EXOTIC QCD ON COMPACTIFIED SPACETIME

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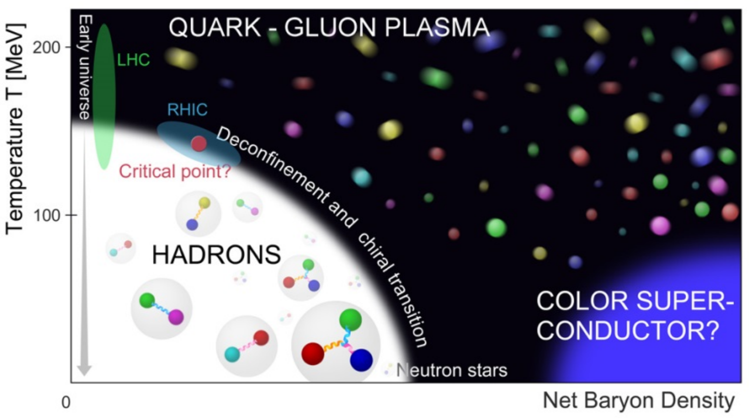
OUTLINE

1. **Motivation**
2. **Center symmetry realization at small S^1**
3. **Monopoles and semiclassical treatment**
4. **Phase diagram of adjoint QCD**
5. **Conclusion**

MOTIVATION (1)



◆**Hard problems in QCD:**

- Chiral symmetry breaking
- Quark confinement
- Nuclear force etc.

◆**Why difficult? → Strongly coupled in IR: $g(\mu)$ grows!**

◆**Similar phenomena also occur in a weakly-coupled regime**

- High-density QCD ･･･ BCS mechanism → ChSB
- SUSY QCD ･･･ Semiclassical confinement via topological objects
- Toroidal QCD (with adjoint quarks)･･･ Semiclassics *without* SUSY

They (may) provide insights into real-world QCD!

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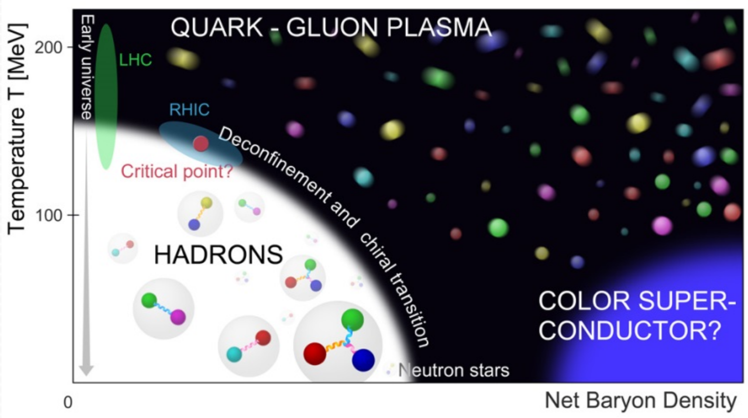

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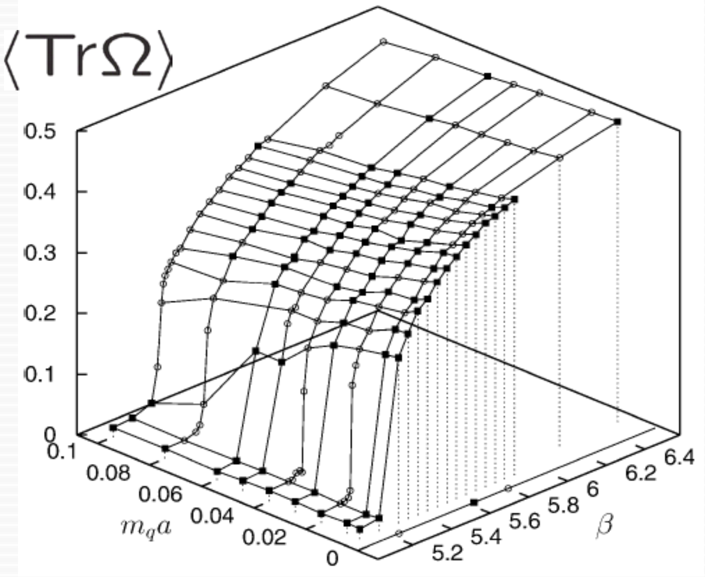
MOTIVATION (2)

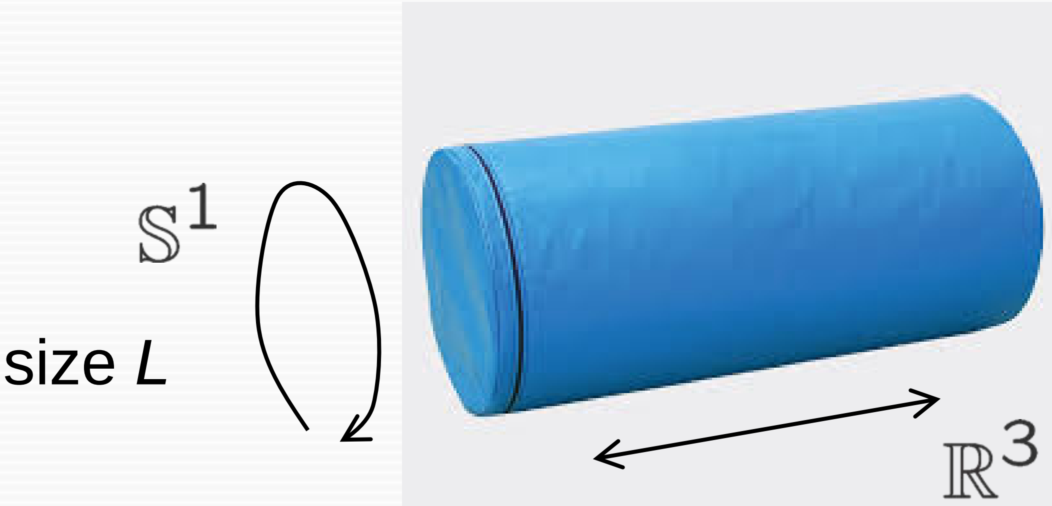
Adjoint QCD has received attention because:

- ✓ Distinct chiral and deconfinement transitions with $T_c \sim 8\,T_d$ for SU(3) — Karsch-Lutgemeier '99
- ✓ Large-N reduction possible?
  Kovtun-Unsal-Yaffe '07
- ✓ The (μ, T) phase diagram
  Kogut et al. '00, Hands et al. '01
- ✓ Conformal/walking behavior?
  Catterall '08, Hietanen '09,
  Del Debbio '09, De Grand '11, …



