \left[ \left( \frac{4\pi}{\sqrt{2a}} \right) \left( mz_{\text{max}} \right) \right]^I, \]

Remember, according to NDA,

\[ |a^{(I)}_{n_1n_2m_1...m_I}|, |b^{(I)}_{n_1n_2m_1...m_I}| \sim \left( \frac{4\pi \beta}{N_c} \right)^I \]
Discussion

Result from holography

\[ |a^{(I)}_{n_1 n_2 m_1 \ldots m_I}|, |b^{(I)}_{n_1 n_2 m_1 \ldots m_I}| \sim \left[ \left( \frac{4\pi}{\sqrt{2a}} \right) (mz_{\text{max}}) \right]^I, \]

Remember, according to NDA,

\[ |a^{(I)}_{n_1 n_2 m_1 \ldots m_I}|, |b^{(I)}_{n_1 n_2 m_1 \ldots m_I}| \sim \left( \frac{4\pi \beta}{N_c} \right)^I \]

1. Scaling by common factor does exist!
Discussion

Result from holography

\[ |a^{(I)}_{n_1n_2m_1...m_I}|, |b^{(I)}_{n_1n_2m_1...m_I}| \sim \left[ \left( \frac{4\pi}{\sqrt{2a}} \right) (mz_{\text{max}}) \right]^I, \]

Remember, according to NDA,

\[ |a^{(I)}_{n_1n_2m_1...m_I}|, |b^{(I)}_{n_1n_2m_1...m_I}| \sim \left( \frac{4\pi \beta}{N_c} \right)^I \]

1, Scaling by common factor does exist!

2, \( N_c \) is naturally generalized into \( \sqrt{a} \)

\[ 4a = \frac{\text{Vol}(S^5)}{\text{Vol}(W)} N_c^2 \geq N_c^2 \]

\[ W = Y^{p,q} \quad 4a \simeq pN_c^2 \quad \text{the dual gauge theory is } (SU(N_c))^p\text{-quiver gauge theory} \]

a: central charge of CFT, which counts dof.s.
Discussion

Result from holography

\[ |a_{n_1n_2m_1...m_I}^{(I)}|, |b_{n_1n_2m_1...m_I}^{(I)}| \sim \left[ \left( \frac{4\pi}{\sqrt{2a}} \right) (m z_{\max}) \right]^I, \]

Remember, according to NDA,

\[ |a_{n_1n_2m_1...m_I}^{(I)}|, |b_{n_1n_2m_1...m_I}^{(I)}| \sim \left( \frac{4\pi \beta}{N_c} \right)^I \]

1. Scaling by common factor does exist!

2. \( N_c \) is naturally generalized into \( \sqrt{a} \)

\[ 4a = \frac{\text{Vol}(S^5)}{\text{Vol}(W)} N_c^2 \geq N_c^2 \]

\[ W = Y^{p,q} \quad 4a \sim p N_c^2 \quad \text{the dual gauge theory is } (SU(N_c))^p\text{-quiver gauge theory} \]

\( a \): central charge of CFT, which counts dof.s.

3. Take care that \( (m z_{\max}) \sim (5\sim6) \) is a little large number!