Is the theoretical prediction for the muon $g - 2$ *really correct* ?

M. Hayakawa

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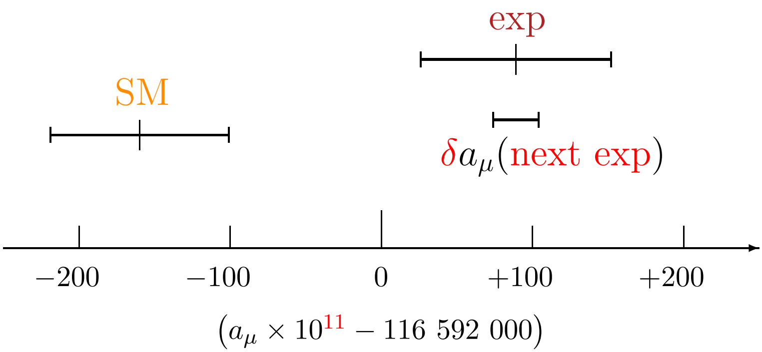
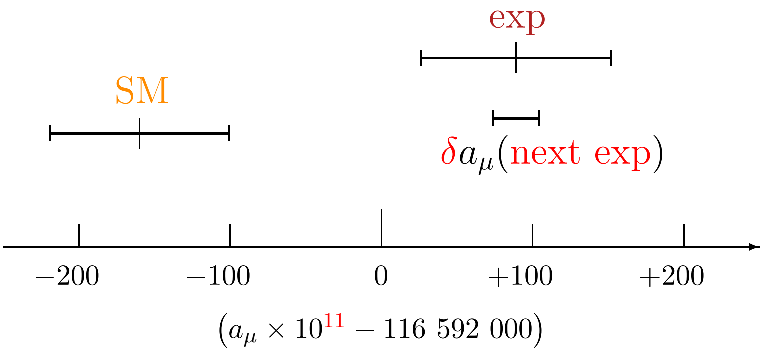
current situation of muon $g-2$



- $a_\mu(\mathrm{exp})$ was measured at **Brookhaven National Laboratory (BNL)**.
- Improvement of $a_\mu(\mathrm{exp})$ with uncertainty $\delta a_\mu(\mathrm{next\ exp})$ at **Fermilab** and **J-PARC**.

reexamination of theory of muon $g-2$



- *Question the validity* of $a_\mu\,(\mathrm{SM})$, and *ask again* **if** $a_\mu\,(\mathrm{SM})$ **really differs from** $a_\mu\,(\mathrm{exp})$.
- Dissect as large contribution to $a_\mu\,(\mathrm{SM})$ as

$$\Delta a_\mu \equiv a_\mu\,(\mathrm{exp}) - a_\mu\,(\mathrm{SM}) \sim 249\ (87) \times 10^{-11}\,.$$

$a_\mu(\mathrm{SM})$

We decompose $a_\mu \equiv (g_\mu - 2)/2$ into three parts :

$$a_\mu(\mathrm{SM}) = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{QCD}) + a_\mu(\mathrm{weak})\,.$$

They are mutually exclusive :

- QED contribution, $a_\mu(\mathrm{QED})$, is calculated by QED *with charged leptons* $(e,\ \mu,\ \tau)$ *only*.
- QCD contribution, $a_\mu(\mathrm{QCD})$, is calculated by $(\mathrm{QCD} + \mathrm{QED})$ *with* $a_\mu(\mathrm{QED})$ *subtracted*.
- $a_\mu(\mathrm{weak})$ consists of all the others, $i.e.$ those from Feynman diagrams with at least one W boson, Z boson or Higgs boson.

Each of them is further expanded as a power series $w.r.t.$ α (+ Yukawa coupling constant $+\lambda_H$ for $a_\mu(\mathrm{weak})$).

theoretical issues regarding $a_\mu(\mathrm{SM})$

I list the contributions that could be *incorrect* and could be *responsible* for $\Delta a_\mu \equiv a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM}) = 249\ (87) \times 10^{-11}$ (10^{-11} used as *universal unit*) :

- Leading-Order Hadronic Vacuum Polarization contribution $\sim O(\alpha^2)$,
  $a_\mu\,(\mathrm{LO\text{-}HVP}) = 6\ 949\ (43) \times 10^{-11}$ (K. Hagiwara, R. Liao, A. D. Martin, D. Nomura and T. Teubner, J. Phys. G **38**, 085003 (2011).) .
- Hadronic Light-by-Light contribution $\sim O(\alpha^3)$,
  $a_\mu\,(\mathrm{HLbL}) = 116\ (40) \times 10^{-11}$.
- 4-loop QED correction $\sim O(\alpha^4)$,
  $a_\mu\,(\mathrm{QED,\ 4\text{-}loop}) = 381.008\ (19) \times 10^{-11}$.

a_μ (LO-HVP)

- a_μ (LO-HVP) $= 6\ 949\ (43) \times 10^{-11}$ dominates a_μ(QCD), which is $O(\alpha^2)$ (Recall $\Delta a_\mu = 249\ (87) \times 10^{-11}$).

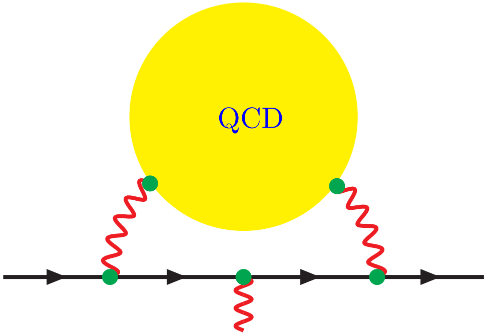


Figure: The $O(\alpha^2)$-diagram in a_μ (LO-HVP). QCD part represents the *renormalized* HVP function.

a_μ (LO-HVP)

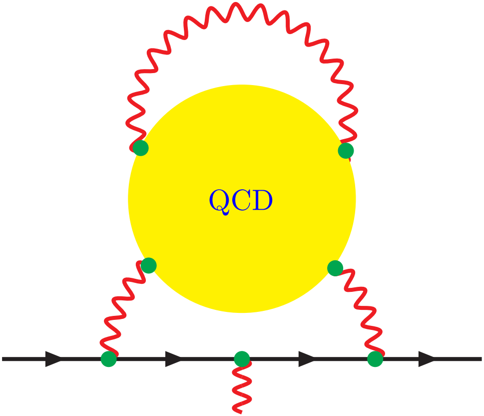


Figure: The $O(\alpha^3)$-diagram in a_μ (LO-HVP), although QCD part is actually the hadronic light-by-light scattering amplitude.

a_μ (LO-HVP)

$$a_\mu(\text{LO-HVP}) = \left(\frac{\alpha}{3\pi}\right)^2 \int_{(m_{\pi^0})^2}^{\infty} \frac{ds}{s} \frac{m_\mu^2}{s} K(s) \left(\frac{\alpha}{\alpha(s)}\right)^2 \frac{\sigma(s)}{\frac{4\pi\alpha^2}{3s}},$$

where

- $\sigma(s) \equiv \sigma\left(e^+e^- \to \gamma^*(s) \to \{\text{hadrons},\ \text{hadrons} + \gamma\}\right)$
- $K(s)$ is a known function of s, which is almost constant.
- The factor $(\alpha/\alpha(s))^2$, with the running QED gauge coupling constant $\alpha(s)$, eliminates a part of the NLO effect in $\sigma(s)$ that should be treated as $a_\mu(\text{NLO-HVP})$ separately.
- The *validity of* $a_\mu(\text{LO-HVP})$ is determined by the *validity of the experimental results for* $\sigma(s)$.

a_μ (LO-HVP)

- $\sigma(s)$ at smaller s dominates a_μ(LO-HVP) :

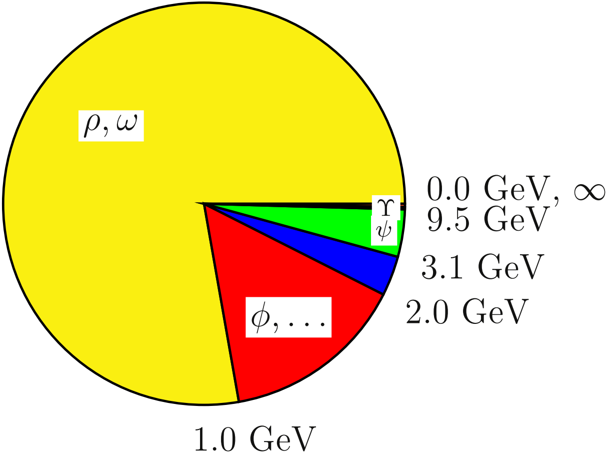


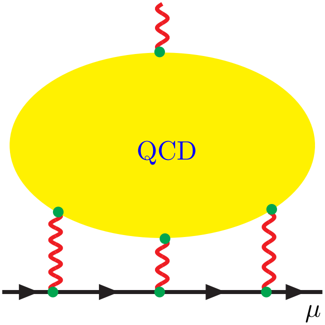
Figure: The distribution of hadronic ingredients in a_μ(LO-HVP).

content of this talk

1. - Why is lattice QCD study is necessary for HLbL contribution,
   - A remark on lattice QCD simulation.
2. - The latest development on 4-loop QED correction (A. Kurz, T. Liu, P. Marquard, A. Smirnov, V. Smirnov and M. Steinhauser, arXiv:1602.02785 [hep-ph] + Phys. Rev. D **92**, no. 7, 073019 (2015).).
   - Impact of completion of 5-loop QED correction on $\delta a_\mu(\text{next exp})$.
3. The current situation regarding lattice QCD simulation for HLbL contribution.

