g-2
- Recent progress on the lattice -

Norikazu Yamada (KEK, GUAS)

Thanks to
Tom Blum, **Masashi Hayakawa**, and **Taku Izubuchi**
for many discussions and providing materials

"PPP2012"@Yukawa Institute, 2012.7.18

- Slides given by Tom Blum and Taku Izubuchi@Lattice 2012
  http://www.physics.adelaide.edu.au/cssm/lattice2012/program.php

- Many talks@2nd Workshop on Muon g-2 and EDM in the LHC Era
  https://indico.in2p3.fr/conferenceOtherViews.py?view=standard&confId=6637

- Endo-san @ this workshop

Introduction

Particle having spin feels potential in the external magnetic field.

$$V(x) = -\vec{\mu}_l \cdot \vec{B}$$

Particle's magnetic moment $\mu_l \propto$ its spin.

$$\vec{\mu}_l = g_l \frac{e}{2m_l} \vec{S}_l$$

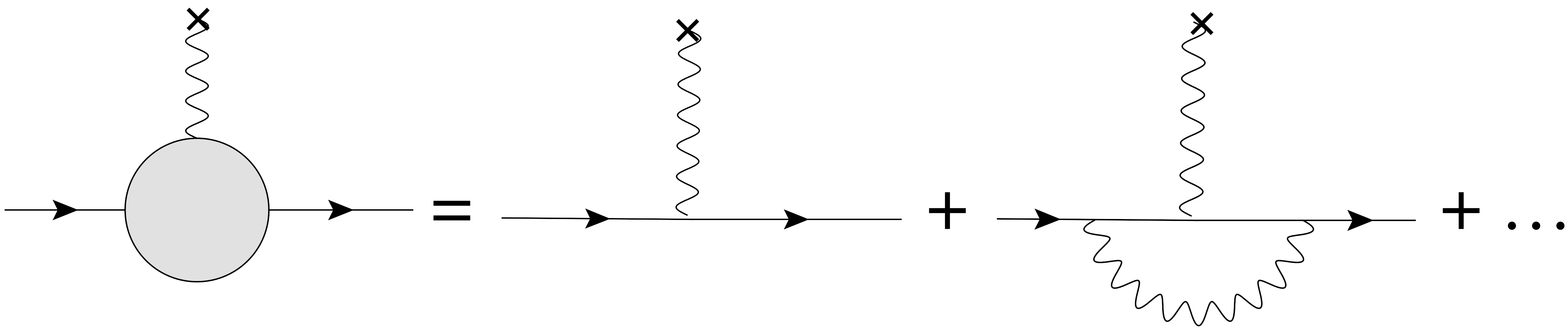
g_l : Landé g-factor (= 2 for elementary fermions@tree level)

Anomalous magnetic moment (*g*-2): Deviation from 2

$$a_l = \frac{g_l - 2}{2}$$

Introduction

$$\text{Form factor}: \Gamma_\mu(q) = \gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2\, m_l} F_2(q^2)$$



At tree level, $F_1(q^2) = 1, \quad F_2(q^2) = 0$

After quantum correction $\Rightarrow a_l = F_2(0)$

Experimental status of $(g\text{-}2)_e$ and $(g\text{-}2)_\mu$

$$
\begin{aligned}
a_e^{\rm EXP} &= (11\ 596\ 521.807\ 6 \ \pm 0.0027) \times 10^{-10} \\
a_\mu^{\rm EXP} &= (11\ 659\ 208.9 \ \pm 6.3) \times 10^{-10}
\end{aligned}
$$

[PDG]

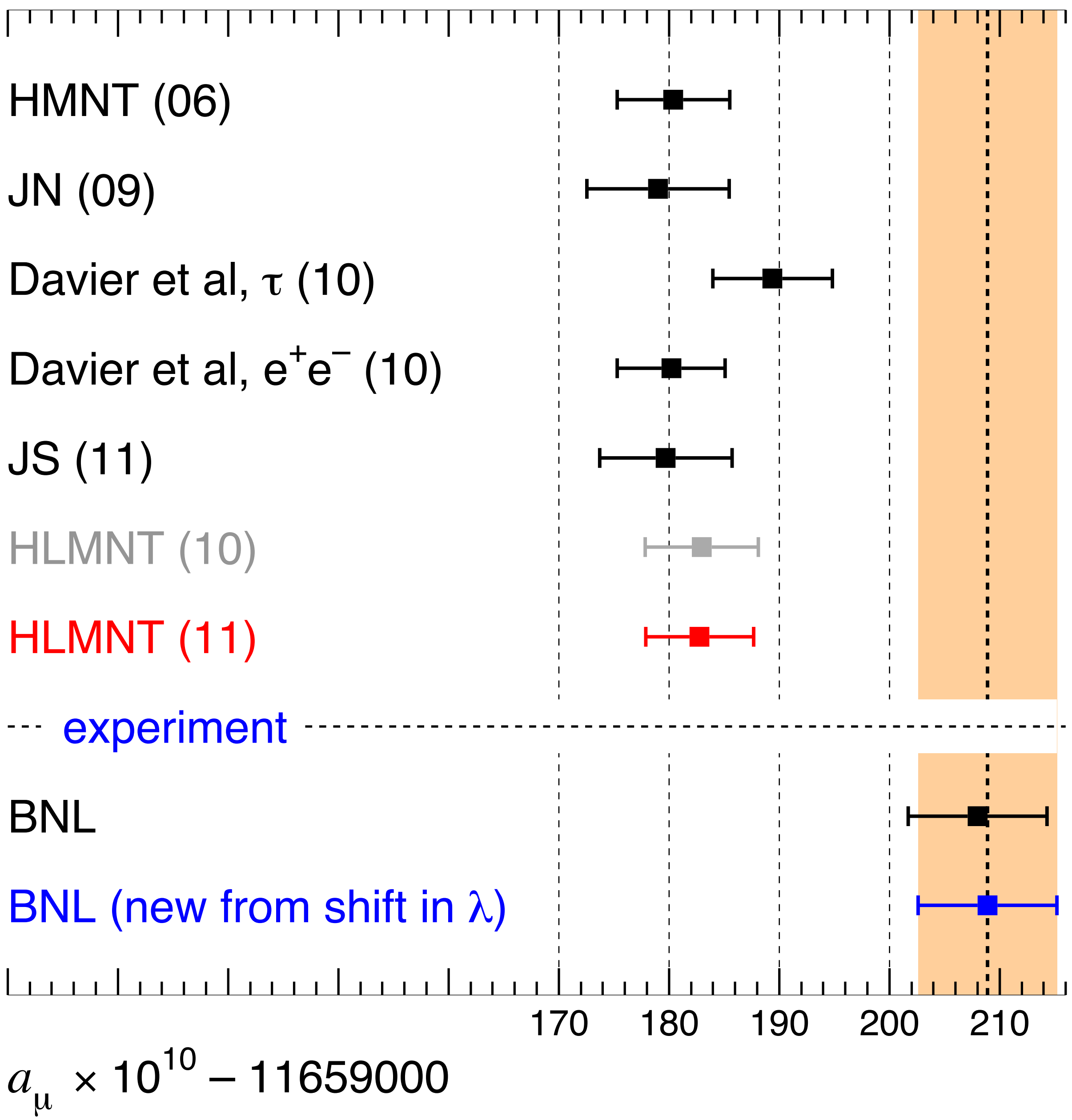
Theoretical calcs. are important because

a_e tests validity of QED or (perturbative) field theory
    Determining α_{QED} (important in EW precision test)

a_μ tests the SM or constraints BSM
    Much more sensitive to heavy dof than a_e by $(m_\mu/m_e)^2$

Current status of $(g\text{-}2)_\mu$

K. Hagiwara, R. Liao, A. D. Martin, D. Nomura and T. Teubner, J. Phys. G: Nucl. Part. Phys. 38 (2011) 085003



$$a_\mu^{\rm SM} = (11\,659\,182.8 \pm 4.9) \times 10^{-10} \quad \text{(using [1])}$$
$$a_\mu^{\rm EXP} = (11\,659\,208.9 \pm 6.3) \times 10^{-10} \quad \text{[PDG]}$$

$$a_\mu^{\rm EXP} - a_\mu^{\rm SM} = (26.1 \pm 8.0) \times 10^{-10}$$

3.3σ discrepancy!

The SM prediction stable.

[1] a_μ(Hlbl)=10.5(2.6) is used.
Based on model estimates.
J. Prades, E. de Rafael and A. Vainshtein, arXiv:0901.0306

5-loop calc. in QED part completed!

T. Aoyama, M. Hayakawa, T. Kinoshita, M. Nio,
arXiv:1205.5370 [hep-ph];arXiv:1205.5368 [hep-ph]

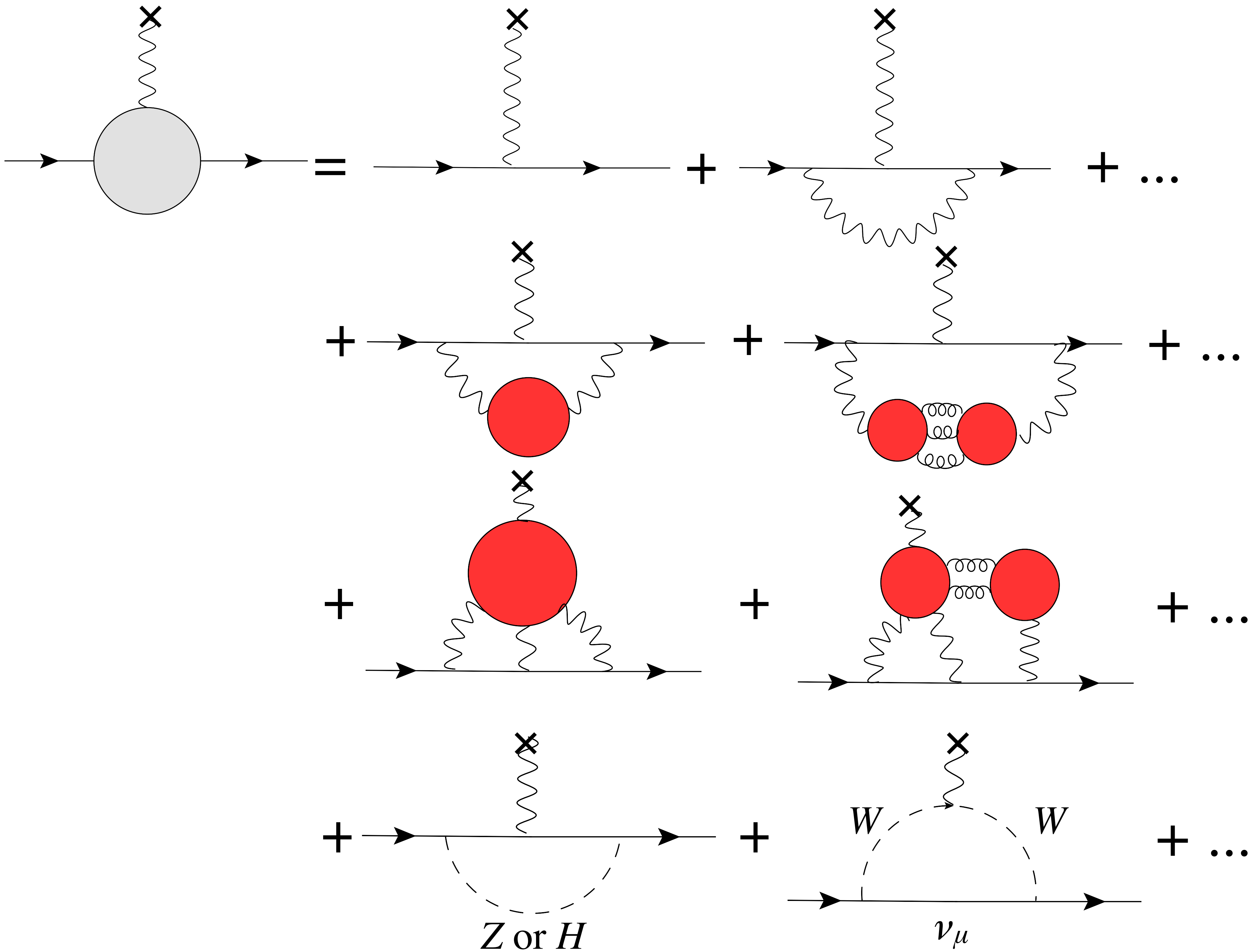
$a_e^{\mathrm{EXP}} - a_e^{\mathrm{THEORY}} = -1.09(83) \times 10^{-12}$

$\alpha^{-1}_{\mathrm{QED}} = 137.035\,999\,166\,(34)$ [0.25 ppb]

$a_\mu(\mathrm{QED\ part}) = 11\,658\,471.885\,3\,(9)(19)(7)(29) \times 10^{-10}$
[using $\alpha^{-1}_{\mathrm{QED}}$ above]

$a_\mu^{\mathrm{SM}} = 11\,659\,184.0\,(5.9) \times 10^{-10}$
[2.9 σ between this and EXP. $a_\mu(\mathrm{Hlbl})=11.6(4.0)$ is used.]

