hadronic interaction and beyond standard model from lattice gauge theory

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素粒子物理学の進展2014 @ 京都大学基礎物理学研究所, July 28–August 1, 2014

hadron interaction from lattice QCD

and

search for walking technicolor from lattice gauge theory

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Hadron interaction from lattice QCD — Review —

- Lüscher's finite volume method
- Scattering lengths
- Scattering phase shifts (Resonances)
- Bound states (Light nuclei)

Hadronic interactions

One of ultimate goals of Lattice QCD

quantitatively understand properties of hadrons

Current status of lattice QCD

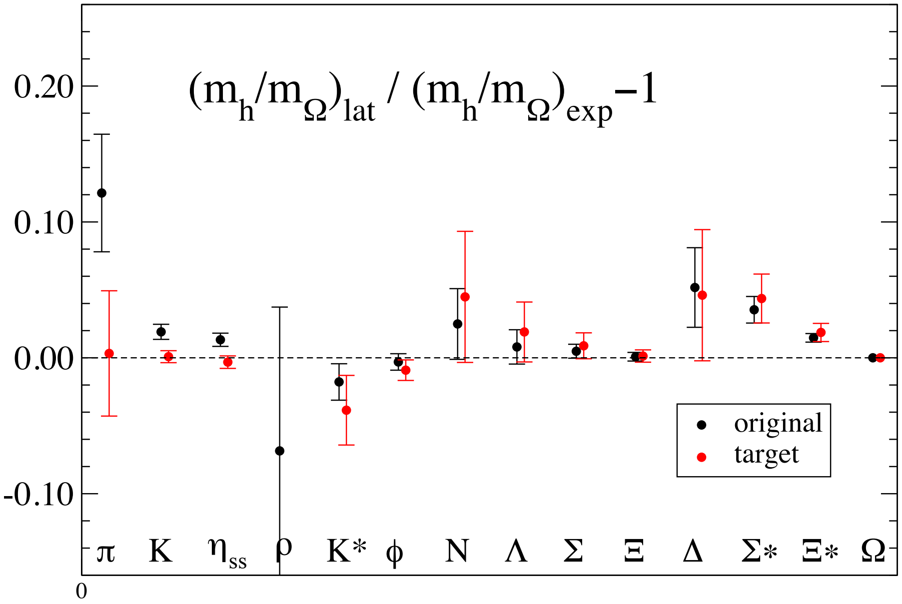
very close to reproduce mass for stable hadrons

Hadron masses from Lattice QCD

$N_f = 2+1 (m_u = m_d \neq m_s)$

'10 PACS-CS

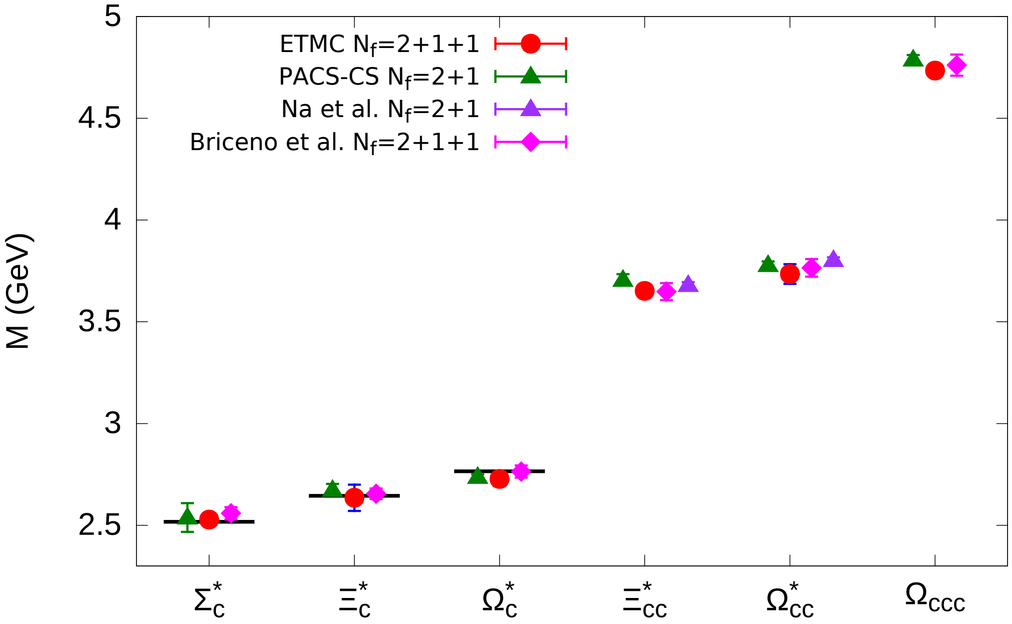
Light hadrons



$N_f = 2+1+1 (m_u \sim m_d \neq m_s \neq m_d)$

'14 Alexandrou *et al.*

Charmed baryons: spin 3/2



target = physical pion mass

Light hadrons consistent within a few%

Predictions in some charmed baryons, also in Ω_{cc}, B_c and B_c^*

Hadronic interactions

One of ultimate goals of Lattice QCD

quantitatively understand properties of hadrons

Current status of lattice QCD

very close to reproduce mass for stable hadrons

Experiment

Many hadrons decay through hadronic interaction,
originating from strong interaction

Next task: hadronic interactions

Decay and scattering

Cannot be treated separately to understand properties of unstable hadrons

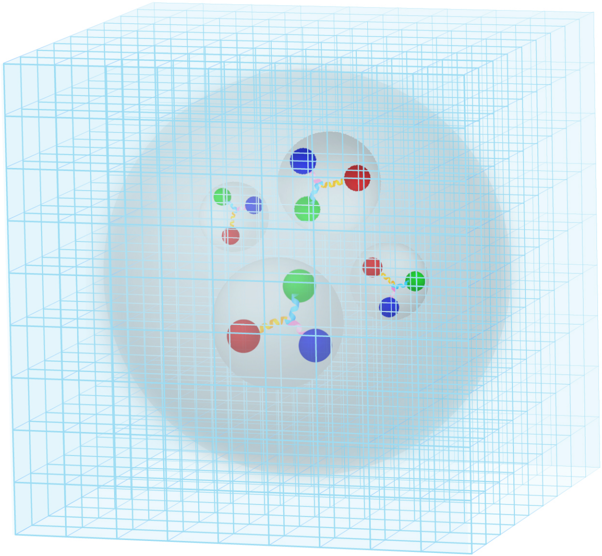
finial states of unstable particle = scattering states

More difficult to calculate, but important for the ultimate goal

Hadronic interactions

One of ultimate goals of Lattice QCD

quantitatively understand properties of hadrons



Famous hadronic interaction

nuclear force : bind nucleons into nucleus

originate from strong interaction

← well known in experiment

Another ultimate goal of lattice QCD

quantitatively understand formation of nuclei

from first principle of strong interaction

http://www.jicfus.jp/jp/promotion/pr/mj/2014-1/

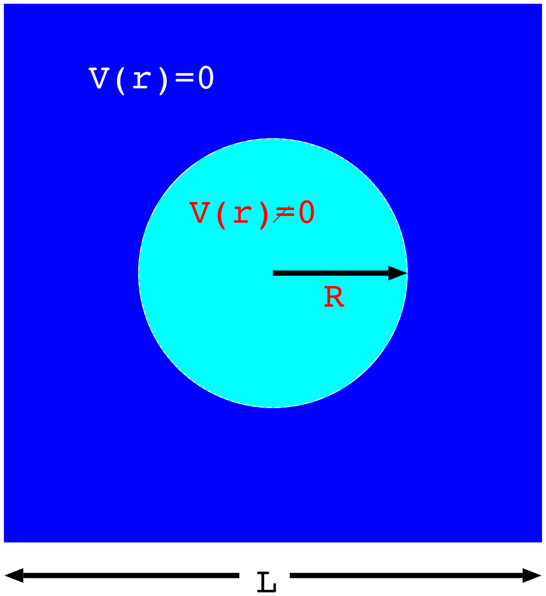
c.f. Nuclear force from lattice QCD, see slide of PPP2013 by S. Aoki

Review recent results related to scatterings, decays, and light nuclei

Lüscher's finite volume method

Lüscher, CMP105:153(1986),NPB354;531(1991)

spinless two-particle elastic scattering in center of mass frame on L^3



Important assumption

1. Two-particle interaction is localized.

   → Interaction range R exists.

$$V(r)\begin{cases} \neq 0 & (r \leq R) \\ = 0\ (\sim e^{-cr}) & (r > R) \end{cases}$$

2. $V(r)$ is not affected by boundary. → $R < L/2$

Two-particle wave function $\phi_p(\vec{r})$ satisfies Helmholtz equation

$$\left(\nabla^2 + p^2\right)\phi_p(\vec{r}) = 0 \text{ in } r > R\ (R < L/2)$$

← Klein-Gordon eq. of free two particles

$$E = 2\sqrt{m^2+p^2},\ p^2 \neq \left(\frac{2\pi}{L}\cdot\vec{n}\right)^2 \text{ in general}$$

Lüscher's finite volume method (cont'd)

Helmholtz equation on L^3

Lüscher, CMP105:153(1986),NPB354;531(1991)

1. Solution of $(\nabla^2 + p^2)\phi_p(\vec{r}) = 0$ in $r > R$

$$\phi_p(\vec{r}) = C \cdot \sum_{\vec{n} \in Z^3} \frac{e^{i\vec{r}\cdot\vec{n}(2\pi/L)}}{\vec{n}^2 - q^2}, \quad q^2 = \left(\frac{Lp}{2\pi}\right)^2 \neq \text{integer}$$

2. Expansion by spherical Bessel $j_l(pr)$ and Noeman $n_l(pr)$ functions

$$\phi_p(\vec{r}) = \beta_0(p) n_0(pr) + \alpha_0(p) j_0(pr) + (l \geq 4)$$

3. S-wave Scattering phase shift $\delta_0(p)$ <u>in infinite volume</u>

$$\frac{\beta_0(p)}{\alpha_0(p)} = \boxed{\tan\delta_0(p) = \frac{\pi^{3/2} q}{Z_{00}(1;q^2)}} \quad Z_{00}(s;q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n}\in Z^3} \frac{1}{(\vec{n}^2 - q^2)^s}, \quad q = \frac{2\pi}{L} p$$

$$\text{Scattering amplitude} = \frac{E}{2p}\frac{1}{2i}\left(e^{2i\delta(p)} - 1\right)$$

Relation between $\boxed{\delta(p) \text{ and } p}$ $\left(E = 2\sqrt{m^2 + p^2}\right)$

Wave function: CP-PACS, PRD70:094504(2005), Sasaki and Ishizuka, PRD78:014511(2008)

Potential: Ishii, Aoki, and Hatsuda, PRL99:022001(2007), $\cdots$

Applications

$K \to \pi\pi$: Lellouch and Lüscher, CMP219:31(2001), $M_{K_L} - M_{K_S}$: RBC+UKQCD, arXiv:1406.0916

Scattering length a_0^I

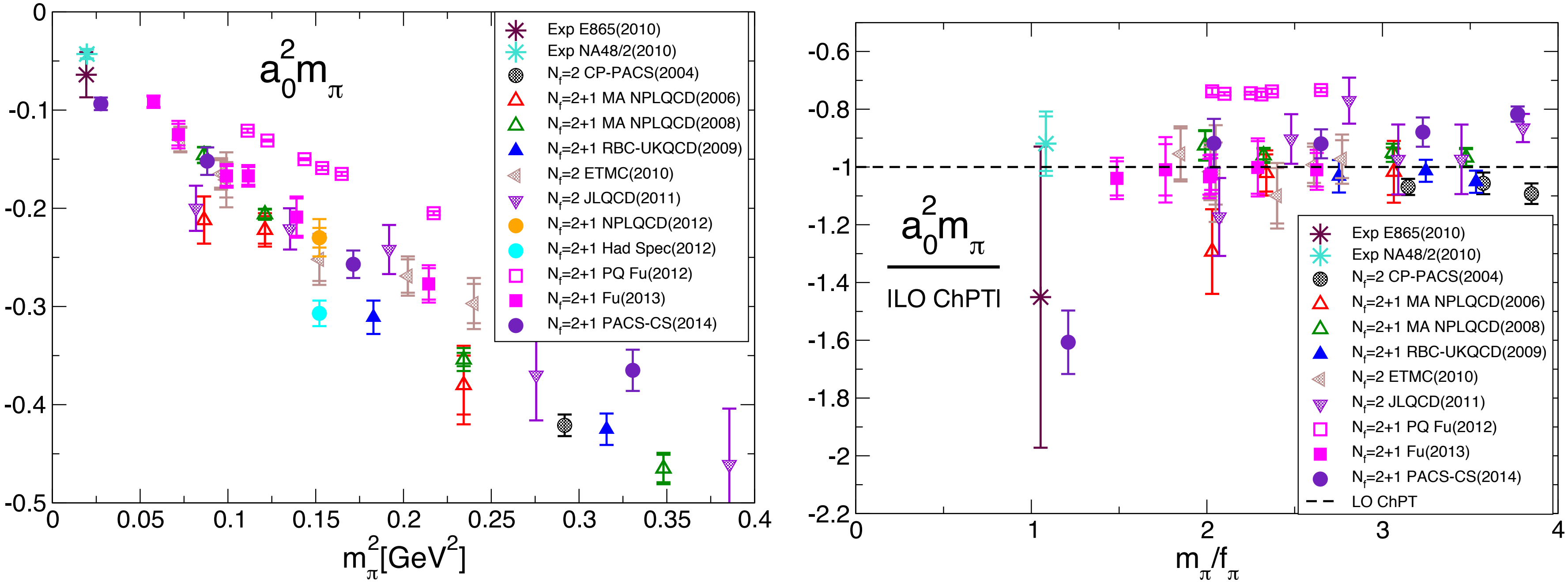
$$a_0 = \lim_{p \to 0} \frac{\tan \delta(p)}{p}$$