Engels et al. '05

QCD ON A CYLINDER (1)



- Boundary condition for bosons: periodic
- B. C. for fermions:

  anti-periodic (thermal compactification) ··· $T > 0$

  or periodic (spatial compactification) ··· $T = 0$

- Wilson line (holonomy)

$$\Omega \equiv \mathcal{P} \exp\left(i \oint dx \;\; A_4(x) \right)$$

QCD ON A CYLINDER (2)

- Pure Yang-Mills: GPY '81

  $\langle \mathrm{Tr}\,\Omega \rangle = 0$ at low *T*, $\langle \mathrm{Tr}\,\Omega \rangle \neq 0$ at high *T*
- Gauge theories w/ not too many flavors (thermal b.c.):

  $\langle \mathrm{Tr}\,\Omega \rangle \simeq 0$ at low *T*, $\langle \mathrm{Tr}\,\Omega \rangle \neq 0$ at high *T*

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Boundary condition makes a difference!

- Super Yang-Mills on $\mathbb{R}^3 \times S^1$ Davies et al. '99, '03
  - ✓ SUSY is respected by periodic b.c. of gluinos
  - ✓ $V_{eff}(\Omega) = 0$ to all orders in pert. theory
  - ✓ At small S^1, the flat direction of Ω is lifted by topological objects (monopoles) → $\langle \mathrm{Tr}\,\Omega \rangle = 0$
  - ✓ *Center symmetry is unbroken for any S^1 radius!*
  - ✓ Semiclassics lead to **mass gap** for gauge fields and **area law** of the Wilson loop

QCD ON A CYLINDER (3)

Generalization: N_f-flavor adjoint QCD
with periodic boundary condition

- $N_f = \frac{1}{2}$: SYM
- $N_f > \frac{1}{2}$: No supersymmetry.

→ The gauge symmetry breaking occurs already *at one loop* (Hosotani mechanism).

& Spontaneous ChSB occurs (in the chiral limit).

Anyway, center symmetry $(\mathbb{Z}_N)$ of SU(N) is stabilized at sufficiently small S^1.

QCD ON A CYLINDER (4)

More generally, …

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Argyres-Unsal ’12

		G	$\rightarrow$	H	
$SU(N+1)$	$\simeq$	A_N	$\rightarrow$	$U(1)^N$	for $N \geq 1$
$SO(2N+1)$	$\simeq$	B_N	$\rightarrow$	$U(1)^{N-1} \times SO(3)$	for $N = 2, 3$
			$\rightarrow$	$SO(4) \times U(1)^{N-3} \times SO(3)$	for $N \geq 4$
$Sp(2N)$	$\simeq$	C_N	$\rightarrow$	$U(1)^N$	for $N \geq 3$
$SO(2N)$	$\simeq$	D_N	$\rightarrow$	$SO(4) \times U(1)^{N-4} \times SO(4)$	for $N \geq 4$
		E_6	$\rightarrow$	$SU(3) \times SU(3) \times SU(3)$	
		E_7	$\rightarrow$	$SU(2) \times SU(4) \times SU(4)$	
		E_8	$\rightarrow$	$SU(2) \times SU(3) \times SU(6)$	
		F_4	$\rightarrow$	$SU(3) \times SU(2) \times U(1)$	
		G_2	$\rightarrow$	$SU(2) \times U(1)$	

Table 1. Perturbative patterns of Higgsing of the gauge group G to an unbroken group H for $n_f > 1$ adjoint fermions with periodic boundary conditions on $\mathbb{R}^3 \times S^1$.

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Complete abelianization

(necessary condition for semiclassics)

GOAL OF THIS WORK

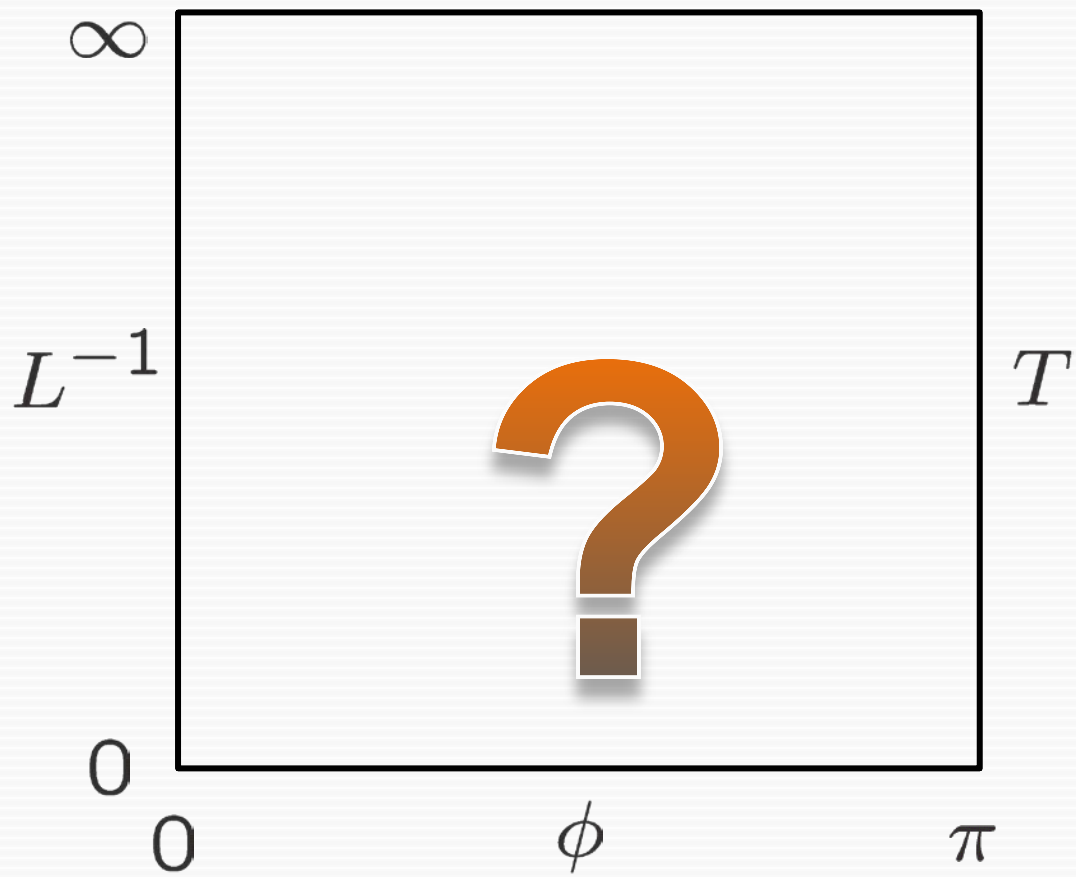
$$\psi(x_4 + L) = \mathrm{e}^{i\phi}\psi(x_4)$$

Adjoint QCD for $\phi = 0$ and $\phi = \pi$ show drastically different phase structures.