Classification of diagrams



QED

Hadronic vacuum polarization (HVP)

Hadronic light-by-light (Hlbl)

Electroweak (EW)

Breakdown

K. Hagiwara, R. Liao, A. D. Martin, D. Nomura and T. Teubner, J. Phys. G: Nucl. Part. Phys. 38 (2011) 085003

$$a_\mu^{\rm SM} = (11\ 659\ 182.8 \pm 4.9) \times 10^{-10}$$

$$a_\mu^{\rm QED} = (11\ 658\ 471.808 \pm 0.015) \times 10^{-10}$$
$$a_\mu^{\rm EW} = (15.4 \pm 0.2) \times 10^{-10}$$
$$a_\mu^{\rm had,LOVP} = (694.91 \pm 4.27) \times 10^{-10}$$
$$a_\mu^{\rm had,HOVP} = (-9.84 \pm 0.07) \times 10^{-10}$$
$$a_\mu^{\rm had,lbl} = (10.5 \pm 2.6) \times 10^{-10}$$

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- Discrepancy between EXP and SM is larger than EW!

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- Currently the dominant uncertainty comes from HVP.
- Theoretical estimate of Hlbl is really under control?
- LQCD ➡ the first principles' estimate for the hadronic parts.

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2. Leading order hadronic contribution (HVP)
3. Hadronic light-by-light contribution (Hlbl)
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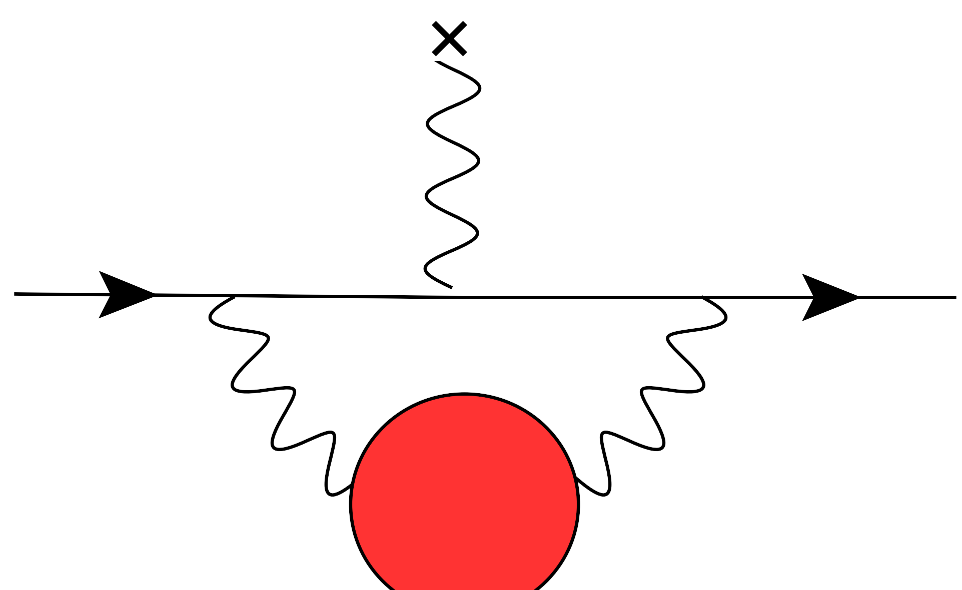
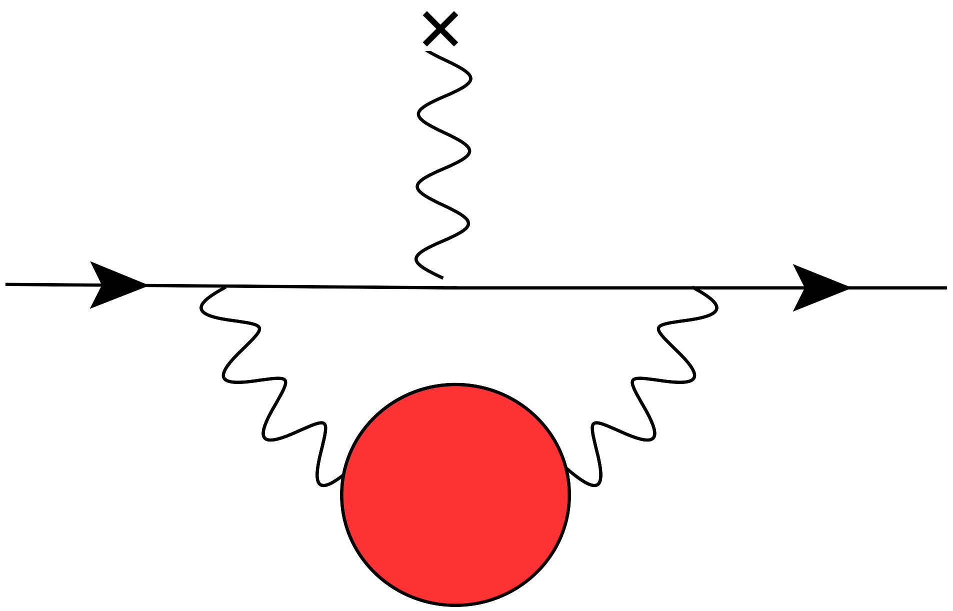
Leading order hadronic contribution (HVP)



Hadronic Vacuum Polarization (HVP)



- Current best estimate using dispersion relation and σ_{total}(e+e-)

$$a_\mu^{\text{HVP}} = \frac{1}{4\pi^2}\int_{4m_\pi^2}^{\infty} ds K(s)\sigma_{\text{total}}(s)$$

$K(s)$: known function

$$a_\mu^{\text{had,LOVP}} = (\quad 694.91 \quad \pm 4.27 \quad) \times 10^{-10}$$
$$a_\mu^{\text{had,HOVP}} = (\quad -9.84 \quad \pm 0.07 \quad) \times 10^{-10}$$

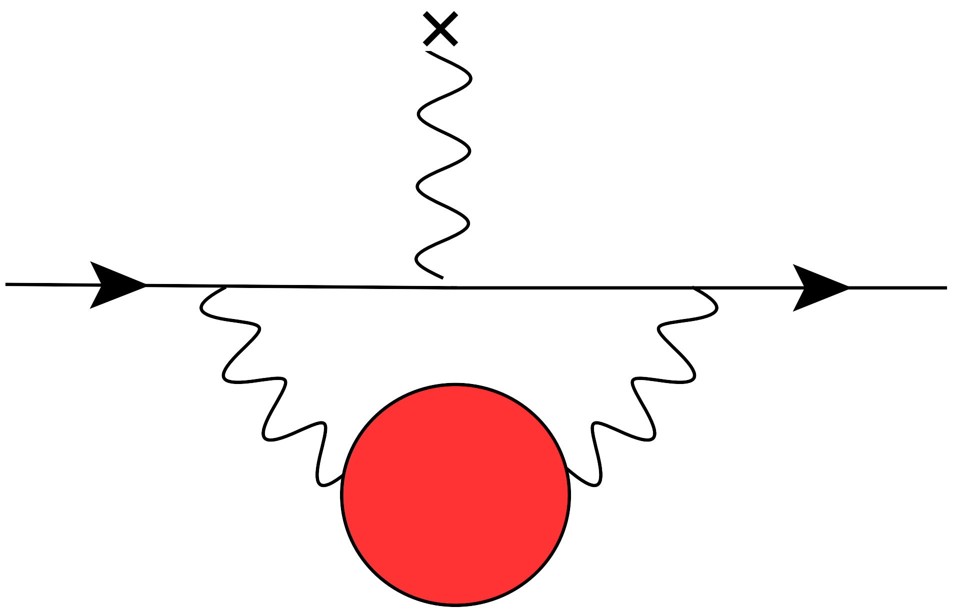
Current uncertainty ~ 0.6 %!

~ 0.3 % in 3-5 years?

depending on upcoming e+e- EXP and existing Belle data.

HVP on the lattice

T. Blum, PRL91(2003)052001



$$\Pi_{\mu\nu}(Q) = i\int d^4x\, e^{iQ\cdot x}\langle 0|T[j_\mu(x)j_\nu(0)]|0\rangle|0\rangle$$
$$= \left(Q_\mu Q_\nu - Q^2 g_{\mu\nu}\right)\Pi(Q^2)$$

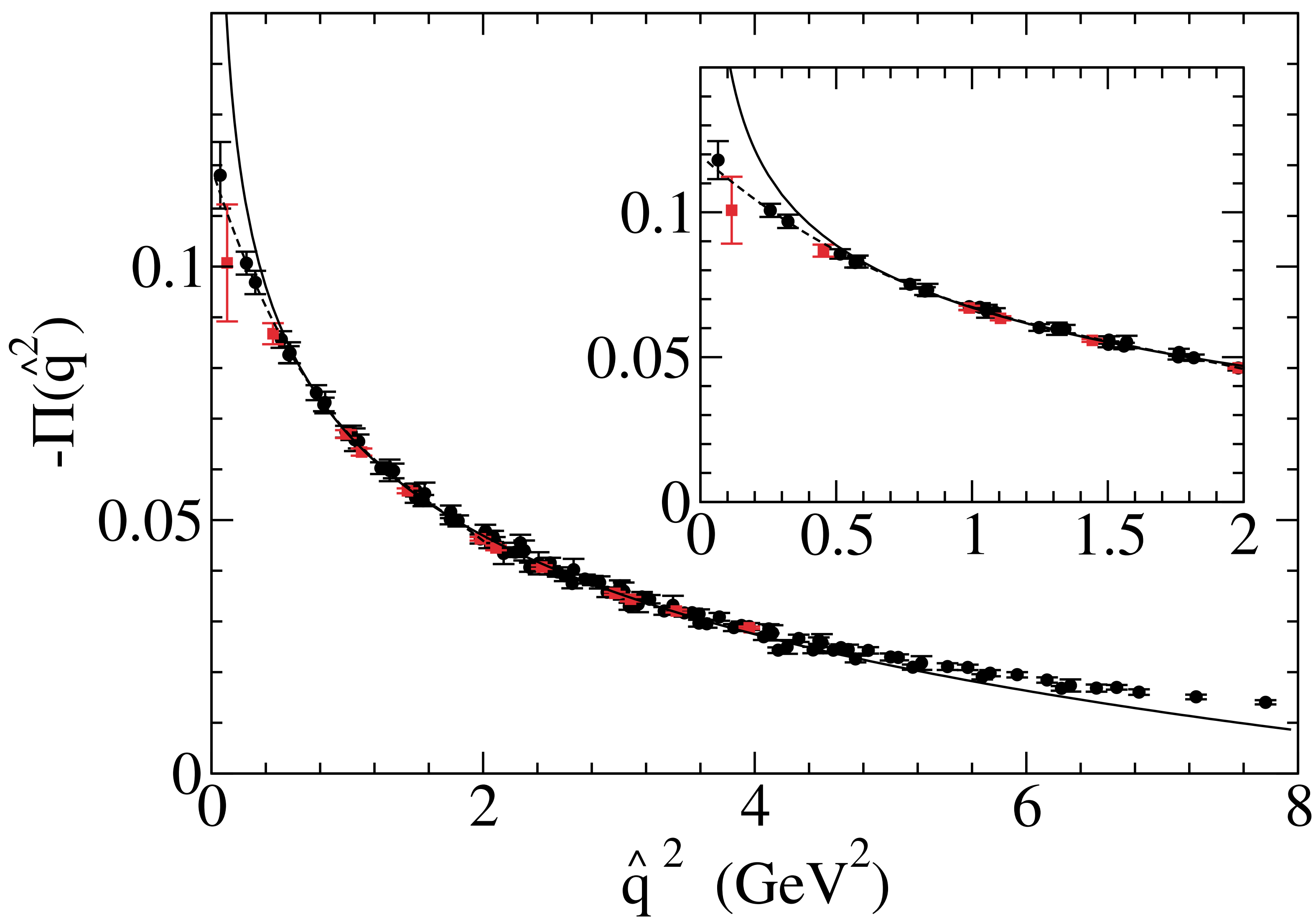
$$a_\mu^{\rm HVP} = \left(\frac{\alpha}{\pi}\right)^2\int_0^\infty dQ^2 f(Q^2)\Pi(Q^2)$$

- $f(Q^2)$ is known and singular toward $Q^2 \to 0$.
- The integral is dominated by small Q^2 region.

