



Searching high and low for traces of inflation

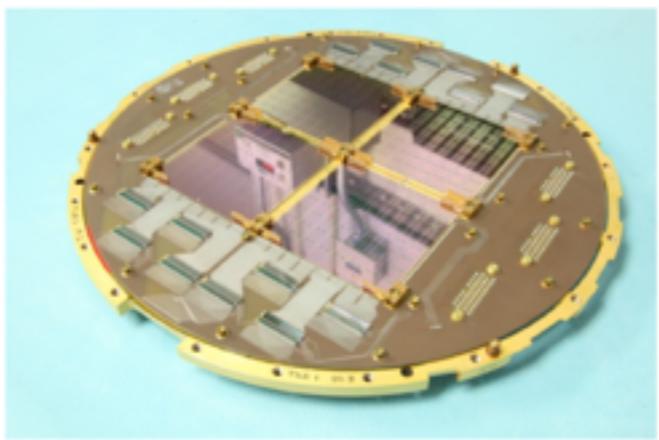
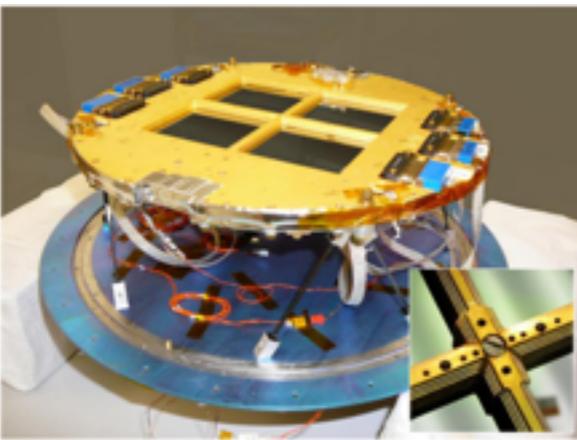
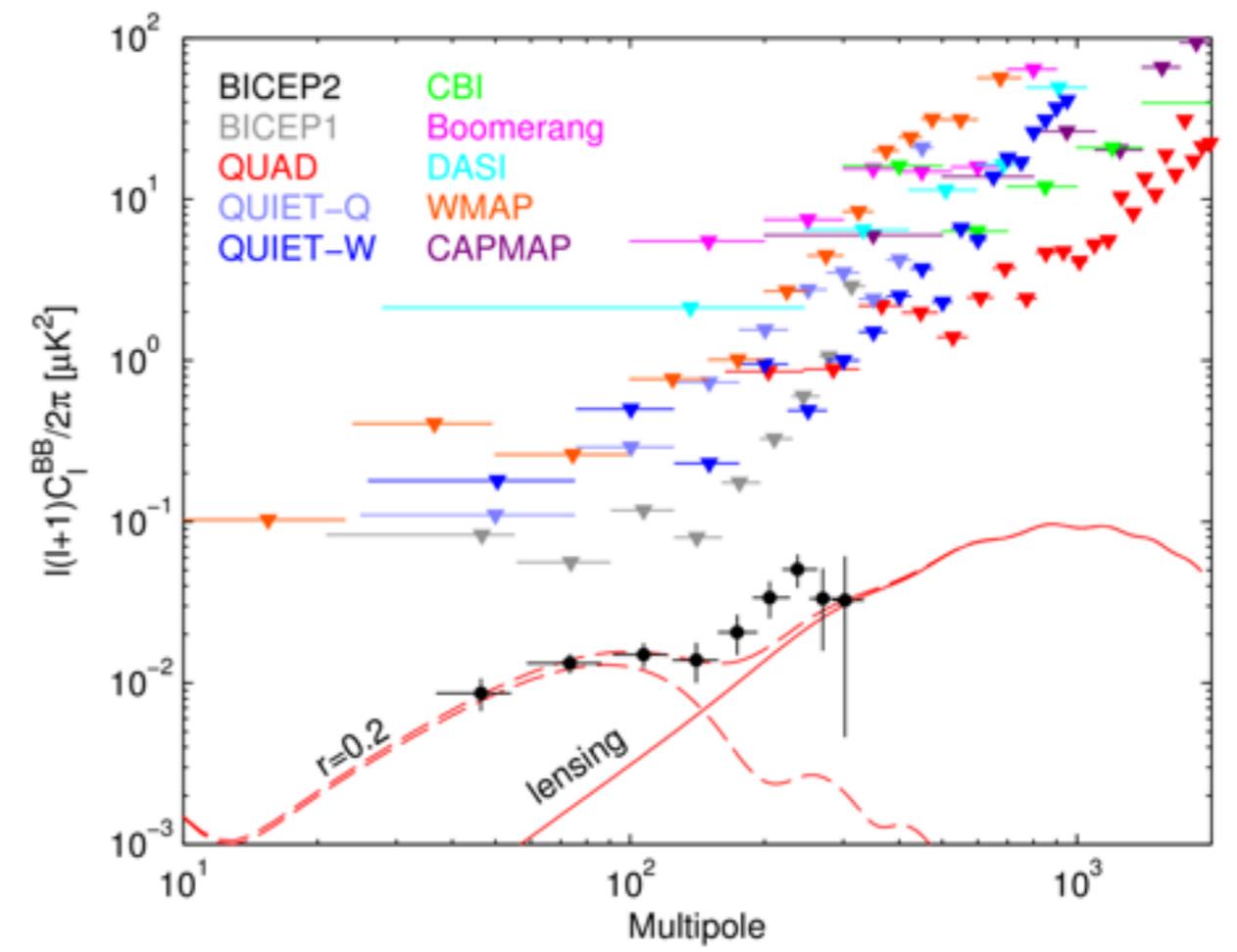
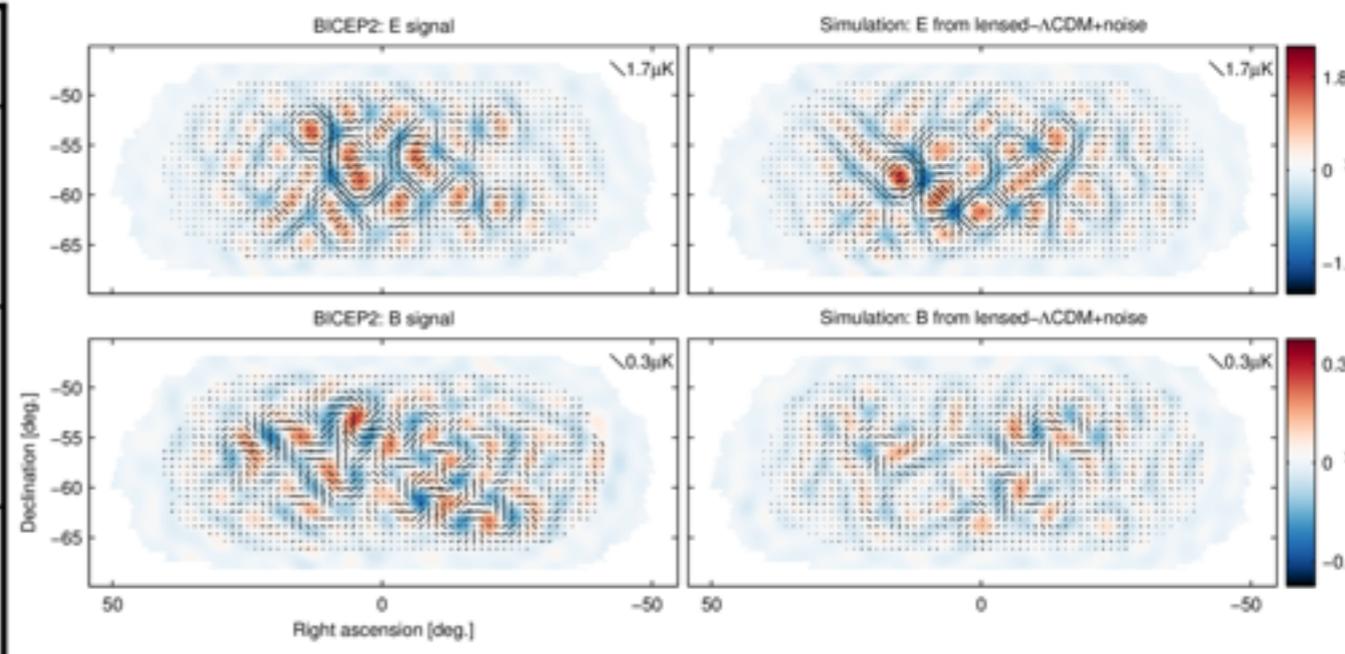
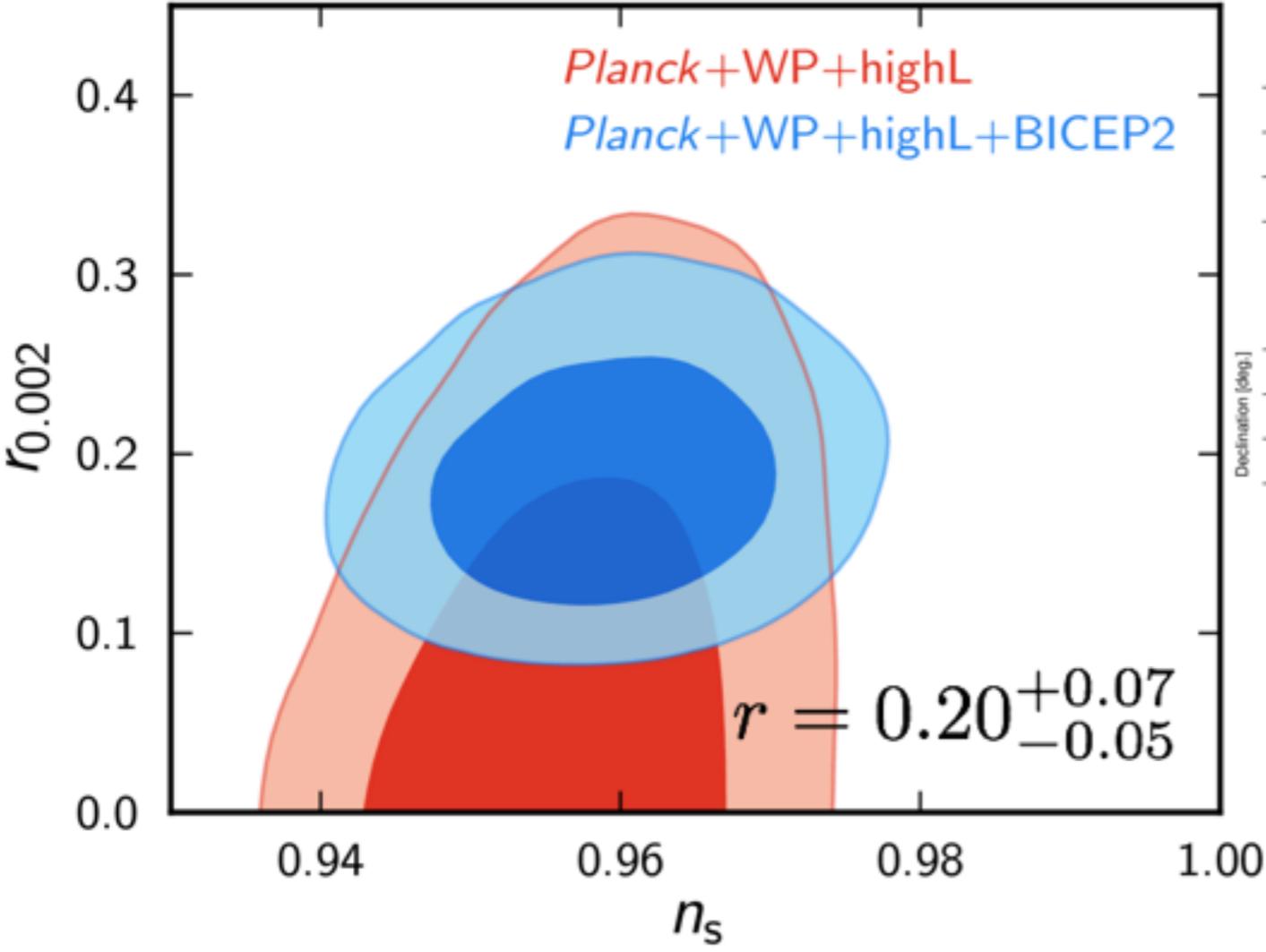
29th July 2014
PPP 2014 @YITP

Fuminobu Takahashi
(Tohoku)



BICEP2

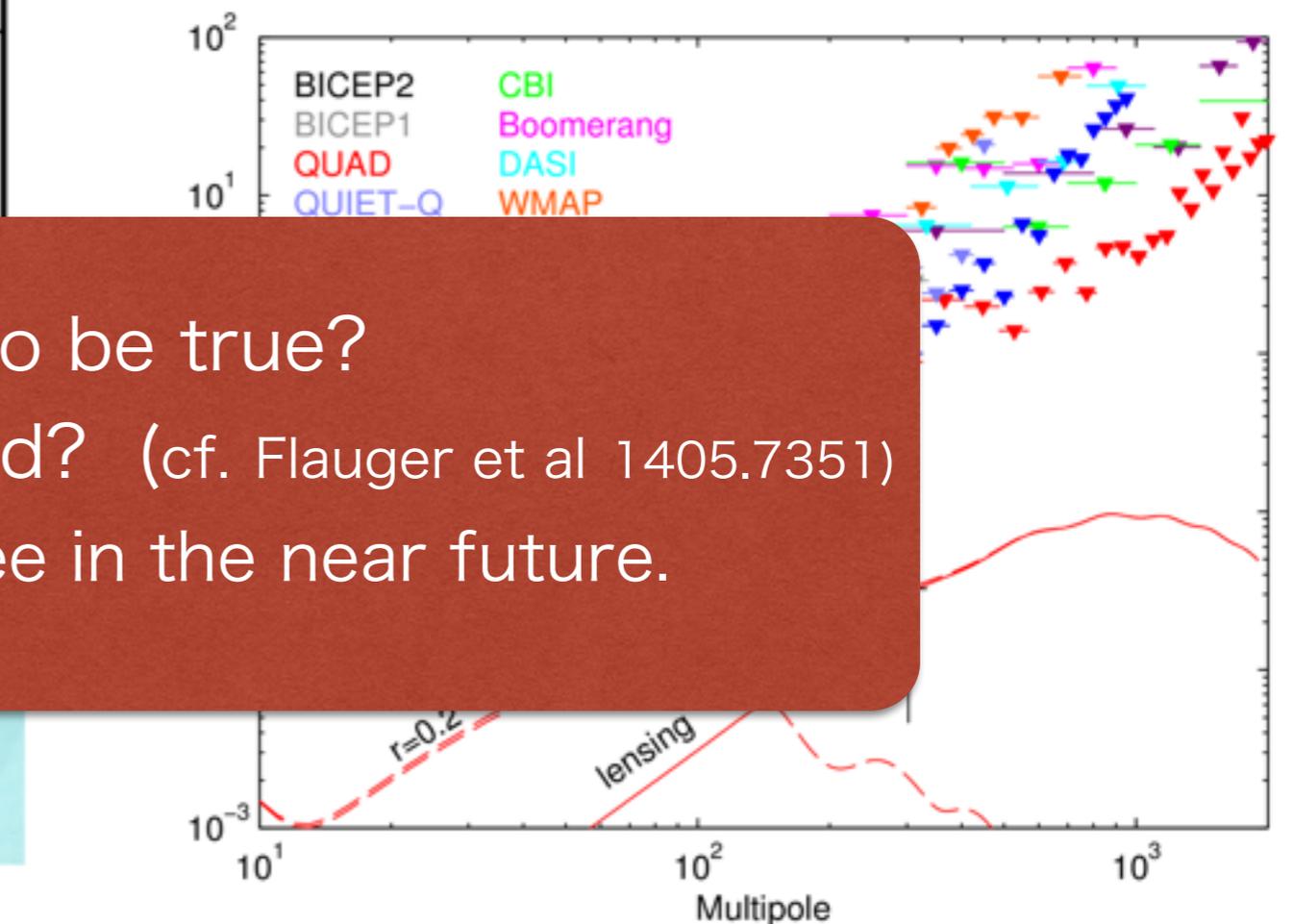
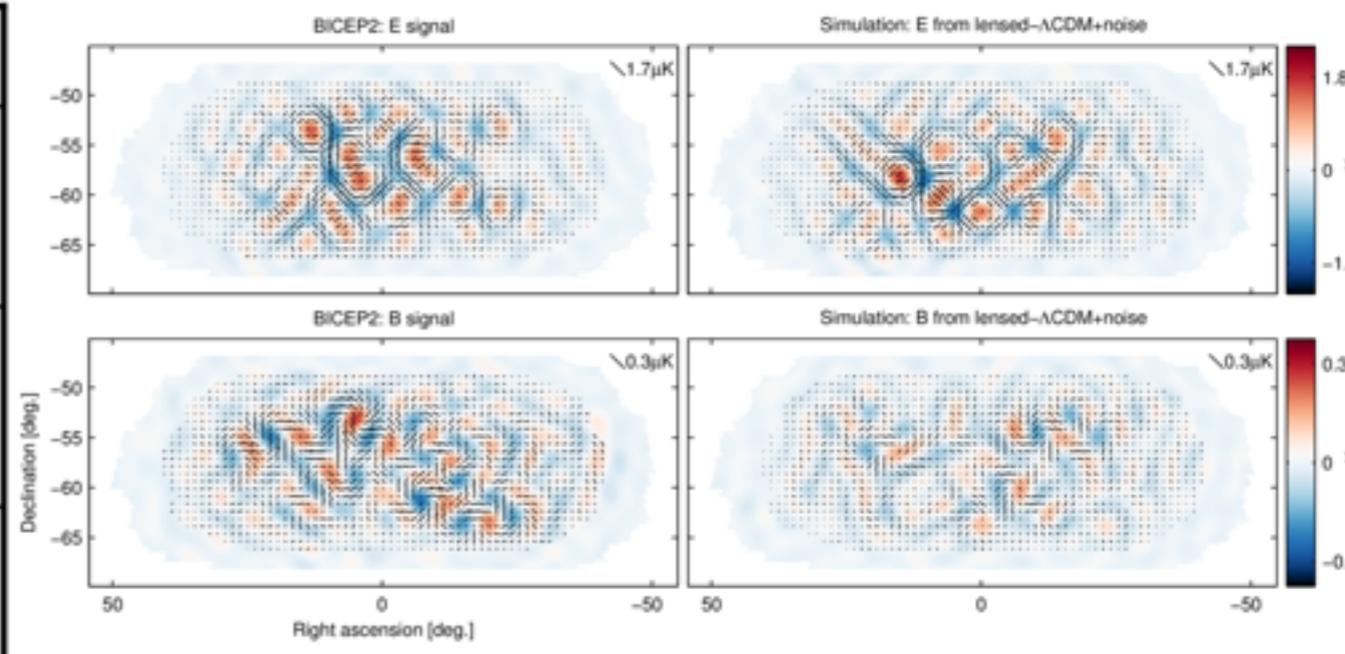
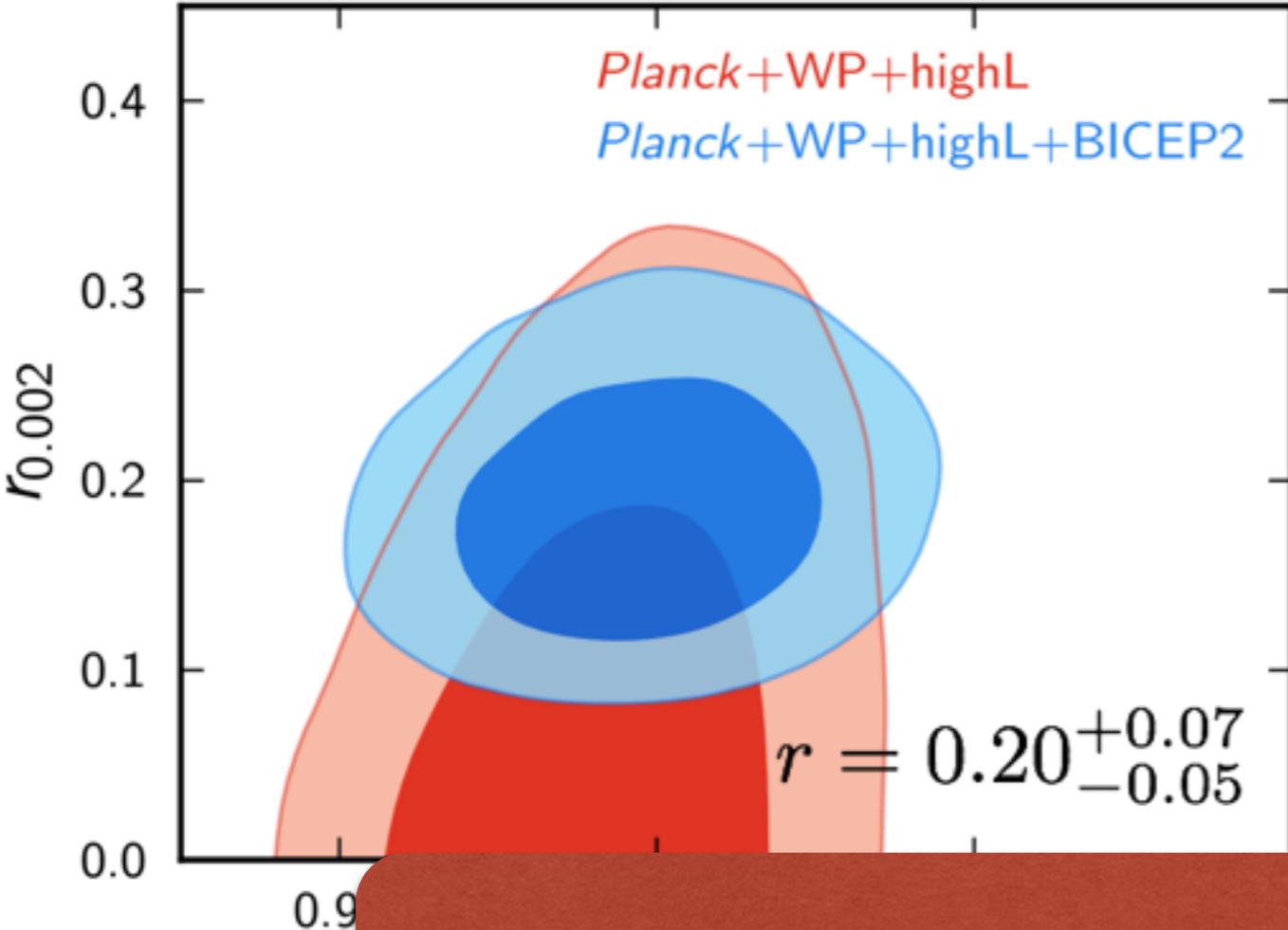
1403.3985



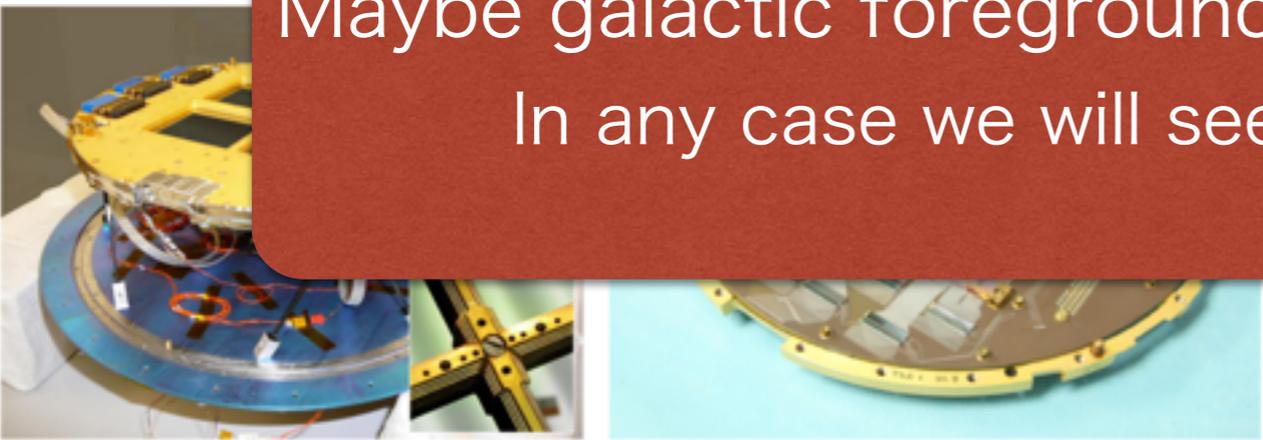


BICEP2

1403.3985



Too good to be true?
Maybe galactic foreground? (cf. Flauger et al 1405.7351)
In any case we will see in the near future.



What if $r = 0(10^{-3}-10^{-1})$?

What can we say about inflation?



It's GUT-scale inflation!

$$V_{\text{inf}} \simeq (2.1 \times 10^{16} \text{ GeV})^4 \left(\frac{r}{0.16} \right)$$

$$H_{\text{inf}} \simeq 1.0 \times 10^{14} \text{ GeV} \left(\frac{r}{0.16} \right)^{\frac{1}{2}},$$

GUT-scale, large-field inflation

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See Kitajima's poster

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 - The QCD axion less likely? PQ symmetry restoration?

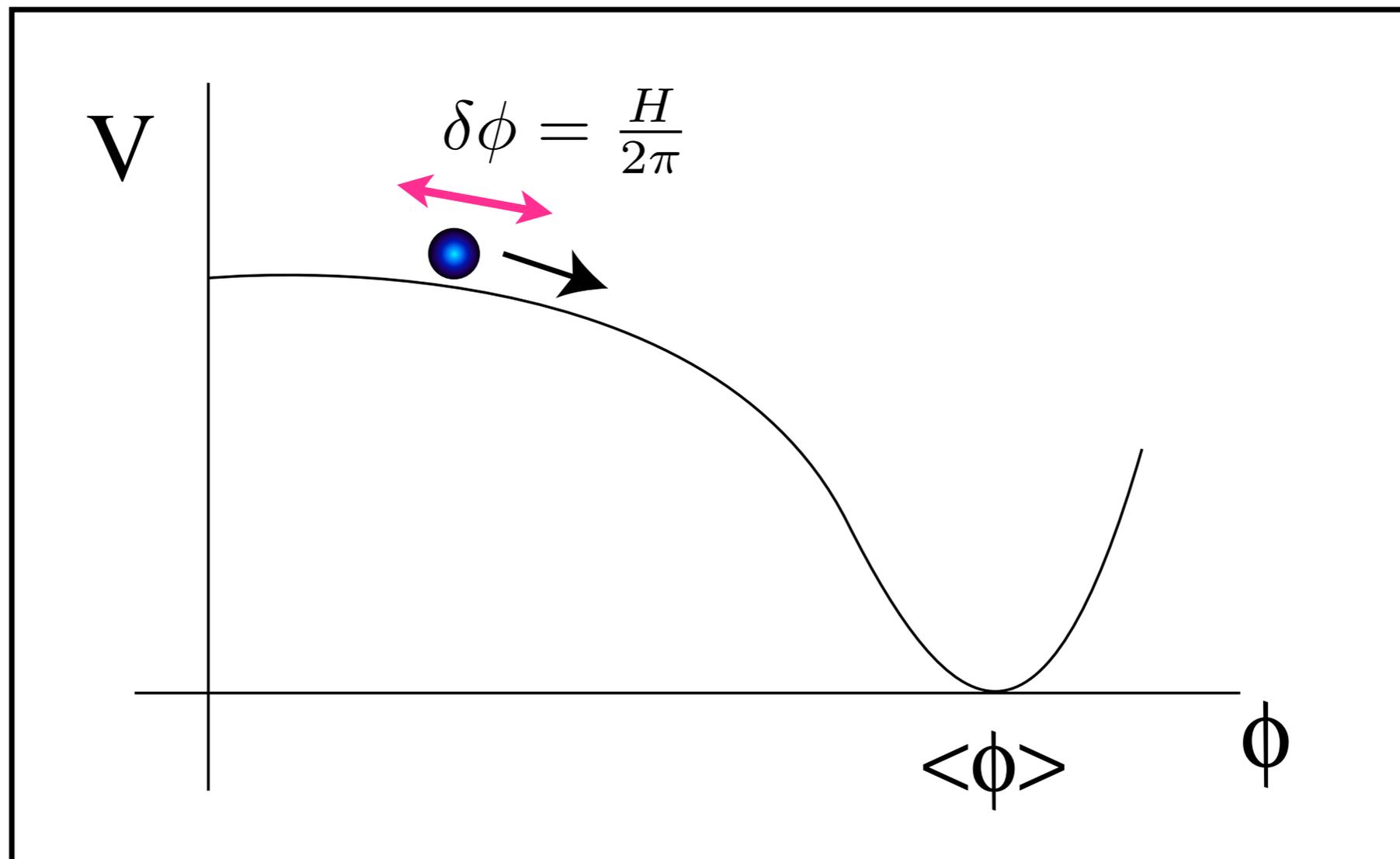
Inflation

Accelerated cosmic expansion solves various theoretical problems of the std. big bang cosmology.