HLbL contribution $a_\mu(\mathrm{HLbL})$

HLbL (Hadronic Light-by-Light scattering contribution) :



- Induced through the **scattering of photons** caused by QCD.
- The yellow part requires *theoretical calculation*.

hadronic model calculation of $a_\mu(\mathrm{HLbL})$

- $a_\mu(\mathrm{HLbL})$ has been *only estimated* according to the **hadronic models** whose dynamical variables are *mesons* $(\pi^\pm,\ \pi^0,\ \cdots)$;

$$10^{11} \times a_\mu(\mathrm{HLbL}) = 116\ (40)$$
$$10^{11} \times \{a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM})\} = 249\ (87)$$

- We cannot help but worry about *significance* of the contribution from the scattering of photons with such q that $500\ \mathrm{MeV} \lesssim |q| \lesssim 1,000\ \mathrm{MeV}$, which we do *not* expect can be described by low energy effective theories of QCD (LET).
- I have called this estimation the *hadronic model calculation*, **not** the calculation due to LET, *i.e.* chiral perturbation theory.
- The question can be studied only through the calculation with *quarks and gluons as the dynamical variables*.
- $a_\mu\left(\mathrm{HLbL};\, m_u = m_d \sim 5\ \mathrm{MeV},\ \alpha_{\mathrm{SU(3)}} = 0\right) \sim 6500 \times 10^{-11}$.
- Crucial to calculate $a_\mu(\mathrm{HLbL})$ by lattice QCD simulation.

content for $a_\mu(\mathrm{HLbL})$, part (1)

- Introduce the terms frequently used by lattice community :
  - connected(-type) diagram,
  - disconnected(-type) diagram.
- Current situation of lattice QCD simulation for $a_\mu(\mathrm{HLbL})$:
  - $a_\mu(c\mathrm{HLbL}) \equiv [a_\mu(\mathrm{HLbL})$ from *connected*-type diagram] has been computed.
  - (" *None* has been done for *disconnected*-type diagrams" till the end of this July $\Rightarrow$)
    Only one of six disconnected-type diagram contributions has been computed.
- Can we examine the *validity of hadronic model calculation* ($\Rightarrow$ *significance of disconnected-type diagram*) ?

introduction to lattice QCD simulation

Consider the VEV of the operator M involving quark fields $w.r.t$ QCD:

$$\left\langle M[U;\psi,\overline{\psi}]\right\rangle_{\mathrm{QCD}} \equiv \frac{1}{Z_{\mathrm{QCD}}}\int dU \int d\psi d\overline{\psi}\, M[U;\,\psi,\,\overline{\psi}]\, e^{-S_{\mathrm{QCD}}[U;\,\psi,\,\overline{\psi}]},$$

where

- Flavor-multiplet of quark fields $\psi = (u,\, d,\, s)^T$.
- Wilson line $U(x,\,\mu) \in \mathrm{SU}(3)_{\mathrm{C}}$ couples the object at $x + a\widehat{\mu}$ to the one at x *in a gauge-invariant manner*.
- In $S_{\mathrm{QCD}}\left[U;\,\psi,\,\overline{\psi}\right] = S_{\mathrm{YM}}\left[U\right] + S_{\mathrm{F}}\left[U;\,\psi,\,\overline{\psi}\right]$,

$$S_{\mathrm{F}}[U;\,\psi,\,\overline{\psi}] = \overline{\psi}\cdot \mathsf{D}[U]\cdot\psi \equiv \sum_x \overline{\psi}(x)\left(\sum_y \mathsf{D}[U](x,\,y)\psi(y)\right),$$

where the **Dirac operator** $\mathsf{D}[U]$ contains the *interaction with gluons* and **quark mass** terms.

introduction to lattice QCD simulation

$$\left\langle M[U;\psi;\overline{\psi}]\right\rangle_{\mathrm{QCD}} \equiv \frac{1}{Z_{\mathrm{QCD}}}\int dU \int d\psi d\overline{\psi}\, M[U;\,\psi,\,\overline{\psi}]\, e^{-S_{\mathrm{QCD}}[U;\psi,\overline{\psi}]}$$

$$= \frac{1}{Z_{\mathrm{QCD}}}\int dU\, e^{-S_{\mathrm{YM}}[U]} \lim_{\eta,\overline{\eta}\to 0}\int d\psi d\overline{\psi}\, \underline{\underline{M[U;\,\psi,\,\overline{\psi}]}}\, e^{-\overline{\psi}\cdot \mathsf{D}[U]\cdot\psi+\overline{\eta}\cdot\psi+\overline{\psi}\cdot\eta}$$

$$= \frac{1}{Z_{\mathrm{QCD}}}\int dU\, e^{-S_{\mathrm{YM}}[U]} \lim_{\eta,\overline{\eta}\to 0} \underline{\underline{M\left[U;\,\frac{\partial^L}{\partial\overline{\eta}},\,\frac{\partial^R}{\partial\eta}\right]}}$$

$$\times \underbrace{\int d\psi d\overline{\psi}\, e^{-\overline{\psi}\cdot\mathsf{D}[U]\cdot\psi+\overline{\eta}\cdot\psi+\overline{\psi}\cdot\eta}}$$

$$= \frac{1}{Z_{\mathrm{QCD}}}\int dU\, e^{-S_{\mathrm{YM}}[U]} \underbrace{\int d\psi' d\overline{\psi}'\, e^{-\overline{\psi}'\cdot\mathsf{D}[U]\cdot\psi'}}$$

$$\times \lim_{\eta,\overline{\eta}\to 0} M\left[U;\,\frac{\partial^L}{\partial\overline{\eta}},\,\frac{\partial^R}{\partial\eta}\right] \underbrace{e^{-\overline{\eta}\cdot\mathsf{D}[U]^{-1}\cdot\eta}}$$

$$= \left\langle \lim_{\eta,\overline{\eta}\to 0} M\left[U;\,\frac{\partial^L}{\partial\overline{\eta}},\,\frac{\partial^R}{\partial\eta}\right] e^{-\overline{\eta}\cdot\mathsf{D}[U]^{-1}\cdot\eta}\right\rangle_{\mathrm{QCD}}.$$

correlation function relevant to determination of m_{P^a}

The correlation function relevant to determination of m_{P^a} :

$$G^a(x_0) \equiv \left\langle \sum_{\mathbf{x}} \overline{\psi}(x)\gamma_5 T^a \psi(x) \cdot \overline{\psi}(0)\gamma_5 T^a \psi(0) \right\rangle_{\rm QCD} ,$$

where T^a are $\mathrm{U}(3)_F$ generators $(\mathrm{tr}\,(T^aT^b) = \delta^{ab}\,;\ T^{a=0} \equiv \mathbb{I}_3/\sqrt{3})$.
In what follows, $m_u = m_d = m_s$ is assumed :

- $\mathrm{SU}(3)_F$ symmetry.
- The ground state in the flavor *non-singlet* channel ($a = 3$) is π, which should be a *pseudo-Goldstone boson*.
- The ground state in the flavor *singlet* channel ($a = 0$) is η', which should *not be* a pseudo-Goldstone boson.

correlation function relevant to determination of m_{P^a}

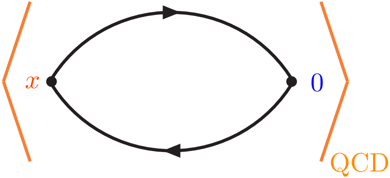
$$
\begin{aligned}
G^a(x_0) &\equiv \left\langle \sum_{\mathbf{x}} \overline{\psi}(x)\gamma_5 T^a \psi(x) \cdot \overline{\psi}(0)\gamma_5 T^a \psi(0) \right\rangle_{\mathrm{QCD}} \\
&= \left\langle \sum_{\mathbf{x}} \lim_{\eta,\overline{\eta}\to 0} \left(\frac{\partial^R}{\partial\eta(x)} \gamma_5 T^a \frac{\partial^L}{\partial\overline{\eta}(x)} \right) \left(\frac{\partial^R}{\partial\eta(0)} \gamma_5 T^a \frac{\partial^L}{\partial\overline{\eta}(0)} \right) e^{-\overline{\eta}\cdot \mathsf{D}[U]^{-1}\cdot\eta} \right\rangle_{\mathrm{QCD}} \\
&= \left\langle \sum_{\mathbf{x}} \left(\frac{\partial^R}{\partial\eta(x)} \gamma_5 T^a \frac{\partial^L}{\partial\overline{\eta}(x)} \right) \left(\frac{\partial^R}{\partial\eta(0)} \gamma_5 T^a \frac{\partial^L}{\partial\overline{\eta}(0)} \right) \right. \\
&\qquad \left. \times \frac{1}{2} \left(\overline{\eta} \cdot \mathsf{D}[U]^{-1} \cdot \eta \right) \left(\overline{\eta} \cdot \mathsf{D}[U]^{-1} \cdot \eta \right) \right\rangle_{\mathrm{QCD}} \\
&= -\mathrm{tr}\left((T^a)^2 \right) \left\langle \sum_{\mathbf{x}} \mathrm{tr}_{s,\,c} \left[\gamma_5 \mathsf{D}[U]^{-1}(x,\,0) \gamma_5 \mathsf{D}[U]^{-1}(0,\,x) \right] \right\rangle_{\mathrm{QCD}} \\
&\quad + \left(\mathrm{tr}\,(T^a) \right)^2 \left\langle \sum_{\mathbf{x}} \mathrm{tr}_{s,\,c} \left[\gamma_5 \mathsf{D}[U]^{-1}(x,\,x) \right] \mathrm{tr}_{s,\,c} \left[\gamma_5 \mathsf{D}[U]^{-1}(0,\,0) \right] \right\rangle_{\mathrm{QCD}} .
\end{aligned}
$$

difference between π and η'

