$g^2$

- Recent progress on the lattice -

Norikazu Yamada (KEK, GUAS)

Thanks to

Tom Blum, Masashi Hayakawa, and Taku Izubuchi

for many discussions and providing materials

"PPP2012"@Yukawa Institute, 2012.7.18
• Slides given by Tom Blum and Taku Izubuchi @ Lattice 2012

• Many talks @ 2nd Workshop on Muon g-2 and EDM in the LHC Era
  https://indico.in2p3.fr/conferenceOtherViews.py?view=standard&confId=6637

• Endo-san @ this workshop
Introduction

Particle having spin feels potential in the external magnetic field.

\[ V(x) = -\vec{\mu}_l \cdot \vec{B} \]

Particle’s magnetic moment \( \mu_l \propto \) its spin.

\[ \vec{\mu}_l = g_l \frac{e}{2m_l} \vec{S}_l \]

\( g_l \): Landé \( g \)-factor (= 2 for elementary fermions@tree level)

**Anomalous magnetic moment (g-2):** Deviation from 2

\[ a_l = \frac{g_l - 2}{2} \]
Introduction

Form factor: $\Gamma_\mu(q) = \gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_l} F_2(q^2)$

At tree level, $F_1(q^2) = 1, \quad F_2(q^2) = 0$

After quantum correction $\Rightarrow a_l = F_2(0)$
Experimental status of \((g-2)\), and \((g-2)\)_\(\mu\)

\[
\begin{align*}
  a_e^{\text{EXP}} &= (11 \, 596 \, 521.807 \, 6 \pm 0.0027) \times 10^{-10} \\
  a_\mu^{\text{EXP}} &= (11 \, 659 \, 208.9 \pm 6.3) \times 10^{-10}
\end{align*}
\]

[PDG]

Theoretical calcs. are important because

\(a_e\) tests validity of QED or (perturbative) field theory
  Determining \(\alpha_{QED}\) (important in EW precision test)

\(a_\mu\) tests the SM or constraints BSM
  Much more sensitive to heavy dof than \(a_e\) by \((m_\mu/m_e)^2\)
Current status of \((g-2)_\mu\)


\[ a_{\mu}^{\text{SM}} = (11 \, 659 \, 182.8 \pm 4.9) \times 10^{-10} \text{ (using [1])} \]

\[ a_{\mu}^{\text{EXP}} = (11 \, 659 \, 208.9 \pm 6.3) \times 10^{-10} \text{ [PDG]} \]

\[ a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}} = (26.1 \pm 8.0) \times 10^{-10} \]

3.3\(\sigma\) discrepancy!

The SM prediction stable.

[1] \(a_{\mu}(Hlbl)=10.5(2.6)\) is used.

Based on model estimates.

J. Prades, E. de Rafael and A. Vainshtein, arXiv:0901.0306
5-loop calc. in QED part completed!

T. Aoyama, M. Hayakawa, T. Kinoshita, M. Nio,

\[ a_e^\text{EXP} - a_e^\text{THEROY} = -1.09(83) \times 10^{-12} \]

\[ \alpha^{-1}_\text{QED} = 137.035\,999\,166\,(34) \, [0.25 \text{ ppb}] \]

\[ a_\mu(\text{QED part}) = 11\,658\,471.885\,3\,(9)(19)(7)(29) \times 10^{-10} \]
[using \( \alpha^{-1}_\text{QED} \) above]

\[ a_\mu^\text{SM} = 11\,659\,184.0\,(5.9) \times 10^{-10} \]
[2.9 \sigma between this and EXP. \( a_\mu(\text{Hlbl})=11.6(4.0) \) is used.]
Classification of diagrams

\[
\begin{align*}
\text{QED} & \quad \text{Hadronic vacuum polarization (HVP)} \\
\text{Hadronic light-by-light (Hlbl)} & \quad \text{Electroweak (EW)}
\end{align*}
\]
Breakdown


\[ a_{\mu}^{\text{SM}} = (11.659 \pm 4.9) \times 10^{-10} \]

\[ a_{\mu}^{\text{QED}} = (11.658 \pm 0.015) \times 10^{-10} \]

\[ a_{\mu}^{\text{EW}} = (15.4 \pm 0.2) \times 10^{-10} \]

\[ a_{\mu}^{\text{had}, \text{LOVP}} = (-9.84 \pm 0.07) \times 10^{-10} \]

\[ a_{\mu}^{\text{had}, \text{HOVP}} = (10.5 \pm 2.6) \times 10^{-10} \]

\[ a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}} = (26.1 \pm 8.0) \times 10^{-10} \]