$I = 2$ $\pi\pi$ a_0^2 and $I = 1/2$ $K\pi$ $a_0^{1/2}$

Scattering length I

$I = 2$ $\pi\pi$ Simplest scattering system

Comparison of dynamical calculations



○ Wilson type; □ ASQTAD; △ DWF; ◁ Twisted; ▽ overlap

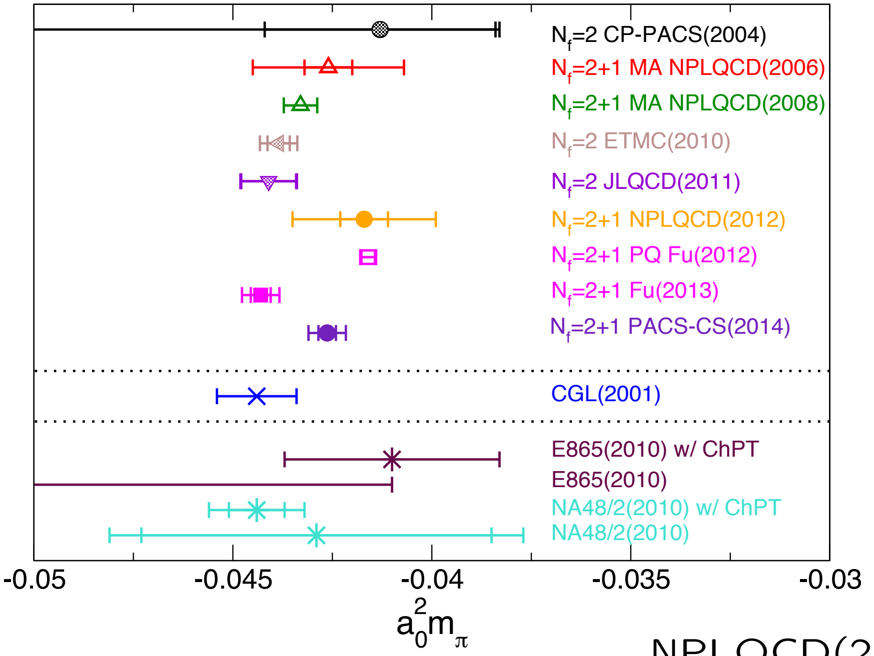
MA:DWF on ASQTAD; PQ:partial quenched

NLO ChPT: $a_0^2 m_\pi = \frac{m_\pi^2}{8\pi f_\pi^2}\left[-1+\frac{32}{f_\pi^2}\left[m_\pi^2 L_{\pi\pi} + \text{analytic} + \text{log}\right]\right]$

Scattering length I

$I = 2$ $\pi\pi$ Simplest scattering system

Comparison of dynamical calculations at physical m_π



NPLQCD(2012): m_π/f_π from MA calc.

○ Wilson type; □ ASQTAD; △ DWF; ◁ Twisted; ▽ overlap

MA:DWF on ASQTAD; PQ:partial quenched

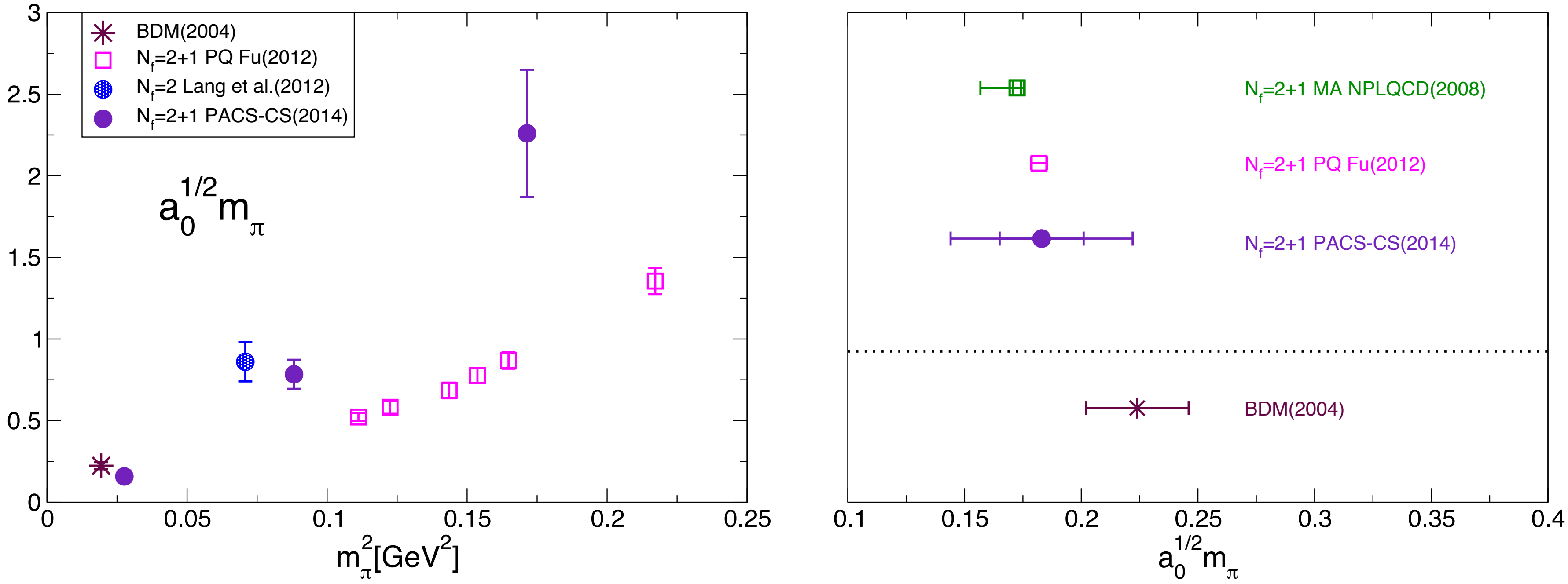
Sources of systematic error: finite volume effects, Δ_{MA}, Δ_{Wilson}, $\cdots$

high precision measurements era

more precise calculation under way

Scattering length II

$I = 1/2\ K\pi$ needs rectangle diagram



○ Wilson type; ☐ ASQTAD PQ:partial quenched

Fu, PRD85:074501(2012), Lang *et al.*, PRD86:054508(2012), PACS-CS, PRD89:054502(2014)
other works: NPLQCD, PRD74:114503(2006)(indirect), Nagata *et al.*, PRC80:045203(2009)

NLO ChPT:
$$a_0^{1/2}\mu_{\pi K} = \frac{\mu_{\pi K}^2}{4\pi f_\pi^2}\left[2+\frac{32}{f_\pi^2}\left[m_\pi m_K L' + \frac{m_\pi^2+m_K^2}{2}L_5 + \text{analytic} + \text{log}\right]\right]$$
$$\mu_{\pi K} = m_\pi m_K/(m_\pi + m_K)$$

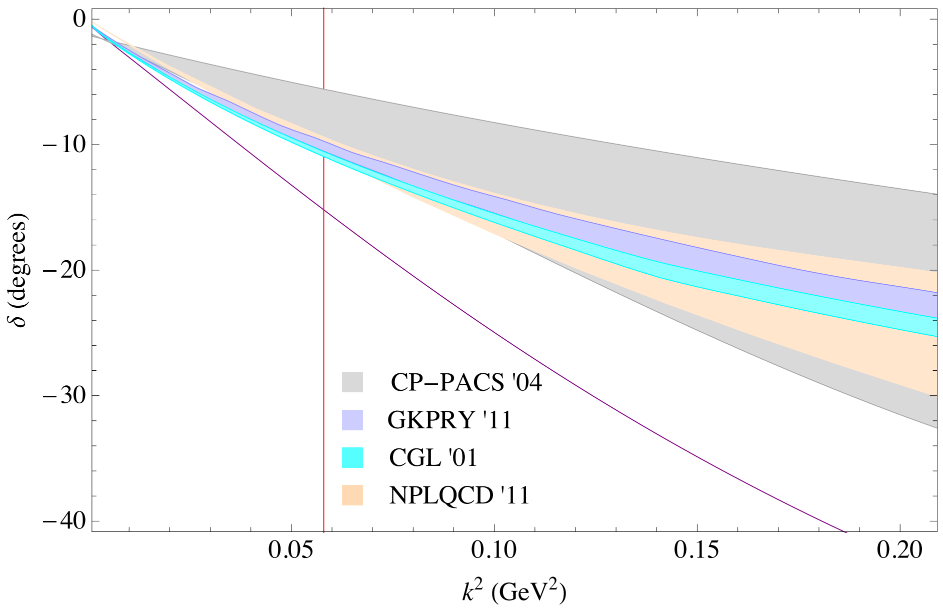
$I = 0\ \pi\pi$ needs disconnected diagram → much more difficult

Scattering phase shift $\delta(p)$

$I = 2$ $\pi\pi$, $I = 1$ $\pi\pi \to \rho$, $I = 1/2$ $K\pi \to K^*$

Phase shift I

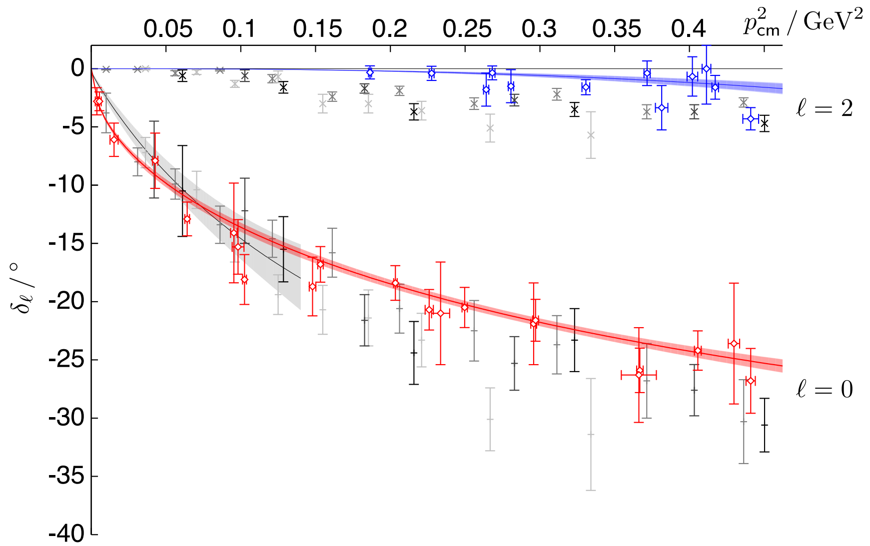
$I = 2$ S-wave $\pi\pi$ Simplest scattering system



NPLQCD, PRD85:034505(2012)

NLO ChPT in $p \neq 0 \to$ physical m_π

from $m_\pi = 0.39$ GeV, m_π/f_π from MA calc.



Hadron Spectrum, PRD86:034031(2012)

S- and D-wave ($l = 0$ and 2)

calc. at $m_\pi = 0.39$ GeV

other works: CP-PACS, PRD67:014502(2003), Kim, NPB(Proc.Suppl.)129:197(2004), CP-PACS, PRD70:074513(2004), CLQCD, JHEP06:053(2007), Sasaki and Ishizuka, PRD78:014511(2008), Kim and Sachrajda, PRD81:114506(2010), Hadron Spectrum, PRD83:071504(R)(2011)

Phase shift II

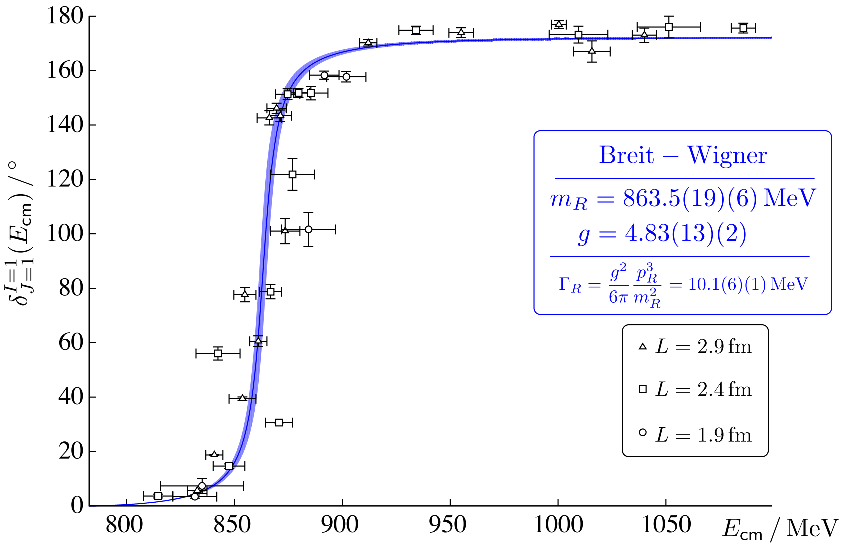
$I = 1$ P-wave $\pi\pi \to \rho$

	1.	2.	3.	4.	5.	6.
$L\lvert P\rvert/2\pi$	1	0,1,$\sqrt{2}$	0,1,$\sqrt{2}$	0,1^2,$\sqrt{2}$	0^*	0,1^2,$\sqrt{2}^3$, $\sqrt{3}^2$,2
N_{mom}	2	5–6	5	6	6	29
m_π[MeV]	320	290-480	270	410, 300	300	390
$m_\pi L$	4.2	$\geq$ 3.7	2.7	6.0, 4.4	$\geq$ 4.6	$\geq$ 3.8

* asymmetric lattice $L^2 \times \eta L$, $\eta = 1, 1.25, 2$

1. CP-PACS, PRD76:094506(2007), 2. ETMC, PRD83:094505(2011),
3. Lang *et al.*, PRD84:054503(2011), 4. PACS-CS, PRD84:094505(2011),
5. Pelissier *et al.*, PRD87:014503(2013), 6. Hadron Spectrum, PRD87:034505(2013)

other works:QCDSF, PoS(LATTICE 2008)136, BMW, PoS(Lattice 2010)139



Hadron Spectrum, PRD87:034505(2013)