We study SU(2) and SU(3) adjoint QCD with generic $0 \leq \phi \leq \pi$ for fermions.

- Perturbative and semiclassical analysis at small S^1
- Non-perturbative model analysis at all S^1

PHASES AT SMALL S^1

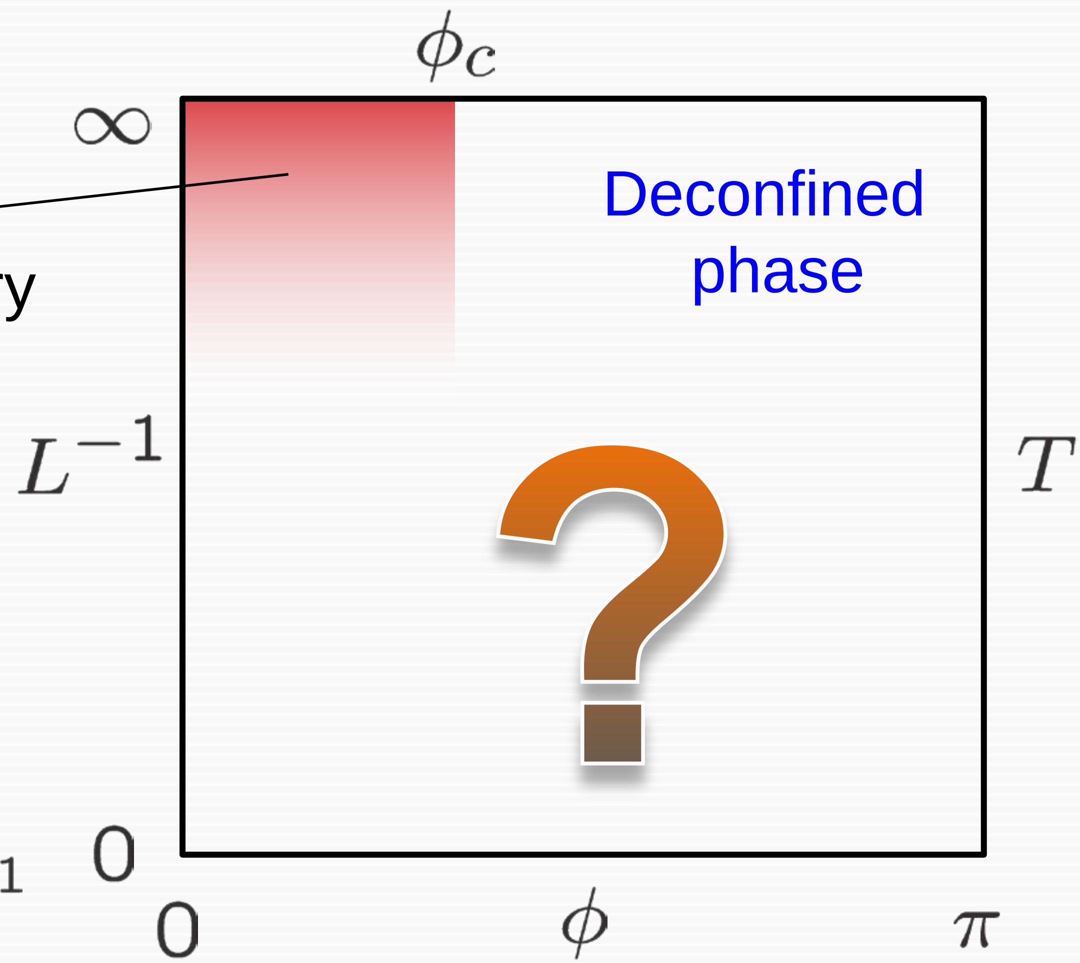


PHASES AT SMALL S^1

Perturbative GPY potential predicts a 1st-order transition

Confining phase
with gauge symmetry breaking

$$SU(2) \to U(1)$$



$\phi_c = 0.326\pi$ for $N_f = 1$

