Gravitational waves from bubble collisions: analytic spectrum

Masahiro Takimoto (KEK→Weizmann)

Based on arXiv:1605.01403 (by Rusuke Jinno & MT)

Introduction & Summary

FIRST DECECTION OF GWS

- LIGO announcement @ 2016/2/11

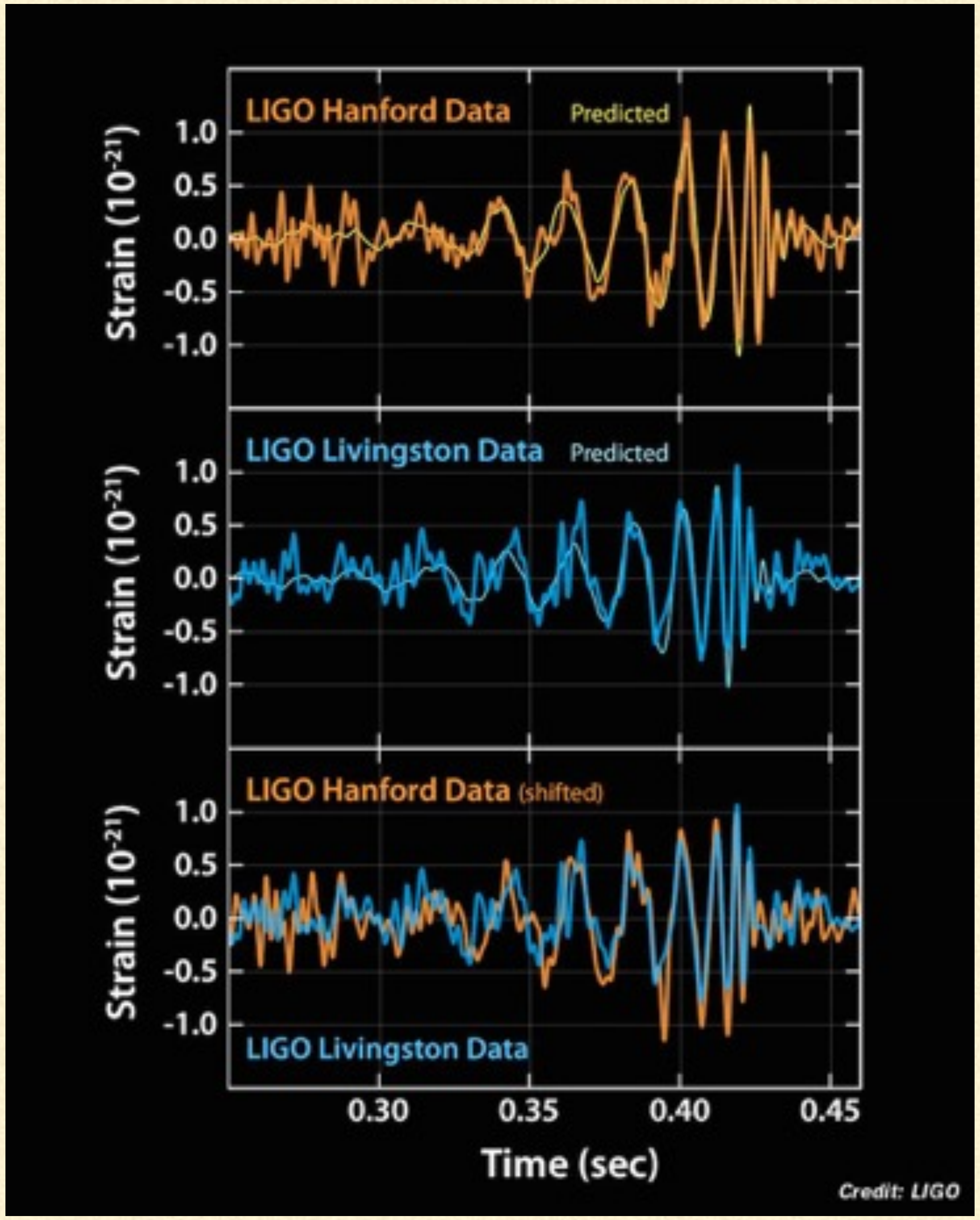
  - Black hole binary

    36M⊙ + 29M⊙ →62M⊙

    with 3.0M⊙ radiated in GWs