Breit-Wigner form fit

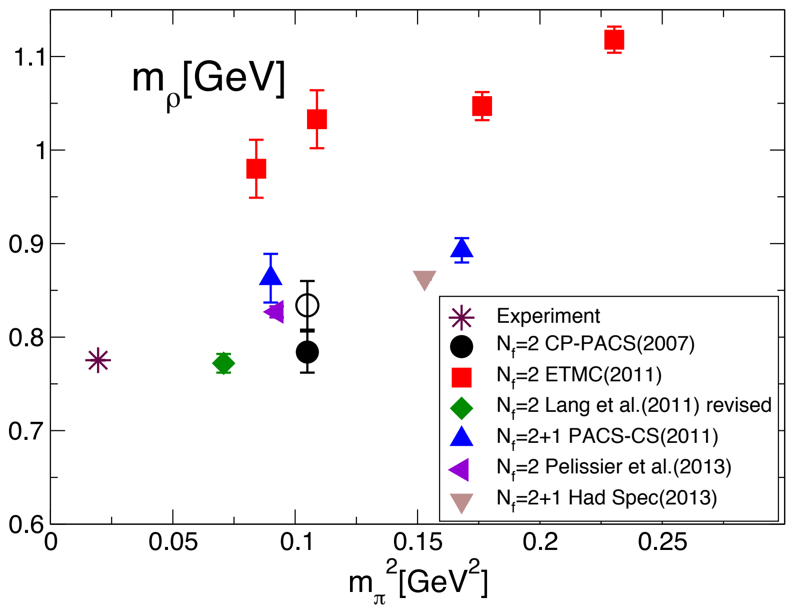
$$\frac{p^3}{\sqrt{s}}\cot\delta(p) = \frac{6\pi}{g^2_{\rho\pi\pi}}(m_\rho^2 - s)$$

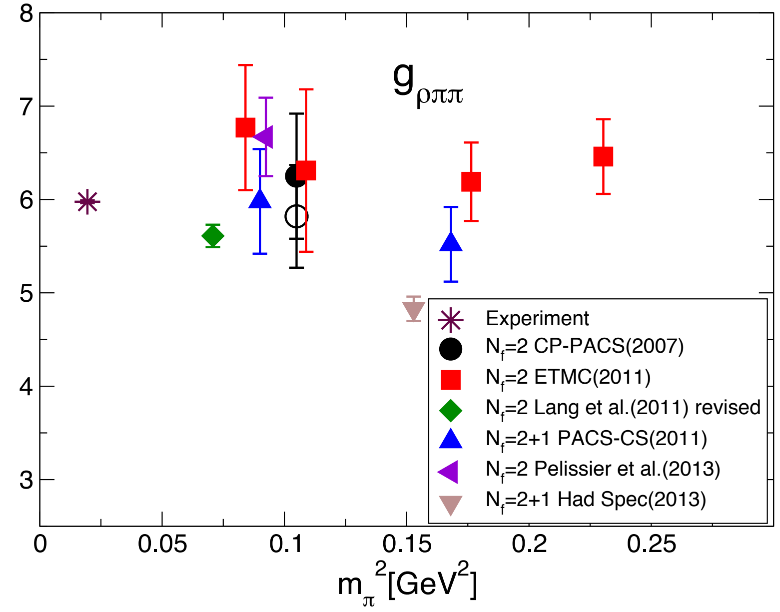
$$s = E^2_{\mathrm{cm}}$$

$$\Gamma_\rho = \frac{p_\rho^3}{m_\rho^2}\frac{g^2_{\rho\pi\pi}}{6\pi}, \quad p_\rho^2 = \frac{m_\rho^2}{4} - m_\pi^2$$

Phase shift II

$I = 1$ P-wave $\pi\pi \to \rho$





open symbol: lattice dispersion relation

CP-PACS, PRD76:094506(2007), ETMC, PRD83:094505(2011), Lang *et al.*, PRD84:054503(2011),
PACS-CS, PRD84:094505(2011), Pelissier *et al.*, PRD87:014503(2013),
Hadron Spectrum, PRD87:034505(2013)

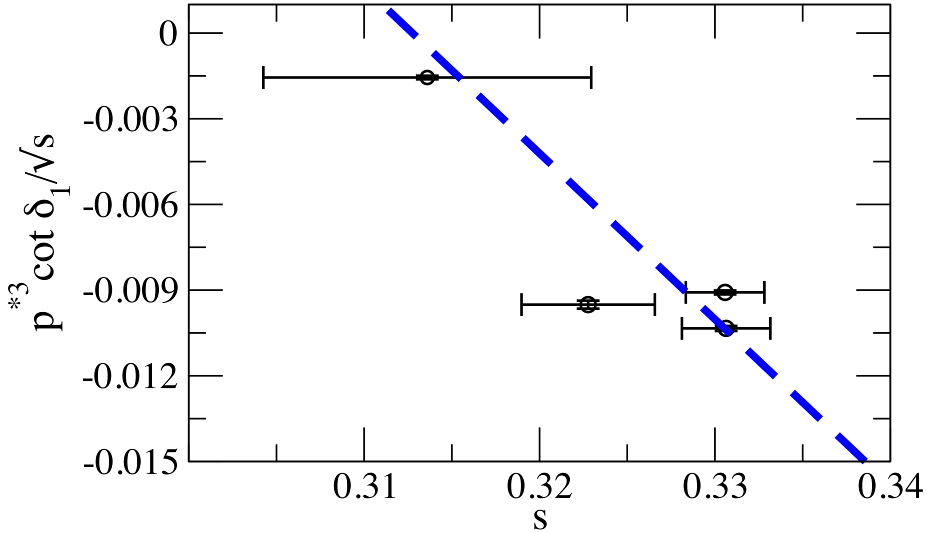
m_ρ scattered due to systematic error from scale (a^{-1}) detemination

Roughly consistent $g_{\rho\pi\pi}$ with experiment

Phase shift III

$I = 1/2$ P-wave $K\pi \to K^*$

Breit-Wigner form fit of $\dfrac{p^3}{\sqrt{s}} \cot \delta(p)$



$N_f = 2$ clover: $L|P|/2\pi = 0, 1, \sqrt{2}$, choose irreps where $l \geq 1$

- $L = 1.9$ fm @ $m_\pi = 0.27$ GeV

Prelovsek *et al.*, PRD88:054508(2013)

- 4 data in resonance region
- $g_{K^*\pi K} = 5.7(1.6) \leftrightarrow g^{\text{exp}}_{K^*\pi K} = 5.65(5)$, $m_{K^*} = 0.891(14)$GeV
- no data in $m_{K^*} < E_\pi(p_{\text{cm}}) + E_K(p_{\text{cm}})$

Bound states

Light nuclei, $X(3872)$

Direct calculation of light nuclei

Traditional method, for example ^{4}He channel

$$\langle 0|O_{^4\mathrm{He}}(t)O^\dagger_{^4\mathrm{He}}(0)|0\rangle = \sum_n \langle 0|O_{^4\mathrm{He}}|n\rangle\langle n|O^\dagger_{^4\mathrm{He}}|0\rangle e^{-E_n t} \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

Problems of multi-nucleon correlation function

1. Statistical error

Statistical error $\propto \exp\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$ in N_N-nucleon system

→ heavier quark mass + large number of measurements

2. Calculation cost PACS-CS, PRD81:111504(R)(2010)

Wick contraction for ^{4}He $= p^2n^2 = (udu)^2(dud)^2$: 518400 → 1107

→ reduction using $p(n) \leftrightarrow p(n)$ $p \leftrightarrow n$, $u(d) \leftrightarrow u(d)$ in $p(n)$

Multimeson: Detmold and Savage, PRD82:014511(2010)

Multibaryon: Doi and Endres, CPC184:117(2013), Detmold and Orginos, PRD87:114512(2013), Günther *et al.*, PRD87:094513(2013)

3. Identification of bound state on finite volume

$\Delta E = E_0 - N_N M_N < 0$: Bound state ↔ Attractive scattering state due to finite volume

→ Volume dependence of ΔE

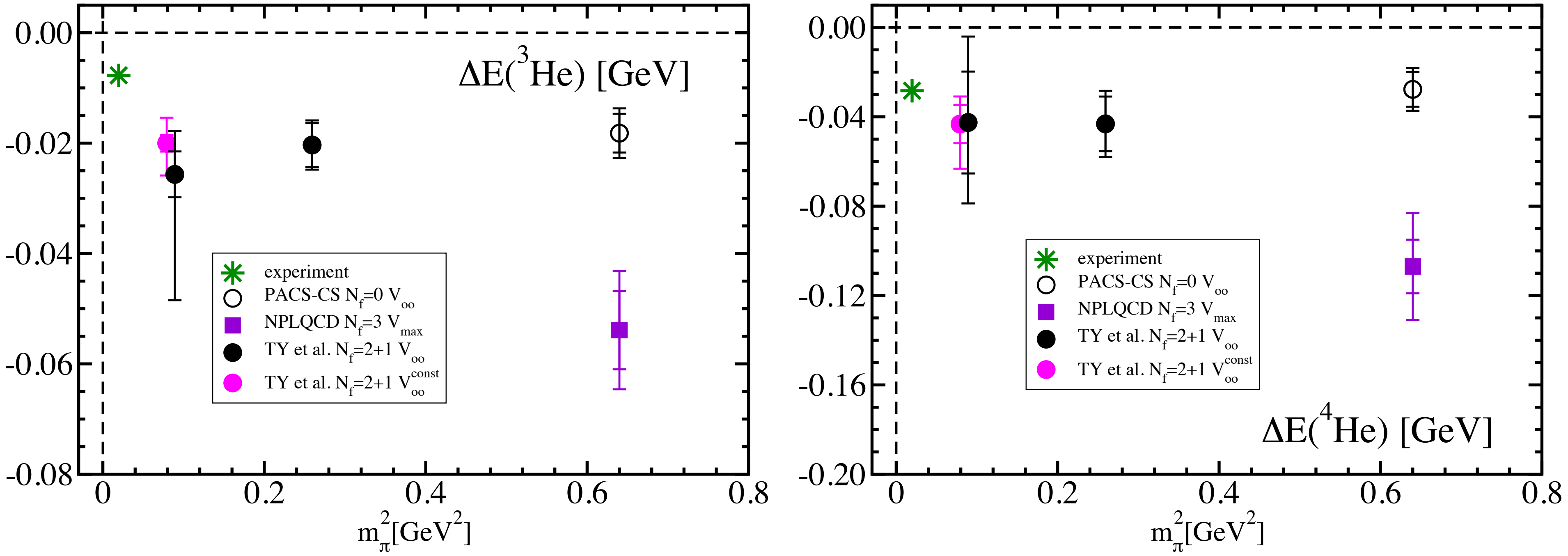
(→ Negative scattering length $a_0 < 0$ in 2-particle system)

Beane *et al.*, PLB585:106(2004); Sasaki and TY, PRD74:114507(2006)

Light nuclei ^{3}He and ^{4}He

First calculation of ^{3}He and ^{4}He PACS-CS, PRD81:111504(R)(2010)

NPLQCD, PRD87:034506(2013), TY *et al.*, PRD86:074514(2012) and preliminary result@$m_\pi = 0.3$GeV



$L^3 \to \infty$ results only

Light nuclei likely formed in 0.3 GeV $\leq m_\pi \leq$ 0.8 GeV

Same order of ΔE to experiments

Can reproduce experimental values?

Investigations of m_π dependence $\to$ $m_\pi = 0.14$ GeV @ $L \sim 8$ fm

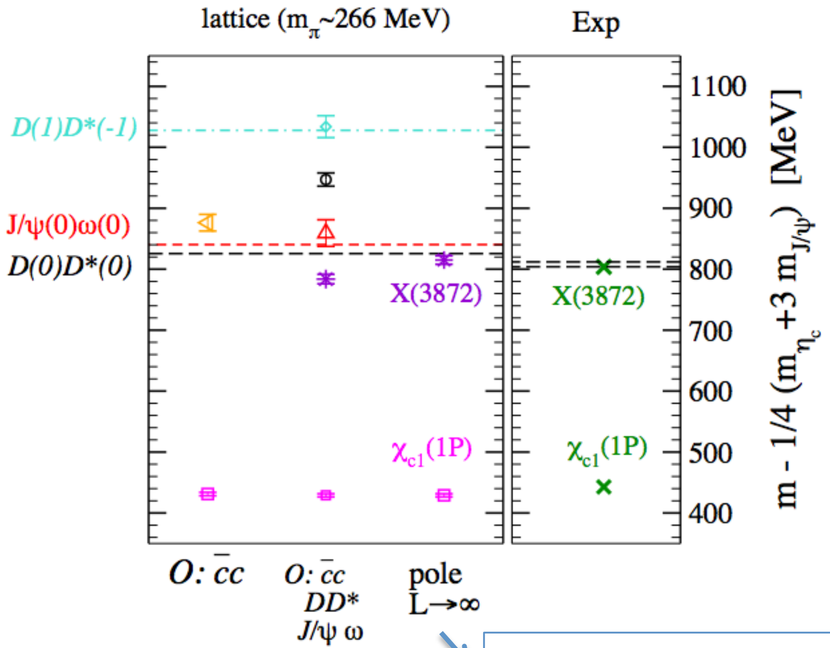
X(3872) $J^{PC} = 1^{++}$ charmonium like state

Belle, PRL91:262001(2003); LHCb, PRL110:222001(2013)

- $M_X = 3871.68(17)$ MeV
- decay to $I = 0(\omega J/\phi)$ and $I = 1(\rho J/\phi)$
- slightly below $D\overline{D}^*$ threshold → molecule of $D\overline{D}^*$?

$N_f = 2$ $I = 0$ Lat14 slide by S. Prelovsek

Prelovsek *et al.*, PRL111:192001(2013)



- δ_0 for DD* extracted using Luscher's rel. and interpolated near threshold
- pole in T-matrix $T \propto [\cot\delta - i]^{-1} = \infty$ found just below DD* threshold.