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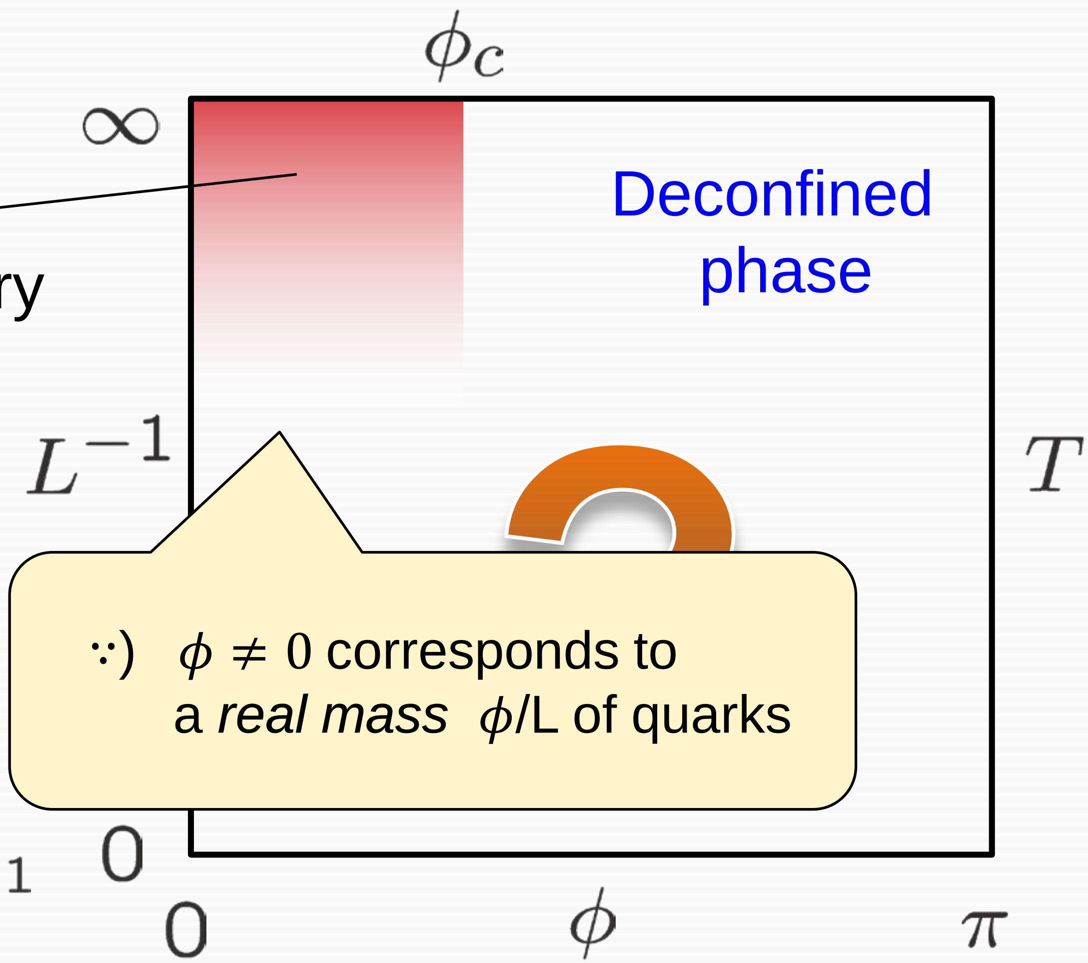
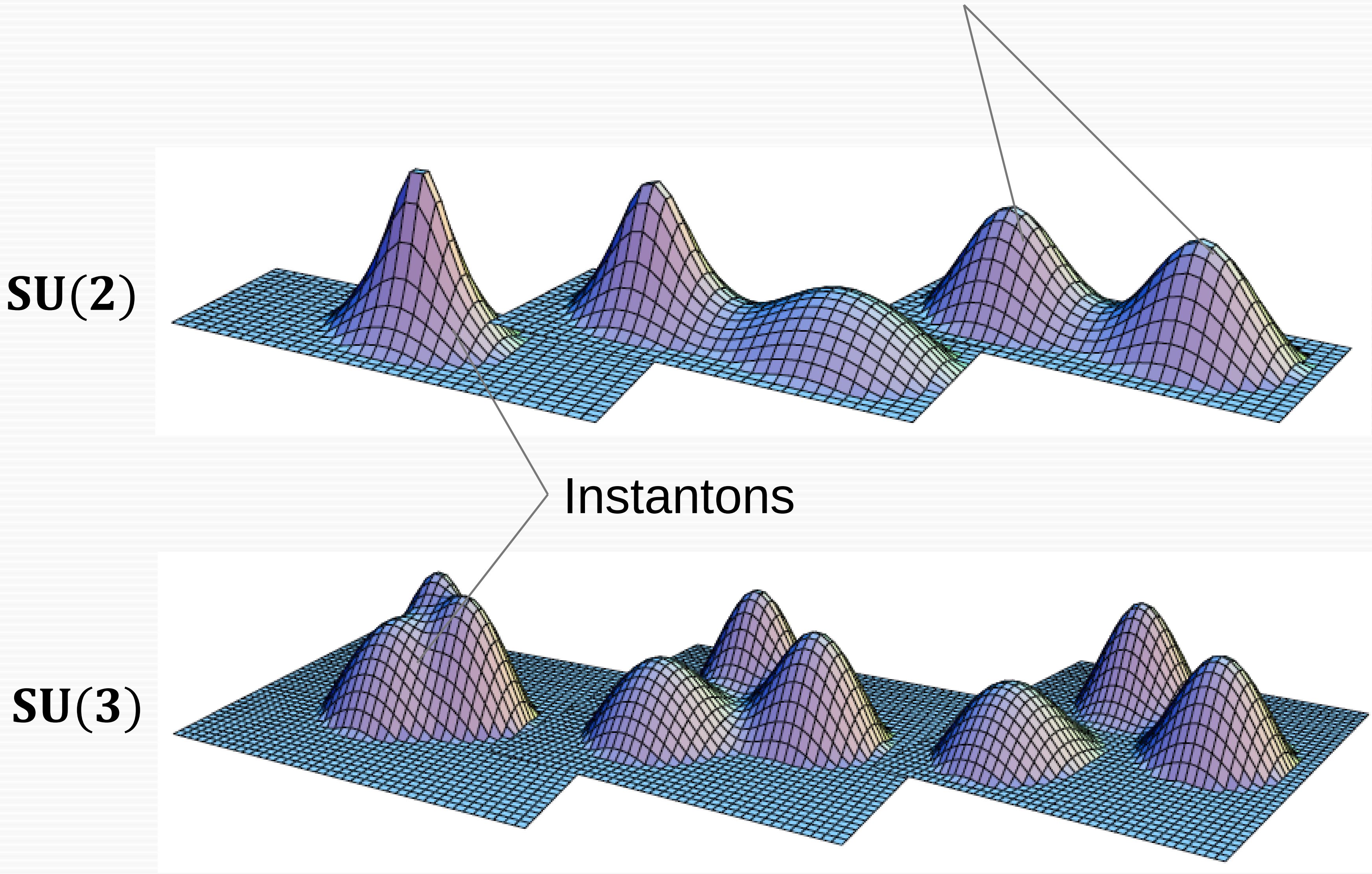


$\phi_c = 0.326\pi$ for $N_f = 1$

$\pi_2(SU(2)/U(1)) = \mathbb{Z}$ → Monopoles



from: Bruckmann et al., hep-th/0309008

INDEX THEOREM (1)

- APS Index theorem on $\mathbb{R}^4$:

  $2N_c$ adjoint zero modes for each BPST instanton.

- Callias Index theorem on $\mathbb{R}^3$:

  2 adjoint zero modes for each PS monopole.