- Discrepancy between EXP and SM is larger than EW!
Breakdown


\[
\begin{align*}
a_{\mu}^{\text{SM}} &= (11 \ 659 \ 182.8 \ \pm 4.9) \times 10^{-10} \\
a_{\mu}^{\text{QED}} &= (11 \ 658 \ 471.808 \ \pm 0.015) \times 10^{-10} \\
a_{\mu}^{\text{EW}} &= (15.4 \ \pm 0.2) \times 10^{-10} \\
a_{\mu}^{\text{had},\text{LOVP}} &= (694.91 \ \pm 4.27) \times 10^{-10} \\
a_{\mu}^{\text{had},\text{HOVP}} &= (-9.84 \ \pm 0.07) \times 10^{-10} \\
a_{\mu}^{\text{had},\text{lbl}} &= (10.5 \ \pm 2.6) \times 10^{-10}
\end{align*}
\]

\[a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}} = (26.1 \pm 8.0) \times 10^{-10}\]

- Discrepancy between EXP and SM is larger than EW!
- Currently the dominant uncertainty comes from HVP.
Breakdown


\[
a_{\mu}^{SM} = (11.659, 182.8, \pm 4.9) \times 10^{-10}
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- Discrepancy between EXP and SM is larger than EW!
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- Theoretical estimate of Hlbl is really under control?
Breakdown


\[ a^\text{SM}_\mu = \left( 11 \ 659 \ 182.8 \ \pm 4.9 \right) \times 10^{-10} \]

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\[ a^\text{EXP}_\mu - a^\text{SM}_\mu = \left( 26.1 \ \pm 8.0 \right) \times 10^{-10} \]

- Discrepancy between EXP and SM is larger than EW!
- Currently the dominant uncertainty comes from HVP.
- Theoretical estimate of HLBL is really under control?
- LQCD ➔ the first principles’ estimate for the hadronic parts.
Contents

1. Introduction

2. Leading order hadronic contribution (HVP)

3. Hadronic light-by-light contribution (Hlbl)

+ ...

4. Summary
Leading order hadronic contribution (HVP)
Hadronic Vacuum Polarization (HVP)

- Current best estimate using dispersion relation and \( \sigma_{\text{total}}(e^+e^-) \)

\[
a_\mu^{\text{HVP}} = \frac{1}{4\pi^2} \int_{4m^2_\pi}^\infty ds K(s) \sigma_{\text{total}}(s)
\]

\( K(s) : \) known function

\[
a_{\mu}^{\text{had,LOVP}} = \left( \begin{array}{cc} 694.91 & \pm 4.27 \end{array} \right) \times 10^{-10}
\]

\[
a_{\mu}^{\text{had,HVVP}} = \left( \begin{array}{cc} -9.84 & \pm 0.07 \end{array} \right) \times 10^{-10}
\]

Current uncertainty \( \sim 0.6 \% \)!

\( \sim 0.3 \% \) in 3-5 years?

depending on upcoming e+e- EXP and existing Belle data.
\[ \Pi_{\mu\nu}(Q) = i \int d^4x \, e^{iQ \cdot x} \langle 0 | T[j_\mu(x)j_\nu(0)] | 0 \rangle | 0 \rangle \\
= (Q_\mu Q_\nu - Q^2 g_{\mu\nu}) \Pi(Q^2) \]

\[ a_\mu^\text{HVP} = \left( \frac{\alpha}{\pi} \right)^2 \int_0^\infty dQ^2 f(Q^2) \Pi(Q^2) \]

- \( f(Q^2) \) is known and singular toward \( Q^2 \to 0 \).
- The integral is dominated by small \( Q^2 \) region.
HVP on the lattice [pioneering work]

T. Blum, PRL91(2003)052001

\[ \Pi_{\mu\nu}(Q) = i \int d^4x \, e^{iQ \cdot x} \langle 0 | T[j_\mu(x)j_\nu(0)] | 0 \rangle = (Q_\mu Q_\nu - Q^2 g_{\mu\nu}) \Pi(Q^2) \]

- Quenched approximation
- \(O(a^2)\) error at large \(Q^2\). ⇒ Use PT for high \(Q^2\).
- \(L\) sets the non-zero minimum momentum, \(q^{\text{lat}} \sim 2\pi/L\).
- \(a_\mu^{\text{LOHVP}} = 460(78) \times 10^{-10}\) (@\(m_q \sim m_s\) is roughly consistent with what is expected in quench.

