A model with radiative electroweak symmetry breaking and dark matter

K.E. and Y. Sumino (Tohoku Univ.), *JHEP* 1505 (2015) 030.
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1. Introduction

Higgs sector in the Standard Model

- "Higgs" boson was discovered in 2012.
- Its properties are <u>consistent with the Higgs boson in SM</u>.

VEV, mass, spin, parity, some couplings etc.

$$V_H^{\rm SM} = -\mu_H{}^2 H^\dagger H + \lambda_H (H^\dagger H)^2$$

Higgs potential of SM

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{\rm H} + h \end{pmatrix}$$

VEV (unitary gauge)

$v_H = 246$ GeV

h :Higgs boson field

EWSB →

$$\text{const.} + \lambda_H v_H{}^2 h^2 + \lambda_H v_H h^3 + \frac{\lambda_H}{4} h^4$$

mass

$$m_h = \sqrt{2\lambda_H} v_H \simeq 125 \text{ GeV}$$

unmeasured

triple coupling

quartic coupling

- Measurements of Higgs self couplings are necessary to determine the Higgs potential.

Motivation

<u>Current situation</u>

Higgs sector is as in the SM. → **OK!** → Consistent with current experimental data.

Consistent with current experimental data. → **OK?** → Higgs sector is as in the SM.

- Is there possibility that ONLY self couplings deviate (substantially) from SM predictions?
- We are interested in information about the vicinity of the vacuum (“shape” of potential).

Motivation

- One candidate: Scale-invariant extension

→ EWSB via CW mechanism (radiative symmetry breaking)

→ Effective potential is like $\phi^4 \log(\phi)$

- irregular at the origin (e.g. QCD, BCS theory)
- realise Higgs VEV 246 GeV and mass 125 GeV,
- large deviations in cubic and quartic self couplings?

We dealt with singlet–extension.

Contents

2. Model

and

Our method of analysis

Model

$$\mathscr{L} = \mathscr{L}_{\rm SM}|_{\mu_{\rm H}\to 0} + \frac{1}{2}(\partial_\mu \vec{S})^2 - \lambda_{\rm HS}(H^\dagger H)(\vec{S}\cdot\vec{S}) - \frac{\lambda_{\rm S}}{4}(\vec{S}\cdot\vec{S})^2$$

H: Higgs doublet

$\vec{S}$: SM singlet (global $O(N)$-multiplet)

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}G^+ \\ v_{\rm H} + h + iG^0 \end{pmatrix}, \quad \vec{S} = \begin{pmatrix} v_{\rm S} + s_1 & s_2 & \cdots & s_N \end{pmatrix}^{\rm T}$$

K. A. Meissner et.al. [hep-ph/0612165],
R. Foot et. al. [0704.1165]
etc.

- We have re-analysed the model and its effective potential,
- and found the consistent vacuum by more precise perturbation.

(cf. R. Dermisck et.al. [1308.0891], C. Tamarit [1404.7673])

Particle content

$$\mathscr{L} = \mathscr{L}_{\rm SM}|_{\mu_{\rm H}\to 0} + \frac{1}{2}(\partial_\mu \vec{S})^2 - \lambda_{\rm HS}(H^\dagger H)(\vec{S}\cdot\vec{S}) - \frac{\lambda_{\rm S}}{4}(\vec{S}\cdot\vec{S})^2$$

H: Higgs doublet

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$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}G^+ \\ v_{\rm H} + h + iG^0 \end{pmatrix}, \quad \vec{S} = \begin{pmatrix} v_{\rm S} + s_1 & s_2 & \cdots & s_N \end{pmatrix}^{\rm T}$$

SM particles

+

N real singlet scalar bosons (s_i)

- mass^2 ~ $\lambda_{HS} v_H^2$
- singlet field VEV <S>=0 eventually
- thus singlet cannot decay