HVP on the lattice [pioneering work]

T. Blum, PRL91(2003)052001

$$\Pi_{\mu\nu}(Q) = i\int d^4x\, e^{iQ\cdot x}\langle 0|T[j_\mu(x)j_\nu(0)]|0\rangle|0\rangle \;= (Q_\mu Q_\nu - Q^2 g_{\mu\nu})\,\Pi(Q^2)$$



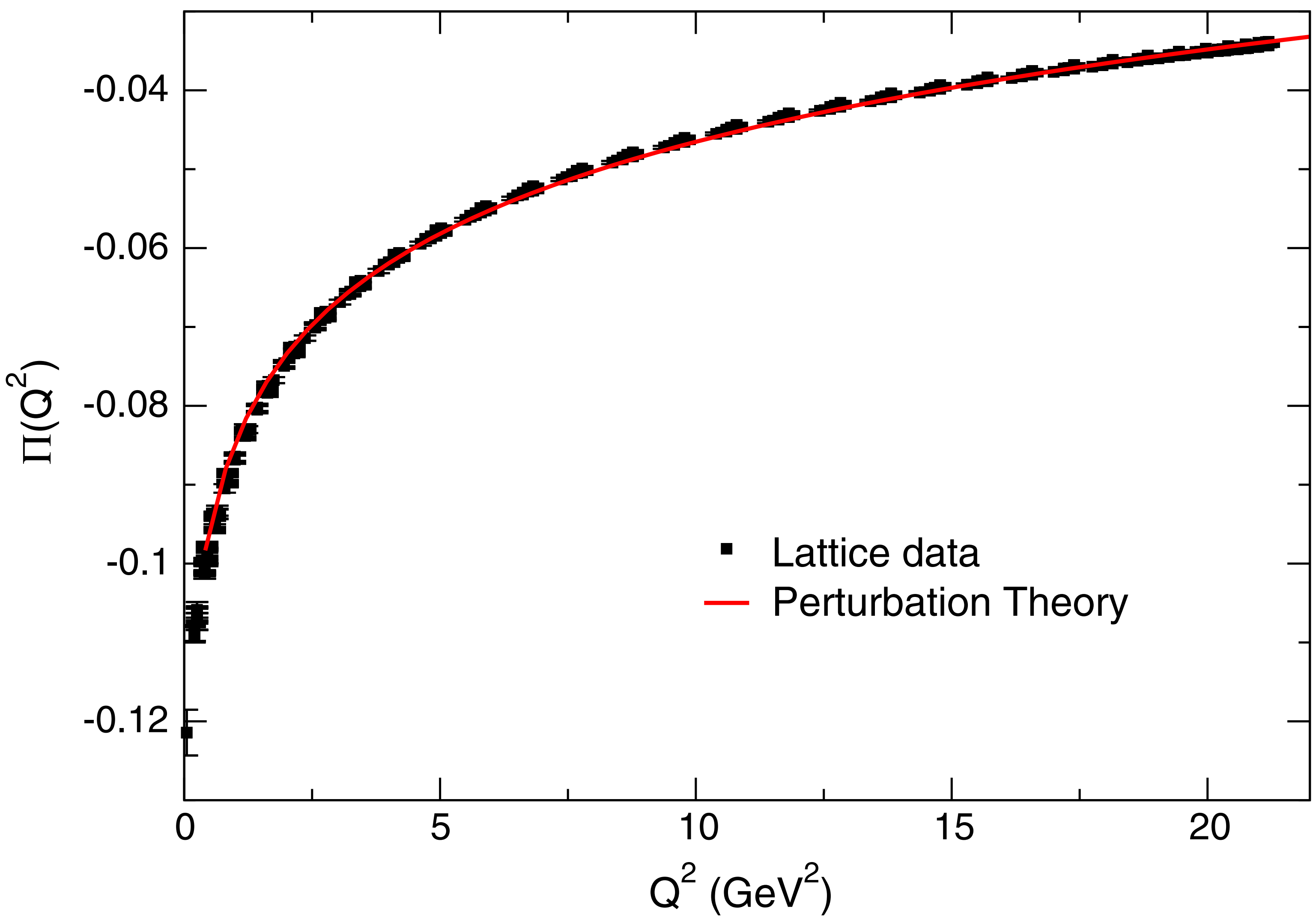
- Quenched approximation
- $O(a^2)$ error at large Q^2.
  ⇒Use PT for high Q^2.
- L sets the non-zero minimum momentum, $q^{\rm lat} \sim 2\pi/L$.
- $a_\mu^{\rm LOHVP} = 460(78)\times 10^{-10}$ (@$m_q \sim m_s$) is roughly consistent with what is expected in quench.

Feasibility was demonstrated!

HVP on the lattice [recent calc.]

ex) P. Boyle, L. Del Debbio, E. Kerrane and J. Zanotti, PRD85, 074504(2012)

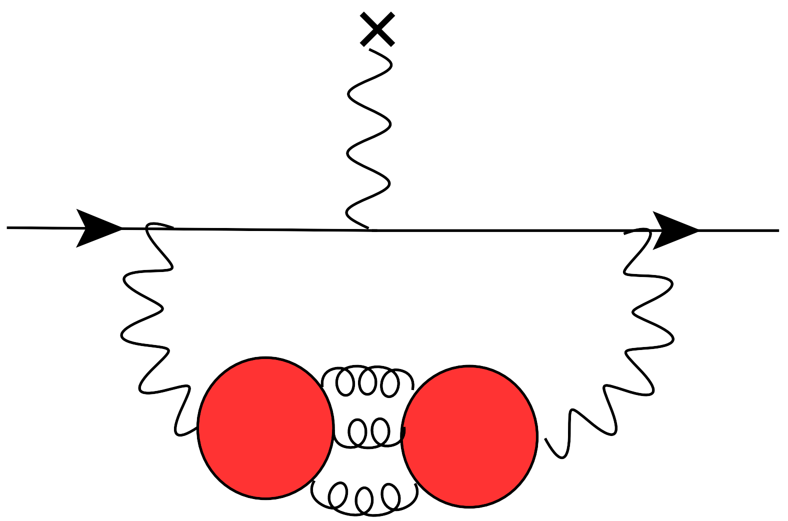
$$\Pi_{\mu\nu}(Q) = i\int d^4x\, e^{iQ\cdot x}\langle 0|T[j_\mu(x)j_\nu(0)]|0\rangle|0\rangle \;=\left(Q_\mu Q_\nu - Q^2 g_{\mu\nu}\right)\Pi(Q^2)$$



- Including the effects of 2+1 flavors of sea quarks
- $m_\pi \sim$ 180 MeV

$a_\mu^{\text{LOHVP}} = 641(31)(32)\times 10^{-10}$

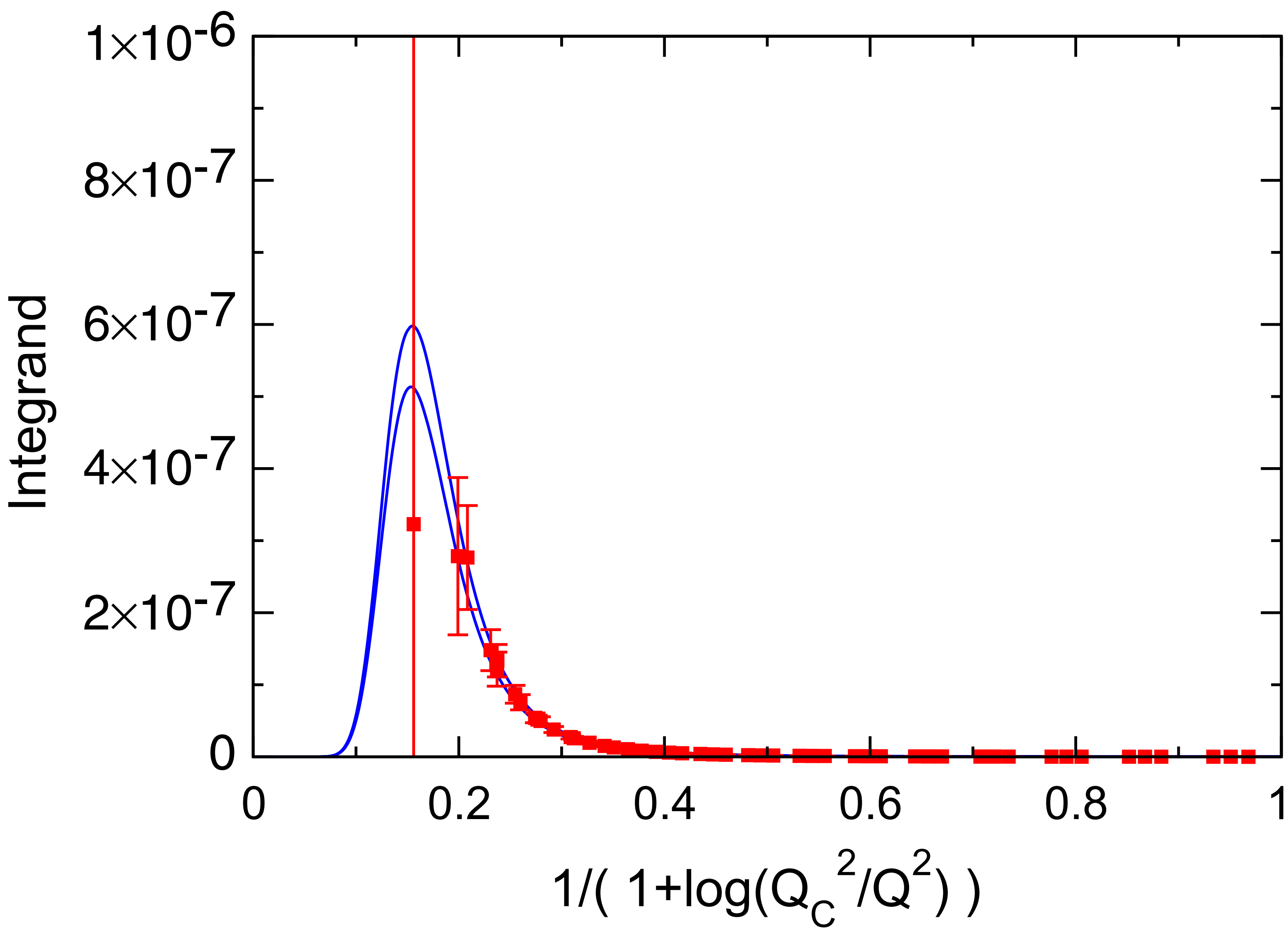
Source of uncertainties



- ~~Quench approximation~~
- Disconnected diagram ($\lesssim$O(10 %)?)
- Finite volume effect (not seen at present accuracy)
- Discretization error (not seen at present accuracy)
- Need more data in small mom. region
- Chiral extrapolation
- Statistical error