- Nye-Singer Index theorem on $\mathbb{R}^3 \times S^1$:

  Index for PBC. Dependent on the holonomy at spatial infinity.

  cf. Poppitz-Unsal '09

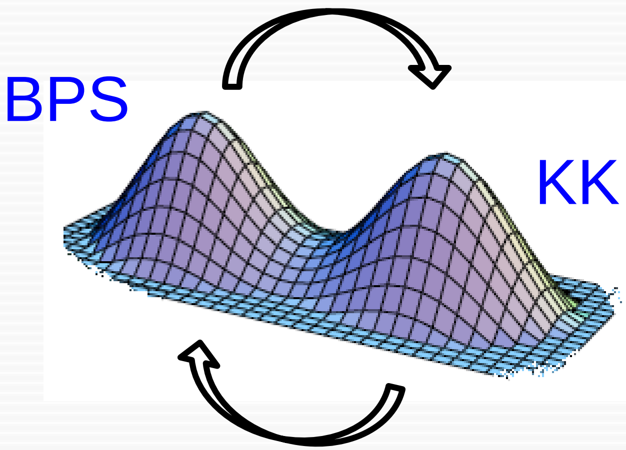
→ This can be extended to twisted boundary conditions ($\phi \neq 0$).

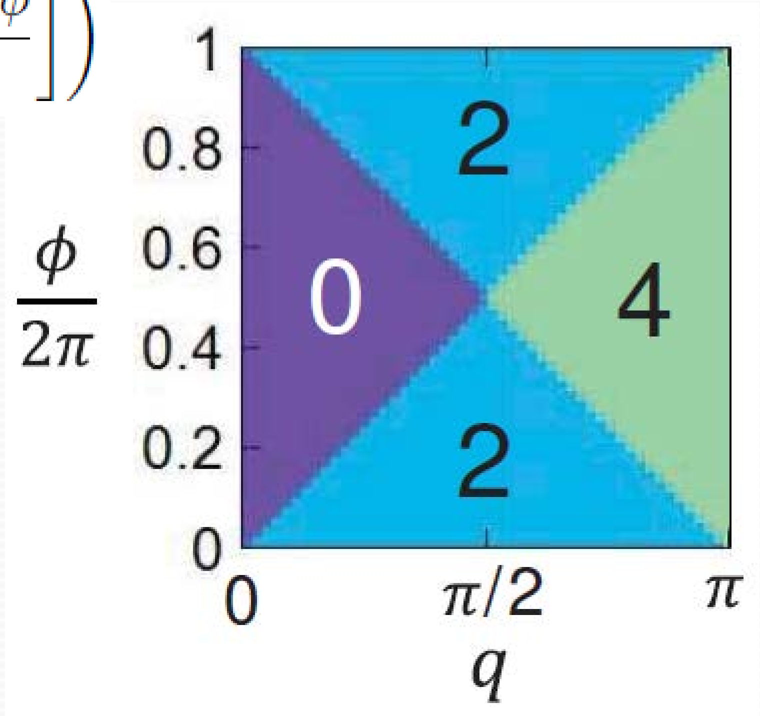
INDEX THEOREM (2)

- For SU(2) adjoint Dirac operator with $A_4\big|_\infty = \frac{1}{L}\begin{pmatrix} -q & 0 \\ 0 & q \end{pmatrix}$,
  the index in the background of
  a BPS monopole reads

$$I_{\rm adj}^{(\phi)}[1,0] = 2\left(\left\lfloor\frac{2q+\phi}{2\pi}\right\rfloor - \left\lfloor\frac{-2q+\phi}{2\pi}\right\rfloor\right)$$

- Zero modes jump !



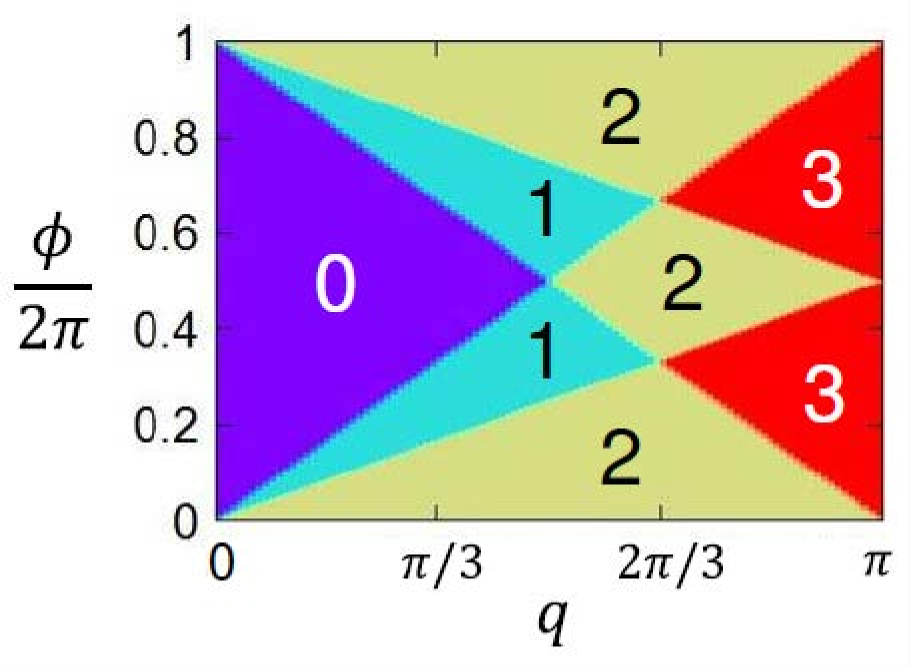


($\phi = 0 \to$ Callias)

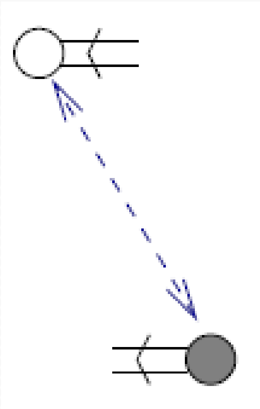
INDEX THEOREM (3)

- For SU(3) with e.g., $A_4\big|_\infty = \frac{1}{L}\begin{pmatrix} -q & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & q \end{pmatrix}$,

$$I^{(\phi)}_{\text{adj}}[1,0,0] = I^{(\phi)}_{\text{adj}}[0,1,0] = \left\lfloor \frac{q+\phi}{2\pi} \right\rfloor - \left\lfloor \frac{-q+\phi}{2\pi} \right\rfloor + \left\lfloor \frac{2q+\phi}{2\pi} \right\rfloor - \left\lfloor \frac{-2q+\phi}{2\pi} \right\rfloor$$



Zero-mode exchange interaction between monopole and anti-monopole leads to mass gap for the U(1) photon: e.g. for $N_c = N_f = 2$,