$M_X - (m_{D^0} + m_{D^{*0}}) = -11(7)$ MeV

c.f. Exp. $-0.14(22)$ MeV

Another lattice calculation $-13(6)$ MeV

DeTar *et al.*, Lat14

Several lattice calcluations of Z_c^+

conclusion is not settled yet

Short summary of hadronic interactions

Hadronic interactions

important to understand properties of hadrons and nuclei

Steadily progressing

Scattering length a_0
High precision calculations in some channels
Rectrangle possilbe, but disconnected diagram needs investigation

Scattering phase shift $\delta(p)$
Resonances possible $\rho \to \pi\pi$ and $K^* \to K\pi$

Bound states
Light nuclei possible, but systematic error study necessary
Charmed exotic meson calculation started

Application to BSM study

$I = 2$ $\pi\pi$ a_0^2 in $N_f = 6$ QCD, nuclei in $N_f = 2$ SU(2) gauge theory, $\cdots$

Search for walking technicolor from lattice gauge theory

LatKMI Collaboration

Y. Aoki, T. Aoyama, E. Bennett, M. Kurachi, T. Maskawa, K. Miura, K.-i. Nagai, H. Ohki, E. Rinaldi, A. Shibata, K. Yamawaki, T. Yamazaki

Refs. PRD86(2012)054506, PRD87(2013)094511,
PRL111(2013)162001, and PRD89(2014)111502(R)

- Recent studies of LatKMI Collaboration
- Results of flavor-singlet scalar

$$N_f = 12 \text{ QCD and } N_f = 8 \text{ QCD}$$

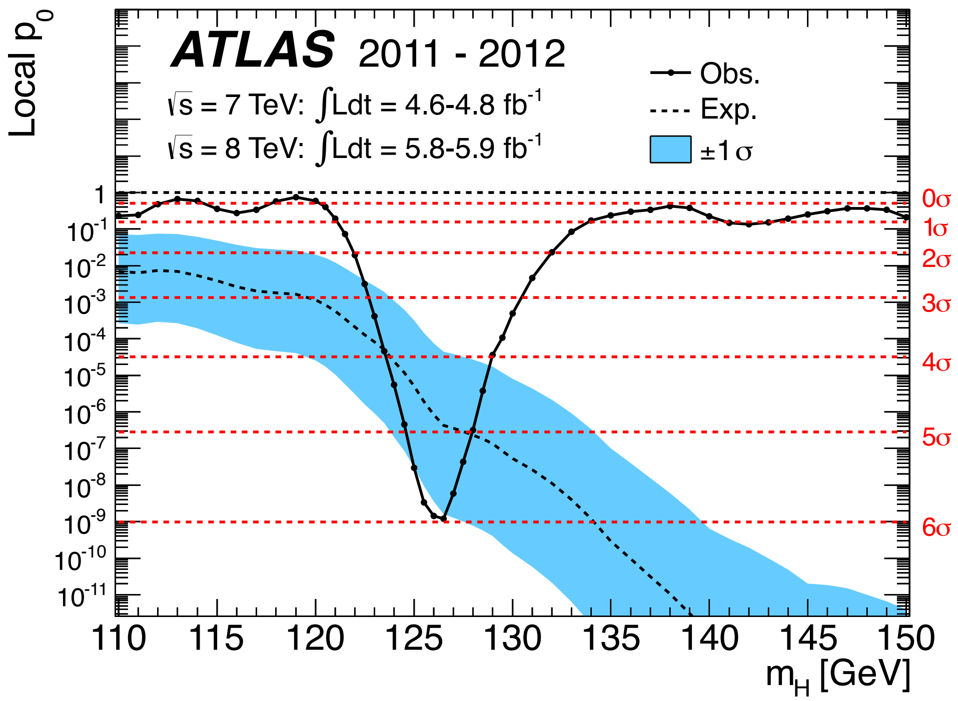
Introduction

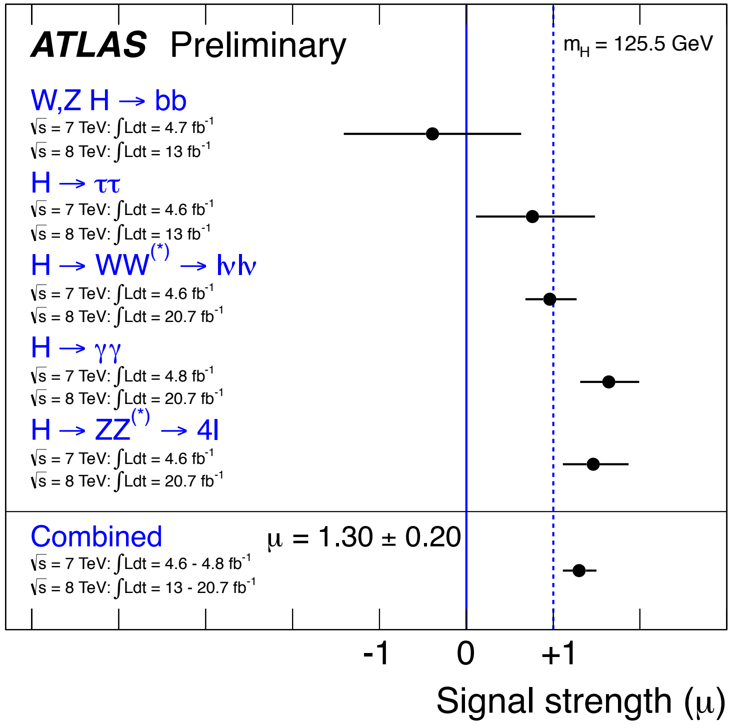
Discovery of Higgs boson @ LHC

$m_H = 125\text{–}126$ GeV

PLB716(2012)

ATLAS NOTE: ATLAS-CONF-2013-034





$\mu = 1$: SM, $\mu = 0$: background

Run2 (2015-) will give improved results.

However, we still have possibility that Higgs boson is not SM Higgs.

Introduction

Discovery of Higgs boson @ LHC

$m_H =$ 125–126 GeV

- Higgs boson

  elementary

- Mechanism of electroweak symmetry breaking

  $\langle H \rangle \neq 0$

- Gauge hierarchy problem

  fine tuning of m_H

Standard Model

Beyond Standard Model: SUSY, Little Higgs, Technicolor, $\cdots$

Hosotani mechanism [Poster: Noaki-san]

Introduction

Discovery of Higgs boson @ LHC

$m_H = 125\text{–}126$ GeV

- Higgs boson

  elementary

  composite

- Origin of electroweak symmetry breaking

  $\langle H \rangle \neq 0$

  VEV from dynamics

- Gauge hierarchy problem

  fine tuning of m_H

  no fine tuning

Standard Model

Technicolor: strongly coupled theory

Beyond Standard Model: SUSY, Little Higgs, Technicolor, $\cdots$

Hosotani mechanism [Poster: Noaki-san]

Technicolor

N_f massless fermions + SU(N_{TC}) gauge at $\mu_{\mathrm{TC}} = O(1)$ TeV

N_f, representation of fermions, N_{TC} not determined

$F^{\mathrm{TC}}, \langle \overline{Q} Q \rangle \neq 0 \to$ similar to QCD

$$F^{\mathrm{TC}} = O(250)\ \mathrm{GeV} \to F_\pi^{\mathrm{QCD}} = 93\ \mathrm{MeV}$$

But, Technicolor $\neq$ scale up of QCD

- FCNC vs quark mass

Inconsistency of constraints

FCNC ($K^0 - \overline{K}^0$ mixing) $\Longleftrightarrow$ large quark mass $m_t = O(100)$ GeV

- Small Higgs mass

$$\frac{m_{\mathrm{Higgs}}}{F^{\mathrm{TC}}} \lesssim 1 \Longleftrightarrow \frac{m^{\mathrm{QCD}}_{f_0(500)}}{F_\pi} = 4 \sim 6$$

Walking technicolor

'86 Yamawaki, Bando, Matumoto

'85 Holdom; '86 Akiba, Yanagida; '86 Appelquist, Karabali, Wijewardhana

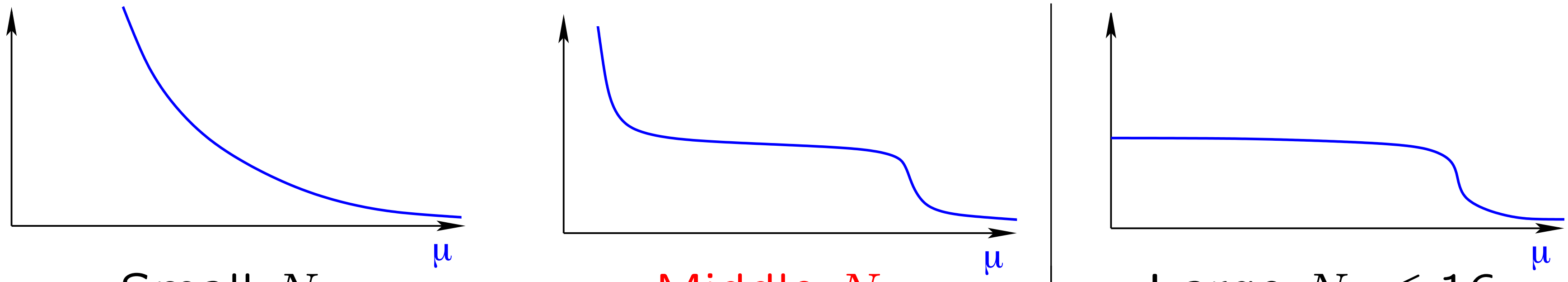
[Talk: Matsuzaki-san]

N_f massless fermions + SU(N_{TC}) gauge at $O(1)$ TeV

Model requirements:

- Spontaneous chiral symmetry breaking
- Slow running (walking) coupling in wide scale range

Running coupling of SU(3) gauge theory



Small N_f Middle N_f Large $N_f \leq 16$

Chiral symmetry breaking ← phase boundary → Conformal

- Large anomalous mass dimension $\gamma \sim 1$ in walking region

Walking technicolor

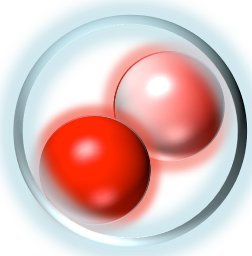
N_f massless fermions + SU(N_{TC}) gauge at $O(1)$ TeV

Model requirements:

- Spontaneous chiral symmetry breaking
- Slow running (walking) coupling in wide scale range
- Large anomalous mass dimension $\gamma \sim 1$ in walking region

- Light composite scalar ≈ Higgs

$$m_{\text{Higgs}}/v_{\text{EW}} \sim 0.5 = m_\sigma/(\sqrt{N_d} f_\pi)$$



f_π : decay constant, N_d : number of weak doublets

usual QCD $m_\sigma/f_\pi \sim$ 4–6

Light composite scalar expected as pNGB (technidilaton)
of scale symmetry breaking

Motivation

- Spontaneous chiral symmetry breaking
- Slow running (walking) coupling in wide scale range
- Large anomalous mass dimension $\gamma \sim 1$ in walking region
- Light composite scalar

Question: Such a theory really exists?

Motivation

- Spontaneous chiral symmetry breaking
- Slow running (walking) coupling in wide scale range
- Large anomalous mass dimension $\gamma \sim 1$ in walking region
- Light composite scalar

Question: Such a theory really exists?

Nonperturbative calculation is important.

→ numerical calculation with lattice gauge theory

study of (approximate) conformal gauge theory

'92 Iwasaki *et al.*, '92 Brown *et al.*, '97 Damgaard *et al.*, '08 Appelquist *et al.*, $\cdots$

	SU(2)	SU(3)	SU(4)
N_f	2, 4, 6, 8	2, 4, 6, 8, 2+N	2

fundamental, adjoint, sextet fermion representations

using running coupling, hadron spectra, finite T phase transition, $\cdots$

Recent studies of LatKMI Collaboration

Purpose in our project

Search for candidate of walking technicolor

Systematic investigation of N_f dependence

SU(3) gauge theory with $N_f = 0, 4, 8, 12, 16$ fermions

$\rightarrow N_f = 0, 4, 8, 12, 16$ QCD

Common setup for all N_f: Improved staggered action (HISQ/Tree)

Cheaper calculation cost + small lattice systematic error

HISQ: '07 HPQCD and UKQCD; HISQ/Tree: '12 Bazakov *et al.*

Basic physical quantities: m_π, F_π, m_ρ, $\langle\overline{\psi}\psi\rangle$

$N_f = 4$: PRD86(2012)054506:PRD87(2013)094511

$N_f = 8$: PRD87(2013)094511

$N_f = 12$: PRD86(2012)054506

Recent studies of LatKMI Collaboration

Search for candidate of walking technicolor

$N_f = 12$: PRD86(2012)054506; $N_f = 8$: PRD87(2013)094511 + updates

$N_f = 4$ QCD: Spontaneous chiral symmetry breaking

$N_f = 12$ QCD: Consistent with conformal phase

hyperscaling $m_\pi, F_\pi \propto m_f^{\frac{1}{1+\gamma^*}}$, $\gamma^* = \gamma$ at infrared fixed point

$N_f = 8$ QCD

- Spontaneous chiral symmetry breaking

  $F_\pi/m_\pi \to \infty$ towards $m_f \to 0$
- Slow running (walking) coupling in wide scale range

  Approximate hyperscaling in F_π
- Large anomalous mass dimension $\gamma \sim 1$ in walking region

  $\gamma = 0.6$–1.0: hyperscaling-like behavior of m_π, F_π, m_ρ

Recent studies of LatKMI Collaboration

Search for candidate of walking technicolor

$N_f = 12$: PRD86(2012)054506; $N_f = 8$: PRD87(2013)094511 + updates

$N_f = 4$ QCD: Spontaneous chiral symmetry breaking

$N_f = 12$ QCD: Consistent with conformal phase

hyperscaling $m_\pi, F_\pi \propto m_f^{\frac{1}{1+\gamma^*}}$, $\gamma^* = \gamma$ at infrared fixed point

$N_f = 8$ QCD may be a candidate of Walking technicolor

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- Large anomalous mass dimension $\gamma \sim 1$ in walking region
  $\gamma = 0.6$–1.0: hyperscaling-like behavior of m_π, F_π, m_ρ
- Light composite flavor-singlet scalar
  ⇐ Important to check, but very difficult
  due to large statistical error (disconnected diagram)

Recent studies of LatKMI Collaboration

Search for candidate of walking technicolor

$N_f = 12$: PRD86(2012)054506; $N_f = 8$: PRD87(2013)094511 + updates

$N_f = 4$ QCD: Spontaneous chiral symmetry breaking

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- Light composite flavor-singlet scalar

⇐ Important to check, but very difficult

noise reduction method + huge number of gauge conf.