Importance of small q^2 region

$$\int_0^{Q_C^2} dQ^2 f(Q^2) \times \hat{\Pi}(Q^2) \rightarrow \int_0^1 dt f(Q^2) \times \hat{\Pi}(Q^2) \times \frac{Q^2}{t^2} \qquad \text{where} \quad t = \frac{1}{1 + \log\frac{Q_C^2}{Q^2}}$$

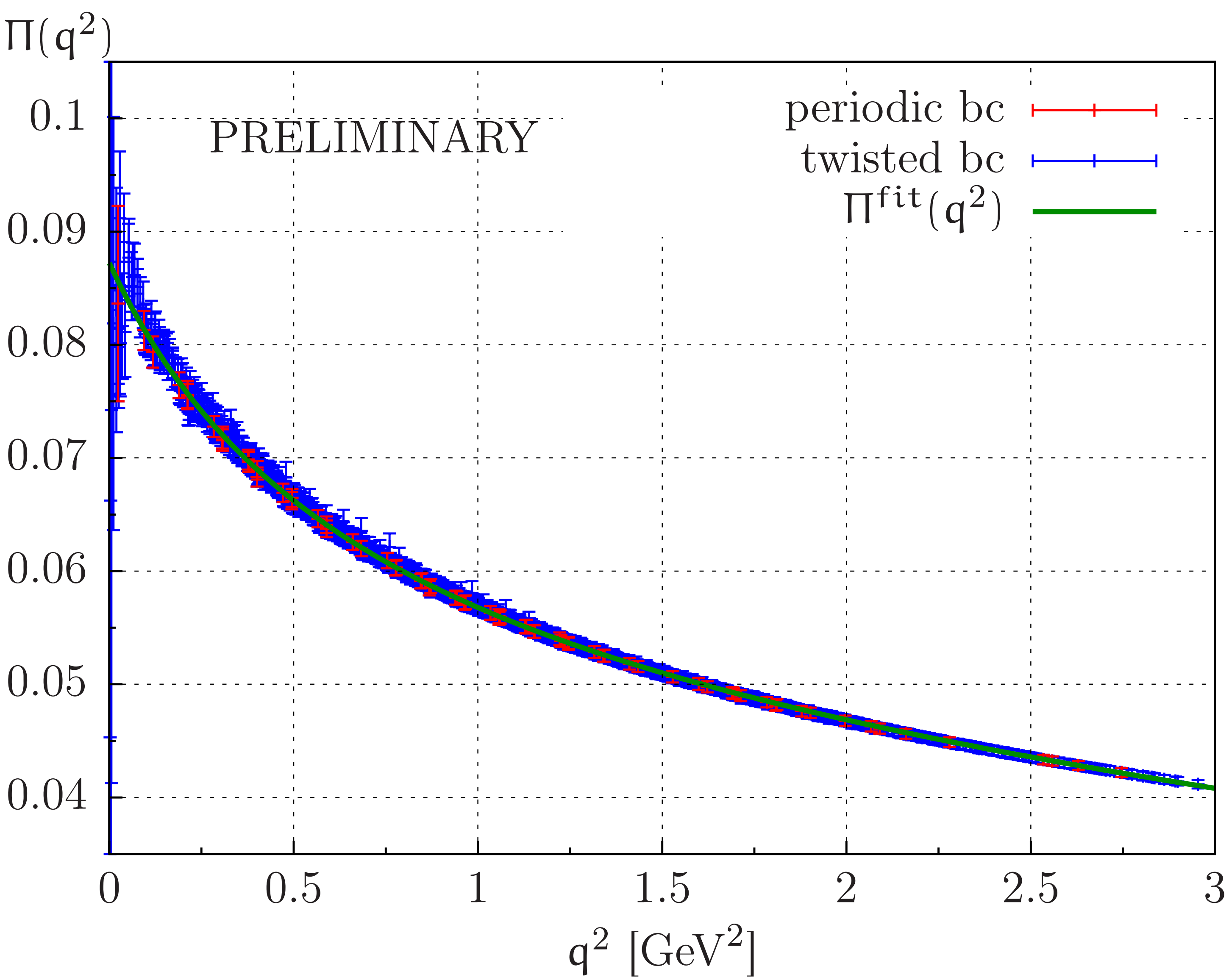


- In dominant region, only a few points exists, and they are inaccurate.
- More accurate data in this region are clearly favorable.

P. Boyle, L. Del Debbio, E. Kerrane and J. Zanotti, PRD85, 074504(2012)

Twisted boundary condition

P.F. Bedaque, PLB 593 (2004) 82; C.T. Sachrajda and G. Villadoro, PLB 609 (2005) 73



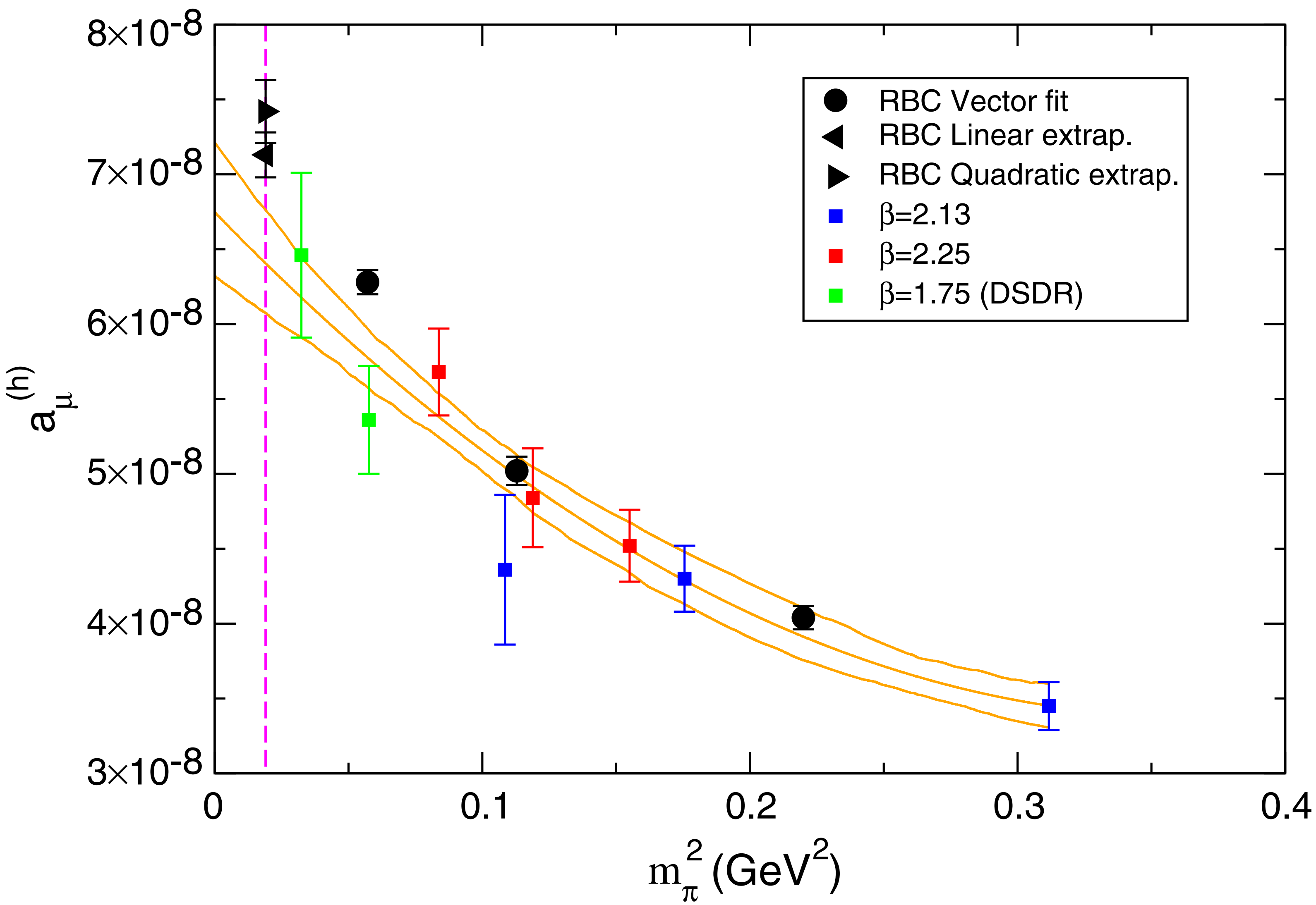
B. Jaeger [Mainz group] @ Lattice 2012

- On a torus, the action must be single-valued, while fields do not have to be.
- Impose the twisted boundary condition on quark fields.

$$q(x+L) = q(x)e^{i\theta}$$
$$(\theta\text{:arbitrary})$$

- q^2 can be arbitrary small.

Chiral extrapolation



- One of the dominant sys. errors ~ 5 %
- Functional form unknown

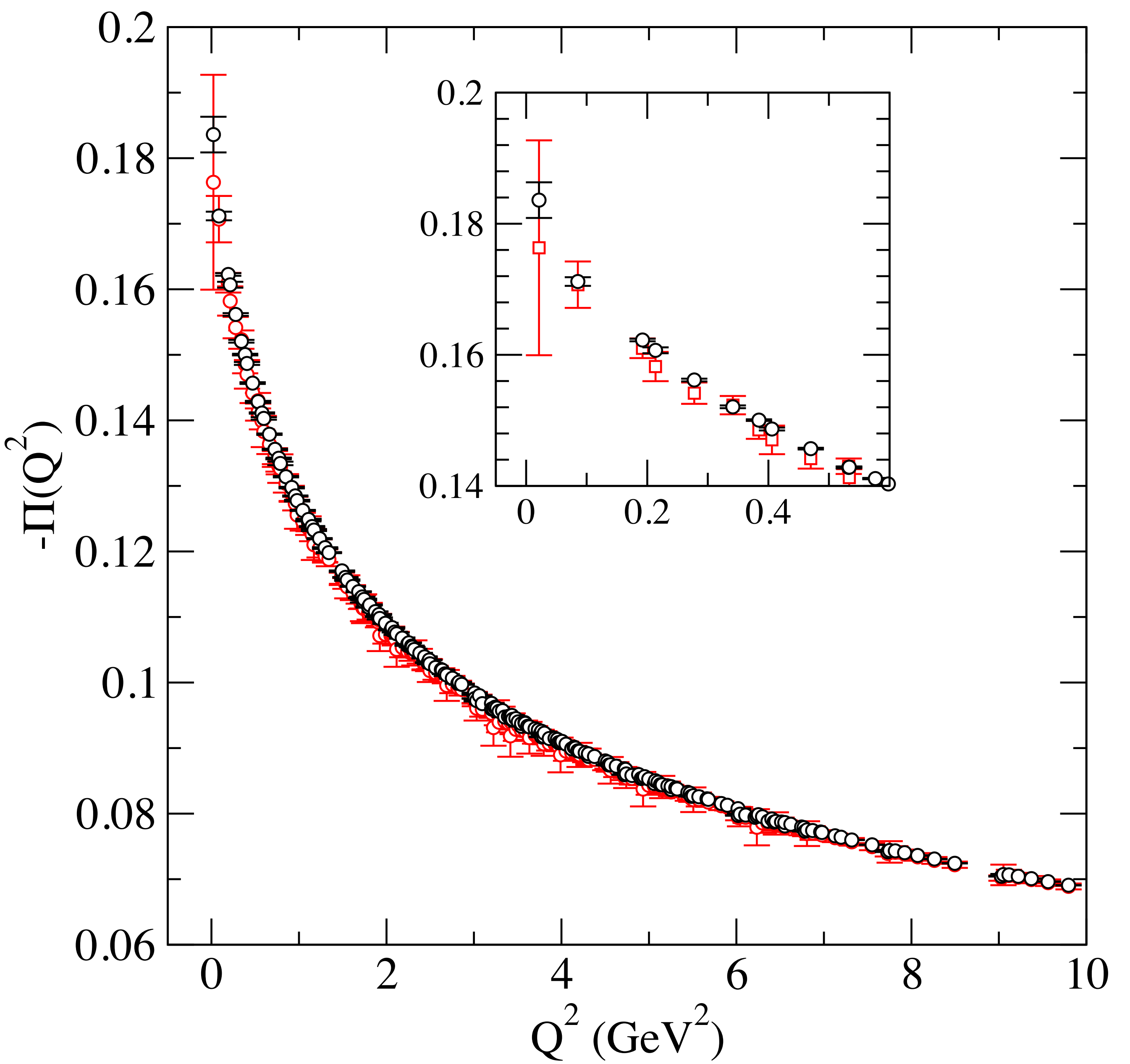
- Simulations@physical m_q are becoming a trend. ⇒no need to extrapolation or only a small extrapolation in future

P. Boyle, L. Del Debbio, E. Kerrane and J. Zanotti, PRD85, 074504(2012)

Statistical error

E. Shintani and T. Izubuchi, poster@Lattice 2012; Blum, Izubuchi, Shintani, et al.,(RBC/UKQCD)

New error reduction technique, All Mode Averaging (AMA), significantly reduces stat. error (× 1/5~1/20)!