Guth '81, Sato '80, Starobinsky '80, Kazanas '80, Brout, Englert, Gunzig, '79

One way to realize the inflationary expansion is the slow-roll inflation.

Linde '82, Albrecht and Steinhardt '82



Perturbations of spacetime

Flat FRW Universe:

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

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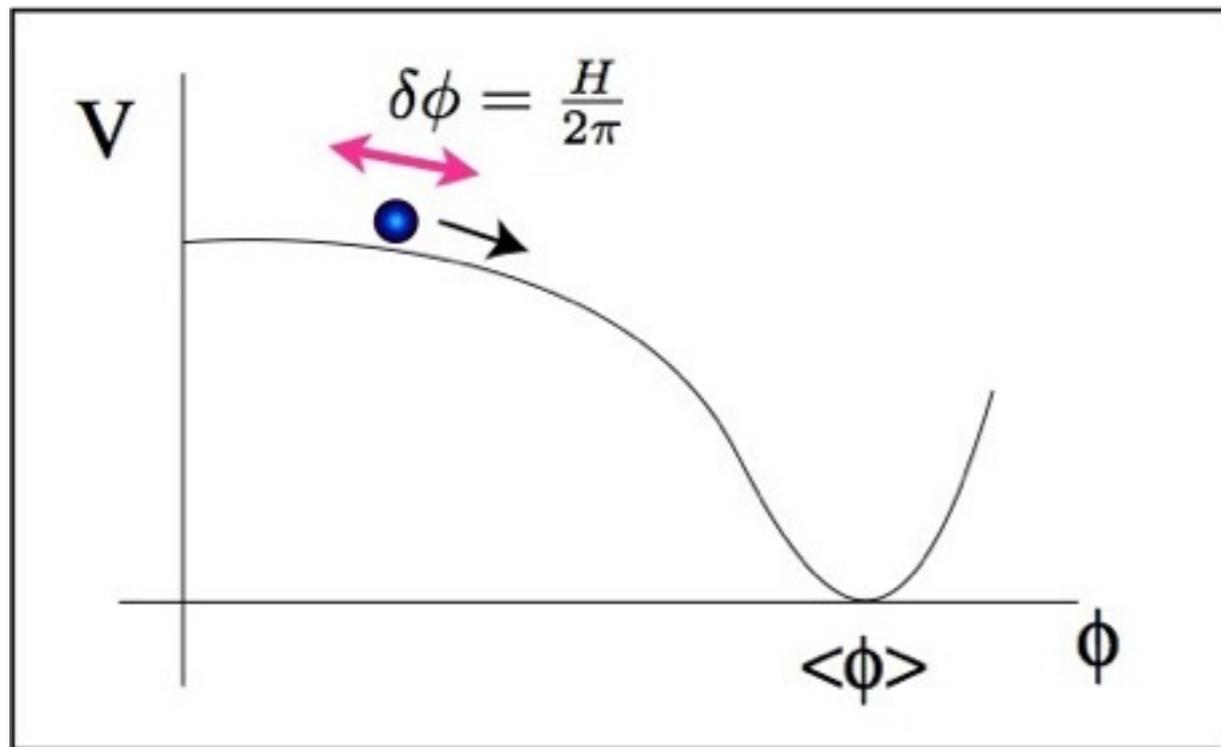
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The perturbations can be decomposed into three types.

1. Scalar $ds^2 = -(1 + 2\Phi)dt^2 + a^2(1 + 2\Psi)d\mathbf{x}^2$ **Inflaton**
2. Vector
3. Tensor $ds^2 = -dt^2 + a^2 (\delta_{ij} + h_{ij}) dx^i dx^j$ **GW**

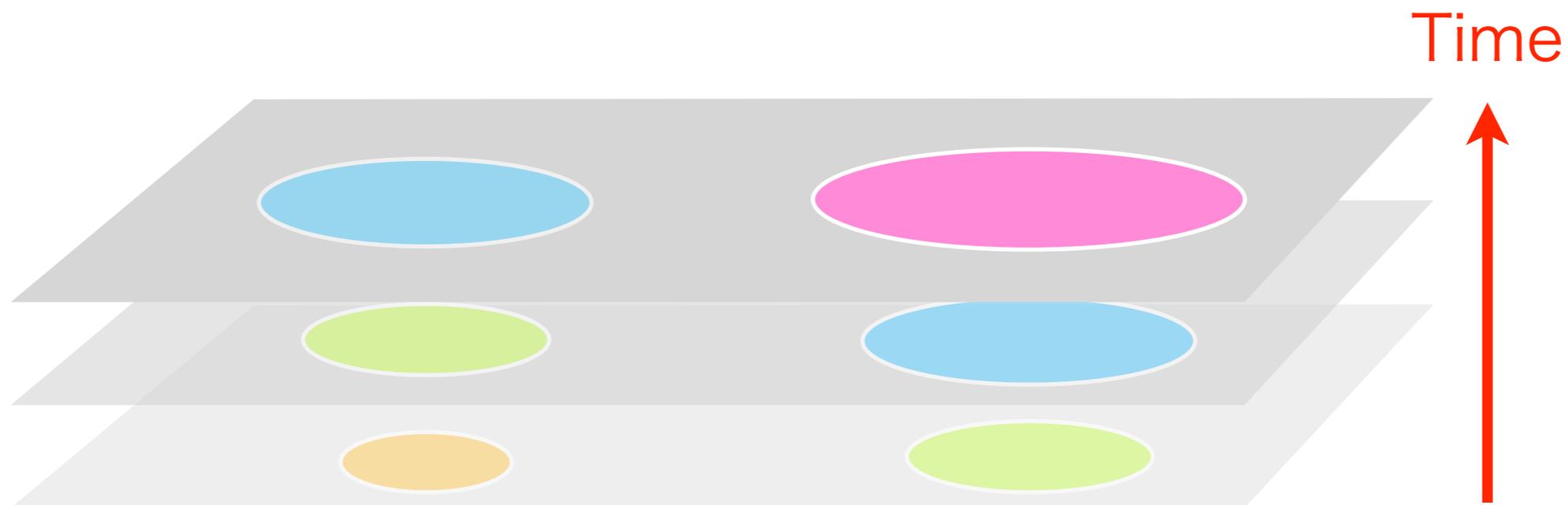
Scalar mode

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(1 + 2\Psi)d\mathbf{x}^2$$



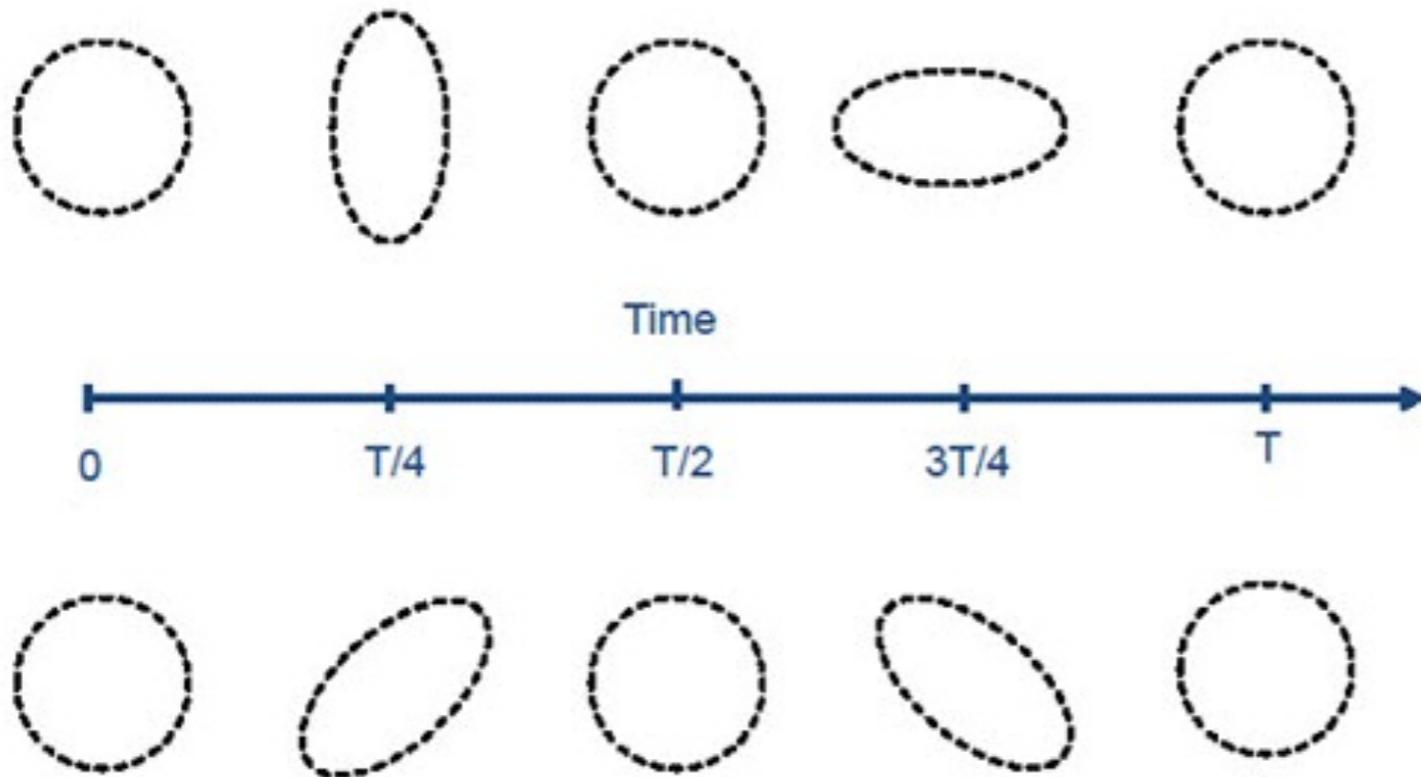
It is due to **fluctuations in time** induced by the inflaton's quantum fluctuation.

$$\Phi \sim \frac{\delta\rho}{\rho} \sim H\delta t \sim H_{\text{inf}} \frac{\delta\phi}{\dot{\phi}} \sim \left| \frac{V^{3/2}}{V' M_P^3} \right|$$



Tensor mode

$$ds^2 = -dt^2 + a^2 (\delta_{ij} + h_{ij}) dx^i dx^j$$



It is due to **fluctuations of graviton itself.**

$$h_{ij} \sim \frac{H_{\text{inf}}}{M_P}$$

Observation vs Theory

Scalar mode

$$P_{\mathcal{R}} = A_s \left(\frac{k}{k_0} \right)^{n_s - 1}$$

$$A_s = \frac{V^3}{2\sqrt{3}V'^2},$$

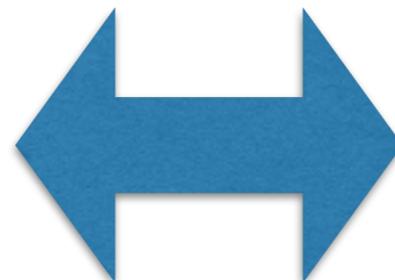
$$n_s = 1 + 2\frac{V''}{V} - 3\left(\frac{V'}{V}\right)^2,$$

Tensor mode

$$P_t = A_t \left(\frac{k}{k_0} \right)^{n_t}$$

$$r = 8\left(\frac{V'}{V}\right)^2$$

$$A_s, n_s, r \equiv \frac{A_t}{A_s}$$

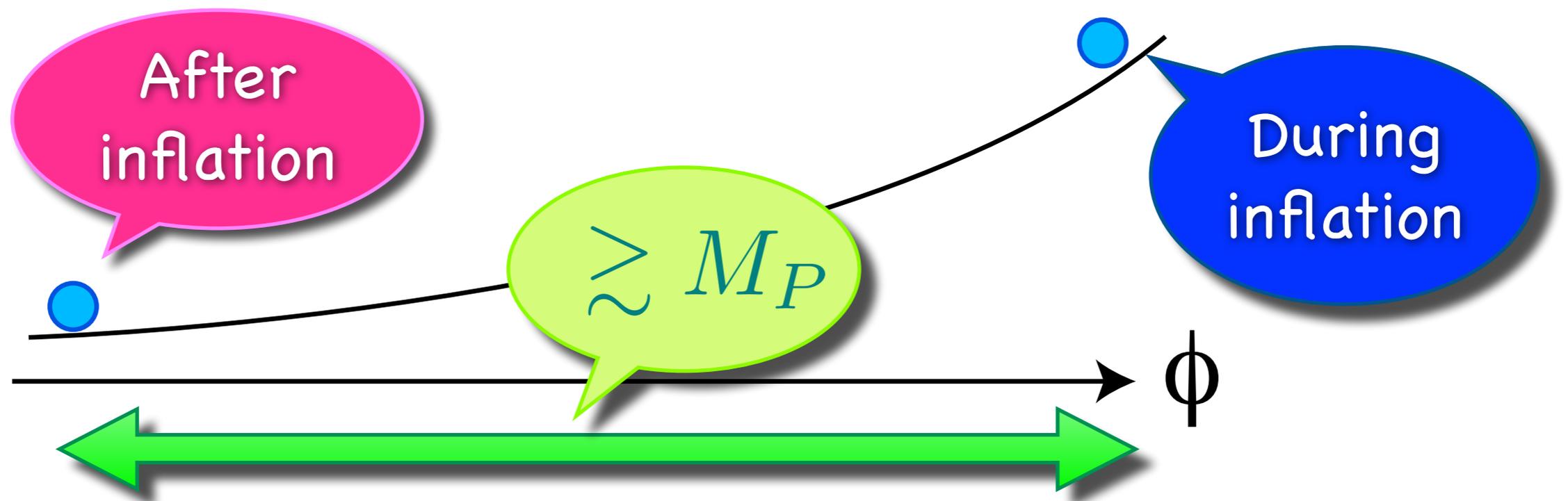


$$V, V', V''$$

V : the inflaton potential

Large-field inflation

The inflaton excursion exceeds the Planck scale.



Lyth bound:
$$\Delta\phi \gtrsim 8M_P \left(\frac{r}{0.2}\right)^{\frac{1}{2}} \left(\frac{N}{50}\right)$$

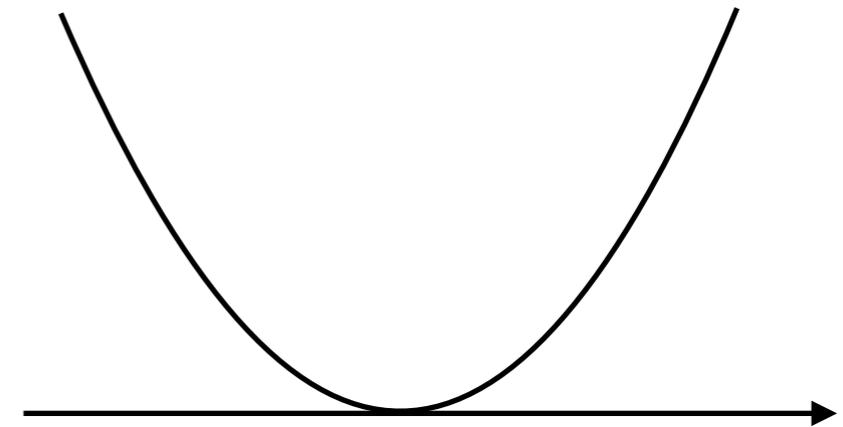
Various large-field inflation models

Quadratic chaotic inflation

Linde '83

$$V = \frac{1}{2}m^2\phi^2$$

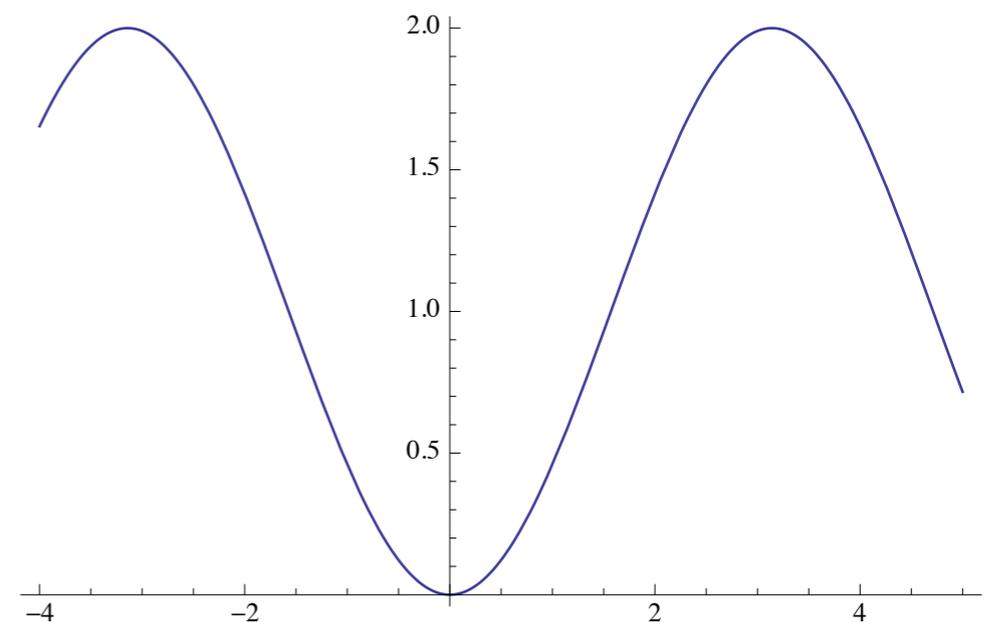
$$m \simeq 2 \times 10^{13} \text{ GeV} \quad \phi_{60} \sim 16M_P$$



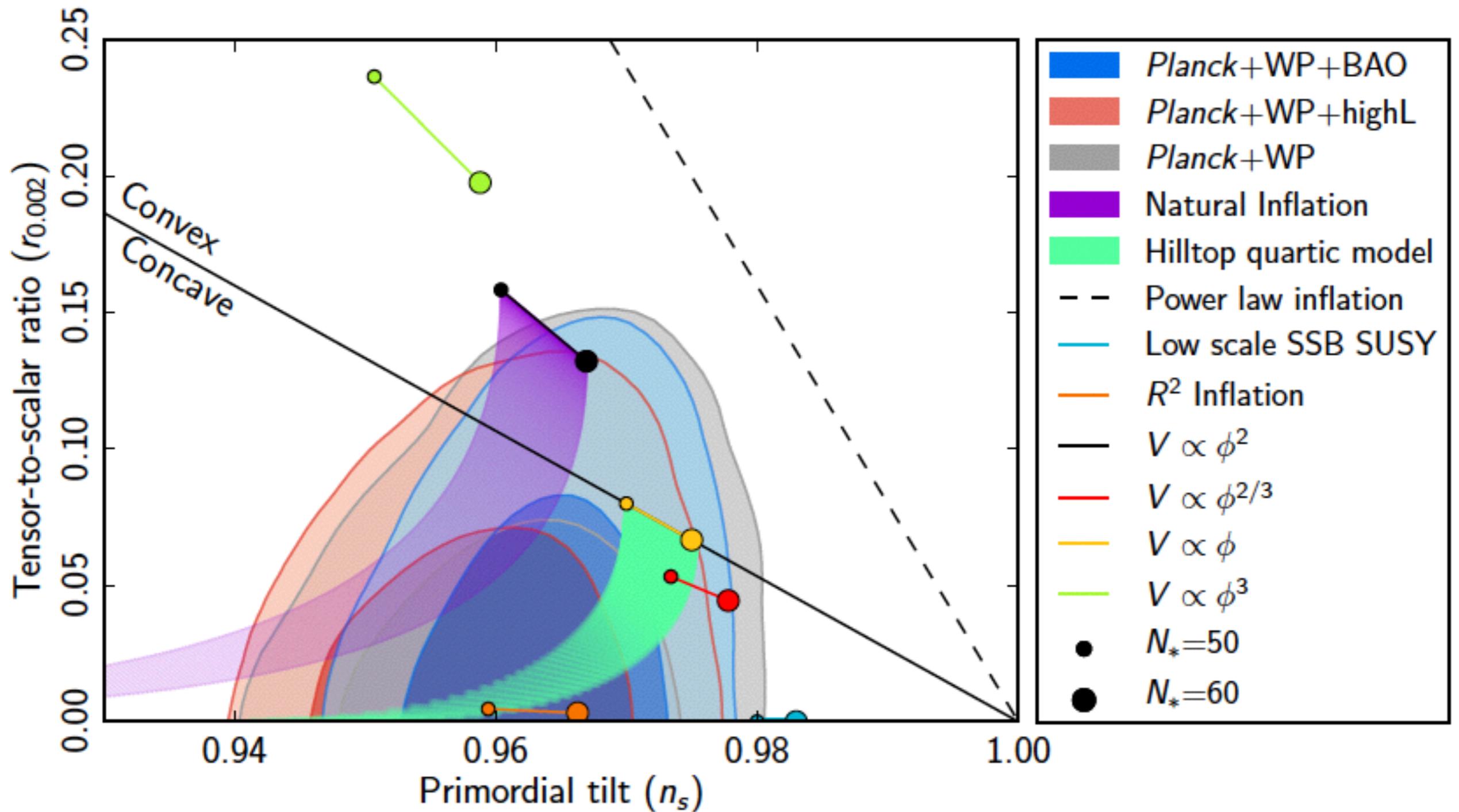
Natural inflation

Freese et al, '90

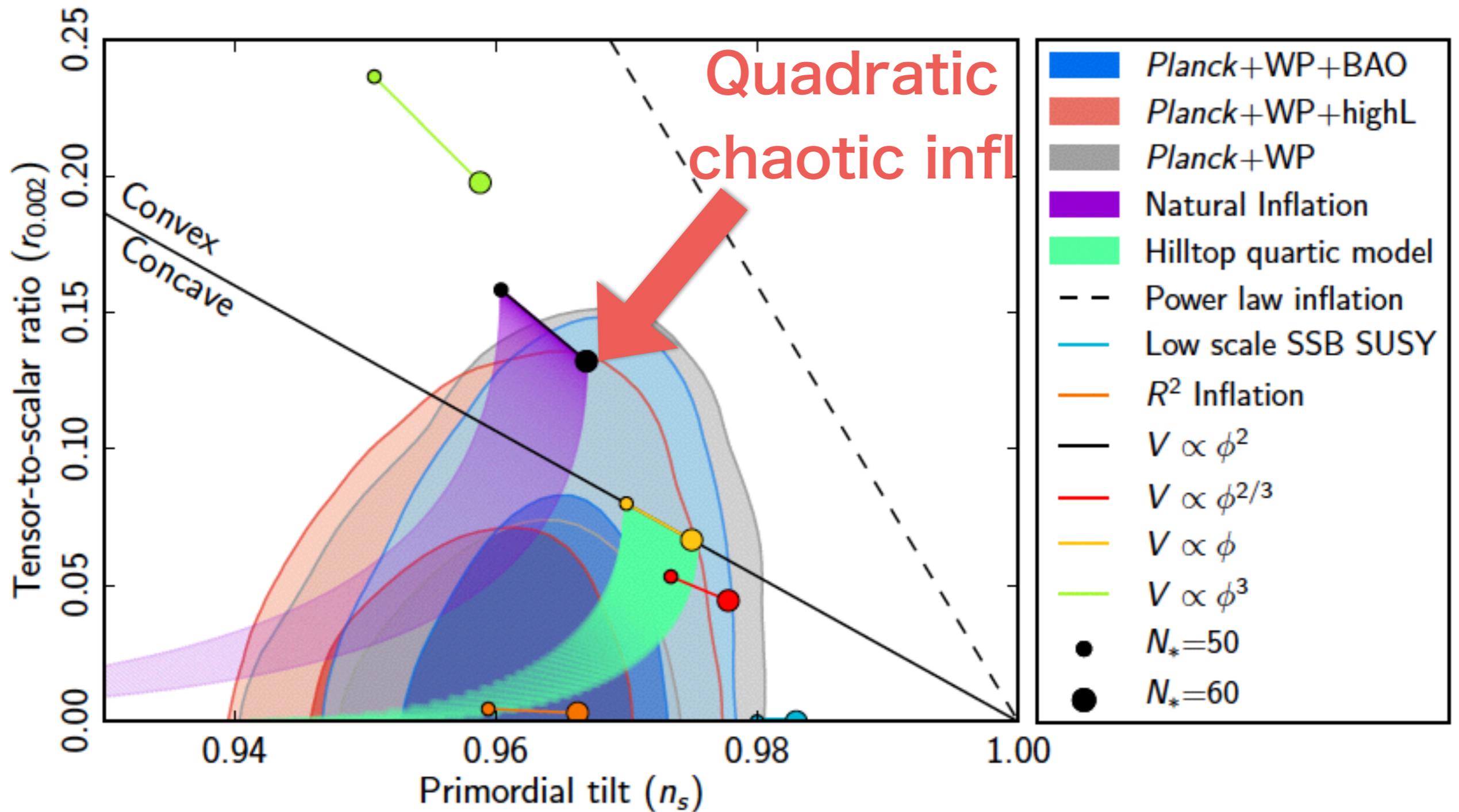
$$V = \Lambda^4 \left(1 - \cos \left(\frac{\phi}{f} \right) \right)$$