Feasibility was demonstrated!
HVP on the lattice [recent calc.]

ex) P. Boyle, L. Del Debbio, E. Kerrane and J. Zanotti, PRD85, 074504(2012)

\[ \Pi_{\mu\nu}(Q) = i \int d^4 x \, e^{iQ \cdot x} \langle 0 | T [j_\mu(x) j_\nu(0)] | 0 \rangle | 0 \rangle = (Q_\mu Q_\nu - Q^2 g_{\mu\nu}) \Pi(Q^2) \]

- Including the effects of 2+1 flavors of sea quarks
- \( m_\pi \sim 180 \text{ MeV} \)

\[ a_\mu^{\text{LOHVP}} = 641(31)(32) \times 10^{-10} \]
Source of uncertainties

- Quench approximation
- Disconnected diagram ($\approx O(10\%)$?)
- Finite volume effect (not seen at present accuracy)
- Discretization error (not seen at present accuracy)
- Need more data in small mom. region
- Chiral extrapolation
- Statistical error
Importance of small $q^2$ region

\[
\int_0^{Q_C^2} dQ^2 f(Q^2) \times \hat{\Pi}(Q^2) \rightarrow \int_0^1 dt f(Q^2) \times \hat{\Pi}(Q^2) \times \frac{Q^2}{t^2} \quad \text{where} \quad t = \frac{1}{1 + \log \frac{Q_C^2}{Q^2}}
\]

- In dominant region, only a few points exists, and they are inaccurate.
- More accurate data in this region are clearly favorable.

On a torus, the action must be single-valued, while fields do not have to be.

- Impose the twisted boundary condition on quark fields.
  
  \[ q(x + L) = q(x)e^{i\theta} \]

  \( \theta \): arbitrary

- \( q^2 \) can be arbitrary small.
Chiral extrapolation

- One of the dominant sys. errors ~ 5 %
- Functional form unknown
- Simulations@physical $m_q$ are becoming a trend.
  ⇒ no need to extrapolation or only a small extrapolation in future

New error reduction technique, **All Mode Averaging (AMA)**, significantly reduces stat. error ($\times 1/5 \sim 1/20$)!

- Full use of translational invariance.
- Wide range of application
- Stat. error won’t limit the accuracy.
- Potentially a game changer?
Summary of recent lattice calc. of HVP

0.5 % accuracy is challenging, but we should be able to come close to it by combining

Twisted b.c. + Simulation@physical $m_q$ + AMA + …
and Supercomputer.

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<th>errors</th>
<th>action</th>
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<td>Wilson</td>
<td>Mainz (2011)</td>
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presented by T. Blum@Lattice 2012
Hadronic light-by-light contribution
Hadronic light-by-light

\[
\Gamma^{(Hlbl)}_{\mu}(p_2, p_1) = \left(\begin{array}{l}
\end{array}\right)
\]

\[
= \frac{i}{\varepsilon} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{\Pi^{(4)}_{\mu\nu\rho\sigma}(q, k_1, k_3, k_2)}{k_1^2 k_2^2 k_3^2}
\times \gamma_\nu S^{(\mu)}(p_2 + k_2) \gamma_\rho S^{(\mu)}(p_1 + k_1) \gamma_\sigma
\]

\[
\Pi^{(4)}_{\mu\nu\rho\sigma}(q, k_1, k_3, k_2) = \int d^4 x_1 d^4 x_2 d^4 x_3 \exp[-i(k_1 \cdot x_1 + k_2 \cdot x_2 + k_3 \cdot x_3)]
\times \langle 0 | T[j_\mu(0) j_\nu(x_1) j_\rho(x_2) j_\sigma(x_3)] | 0 \rangle
\]

Form factor : \[
\Gamma_\mu(q) = \gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2 m_l} F_2(q^2)
\]

In contrast to the VP case, no experimental input is available.
Two known results, theory does not work in this case. To see that this is indeed what
is suppressed:
π model dependent than the previous chiral perturbation theory adopted a dynamical framework which takes into account explicit
Model dependent estimates of the single logarithmic term as
coefficient of the $\ln(\rho/m)$ with an additional assumption about dominance of one of the factors. Exchanges with
momenta but one can keep the ratio

$$\frac{N_{\rho}}{N_{\pi}}$$

Note that the overall sign of the pion exchange, for physical
Although the coefficient of the $\ln(\rho/m)$ under similar assumptions, but they are much smaller.