Composite flavor-singlet scalar in $N_f = 12$ QCD

Purpose of $N_f = 12$ QCD calculation

Why $N_f = 12$

- Investigated by many groups

  '08,'09 Appelquist *et al.*, '10 Deuzeman *et al.*, '10,'12,'13,'14 Hasenfratz,
  '11 Fodor *et al.*, '11 Appelquist *et al.*, '11 DeGrand, '11 Ogawa *et al.*,
  '12 Lin *et al.*, '12,'13 Iwasaki *et al.*, '12,'13 Itou, '12 Jin and Mawhinney, and $\cdots$

In our work PRD86(2012)054506

consistent behavior with conformal phase

- A few studies of flavor-singlet scalar in conformal theory
  1. SU(2) Adjoint $N_f = 2$ glueball: '09 Del Debbio *et al.*
  2. SU(3) $N_f = 12$ meson: '12 Jin and Mawhinney

Purpouse of this work

Understand properties of flavor-singlet scalar in $N_f = 12$

regarded as pilot study of $N_f = 8$ theory

Flavor-singlet scalar in $N_f = 12$ QCD

PRL111(2013)162001

Simulation parameters

- $\beta = 4$ HISQ/Tree action

  calculation of m_σ

- Huge number of configurations

  measuring every 2 tarj.

- Four m_f on more than two volumes

- Noise reduction method with $N_r = 64$

- Local meson operator of $(1 \otimes 1)$

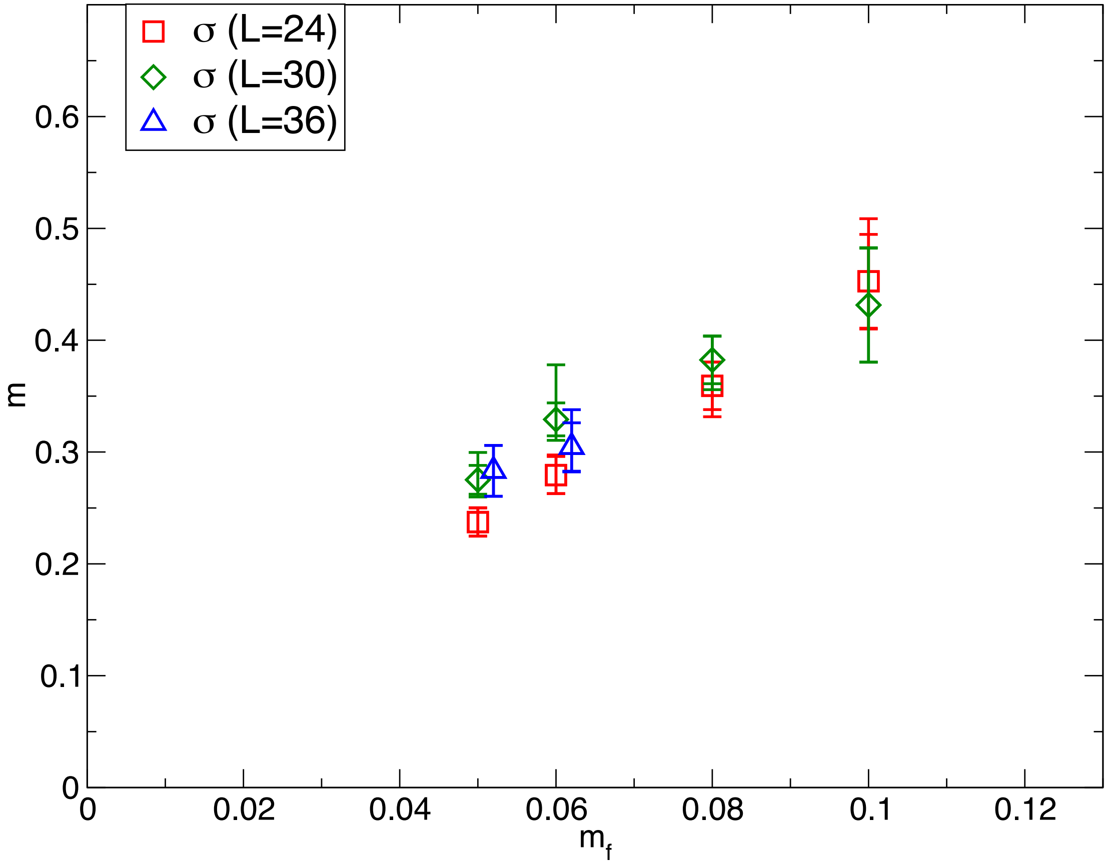
L, T	m_f	confs
24,32	0.05	11000
	0.06	14000
	0.08	15000
	0.10	9000
30,40	0.05	10000
	0.06	15000
	0.08	15000
	0.10	4000
36,48	0.05	5000
	0.06	6000

Machines: φ at KMI, CX400 at Kyushu Univ.

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4$–8



Reasonable signals with almost 10% statistical error

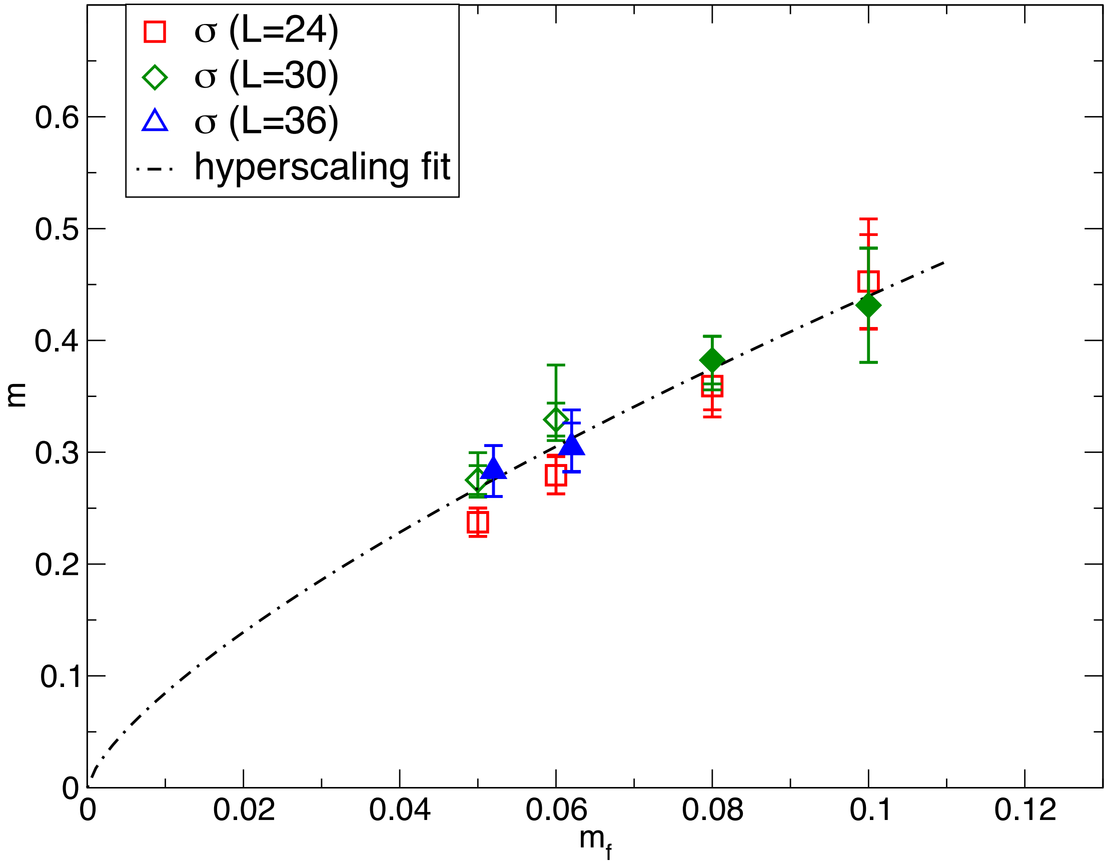
Systematic error from fit range dependence of m_σ

Finite volume effect under control ← 2 larger volumes agree

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t =$ 4–8



Hyperscaling test with fixed γ using larget volume at each m_f

$$m_\sigma = C m_f^{1/(1+\gamma)}$$ with $\gamma = 0.414$ from hyperscaling of m_π