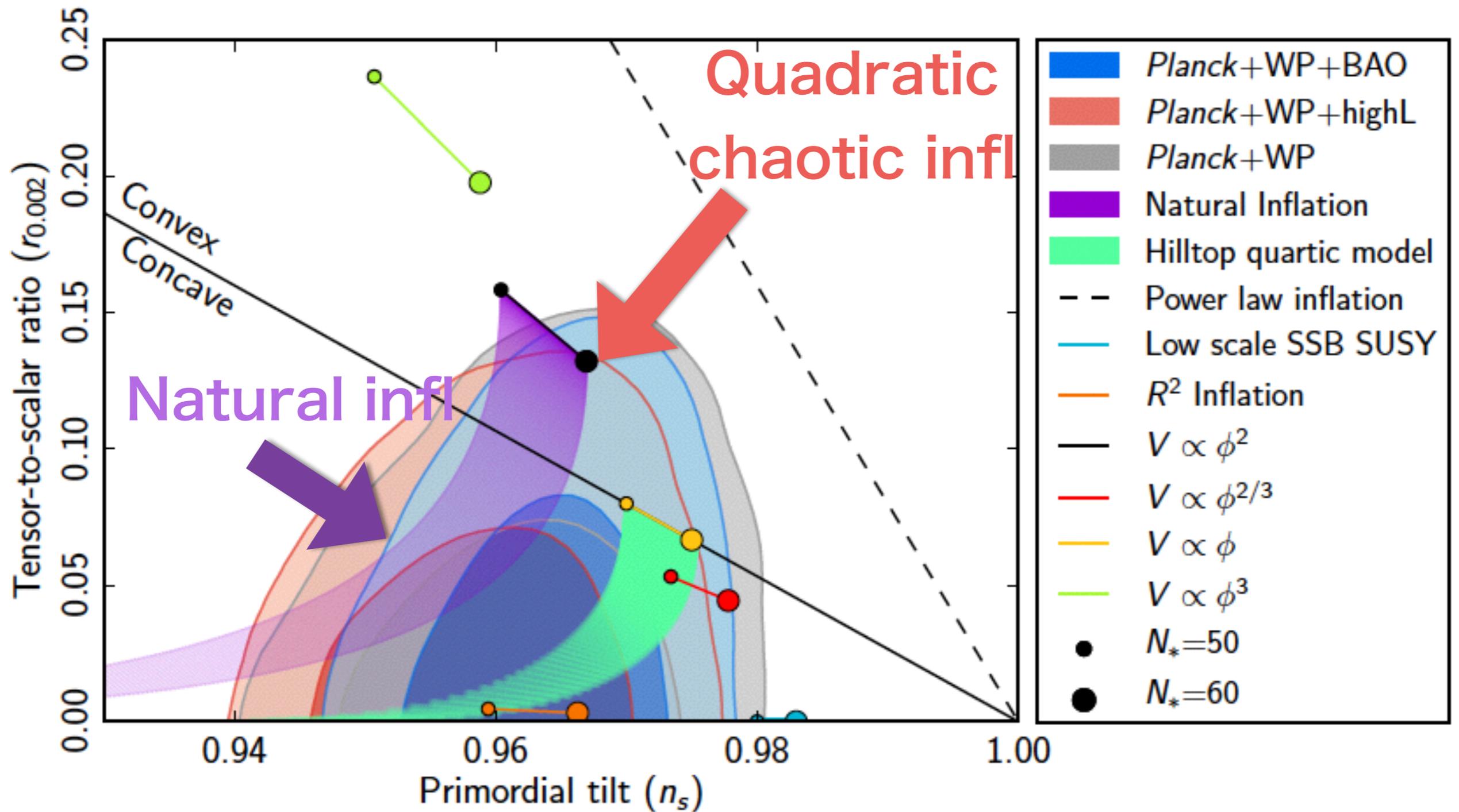
Predicted values of (n_s, r)



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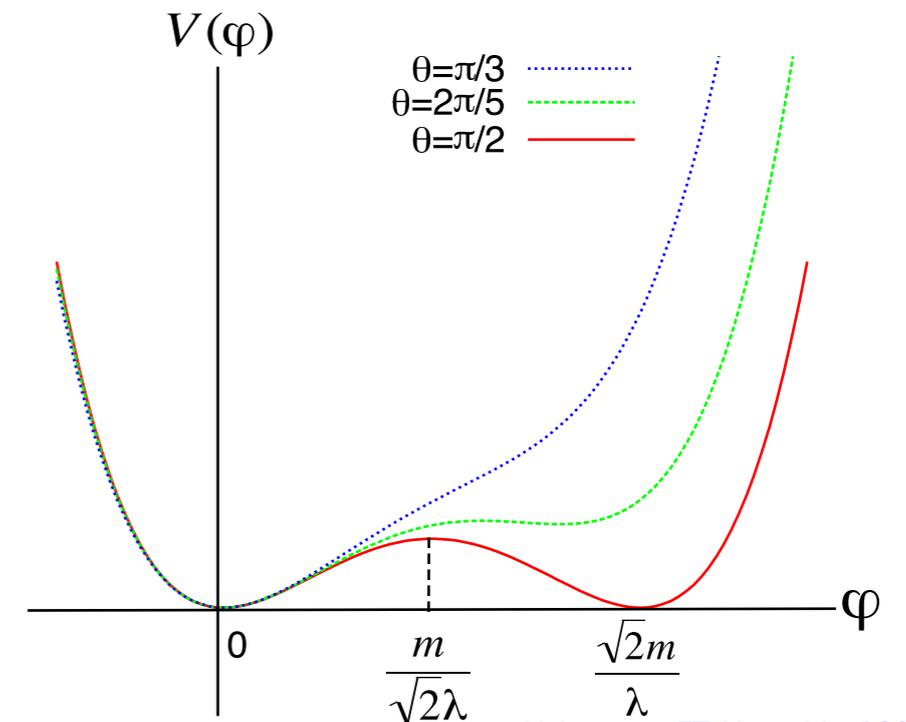


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Polynomial chaotic inflation

Destri, de Vega, Sanchez [astro-ph/0703417]
 Nakayama, FT, Yanagida 1303.7315
 (see also Kobayashi, Seto 1403.5055
 Kallosh, Linde, Wesphal 1405.0270)

$$V = \frac{1}{2}m^2\phi^2 + \frac{\kappa}{3}\phi^3 + \frac{\lambda}{4}\phi^4 + \dots$$



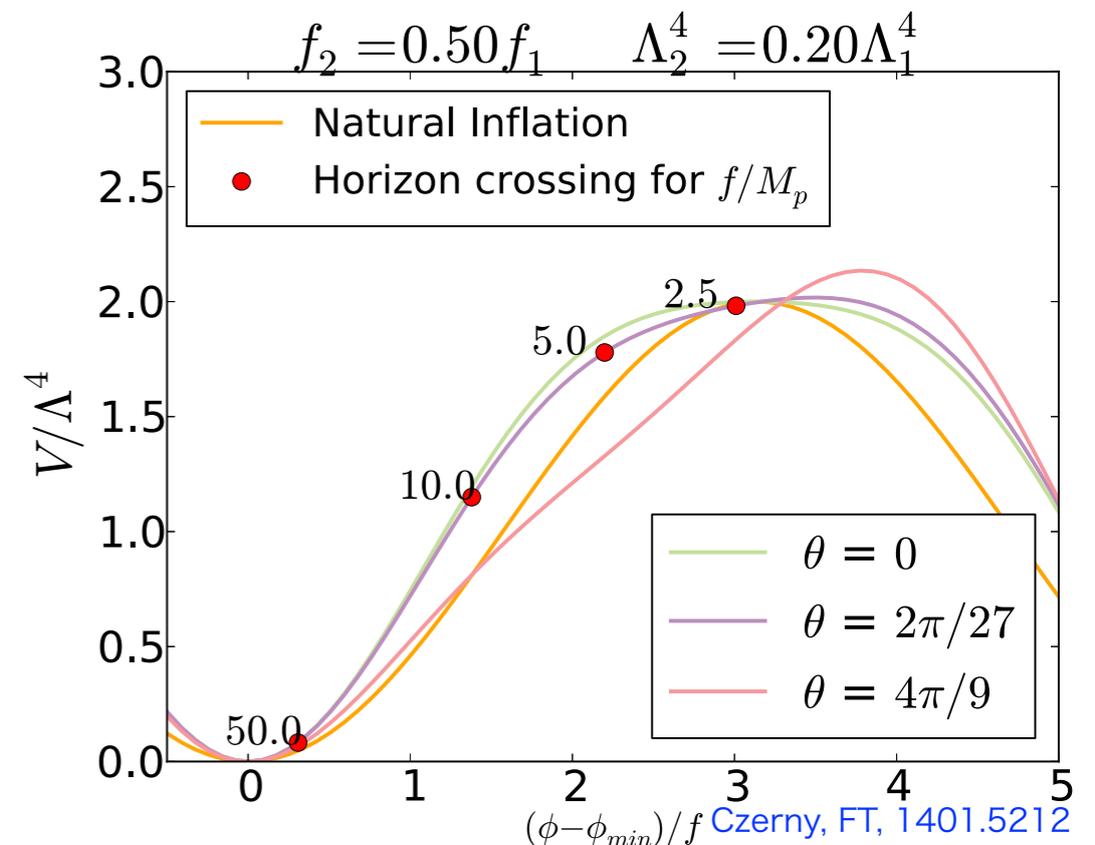
Nakayama, FT, Yanagida, 1303.7315

Multi-Natural inflation (MNI)

Czerny, FT 1401.5212
 Czerny, Higaki FT 1403.0410, 1403.5883

$$V(\phi) = C - \Lambda_1^4 \cos(\phi/f_1) - \Lambda_2^4 \cos(\phi/f_2 + \theta),$$

Sub-Planckian decay constants are allowed as hilltop inflation can be realized.



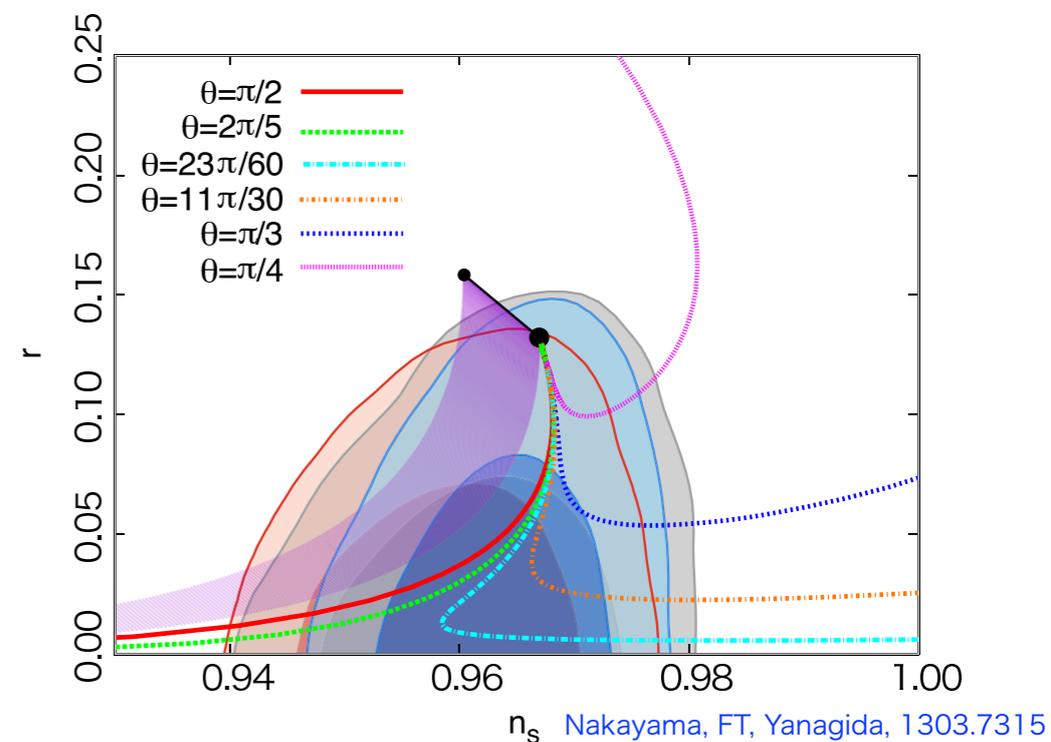
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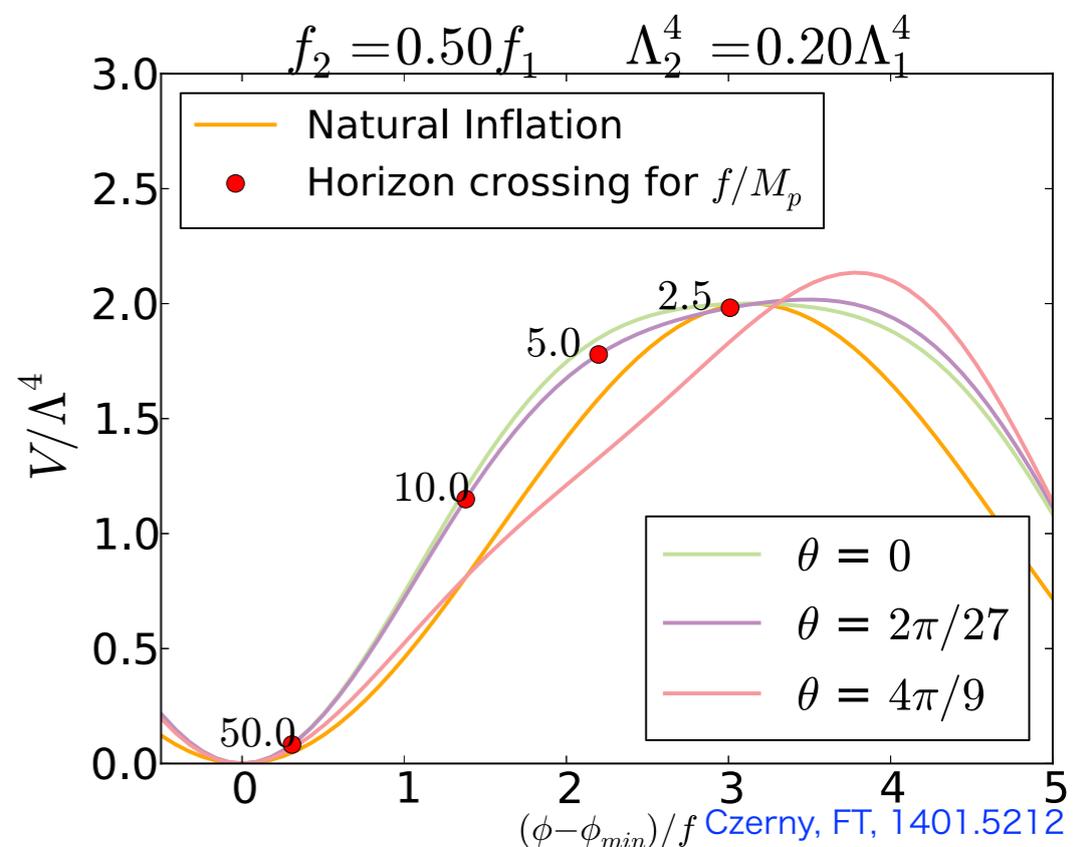


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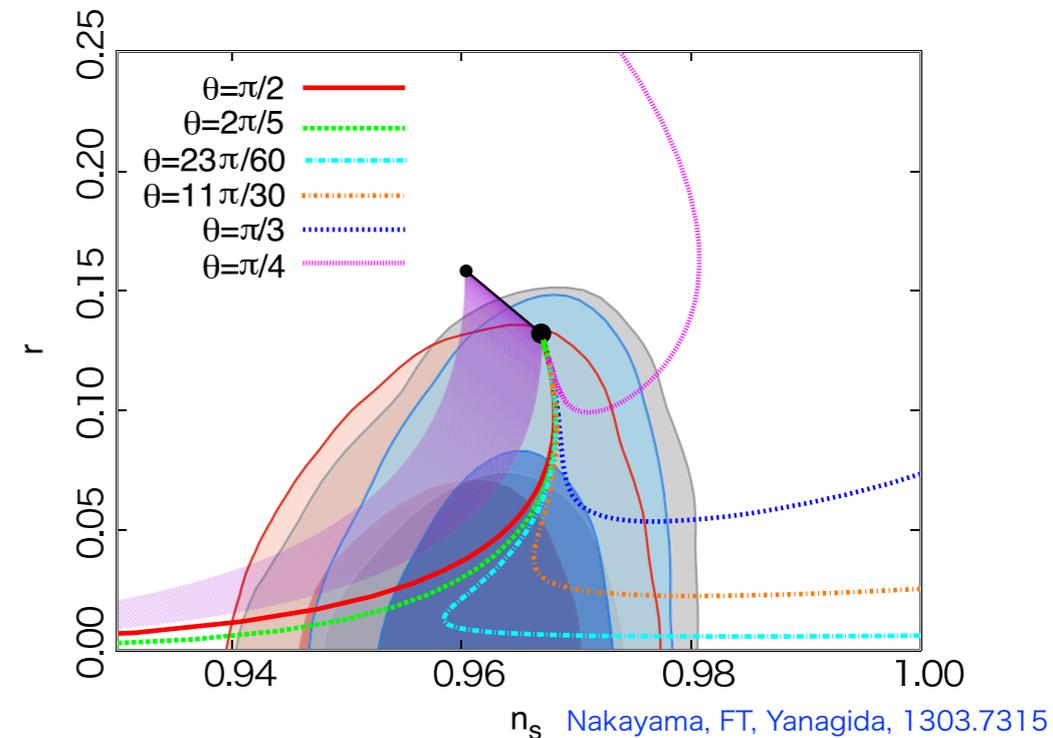


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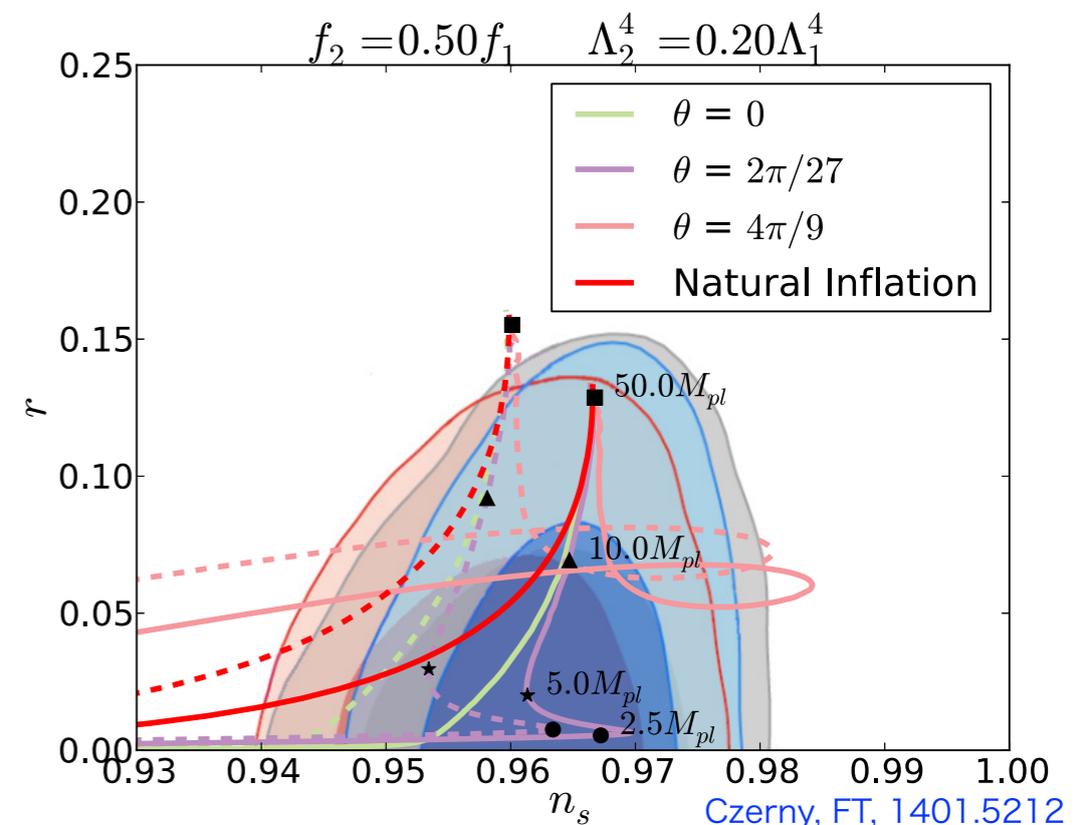


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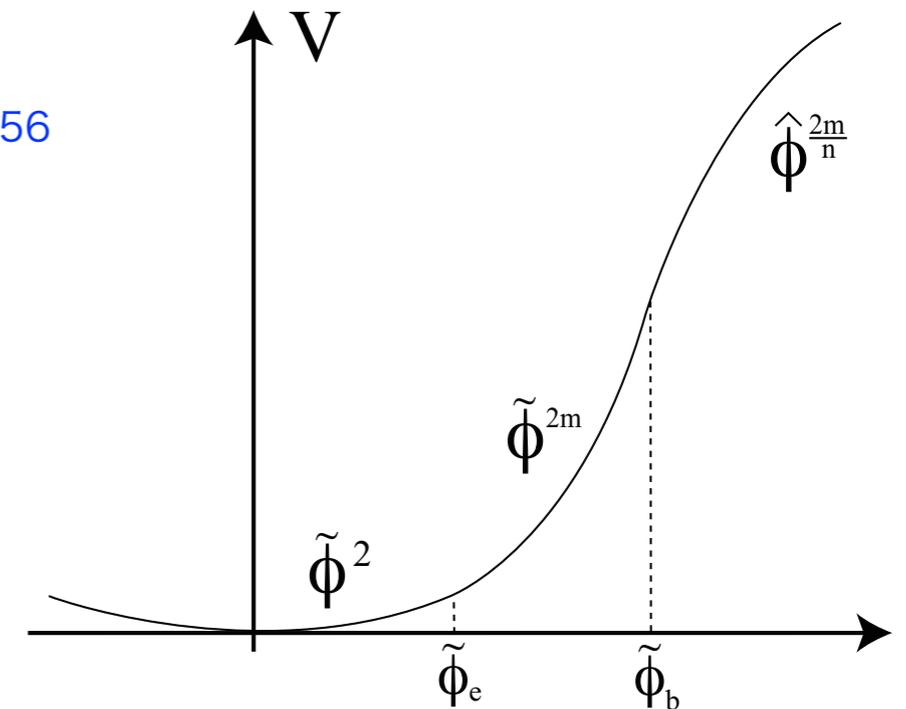
Running kinetic inflation

FT 1006.2801 Nakayama, FT 1008.2956

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \xi(\partial\phi^n)^2 - V(\phi)$$

Linear or fractional-power potential can be realized.

cf. Harigaya, Ibe, Schmitz, Yanagida, 1211.6241 for dynamical realization.



Axion monodromy inflation

$$V = \mu^3 \phi + \Lambda^4 \cos\left(\frac{\phi}{f_a}\right)$$

Silverstein, Westphal, 0803.3085

McAllister, Silverstein, Westphal, 0808.0706

Inflation and particle physics

Since early 80's, inflation models have been often designed in response to cosmological issues, not particle physics ones.

However, they are still closely related to each other;

- Realization of inflation in SUGRA/string theory
- Successful reheating (baryogenesis, dark matter, unwanted relics, etc.)
- Inflation dynamics and SUSY breaking

Chaotic inflation in SUGRA

Kawasaki, Yamaguchi, Yanagida, hep-ph/0004243 , hep-ph/0011104

To have a good control over the inflaton field values greater than the Planck scale, we impose a shift symmetry;

$$\phi \rightarrow \phi + iC,$$

which is explicitly broken by the superpotential.

$$K_{\text{inf}} = c(\phi + \phi^\dagger) + \frac{1}{2}(\phi + \phi^\dagger)^2 + |X|^2 - k|X|^4 + \dots$$

$$W_{\text{inf}} = mX\phi,$$

$$V_{\text{sugra}} = e^K \left((D_i W) K^{i\bar{j}} (D_{\bar{j}} W)^* - 3|W|^2 \right).$$

$$V \simeq \frac{1}{2} m^2 \varphi^2$$

$$\varphi \equiv \sqrt{2} \text{Im}[\phi]$$

even for $\varphi \gg M_p$

Z₂ or not Z₂?

- One can impose a Z₂ symmetry on the inflaton and X.

$$Z_2: \quad \phi \rightarrow -\phi \quad X \rightarrow -X$$

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The inflaton might be right-handed sneutrino.

Murayama, Nakayama, FT, Yanagida, 1404.3857

$$W = \underset{(-)(-)}{\phi LH_u} \quad \rightarrow \quad \mathcal{L} \sim \frac{(LH_u)^2}{M} \quad \text{Neutrino mass is a low-E consequence of the inflaton!}$$

Sneutrino Chaotic inflation

Murayama, Nakayama, FT, Yanagida, 1404.3857

We impose an approximate shift symmetry on one of N_i

$$K = |N_1|^2 + |N_2|^2 + \frac{1}{2}(N_3 + N_3^\dagger)^2 + \dots$$

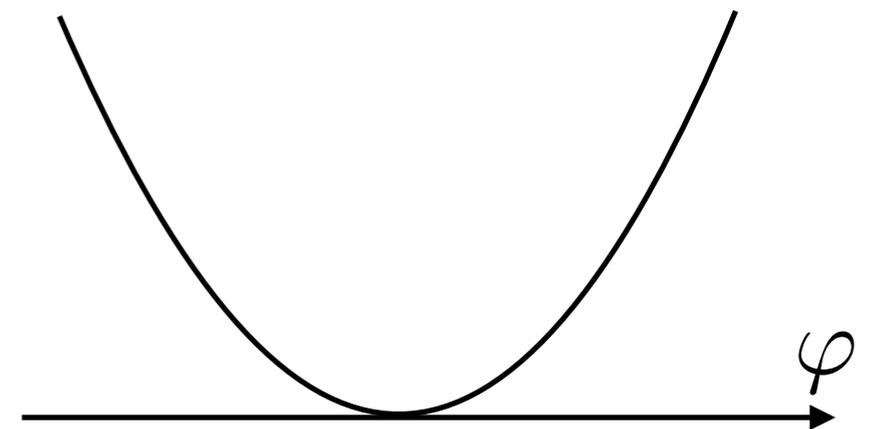
$$W = \frac{1}{2}M_{ij}N_iN_j + h_{i\alpha}N_iL_\alpha H_u$$

with

$$M_{ij} = \begin{pmatrix} m & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix}$$

The inflaton is $\varphi = \sqrt{2}\text{Im}N_3$

$$V = \frac{1}{2}M^2\varphi^2$$



All the other directions can be stabilized during inflation.

Chaotic inflation w/o Z_2

$$K = c(\phi + \phi^\dagger) + \dots \quad \rightarrow \quad \langle K_\phi \rangle = c = \mathcal{O}(1)$$

- **Pros**

The inflaton automatically decays into the visible sector w/o introducing ad hoc couplings.

[Endo, Kawasaki, FT, Yanagida, hep-ph/0607170](#)

[Endo, FT, Yanagida, hep-ph/0701042](#)

$$\Gamma \sim \frac{m^3}{M_P^2}, \quad T_R \sim 10^9 \text{ GeV}$$

Thermal leptogenesis is likely

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- **Cons**

The inflaton decays into hidden sectors, producing too many gravitinos.

[Endo, Hamaguchi, FT, `06](#)

[Nakamura, Yamaguchi, `06](#)

[Kawasaki, FT, Yanagida, `06](#)

[Dine, Kitano, Morisse, Shirman, `06](#)

[Endo, FT, Yanagida, `06, `07](#)

$$\Gamma(\phi \rightarrow 2\psi_{3/2}) \sim \frac{m^3}{M_P^2}$$

Gravitino production in chaotic inflation w/o Z_2

Nakayama, FT, Yanagida, 1404.2472

Let us add a SUSY breaking field z ;

$$K = K_{\text{inf}} + |z|^2 - \frac{|z|^4}{\Lambda^2}, \quad m_z^2 \simeq \frac{12m_{3/2}^2}{\Lambda^2}.$$
$$W = W_{\text{inf}} + \mu^2 z + W_0, \quad \langle z \rangle \simeq 2\sqrt{3} \left(\frac{m_{3/2}}{m_z} \right)^2 \simeq \frac{m_{3/2}}{m_z} \Lambda.$$

There are various sources for gravitino production;

- Thermal production
- Non-thermal production
 - Inflaton decays into gravitinos
 - Inflaton decays into z .
 - The z coherent oscillations.

