Phase locked inflation
~ Effectively trans-Planckian natural inflation~

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Harigaya and Ibe, arXiv: 1404.3511, 1407.4893

Today's Plan

- Inflation by a scalar field with flat potential
- Natural inflation and its demand
- Phase locking

Inflation by scalar with flat potential

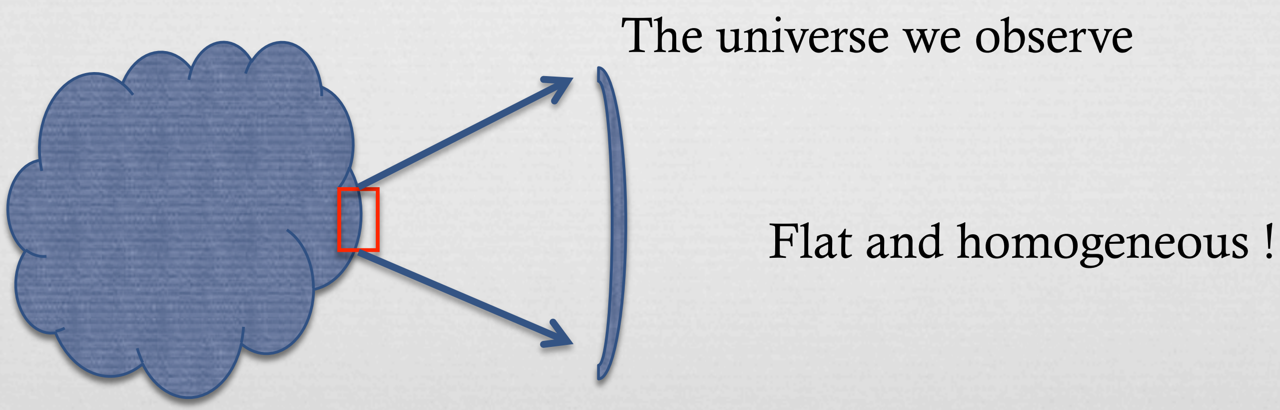
Inflaton should have a flat potential

Cosmic inflation

Guth (1981), Sato (1981), Starobinsky (1980)

Quasi- exponential expansion of the universe at the very early universe

- Solve the Horizon problem
- Solve the Flatness problem
- Generate the cosmic perturbation



Cosmic inflation

Guth (1981), Sato (1981), Starobinsky (1980)

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- Solve the Horizon problem
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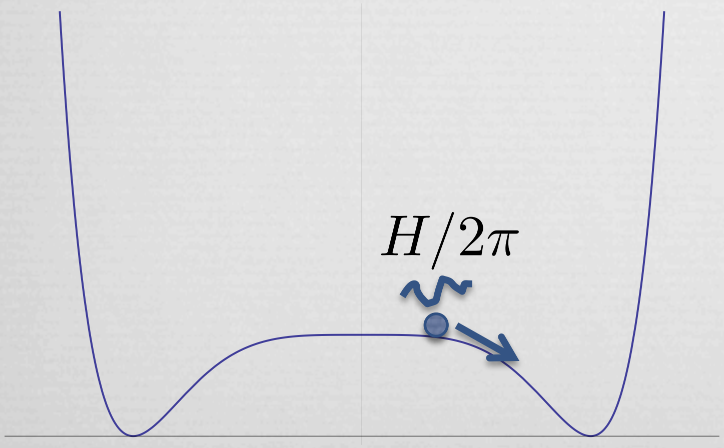
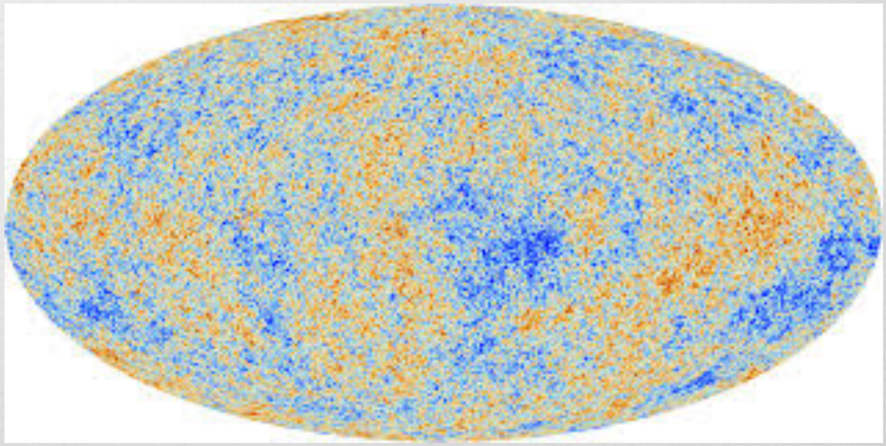
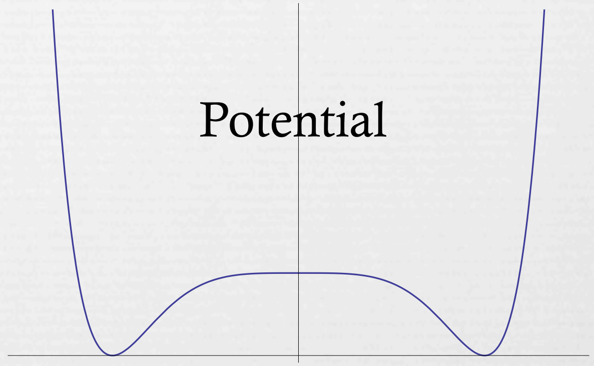
Mukhanov and Chibisov (1981)