Effective potential

H: Higgs doublet

$\vec{S}$: SM singlet (global $O(N)$-multiplet)

$$\langle H \rangle_J = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \qquad \langle \vec{S} \rangle_J = (\varphi,\, 0,\, \cdots,\, 0)\,; \qquad \phi,\, \varphi \in \mathbb{R}, \quad \phi,\, \varphi \neq 0$$

$$V_{\text{eff}}(\phi,\, \varphi) = V_{\text{tree}}(\phi,\, \varphi) + V_{\text{1-loop}}(\phi,\, \varphi)$$

$$V_{\text{tree}}(\phi,\, \varphi) = \frac{\lambda_{\text{H}}}{4}\phi^4 + \frac{\lambda_{\text{HS}}}{2}\phi^2\varphi^2 + \frac{\lambda_{\text{S}}}{4}\varphi^4$$

$$V_{\text{1-loop}}(\phi,\, \varphi) = \sum_i \frac{n_i}{64\pi^2} {M_i}^4(\phi,\, \varphi)\left[\ln\frac{{M_i}^2(\phi,\, \varphi)}{\mu^2} - c_i\right](\overline{\text{MS}}\text{ scheme})$$

${M_i}^2(\phi,\, \varphi)$: mass-squared eigenvalues ← determined in tree level

μ: renormalisation scale → set to Higgs VEV 246GeV

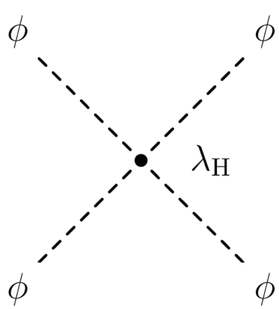
c_i: scheme-dependent constants

Order counting in perturbation

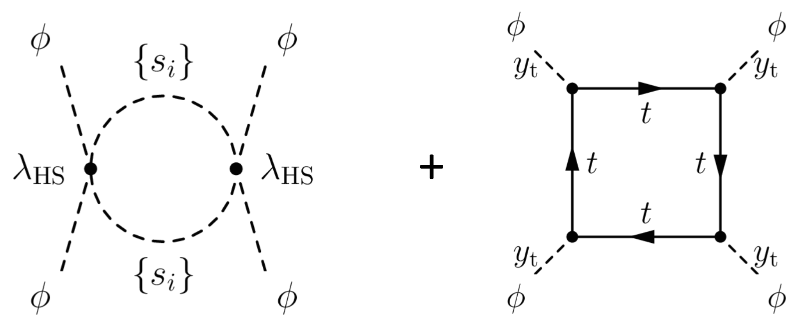
It is necessary that tree and 1-loop effects balance each other to occur correctly EWSB via CW mechanism.

tree effects

1-loop effects



vs



$$|\lambda_{\rm H}| \sim \frac{N{\lambda_{\rm HS}}^2}{(4\pi)^2} - \frac{N_{\rm C}{y_{\rm t}}^4}{(4\pi)^2} \ll 1$$

(massless scalar QED: $\lambda \sim \dfrac{e^4}{(4\pi)^2} \ll 1$)

(Roughly $|\lambda_{\rm H}| \ll |\lambda_{\rm HS}|$. $N_{\rm C}$ is the colour factor.)