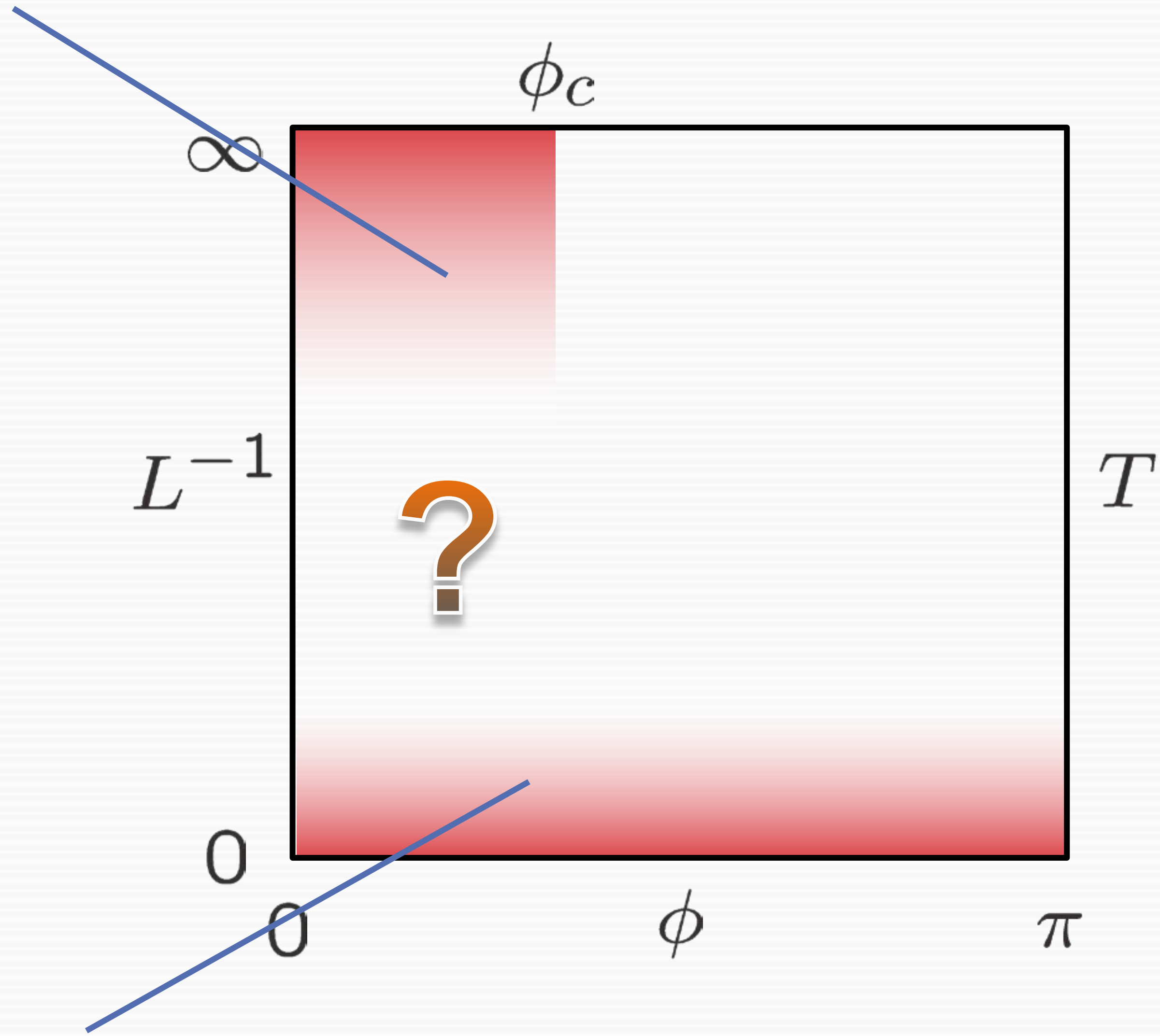
$$\mathcal{M} \sim \left(\log \frac{1}{L\Lambda}\right)^{\frac{65}{8}} \exp\left(-\left(2\pi + \sqrt{\frac{8\phi}{\pi}}\right)\sqrt{\log \frac{1}{L\Lambda}}\right)$$

and area law of the Wilson loop with the string tension $\gamma \sim \frac{g^2}{L}\mathcal{M}$.

⋯ Direct extension of Unsal '07 ($\phi = 0$) and Polyakov '77 (2+1 dim.)

Valid only for $0 \le \phi \le \phi_c$ (confining phase)