Condition for Inflaton

Potential energy >> kinetic energy



Let us check the consistency

$$\cancel{\ddot{\phi}} + 3H\dot{\phi} + V_\phi(\phi) = 0$$

$$3H^2 M_{\rm pl}^2 \simeq V(\phi)$$

$$\Rightarrow \quad \dot{\phi}^2 \simeq M_{\rm pl}^2 \frac{V_\phi^2}{3V} \ll V$$

Time derivative

$$\epsilon \equiv \frac{1}{2} M_{\rm pl}^2 (V_\phi / V)^2 \ll 1$$

$$\eta \equiv M_{\rm pl}^2 V_{\phi\phi} / V \ll 1$$

Slow-roll conditions

Flat potential

$$\epsilon \equiv \frac{1}{2} M_{\rm pl}^2 \left(V_\phi / V \right)^2 \ll 1$$

$$\eta \equiv M_{\rm pl}^2 V_{\phi\phi} / V \ll 1$$

Slow-roll conditions

What is the origin of the almost flat potential ?
How is the potential controlled ?
Is quantum effect negligible ?

Natural inflation and its demand

Freese, Frieman and Olinto (1990)

Inflaton is a NGB

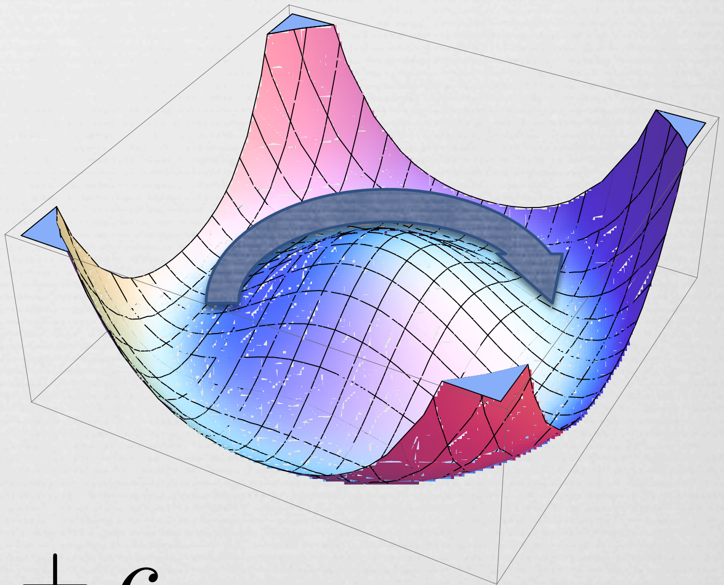
Decay constant must be larger than the Planck scale