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Let us add a SUSY breaking field z ;

$$K = K_{\text{inf}} + |z|^2 - \frac{|z|^4}{\Lambda^2}, \quad m_z^2 \simeq \frac{12m_{3/2}^2}{\Lambda^2}.$$
$$W = W_{\text{inf}} + \mu^2 z + W_0, \quad \langle z \rangle \simeq 2\sqrt{3} \left(\frac{m_{3/2}}{m_z} \right)^2 \simeq \frac{m_{3/2}}{m_z} \Lambda.$$

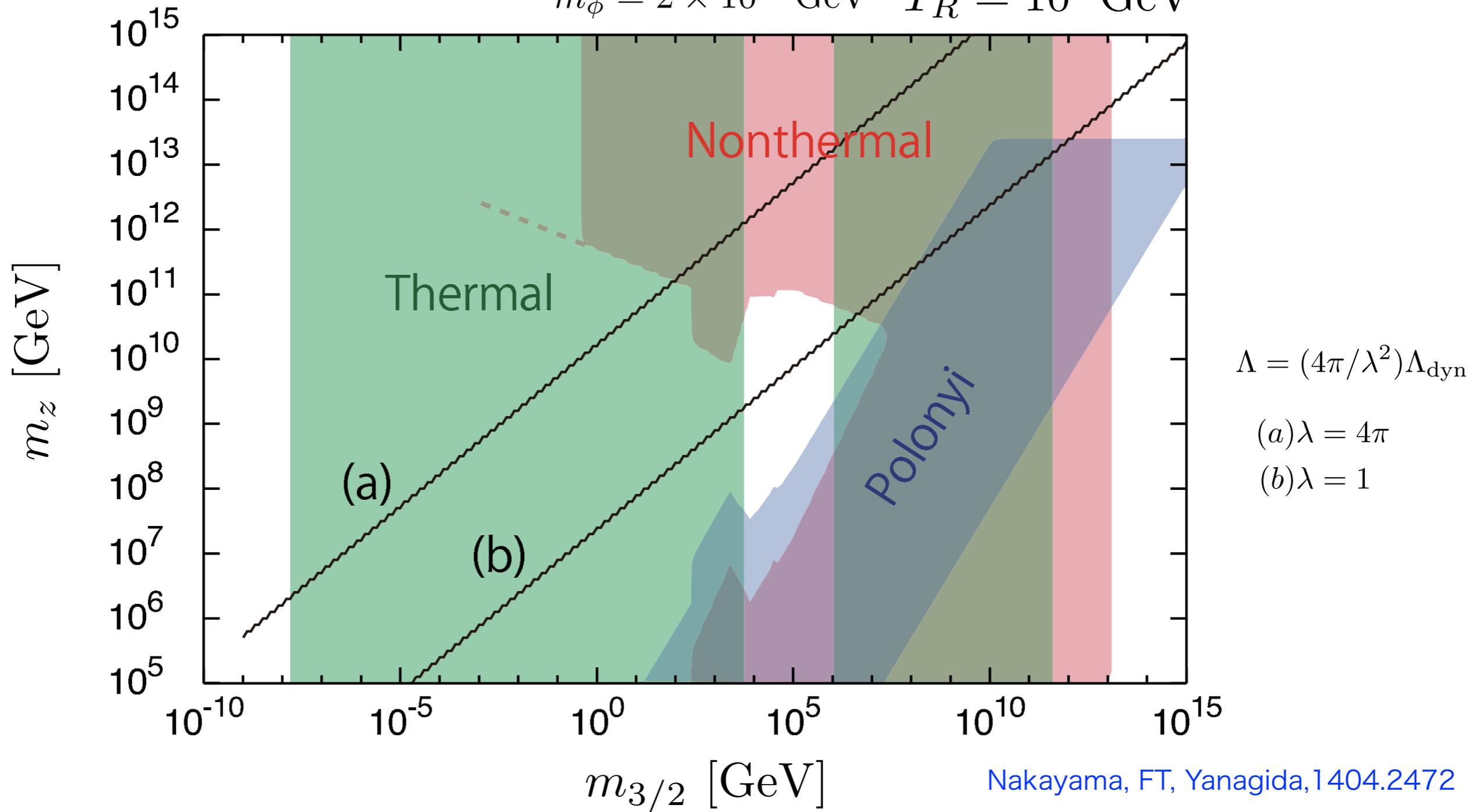
There are various sources for gravitino production;

$$Y_{3/2} = Y_{3/2}^{(\text{th})} + Y_{3/2}^{(\phi)} + Y_{3/2}^{(z)}.$$

$$Y_{3/2}^{(\text{th})} \simeq \begin{cases} \min \left[2 \times 10^{-12} \left(1 + \frac{m_{\tilde{g}}^2}{3m_{3/2}^2} \right) \left(\frac{T_{\text{R}}}{10^{10} \text{ GeV}} \right), \frac{0.42}{g_{*s}(T_{3/2})} \right] & \text{for } T_{\text{R}} \gtrsim m_{\text{SUSY}}, \\ 0 & \text{for } T_{\text{R}} \lesssim m_{\text{SUSY}}, \end{cases}$$

$$Y_{3/2}^{(\phi)} = \frac{3T_{\text{R}}}{4m} \frac{2\Gamma(\Phi \rightarrow \tilde{z}\tilde{z}) + 4\Gamma(\Phi \rightarrow zz^\dagger)}{\Gamma_{\text{tot}}}, \quad Y_{3/2}^{(z)} \simeq \frac{2}{m_z} \frac{\rho_z}{s}.$$

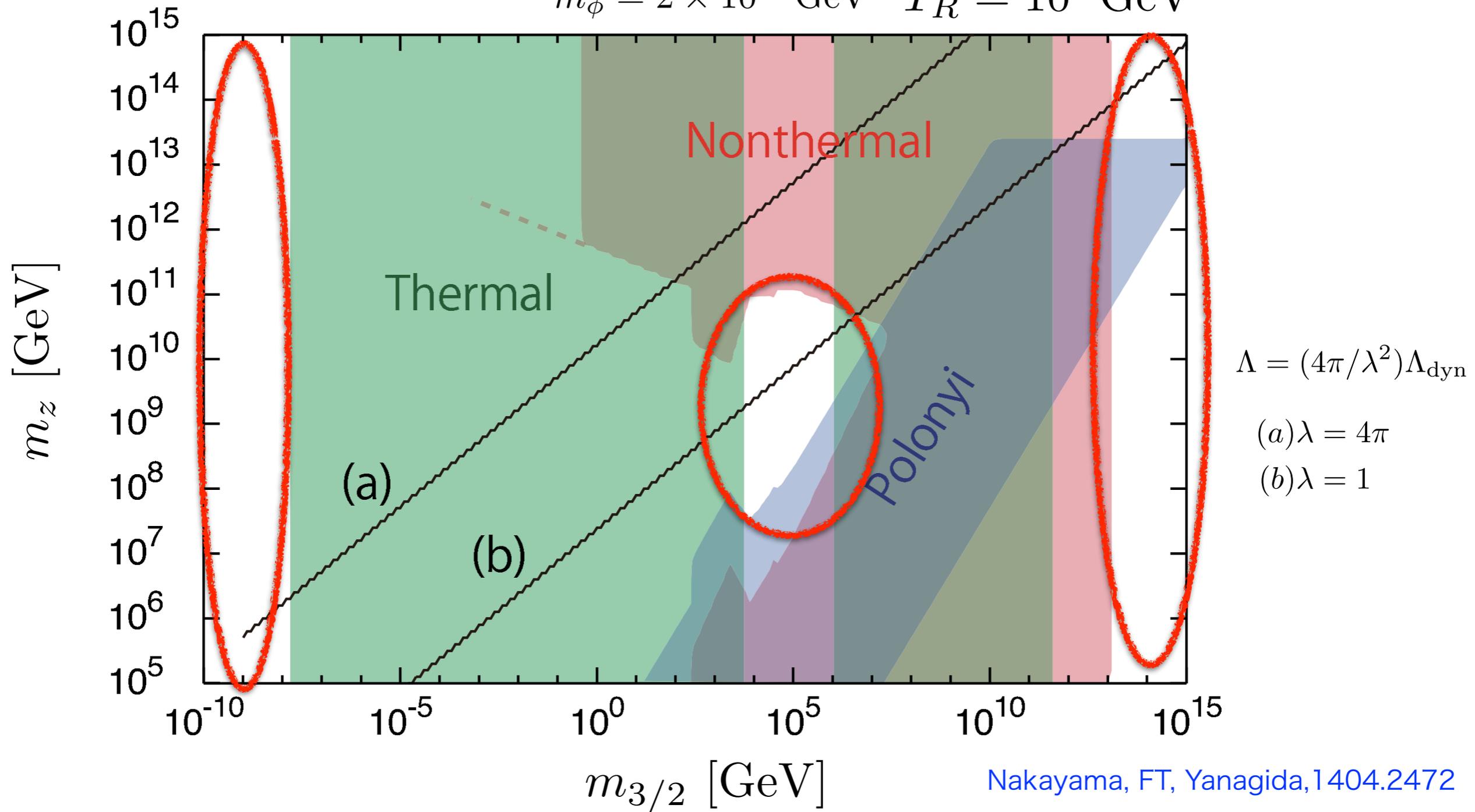
$$m_\phi = 2 \times 10^{13} \text{ GeV} \quad T_R = 10^9 \text{ GeV}$$



3 allowed regions:

- (i) $m_{3/2} \lesssim 16\text{eV}$
- (ii) $m_{3/2} = 10 - 1000\text{TeV}$
- (iii) $m_{3/2} \gtrsim 10^{13} \text{ GeV}$

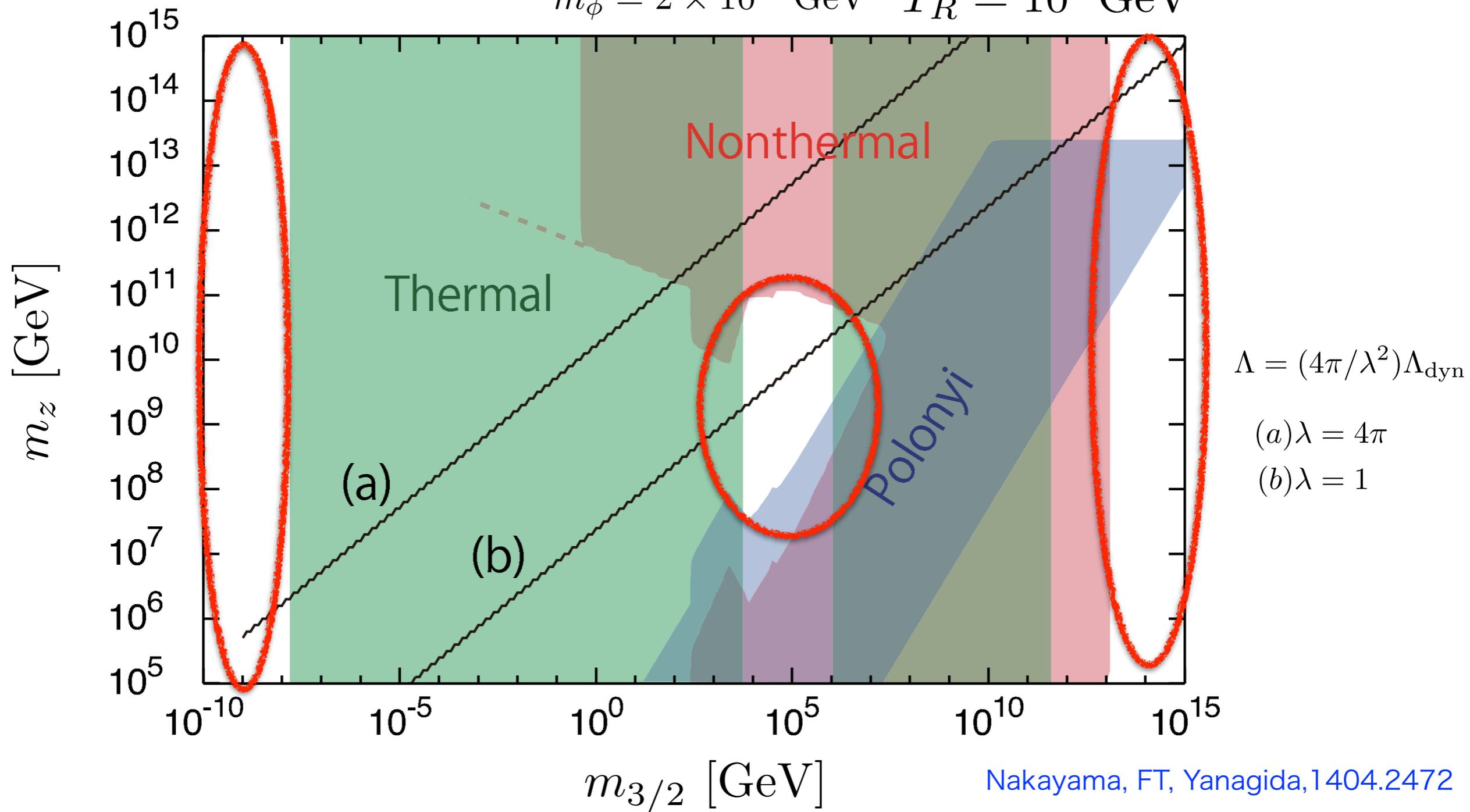
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consistent with
126 GeV Higgs.

Polynomial chaotic inflation in SUGRA

Nakayama, FT, Yanagida 1303.7315,1305.5099

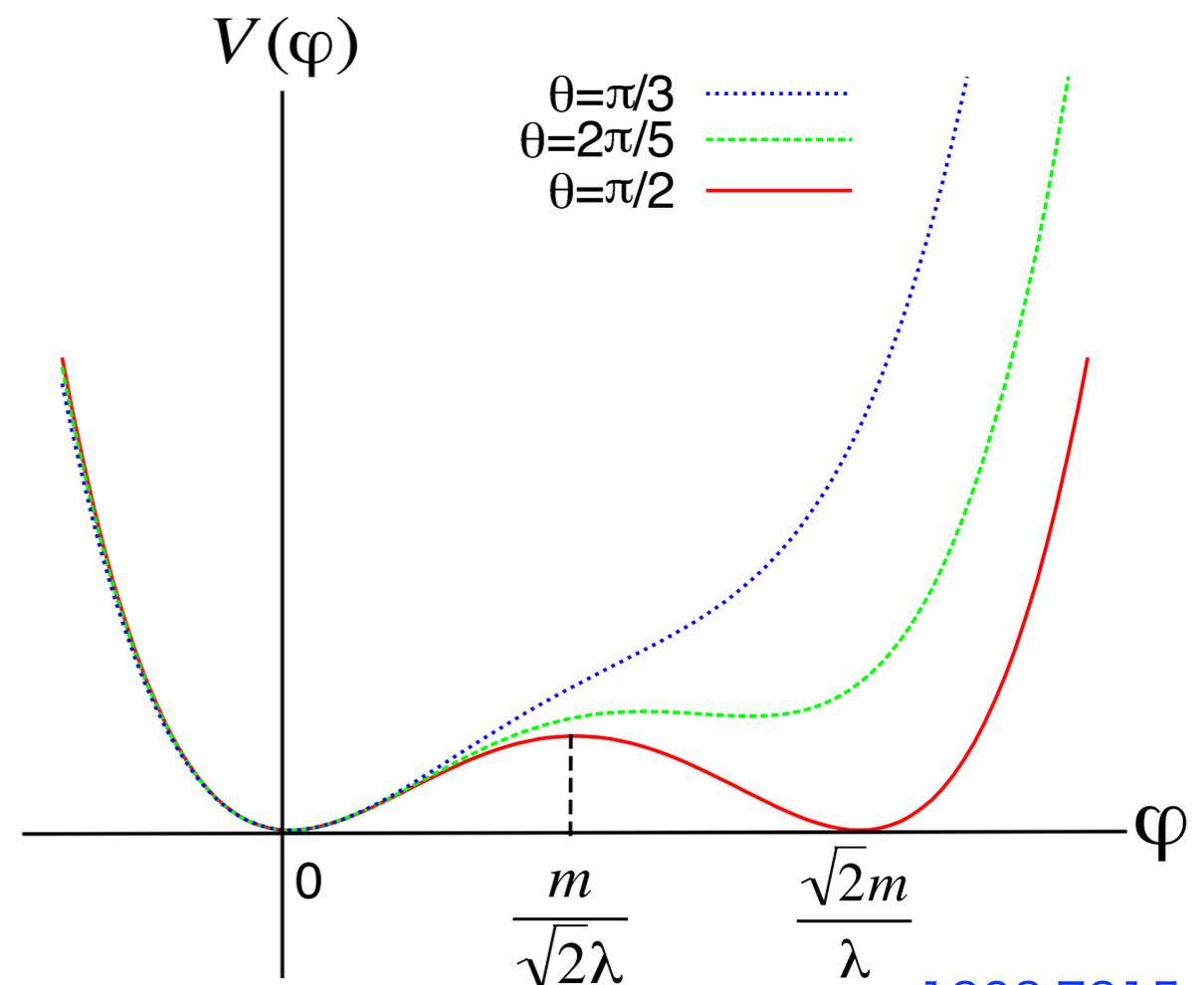
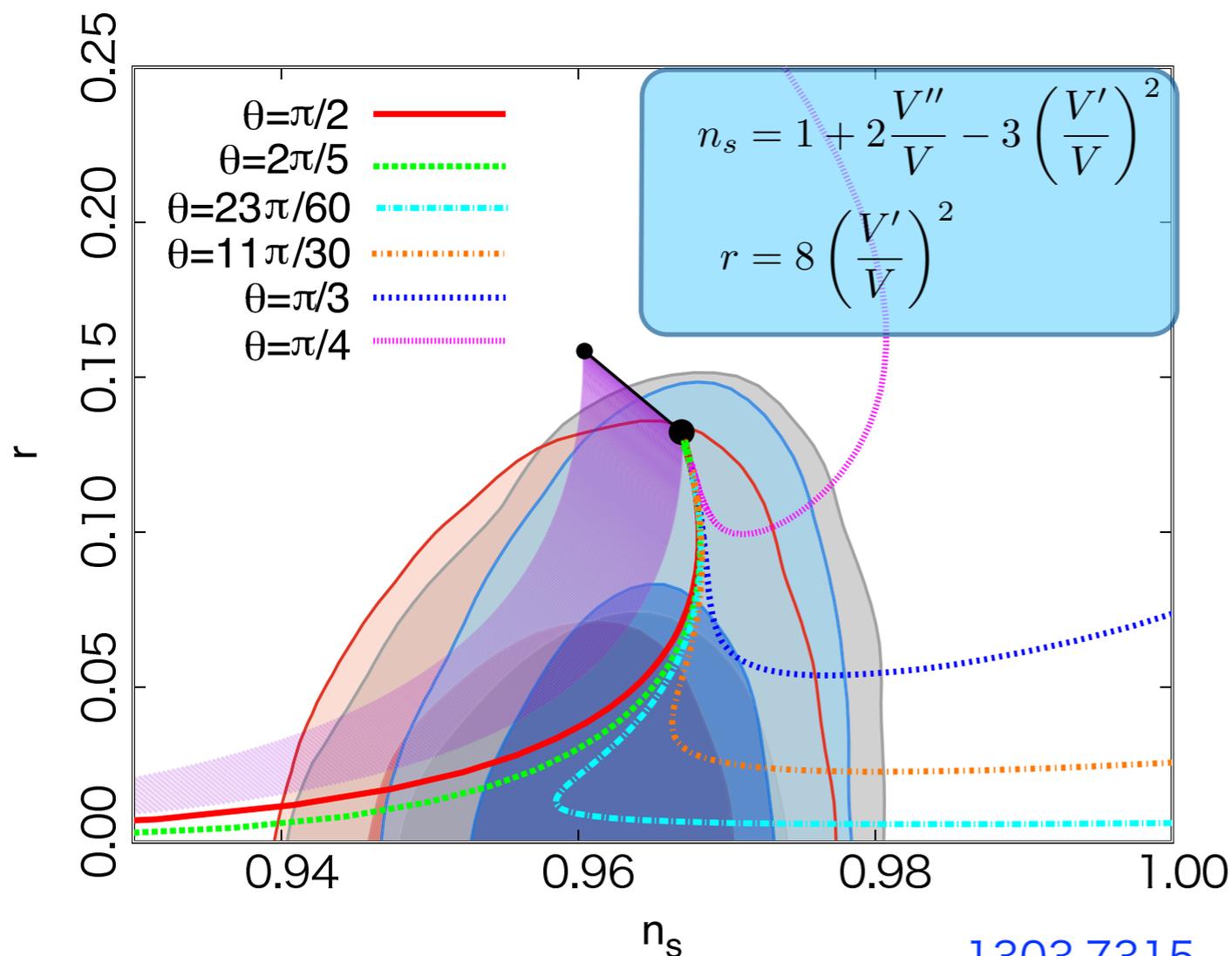
(cf. Kallosh, Linde, Westphal 1405.0270)

$$K = \frac{1}{2}(\phi + \phi^\dagger)^2 + |X|^2 + \dots,$$

$$W = X(m\phi + k_2\phi^2 + \dots),$$

$$V \simeq \frac{1}{2}\varphi^2 \left(m^2 - \sqrt{2}m\lambda \sin\theta \varphi + \frac{\lambda^2}{2}\varphi^2 \right)$$

$$\lambda = |k_2| \quad \theta = \arg[k_2] \quad \text{Re}[\phi] \lesssim 1$$



1303.7315

1303.7315

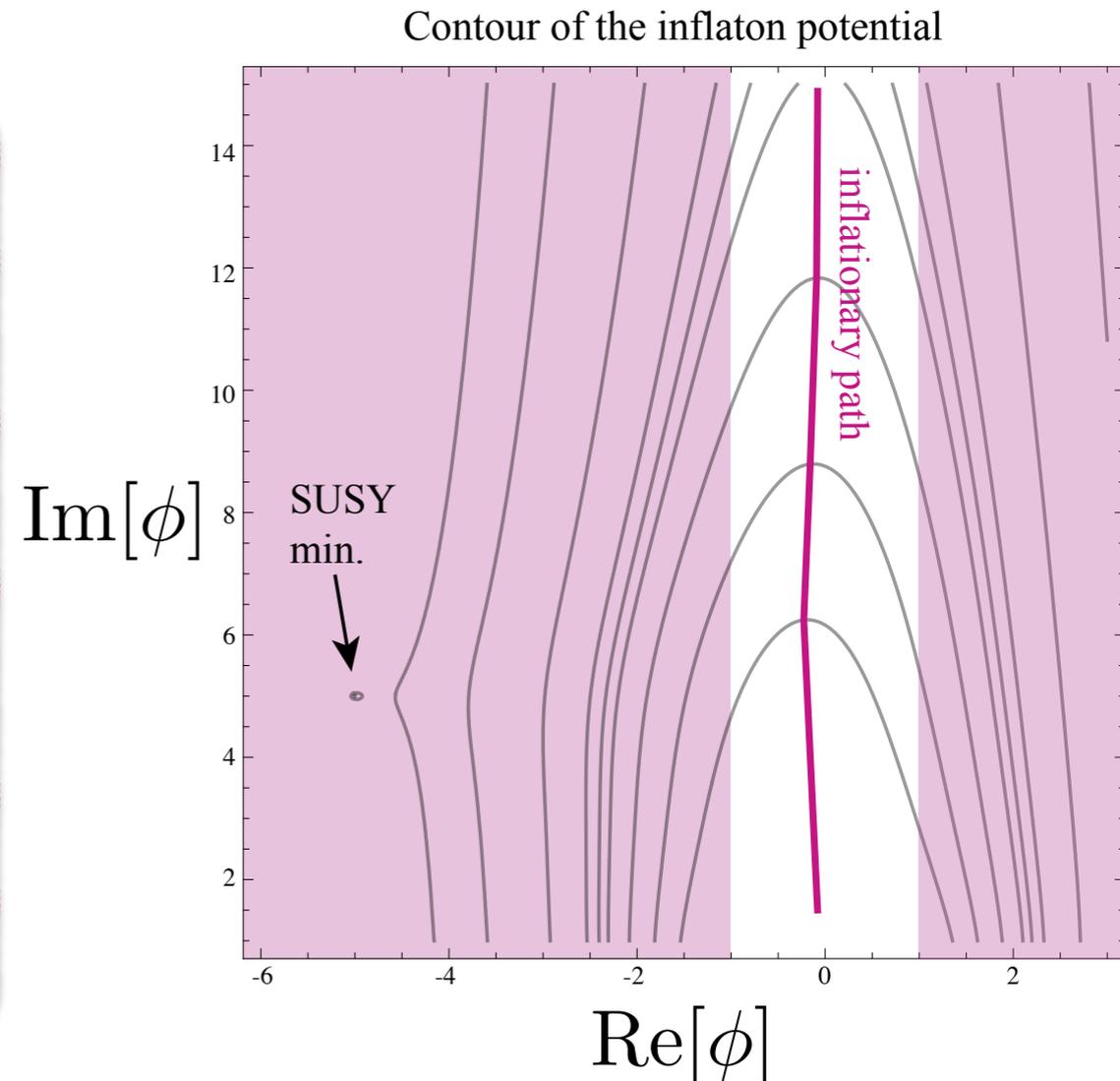
Side remark

Very precisely speaking, the inflaton potential is not exactly polynomial, because $\text{Re}[\phi] \neq 0$ for $\theta \neq 0$.

There is also a SUSY min. at $\text{Re}[\phi] = -\frac{m}{\lambda} e^{-i\theta}$.

Linde 1402.0526

- However, the inflaton dynamics can be well approximated by the polynomial potential.
- Numerically confirmed that (n_s, r) remain intact even if this effect is taken into account.