Figure 2: Next–to–leading terms in the large momentum limit . The mass of the

$$a_{\text{HLbl}} \approx \frac{\gamma_{\text{HLbl}}}{N_{\rho}}$$

means that the bulk of the contribution

$$\frac{N_{\rho}}{N_{\pi}}$$

can be contrasted with the one of the coefficient of the $\ln(\rho/m)$. The rather small value of

$$\frac{N_{\rho}}{N_{\pi}}$$

can be interpreted as a rather small value of the $\rho$ mass. Quark loop with $\sim 300$ MeV constituent mass

- More or less justified by large $N_c$ argument.
- All existing results fall into

$$a_{\text{HLbl}} = (11 \pm 4) \times 10^{-10}$$

- Current best estimate based on models:

$$a_{\text{HLbl}} = (10.5 \pm 2.6) \times 10^{-10}$$

Uncertainty is uncertain.
Lattice calculation favorable!
Conventional approach on the lattice

Calculate 4-point function

\[ \Pi_{\mu\nu\rho\sigma}^{(4)}(q, k_1, k_3, k_2) = \int d^4x_1 d^4x_2 d^4x_3 \exp[-i(k_1 \cdot x_1 + k_2 \cdot x_2 + k_3 \cdot x_3)] \times \langle 0|T[j_\mu(0)j_\nu(x_1)j_\rho(x_2)j_\sigma(x_3)]|0\rangle \]

Then, one will obtain a single set of \((q, k_1, k_3, k_2)\).

Calc. requires integration over \(k_1\) and \(k_2\),

\[ \Gamma_{\mu}^{(Hlbl)}(p_2, p_1) = i e^6 \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{\Pi_{\mu\nu\rho\sigma}^{(4)}(q, k_1, k_3, k_2)}{k_1^2 k_2^2 k_3^2} \times \gamma_\nu S^{(\mu)}(p_2 + k_2) \gamma_\rho S^{(\mu)}(p_1 + k_1) \gamma_\sigma \]

Need to repeat \((Volume)^2\) times \(\sim 10^{10-11}\) times.

With this approach, the calculation won’t end...
Let lattice calculate the form factor itself

\[ \Gamma_{\mu}^{(H\bar{l}l)}(p_2, p_1) = i e^6 \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{\Pi_{\mu\nu\rho\sigma}^{(4)}(q, k_1, k_3, k_2)}{k_1^2 k_2^2 k_3^2} \times \gamma_\nu S^{(\mu)}(p_2 + k_2) \gamma_\rho S^{(\mu)}(p_1 + k_1) \gamma_\sigma \]
Alternative approach

M. Hayakawa, T. Blum, T. Izubuchi and NY, hep-lat/0509016

Let lattice calculate the form factor itself

\[ \Gamma_{\mu}^{(\text{Hlbl})}(p_2, p_1) = i e^6 \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{\Pi_{\mu\nu\rho\sigma}^{(4)}(q, k_1, k_3, k_2)}{k_1^2 k_2^2 k_3^2} \times \gamma_\nu S^{(\mu)}(p_2 + k_2) \gamma_\rho S^{(\mu)}(p_1 + k_1) \gamma_\sigma \]

- Standard method in other form factor calculations
- Need to incorporate QED on the lattice.
Motivation (other than light-by-light)
- Ordinary lattice calculations are done in the iso-spin symmetric limit. ($m_u=m_d$ and no EM interaction).
- Lattice calc. are being precise, and iso-spin breaking effects start to be visible.
- In order to determine the most poorly known quark mass, $m_u$ and $m_d$, QED must be taken into account!
- One should be able to exclude $m_u=0$ and to reproduce

\[
\begin{align*}
    m_N - m_P &= 1.2933321(4)\text{MeV} \\
    m_{\pi^\pm} - m_{\pi^0} &= 4.5936(5)\text{MeV} , \\
    m_{K^\pm} - m_{K^0} &= -3.937(28)\text{MeV} ,
\end{align*}
\]
QCD+QED lattice simulation

- Quenched approximation
  Duncan, Eichten, Thacker PRL76(1996) 3894;

- Two-flavor QCD

- Three-flavor QCD

- ...