More detail : See Takahashi-san's talk

NGB

Assume spontaneous breaking of a global symmetry

Associated flat direction :
Nambu-Goldstone Boson ϕ

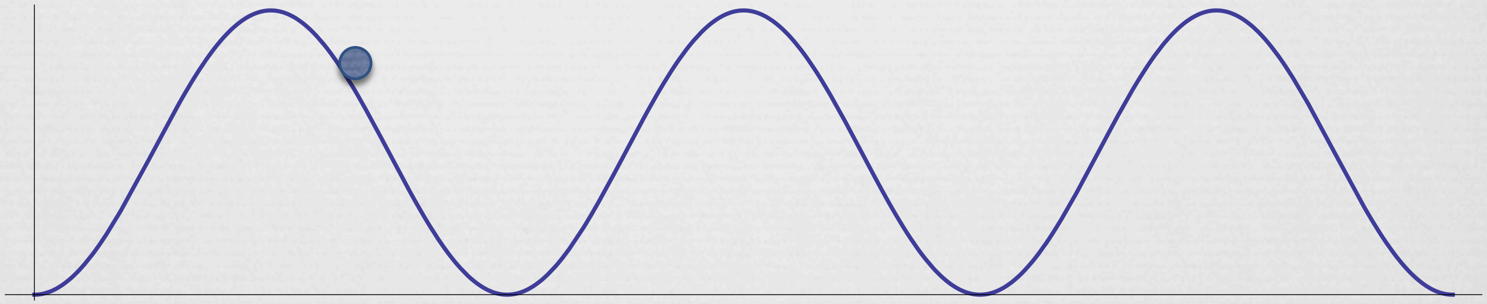


Shift symmetry $\phi \to \phi + c$

Pseudo-NGB

Assume explicit breaking of the global symmetry
to a discrete one by a small amount

$$V(\phi) = \Lambda^4(1 - \cos(\phi/F))$$



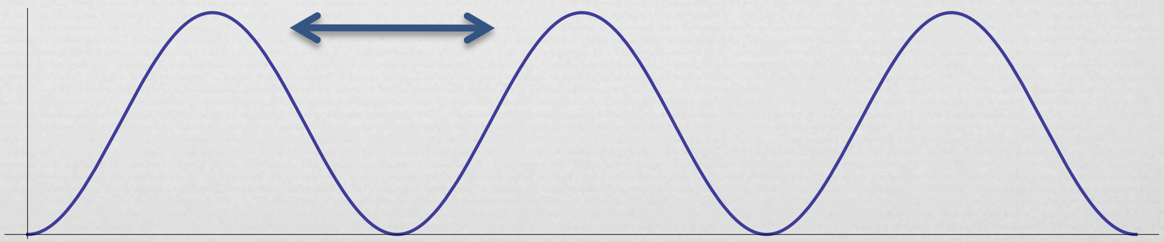
Flatness is controlled by symmetry and smallness of its breaking
Flatness is natural in 't Hooft's sense

Slow-roll conditions

$$\epsilon = \frac{1}{2M_{pl}^2}\left(\frac{V'}{V}\right)^2 = \frac{M_{pl}^2}{2F^2}\frac{\sin^2(\phi/F)}{(1-\cos(\phi/F))^2},$$

$$\eta = \frac{M_{pl}^2 V''}{V} = \frac{M_{pl}^2}{F^2}\frac{\cos(\phi/F)}{(1-\cos(\phi/F))}$$

$F \gg M_{\rm pl}$ is required



Toy model

Assume a global U(1) symmetry

$$V = V(XX^*)$$

$$X \to |<X>| \exp(i\phi/F)$$

$$F = \sqrt{2}|<X>|$$

$$|<X>| \gg M_{\rm pl}$$

Expicit breaking

$$\Delta V = M^3 X + \text{h.c.}$$

$$V(\phi) = \Lambda^4(1 - \cos(\phi/F))$$

Higher dimensional term

Gravity → Theory with Planck scale suppressed interactions

Without any reason, we expect

$$\Delta V = M^3 X(1 + c_1 \frac{XX^*}{M_{\rm Pl}^2} + c_2 \frac{(XX^*)^2}{M_{\rm Pl}^4} \cdots)$$

$$c_n = O(1)$$

Higher dimensional term

$$\Delta V = M^3 X (1 + c_1 \frac{XX^*}{M_{\rm Pl}^2} + c_2 \frac{(XX^*)^2}{M_{\rm Pl}^4} \cdots)$$

