Naïve dimensional analysis in holography

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Motivation

- Naïve Dimensional Analysis (NDA)
  - an ansatz for coupling constants among composite states (hadrons) in (arbitrary) strongly coupled theories
  - widely used not only in QCD, but also in many models beyond standard models

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However, there is no clear justification for NDA.

We examine the NDA ansatz from gauge/gravity duality, by estimating glueball couplings

Statement of NDA ansatz

Manohar, Georgi '84
Georgi, Randall '86
Georgi '92
Luty '97
Cohen, Kaplan, Nelson '97

NDA ansatz claims that the effective action of an SU(Nc) gauge theory is

$$S = \int d^4x \, \frac{N_c^2}{(4\pi\beta)^2} \, \mathcal{L}(\phi(x), \partial_\mu, \Lambda_{\rm NDA}),$$

ϕ : fields of composite states (glueball)

Beta is order 1.

Dimensionless coefficients of every terms in L are order 1.

Essential point:

overall 4 pi (~ 13) factor which is sizable compared with 1.

Coupling constants from NDA ansatz

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ϕ : fields of composite states (glueball)

Λ_{NDA} is mass scale of hadrons.

After rescaling so that phi's are canonical normalized,

(I+2)-point coupling terms are given by

$$\left(\frac{4\pi\beta}{N_c}\right)^I \frac{\Lambda_{\mathrm{NDA}}^{-I+2}}{(I+2)!} \phi^{I+2} \qquad \left(\frac{4\pi\beta}{N_c}\right)^I \frac{\Lambda_{\mathrm{NDA}}^{-I}}{2!I!} \phi^I (\partial_\mu \phi \partial^\mu \phi)$$

$$(I+2)\text{-point coupling} = \left(\frac{4\pi\beta}{N_c}\right)^I \text{ in units of } \Lambda_{\mathrm{NDA}}$$

QCD seems to be consistent with NDA

From chiral Lagrangian, $\mathcal{L}_{\pi\pi\pi} \simeq f_\pi^{-1}(\partial\pi)^2\pi$

$$g_{\pi\pi\pi} = f_\pi^{-1} \simeq (93\ \mathrm{MeV})^{-1}$$

On the other hand, according to NDA ,

$$g_{\pi\pi\pi} \simeq \frac{4\pi}{\sqrt{N_c}}\frac{1}{M_\rho} \simeq (110\ \mathrm{MeV})^{-1}$$

From decay of vector mesons,

$$g_{\rho\pi\pi} \simeq \frac{4\pi}{\sqrt{N_c}} \times (0.83) \qquad g_{\phi K^+K^-} \simeq \frac{4\pi}{\sqrt{N_c}} \times (0.87)$$

Argument in favor of NDA ansatz

Manohar, Georgi ’84
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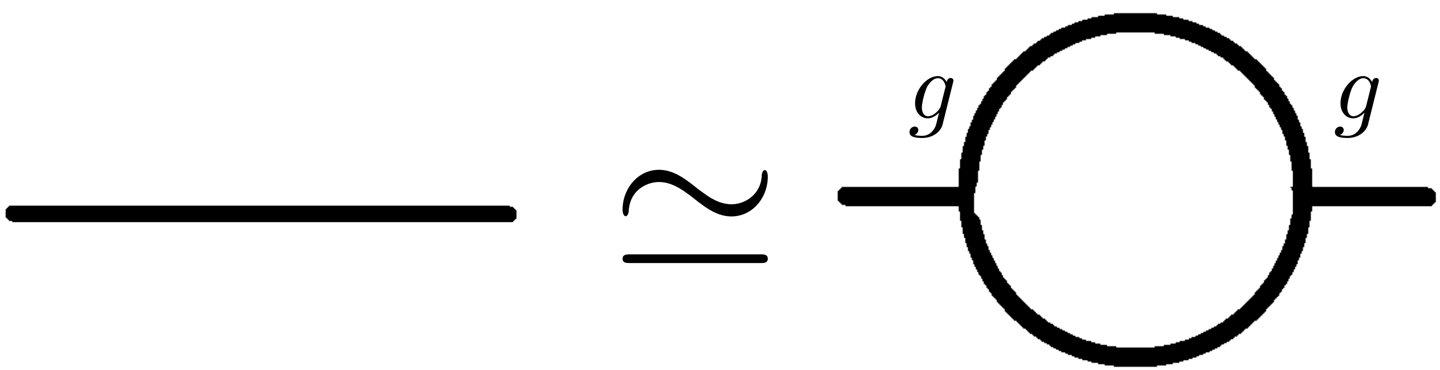
ansatz of “loop saturation”