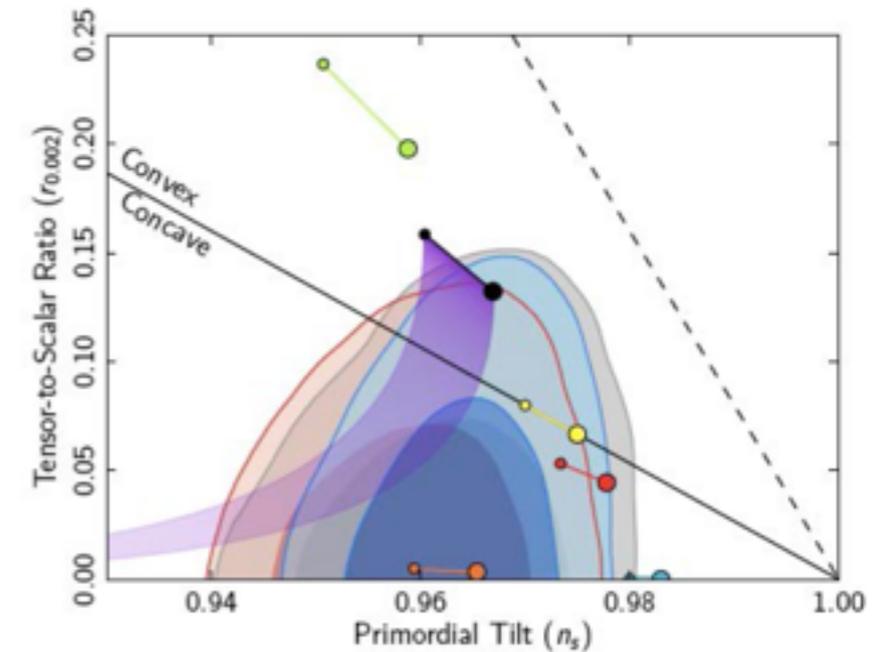


Natural and Multi-Natural Inflation

- Natural inflation [Freese, Frieman, Olinto '90](#)

$$V = \Lambda^4 \left(1 - \cos \left(\frac{\phi}{f} \right) \right)$$

Only large-field inflation is possible,
and f is bounded below: $f \gtrsim 5M_P$

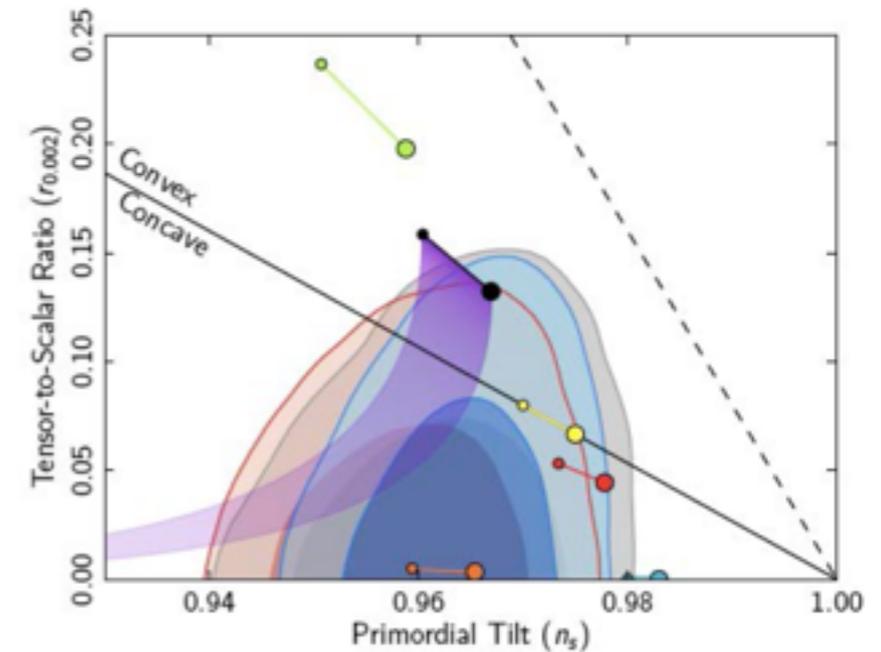


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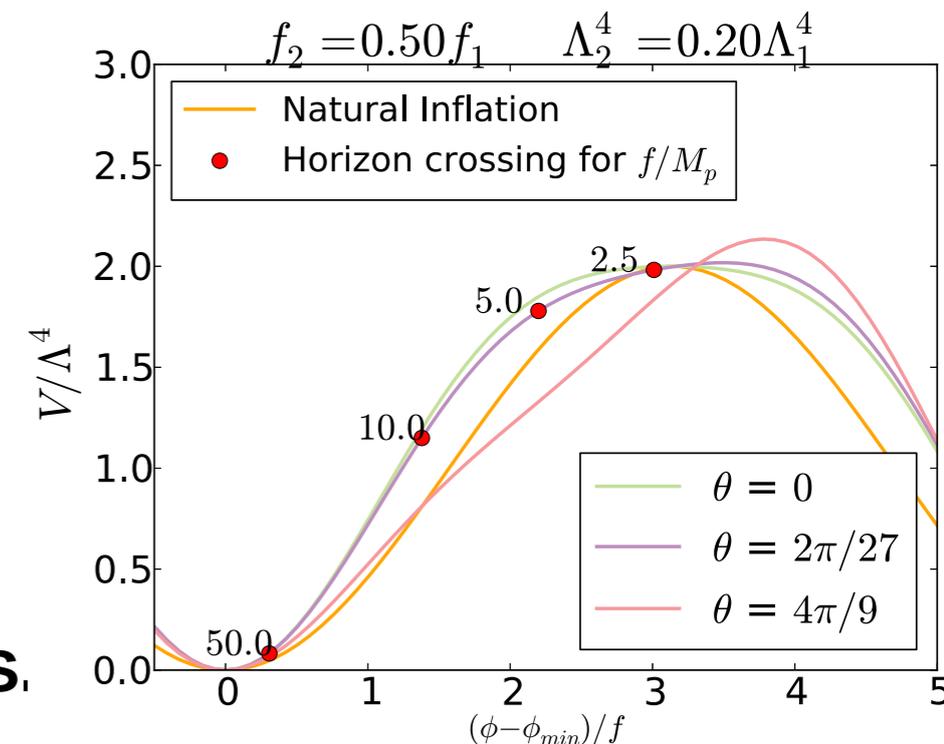
- Multi-Natural inflation

[Czerny, FT 1401.5212](#), [Czerny, Higaki, FT 1403.0410](#), [1403.5883](#)

$$V = \sum_{i=1}^{N_{\text{source}}} \Lambda_i^4 \cos \left(\frac{\phi}{f_i} + \theta_i \right) + \text{const.}$$

For $N_{\text{source}} = 2$, various values of (n_s, r) are possible as in the polynomial chaotic inf.

No lower bound on the decay constants.

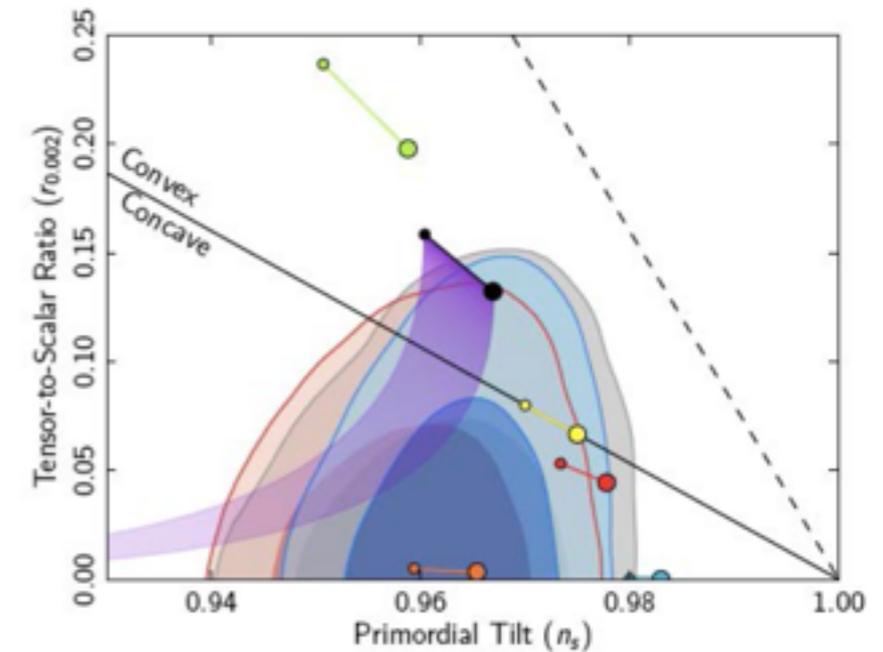


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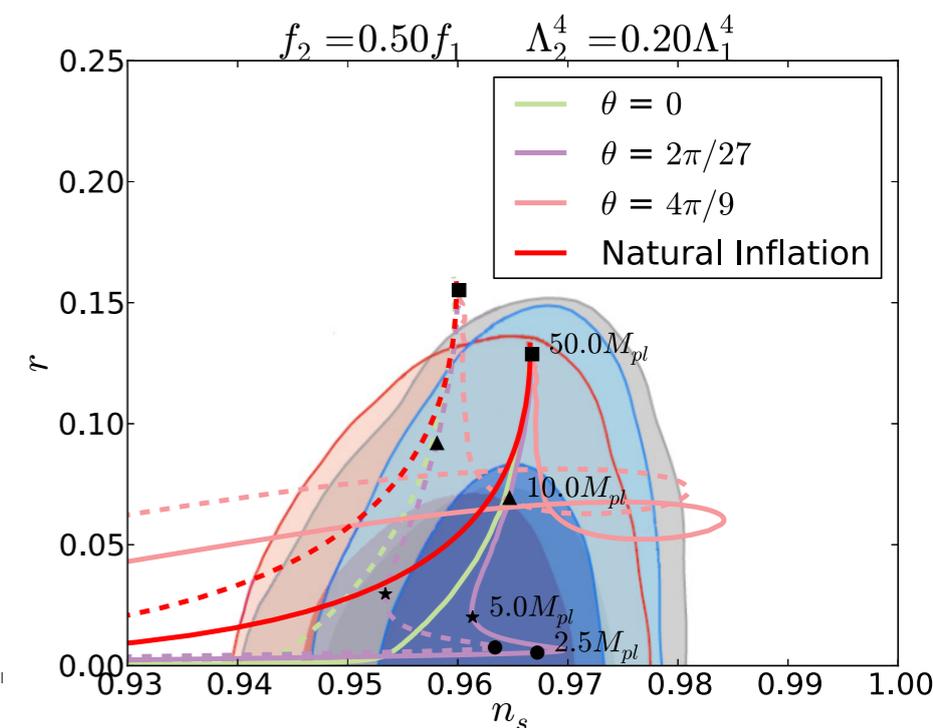
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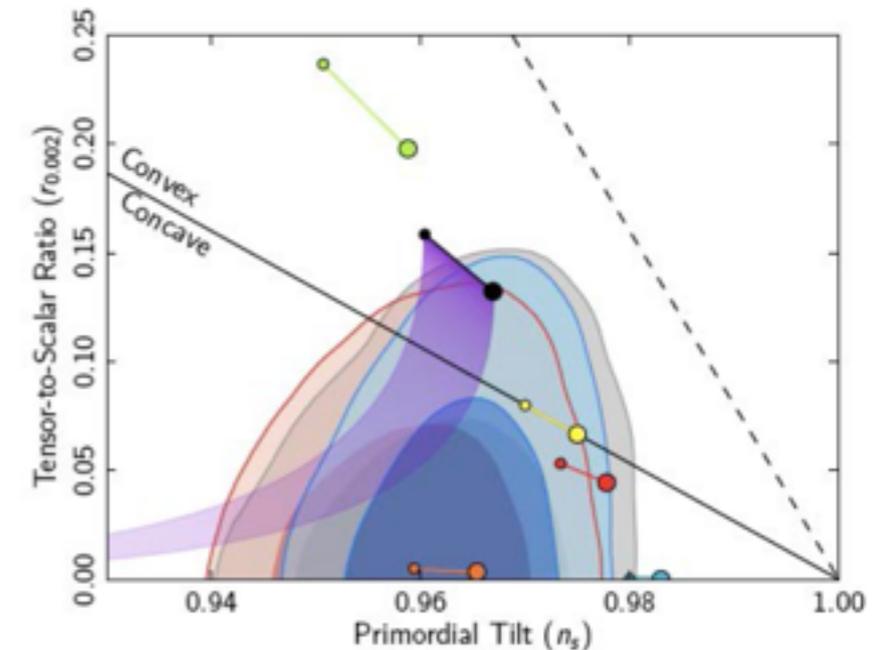


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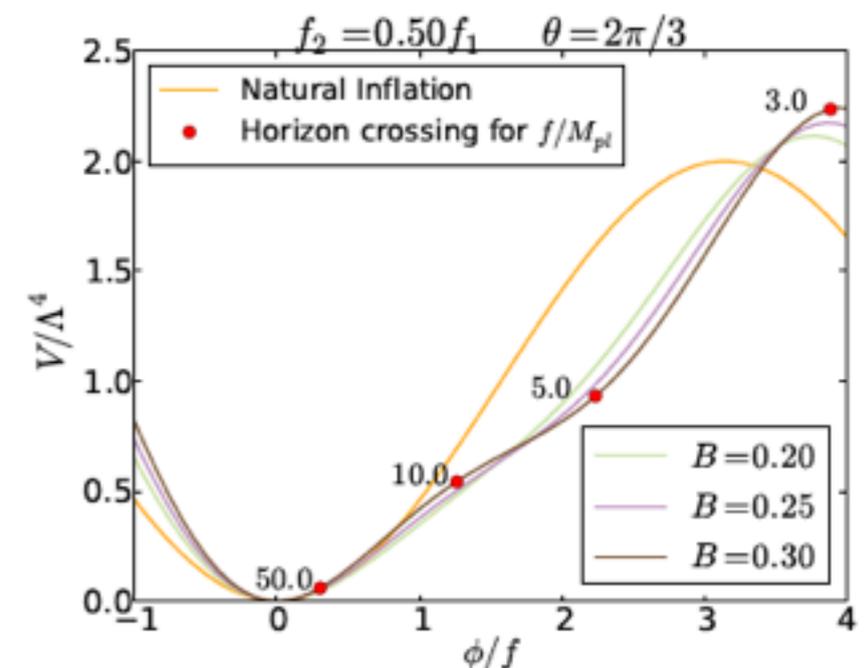
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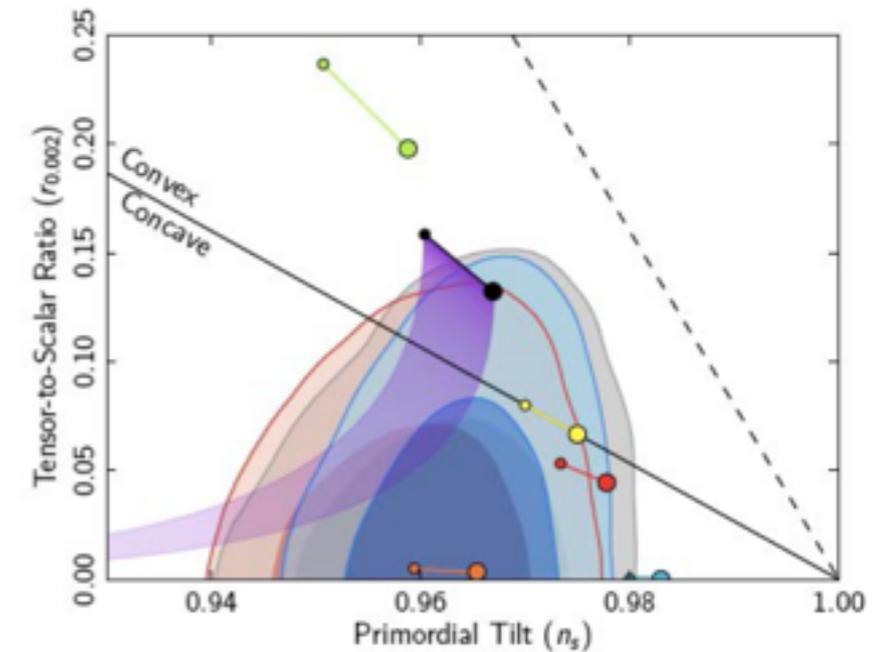


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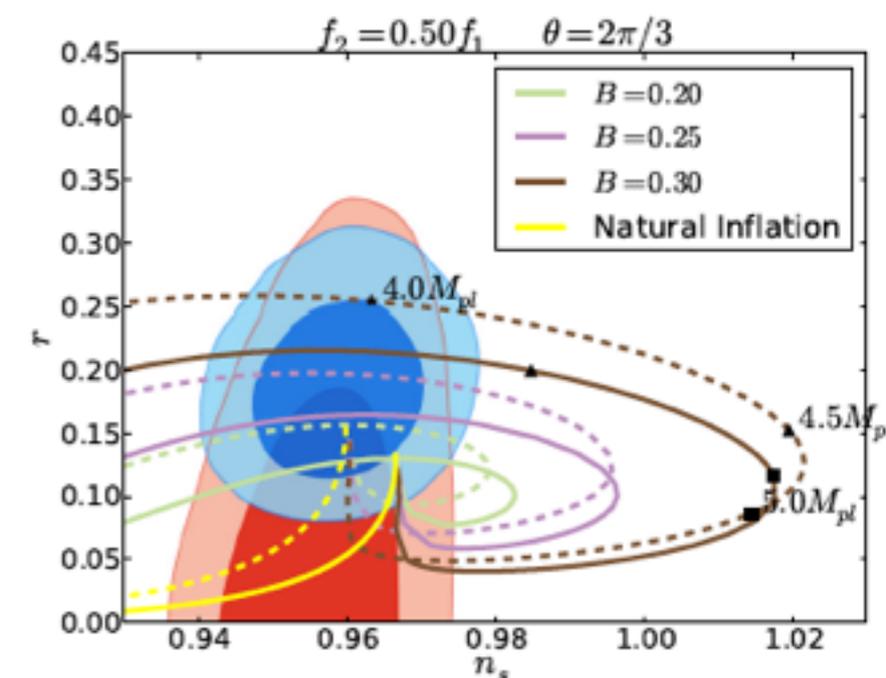
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Axion hilltop inflation

(Small-field Multi-Natural inflation)

Czerny, FT 1401.5212

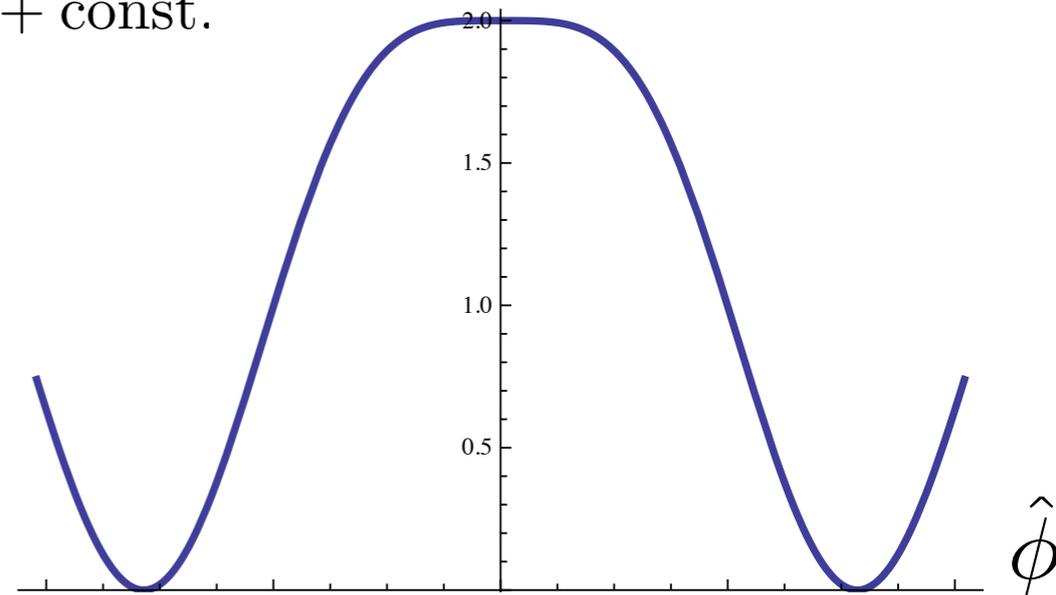
Czerny, Higaki FT 1403.0410

Hilltop quartic inflation (new inflation) can be realized by requiring a flat-top potential in multi-natural inflation.

$$V(\phi) = \Lambda_1^4 \left(1 - \cos \left(\frac{\phi}{f_1} \right) \right) + \Lambda_2^4 \left(1 - \cos \left(\frac{\phi}{f_2} + \theta \right) \right) + \text{const.}$$

$$\simeq V_0 - \lambda \hat{\phi}^4 + \dots \quad \hat{\phi} \equiv \phi - \pi f_1$$

$$\text{for } \frac{\Lambda_1^2}{f_1} = \frac{\Lambda_2^2}{f_2} \text{ and } \theta = -\pi \frac{f_1}{f_2}$$



Axion hilltop inflation is possible for $f < M_{\text{P}}$.

- Simple realization of hilltop inflation by axion.
- The potential shape is under control.
- Spectral index can give a better fit to the Planck data by a slight shift of the phase.

Axion hilltop inflation

(Small-field Multi-Natural inflation)

Czerny, FT 1401.5212

Czerny, Higaki FT 1403.0410

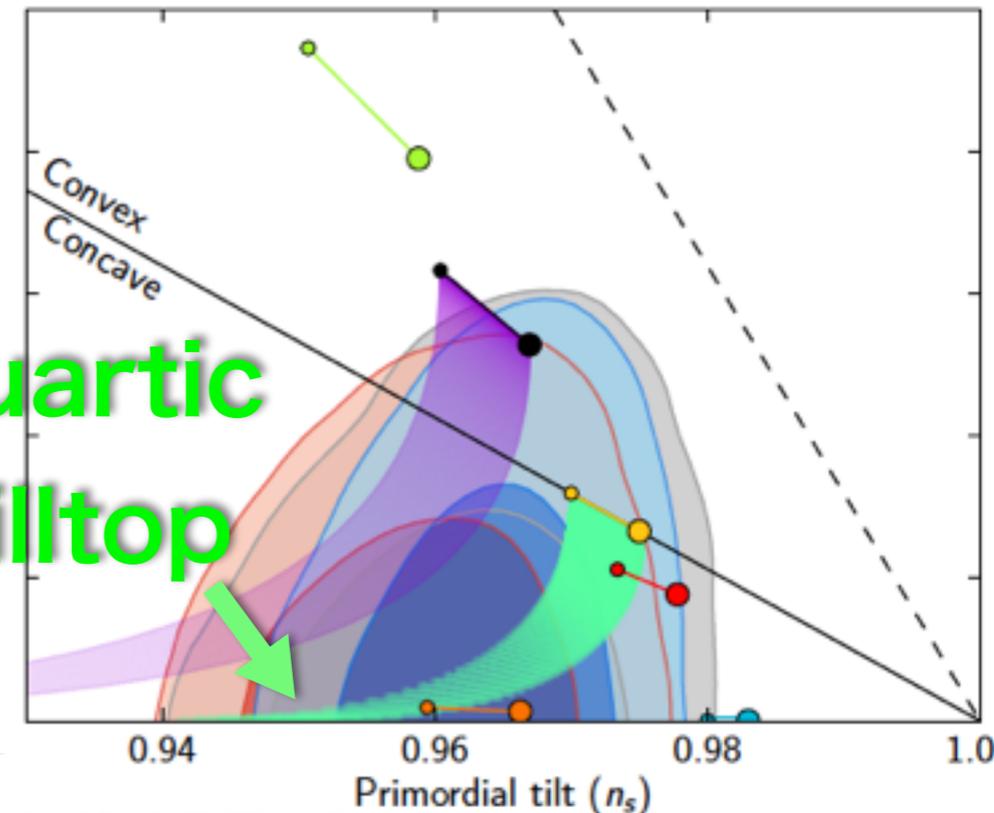
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Quartic
hilltop

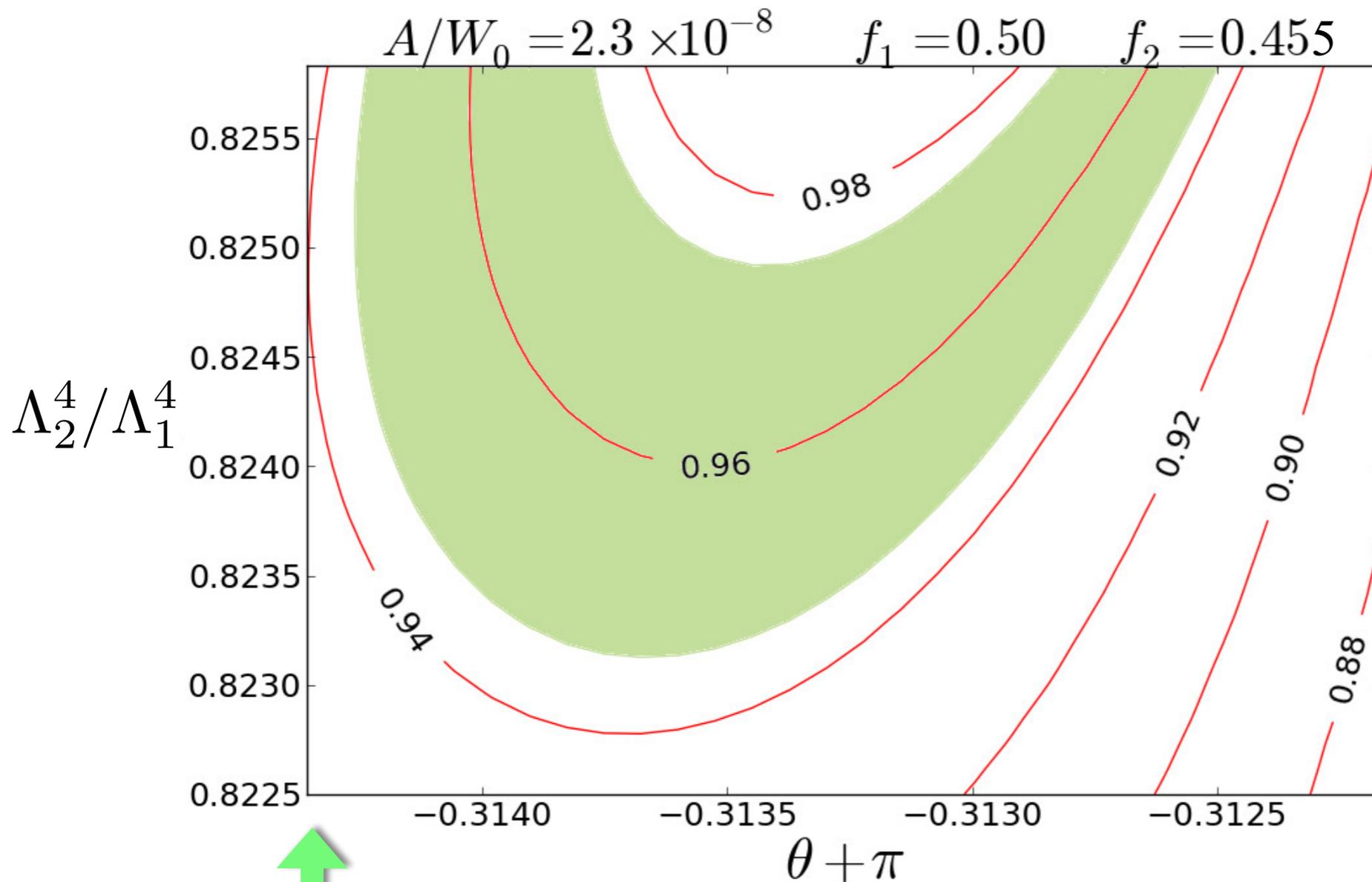


Axion hilltop inflation is possible for $f < M_{\text{Pl}}$.

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- Spectral index can give a better fit to the Planck data by a slight shift of the phase.