- Three-flavor QCD with charged sea quarks
**$m_u$ and $m_d$**

**PDG2012**

Values above of weighted average, error, and scale factor are based upon the data in this ideogram only. They are not necessarily the same as our 'best' values, obtained from a least-squares constrained fit utilizing measurements of other (related) quantities as additional information.

**$m_u$**

**$m_d$**

**VALUES**

$\chi^2$

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$\chi^2 = 26.3$ (Confidence Level = 0.0002)

**VALUES**

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$\chi^2 = 4.1$ (Confidence Level = 0.665)

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**Citation:** K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010) and 2011 partial update for the 2012 edition (URL: http://pdg.lbl.gov)
Calculate

This includes unwanted diagrams, while what we want is lbl only. Easier way to get rid of unwanted one?
One of photon propagators, whose analytical expression is known, is attached by hand. This still includes unwanted diagrams.
Alternative approach

M. Hayakawa, T. Blum, T. Izubuchi and NY, hep-lat/0509016

How to subtract unwanted one?
Subtract the similar expectation value, but there the configuration average of the quark part and the muon part are taken separately.

⇒ No photon connecting quark and muon in the 2nd term.
⇒ Only lbl (and higher order terms) survives
Numerical tests

• Tests with only QED
  - T. Blum and S. Chowdhury, NP(PS)189, 251 (2009)
  Consistent with PT if the spatial volume is as large as $V=24^3$.

• Tests with (2+1)f QCD+QED
  - $V=16^3 \times 32$
  - $m_\pi \approx 420\text{MeV}$, $m_\mu \approx 190, 692 \text{MeV}$ ($m_\mu^{\text{phys}} \sim 105\text{MeV}$)
  - Hlbl amplitude behaves as $\sim \epsilon^4$, while un-subtracted amplitude stays the same.
Numerical tests

- Tests with (2+1)f QCD+QED (preliminary)
  - $V=24^3\times48$ ($\sim 2.7$ fm)
  - $m_\pi \approx 329$ MeV, $m_\mu \approx 190$ MeV
  - Two lowest values of $Q^2$ (0.11 and 0.18 GeV$^2$)
  - All Mode Averaging (AMA)

- $F_2(0.18$ GeV$^2$) = $(0.142 \pm 0.067) \times \left(\frac{\alpha}{\pi}\right)^3$
- $F_2(0.11$ GeV$^2$) = $(0.038 \pm 0.095) \times \left(\frac{\alpha}{\pi}\right)^3$
- $a_\mu$(HLbL/model) = $(0.084 \pm 0.020) \times \left(\frac{\alpha}{\pi}\right)^3$

Signal may be emerging in the model ballpark.
O(100 %) stat. error is encouraging!
Sources of systematic uncertainties

- The hadronic vacuum polarization (HVP) contribution ($O(\alpha^2)$)
- The hadronic light-by-light (HLbL) contribution ($O(\alpha^3)$)

Implications for new physics

Summary/Outlook

HLbL systematic error

- "Disconnected" diagrams (quark loops connected by gluons) not calculated yet (not suppressed).

Several possibilities,

1. Use multiple valence quark loops (qQED)
2. Re-weight in $(T. Ishikawa)$ or dynamical QED in HMC
3. "A source" (see Izubuchi's talk) (no subtraction)

Tom Blum (UConn and RIKEN BNL Research Center)

The muon anomalous magnetic moment
Sources of systematic uncertainties

Many improvements remain to be done.

• Disconnected diagrams
  Not easy. But several promising methods almost ready to test.
• \( q^2 \rightarrow 0 \) [Twisted b.c. applicable?]
• \( m_q \rightarrow m_{q,\text{phys}}, m_\mu \rightarrow m_{\mu,\text{phys}} \)
• Finite volume. Excited states/“around the world” effects
• \( a \rightarrow 0 \)
• QED renormalization
• …

Personally, even 50% uncertainty is sensible, and such an accuracy will be possible in 5 years. (10% error may not be too optimistic.)
Summary
Prospect: Experiment on $(g-2)_\mu$

\[
\alpha_\mu^{\text{EXP}} = (11 659 208.9 \pm 6.3) \times 10^{-10}
\]

Two independent measurements@J-PARC and FNAL are planned.