Tree level diagram ~ loop diagram @ energy scale ~ hadron mass



$$1 \simeq \frac{1}{q^2 + M^2} g^2 \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 + M^2)((q-p)^2 + M^2)}$$

$$\simeq \frac{g^2}{M^2} \frac{\mathrm{Vol}(S^3)}{2(2\pi)^4} = \left(\frac{g}{4\pi M}\right)^2$$

4pi : loop factor

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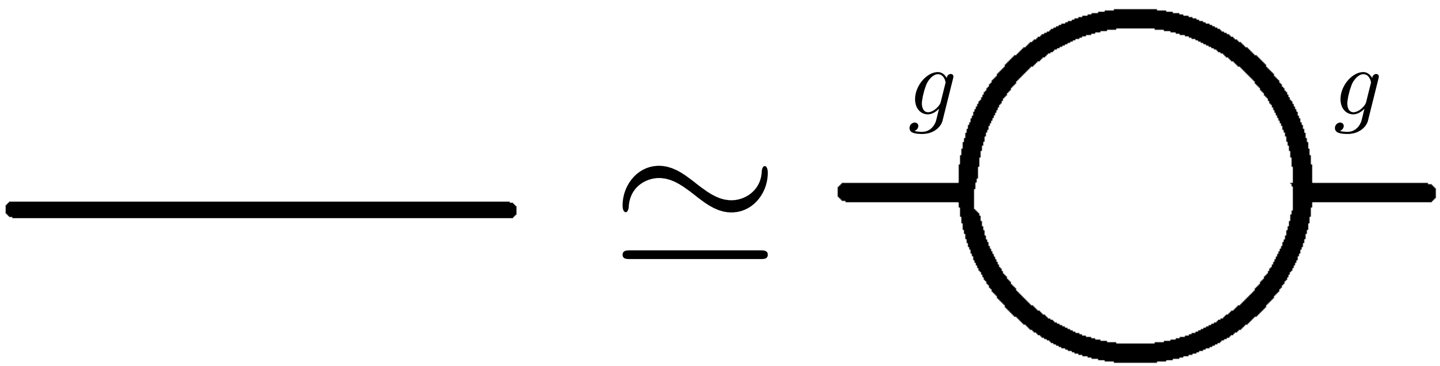

Loop saturation is also an ansatz.

Loop saturation seems to be inconsistent with large N_c.

Hadron coupling in holography

- Today, it is well-known and straightforward to calculate the mass spectra and coupling constants of hadrons in a given gravity background.
  - ex)
    - Csaki, Ooguri, Oz, Terning '98 — glueball mass spectra
    - Hong, Yoon, Strassler '04, '05 — three point coupling of vector meson
    - Sakai, Sugimoto '04, '05 — pions, decay of meson
- More than 3-point couplings were little studied, because it looked merely tired calculation.
- However, it is interesting and physically important to examine whether the NDA rule exist, which governs a lot of coupling constants.

Setup of gravitational dual

The gravity dual of conformal gauge theory: AdS_5 x W

$$ds^2 = R^2/z^2(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) + R^2 ds_W^2, \quad 0 < z < \infty,$$

$$W = S^5, T^{1,1}, Y^{p,q}$$

The gravity dual of confining gauge theory:

$$ds^2 = (f(z))^{-2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) + R^2 ds_{W_z}^2, \quad 0 < z < z_{\max}$$

We consider a confining gauge theory which is conformal in UV limit

$$(f(z))^{-2} \to R^2/z^2, \quad W_z \to W, \quad (z \to 0)$$

Effective 4d action from gravity description

Gauge/gravity duality implies that we can obtain 4d hadron action by dimensional reduction of 10d sugra action on the corresponding background.