Spectral index of axion hilltop inflation

$$V(\phi) = \Lambda_1^4 \left(1 - \cos \left(\frac{\phi}{f_1} \right) \right) + \Lambda_2^4 \left(1 - \cos \left(\frac{\phi}{f_2} + \theta \right) \right) + \text{const.}$$



Relative phase leads to the effective linear term, which affects n_s in hilltop inflation.

cf. FT 1308.4212

Czerny, Higaki FT 1403.0410

Quartic hilltop inflation

Natural and Multi-Natural Inflation in SUGRA

Czerny, Higaki, FT 1403.0410 (before BICEP2) , 1403.5883 (after BICEP2)

$$K = \frac{f^2}{2} (\Phi + \Phi^\dagger)^2 + \dots$$

$$W = W_0 + Ae^{-a\Phi} + Be^{-i\theta} e^{-b\Phi}$$

Natural inflation if $B=0$.
Kalosh, hep-th/0702059

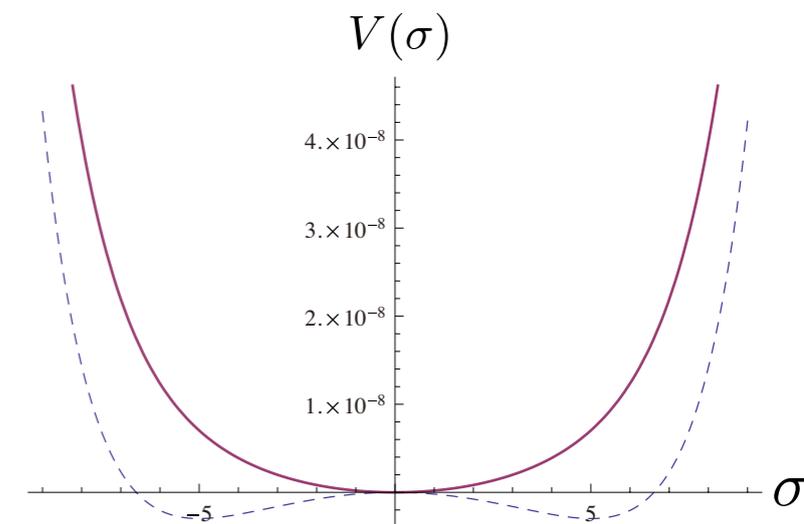
where $A, B \ll W_0 < 1, a > 0, b > 0, a \neq b$

$$\Phi = \sigma + i\varphi$$

The saxion is stabilized around the origin by SUSY breaking effect.

$$V = e^{2f^2\sigma^2} \left(4f^2\sigma^2 - 3 \right) |W_0|^2 + \Delta V_{\text{up-lift}}$$
$$\simeq 2f^2 |W_0|^2 \sigma^2 + \dots,$$

The saxion mass: $m_\sigma \simeq \sqrt{2} m_{3/2}$



Natural and Multi-Natural Inflation in SUGRA

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where $A, B \ll W_0 < 1$, $a > 0, b > 0, a \neq b$

The axion potential:

$$V_{\text{axion}}(\phi) \simeq 6AW_0 \left[1 - \cos \left(\frac{\phi}{f_1} \right) \right] + 6BW_0 \left[1 - \cos \left(\frac{\phi}{f_2} + \theta \right) \right] + \text{const}$$
$$f_1 \equiv \frac{\sqrt{2}f}{a}, \quad f_2 \equiv \frac{\sqrt{2}f}{b},$$

Large-field NI/MNI requires super-Planckian f_1 and/or f_2 .

Small-field MNI possible if $A/B \approx f_1^2/f_2^2$ and $\theta \approx -\pi f_1/f_2$

Natural and Multi-Natural Inflation in SUGRA

Czerny, Higaki, FT 1403.0410 (before BICEP2) , 1403.5883 (after BICEP2)

Effective large decay constant can be realized by the alignment of (more than) two axions.

Kim, Nilles, Peloso, hep-ph/0409138

$$K = \frac{f^2}{2} (\Phi_1 + \Phi_1^\dagger)^2 + \frac{f^2}{2} (\Phi_2 + \Phi_2^\dagger)^2,$$

$$W = W_0 + C e^{-c(\Phi_1 + \Phi_2)} + A e^{-a(\Phi_1 + (1 + \Delta_1)\Phi_2)} + B e^{-b(\Phi_1 + (1 + \Delta_2)\Phi_2) - i\theta},$$

$$A \sim B \ll C \ll W_0$$

Heavier moduli

$$\sqrt{2}\xi \equiv \phi_1 + \phi_2$$

Lighter one appears with small coefficient due to the alignment.

$$\sqrt{2}\phi \equiv -\phi_1 + \phi_2$$

$$V_{\text{axion}}^{(\text{eff})}(\phi) \approx -6W_0 A \cos\left[\frac{\phi}{f_1}\right] - 6W_0 B \cos\left[\frac{\phi}{f_2} + \theta\right] + \text{const.},$$

$$f_1 \equiv \frac{2}{a\Delta_1} f, \quad f_2 \equiv \frac{2}{b\Delta_2} f. \quad f_1 = \mathcal{O}(10) \text{ for e.g. } a = \frac{2\pi}{n_1}, \quad n_1 = \mathcal{O}(10),$$

$$\Delta_1 = \mathcal{O}(0.1), \quad f = \mathcal{O}(0.1)$$

Aligned Natural Inflation

Kim, Nilles, Peloso, hep-ph/0409138

Czerny, Higaki, FT 1403.5883, Harigaya and Ibe 1404.3511, Choi, Kim, Yun, 1404.6209, Higaki, FT, 1404.6923, Tye, Won, 1404.6988, Kappl, Krippendorf, Nilles, 1404.7127, Bachlechner et al, 1404.7496, Ben-Dayan, Pedro, Westphal, 1404.7773, Long, McAllister, McGuirk 1404.7852

The effectively large decay constant can be realized by the alignment of two (or more) axion potentials.

- Two axions: $\phi_1 \rightarrow \phi_1 + 2\pi f_1$ $\phi_2 \rightarrow \phi_2 + 2\pi f_2$

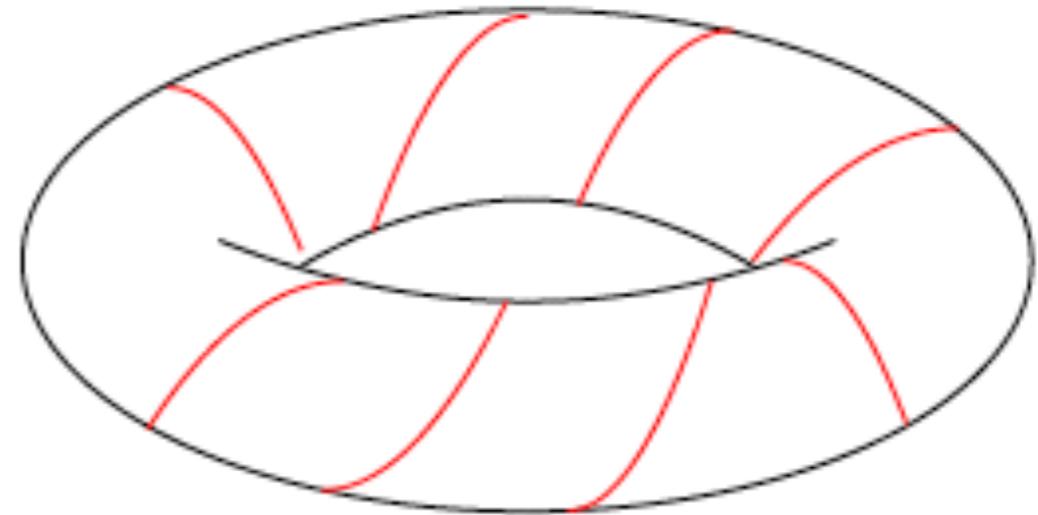
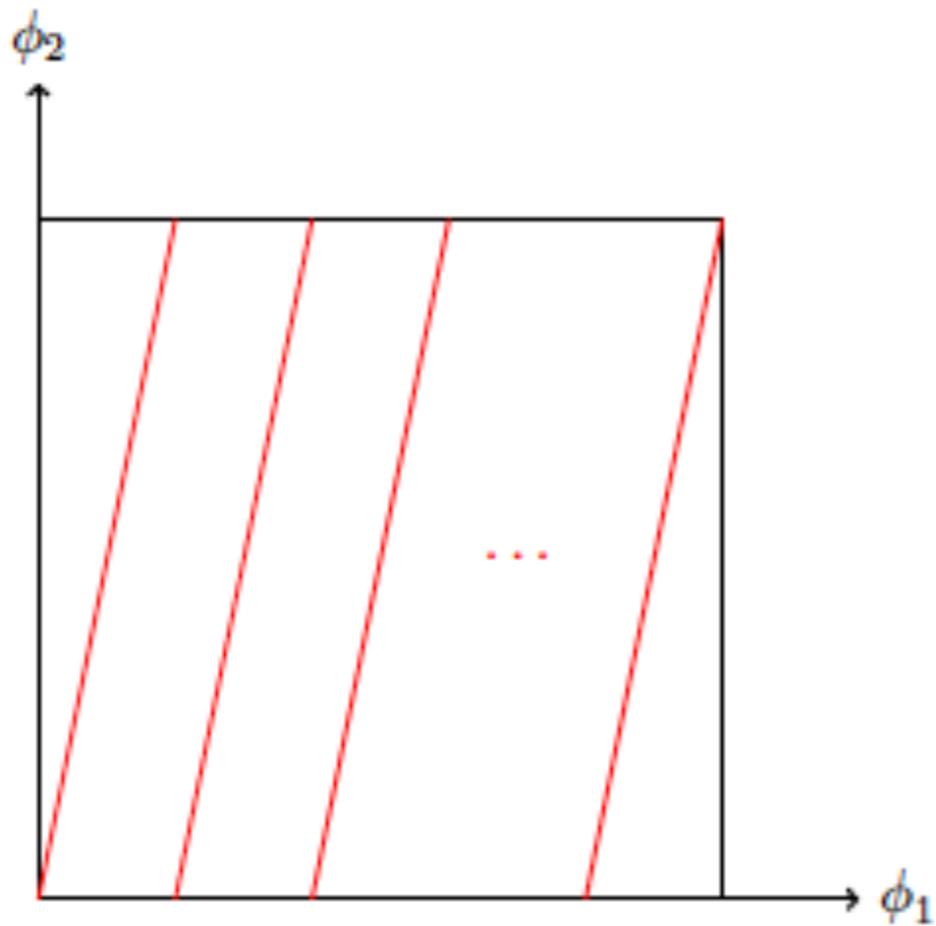
$$V(\phi_i) = \Lambda_1^4 \left[1 - \cos \left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2} \right) \right] + \Lambda_2^4 \left[1 - \cos \left(m_1 \frac{\phi_1}{f_1} + m_2 \frac{\phi_2}{f_2} \right) \right]$$

If $n_1/n_2 = m_1/m_2$, there is a flat direction; the corresponding decay constant would be infinite.

If $n_1/n_2 \approx m_1/m_2$, there is a relatively light direction; the corresponding decay constant can be larger than f_1 or f_2 .

Aligned Natural Inflation

Kim, Nilles, Peloso, hep-ph/0409138



Taken from 1404.6209 by Choi, Kim, Yun

Aligned Natural Inflation

e.g.) $n_2 \gg n_1 \sim m_2 \sim \mathcal{O}(1)$ and $m_1 = 0$

$$f_1 = f_2, \Lambda_1 = \Lambda_2$$

Tye, Won, 1404.6988

Ben-Dayan, Pedro, Westphal, 1404.7773

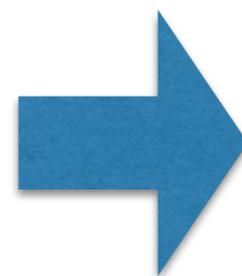
See also Harigaya's talk

$$V(\phi_i) = \Lambda^4 \left[2 - \cos \left(n_1 \frac{\phi_1}{f} + n_2 \frac{\phi_2}{f} \right) - \cos \left(m_2 \frac{\phi_2}{f} \right) \right]$$

$$M_{ij}^2 = \frac{\Lambda^4}{f^2} \begin{pmatrix} n_1^2 & n_1 n_2 \\ n_1 n_2 & n_2^2 + m_2^2 \end{pmatrix}$$

$$\sqrt{\det M^2} = n_1 m_2 \frac{\Lambda^4}{f^2}$$

$$\text{Tr}[M^2] = (n_1^2 + n_2^2 + m_2^2) \frac{\Lambda^4}{f^2}$$



$$M_{\text{light}} \sim \frac{\Lambda^2}{n_2 f}$$

$$f_{\text{eff}} \sim n_2 f$$

Aligned Natural Inflation

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Tye, Won, 1404.6988

Ben-Dayan, Pedro, Westphal, 1404.7773

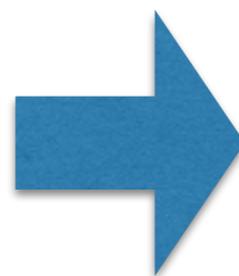
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Tye, Won, 1404.6988

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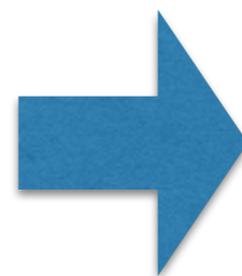
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Tye, Won, 1404.6988

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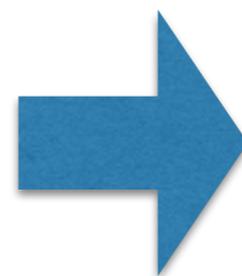
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Aligned Natural Inflation

Kim, Nilles, Peloso, hep-ph/0409138

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$$V(\phi_i) = \Lambda_1^4 \left[1 - \cos \left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2} \right) \right] + \Lambda_2^4 \left[1 - \cos \left(m_1 \frac{\phi_1}{f_1} + m_2 \frac{\phi_2}{f_2} \right) \right]$$

For $\Lambda_1 \gg \Lambda_2$, the effective decay constant is

$$f_{\text{eff}} = \frac{\sqrt{n_1^2 f_2^2 + n_2^2 f_1^2}}{|n_1 m_2 - n_2 m_1|}$$

Some hierarchy among the anomaly coefficients are needed to realize a large enhancement of $O(100)$.

Aligned Natural Inflation

- Multiple axions: $\phi_i \equiv \phi_i + 2\pi f_i \quad (i = 1, \dots, N)$

$$V(\phi_i) = \sum_{i=1}^N \Lambda_i^4 \left[1 - \cos \left(\sum_{j=1}^N \frac{n_{ij} \phi_j}{f_j} \right) \right]$$

For a moderately large N (> 5 or so), the effective decay constant can be enhanced w/o hierarchy among the anomaly coefficients.

[Choi, Kim, Yun, 1404.6209](#)

Prob. dist. was studied in detail for various cases incl. $N_{\text{source}} \neq N_{\text{axion}}$

[Higaki, FT, 1404.6923](#)

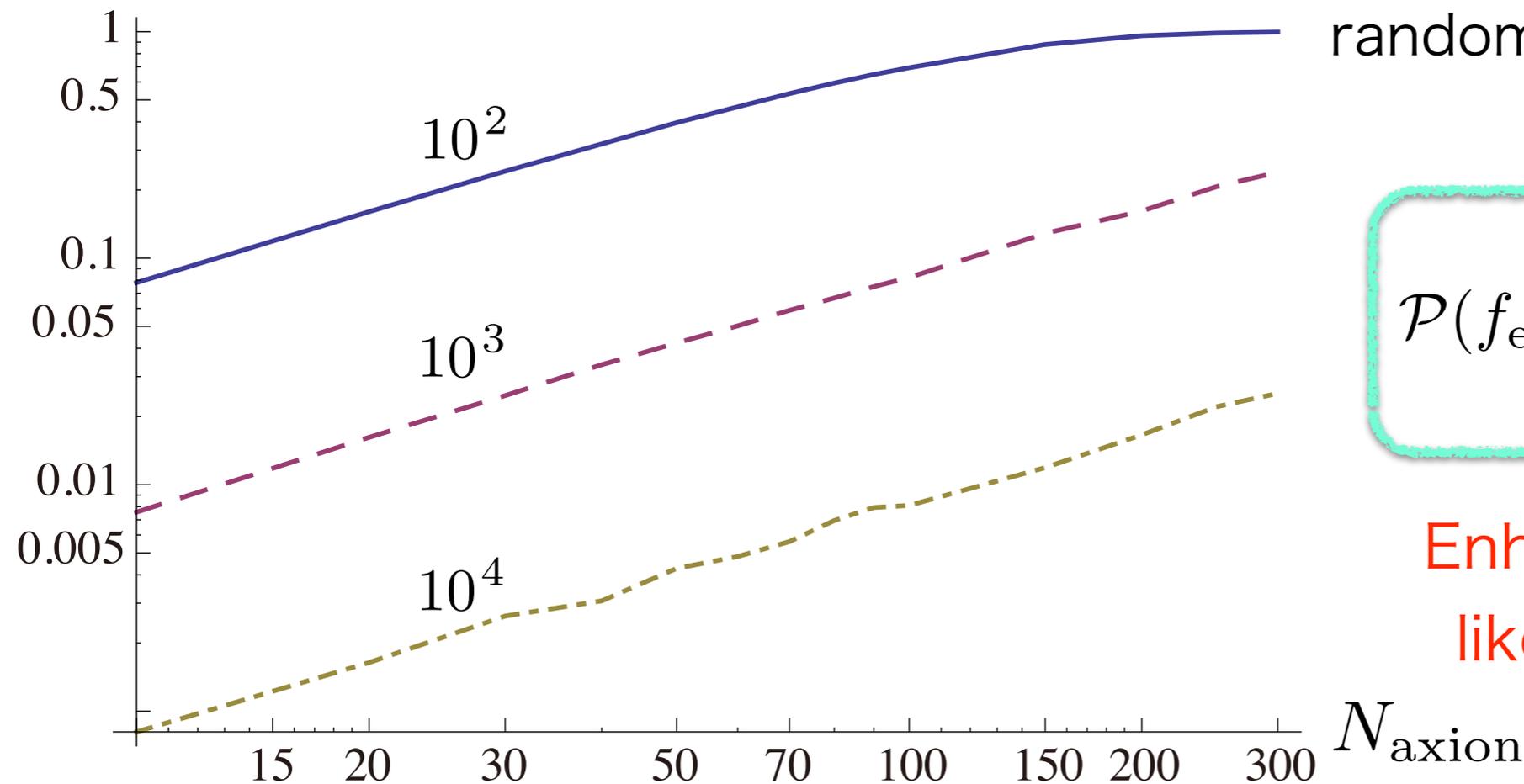
Aligned Natural Inflation

$$V(\phi_i) = \sum_{i=1}^{N_{\text{source}}} \Lambda_i^4 \cos \left(\sum_{j=1}^{N_{\text{axion}}} a_{ij} \frac{\phi_j}{f_j} + \theta_i \right) + V_0$$

- Prob dist for the enhancement of the decay constant

$$\mathcal{P}(f_{\text{eff}}/f_i > 10^x)$$

$$N_{\text{axion}} = N_{\text{source}}, \quad n = 2$$



We generated integer-valued random matrix $-n \leq a_{ij} \leq n$

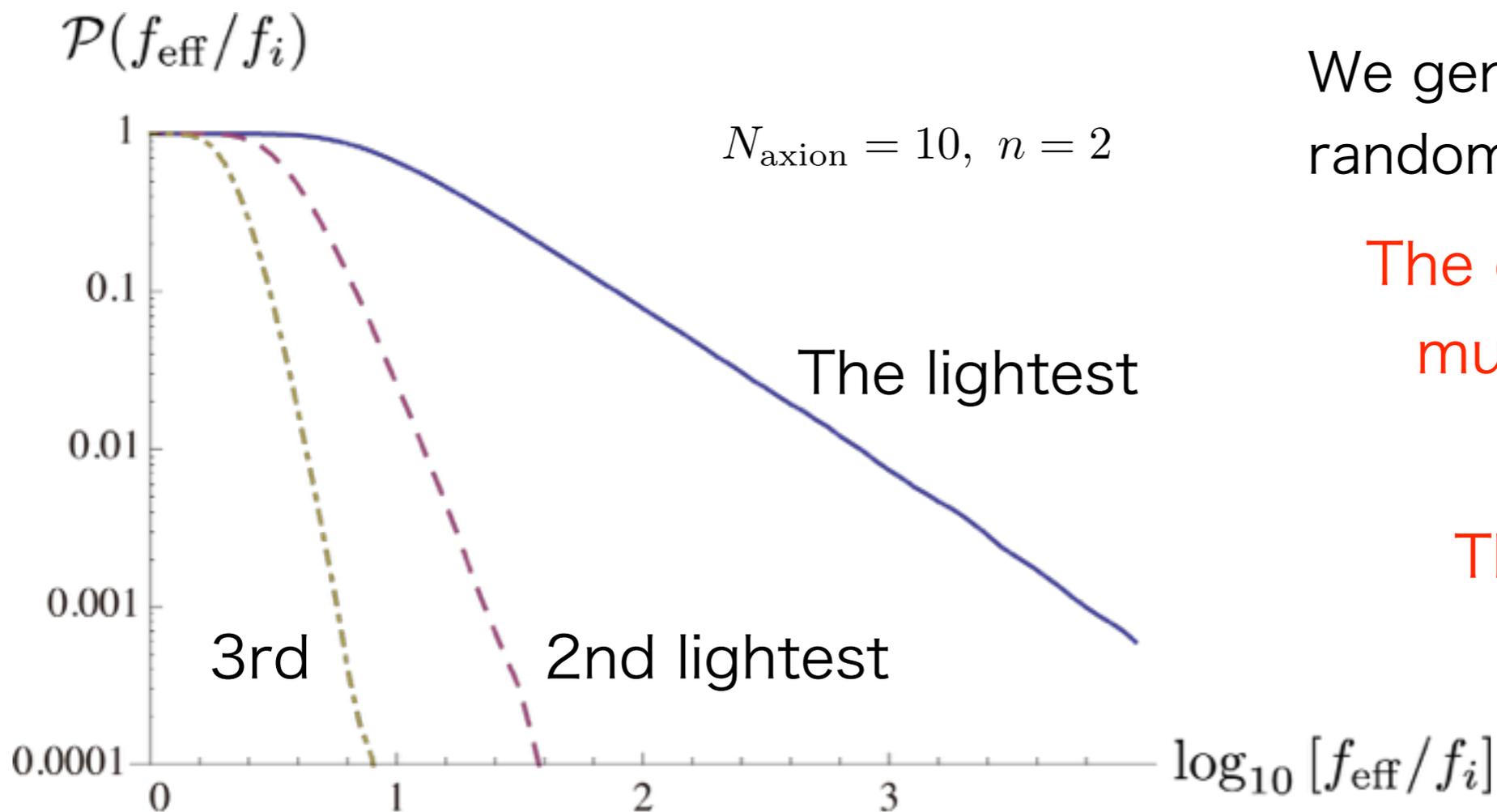
$$\mathcal{P}(f_{\text{eff}}/f_i) \sim N_{\text{axion}} \left(\frac{f_i}{f_{\text{eff}}} \right)$$

Enhancement becomes likely for larger N_{axion} .

Aligned Natural Inflation

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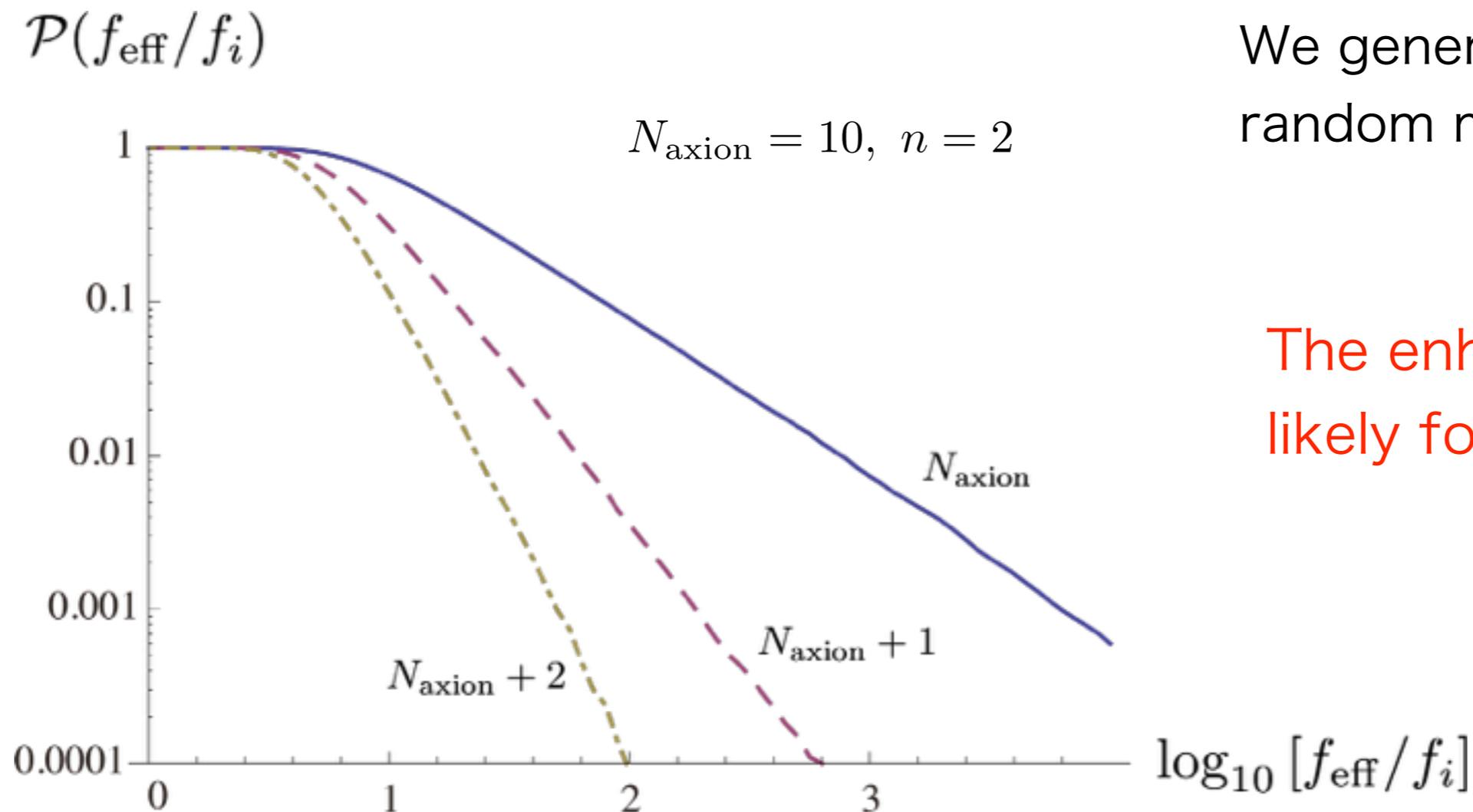
The enhancement along multiple directions is highly unlikely.

The inflaton is the lightest axion!

Aligned Natural Inflation

$$V(\phi_i) = \sum_{i=1}^{N_{\text{source}}} \Lambda_i^4 \cos \left(\sum_{j=1}^{N_{\text{axion}}} a_{ij} \frac{\phi_j}{f_j} + \theta_i \right) + V_0$$

- Prob dist for the enhancement of the decay constant



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The enhancement is less likely for larger N_{source} .