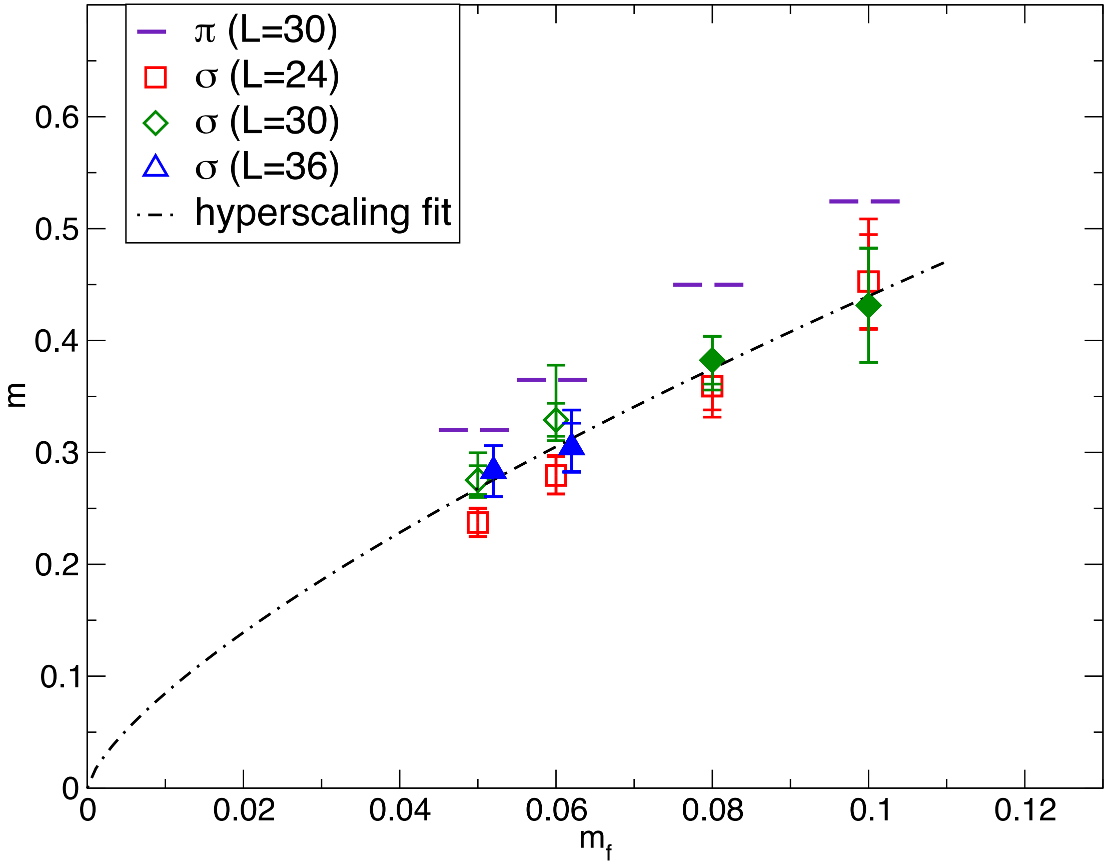
PRD86(2012)054506

Consistent hyperscaling as m_π

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4$–8

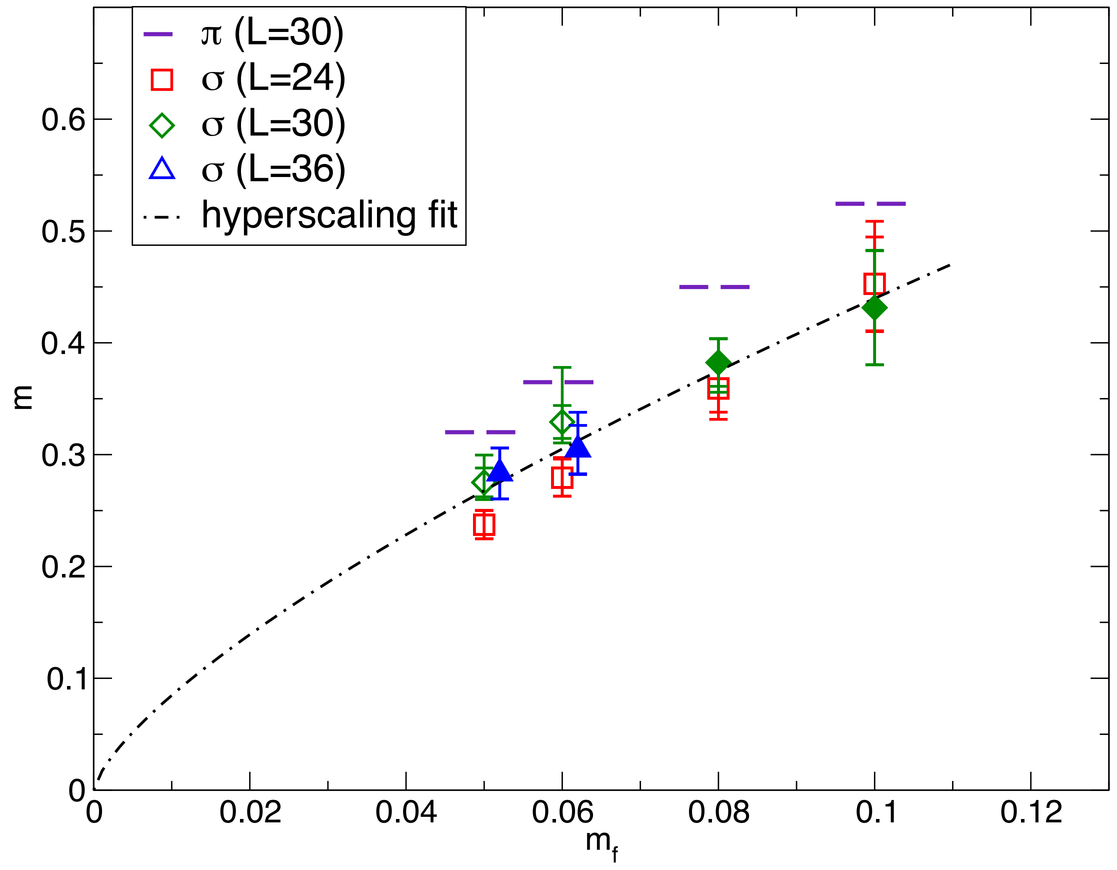


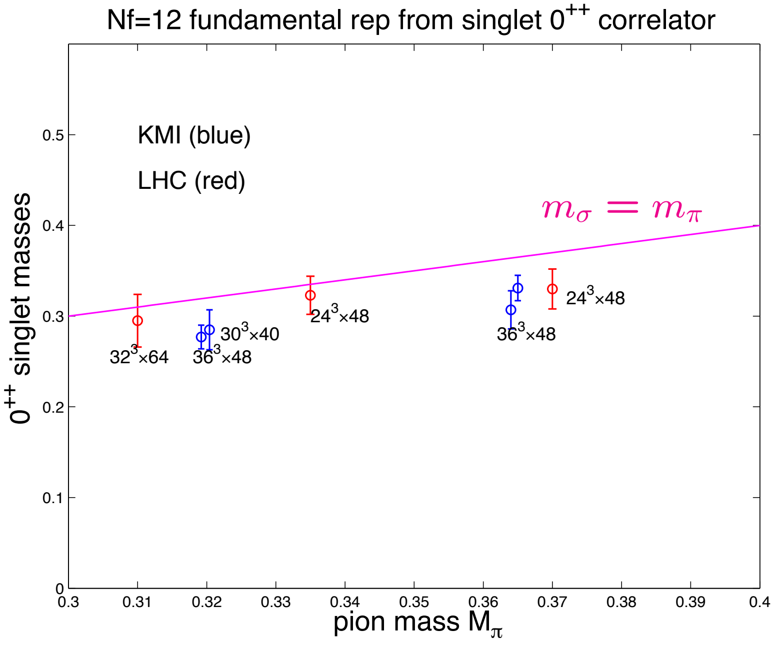
Lighter than π in all m_f

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4$–8





Lighter than π in all m_f

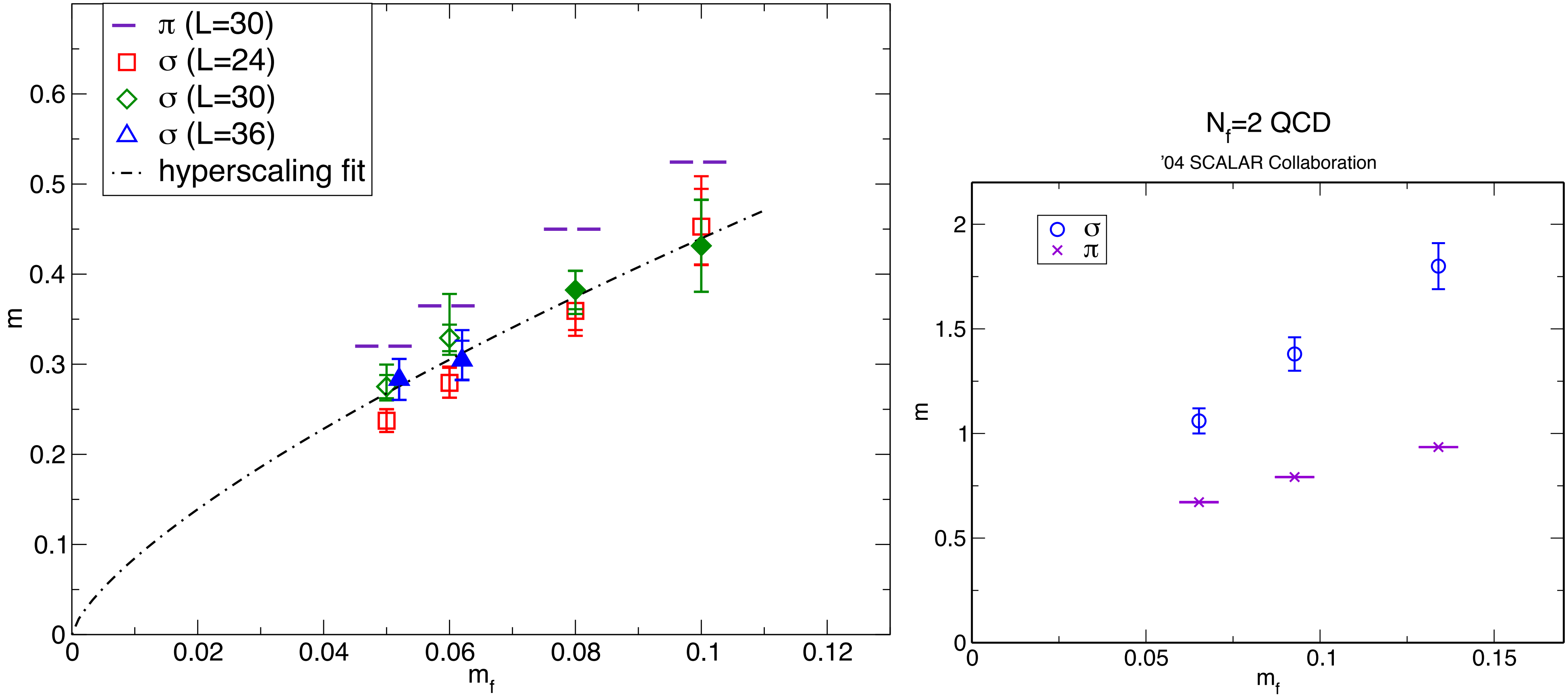
same trend observed by another group after our work

'14 LH collaboration

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4$–8



Lighter than π in all m_f

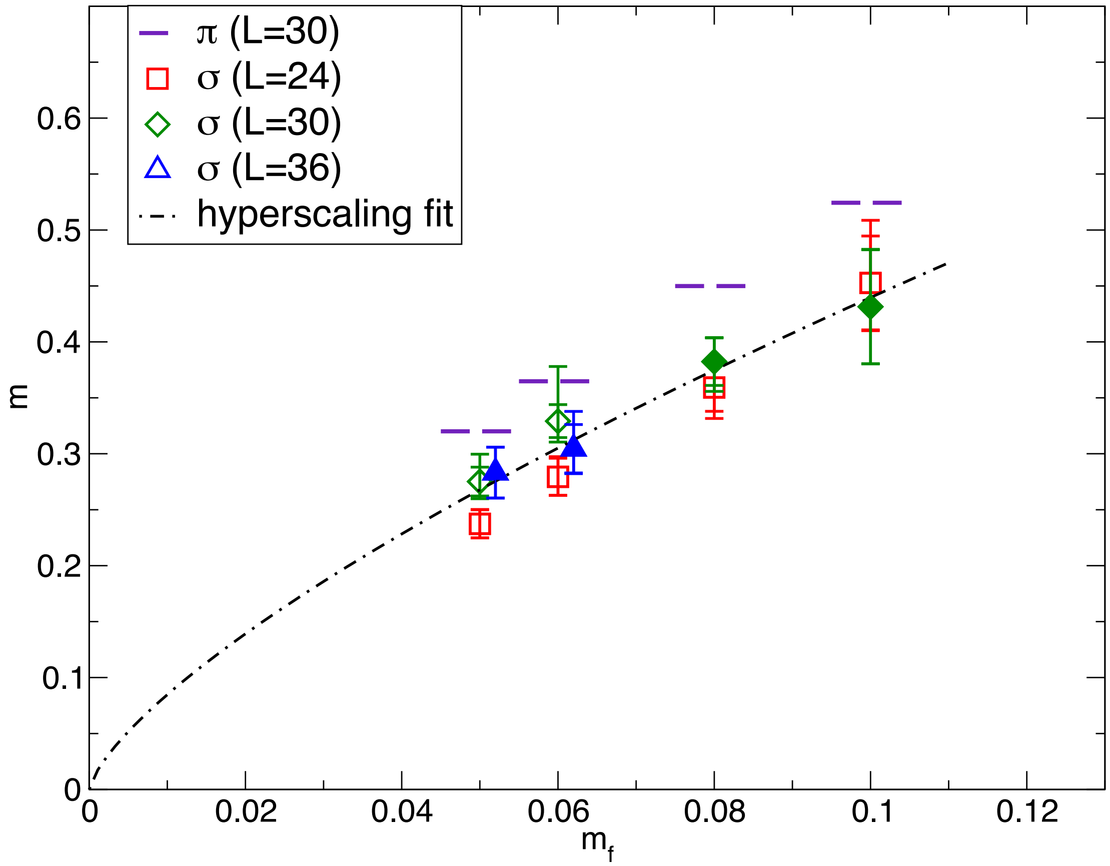
Much different from usual QCD

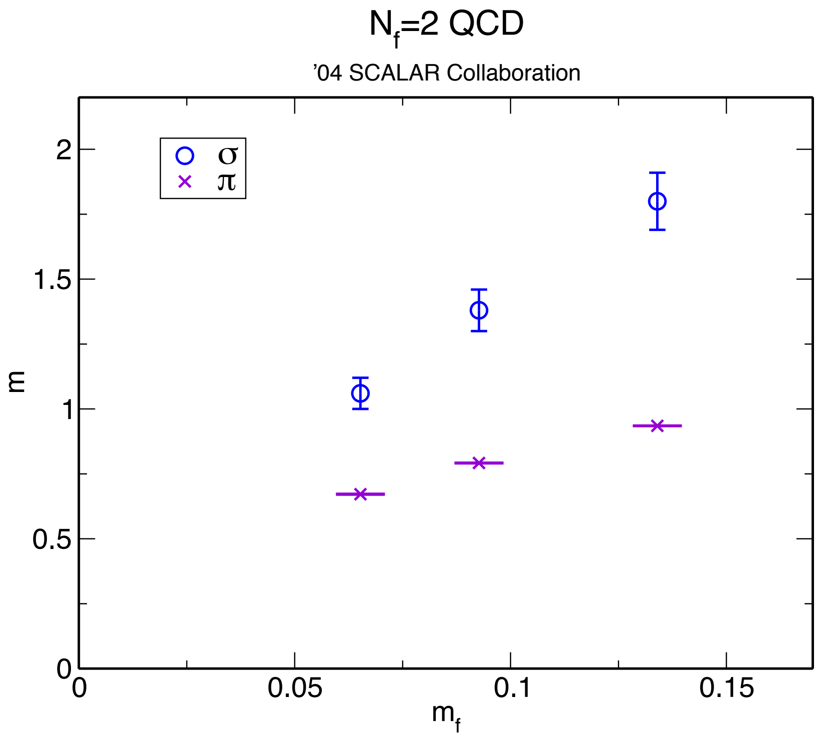
'04 SCALAR Coll.; '07 Bernard *et al.* $m_\sigma = 0.74(15)$ GeV at $m_\pi = 0.26$ GeV

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4$–8




N_f=2 QCD

'04 SCALAR Collaboration

Conformal symmetry may make σ light

Encouranging for observing light scalar
in approximate conformal theory

Composite flavor-singlet scalar in $N_f = 8$ QCD

Flavor-singlet scalar in $N_f = 8$ QCD

$N_f = 8$ QCD may be candidate of walking theory; PRD87(2013)094511

If flavor-singlet scalar is light

→ candidate of walking theory

→ possibility of composite Higgs (technidilaton)

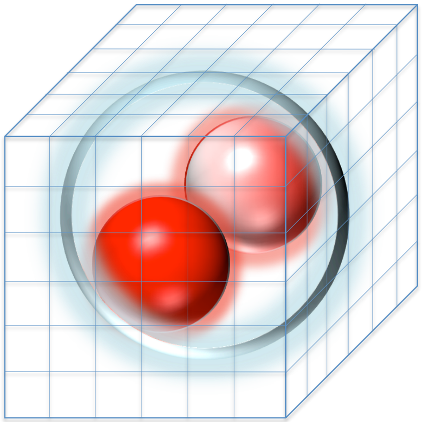
Required conditon to explain $m_{\mathrm{Higgs}}/v_{\mathrm{EW}} \sim 0.5$

$$m_\sigma/f_\pi \sim 1 \text{ in } m_f = 0 \text{ limit}$$

c.f. usual QCD $m_\sigma/f_\pi \sim$ 4–5

Purpose

1. Different from usual QCD?
2. Estimate $m_\sigma/(F/\sqrt{2})$ in $m_f = 0$ lim



Flavor-singlet scalar in $N_f = 8$ QCD

PRD89(2014)111502(R)

Simulation parameters

- $\beta = 3.8$ HISQ/Tree action
  calculation of m_σ
- Huge number of configurations
  measuring every 2 tarj.
- Five m_f with three volumes
- Noise reduction method with $N_r = 64$
- Local meson operator of $(1 \otimes 1)$