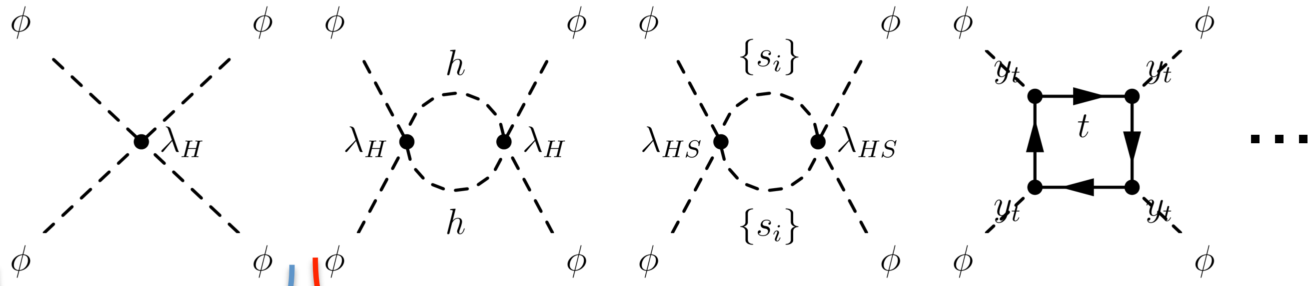
↓

<u>Using this relation gives us specific order-counting.</u>

Order counting in perturbation

only for Higgs field ϕ, for simplicity

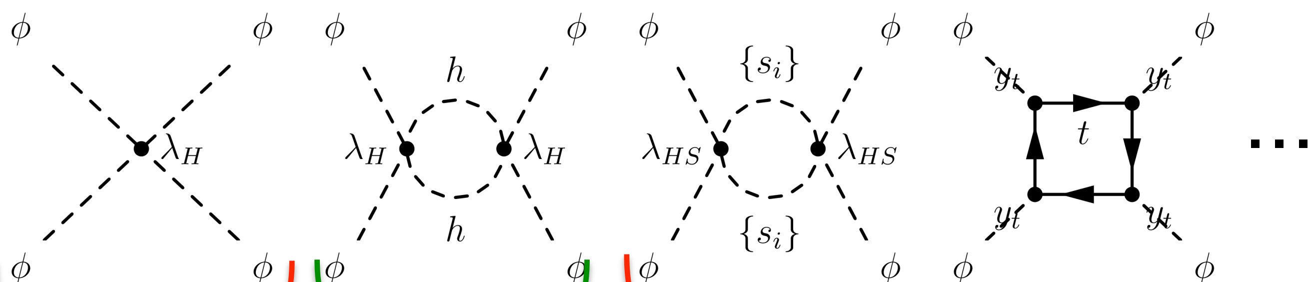
[Usual perturbation]



LO (tree level)

NLO (1-loop level)

[This prescription]



NLO of order-counting

NNLO of order-counting

NLO of order-counting

Order counting in perturbation

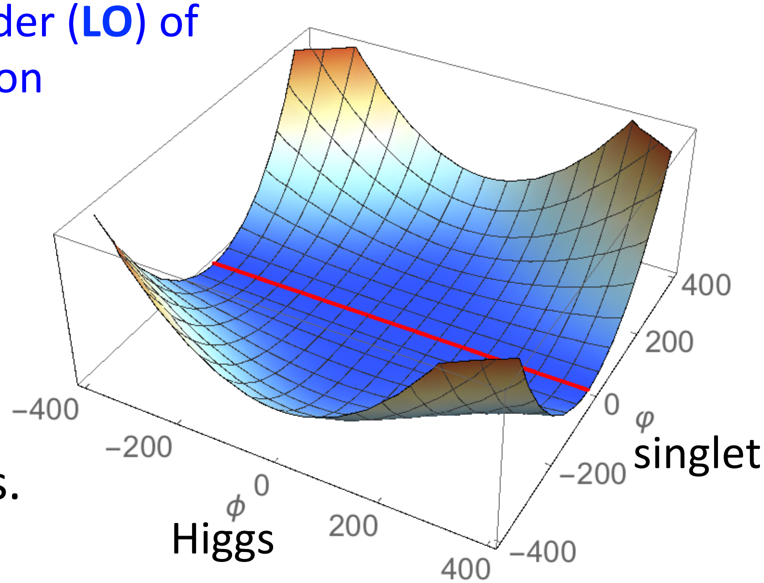
[For the tree potential]

$$|\lambda_{\rm H}| \ll |\lambda_{\rm HS}|$$

$$\frac{\lambda_{\rm H}}{4}\phi^4 + \frac{\lambda_{\rm HS}}{2}\phi^2\varphi^2 + \frac{\lambda_{\rm S}}{4}\varphi^4$$

next-to-leading order (**NLO**) of perturbation

leading order (**LO**) of perturbation



- The **LO** potential is $\frac{\lambda_{\rm HS}}{2}\phi^2\varphi^2 + \frac{\lambda_{\rm S}}{4}\varphi^4$
- and it has degenerate minima on the ϕ-axis.

Order counting in perturbation

[For the 1-loop corrections]

$$\frac{1}{64\pi^2}\mathrm{STr}M^4(\phi)\left[\ln\left(\frac{M^2(\phi)}{\mu^2}\right)-C\right]$$

$$|\lambda_\mathrm{H}| \ll |\lambda_\mathrm{HS}|$$

- $$\frac{1}{64\pi^2}(\lambda_\mathrm{H}\phi^2+\lambda_\mathrm{HS}\varphi^2)^2\left[\ln\left(\frac{\lambda_\mathrm{H}\phi^2+\lambda_\mathrm{HS}\varphi^2}{\mu^2}\right)-\frac{3}{2}\right]$$
  $$\simeq \frac{1}{64\pi^2}(\lambda_\mathrm{HS}\varphi^2)^2\left[\ln\left(\frac{\lambda_\mathrm{HS}\varphi^2}{\mu^2}\right)-\frac{3}{2}\right]$$
- $$\frac{1}{64\pi^2}{F_\pm}^2(\phi,\,\varphi)\left[\ln\left(\frac{F_\pm(\phi,\,\varphi)}{\mu^2}\right)-\frac{3}{2}\right]\simeq\frac{1}{64\pi^2}{F_{\pm\mathrm{app}}}^2(\phi,\,\varphi)\left[\ln\left(\frac{F_{\pm\mathrm{app}}(\phi,\,\varphi)}{\mu^2}\right)-\frac{3}{2}\right],$$