- Full use of translational invariance.
- Wide range of application
- Stat. error won't limit the accuracy.
- Potentially a game changer?

Summary of recent lattice calc. of HVP

a_μ	N_f	errors	action	group
713(15)	2+1	stat.	Asqtad	Aubin, Blum (2006)
748(21)	2+1	stat.	Asqtad	Aubin, Blum (2006)
641(33)(32)	2+1	stat., sys.	DWF	UKQCD (2011)
572(16)	2	stat.	TM	ETMC (2011)
618(64)	2+1[1]	stat., sys.	Wilson	Mainz (2011)

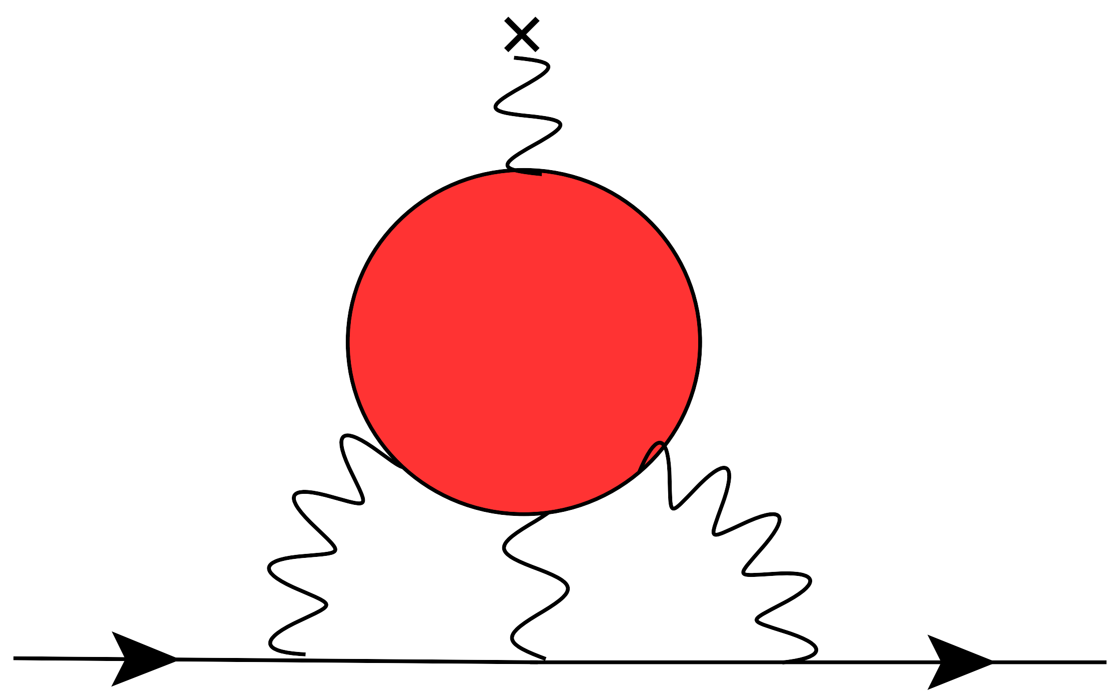
presented by T. Blum@Lattice 2012

0.5 % accuracy is challenging, but we should be able to come close to it by combining

Twisted b.c. + Simulation@physical m_q + AMA + …

and Supercomputer.

Hadronic light-by-light contribution



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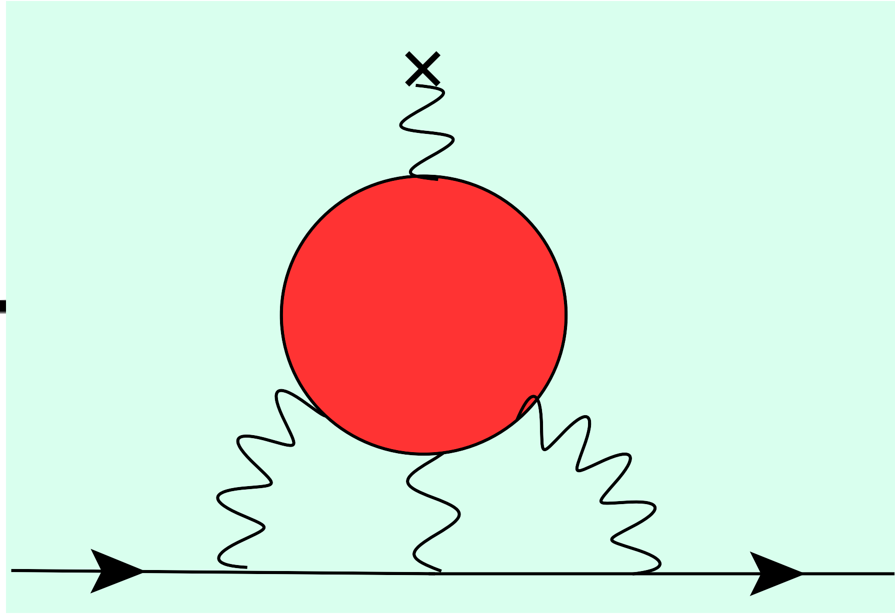


$$\Gamma_\mu^{(\mathrm{Hlbl})}(p_2,p_1) = ie^6\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_3,k_2)}{k_1^2\,k_2^2\,k_3^2} \times\gamma_\nu S^{(\mu)}(p_2+k_2)\gamma_\rho S^{(\mu)}(p_1+k_1)\gamma_\sigma$$

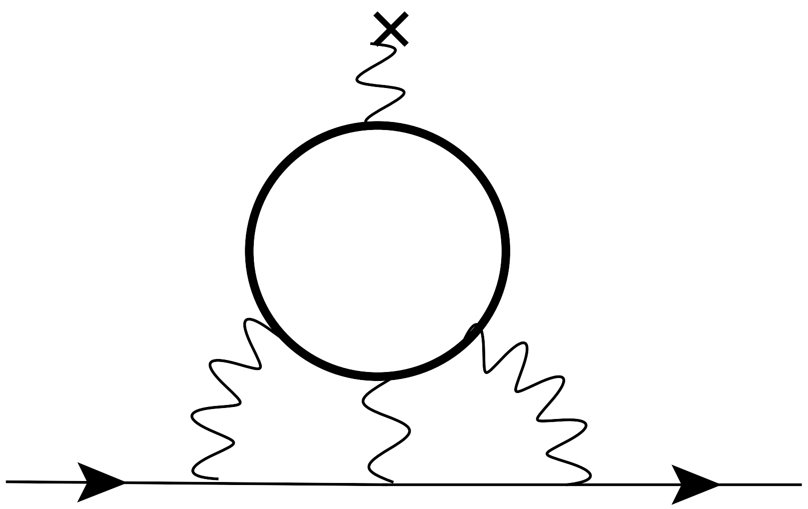
$$\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_3,k_2) = \int d^4x_1\,d^4x_2\,d^4x_3\,\exp[-i(k_1\cdot x_1+k_2\cdot x_2+k_3\cdot x_3)] \times\underline{\langle 0|T[j_\mu(0)j_\nu(x_1)j_\rho(x_2)j_\sigma(x_3)]|0\rangle}$$

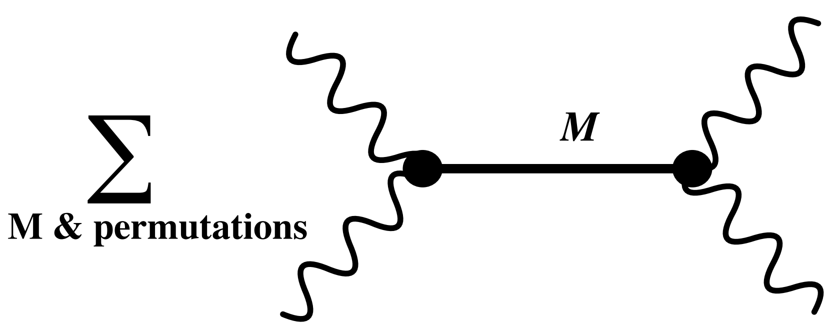
Form factor : $\Gamma_\mu(q)=\gamma_\mu\,F_1(q^2)+\dfrac{i\sigma^{\mu\nu}q_\nu}{2\,m_l}\,F_2(q^2)$

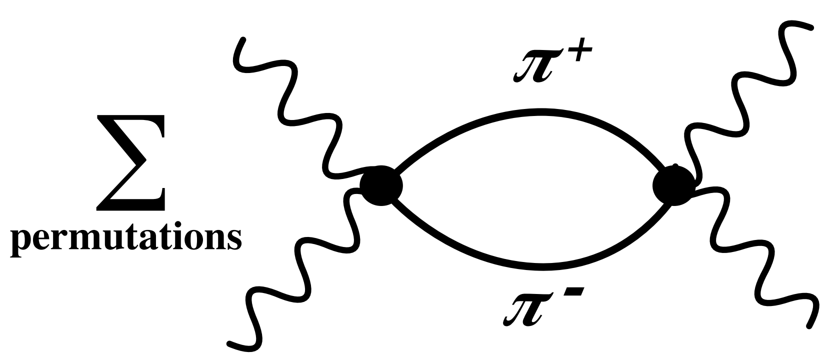
In contrast to the VP case , no experimental input is available.

Hadronic light-by-light : model estimates

Summarized in J. Prades, E. de Rafael and A. Vainshtein, arXiv:0901.0306 [hep-ph]







- Quark loop with ~300 MeV constituent mass
- ChPT (Meson exchange/loop diagrams)
- Extended NFL
- ...

- More or less justified by large N_c argument.
- All existing results fall into

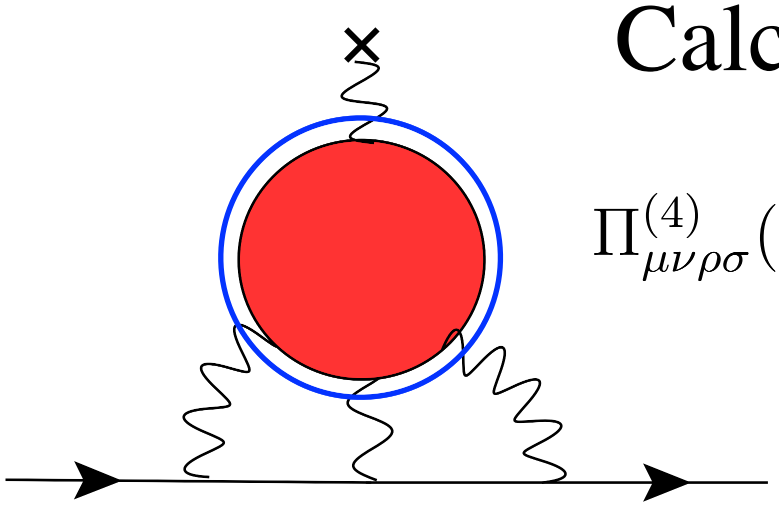
$$a^{\mathrm{Hlbl}} = (11 \pm 4) \times 10^{-10}$$

- Current best estimate based on models:

$$a^{\mathrm{Hlbl}} = (10.5 \pm 2.6) \times 10^{-10}$$

Uncertainty is uncertain.
Lattice calculation favorable!