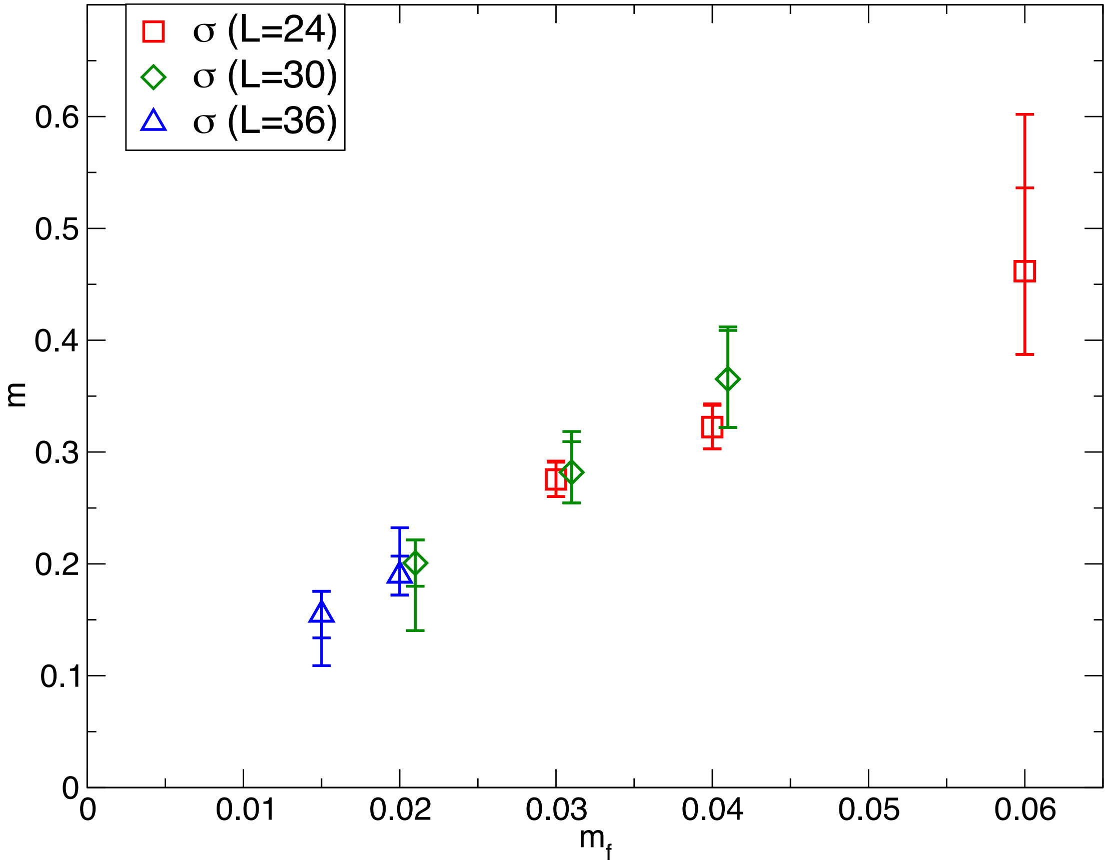
L, T	m_f	confs
24,32	0.03	36000
	0.04	50000
	0.06	18000
30,40	0.02	8000
	0.03	16500
	0.04	12900
36,48	0.02	5000
	0.015	3200

Machines: φ at KMI, CX400 at Nagoya Univ.,
CX400 and HA8000 at Kyushu Univ.

m_f dependence in $N_f = 8$

PRD89(2014)111502(R)

m_σ from fit of $2D(t)$ with $t = 6$–11



Reasonable signals with statistical error $< 20\%$

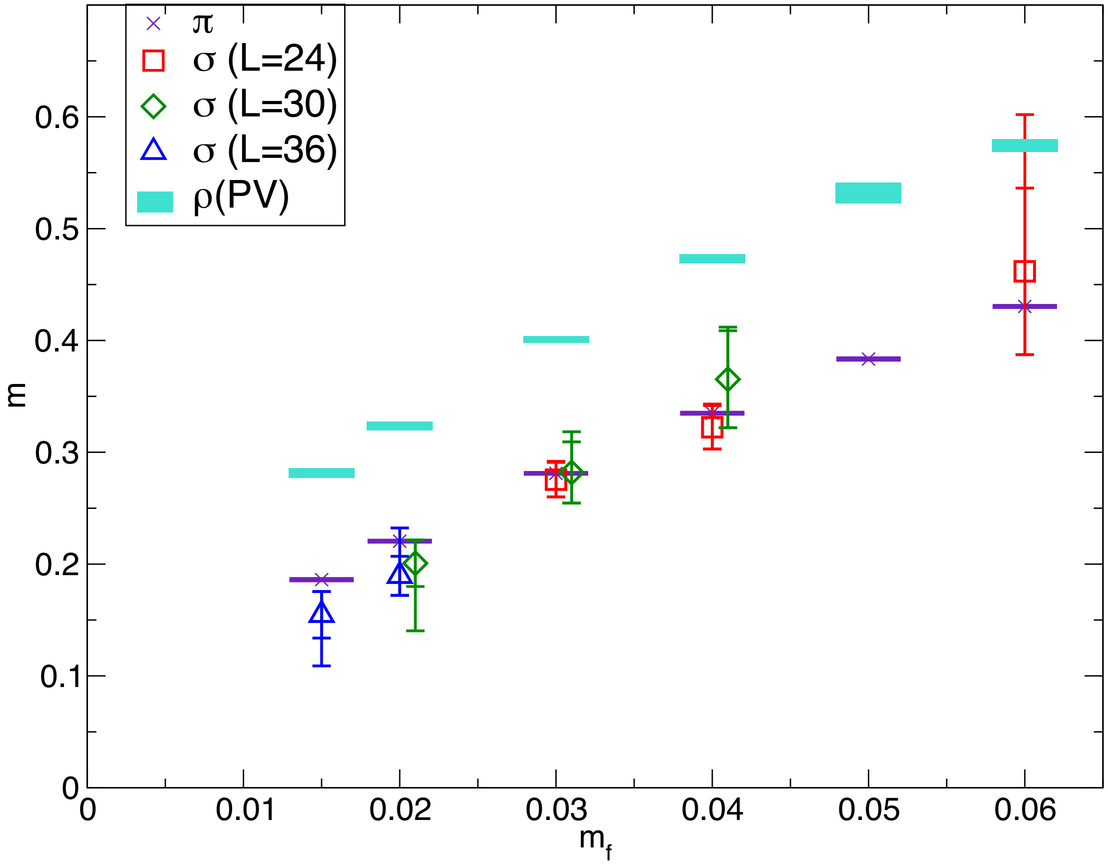
Systematic error from fit range dependence of m_σ

Finite volume effect seems under control

m_f dependence in $N_f = 8$

PRD89(2014)111502(R)

m_σ from fit of $2D(t)$ with $t =$ 6–11



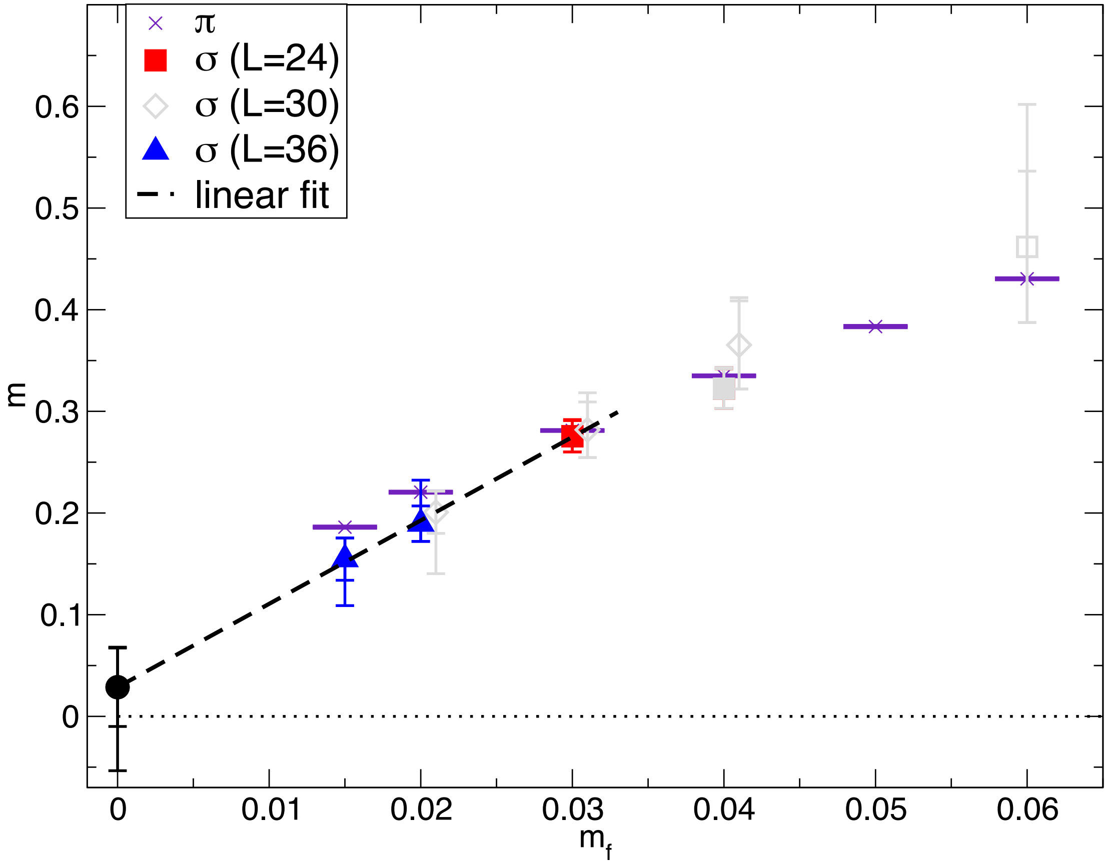
Reasonable signals with statistical error $< 20\%$

Systematic error from fit range dependence of m_σ

$m_\sigma \sim m_\pi$ in all m_f

Different from usual QCD, but similar to $N_f = 12$ QCD

Chiral extrapolation (1) in $N_f = 8$ PRD89(2014)111502(R)



$$m_\sigma = m_0 + A m_f\text{:}\ m_0 = 0.029(39)\left(^{8}_{72}\right) \to \frac{m_\sigma}{F/\sqrt{2}} = 2.0(2.7)\left(^{0.8}_{5.1}\right)$$

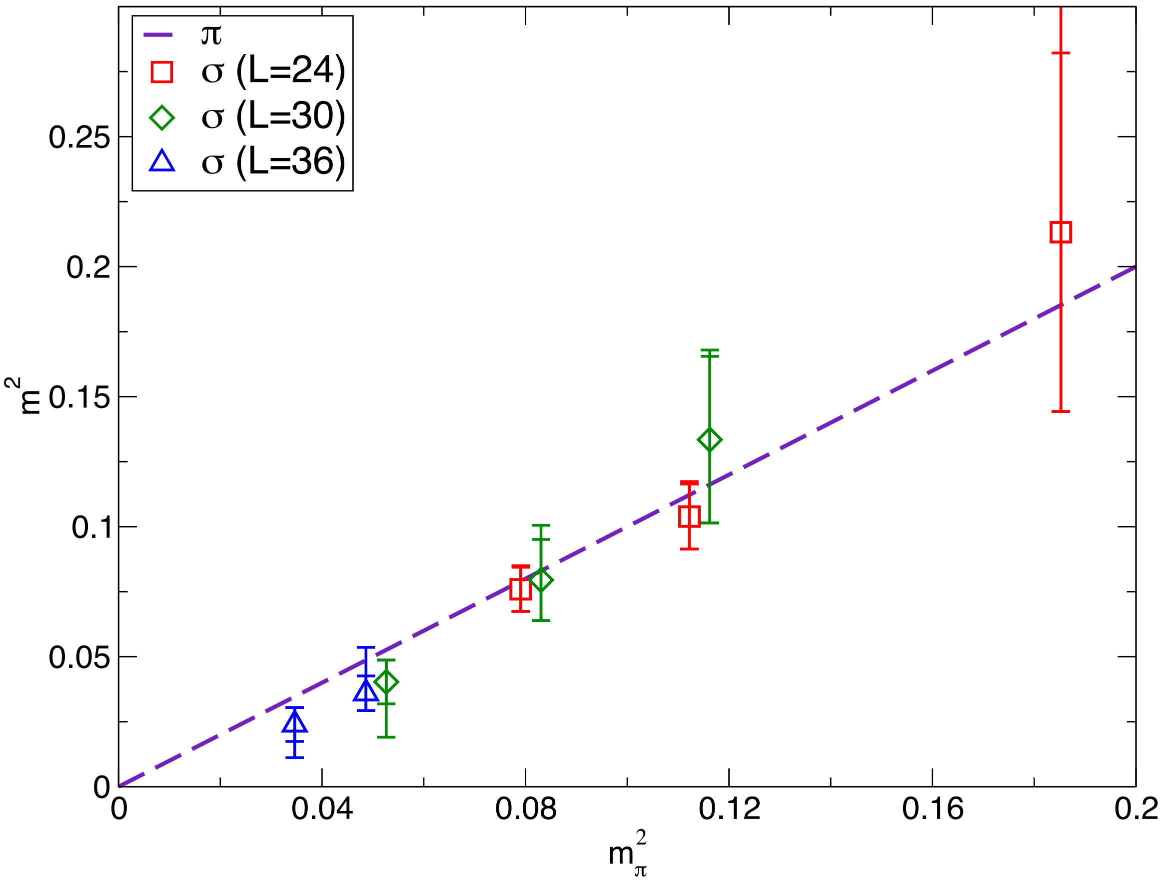
$F = 0.0202(13)\left(^{54}_{67}\right)$ updated from PRD87(2013)094511

$f_\pi = F/\sqrt{2} \sim 93$ MeV in usual QCD

Chiral extrapolation (2) in $N_f = 8$ PRD89(2014)111502(R)