  where

  $$F_\pm(\phi,\,\varphi)\simeq F_{\pm\mathrm{app}}(\phi,\,\varphi)$$
  $$\equiv\frac{\lambda_\mathrm{HS}}{2}\phi^2+\frac{\lambda_\mathrm{HS}+3\lambda_\mathrm{S}}{2}\varphi^2\pm\sqrt{\left[-\frac{\lambda_\mathrm{HS}}{2}\phi^2+\frac{\lambda_\mathrm{HS}-3\lambda_\mathrm{S}}{2}\varphi^2\right]^2+4{\lambda_\mathrm{HS}}^2\phi^2\varphi^2}$$

Order counting in perturbation

$$
\begin{aligned}
V_{\rm LO} =& \frac{\lambda_{\rm HS}}{2}\phi^2\varphi^2 + \frac{\lambda_{\rm S}}{4}\varphi^4 \\
V_{\rm NLO} =& \frac{\lambda_{\rm H}}{4}\phi^4 + \frac{{F_{+\rm app}}^2(\phi,\varphi)}{64\pi^2}\left[\ln\left(\frac{F_{+\rm app}(\phi,\varphi)}{\mu^2}\right) - \frac{3}{2}\right] \\
&+ \frac{3}{64\pi^2}(\lambda_{\rm HS}\varphi^2)^2\left[\ln\left(\frac{\lambda_{\rm HS}\varphi^2}{\mu^2}\right) - \frac{3}{2}\right] \\
&+ \frac{N-1}{64\pi^2}(\lambda_{\rm HS}\phi^2 + \lambda_{\rm S}\varphi^2)^2\left[\ln\left(\frac{\lambda_{\rm HS}\phi^2 + \lambda_{\rm S}\varphi^2}{\mu^2}\right) - \frac{3}{2}\right] \\
&+ (\text{vector boson parts}) + (\text{fermion parts})
\end{aligned}
$$

@ **LO**

- Higgs field has degenerate minima on the ϕ -axis.
- Singlet field does not have VEV (*O*(*N*)-symmetry is not broken).

@ **NLO**

- Higgs VEV determines and Higgs boson gets its mass.
- Higgs becomes lighter than singlet(s).
  (cf. E. Gildener and S. Weinberg [*PRD*13, 3333 (1976)])

RG-improved potential

We used the two kinds of RG-improved potential.

[M. Bando et. al., *PLB* 301, 83 (1993)]

$$V_{\rm eff}^{\rm (LL)}(\phi,\,\varphi) = \frac{\lambda_{\rm H}(t)}{4}\phi^4(t) + \frac{\lambda_{\rm HS}(t)}{2}\phi^2(t)\varphi^2(t) + \frac{\lambda_{\rm S}(t)}{4}\varphi^4(t)$$

$$V_{\rm eff}^{\rm (imp\text{-}NLO)}(\phi,\,\varphi) = \frac{\lambda_{\rm H}(t)}{4}\phi^4(t) + \frac{\lambda_{\rm HS}(t)}{2}\phi^2(t)\varphi^2(t) + \frac{\lambda_{\rm S}(t)}{4}\varphi^4(t) + \sum_i \frac{n_i}{64\pi^2} {M_i}^4(\phi(t),\,\varphi(t)) \left[\ln\frac{{M_i}^2(\phi(t),\,\varphi(t))}{\mu^2(t)} - c_i\right]$$

(t: running parameter)

<u>Using both</u>, we can compare the "shape" around the vacuum and can confirm the validity of perturbation there.