Mass gap & confinement at weak coupling

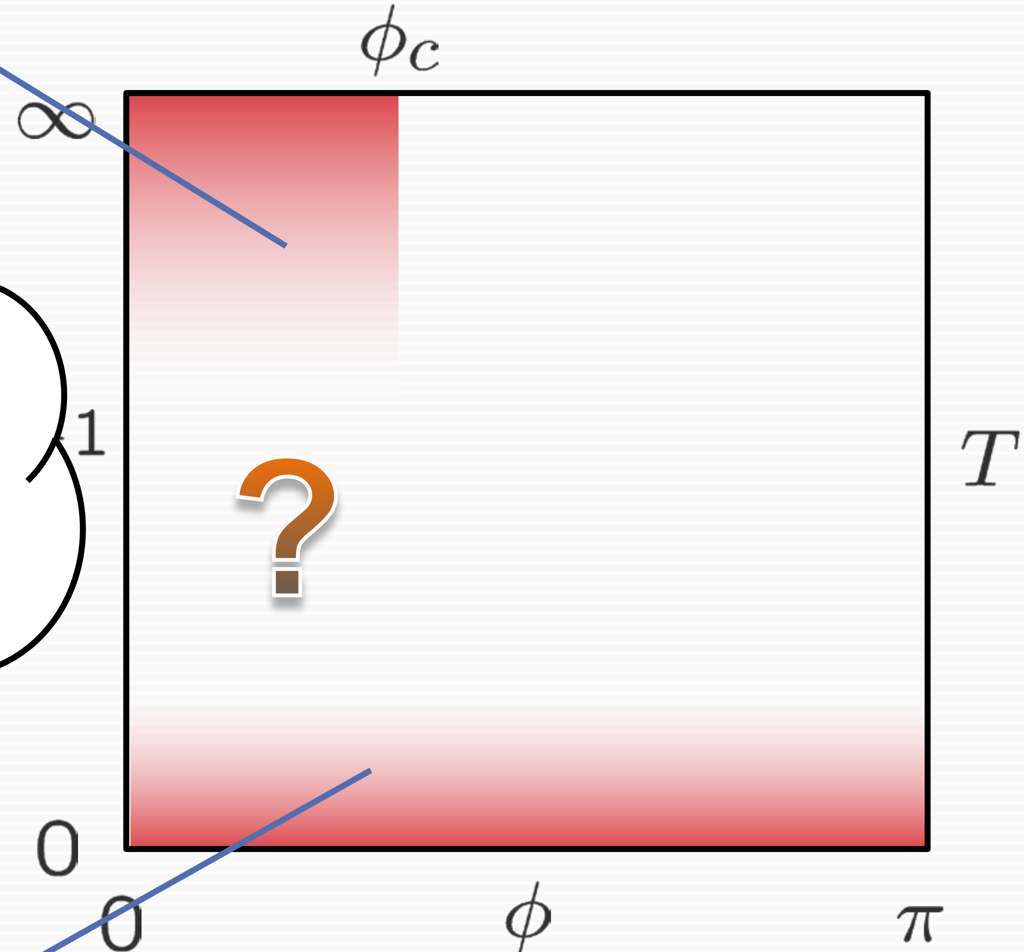


Mass gap & confinement at strong coupling

Mass gap & confinement at weak coupling

Do they form a single phase?

(modulo chiral transition)



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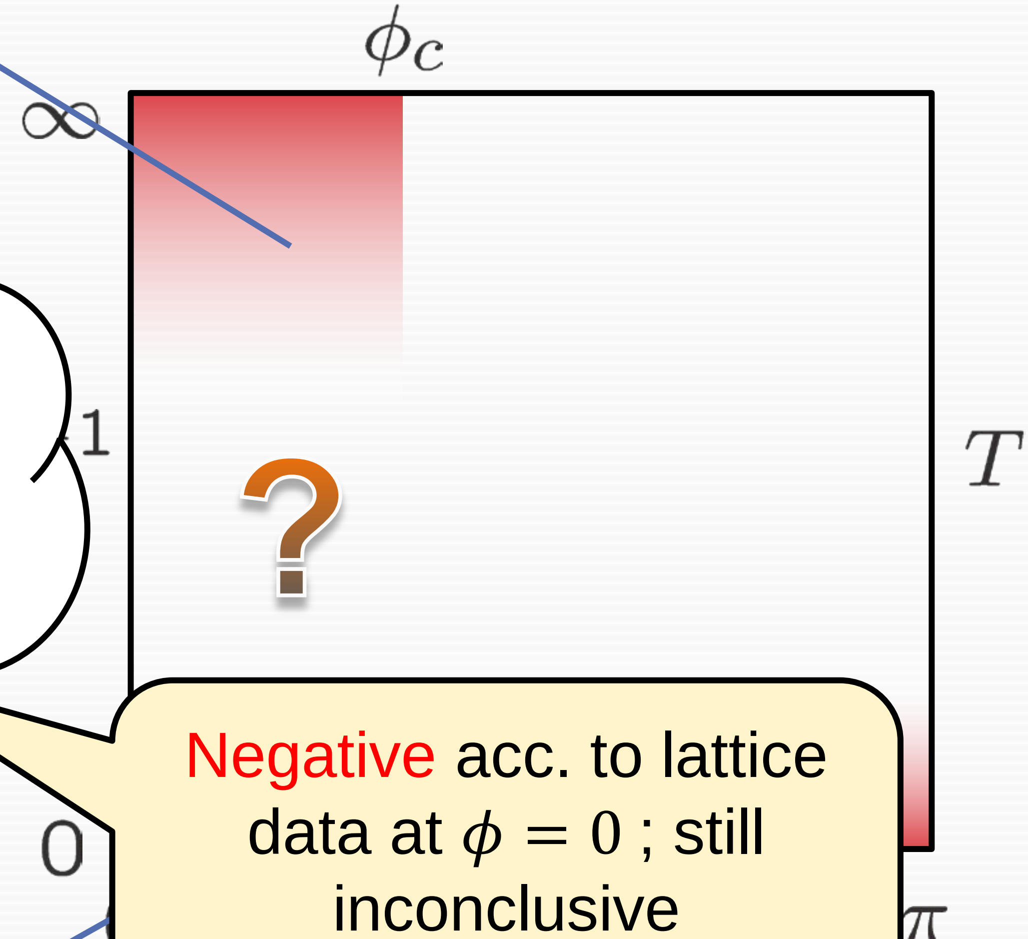
Mass gap & confinement **at weak coupling**



Do they form a single phase?

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Negative acc. to lattice data at $\phi = 0$; still inconclusive

cf. Cossu-D'Elia '09

Mass gap & confinement **at strong coupling**

CHIRAL SYMMETRY

- Dynamical ChSB strongly affects center symmetry

Nishimura-Ogilvie '10

- 4-d NJL model

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G\left\{(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right\}$$

- What happens for quarks with PBC ($\phi = 0$)?

$L \to 0$: Dimensional reduction $\mathbb{R}^3 \times S^1 \to \mathbb{R}^3$

4-d NJL $\longrightarrow$ 3-d NJL with $G \to G_{3d} = \frac{G}{L}$

The 3-d coupling gets large as $L \to 0$

➔ ~~NO~~ *Delayed chiral restoration for PBC?*

(qualitatively consistent with lattice QCD; Cossu et al. '09)

PHASE STRUCTURE (1)

- Gauge field sector:
  Use a phenomenological model of Ogilvie et al. ’02

$$V_g(\Omega) = -\frac{2}{\pi^2 L^4}\sum_{n=1}^{\infty}\frac{\mathrm{Tr}_A \Omega^n}{n^4} + \frac{M^2}{2\pi^2 L^2}\sum_{n=1}^{\infty}\frac{\mathrm{Tr}_A \Omega^n}{n^2}$$