Conventional approach on the lattice



Calculate 4-point function

$$\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_3,k_2) = \int d^4x_1\, d^4x_2\, d^4x_3\, \exp[-i(k_1\cdot x_1 + k_2\cdot x_2 + k_3\cdot x_3)] \times \langle 0|T[j_\mu(0)j_\nu(x_1)j_\rho(x_2)j_\sigma(x_3)]|0\rangle$$

Then, one will obtain a single set of (q, k_1, k_3, k_2).

Calc. requires integration over k_1 and k_2,

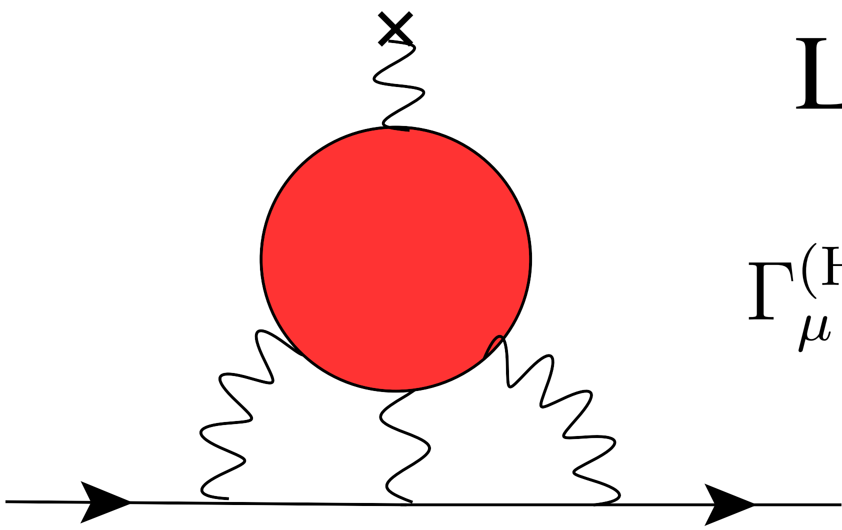
$$\Gamma^{(\mathrm{Hlbl})}_\mu(p_2,p_1) = ie^6 \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_3,k_2)}{k_1^2\,k_2^2\,k_3^2} \times \gamma_\nu S^{(\mu)}(p_2+k_2)\gamma_\rho S^{(\mu)}(p_1+k_1)\gamma_\sigma$$

Need to repeat (*Volume*)^2 times ~ $10^{10\text{-}11}$ times.

With this approach, the calculation won't end...

Alternative approach

M. Hayakawa, T. Blum, T. Izubuchi and NY, hep-lat/0509016



Let lattice calculate the form factor itself

$$\Gamma_\mu^{(\mathrm{Hlbl})}(p_2,p_1) = ie^6 \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_3,k_2)}{k_1^2\,k_2^2\,k_3^2} \times \gamma_\nu S^{(\mu)}(p_2+k_2)\gamma_\rho S^{(\mu)}(p_1+k_1)\gamma_\sigma$$

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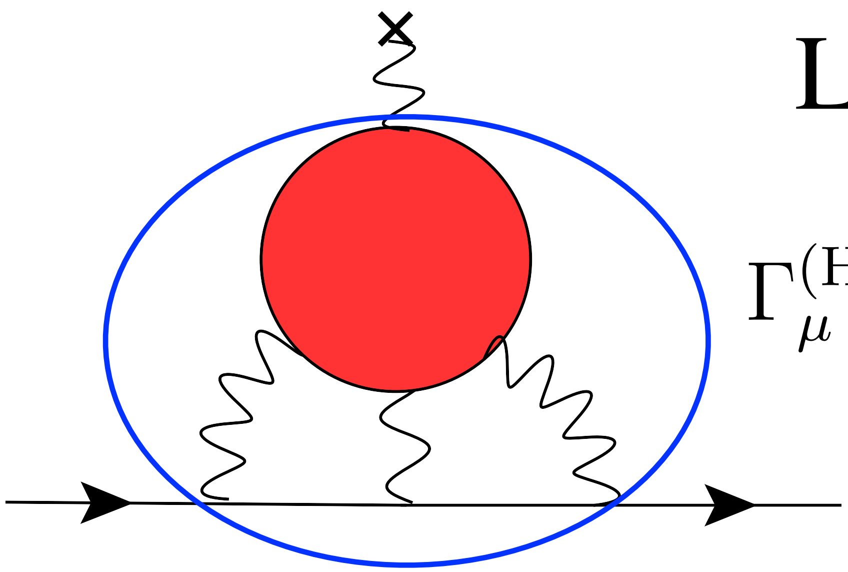

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- Standard method in other form factor calculations
- Need to incorporate **QED** on the lattice.

Lattice QCD + QED

Motivation (other than light-by-light)

- Ordinary lattice calculations are done in the iso-spin symmetric limit. (m_u=m_d and no EM interaction).
- Lattice calc. are being precise, and iso-spin breaking effects start to be visible.
- In order to determine the most poorly known quark mass, m_u and m_d, QED must be taken into account!
- One should be able to exclude m_u=0 and to reproduce

$$m_N - m_P = 1.2933321(4)\text{MeV}$$

$$m_{\pi^\pm} - m_{\pi^0} = 4.5936(5)\text{MeV},$$

$$m_{K^\pm} - m_{K^0} = -3.937(28)\text{MeV},$$

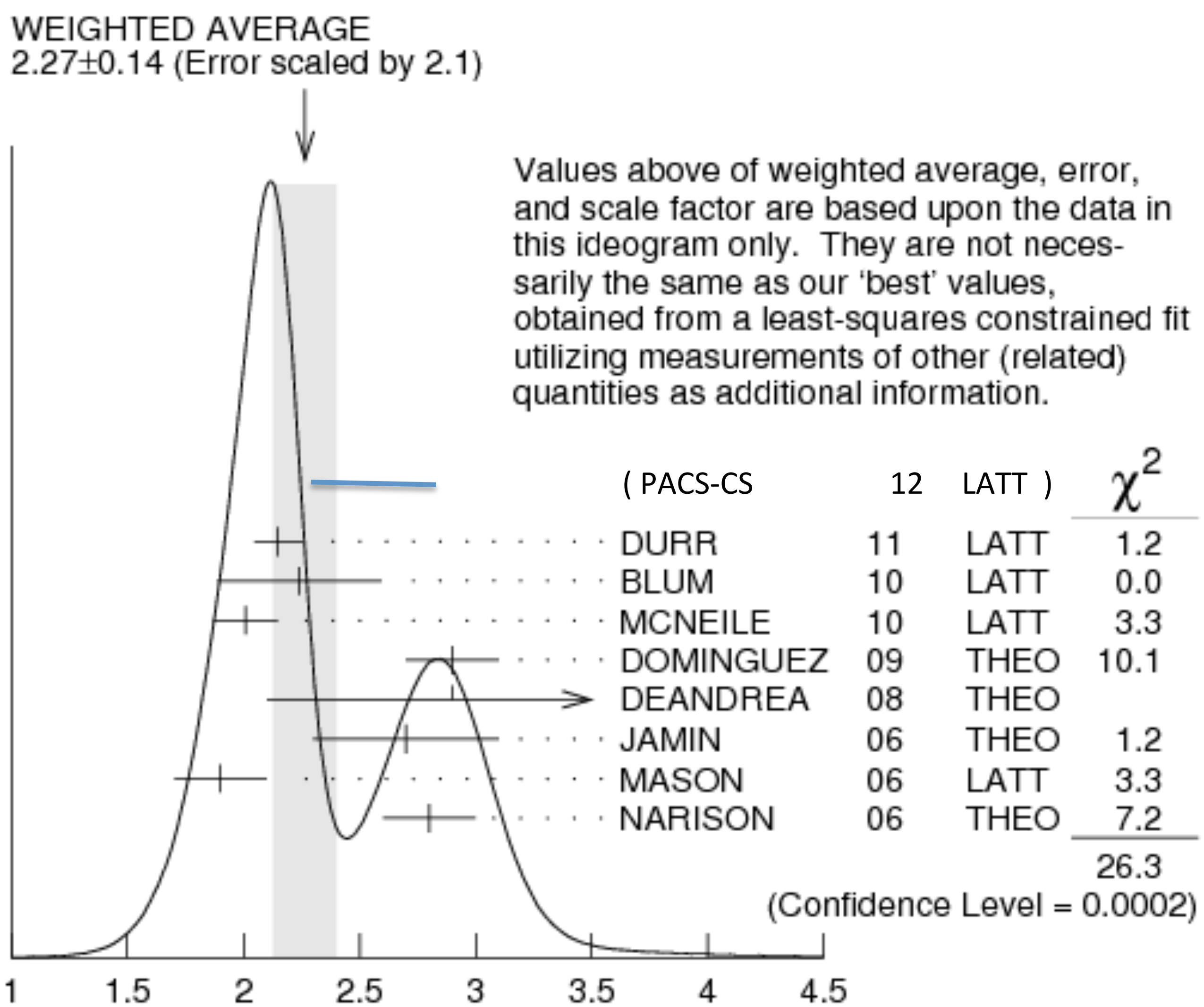
QCD+QED lattice simulation

- Quenched approximation
  Duncan, Eichten, Thacker PRL76(1996) 3894;
  Y. Namekawa and Y. Kikukawa, PoS LAT 2005, 090 (2006)
- Two-flavor QCD
  T. Blum, T. Doi, M. Hayakawa, T. Izubuchi and NY, PRD76, 114508 (2007)
- Three-flavor QCD
  T. Blum, R. Zhou, T. Doi, M. Hayakawa, T. Izubuchi, S. Uno and NY, PRD82, 094508 (2010)
- ...
- Three-flavor QCD with charged sea quarks
  - T. Ishikawa, T. Blum, M. Hayakawa, T. Izubuchi, C. Jung and R. Zhou, arXiv:1202.6018 [hep-lat];
  - S. Aoki, K.-I. Ishikawa, N. Ishizuka, K. Kanaya, Y. Kuramashi, Y. Nakamura, Y. Namekawa and M. Okawa, Y. Taniguchi, A. Ukawa, N. Ukita and T. Yoshi ´e [PACS-CS Collaboration], arXiv:1205.2961 [hep-lat]