- In the flavor-non-singlet channel, $\mathrm{tr}\, T^3 = 0$ so that

$$G^{a=3}(x_0) = -\left\langle \sum_{\mathbf{x}} \underset{\mathrm{s,\,c}}{\mathrm{tr}} \left(\gamma_5 \mathsf{D}[U]^{-1}(x,\,0)\gamma_5 \mathsf{D}[U]^{-1}(0,\,x)\right) \right\rangle_{\mathrm{QCD}} ,$$

which is represented by the *connected*-type diagram



- Each line denotes $\mathsf{D}[U]^{-1}$, **not** *a free quark propagator*.
- The connected-type diagram is responsible for generating the pole of *pseudo-Goldstone boson*.

difference between π and η'

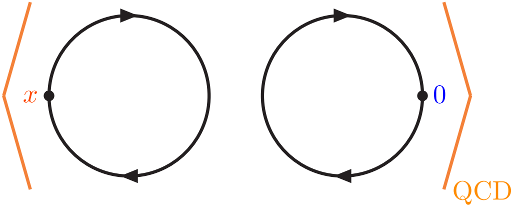
- In the continuum theory ($G_\mu(x)$ is gluon field),

$$
\begin{aligned}
\mathsf{D}^{-1} &= \frac{1}{\gamma_\mu\partial_\mu + m - \mathrm{i}\gamma_\mu G_\mu} \\
&= \frac{1}{\gamma_\mu\partial_\mu + m} + \frac{1}{\gamma_\mu\partial_\mu + m}(\mathrm{i}\gamma_\alpha G_\alpha)\frac{1}{\gamma_\nu\partial_\nu + m} \\
&\quad + \frac{1}{\gamma_\mu\partial_\mu + m}(\mathrm{i}\gamma_\alpha G_\alpha)\frac{1}{\gamma_\nu\partial_\nu + m}(\mathrm{i}\gamma_\beta G_\beta)\frac{1}{\gamma_\lambda\partial_\lambda + m} \\
&\quad + \cdots .
\end{aligned}
$$

- The QCD average $\langle\mathcal{A}[U]\rangle_{\mathrm{QCD}}$ includes the effect of the virtual quark-antiquark pair creation/annihilation through gluons.

difference between π and η'

- To the flavor-singlet channel the *disconnected*-type diagram



also contributes :

$$G^{0}(x_0) = \underbrace{G^{3}(x_0)}_{\text{connected}\,\to\,\text{pion mass}}$$

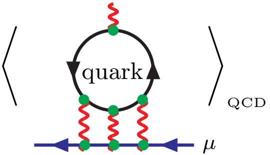
$$+\underbrace{3\left\langle \sum_{\mathbf{x}} \mathop{\mathrm{tr}}_{\mathrm{s,\,c}} \left(\gamma_5 \mathsf{D}[U]^{-1}(x,\, x)\right) \mathop{\mathrm{tr}}_{\mathrm{s,\,c}} \left(\gamma_5 \mathsf{D}[U]^{-1}(0, 0)\right)\right\rangle_{\mathrm{QCD}}}_{\text{disconnected}}$$

- Disconnected-type diagram accounts for $m_{\eta'} \sim 1,000$ MeV .

HLbL diagrams

Each contribution to $a_\mu(\mathrm{HLbL})$ can be expressed as a diagram written by $\mathsf{D}[U]^{-1}$:

- *Connected-type diagram*, in which **all** of four electromagnetic (**EM**) vertices lie on a **single** quark loop :

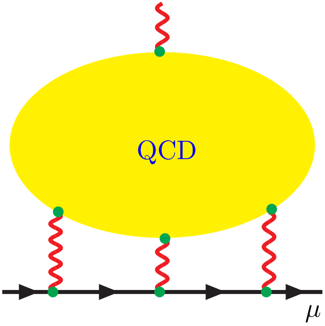


$$+\ (\text{permutations of QED vertices}(\bullet)\ \text{on muon side})\,,$$

  where each **black line** denotes $\mathsf{D}[U]^{-1}$ for a given U.

- **Six** *disconnected-type* diagrams, in each of which **four EM** vertices are *distributed over more than one* quark loops.

HLbL diagrams



$(2_E,\, 2)$-type diagrams

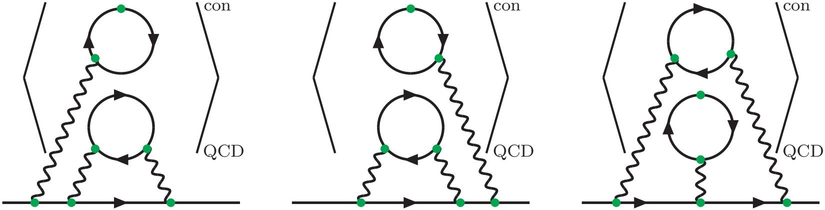


FIG. 1: $(2_E,\, 2)$-type diagrams

$(3_E,\, 1)$-type diagrams

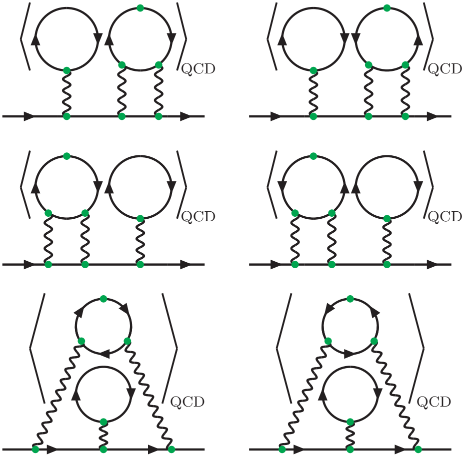


FIG. 1: $(3_E,\, 1)$-type diagrams. The diagrams with $O(a)$ local QED vertices are not shown.

$(1_E,\, 3)$-type diagrams

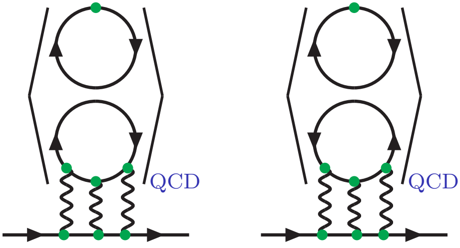


Figure: $(1_E,\, 3)$-type diagrams

Diagrams of $(1_E,\, 1,\, 2)$-type

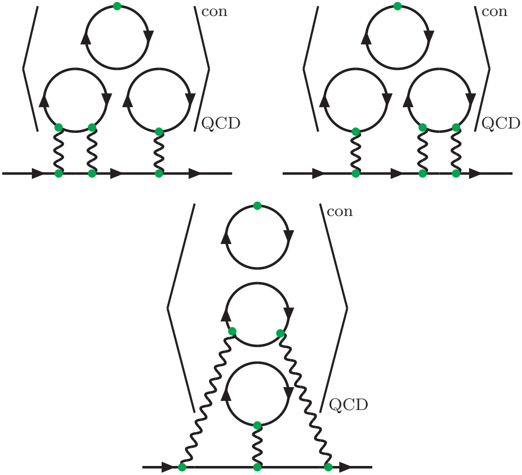


FIG. 1: $(1_E,\, 1,\, 2)$-type diagrams

Diagrams of $(2_E,\ 1,\ 1)$-type

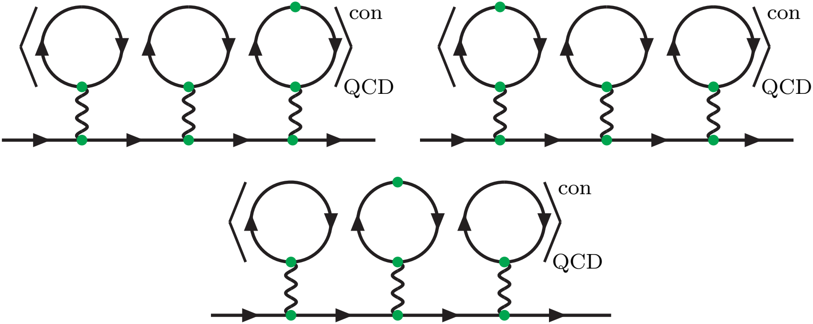


Figure: $(2_E,\ 1,\ 1)$-type diagrams

Diagram of $(1_E, 1, 1, 1)$-type

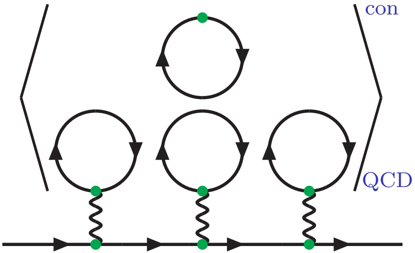


Figure: $(1_E, 1, 1, 1)$-type diagrams

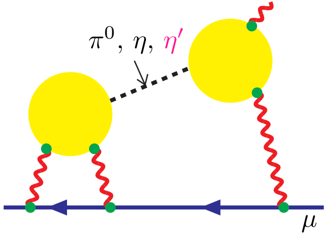
possible significance of disconnected-type diagrams