  - Frequency ~ 35 to 250 Hz

  - Significance > 5.1σ



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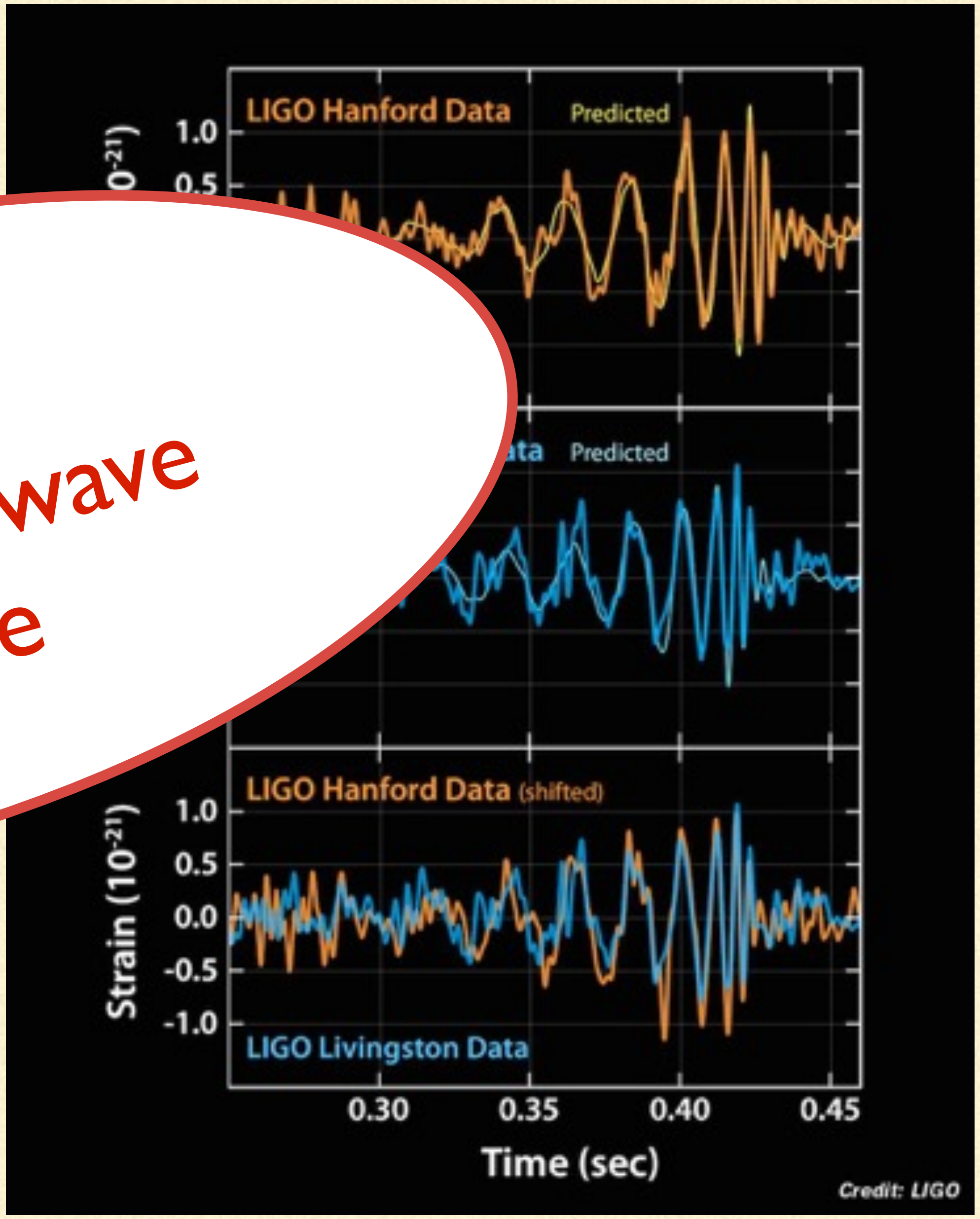
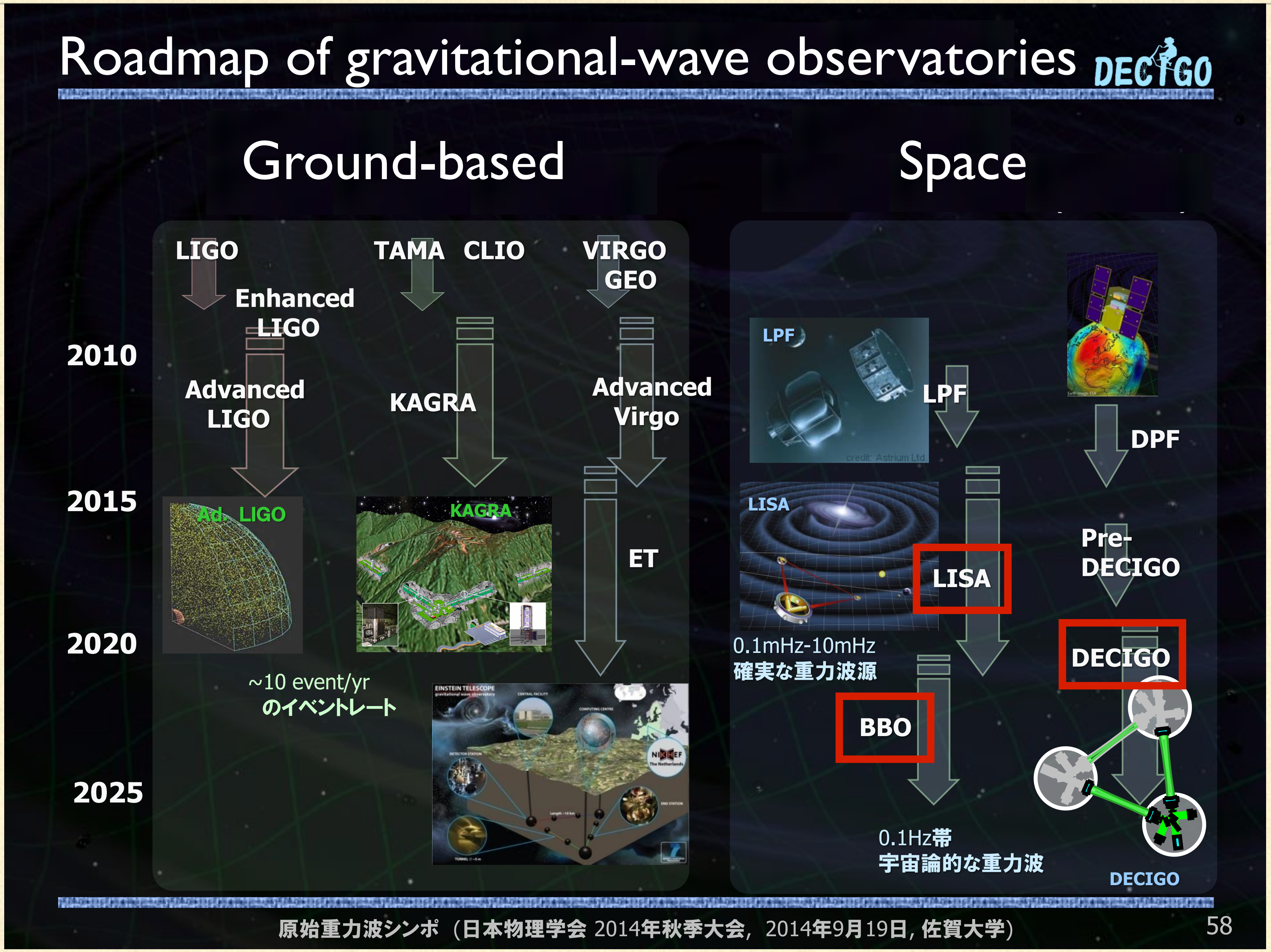