But $c_n \ll 1$ is required to suppress the potential

4-dim QFT with an approximate U(1) symmetry seems not enough.
We need to understand how higher dimensional terms are controlled.
i.e. knowledge about beyond the Planck scale is necessary.

Phase locking

Large decay constant from small energy scale

Hierarchical U(1) charge

Harigaya and Ibe (2014)
arXiv: 1404.3511

	ϕ	S
U(1)	N	1

Assume SSB of the U(1)

$$\phi \to v_\phi \exp\left(iNa/F\right)$$

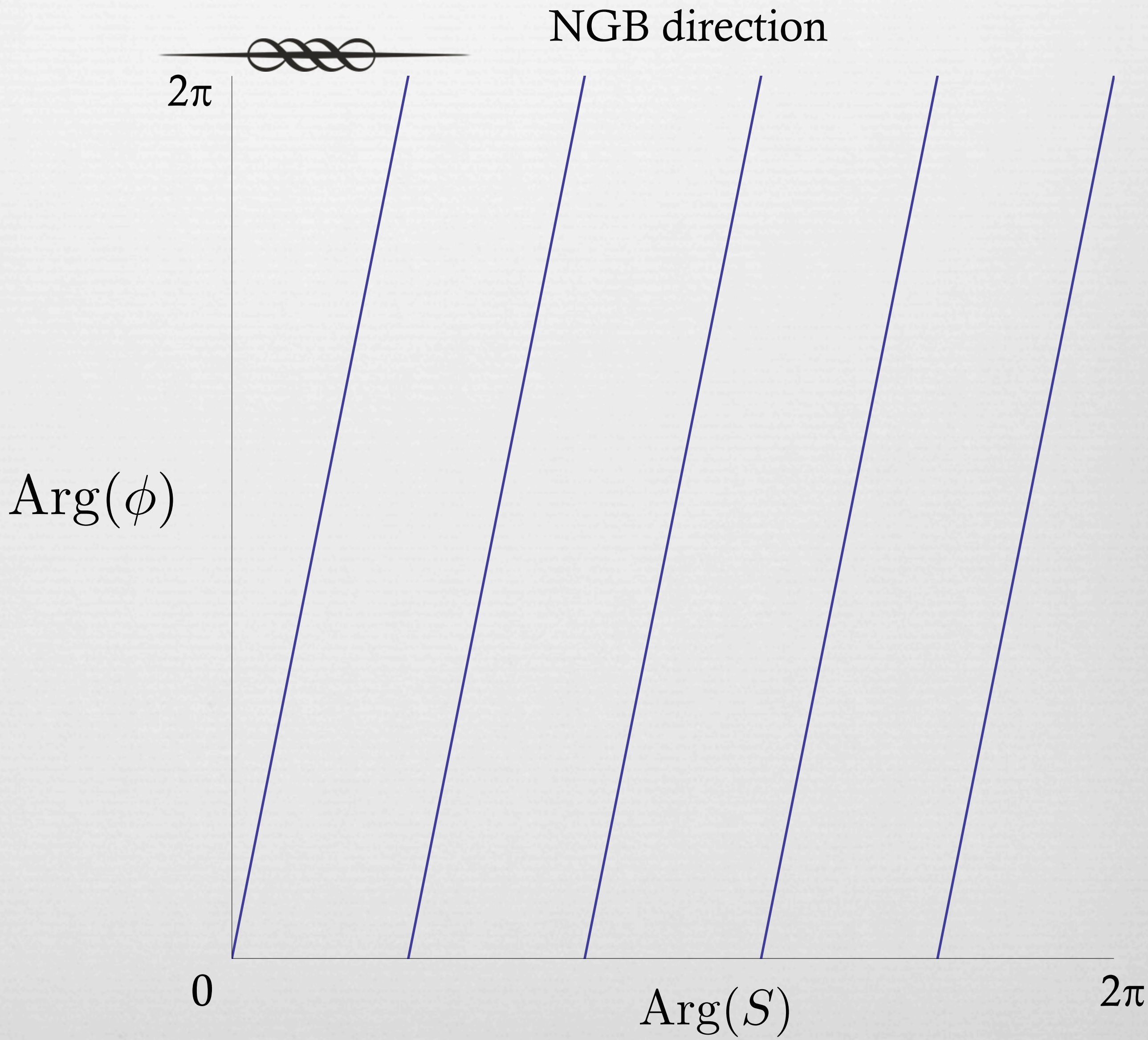
$$S \to v_S \exp\left(ia/F\right)$$

$$F = \sqrt{2(N^2 v_\phi^2 + v_S^2)}$$

a: NGB field

The decay constant is enhanced by N

Phase Locking



Breaking U(1)

Just by hand softly

$$V_{\text{breaking}} = v^3 S + \text{h.c.}$$

By anomaly

$$\mathcal{L} = ySQ\bar{Q}$$

Q : hidden matter in hidden QCD-like theory

Breaking U(1)

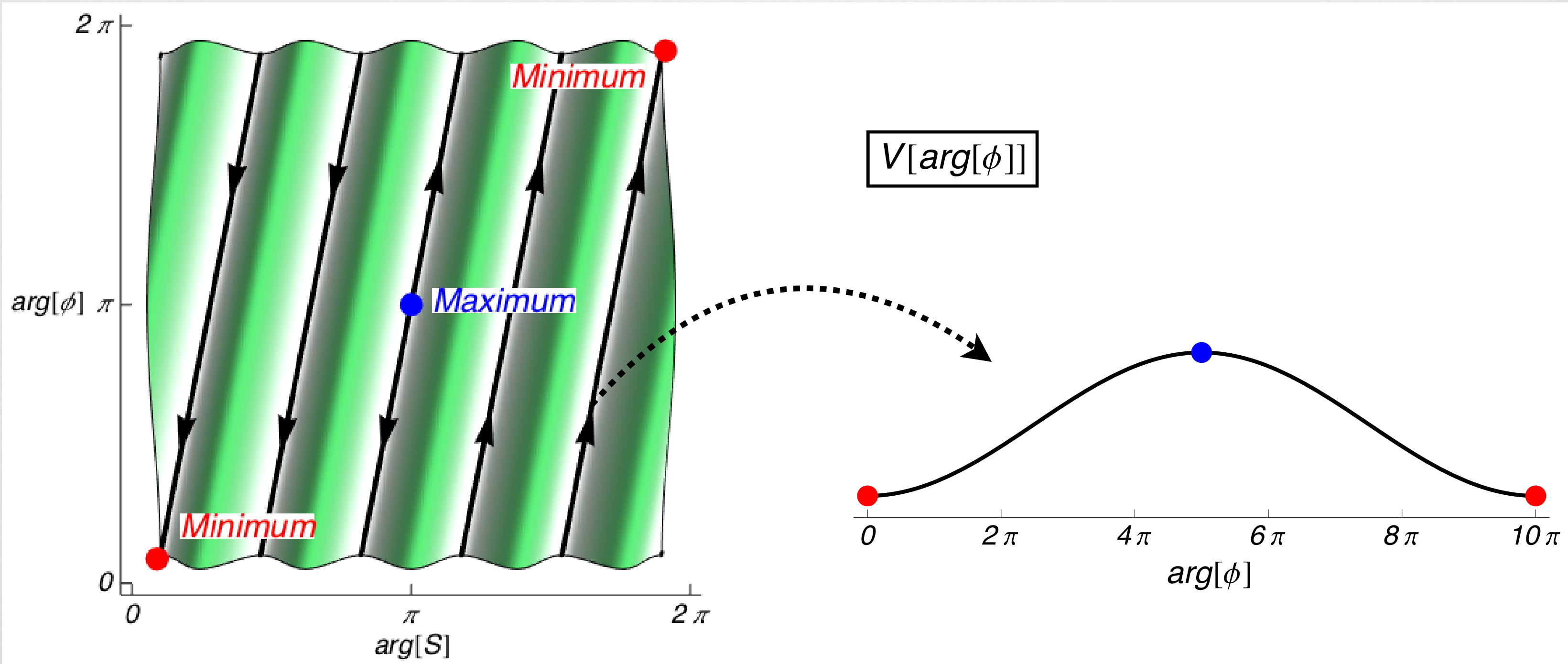
$$\phi \to v_\phi \exp\left(iNa/F\right)$$