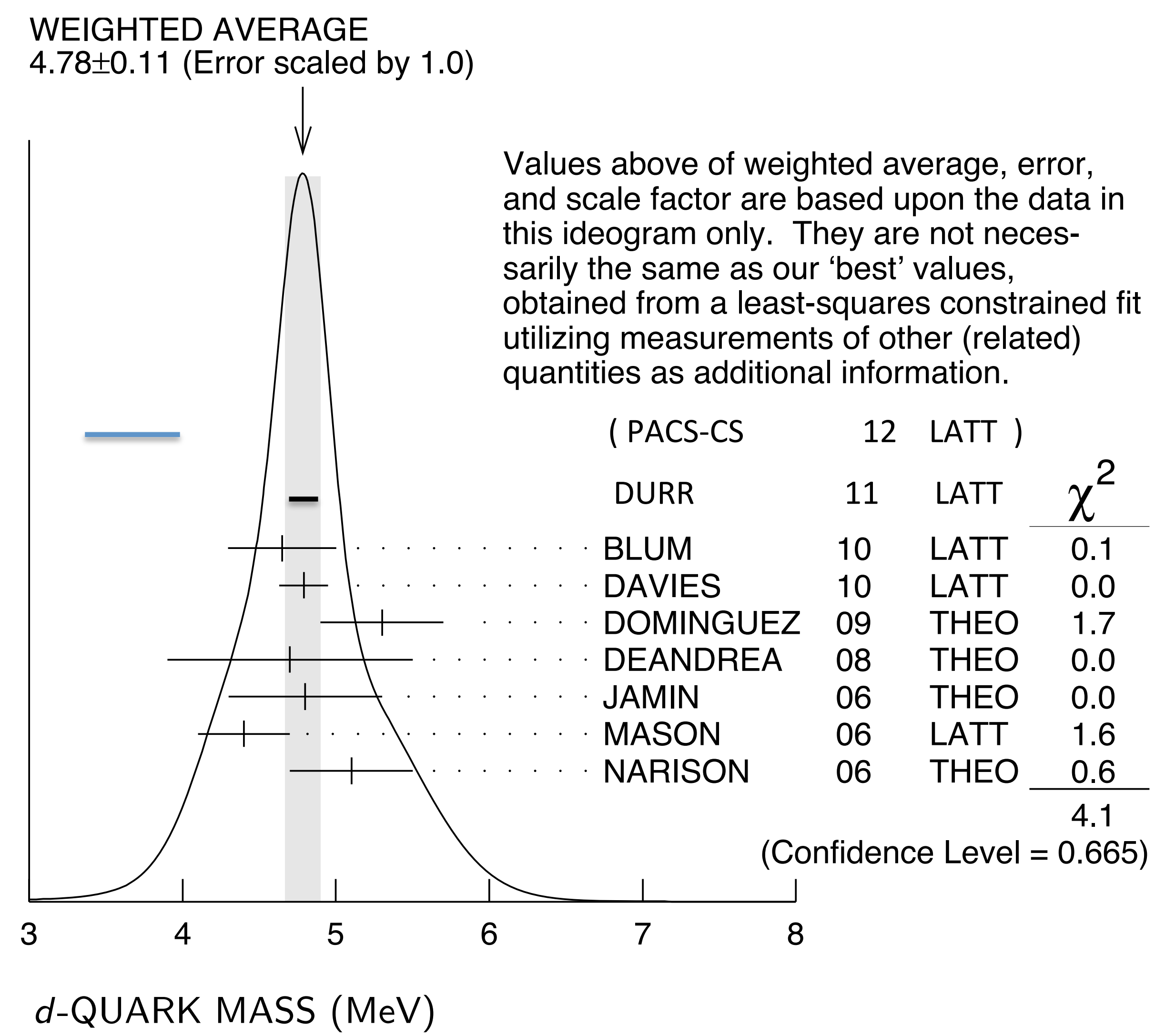
m_u and m_d

T. Izubuchi@Lat2012

PDG2012



m_u

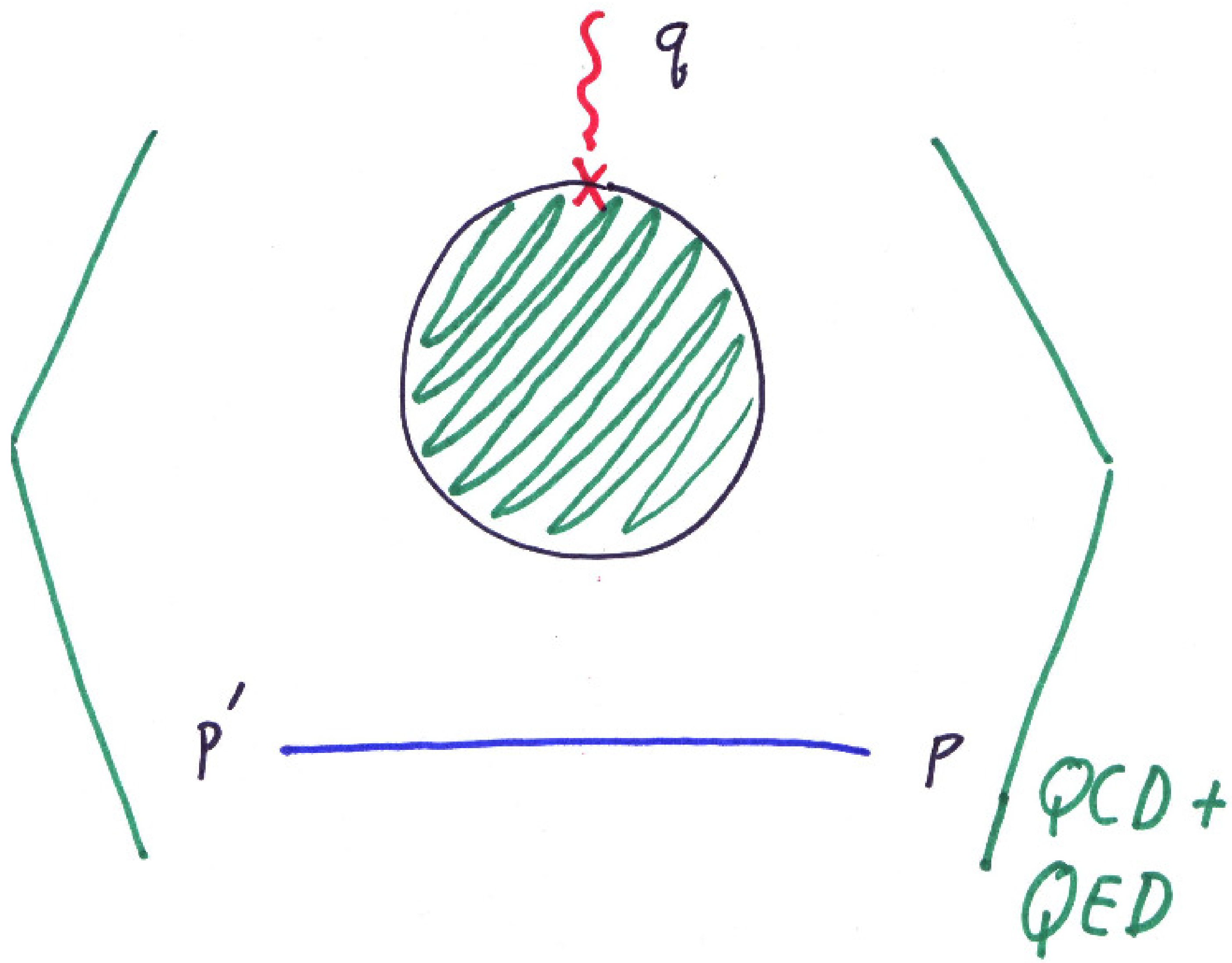


m_d

Alternative approach

M. Hayakawa, T. Blum, T. Izubuchi and NY, hep-lat/0509016

Calculate

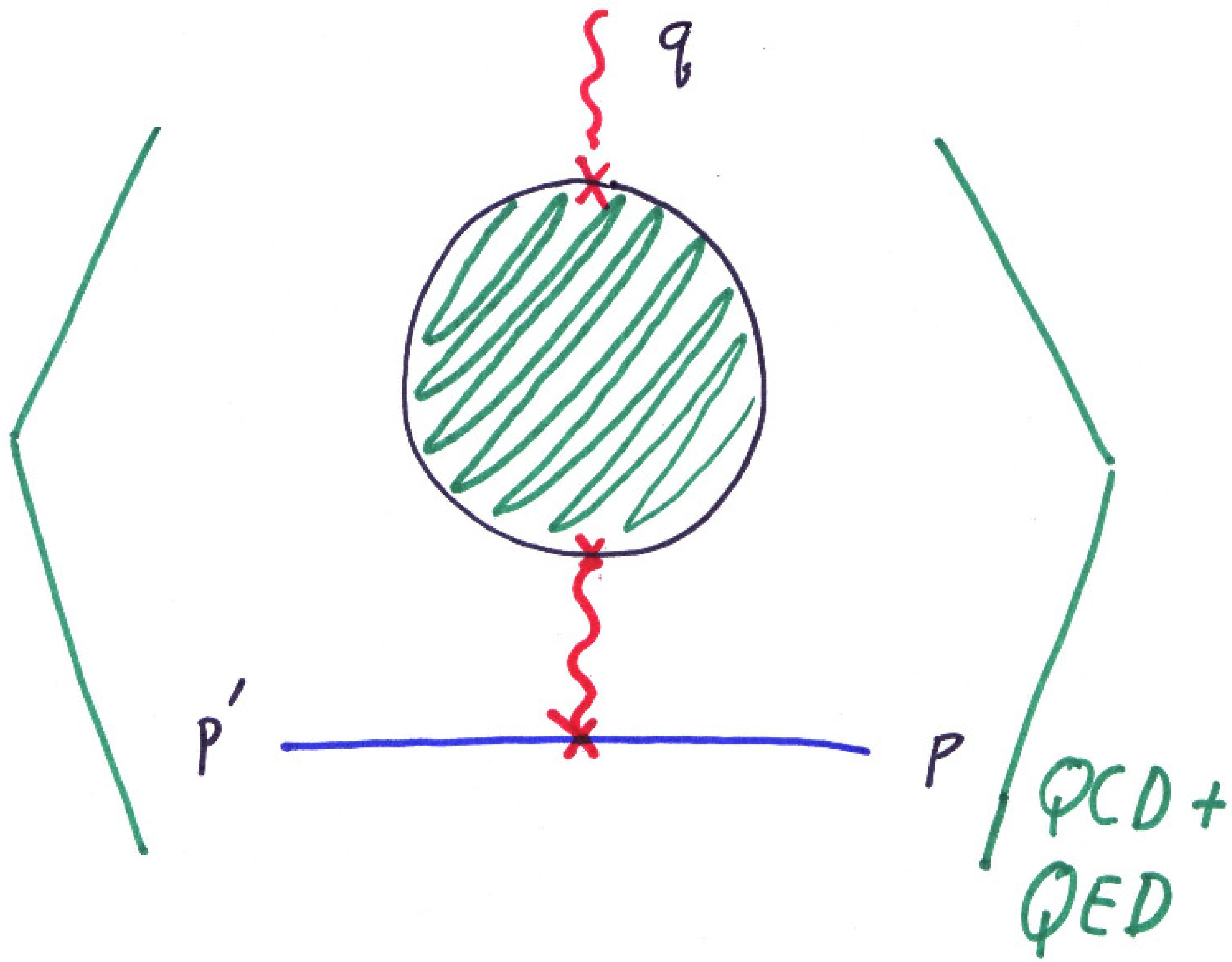


This includes unwanted diagrams, while what we want is lbl only.
Easier way to get rid of unwanted one?