Axion Landscape

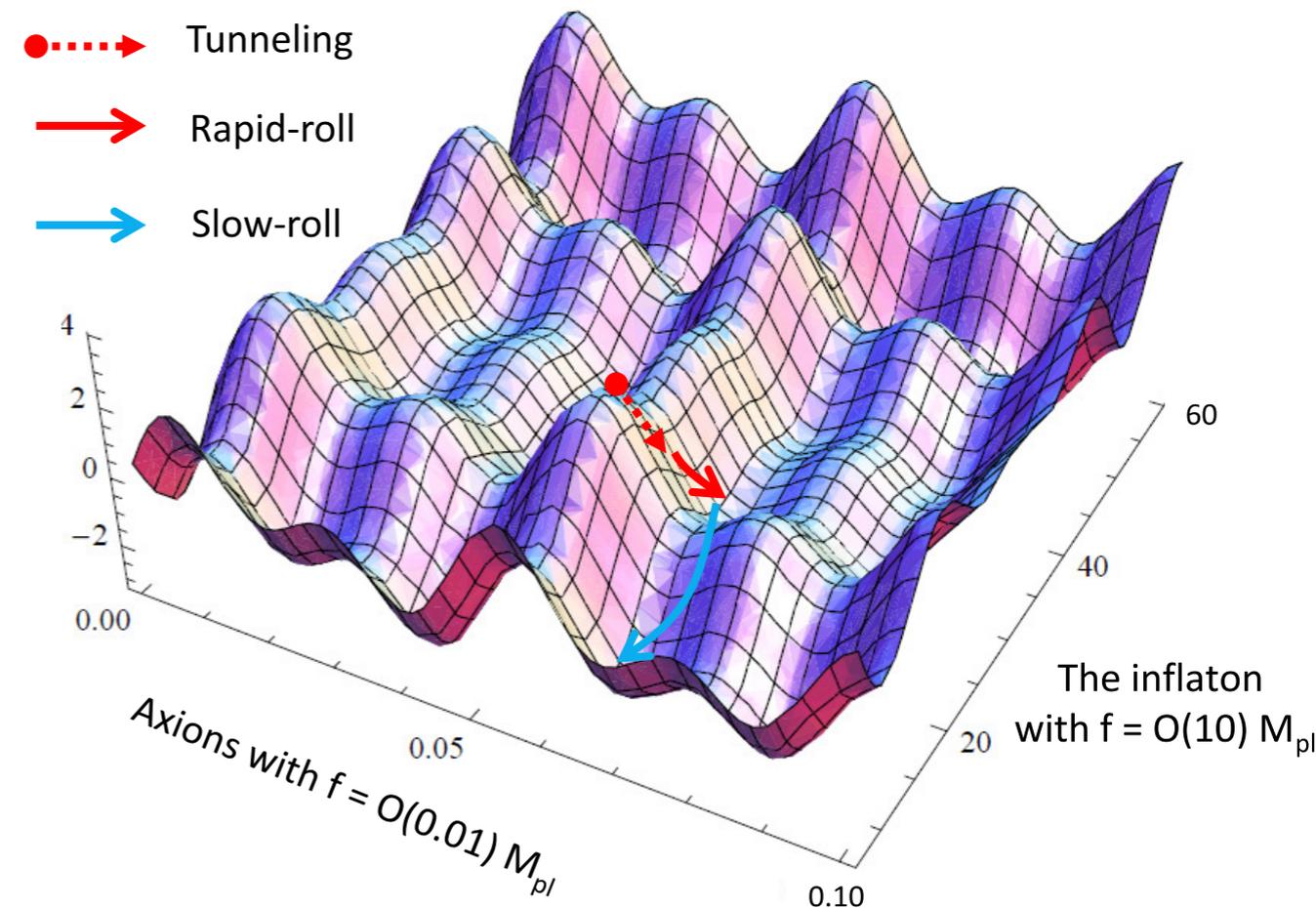
Higaki, FT 1404.6923

For $N_{\text{source}} > N_{\text{axion}}$, many axions may form a mini-landscape.

$$V(\phi_i) = \sum_{i=1}^{N_{\text{source}}} \Lambda_i^4 \cos \left(\sum_{j=1}^{N_{\text{axion}}} a_{ij} \frac{\phi_j}{f_j} + \theta_i \right) + V_0$$

- Eternal inflation takes place in a local minimum.
- A flat direction arises by the KNP mechanism.
- Slow-roll inflation starts along the flat direction after the tunneling event.
- Negative curvature/suppression at large scales if the total e-folding is just 50-60.

Linde '95, Freivogel et al '05,
Yamauchi et al '11, Bousso et al '13



Any little something extra?

- Running spectral index
- Isocurvature perturbations

Running spectral index

The spectral index depends on scales, but its running is too small to be detected in many cases.

Spectral index $n_s - 1 = \frac{d \ln P_\zeta}{d \ln k} \simeq 2\eta - 6\epsilon$

Running of spectral index $\frac{dn_s}{d \ln k} = -24\epsilon^2 + 16\epsilon\eta - 2\xi$

$$\epsilon \equiv \frac{M_p^2}{2} \left(\frac{V'}{V} \right)^2, \quad \eta \equiv M_p^2 \frac{V''}{V}, \quad \xi \equiv M_p^4 \frac{V'V'''}{V^2}.$$

$$\frac{dn_s}{d \ln k} = -0.0134 \pm 0.0090$$

Planck 1303.5802

If the running is $O(0.01)$, the inflation soon ends with $N_e < 30$.

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Large-field inflation with modulations

Let us add small modulations to the inflaton potential,

$$V(\phi) = V_0(\phi) + V_{mod}(\phi),$$

s.t.

$$|V_0(\phi)| \gg |V_{mod}(\phi)|,$$

$$|V'_0(\phi)| > |V'_{mod}(\phi)|.$$

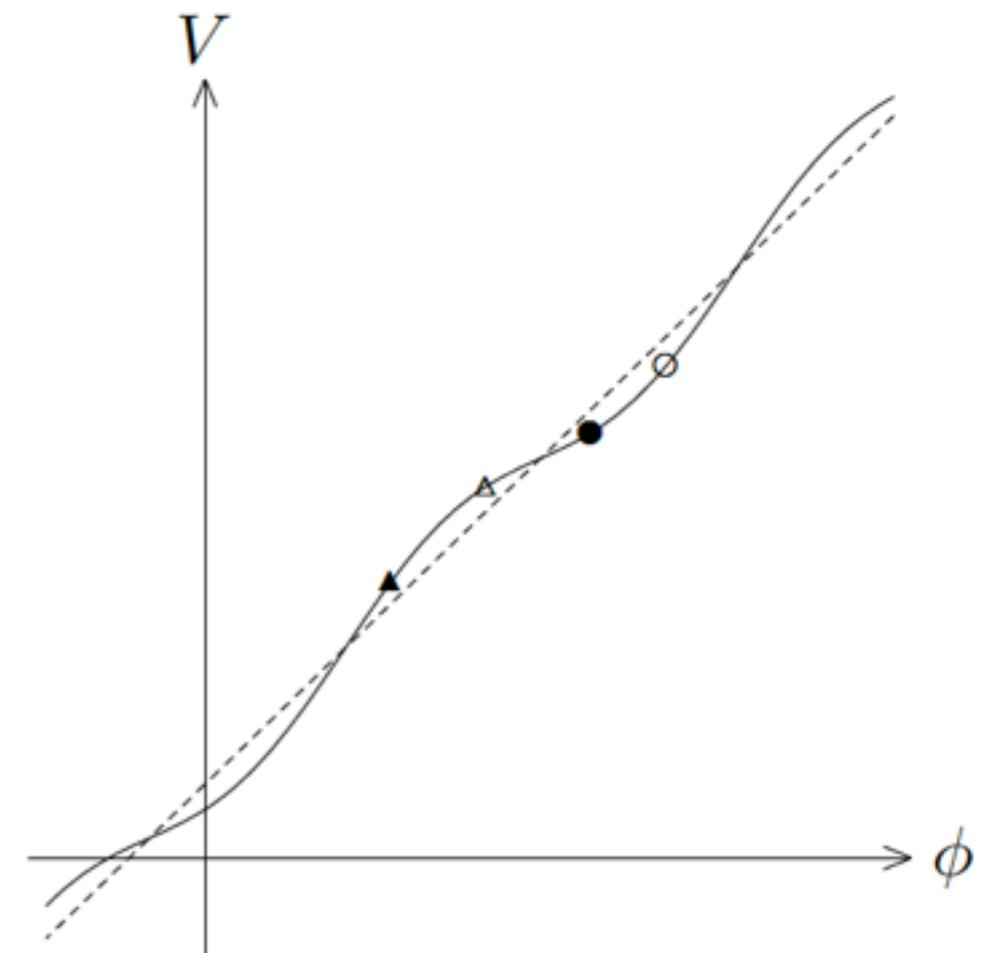
$$|V''_0(\phi)| \lesssim |V''_{mod}(\phi)|,$$

$$|V'''_0(\phi)| \ll |V'''_{mod}(\phi)|.$$

(Both sides represent typical values)

Then the spectral index and its running are significantly affected by modulations, while the inflaton dynamics and the normalization of density perturbations remain intact.

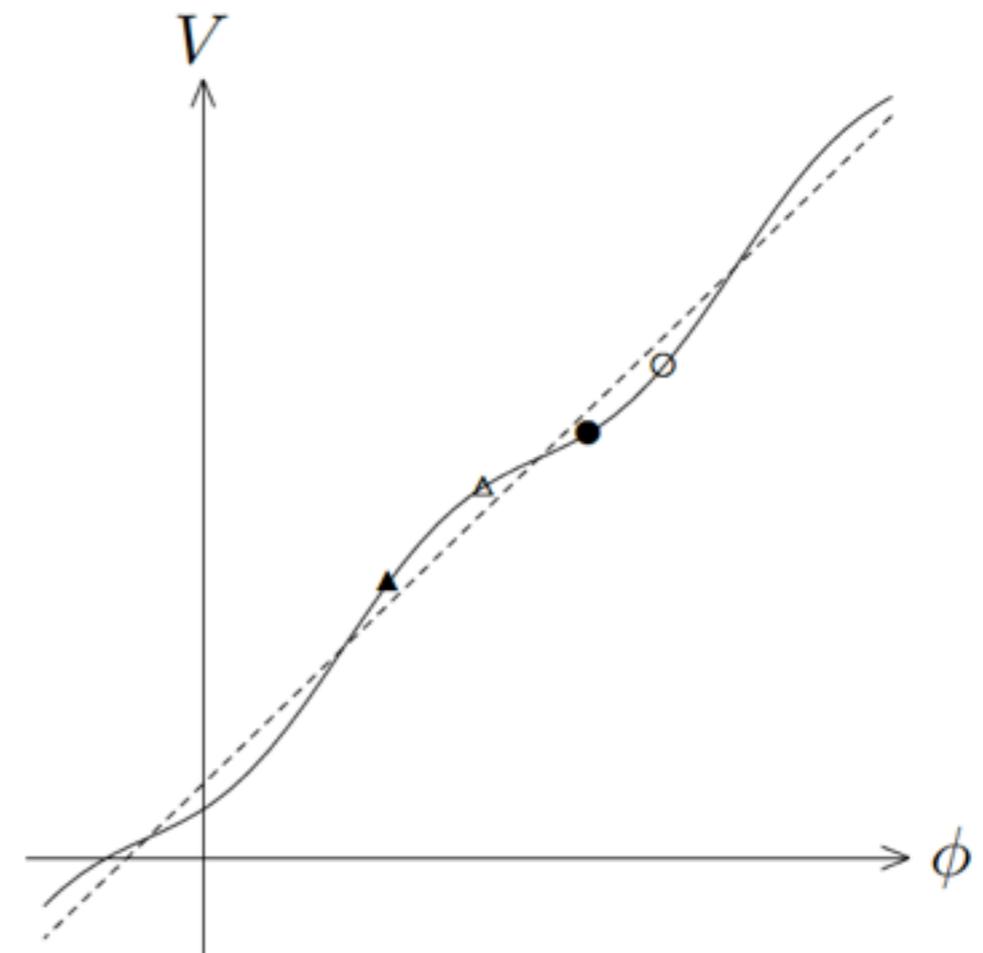
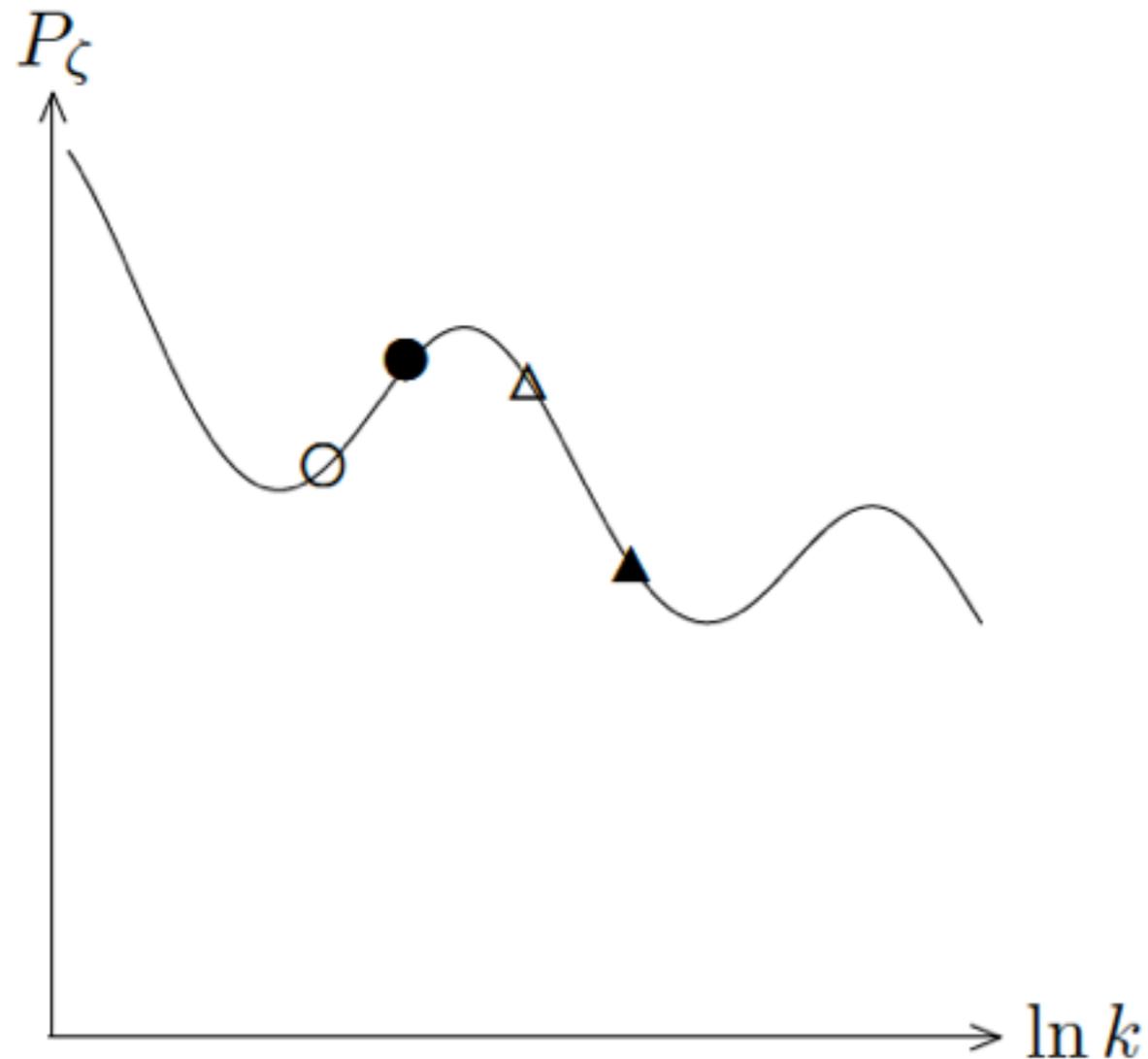
Kobayashi, FT, 1011.3988
Czerny, Kobayashi, FT, 1403.4589



Large-field inflation with modulations

Let us add small modulations to the inflaton potential.

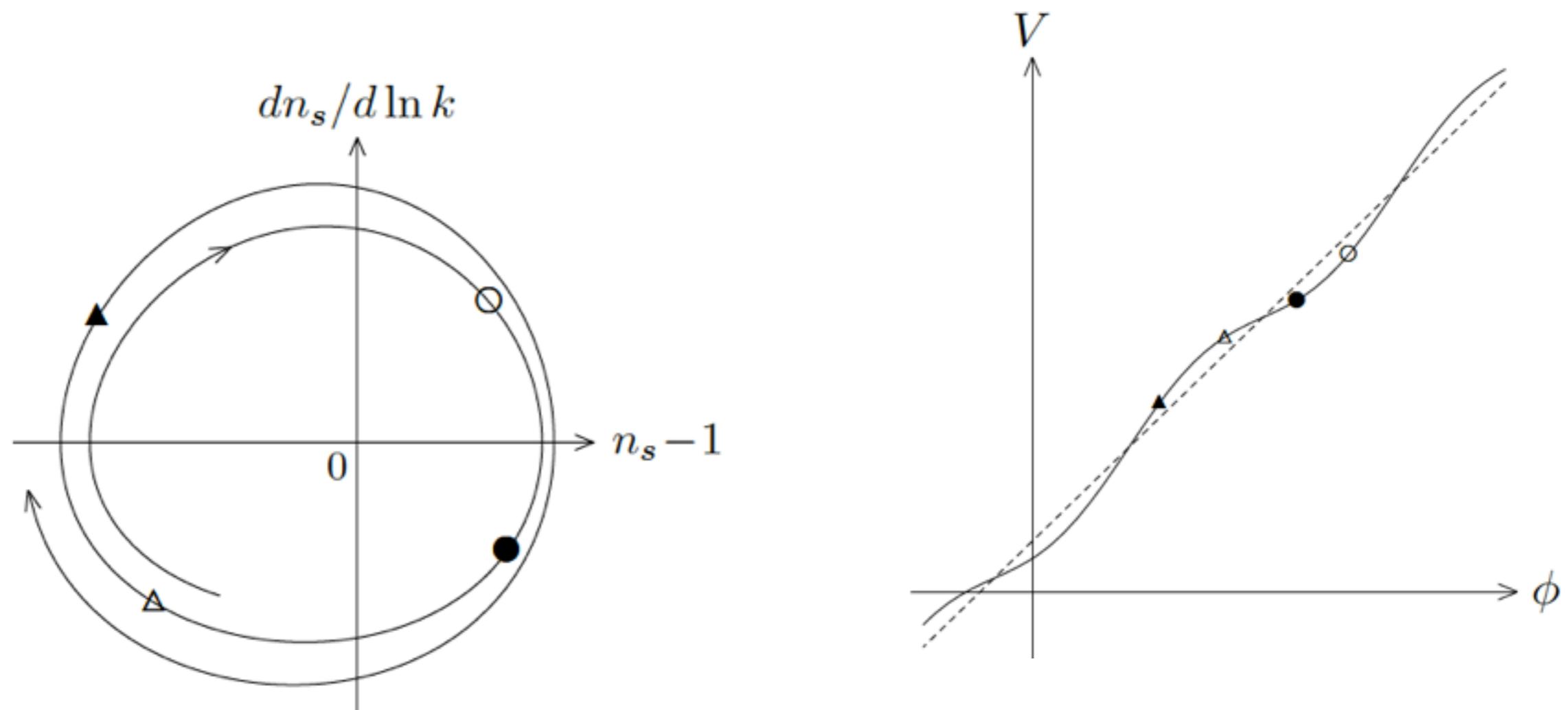
Kobayashi, FT, 1011.3988
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Large-field inflation with modulations

Let us add small modulations to the inflaton potential.

Kobayashi, FT, 1011.3988
Czerny, Kobayashi, FT, 1403.4589



Such small modulations are present in the axion monodromy inflation, and built-in feature of the multi-natural inflation model.

McAllister, Silverstein, Westphal, 0808.0706

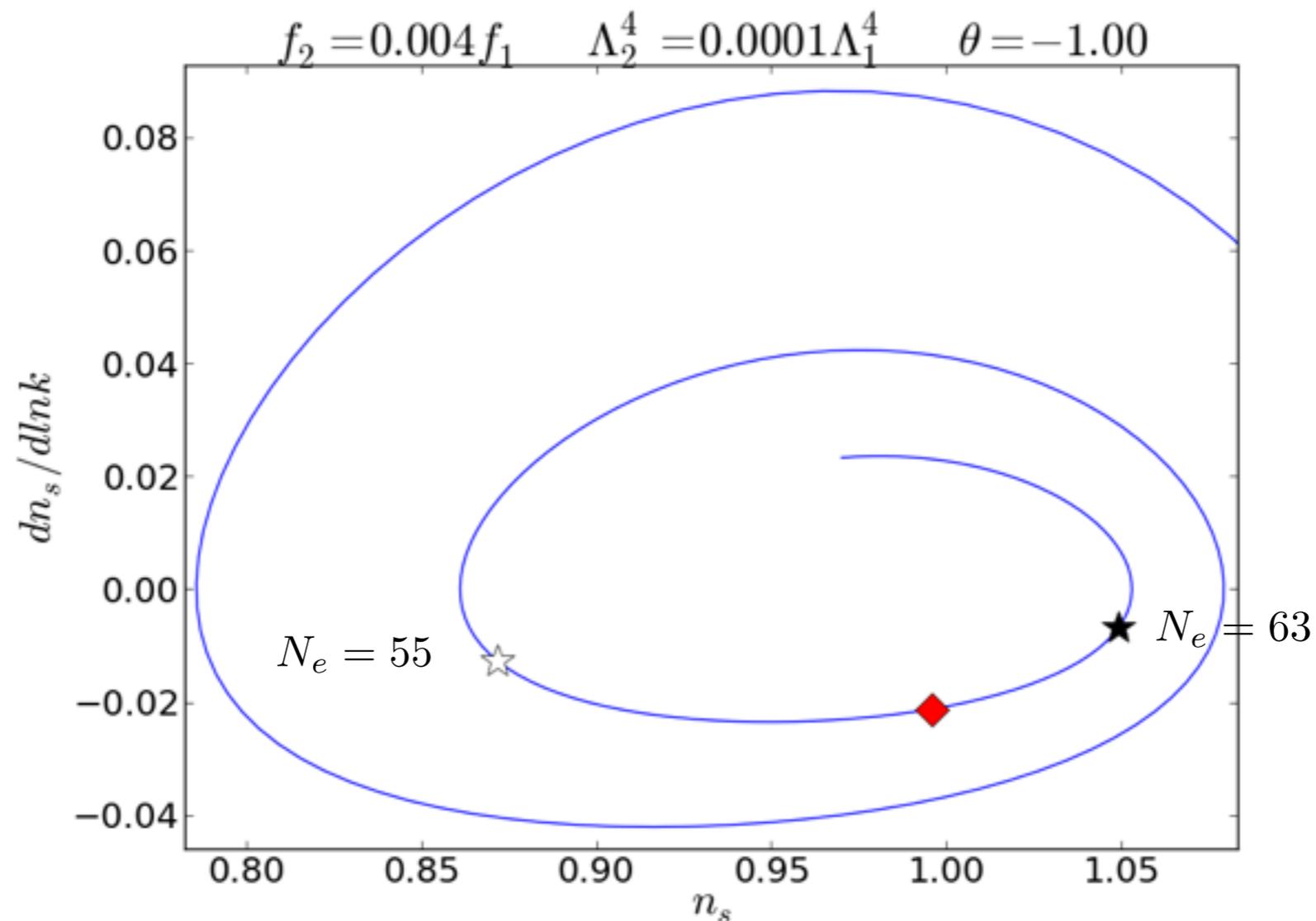
Large-field inflation with modulations

Kobayashi, FT, 1011.3988

Czerny, Kobayashi, FT, 1403.4589

e.g. Multi-natural inflation

$$V(\phi) = C - \Lambda_1^4 \cos(\phi/f_1) - \Lambda_2^4 \cos(\phi/f_2 + \theta),$$



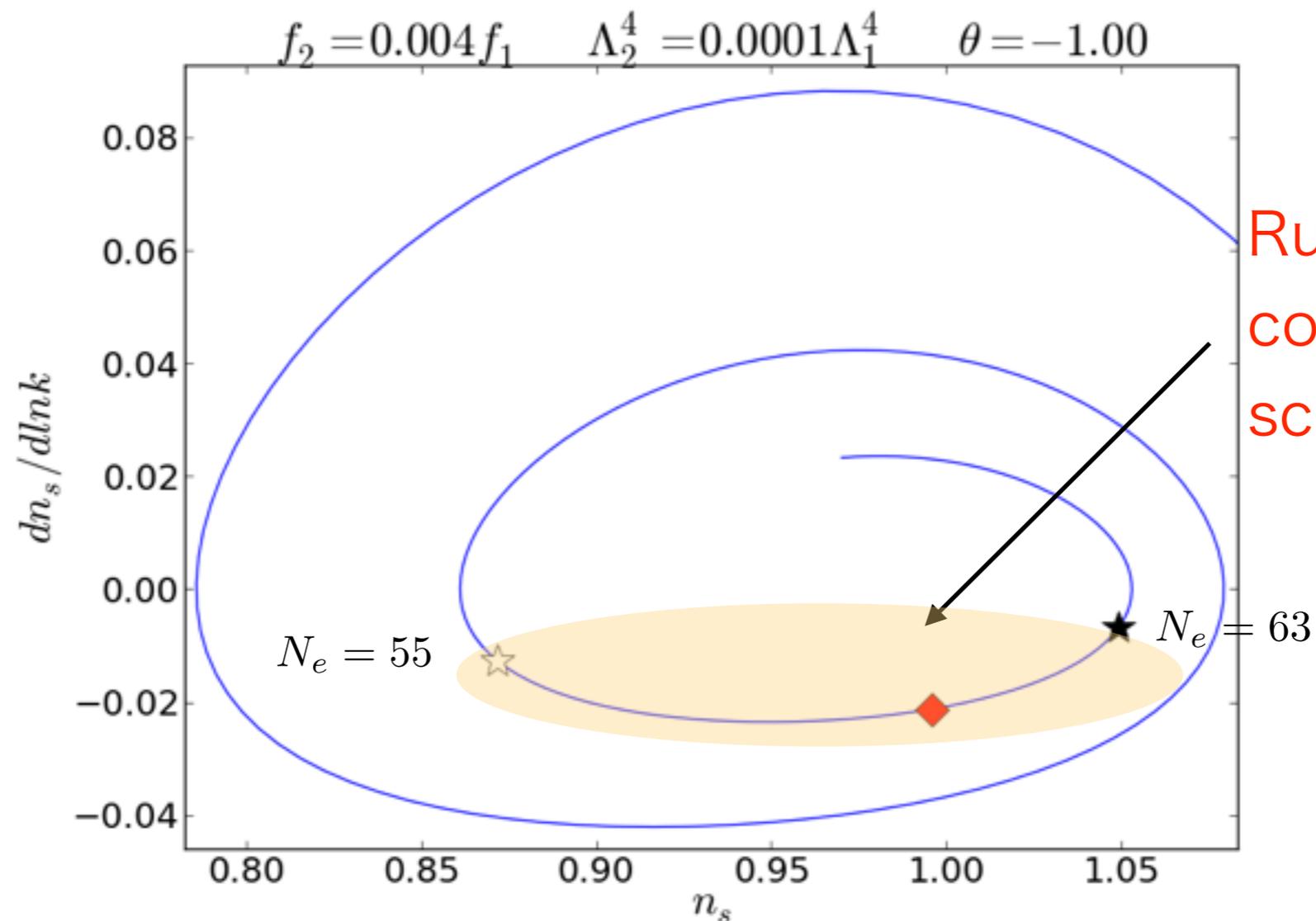
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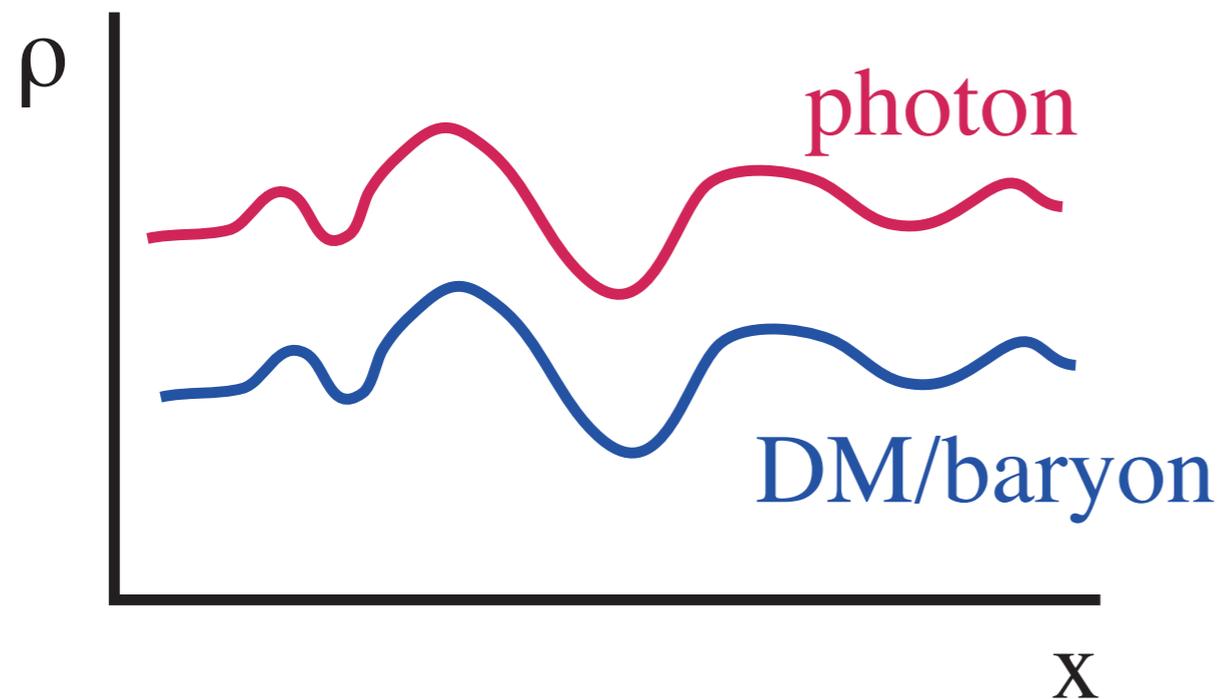
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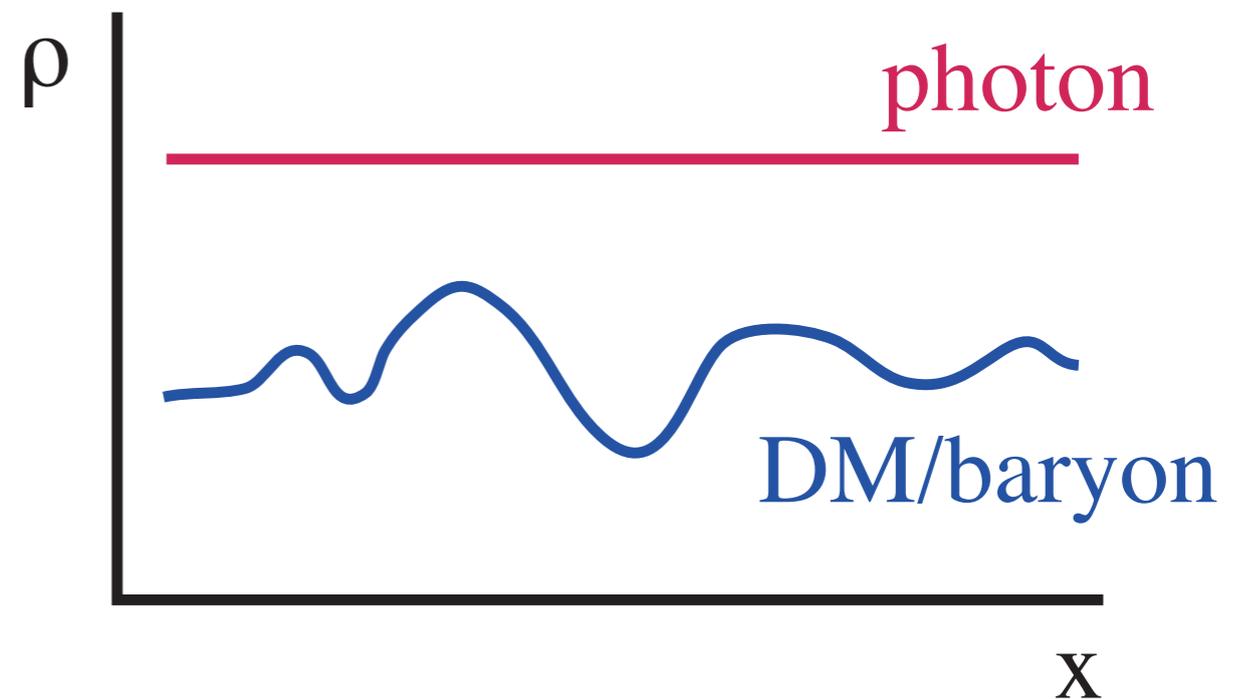
Running is almost constant over CMB scales.