  
  Exp uncertainty reduced by factor 4 in 5 years (by ~2017).
  
  \[6.3 \times 10^{-10} \Rightarrow 1.4 \times 10^{-10}\]

- FNAL: similar
### Prospect: Theory

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</thead>
<tbody>
<tr>
<td>$a_{\mu}^{\text{EW}}$</td>
<td>$(15.4 \pm 0.2) \times 10^{-10}$</td>
</tr>
<tr>
<td>$a_{\mu}^{\text{had,LOVP}}$</td>
<td>$(694.91 \pm 4.27) \times 10^{-10}$</td>
</tr>
<tr>
<td>$a_{\mu}^{\text{had,HQVP}}$</td>
<td>$(-9.84 \pm 0.07) \times 10^{-10}$</td>
</tr>
<tr>
<td>$a_{\mu}^{\text{had,lbl}}$</td>
<td>$(10.5? \pm 2.6?) \times 10^{-10}$</td>
</tr>
</tbody>
</table>

Assuming that improvements occur without changing the central values and model estimate of Hlbl is confirmed by lattice calc. at 40 % level.
Prospect: Theory

\[
\begin{align*}
\alpha_{\mu}^{\text{EXP}} &= (11 \, 659 \, 208.9 \pm 1.4) \times 10^{-10} \\
\alpha_{\mu}^{\text{SM}} &= (11 \, 659 \, 182.8 \pm 4.9) \times 10^{-10} \\
\alpha_{\mu}^{\text{QED}} &= (11 \, 658 \, 471.885 \pm 0.004) \times 10^{-10} \\
\alpha_{\mu}^{\text{EW}} &= (15.4 \pm 0.2) \times 10^{-10} \\
\alpha_{\mu}^{\text{had,LOVP}} &= (694.91 \pm 2.1) \times 10^{-10} \\
\alpha_{\mu}^{\text{had,H0VP}} &= (-9.84 \pm 0.07) \times 10^{-10} \\
\alpha_{\mu}^{\text{had,lbl}} &= (10.5 \pm 2.6) \times 10^{-10}
\end{align*}
\]

Assuming that improvements occur without changing the central values and model estimate of Hlbl is confirmed by lattice calc. at 40 % level.
Prospect: Theory

\[
\begin{array}{c}
a_\mu^{\text{EXP}} = (11\ 659\ 208.9\ \pm\ 1.4) \times 10^{-10} \\
a_\mu^{\text{SM}} = (11\ 659\ 182.8\ \pm\ 4.9) \times 10^{-10} \\
\end{array}
\]

\[
\begin{array}{c}
a_\mu^{\text{QED}} = (11\ 658\ 471.885\ \pm\ 0.004) \times 10^{-10} \\
a_\mu^{\text{EW}} = (15.4\ \pm 0.2) \times 10^{-10} \\
a_\mu^{\text{had,LOVP}} = (694.91\ \pm\ 2.1) \times 10^{-10} \\
a_\mu^{\text{had,HOVP}} = (-9.84\ \pm\ 0.07) \times 10^{-10} \\
a_\mu^{\text{had,lbl}} = (10.5\ \pm\ 4.2) \times 10^{-10} \\
\end{array}
\]

Assuming that improvements occur without changing the central values and model estimate of Hlbl is confirmed by lattice calc. at 40 % level.
### Prospect: Theory

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{\mu}^{\text{EXP}}$</td>
<td>$(11,659,208.9 \pm 1.4) \times 10^{-10}$</td>
</tr>
<tr>
<td>$a_{\mu}^{\text{SM}}$</td>
<td>$(11,659,182.86 \pm 4.7) \times 10^{-10}$</td>
</tr>
<tr>
<td>$a_{\mu}^{\text{QED}}$</td>
<td>$(11,658,471.885 \pm 0.004) \times 10^{-10}$</td>
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$3.3\sigma \Rightarrow 5.3\sigma$

Assuming that improvements occur without changing the central values and model estimate of Hlbl is confirmed by lattice calc. at 40% level.
Summary

• Lattice QCD can play an important role in \((g-2)_\mu\), through the determinations of HVP and Hlbl contribution.

• HVP:
  Currently O(10) % level ⇒ O(a few %) using various techniques simultaneously
  Cross check against dispersion + \(e^+e^-\) cross section.

• Hlbl:
  Numerical tests are encouraging.
  Many things to do (disconnected diagrams, physical masses, ...) \(O(100 \%) \Rightarrow 40-50 \% \) seems doable.

• After all, clear evidence for BSM might emerge.
Acknowledgement

• US DOE, RIKEN BNL Research Center, USQCD Collaboration
• Machines: QCDOC at BNL, Ds cluster at FNAL, q-series clusters at JLAB