Alternative approach

M. Hayakawa, T. Blum, T. Izubuchi and NY, hep-lat/0509016

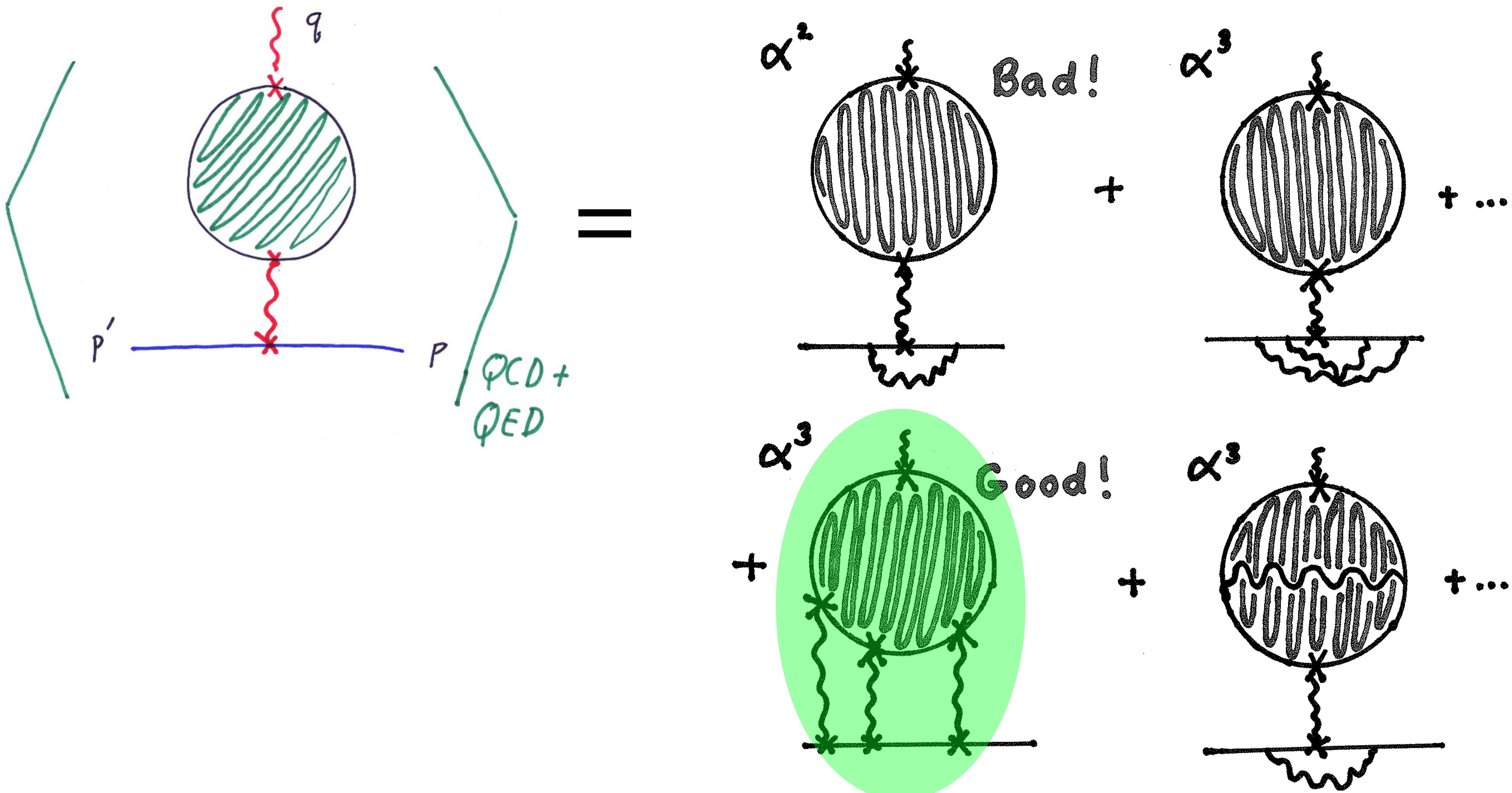
Calculate



One of photon propagators, whose analytical expression is known, is attached by hand.
This still includes unwanted diagrams.

Alternative approach

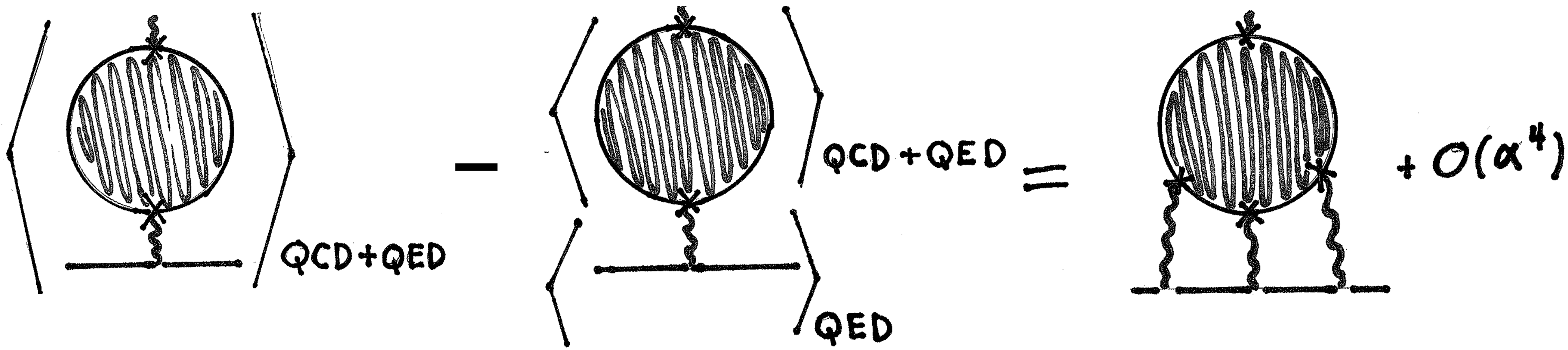
M. Hayakawa, T. Blum, T. Izubuchi and NY, hep-lat/0509016



How to subtract unwanted one?

Alternative approach

M. Hayakawa, T. Blum, T. Izubuchi and NY, hep-lat/0509016



Subtract the similar expectation value, but there the configuration average of the quark part and the muon part are taken separately.

⇒ No photon connecting quark and muon in the 2nd term.

⇒ Only lbl (and higher order terms) survives

Numerical tests

- Tests with only QED
  - S. Chowdhury, T. Blum, T. Izubuchi, M. Hayakawa, NY and T. Yamazaki, PoS LATTICE 2008, 251 (2008)
  - T. Blum and S. Chowdhury, NP(PS)189, 251 (2009)
  - Chowdhury Ph. D. thesis, UConn, 2009

  Consistent with PT if the spatial volume is as large as V=24^3.

- Tests with (2+1)f QCD+QED
  - $V = 16^3 \times 32$
  - $m_\pi \approx 420$MeV, $m_\mu \approx 190, 692$ MeV($m_\mu^{\mathrm{phys}} \sim$105MeV)
  - Hlbl amplitude behaves as $\sim e^4$, while un-subtracted amplitude stays the same.

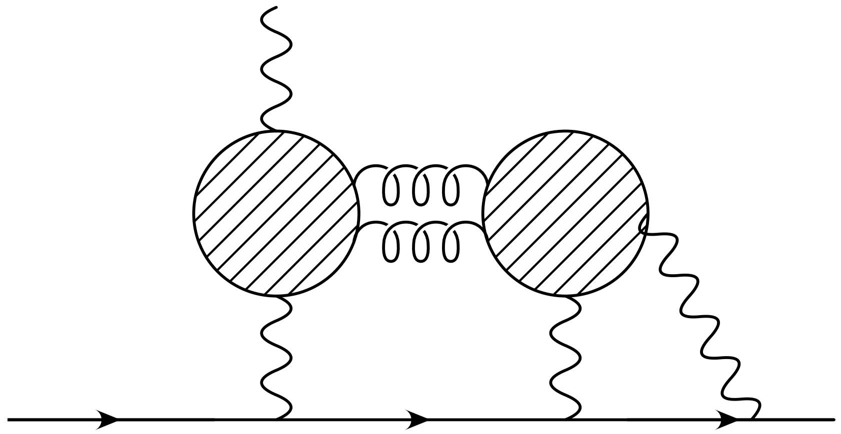
Numerical tests

- Tests with (2+1)f QCD+QED (preliminary)
  - $V=24^3\times 48$ (~2.7 fm)
  - $m_\pi \approx 329$ MeV, $m_\mu \approx 190$ MeV
  - Two lowest values of Q^2 (0.11 and 0.18 GeV^2)
  - All Mode Averaging (AMA)

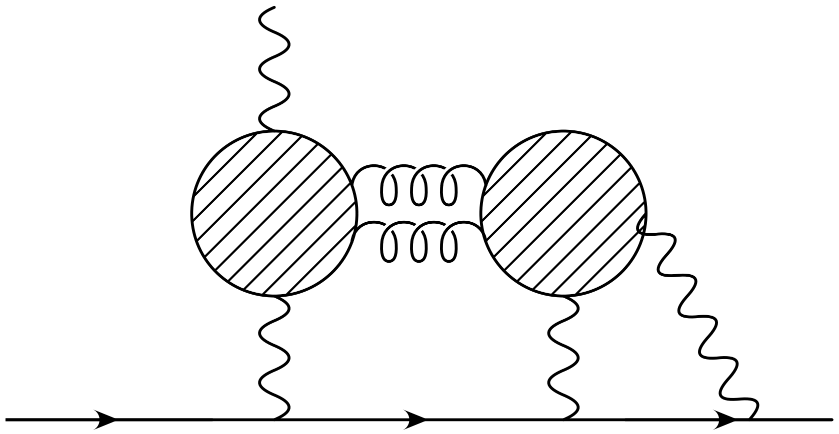
- $F_2(0.18\ \mathrm{GeV}^2) = (0.142 \pm 0.067) \times \left(\frac{\alpha}{\pi}\right)^3$
- $F_2(0.11\ \mathrm{GeV}^2) = (0.038 \pm 0.095) \times \left(\frac{\alpha}{\pi}\right)^3$
- $a_\mu(\mathrm{HLbL/model}) = (0.084 \pm 0.020) \times \left(\frac{\alpha}{\pi}\right)^3$

Signal may be emerging in the model ballpark.
O(100 %) stat. error is encouraging!

Sources of systematic uncertainties



Sources of systematic uncertainties



Many improvements remain to be done.

- Disconnected diagrams
  Not easy. But several promising methods almost ready to test.
- $q^2 \to 0$ [Twisted b.c. applicable?]
- $m_q \to m_{q,\mathrm{phys}}$, $m_\mu \to m_{\mu,\mathrm{phys}}$
- Finite volume. Excited states/"around the world" effects
- $a \to 0$
- QED renormalization
- $\cdots$

Personally, even 50 % uncertainty is sensible,
and such a accuracy will be possible in 5 years.
(10 % error may not be too optimistic.)

Summary

Prospect: Experiment on $(g\text{-}2)_\mu$

$$a_\mu^{\mathrm{EXP}} = (11\ 659\ 208.9 \pm 6.3) \times 10^{-10}$$

Two independent measurements@J-PARC and FNAL are planned.

- J-PARC: start data taking in 2016.
  Exp uncertainty reduced by factor 4 in 5 years (by ~2017).
  $6.3\times10^{-10} \Rightarrow 1.4\times10^{-10}$
- FNAL: similar

Prospect: Theory

$$a_\mu^{\text{EXP}} = (11\ 659\ 208.9 \quad \pm 1.4) \times 10^{-10}$$

$$a_\mu^{\text{SM}} = (11\ 659\ 182.8 \quad \pm 4.9) \times 10^{-10}$$

$$a_\mu^{\text{QED}} = (11\ 658\ 471.885 \quad \pm 0.004) \times 10^{-10}$$
$$a_\mu^{\text{EW}} = (15.4 \quad \pm 0.2) \times 10^{-10}$$
$$a_\mu^{\text{had,LOVP}} = (694.91 \quad \pm 4.27) \times 10^{-10}$$
$$a_\mu^{\text{had,HOVP}} = (-9.84 \quad \pm 0.07) \times 10^{-10}$$
$$a_\mu^{\text{had,lbl}} = (10.5\,? \quad \pm 2.6\,?) \times 10^{-10}$$

Assuming that improvements occur without changing the central values and model estimate of Hlbl is confirmed by lattice calc. at 40 % level.

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$$a_\mu^{\mathrm{had,lbl}} = (10.5 \pm 4.2) \times 10^{-10}$$

$$3.3\sigma \Rightarrow 5.3\ \sigma$$

Assuming that improvements occur without changing the central values and model estimate of Hlbl is confirmed by lattice calc. at 40 % level.

Summary

- Lattice QCD can play an important role in $(g\text{-}2)_\mu$, through the determinations of HVP and Hlbl contribution.

- HVP:
  Currently O(10) % level ⇒ O(a few %) using various techniques simultaneously
  Cross check against dispersion + e^+e^- cross section.

- Hlbl:
  Numerical tests are encouraging.
  Many things to do (disconnected diagrams, physical masses, ...)
  O(100 %) ⇒ 40-50 % seems doable.

- After all, clear evidence for BSM might emerge.

Acknowledgement

- US DOE, RIKEN BNL Research Center, USQCD Collaboration
- Machines:QCDOC at BNL, Ds cluster at FNAL, q-series clusters at JLAB