3. Couplings of the Higgs and singlets

Results: N = 1 case

$N = 1$	
μ	$v_H = 246$ GeV
$y_t(v_H)$	0.919
$g(v_H)$	0.644
$g'(v_H)$	0.359
$\lambda_H(v_H)$	-0.059
$\lambda_{HS}(v_H)$	4.5
$\lambda_S(v_H)$	0.10
$\langle H \rangle$	246 GeV
m_h	126 GeV
$\langle S \rangle$	0
m_s	527 GeV
Landau pole	4.1 TeV

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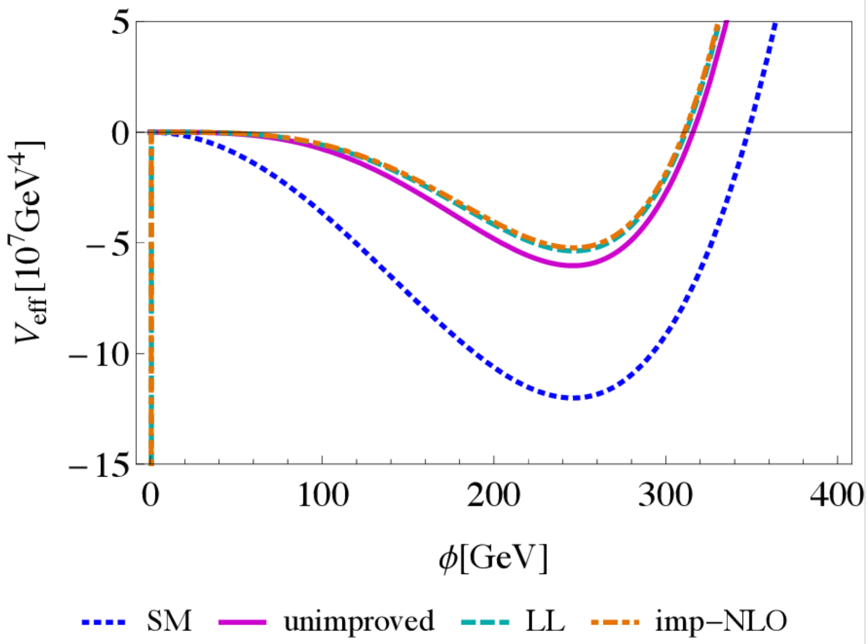

Almost the same as the previous work (R. Dermisck et.al. [1308.0891]).

→ Our analysis is valid around the VEV $\langle \phi \rangle = 246{\rm GeV}$.

Results: N = 4 and 12 cases

$N = 4$	
μ	$v_H = 246$ GeV
$y_t(v_H)$	0.919
$g(v_H)$	0.644
$g'(v_H)$	0.359
$\lambda_H(v_H)$	-0.061
$\lambda_{HS}(v_H)$	2.3
$\lambda_S(v_H)$	0.10
$\langle H \rangle$	246 GeV
m_h	126 GeV
$\langle S \rangle$	0
m_s	378 GeV
Landau pole	19 TeV

$N = 12$	
μ	$v_H = 246$ GeV
$y_t(v_H)$	0.919
$g(v_H)$	0.644
$g'(v_H)$	0.359
$\lambda_H(v_H)$	-0.063
$\lambda_{HS}(v_H)$	1.4
$\lambda_S(v_H)$	0.10
$\langle H \rangle$	246 GeV
m_h	126 GeV
$\langle S \rangle$	0
m_s	293 GeV
Landau pole	37 TeV

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As N increases,

- portal coupling λ_{HS} decreases
- and Landau pole goes far away,
- singlet mass decreases (${m_s}^2 = \lambda_{HS} {v_H}^2$).

Results:
Couplings of the Higgs and Singlets

	$N=1$	$N=4$	$N=12$
$\lambda_{hhh}/\lambda_{hhh}^{(\mathrm{SM})}$	1.8	1.7	1.6
$\lambda_{hhhh}/\lambda_{hhhh}^{(\mathrm{SM})}$	4.3	3.2	2.8
λ_{hss}	20	10	5.9
λ_{hhss}	13	5.7	3.2
λ_{ssss}	8.3	1.9	0.9

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$$V_{\mathrm{eff}} = \mathrm{const.} + \frac{1}{2}m_{\mathrm{h}}^2 h^2 + \frac{1}{2}m_{\mathrm{s}}^2\,\vec{s}\cdot\vec{s} + \frac{\lambda_{\mathrm{hhh}}}{3!}\,v_{\mathrm{H}}h^3 + \frac{\lambda_{\mathrm{hhhh}}}{4!}h^4 + \frac{\lambda_{\mathrm{hss}}}{2}\,v_{\mathrm{H}}\,h\,\vec{s}\cdot\vec{s} + \frac{\lambda_{\mathrm{hhss}}}{4}h^2\,\vec{s}\cdot\vec{s} + \frac{\lambda_{\mathrm{ssss}}}{4!}\,(\vec{s}\cdot\vec{s})^2 + \ldots,$$