In most cases, calculation of disconnected-type diagrams is *much more difficult* than that of connected one:

- *If we can speculate* that disconnected-type diagrams is negligibly small compared to the connected one, we can concentrate on calculation of connected-type diagram.
- But, we are unable to do so.
- Lattice QCD simulation is necessary for the disconnected-type diagram.
- Disconnected-type diagrams can be important *if* hadronic model calculation (HMc) captures bulk of dynamics for $a_\mu(\mathrm{HLbL})$.

more on hadronic model calculation

- In HMc, the pseudoscalar-pole contribution is the most dominant (M. H., T. Kinoshita and A. I. Sanda, Phys. Rev. Lett. **75**, 790 (1995); J. Bijnens, E. Pallante and J. Prades, Phys. Rev. Lett. **75**, 1447 (1995).):



- $a_\mu(\eta') \simeq 0.2 \times a_\mu(\pi^0)$.
- Suppose that HMc is valid. *Unless* disconnected-type diagrams *are included*,
  - η' propagates as a *pseudo-Goldstone boson*.
  - η'-contribution will be *over-estimated* :

$$[true\ a_\mu(\mathrm{HLbL})] < a_\mu(\mathrm{cHLbL}).$$

possible significance of disconnected-type diagrams

We are calculating $(2_E,\, 2)$-type diagram (the *only* disconnected-type diagram that survives in the $\mathrm{SU}(3)_F$ limit):

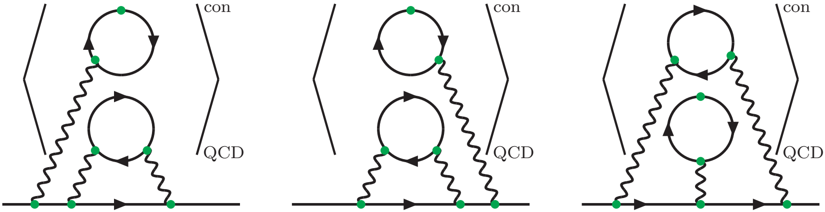


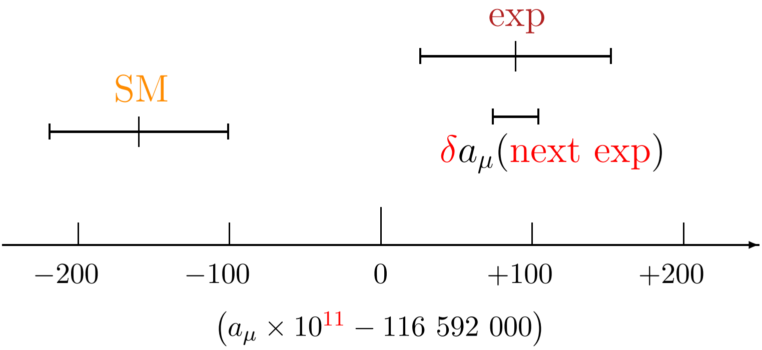
FIG. 1: $(2_E,\, 2)$-type diagrams

possible significance of disconnected-type diagrams

Our lattice calculation implies that $a_\mu(\mathrm{cHLbL}) \sim [a_\mu(\mathrm{HLbL})$ by HMc$]$.
If the disconnected-type diagram contribution, $a_\mu(\mathrm{dHLbL})$, turns out

1) negligibly small, $|a_\mu(\mathrm{dHLbL})| \ll |a_\mu(\mathrm{cHLbL})|$,
   - HMc *cannot be trustworthy*,
   - but *had given a correct value accidentally*.
2) non-negligible and *negative*,
   - the picture in HMc may be **valid**,
   - $a_\mu(\mathrm{HLbL}) \equiv a_\mu(\mathrm{cHLbL}) + a_\mu(\mathrm{dHLbL}) < [a_\mu(\mathrm{HLbL})$ by HMc$]$,
   - *amplifying* the discrepancy $\Delta a_\mu \equiv a_\mu(\exp) - a_\mu(\mathrm{SM}) > 0$.
3) non-negligible and *positive*,
   - the picture in HMc will be **wrong**,
   - $a_\mu(\mathrm{HLbL}) \equiv a_\mu(\mathrm{cHLbL}) + a_\mu(\mathrm{dHLbL}) > [a_\mu(\mathrm{HLbL})$ by HMc$]$.
   - $a_\mu(\mathrm{SM})$ will become *closer to* $a_\mu(\exp)$.

current situation of muon $g-2$



perturbative QED dynamics in lepton $g-2$

- Perturbative series expansion of the QED correction $(\psi = e,\, \mu)$:

$$a_\psi(\mathrm{QED}) = a_\psi^{(1)} \times \left(\frac{\alpha}{\pi}\right) + a_\psi^{(2)} \times \left(\frac{\alpha}{\pi}\right)^2 + a_\psi^{(3)} \times \left(\frac{\alpha}{\pi}\right)^3 + \cdots .$$

- **Perturbative dynamics** is contained in $a_\psi^{(n)}$:

$$a_e(\mathrm{QED}) = 0.5 \times \left(\frac{\alpha}{\pi}\right) + O(1) \times \left(\frac{\alpha}{\pi}\right)^2 + O(1) \times \left(\frac{\alpha}{\pi}\right)^3 + O(1) \times \left(\frac{\alpha}{\pi}\right)^4 + O(1) \times \left(\frac{\alpha}{\pi}\right)^5 + \cdots ,$$

$$a_\mu(\mathrm{QED}) = 0.5 \times \left(\frac{\alpha}{\pi}\right) + O(1) \times \left(\frac{\alpha}{\pi}\right)^2 + O(10) \times \left(\frac{\alpha}{\pi}\right)^3 + \underbrace{O(100) \times \left(\frac{\alpha}{\pi}\right)^4} + O(1,000) \times \left(\frac{\alpha}{\pi}\right)^5 + \cdots ,$$

where $O(100) \times \left(\frac{\alpha}{\pi}\right)^4 \sim O(300) \times 10^{-11} \sim a_\mu(\mathrm{SM}) - a_\mu(\mathrm{exp})$.

perturbative QED dynamics in lepton $g-2$

The difference between $a_\mu(\mathrm{QED})$ and $a_e(\mathrm{QED})$ arises because

- **For** e, μ and τ are both heavier than itself. Therefore, the **mass-independent term** $A_1^{(n)}$ dominates $a_e^{(n)}$:

$$a_e^{(n)} = \underline{\underline{A_1^{(n)}}} + A_2^{(n)}\left(\frac{m_e}{m_\mu}\right) + A_2^{(n)}\left(\frac{m_e}{m_\tau}\right) + A_3^{(n)}\left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right) .$$

- **For** μ, e is much *lighter* than itself. Therefore, the terms caused by the diagrams **with electron loops** dominate $a_\mu^{(n)}$ $(n > 1)$:

$$a_\mu^{(n)} = A_1^{(n)} + \underline{\underline{A_2^{(n)}\left(\frac{m_\mu}{m_e}\right)}} + A_2^{(n)}\left(\frac{m_\mu}{m_\tau}\right) + \underline{A_3^{(n)}\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right)} .$$

- Mathematically, $a_\psi^{(n)}$ $(\psi = e,\, \mu,\, \tau)$ are just given by a magic number $A_1^{(n)}$, and two functions $A_2^{(n)}(x)$ and $A_3^{(n)}(x, y)$.

situation of a_μ (QED, 4-loop) before 2016

- Full calculation of $a_\mu^{(4)}$, which consists of 891 Feynman diagrams, has been done *by only one* group.
- **Human error** in the present result of $a_\mu^{(4)}$?
- T. Aoyama, M. H., T. Kinoshita and M. Nio, Phys. Rev. Lett. **99**, 110406 (2007) found that a part of previous calculation of $a_{e,\mu}^{(4)}$ had been incorrect :

$$a_e^{(4)} = \underbrace{A_1^{(4)}}_{\text{corrected}} + A_2^{(4)}\left(\frac{m_e}{m_\mu}\right) + A_2^{(4)}\left(\frac{m_e}{m_\tau}\right) + A_3^{(4)}\left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right),$$

affecting a_e and $\alpha(a_e)$ *significantly*.