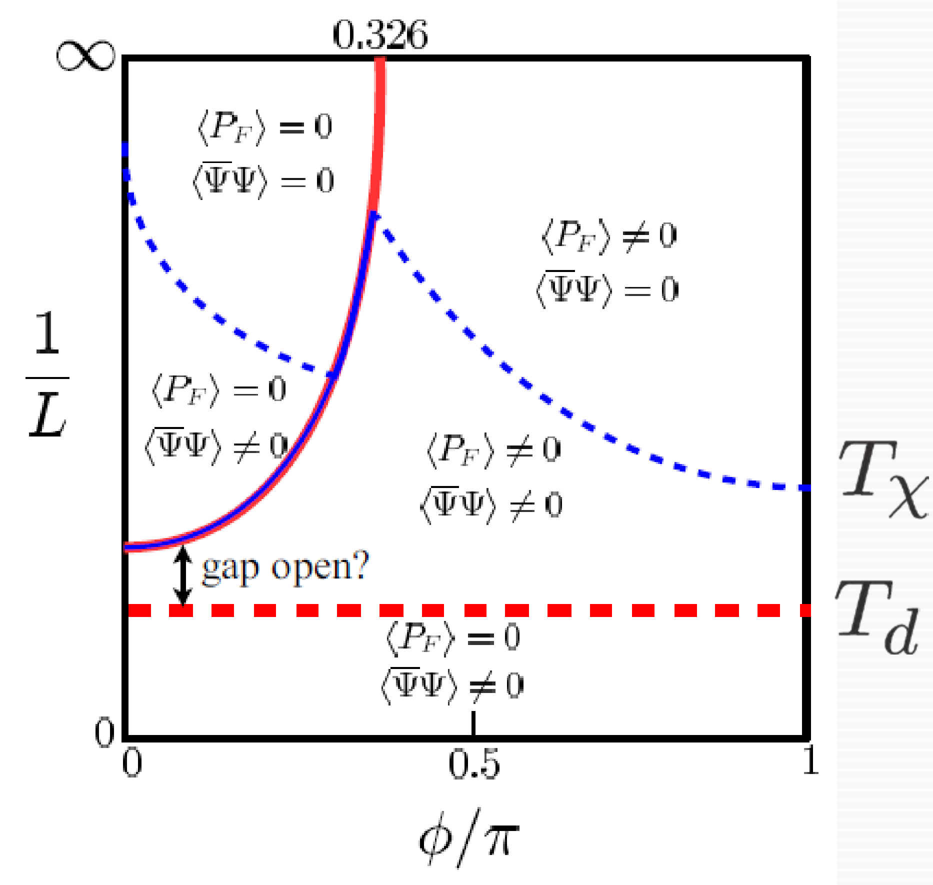
- Quark sector:
  Use the NJL model coupled to *holonomy* Ω ;
  Solved in the mean-field approximation

- **NJL is a cutoff theory;**

  Predictions for $L^{-1} > \Lambda_{UV}$ are meaningless!

PHASE STRUCTURE (2)

A **conjecture** for the phase diagram of (massless) SU(2) adjoint QCD, motivated by models and lattice data at $\phi = \pi$



(cf. No Roberge-Weiss periodicity for *adjoint* quarks)

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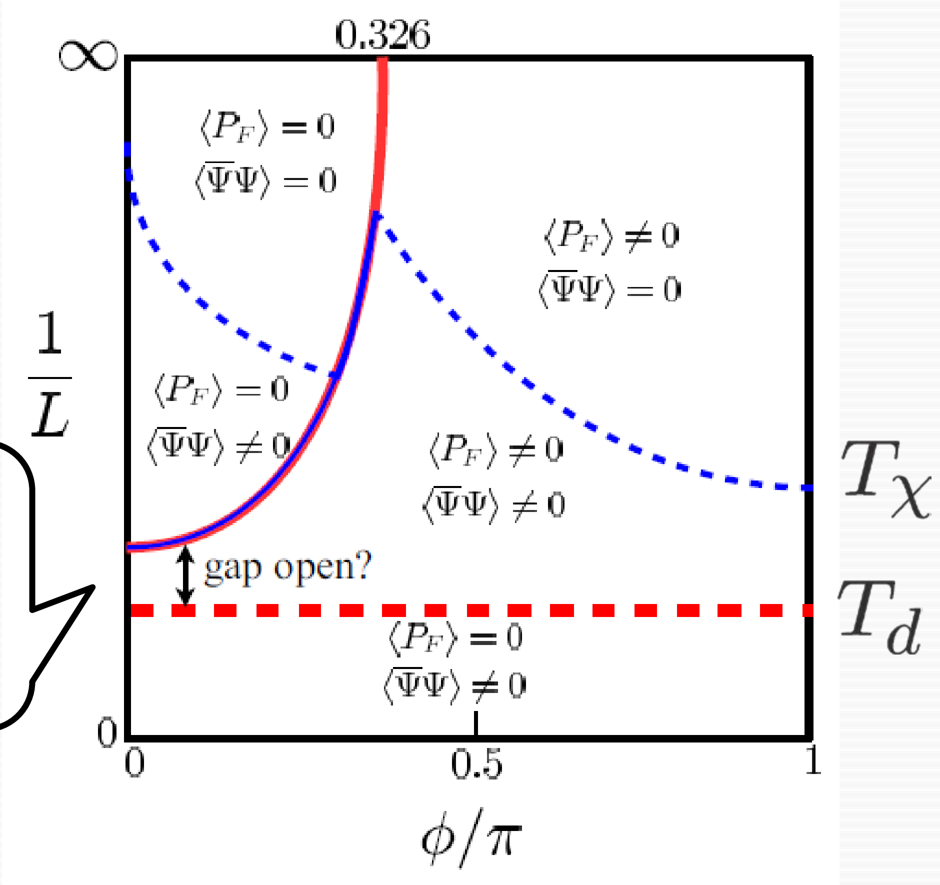

Due to large constituent quark mass

(cf. No Roberge-Weiss periodicity for *adjoint* quarks)

CONCLUSION

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- Adjoint quarks with PBC induces gauge symmetry breaking: $SU(N) \to U(1)^{N-1}$ Hosotani '89~
- Dilute monopole gas

  ➔ Area law & mass gap at small S^1
- Index theorem with general $\phi \in [0, \pi]$
- Complex phase structure on $(L^{-1},\ \phi)$ plane
  - Weak-strong continuity may *not* hold
  - Chiral & deconf. transitions coalesce into a single 1st-order phase transition line !
- Awaits check by future lattice simulations

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Quantitative (rigorous)

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  Qualitative

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QUESTIONS

- What occurs at large N_c?
- Relevant for beyond-SM physics?
- Physics at small S^1 with incomplete Abelianization?
- Multiple compact directions?
- Relation to the dual-super picture of confinement on $\mathbb{R}^4$?
- Instanton-liquid model vs. monopole-liquid model?
- Representations other than adjoint (e.g., fundamental)?
- Implication for resurgence and Borel flow?
- …

More questions than answers!