$$S_{\rm sugra} = \int d^{10}X \mathcal{L}[\Phi(X), \dots]$$

Mode decomposition of sugra field

$$\Phi(X) = \sum_i \chi_i(x)\psi_i(z,\theta)$$

Integrate out W and z directions

$$S_{\rm SUGRA} \to \int d^4x \mathcal{L}[\chi_i(x)] = S_{\rm hadron}$$

Intermediate 5d description

Integrating out the compact W direction

for simplicity, consider only two scalars, dilaton and RR scalar, and moreover only consider the constant mode on W

In the case of conformal gauge theory,

$$S = \frac{(R^5 \mathrm{Vol}(W)) \times R^3}{2\kappa_{10}^2 g_s^2} \int d^4x \int_0^\infty \frac{dz}{z^3} \left(-\frac{1}{2}((\partial\phi)^2 + (\partial_z\phi)^2) - \frac{1}{2} e^{2\phi}((\partial c)^2 + (\partial_z c)^2) \right) + \ldots,$$

backgrounds

$$ds^2 = R^2/z^2(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) + R^2 ds_W^2, \quad 0 < z < \infty,$$

$$e^\Phi = g_s \quad F_5 = \frac{(2\pi\sqrt{\alpha'})^4 N_c}{\mathrm{Vol}(W)}(1+*)\mathrm{vol}(W) \quad R^4 = 4\pi g_s N_c \alpha'^2 \frac{\mathrm{Vol}(S^5)}{\mathrm{Vol}(W)}$$

The overall factor (coupling const.) in 5d action

$$S = \frac{(R^5 \mathrm{Vol}(W)) \times R^3}{2\kappa_{10}^2 g_s^2} \int d^4x \int_0^\infty \frac{dz}{z^3} \left(-\frac{1}{2}((\partial\phi)^2 + (\partial_z\phi)^2) - \frac{1}{2} e^{2\phi}((\partial c)^2 + (\partial_z c)^2) \right) + \ldots,$$

$$\frac{1}{2\kappa_{10}^2 g_s^2} \times (R^5 \mathrm{Vol}(W)) \times R^3 = \frac{4a}{8\pi^2}, \qquad 4a \equiv p' N_c^2, \quad p' \equiv \frac{\mathrm{Vol}(S^5)}{\mathrm{Vol}(W)}.$$

by rescaling 5d fields,

$$S = \int d^4x \int_0^\infty \frac{dz}{z^3} \left[-\frac{1}{2}((\partial\phi')^2 + (\partial_z\phi')^2) - \frac{1}{2}((\partial c')^2 + (\partial_z c')^2) - \sum_{I=1}^{\infty} \left(\frac{4\pi}{\sqrt{2a}} \right)^I \frac{1}{I!2!} (\phi')^I ((\partial c')^2 + (\partial_z c')^2) \right].$$

5d action in confining geometry

Simply insert extra factor Y(z).

Y(z) represents the deviation from CFT.

$$S = \int d^4x \int_0^{z_{\max}} \frac{dz}{z^3} Y(z) \left[-\frac{1}{2}((\partial\phi')^2 + (\partial_z\phi')^2) - \frac{1}{2}((\partial c')^2 + (\partial_z c')^2) \right.$$
$$\left. - \frac{1}{2!I!} \sum_{I=1}^{\infty} \left(\frac{4\pi}{\sqrt{2a}} \right)^I (\phi')^I ((\partial c')^2 + (\partial_z c')^2) \right] + \ldots,$$

$$Y(z) = \frac{(f(z))^{-3}}{(R/z)^3} \frac{\mathrm{Vol}(W_z)}{\mathrm{Vol}(W_{z=0})},$$

$$Y(z) \to 1, \quad (z \to 0)$$

Effective action in 4d

$$S = \int d^4x \int_0^{z_{\max}} \frac{dz}{z^3} Y(z) \left[-\frac{1}{2}((\partial\phi')^2 + (\partial_z\phi')^2) - \frac{1}{2}((\partial c')^2 + (\partial_z c')^2) \right.$$
$$\left. - \frac{1}{2!I!} \sum_{I=1}^{\infty} \left(\frac{4\pi}{\sqrt{2a}} \right)^I (\phi')^I ((\partial c')^2 + (\partial_z c')^2) \right] + \ldots,$$