$$S \to v_S \exp\left(ia/F\right)$$

$$F = \sqrt{2(N^2 v_\phi^2 + v_S^2)}$$

$$V_{\text{breaking}} = v^3 S + \text{h.c.}$$

$$V(a) = \Lambda^4 (1 - \cos(a/F))$$

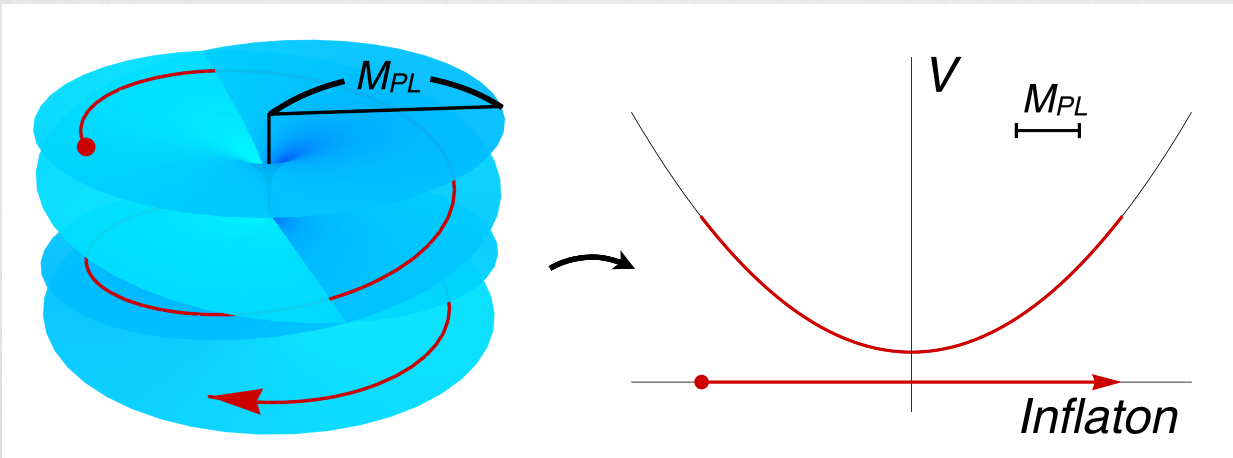


Monodromy inflation

Silverstein and Westpal (2008)
in string theory

	ϕ	S
U(1)	N	1

$$V \sim S \sim \phi^{1/N}$$



Summary and discussion

- Inflation requires flat potential
- In natural inflation, inflaton is identified with a pseudo-NGB
- Large decay constant is obtained from a field theory with small energy scale by phase locking
- Supersymmetrization, initial condition problem : see the second paper