Axion isocurvature perturbations

Adiabatic perturbation



Isocurvature perturbation

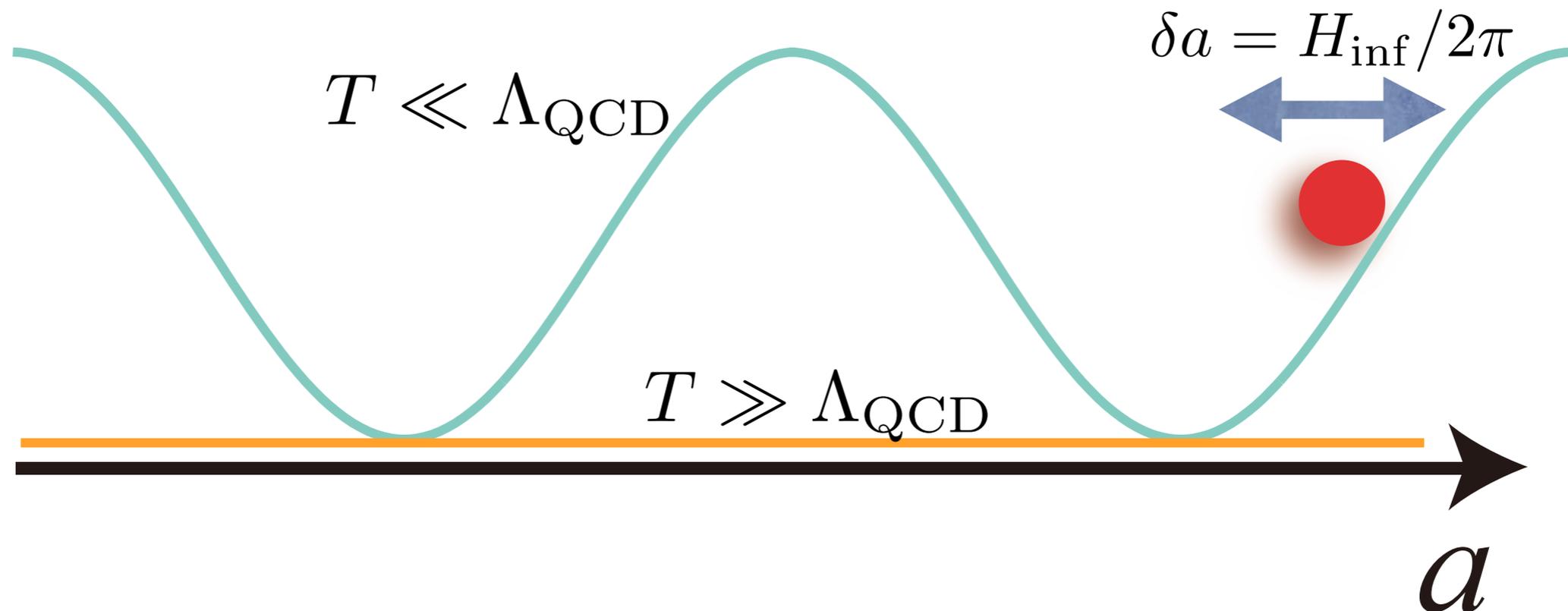


$$\alpha \equiv \frac{P_S}{P_{\mathcal{R}}} \lesssim 0.041 \quad (95\% \text{CL})$$

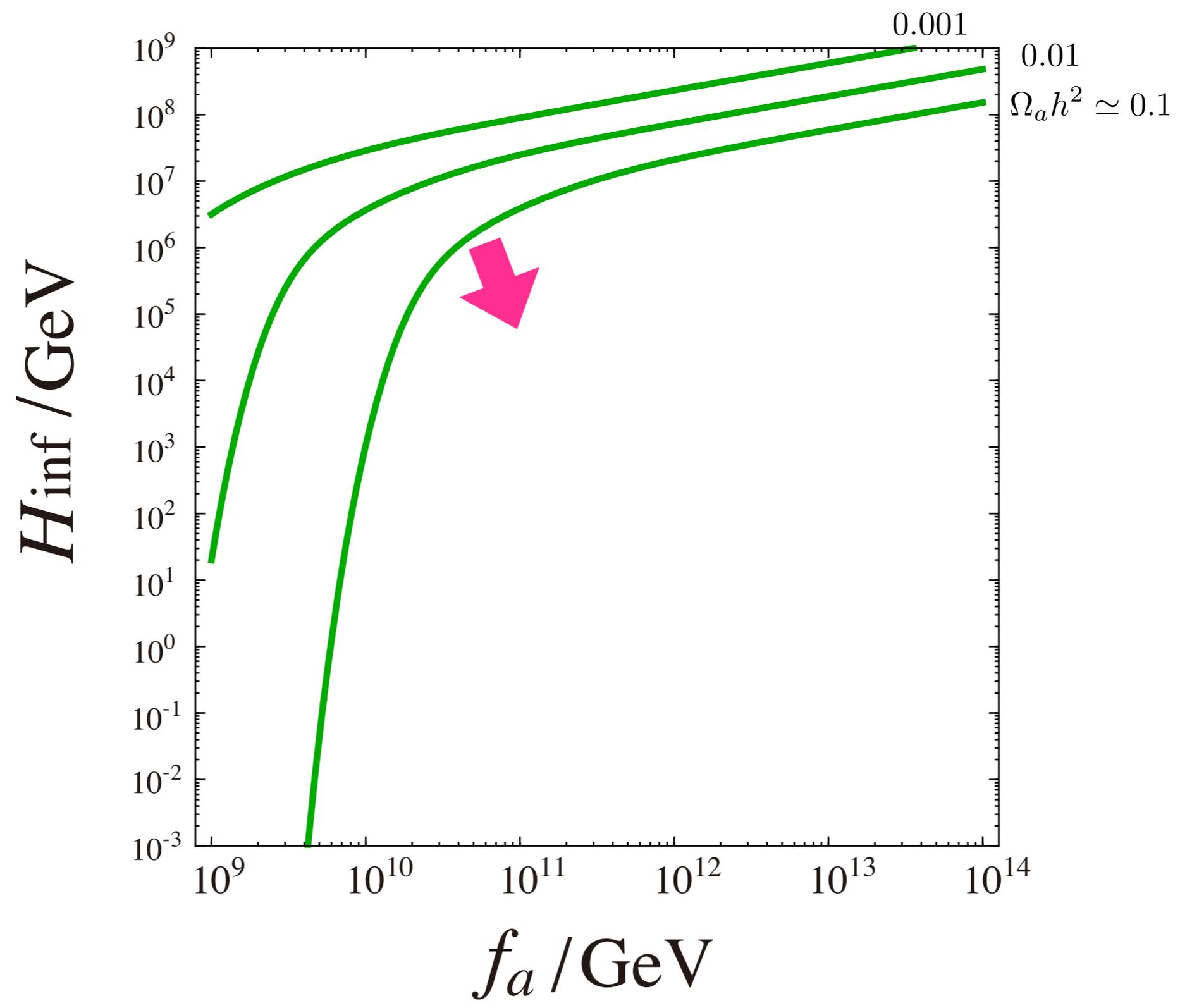
(Planck+WMAP polarization)

- The QCD axion is a plausible candidate for DM with isocurvature perturbations.

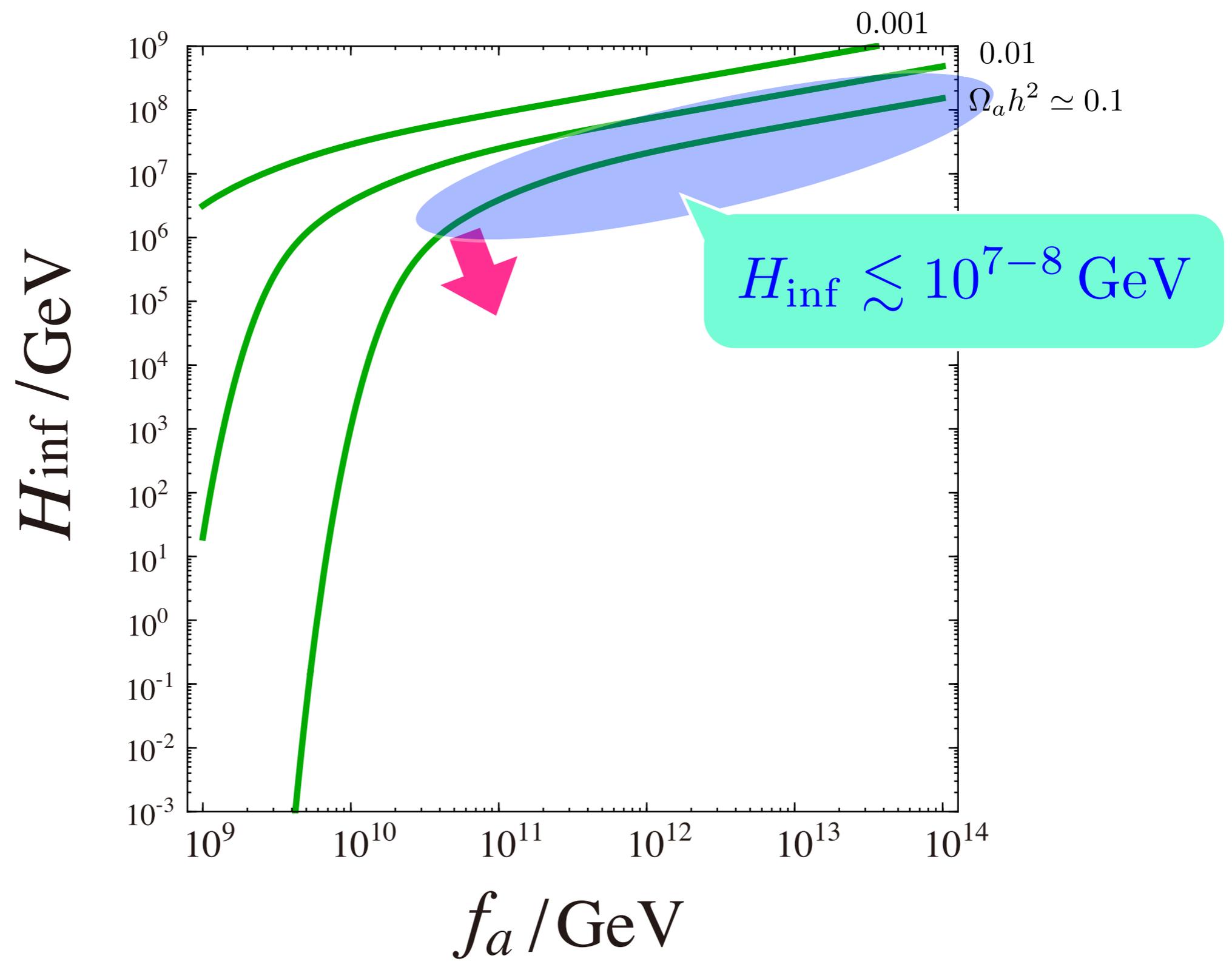
$$\mathcal{L} = \left(\theta + \frac{a}{f_a} \right) \frac{g_s^2}{32\pi^2} G^{\mu\nu} \tilde{G}_{\mu\nu}$$



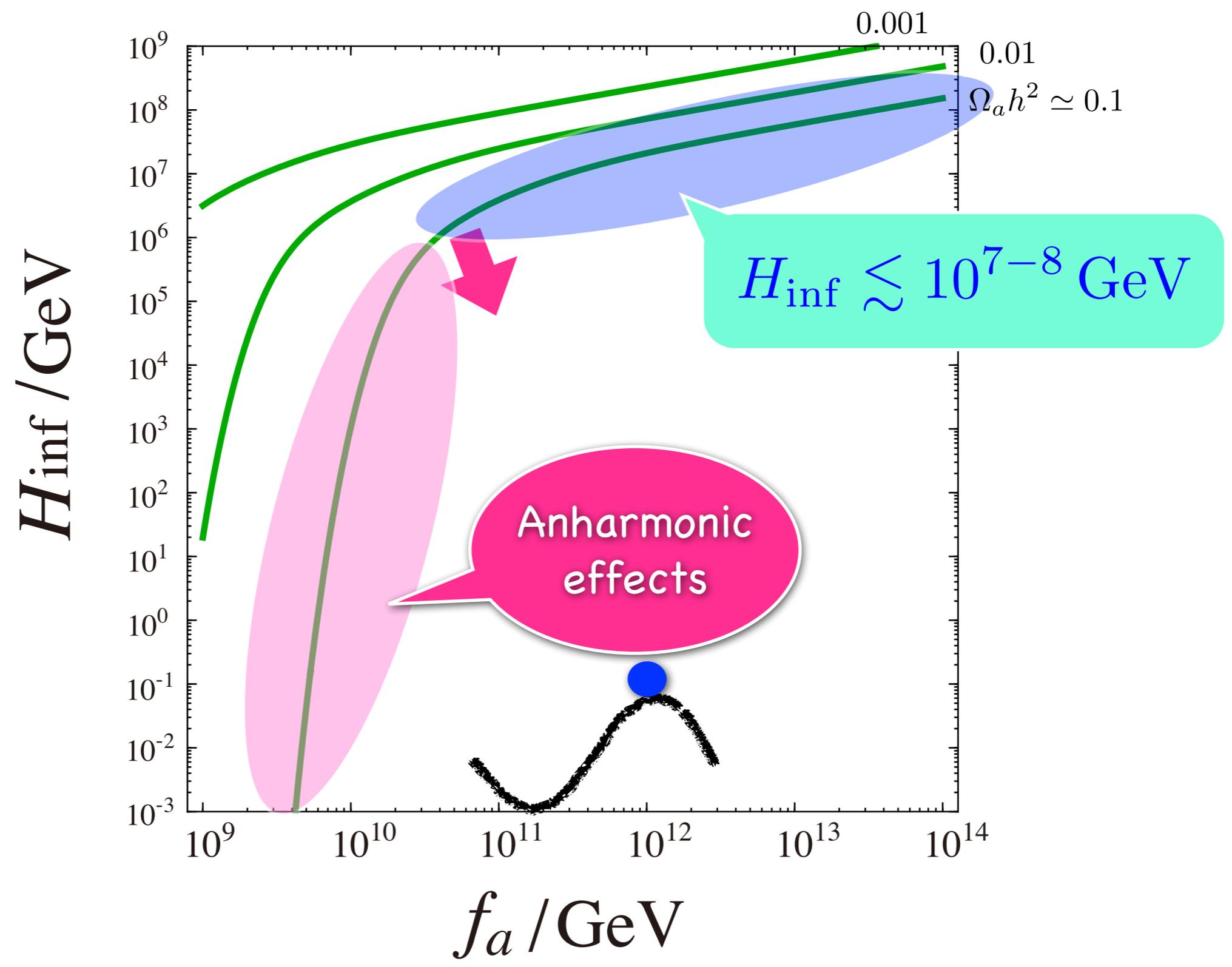
Isocurvature constraint on H_{inf}



Isocurvature constraint on H_{inf}



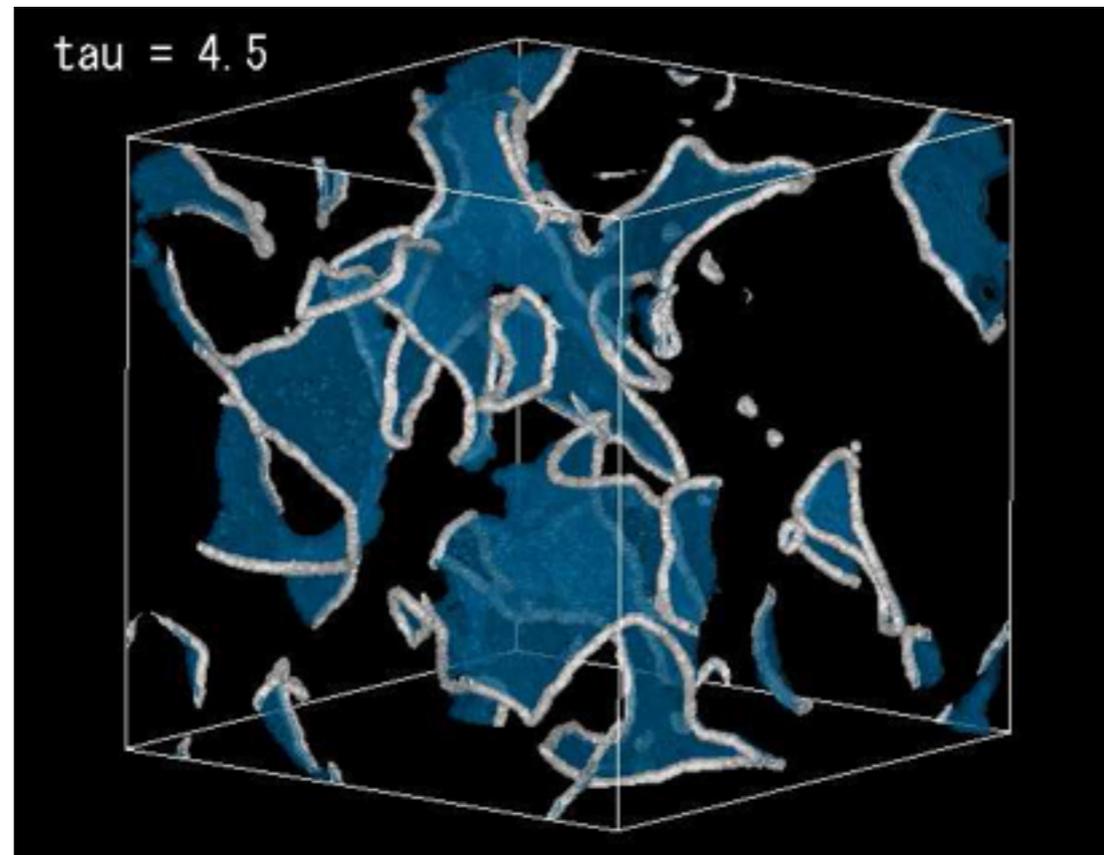
Isocurvature constraint on H_{inf}



Solutions

- Restoration of Peccei-Quinn symmetry during inflation.
- Axions are produced from domain walls and axion DM is possible for $f_a = 10^{10}\text{GeV}$.

[Hiramatsu, Kawasaki, Saikawa and Sekiguchi, 1202.5851,1207.3166](#)

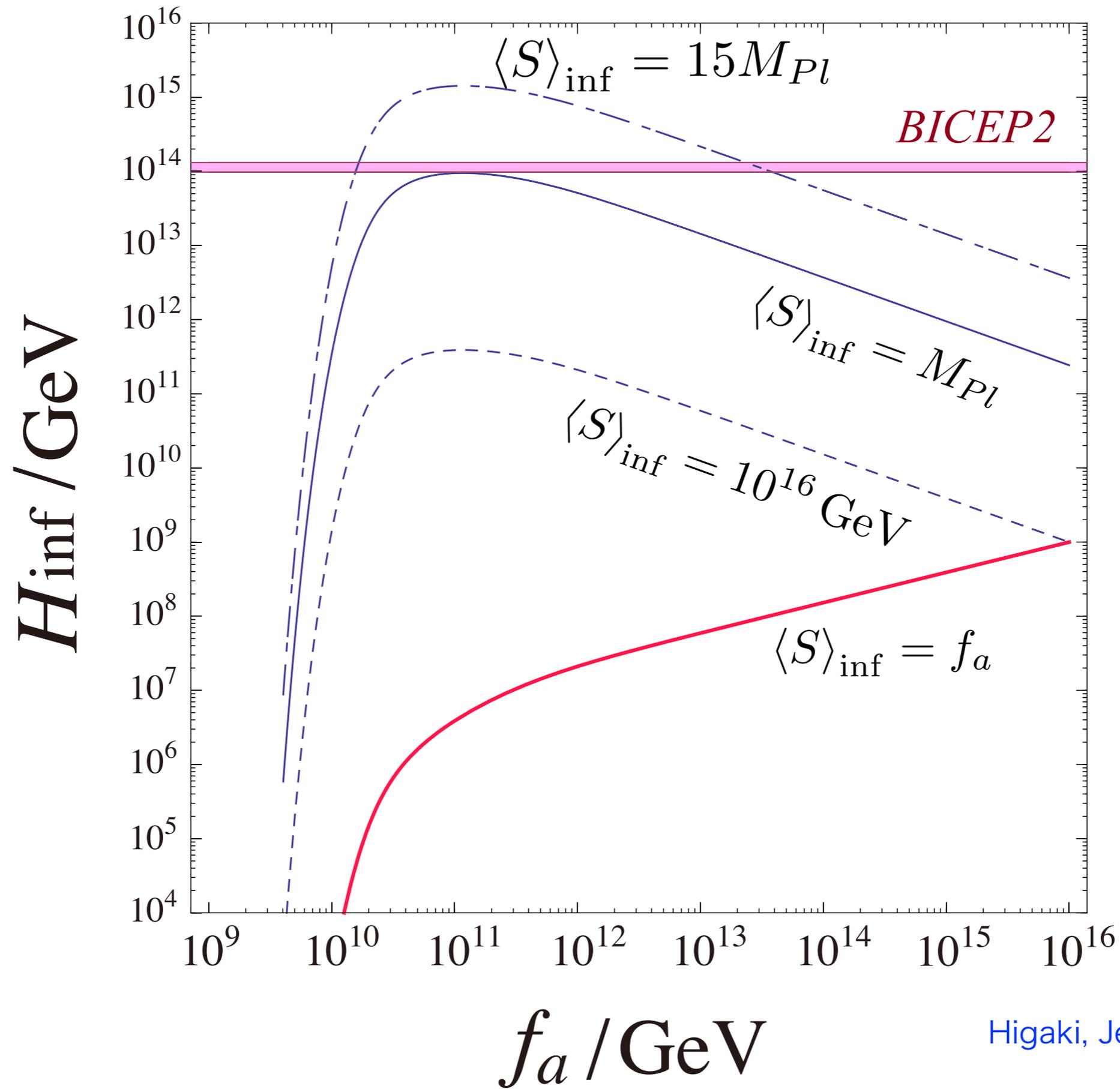


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[Hiramatsu, Kawasaki, Saikawa and Sekiguchi, 1202.5851, 1207.3166](#)

- (Super-)Planckian saxion field value during inflation. (Saxion could be the inflaton)



axion CDM
is assumed.

Solutions

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[Hiramatsu, Kawasaki, Saikawa and Sekiguchi, 1202.5851,1207.3166](#)

- Super-Planckian saxion field value during inflation. (Saxion could be the inflaton)
- Heavy axions during inflation. $m_a^2 \gtrsim H_{\text{inf}}^2$

- Stronger QCD during inflation [Jeong, FT 1304.8131](#)

- Enhanced explicit PQ breaking [Higaki, Jeong, FT, 1403.4186](#)

Conclusions

- **If $r = \mathcal{O}(0.001-0.1)$** , we can get information of the very early Universe at the GUT-scale.
- **Large-field inflation** realized by shift symmetry.
 - Polynomial chaotic inflation/multi-natural inflation lead to various values of (n_s, r) .
- **Axion landscape**
 - Eternal inflation and subsequent slow-roll inflation realized in a unified manner.
 - (Multi-)natural inflation by the KNP mechanism
 - Just 50-60 e-foldings may lead to negative curvature.
- **Anything extra?**
 - Running spectral index realized by small modulations.
 - Isocurvature/non-Gaussianity/spatial curvature, etc.