Comparison with GW's method

E. Gildener and S. Weinberg [*PRD*13, 3333 (1976)]

$N = 1$	λ_{hhh}	λ_{hhhh}	λ_{hss}	λ_{hhss}	λ_{ssss}
SM prediction	0.78	0.78	none	none	none
GW's framework	1.3	2.9	$2\lambda_{HS} = 9.6$	$2\lambda_{HS} = 9.6$	$6\lambda_S = 0.6$
our analysis (LL)	1.4	3.4	10.2	13.0	6.5

$N = 4$	λ_{hhh}	λ_{hhhh}	λ_{hss}	λ_{hhss}	λ_{ssss}
SM prediction	0.78	0.78	none	none	none
GW's framework	1.3	2.9	$2\lambda_{HS} = 4.8$	$2\lambda_{HS} = 4.8$	$6\lambda_S = 0.6$
our analysis (LL)	1.3	2.5	5.0	5.7	1.9

$N = 12$	λ_{hhh}	λ_{hhhh}	λ_{hss}	λ_{hhss}	λ_{ssss}
SM prediction	0.78	0.78	none	none	none
GW's framework	1.3	2.9	$2\lambda_{HS} = 2.8$	$2\lambda_{HS} = 2.8$	$6\lambda_S = 0.6$
our analysis (LL)	1.3	2.2	3.0	3.2	0.92

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4. Veltman's condition for the Higgs mass

Is Veltman's condition favoured?

[Veltman's condition]
Coefficient of quadratic divergence
in quantum corrections for Higgs mass vanishes.
→ If it is, there is no fine-tuning.

Cutoff-regularised quantum correction:

$$V_1(\phi) = \frac{1}{64\pi^2}\mathrm{STr}\left[\Lambda^4\left(\ln\Lambda^2 - \frac{1}{2}\right) + 2M^2(\phi)\Lambda^2 + M^4(\phi)\left(\ln\frac{M^2(\phi)}{\Lambda^2} - \frac{1}{2}\right)\right] + \mathrm{c.t.}$$

[O. Antipin et. al, 1310.0957v3] etc.

Coefficient of quadratic divergence for Higgs mass:

$$\frac{1}{2}\frac{\partial^2}{\partial h^2}\mathrm{STr}M^2(h)\Big|_{\mu=\mu_0} = \frac{1}{{v_H}^2}\left(6{m_W}^2 + 3{m_Z}^2 - 12{m_t}^2 + N{m_s}^2\right)$$

Is Veltman's condition favoured?

SM case

$$\frac{1}{{v_H}^2}\left[6{m_W}^2 + 3{m_Z}^2 - 12{m_t}^2 + {m_h}^2\right] \simeq -4$$

Our model

$$\frac{1}{{v_H}^2}\left[6{m_W}^2 + 3{m_Z}^2 - 12{m_t}^2 + N{m_s}^2\right] \simeq \begin{cases} 0.3\text{-}0.8 & \text{for} \quad N = 1 \\ 4.9\text{-}5.1 & \text{for} \qquad\ 4 \\ 12\text{-}13 & \text{for} \qquad 12 \end{cases}$$

based on K.E. and Y. Sumino

N = 1 case seems to be relatively and approximately favoured, so that Fine-tuning seems to be relaxed.

5. Singlets as dark matter?

Singlets as dark matter?

- Singlets can be the dark matter candidates.

- Their thermal relic abundance are very small.

- Spin-independent cross sections with proton are predicted.
  Precision has been improved recently.

We used the framework in *JHEP* 1506 (2015) 097
(Hisano, Ishiwata, Nagata).