ChPT with scale symmetry breaking '13 Matsuzaki and Yamawaki, PRLXXX

$$m_\sigma^2 = m_0^2 + C \cdot m_\pi^2 + (\text{chiral log of } m_\pi)$$

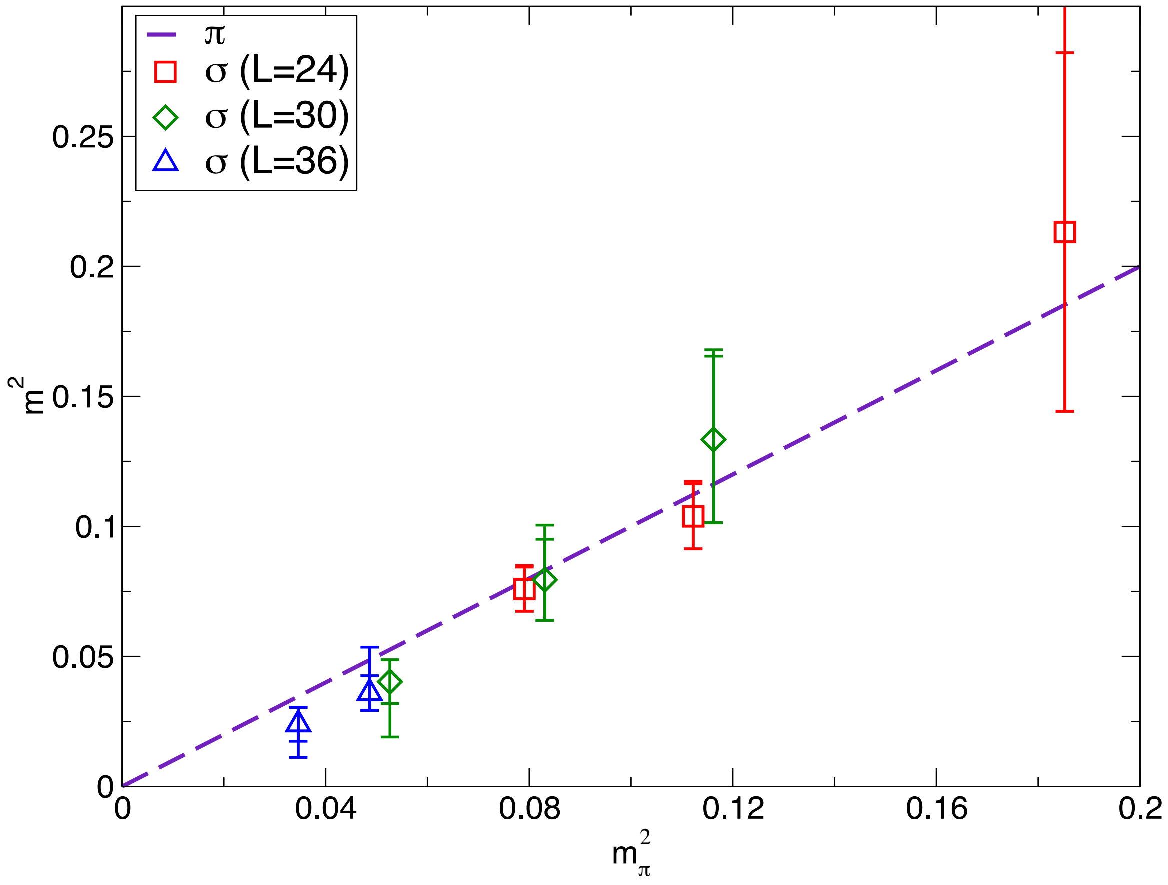


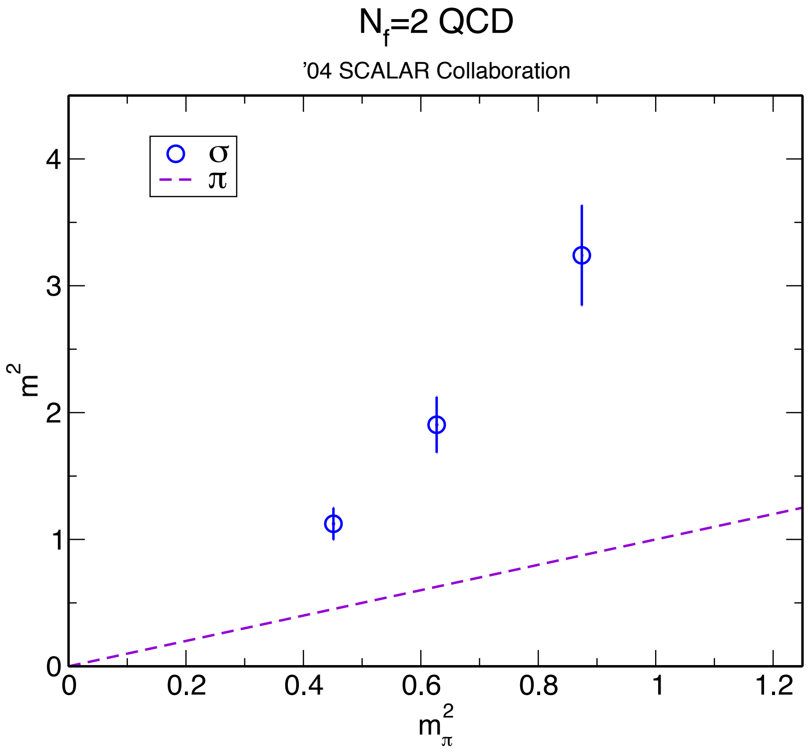
$$m_\sigma \sim m_\pi \to C \sim 1$$

Chiral extrapolation (2) in $N_f = 8$ PRD89(2014)111502(R)

ChPT with scale symmetry breaking ’13 Matsuzaki and Yamawaki, PRLXXX

$$m_\sigma^2 = m_0^2 + C \cdot m_\pi^2 + (\text{chiral log of } m_\pi)$$




N_f=2 QCD

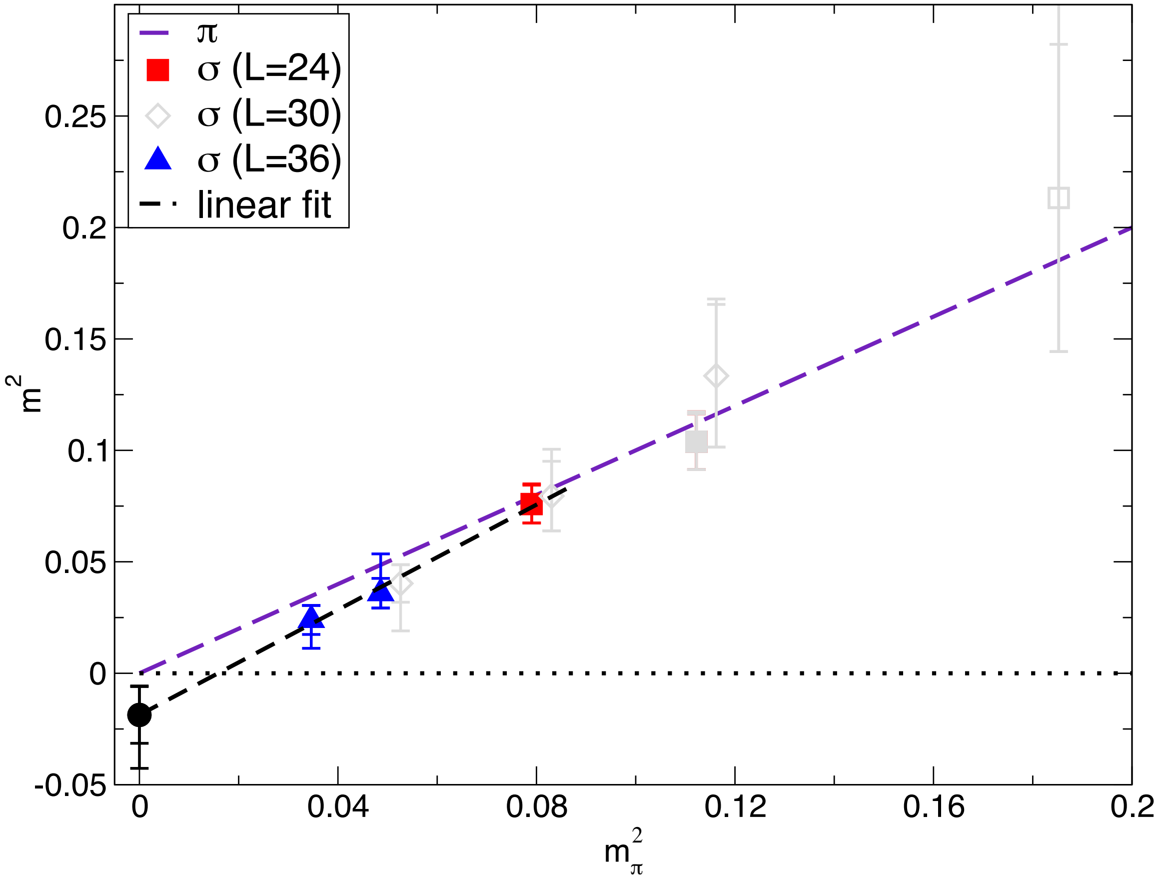
’04 SCALAR Collaboration

$m_\sigma \sim m_\pi \to C \sim 1$: different from $N_f = 2$ QCD

Chiral extrapolation (2) in $N_f = 8$ PRD89(2014)111502(R)

ChPT with scale symmetry breaking '13 Matsuzaki and Yamawaki, PRLXXX

$$m_\sigma^2 = m_0^2 + C \cdot m_\pi^2 + (\text{chiral log of } m_\pi)$$



$m_0^2 < 0$: data not in $m_\sigma > m_\pi$ region

Need to check $m_\sigma > m_\pi$ at smaller m_f as in usual QCD

Comparison of m_σ in $N_f = 8$ with m_{Higgs}

PRD89(2014)111502(R)

$\sqrt{N_d} f_\pi = v_{EW} \rightarrow f_\pi = 123$ GeV $(F = 0.0202(13)(^{54}_{67}), f_\pi = F/\sqrt{2})$

One-family model (four-doublet fermions, $N_d = 4$)

- Simple linear fit

$$\frac{m_\sigma}{F/\sqrt{2}} = 2.0(2.7)(^{0.8}_{5.1})$$

consistent with $m_{\text{Higgs}} = 125$ GeV $\sim F/\sqrt{2}$ within lower error

- ChPT with spontaneous scale symmetry breaking

$$m_\sigma^2 = -0.019(13)(^{3}_{20})$$

consistent with $m_{\text{Higgs}}^2 \sim F^2/2 = 0.0002$ within 1.5 standard deviation

- Several other fits, *e.g.*, $m_\sigma^2/(F/\sqrt{2})^2 = d_0 + d_1 m_\pi^2$

consistent results within large error

Possibility to reproduce m_{Higgs}

Summary of walking technicolor study

$N_f = 12$ QCD consistent behaviors with (mass-deformed) conformal phase
$N_f = 8$ QCD maybe candidate of walking technicolor

$$M_\rho/(F/\sqrt{2}) = 8.5(2.1) \text{ (statistical error only)}$$

Flavor-singlet scalar
Difficulty $\Rightarrow$ Noise reduction method and large N_{conf} $O(10000)$

Results of $N_f = 12$ QCD

- $m_\sigma < m_\pi$; much different from small N_f QCD
- Conformal symmetry may make σ light

Results of $N_f = 8$ QCD

- $m_\sigma \sim m_\pi$; much different from small N_f QCD
- Might be reflection of approximate conformal symmetry
- Need more data at smaller m_f for reliable chiral extrapolation
  but several fit results suggest

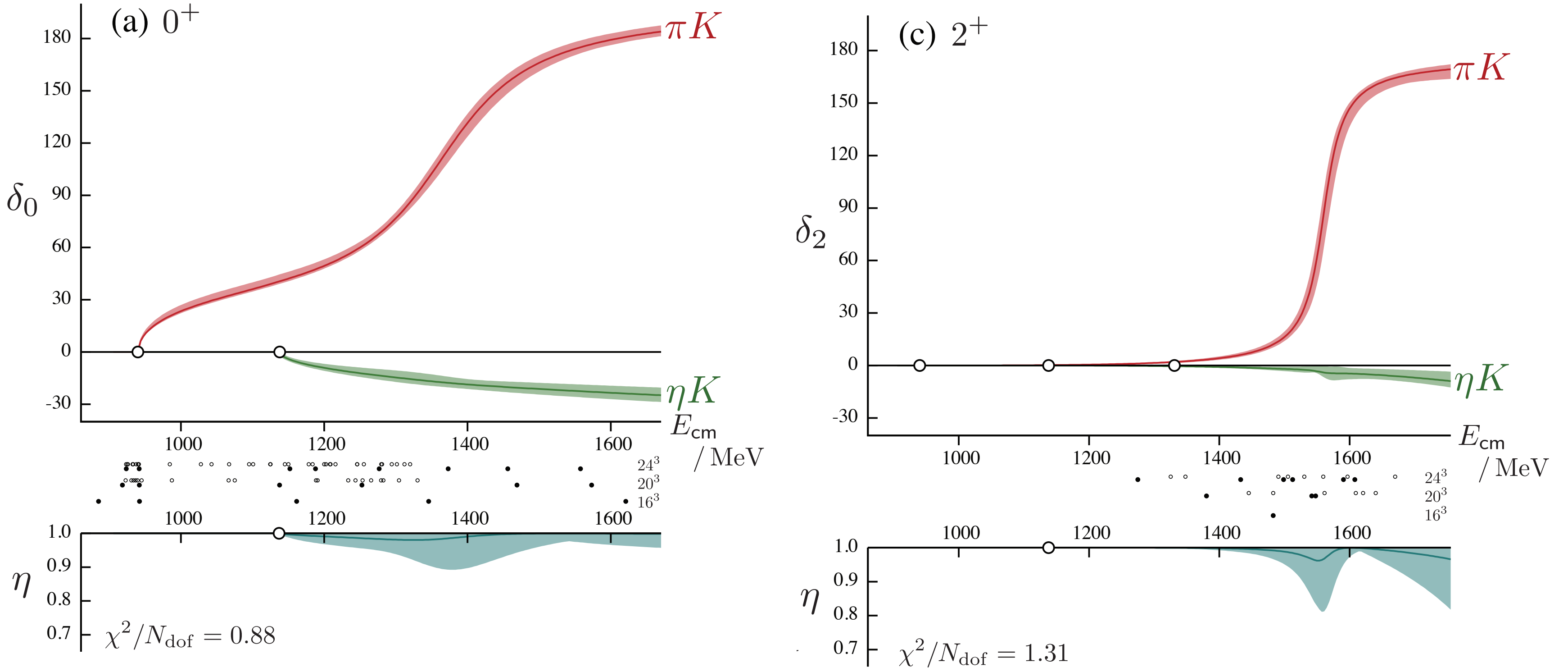
Possibility of light composite scalar $\rightarrow m_{\text{Higgs}} \sim v_{EW}$
(technidilaton)

Future direction: F_σ, S paramter, $\cdots$

Back up

Phase shift IV

$I = 1/2\ K\pi$ S-wave and D-wave



$N_f = 2+1$ aniso. clover: $L|P|/2\pi = 0, 1, \sqrt{2}, \sqrt{3}, 2$, choose irreps

- $K\pi$, $K\eta$ coupled channel analysis
- $\delta_{K\pi}$, $\delta_{K\eta}$, η inelasticity
- $m_\kappa < m_\pi + m_K$
- resonances corresponding to $K_0^*(K_2^*)$ in $l = 0(2)$

Hadron Spectrum, arXiv:1406.4158