- Such a modification did *not* affect $a_\mu^{(4)}$ so much (, but affect a_μ a little bit through $\alpha(a_e)$).

"modern history" of $\alpha^{-1}(a_e)$

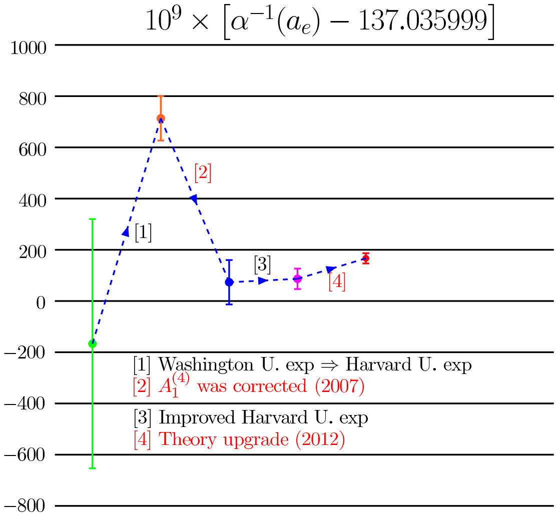


Figure: History of $\alpha^{-1}(a_e)$ derived from $a_e(\alpha) = a_e(\exp)$.

current situation on 4-loop QED correction

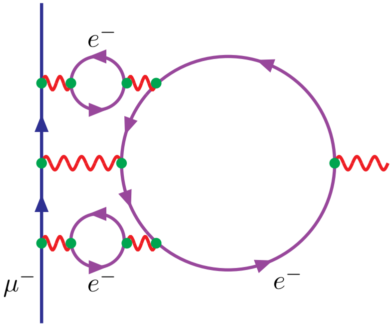
- Situation changed *dramatically* due to the following works by $Steinhauser,\ et.\ al$;
  - A. Kurz, T. Liu, P. Marquard, A. Smirnov, V. Smirnov and M. Steinhauser, arXiv:1602.02785 [hep-ph],
  - A. Kurz, T. Liu, P. Marquard, A. Smirnov, V. Smirnov and M. Steinhauser, Phys. Rev. D **92**, no. 7, 073019 (2015).
- They focus on the dominant contribution only

$$a_\mu^{(4)} = A_1^{(4)} + \underline{\underline{A_2^{(4)}\left(\frac{m_\mu}{m_e}\right)}} + A_2^{(4)}\left(\frac{m_\mu}{m_\tau}\right) + \underline{A_3^{(4)}\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right)}.$$

- They found **complete agreement** with the result by T. Aoyama, M. H, T. Kinoshita and M. Nio, PTEP **2012**, 01A107 (2012) for these !
- **Mistake no longer exists** in the calculation of 4-loop QED correction that *matters significantly to* a_μ.

impact of complete 5-loop QED correction

- *Dominant* 5-loop QED correction would be given by



- Rough estimation signifies
  5-loop QED correction $\sim O(1) \times 10^{-11} = \delta a_\mu(\text{next exp})$

impact of complete 5-loop QED correction

- Recall that the combined uncertainty

$$\delta a_\mu \cong \sqrt{\left(\delta a_\mu\left(\mathrm{SM}\right)\right)^2 + \left(\delta a_\mu\left(\exp\right)\right)^2} \geq \delta a_\mu\left(\mathrm{SM}\right) \ .$$

  roughly quantifies **likelihood of** the difference between $a_\mu\left(\mathrm{SM}\right)$ and $a_\mu\left(\exp\right)$.

- The comparison between theory and experiment with such improved accuracy requires **solid result** for the 5-loop QED correction.

- We **completed full** calculation of 5-loop QED correction, which consists of $12,678$ Feynman diagrams (T. Aoyama, M. H., T. Kinoshita and M. Nio, Phys. Rev. Lett. **109**, 111808 (2012).)

impact of complete 5-loop QED correction

Table: $a_\mu(\mathrm{QED})$ at each loop, *scaled by* 10^{11}

loop n	using $\alpha(\mathrm{Rb})$	using $\alpha(a_e)$
1	116 140 973.318 (77)	116 140 973.213 (30)
2	413 217.6291 (90)	413 217.6284 (89)
3	30 141.902 48 (41)	30 141.902 39 (40)
4	381.008 (19)	381.008 (19)
5	5.0938 (70)	5.0938 (70)
sum	116 584 718.951 (80)	116 584 718.846 (37)

Complete calculation of $a_\mu^{(5)}$ eliminated

the uncertainty $\sim O(1) \times 10^{-11} = \delta a_\mu(\text{next exp})$,

which *would have persisted without being done* !

Now, the uncertainty in $a_\mu(\mathrm{QED})$ comes mostly from

- **one-loop** $(n = 1)$ contribution *through the uncertainty in* α,

$a_\mu(\mathrm{HLbL})$, part 2

- Why has $a_\mu(\mathrm{HLbL})$ not been calculated by lattice QCD ?
- Attempt in collaboration with

Thomas Blum, Saumitra Chowdury (*Connecticut U.*)

Norman Christ (*Columbia U.*)

Luchang Jin (*Columbia U.* ⇒ *BNL*)

Taku Izubuchi (*BNL & RIKEN-BNL Research Center*)

Christoph Lehner, Chulwoo Jung (*BNL*)

Peter Boyle (*Edinburgh U.*)

Andress Jüttner (*Southampton U.*)

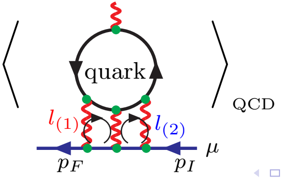
Norikazu Yamada (*KEK & Sokendai*)

- Preliminary results

straightforward method for $a_\mu(\mathrm{cHLbL})$

Initialize $f = 0$, and *repeat the following computation* with spinor-color-valued field $b^{[x_{\rm ex},\, s_0,\, c_0]}(x)_{s,\,c} \equiv \delta^{x_{\rm ex}}_{\;x}\, \delta^{s_0}_{\;s}\, \delta^{c_0}_{\;c}$ (for a fixed position $x_{\rm ex}$ of the external QED vertex) for all spinor components s_0 and color components c_0 (and $\mu \perp q \equiv p_{(F)} - p_{(I)}$):

1. **Solve** $\mathrm{D}[U]v = \gamma_\mu\, b^{[x_{\rm ex},\, s_0,\, c_0]}$. **Solve** $\mathrm{D}[U]w = \gamma_5 b^{[x_{\rm ex},\, s_0,\, c_0]}$.
2. Calculate $b_{(1)}(x) \equiv e^{\mathrm{i} l_{(1)}\cdot x} v(x)$ and $u(x) \equiv \gamma_5 e^{\mathrm{i} l_{(2)}\cdot x} w(x)$.
3. **Solve** $\mathrm{D}[U]v_{(1)} = b_{(1)}$.
4. Calculate $b_{(2)}(x) \equiv e^{\mathrm{i}\left(l_{(2)}+q-l_{(1)}\right)\cdot x} v_{(1)}(x)$.
5. **Solve** $\mathrm{D}[U]v_{(2)} = b_{(2)}$.
6. Take the inner product $h \equiv \left(u,\, v_{(2)}\right)$, and add it to f; $f \leftarrow f + h$.



impossibility of calculation of $a_\mu(\mathrm{HLbL})$

- Calculation of connected-type hadronic light-by-light scattering diagram (cHLbL) requires the following number of repetition of solving the linear problems :

$$
\begin{aligned}
&(V_4)^2 && \Leftrightarrow (l_{(1)},\, l_{(2)}) \\
&\times 2 && \Leftrightarrow v \text{ and } w \text{ at the step 1 can be reused} \\
&\times 3 && \Leftrightarrow (\mu \perp q) \\
&\times \underbrace{\{\#(\text{spinors}) \times \#(\text{colors}) = 4 \times 3 = 12\}}_{\textbf{determination of pion mass}},
\end{aligned}
$$

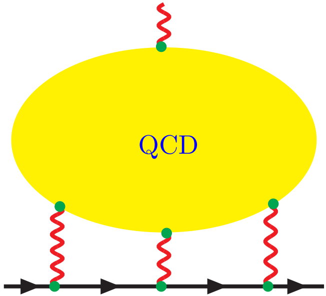
per $\{\text{flavor}, U\}$.

- $\#\,(\textbf{independent momenta}) = \textit{lattice volume}, V_4$.
- For the lattice geometry $48^3 \times 96$, $V_4 \sim 10^7$, so that $(V_4)^2 \sim 10^{14}$.
- It would require more than 100 years even if we were allowed to use full resource of KEI computer !

reexamination of the problem

To *make* the impossible *possible*, we scrutinize the problem itself:

- It is *not* certain that the contribution from the HLbL of **hard photons** ($|q| \gtrsim 0.8$ GeV) is negligible compared to $O(10) \times 10^{-11}$.
- It is sure that the contribution from the HLbL of **harder photons** is *smaller than* that of **soft photons**:
  - Photon propagator $\sim 1/q^2$ *damps hard photon* contribution.
  - The size of the HLbL amplitude *itself* is *smaller for hard photons*, $|q| \gtrsim 0.8$ GeV.



reexamination of the problem

- The **straightforward method** would *consume most of the run-time* on the supercomputer for the calculation of **insignificant** contribution.
- Tempted to **explicitly cut off** the internal photon momenta.
- But, the momentum cutoff **breaks gauge symmetry explicitly**, which could *instabilize the result*.