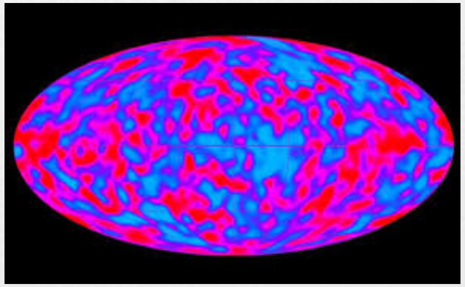
arXiv: 1404.3511, 1407.4893

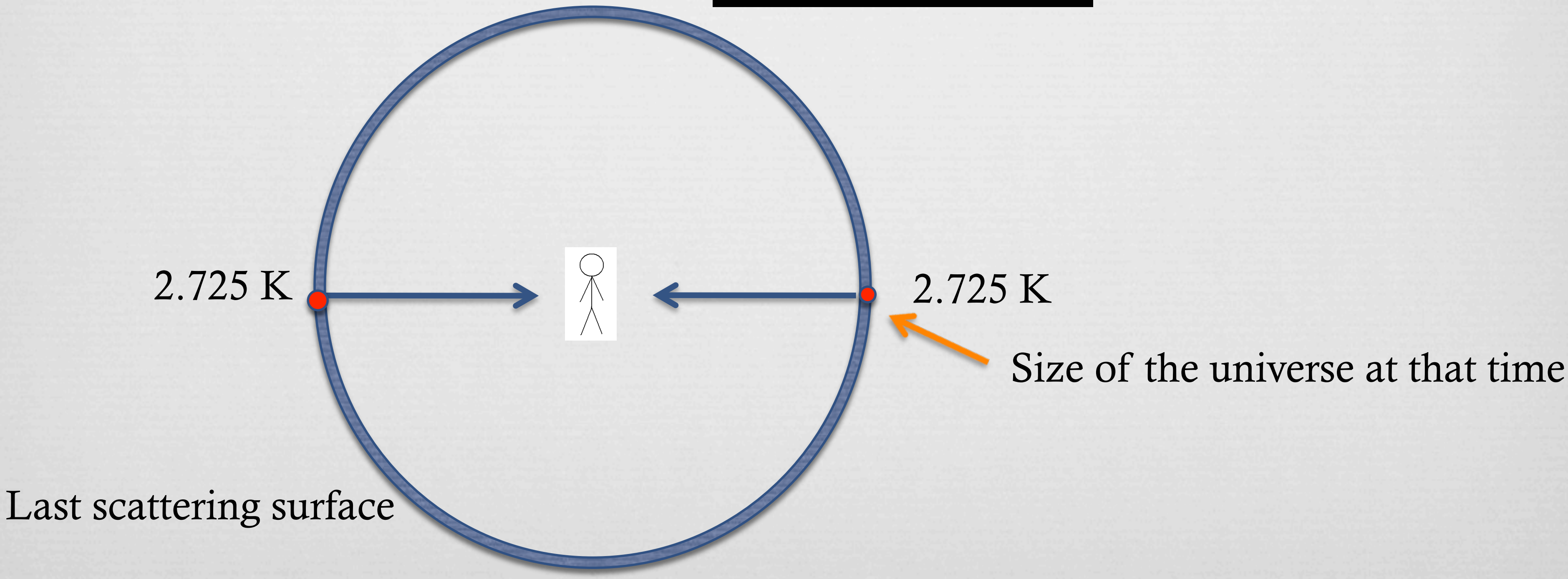
Back up

Horizon problem

Why is the universe almost homogeneous ?

Ex. Cosmic microwave background





Flatness problem

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + x^2\left(d\theta^2 + sin^2\theta d\phi^2\right)\right]$$

Observation :

$$K < 10^{-2} H_0^2 \quad (a_0 = 1)$$

$$K/a^2 \simeq -H^2 + \rho/3$$

In the early universe, $a << 1 \quad |K/a^2| \ll H^2$

The energy density must be extremely tuned

Can ordinary expansion explain the flatness or the horizon problem?

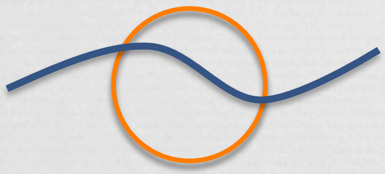
No.