N	1	4	12
$\Omega_{s_i}/\Omega_{\rm DM}$	2.01×10^{-4}	4.54×10^{-4}	8.07×10^{-4}
$\tilde{\sigma}_{\rm SI}^{(p)}\ [10^{-46}\,{\rm cm}^2]$	6.77	25.6	74.5

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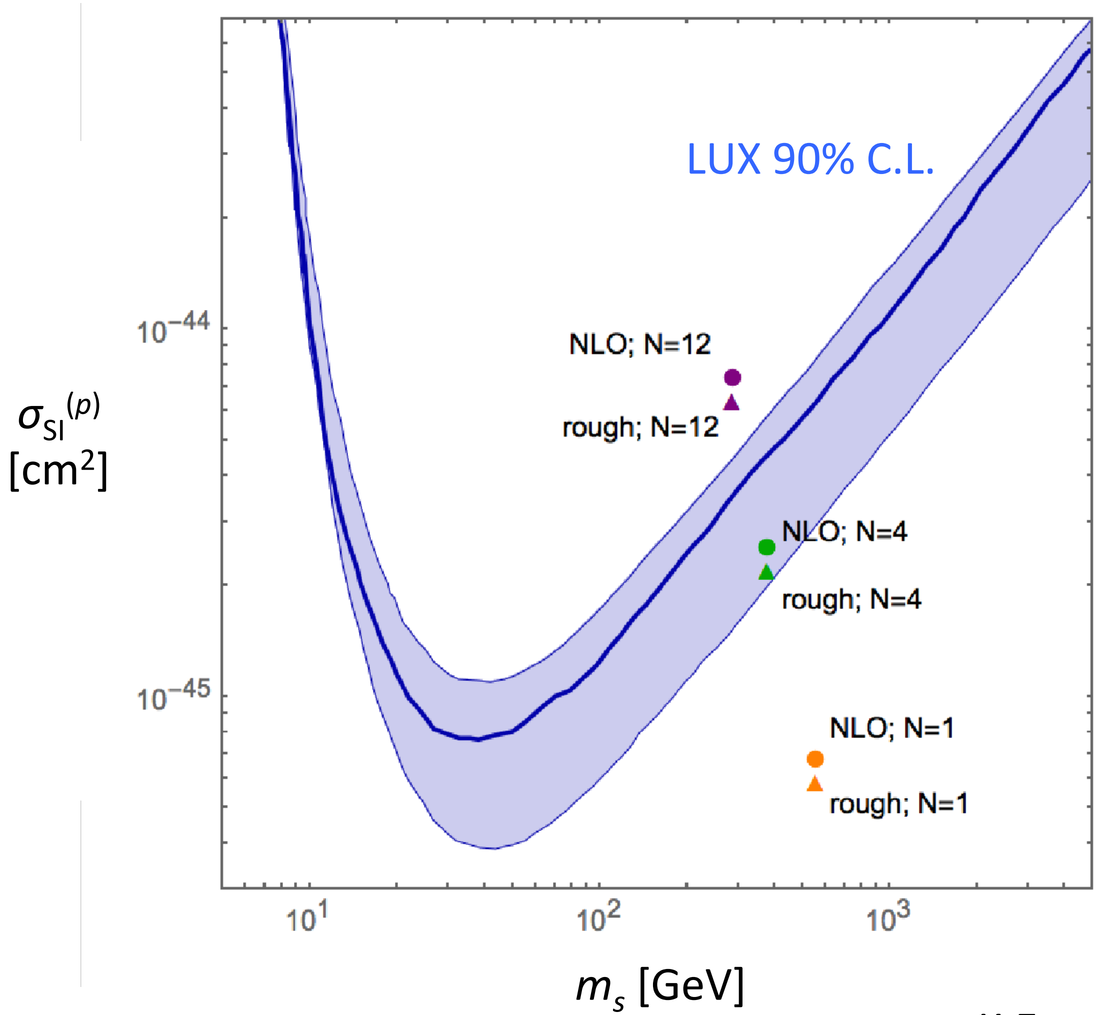
Singlets as dark matter?



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6. Summary

Summary

・Can only Higgs self couplings deviate (significantly) from SM predictions? → classically scale-inv. model

・We found perturbatively valid vacuum taking into account consistent order counting.

・Higgs self couplings: $\lambda_{3h}/\lambda_{3h}^{\rm SM} \sim 1.7,\ \lambda_{4h}/\lambda_{4h}^{\rm SM} \sim 3.0 - 4.0$

・Singlet scalars get mass around 300-500GeV
and are only pair-produced.

・*N* = 1 case

・・ Singlet can be dark matter;
although relic abundance is small, it is detectable.

・・ may approximately satisfy Veltman's condition.

Thank you for your attention.