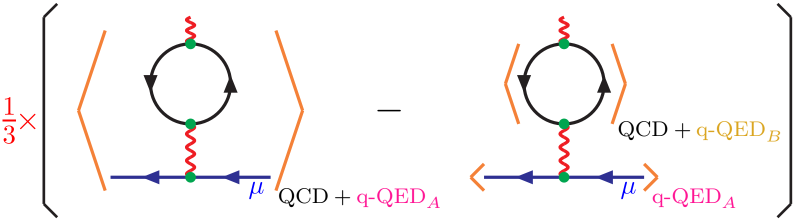
strategy for lattice QCD simulation of $a_\mu(\mathrm{HLbL})$

Gauge-invariant **importance sampling** is the *only* strategy which may enable us to calculate $a_\mu(\mathrm{HLbL})$ by lattice QCD simulation :

- **Importance sampling** implies that the contribution from the scattering with **softer** photons is *more preferentially* calculated in some **stochastic manner**, and is summed up.
- The **hard**-photon contribution is also computed, but sampled *less frequently* according to its importance.
- The **optimal choice** for the function evaluating the importance requires *full knowledge* on the HLbL amplitude, and *cannot* be made.
- With a *presumed* probability $p(a)$ estimating importance, we do **reweighting** $\sum_{a\in A} f(a) = \sum_{a\in A} p(a)\,\frac{f(a)}{p(a)} \sim \sum_{j=1}^{N} \frac{f(a_{(j)})}{p(a_{(j)})}$ with a sample $\{a_{(j)}\}_{j=1,\cdots,N}$ chosen according to $p(a)$, or its variants.

nonperturbative QED method for $a_\mu(\mathrm{cHLbL})$

Consider the following quantity (hep-lat/0509016, arXiv:1407.2923 [hep-lat])



where D $\left[U e^{-\mathrm{i}QeA}\right]^{-1}$ for quark line, D $\left[e^{\mathrm{i}eA}\right]^{-1}$ for muon line, and

$$\langle O[A]\rangle_{\text{q-QED}} \equiv \frac{1}{Z_{\text{q-QED}}}\int dA\, \exp\left[-\sum_x \left\{\frac{1}{4}\left(F_{\mu\nu}(x)\right)^2 + \text{gauge-fixing}\right\}\right] O[A]\,.$$

(quenched QED $\Leftrightarrow$ A does *not* couple to **sea** quarks)

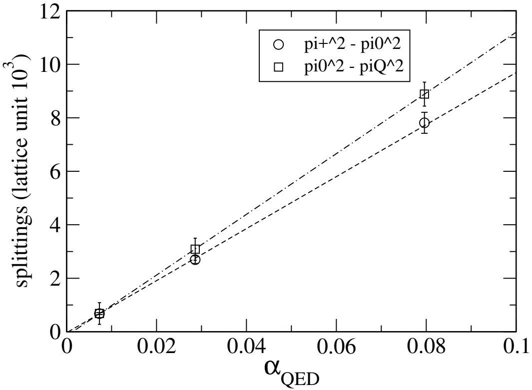
nonperturbative QED method for $a_\mu(\mathrm{cHLbL})$

Nonperturbative QED method assumes

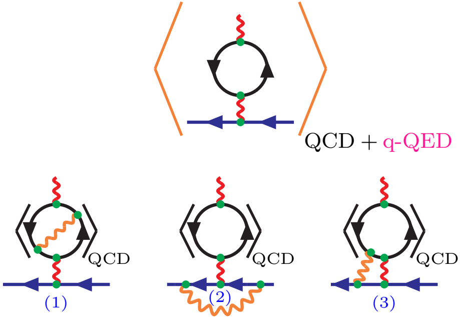
- The quantity nonperturbatively computed w.r.t. QED can be expanded in a power series of α.

The result for the nonperturbative (QCD + q-QED) calculation of pion mass supports it

(N. Yamada *et al.* [RBC Collaboration], PoS LAT **2005**, 092 (2006)) :

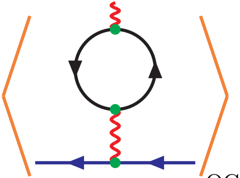


nonperturbative QED method for a_μ(cHLbL)



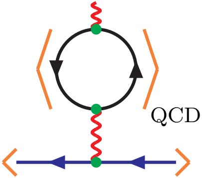
- Perturbatively, q-QED average supplies *only virtual photons*, *i.e.* *no* creation/annihilation of quark-antiquark pairs *via photon*.
- Each photon line thus created has *two* end points **in either one of the following three ways**:
  (1) both on the valence quark part,
  (2) both on the muon part,
  (3) one on the valence quark part, but the other on the muon part.

nonperturbative QED method for a_μ(cHLbL)



QCD + q-QED

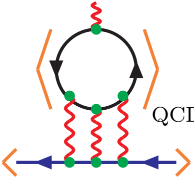
$=$



QCD + q-QED

q-QED

$+\,3\times$



QCD + q-QED

q-QED

+ (5 permutations of vertices on muon side)

+ (diagrams intervened by 5 or more photons)

nonperturbative QED method for $a_\mu(\mathrm{cHLbL})$

The basic idea behind (M. H., T. Blum, T. Izubuchi and N. Yamada, PoS LAT **2005**, 353 (2006) ; T. Blum, S. Chowdhury, M. H. and T. Izubuchi, Phys. Rev. Lett. **114**, 012001 (2015).) is as follows :

- *Remove two photons* connecting quark part and muon part, and *reproduce* them *stochastically*.
- *Smoother* $A_\mu(x)$ would be *preferentially sampled* at weak coupling.
- **Couple quarks** to such $A_\mu(x)$ and construct quark loop in that background.
- As a *smoother* $A_\mu(x)$ would be *more abundant in low frequency modes*, the contribution from the scattering of **softer** photons would be *preferentially sampled*.

nonperturbative QED method for a_μ(cHLbL)

- Nonperturbative QED method requires subtraction of $O(\alpha^2)$ contribution to the magnetic form factor.
- Subtraction will be realized only if the subtraction term is *strongly correlated* with the 1st term.
- Use of the same q-QED configurations for the *muon part* in the subtraction term as the 1st term will be *beneficial*, because

$$\frac{1}{\#(U)}\sum_U \frac{1}{\#(A)}\sum_A \sum_{x,\,y}\sum_\lambda \left(\mathcal{Q}_{\mu,\,\lambda}[U,\,A](x) - \frac{1}{\#(B)}\sum_B \mathcal{Q}_{\mu,\,\lambda}[U,\,B](x)\right) \times \sum_\rho e^2\, D_{\lambda\rho}(x-y) \times \mathcal{L}_\rho[A](y\,;t_F,\,t_I)\,,$$

at $A=0$, which is $O(\alpha^2)$, has *no contribution to the magnetic form factor*.

nonperturbative QED method for $a_\mu(\mathrm{cHLbL})$

- For $a_\mu(\mathrm{cHLbL})$,
  - Iwasaki gauge action with $(2+1)$ dynamical quarks on $24^3 \times 64$ with $a^{-1} = 1.73$ GeV.
  - Shamir-type *domain wall fermion* with $L_5 = 16$, $M_5 = 1.80$ and $m_\pi = 329$ MeV, and $am_\mu = 0.1$ and $M_{5,\,\mu} = 0.99$.
  - **noncompact** lattice QED in Feynman gauge with $e_0 = 1$.
- We measure $F_2(Q^2)$ at momenta Q_μ with the minimum size in the space with finite volume, $|Q| = \frac{2\pi}{L}$.

nonperturbative QED method for $a_\mu(\mathrm{cHLbL})$

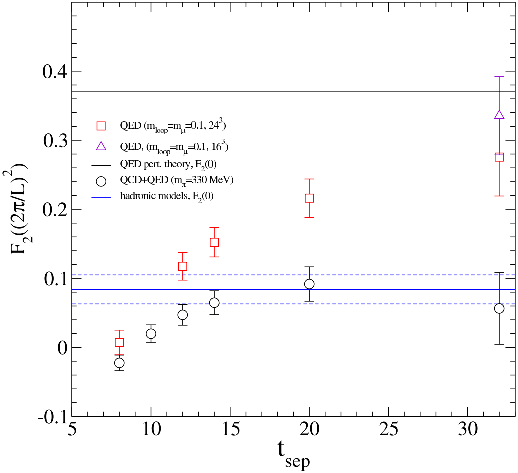


Figure: $F_2(Q^2 = (2\pi/L)^2)/(\alpha/\pi)^3$ from cHLbL (Black). $t_{\mathrm{sep}} \equiv t_F - t_I$. The external QED vertex is put at $(t_I + t_{\mathrm{sep}}/2)$. Blue line denotes $F_2(0)$ from hadronic model calculation.