Physical size $\propto a$

Hubble horizon $\propto a^2(\mathrm{RD}),$

$$a^{3/2}(\mathrm{MD}),$$

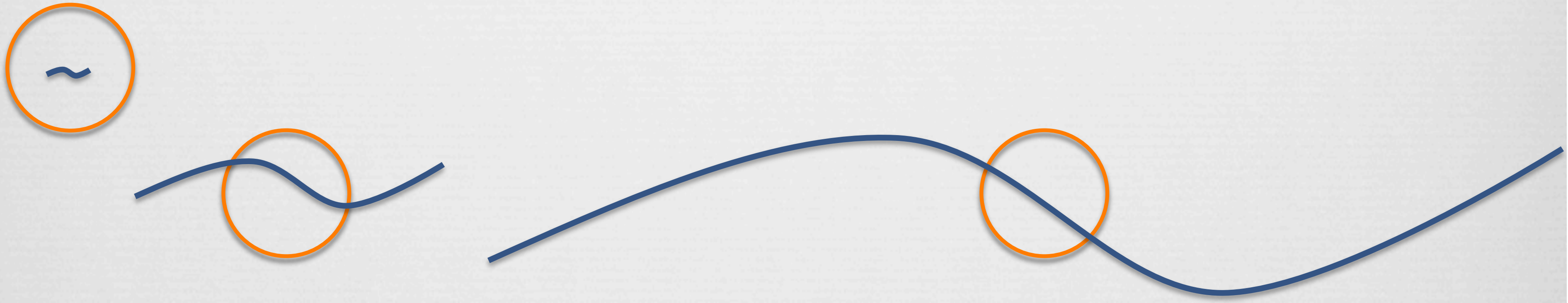
$$a(\text{negative curvature})$$



For a given scale (e.g. CMB scale),
the horizon used to be relatively smaller

Constant energy!

Hubble horizon = constant



The horizon used to be relatively larger

All the scale we observe used be within a Hubble radius

Closed universe

In FRW metric,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + x^2\left(d\theta^2 + sin^2\theta d\phi^2\right)\right]$$

K > 0

$$\int_0^{1/\sqrt{K}} \frac{dr}{\sqrt{1-Kr^2}} = \frac{\pi}{2\sqrt{K}}$$

Finite universe

Finite Life time

Open universe

In FRW metric,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + x^2\left(d\theta^2 + sin^2\theta d\phi^2\right)\right] \qquad K < 0$$

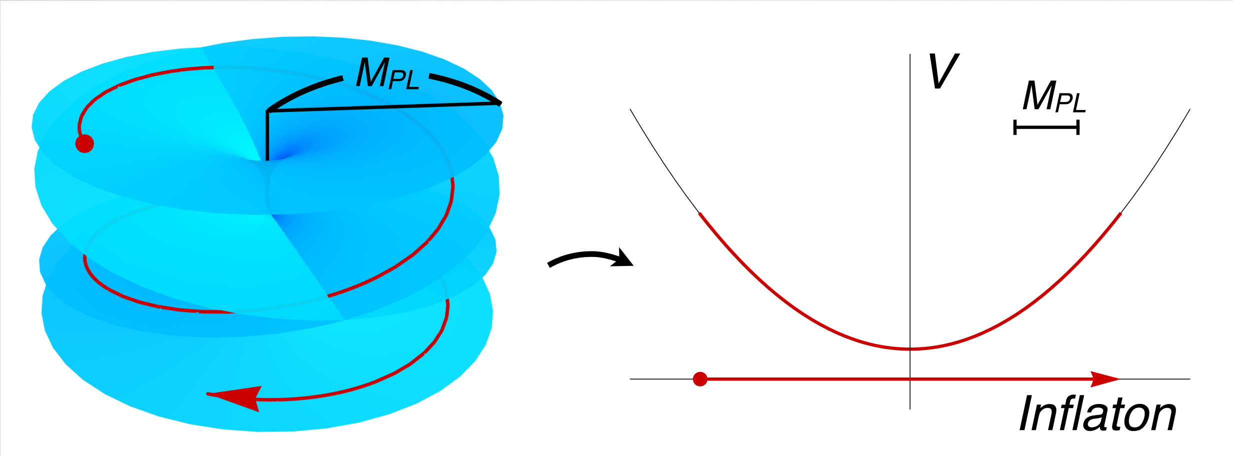
Infinite size,
infinite life time

Riemann surface

	ϕ	S
U(1)	N	1

$$S \sim \phi^{-1/N}$$

$$V_{\text{breaking}} = v^3 S \sim \phi^{-1/N}$$



Inflaton potential on a Riemann surface