Substitute mode decompositions,

$$\phi'(x,z) = \sum_{m=1}^{\infty} \psi_m(z) \tilde{\phi}_m(x), \quad c'(x,z) = \sum_{n=1}^{\infty} \psi_n(z) \tilde{c}_n(x).$$

$\tilde{\phi}_m(x)$: m-th excited glueball corresponding to dilaton

$\tilde{c}_m(x)$: m-th excited glueball corresponding to RR scalar

Def. of mode functions

$$z^3 Y^{-1}(z) \partial_z \left(z^{-3} Y(z) \partial_z \psi_n(z) \right) = m_n^2 \psi_n(z), \qquad \int_0^{z_{\max}} \frac{dz}{z^3} Y(z) \psi_n(z) \psi_m(z) = \delta_{nm},$$

4d coupling constants among glueballs

$$\mathcal{L}_{\rm int} = -\sum_{I=1}^{\infty} \sum_{n_1 n_2 m_1 \ldots m_I} \left[a^{(I)}_{n_1 n_2, m_1 \ldots m_I} \frac{\Lambda_{\rm NDA}^{-I}}{2! I!} \tilde{\phi}_{m_1} \ldots \tilde{\phi}_{m_I} (\partial_\mu \tilde{c}_{n_1})(\partial^\mu \tilde{c}_{n_2}) \right.$$
$$\left. + b^{(I)}_{n_1 n_2, m_1 \ldots m_I} \frac{\Lambda_{\rm NDA}^{2-I}}{2! I!} \tilde{\phi}_{m_1} \ldots \tilde{\phi}_{m_I} \tilde{c}_{n_1} \tilde{c}_{n_2} \right].$$

Dimensionless (I+2)-point coupling constant

$$a^{(I)}_{n_1 n_2 m_1 \ldots m_I} = \left(\frac{4\pi}{\sqrt{2a}} \right)^I \times \Lambda_{\rm NDA}^{I} \int_0^{z_{\rm max}} \frac{dz}{z^3} Y(z) \psi_{n_1} \psi_{n_2} \psi_{m_1} \ldots \psi_{m_I},$$

$$b^{(I)}_{n_1 n_2 m_1 \ldots m_I} = \left(\frac{4\pi}{\sqrt{2a}} \right)^I \times \Lambda_{\rm NDA}^{I-2} \int_0^{z_{\rm max}} \frac{dz}{z^3} Y(z) (\partial_z \psi_{n_1})(\partial_z \psi_{n_2}) \psi_{m_1} \ldots \psi_{m_I}.$$

Rough estimation of overlap integral

(if n and I are not large)

$$\psi_n \sim z_{\rm max}, \quad \partial_z \psi_n \sim m z_{\rm max}$$

m: typical glueball mass

$$|a^{(I)}_{n_1 n_2 m_1 \ldots m_I}| \sim \left(\frac{4\pi}{\sqrt{2a}} \Lambda_{\rm NDA} z_{\rm max} \right)^I$$

$$|b^{(I)}_{n_1 n_2 m_1 \ldots m_I}| \sim \left(\frac{m}{\Lambda_{\rm NDA}} \right)^2 \left(\frac{4\pi}{\sqrt{2a}} \Lambda_{\rm NDA} z_{\rm max} \right)^I$$

If we take $\Lambda_{\rm NDA} \simeq m$

$$|a^{(I)}_{n_1 n_2 m_1 \ldots m_I}|, |b^{(I)}_{n_1 n_2 m_1 \ldots m_I}| \sim \left[\left(\frac{4\pi}{\sqrt{2a}} \right) (m z_{\rm max}) \right]^I ,$$

Rough estimation of overlap integral

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Remember, according to NDA,

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$$|a^{(I)}_{n_1 n_2 m_1 \dots m_I}|, |b^{(I)}_{n_1 n_2 m_1 \dots m_I}| \sim \left[\left(\frac{4\pi}{\sqrt{2a}}\right)(m z_{\rm max})\right]^I,$$