moment method

With $r/2, u,$ $(-r/2)$ and h the positions of *four* QED vertices on the quark loop (T. Blum, N. Christ, M. H., T. Izubuchi, L. Jin and C. Lehner, Phys. Rev. D **93**, 014503 (2016) [arXiv:1510.07100 [hep-lat]])

$$a_\mu(\mathrm{cHLbL}) = \frac{1}{\#(U)} \sum_U \sum_r p(|r|) \left[\frac{1}{p(|r|)} \times \text{Magnetic part of} \sum_{u,\,h} \mathcal{K}_\nu\left(\frac{r}{2},\, u,\, -\frac{r}{2},\, h;\, U\right) \right] .$$

We perform *important sample* w.r.t. r using a **presumed weight** $p(|r|)$, and inversions with sources at $\frac{r}{2}$ and $-\frac{r}{2}$:

$$p(|r|) \propto \begin{cases} 1 & |r| < r_{(0)} \\ |r|^{-3.5} & |r| \geq r_{(0)} \end{cases} .$$

a_μ(cHLbL) by moment method

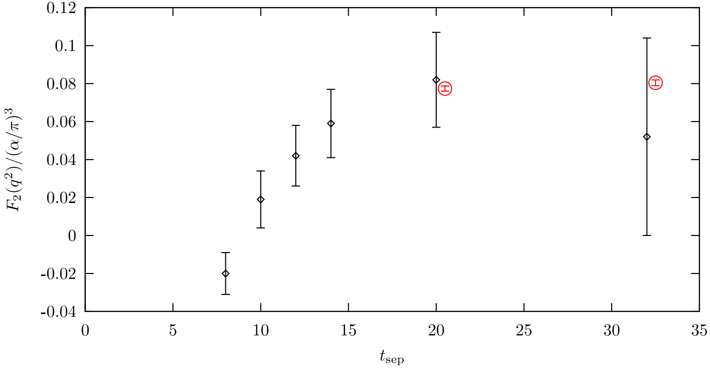


Figure: cHLbL contribution to $F_2(q^2)$ by the **moment method** (red dot, $q = 0$) (T. Blum, N. Christ, M. H., T. Izubuchi, L. Jin and C. Lehner, arXiv:1510.07100 [hep-lat]) and by nonperturbative QED method (black dot, $|q| = 2\pi/(24\,a) = 457$ MeV).

physical pion simulation on $48^3 \times 96$

- $m_\pi = 139$ MeV, physical m_s, $m_\mu = 106$ MeV.
- $a^{-1} = 1.79$ GeV, $L = 5.5$ fm.
- $\#(U) = 69$.
- For every U,
  - for connected diagram, measurement is performed for $112(|r| \leq 5) + 256(|r| > 5)$ pairs of points;
  - for $(2_E, 2)$-type, *each* quark loop is constructed for $N_{\rm meas} = 1024(u, d) + 512(s)$ positions of one EM vertex on that loop, providing $(N_{\rm meas})^2$ combinations.

$(2_E,\ 2)$-type disconnected diagram

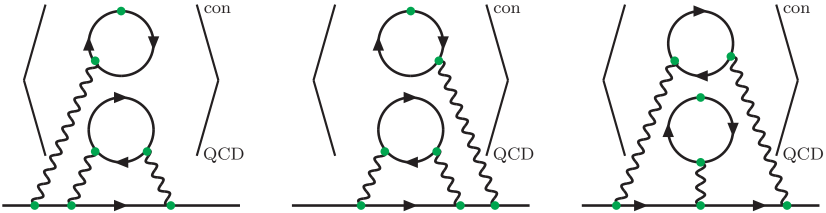


FIG. 1: $(2_E,\ 2)$-type diagrams

Linear problem with local source

To get, for a fixed $x_s \in \Gamma$,

$$\left\{ \mathsf{D}\left[U\right]^{-1}(x,\, x_s) \right\}_{x \in \Gamma} \,,$$

we solve the following linear problem for every $\{s_0,\, c_0\}_{s_0=1,\cdots,4;\, c_0=1,\cdots,3}$

- Prepare local source vector $b^{[x_s,\, s_0,\, c_0]}(x) \equiv \delta^{x_s}{}_{x}\, \eta^{[s_0,\, c_0]}$ with $\left(\eta^{[s_0,\, c_0]}\right)_{s,\, c} = \delta^{s_0}_{s}\, \delta^{c_0}_{c}$.
- Solve $\mathsf{D}[U] v^{[x_s,\, s_0,\, c_0]} = b^{[x_s,\, s_0,\, c_0]}$ to get $v^{[x_s,\, s_0,\, c_0]}$.
- $\left[\mathsf{D}\left[U\right]^{-1}(x,\, x_s)\right]_{(s,\, c),\, (s_0,\, c_0)} \equiv \left(v^{[x_s,\, s_0,\, c_0]}(x)\right)_{(s,\, c)}$.

Recall that, from $\mathsf{D}\left[U\right]^{-1}(0,\, x) = \gamma_5 \mathsf{D}\left[U\right]^{-1}(x,\, 0)^{\dagger}\gamma_5$,

$$G^3(x_0) \equiv \left\langle \sum_{\mathbf{x}} P^3((x_0,\, \mathbf{x}))\, P^3(0) \right\rangle_U = -\left\langle \sum_{\mathbf{x}} \left| \mathsf{D}\left[U\right]^{-1}(x,\, 0) \right|^2 \right\rangle_U$$

for the equality in $\mathrm{SU}(3)_F$-limit.

possible significance of disconnected-type diagrams

Our lattice calculation implies that $a_\mu(\mathrm{cHLbL}) \sim [a_\mu(\mathrm{HLbL})$ by HMc$]$.
If the disconnected-type diagram contribution, $a_\mu(\mathrm{dHLbL})$, turns out

1) negligibly small, $|a_\mu(\mathrm{dHLbL})| \ll |a_\mu(\mathrm{cHLbL})|$,
   - HMc *cannot be trustworthy*,
   - but *had given a correct value accidentally*.

2) non-negligible and *negative*,
   - the picture in HMc may be **valid**,
   - $a_\mu(\mathrm{HLbL}) \equiv a_\mu(\mathrm{cHLbL}) + a_\mu(\mathrm{dHLbL}) < [a_\mu(\mathrm{HLbL})$ by HMc$]$,
   - *amplifying* the discrepancy $\Delta a_\mu \equiv a_\mu(\exp) - a_\mu(\mathrm{SM}) > 0$.

3) non-negligible and *positive*,
   - the picture in HMc will be **wrong**,
   - $a_\mu(\mathrm{HLbL}) \equiv a_\mu(\mathrm{cHLbL}) + a_\mu(\mathrm{dHLbL}) > [a_\mu(\mathrm{HLbL})$ by HMc$]$.
   - $a_\mu(\mathrm{SM})$ will become *closer to* $a_\mu(\exp)$.

physical pion simulation on $48^3 \times 96$

With statistical uncertainty only,

$$\begin{aligned} a_\mu(\mathrm{cHLbL}) &= (116.0 \pm 9.6) \times 10^{-11} \\ a_\mu((2_E, 2)) &= (-62.5 \pm 8.0) \times 10^{-11} \\ a_\mu(\mathrm{cHLbL} + (2_E, 2)) &= (53.5 \pm 13.5) \times 10^{-11}, \end{aligned}$$

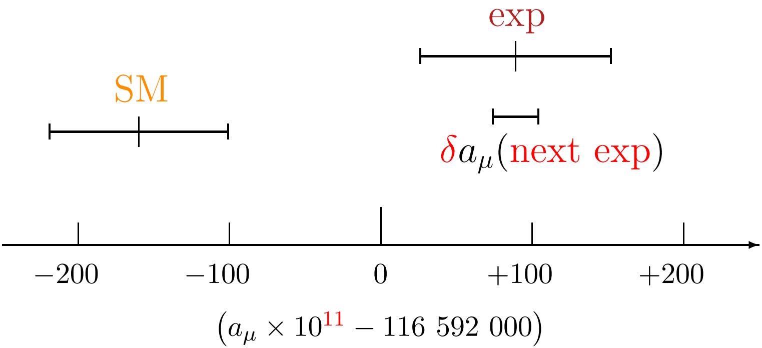
while the hadronic model calculation (HMc) gave

$$a_\mu(\mathrm{HLbL})|_{\mathrm{HMc}} = (116 \pm 40) \times 10^{-11}.$$

We need

- Intensive study on systematic uncertainty ($V < \infty$, $a \neq 0$).
- Calculation of other disconnected-type diagrams.

current situation of muon $g-2$ with $a_\mu(\mathrm{HLbL})|_{\mathrm{HMc}}$



Nonperturbative QED method for full $a_\mu(\mathrm{HLbL})$

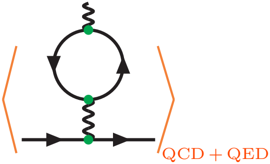
- Nonperturbative QED method to compute full $a_\mu(\mathrm{HLbL})$, $i.e.$ (connected + 6 disconnected), was proposed in arXiv:1301.2607 [hep-lat] , which has one overlooked point.
- The term $(-\mathcal{K}_D)$ is introduced to remove unwanted contribution (M. H., T. Blum, N. H. Christ, T. Izubuchi, L. C. Jin and C. Lehner, arXiv:1511.01493 [hep-lat]).

$$\frac{1}{3}\left\{(\mathcal{M}_C - \mathcal{S}_C) + (\mathcal{M}_{C'} - \mathcal{S}_{C'}) + (\mathcal{M}_D - \mathcal{S}_D) - \mathcal{K}_D\right\}$$

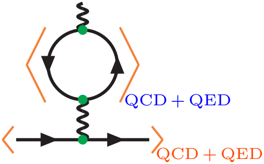
Nonperturbative QED method for full $a_\mu(\mathrm{HLbL})$

$$\frac{1}{3}\left\{(\mathcal{M}_C - \mathcal{S}_C) + (\mathcal{M}_{C'} - \mathcal{S}_{C'}) + (\mathcal{M}_D - \mathcal{S}_D) - \mathcal{K}_D\right\}$$

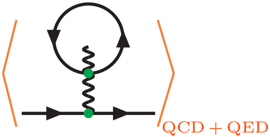
$\mathcal{M}_C =$



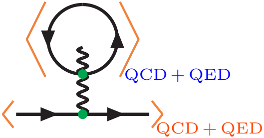
$\mathcal{S}_C =$



$\mathcal{M}_{C'} =$



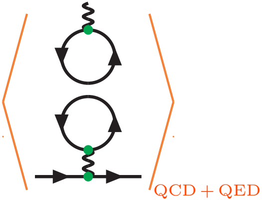
$\mathcal{S}_{C'} =$



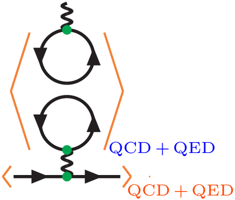
Nonperturbative QED method for full $a_\mu(\text{HLbL})$

$$\frac{1}{3}\left\{(\mathcal{M}_C - \mathcal{S}_C) + (\mathcal{M}_{C'} - \mathcal{S}_{C'}) + (\mathcal{M}_D - \mathcal{S}_D) - \mathcal{K}_D\right\}$$

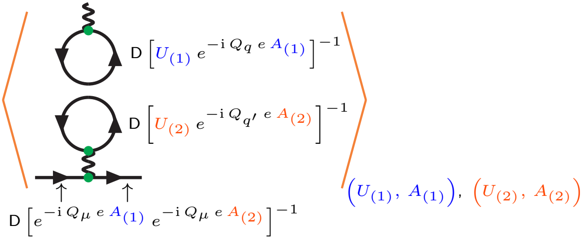
$\mathcal{M}_D =$



$\mathcal{S}_D =$



$\mathcal{K}_D =$



Nonperturbative QED method for full $a_\mu(\mathrm{HLbL})$

Every HLbL diagram is generated with *triplicate redundancy*:

Table: The multiplicity provided by each term in nonperturbative QED method. C, say, denotes $\mathcal{M}_C - \mathcal{S}_C$.

	$C + C'$	D
4_E	3	0
$(3_E, 1)$	2	1
$(1_E, 3)$	0	3
$(2_E, 2)$	1	2
$(2_E, 1, 1)$	1	2
$(1_E, 1, 2)$	0	3
$(1_E, 1, 1, 1)$	0	3

Nonperturbative QED method for full $a_\mu(\mathrm{HLbL})$

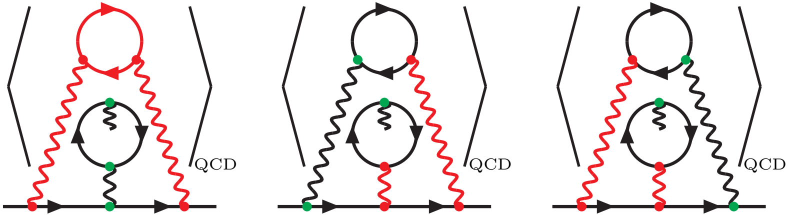


Figure: An identical diagram of $(2_E, 2)$-type is generated in three ways from $\mathcal{M}_C$ (left) and $\mathcal{M}_D$ (middle, right). The red lines and vertices are generated by the ensemble average of $(\mathrm{QCD} + \mathrm{QED})$.

Structure of unwanted HVP

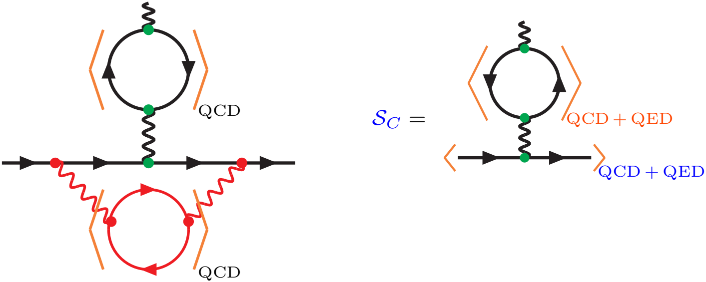


Figure: The left diagram is the disconnected component involved in a diagram of type $(2_E, 2)$ induced from $\mathcal{M}_C$. It is canceled by $(-\mathcal{S}_C)$.

- The disconnected contribution with ($bare$ HVP function + the two photons attached to it) generated $entirely$ by ensemble average is canceled by $(-\mathcal{S}_C)$, $(-\mathcal{S}_{C'})$ or $(-\mathcal{S}_D)$.

Structure of unwanted HVP

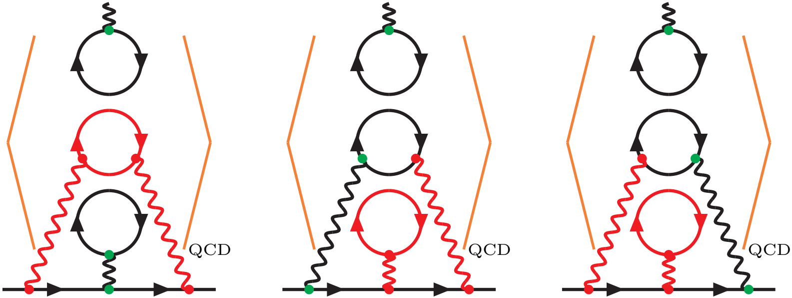


Figure: An identical diagram of $(1_E, 1, 2)$-type is generated from $\mathcal{M}_D$ in three ways. The disconnected component of the left diagram is canceled by $(-\mathcal{S}_D)$. However, *the other two disconnected components survive without being subtracted*.

Structure of unwanted HVP

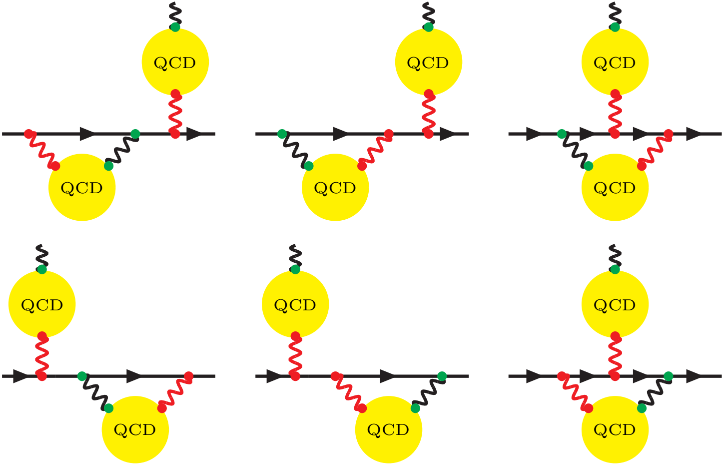


Figure: Summary of unwanted diagrams, showing that every diagram with the same topology appears exactly *twice*.

Structure of unwanted HVP

Here, bare HVP function consists of connected-type contribution and disconnected-type contribution in terms of lattice field theory (diagrams with $O(a)$ QED vertices are not shown here)

where the red lines and vertices are generated by the ensemble average of $(\mathrm{QCD} + \mathrm{QED})$.

HLbL *amplitude* by Mainz lattice group

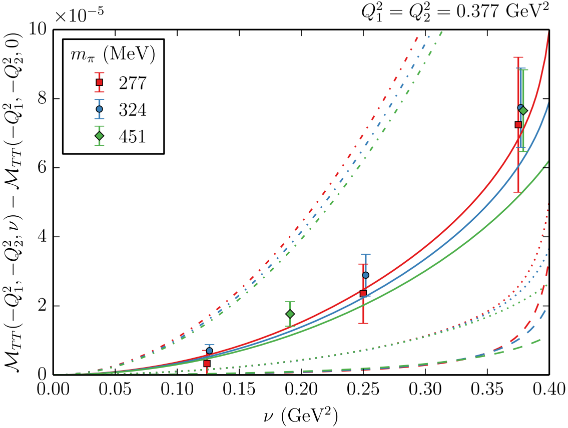


Figure: cHLbL amplitude in two-flavor QCD (J. Green, O. Gryniuk, G. von Hippel, H. B. Meyer and V. Pascalutsa, Phys. Rev. Lett. **115**, no. 22, 222003 (2015)), together with the one by π^0-propagation (dashed line) and the one by $\pi^0 + \eta'$(singlet) (dotted line) in HMc.