Numerical result in hard wall model (Y(z)=1)

	$a_{\rm typ}^{(I)}\left(\frac{4\pi}{\sqrt{2a}}\Lambda_{\rm NDA}z_{\rm max}\cdot 0.8\right)^{-I}$	$\sigma_{\ln a}^{(I)}$	$b_{\rm typ}^{(I)}\left(\frac{4\pi}{\sqrt{2a}}\Lambda_{\rm NDA}z_{\rm max}\cdot 0.8\right)^{-I}$	$\sigma_{\ln b}^{(I)}$
$I=1$	0.3	$\ln[2.]$	$\left(\frac{m_1}{\Lambda_{\rm NDA}}\right)^2\times 0.4$	$\ln[3.]$
$I=2$	0.3	$\ln[2.]$	$\left(\frac{m_1}{\Lambda_{\rm NDA}}\right)^2\times 0.3$	$\ln[3.]$
$I=3$	0.4	$\ln[2.]$	$\left(\frac{m_1}{\Lambda_{\rm NDA}}\right)^2\times 0.2$	$\ln[3.]$
$I=4$	0.4	$\ln[2.]$	$\left(\frac{m_1}{\Lambda_{\rm NDA}}\right)^2\times 0.4$	$\ln[4.]$

$a_{\rm typ}^{(I)}, b_{\rm typ}^{(I)}$: geometric means of $(I+2)$-point couplings

up to 3rd excited mode

$$a_{\rm typ}^{(I)}, b_{\rm typ}^{(I)} \simeq (0.2\sim 0.4)\left[\left(\frac{4\pi}{\sqrt{2a}}\right)m_1 z_{\rm max}0.8\right]^I$$

$$m_1 z_{\rm max}\simeq 5.1$$

Numerical result using Klebanov-Strassler metric

	$a_{\rm typ}^{(I)}\left(\frac{4\pi}{\sqrt{2a_{\rm eff}}}\Lambda_{\rm NDA}z_{\rm max}\cdot 1.5\right)^{-I}$	$\sigma_{\ln a}^{(I)}$	$b_{\rm typ}^{(I)}\left(\frac{4\pi}{\sqrt{2a_{\rm eff}}}\Lambda_{\rm NDA}z_{\rm max}\cdot 1.5\right)^{-I}$	$\sigma_{\ln b}^{(I)}$
$I=1$	0.3	$\ln[2.]$	$\left(\frac{m_1}{\Lambda_{\rm NDA}}\right)^2\times 0.3$	$\ln[3.]$
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$I=3$	0.4	$\ln[2.]$	$\left(\frac{m_1}{\Lambda_{\rm NDA}}\right)^2\times 0.2$	$\ln[4.]$
$I=4$	0.5	$\ln[2.]$	$\left(\frac{m_1}{\Lambda_{\rm NDA}}\right)^2\times 0.2$	$\ln[3.]$

a(z) can be defined in non-conformal geometry. a_eff is its typical value.

Discussion

Result from holography

$$|a^{(I)}_{n_1 n_2 m_1 \dots m_I}|, |b^{(I)}_{n_1 n_2 m_1 \dots m_I}| \sim \left[\left(\frac{4\pi}{\sqrt{2a}}\right)(m z_{\max})\right]^I,$$

Remember, according to NDA,

$$|a^{(I)}_{n_1 n_2 m_1 \dots m_I}|, |b^{(I)}_{n_1 n_2 m_1 \dots m_I}| \sim \left(\frac{4\pi\beta}{N_c}\right)^I$$

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2, N_c is naturally generalized into $\sqrt{a}$ $\qquad 4a = \frac{\mathrm{Vol}(S^5)}{\mathrm{Vol}(W)} N_c^2 \geq N_c^2$

$W = Y^{p,q} \quad 4a \simeq p N_c^2$ the dual gauge theory is $(SU(N_c))^p$-quiver gauge theory

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3, Take care that (m z_max) ~ (5~6) is a little large number!