The era of
Gravitational wave
has come

FROM GROUND TO SPACE



From Ando-san's talk @ JPS meeting 2014

GRAVITATIONAL WAVES AS A PROBE TO HIGH-ENERGY PHYSICS

- Inflationary quantum fluctuations (“Primordial GWs”)
- Preheating
- Cosmic strings, domain walls
- First-order phase transition

GRAVITATIONAL WAVES AS A PROBE TO HIGH-ENERGY PHYSICS

- Inflationary quantum fluctuations ("Primordial GWs")
- Preheating
- Cosmic strings, domain walls
- First-order phase transition can occur in many physics models

  - Electroweak symmetry breaking
  - Peccei-Quinn symmetry breaking
  - SUSY breaking
  - Breaking of GUT group

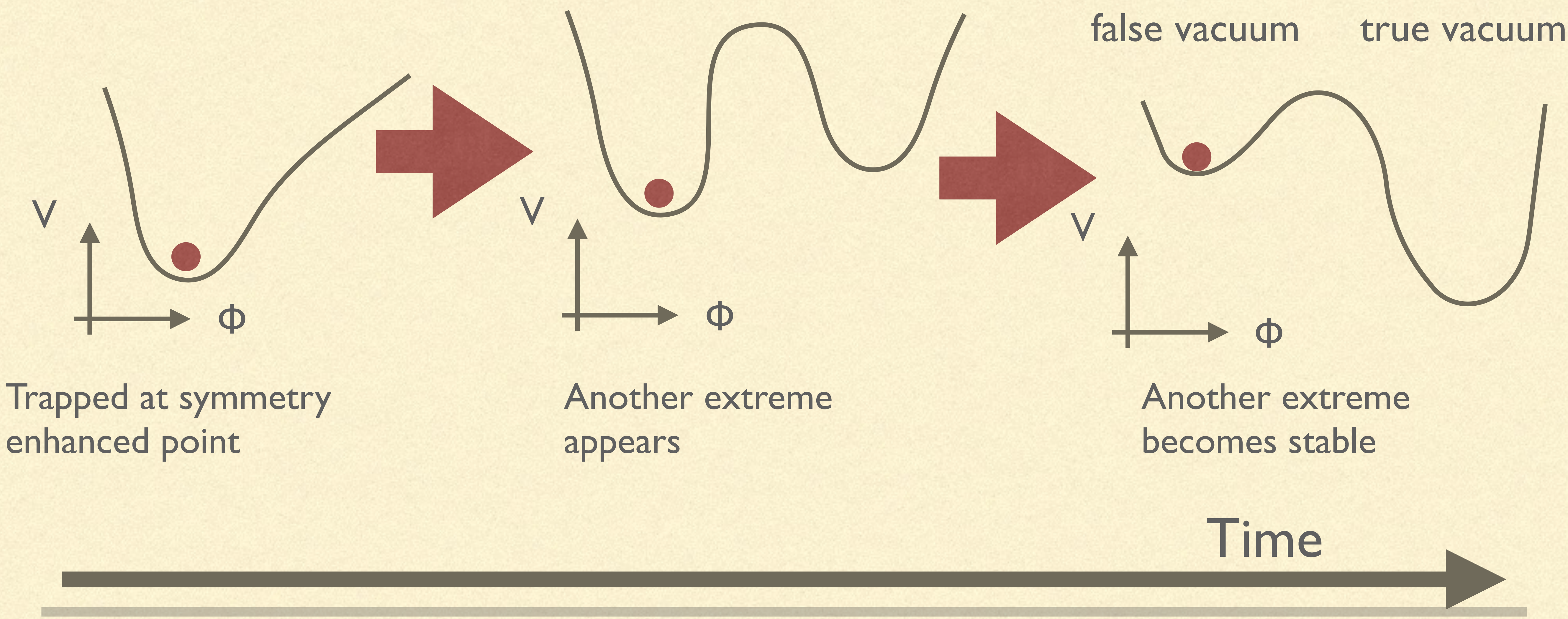
  ... and so on

GRAVITATIONAL WAVES AS A PROBE TO PHASE TRANSITION

- How (first order) phase transition occurs

- High temperature

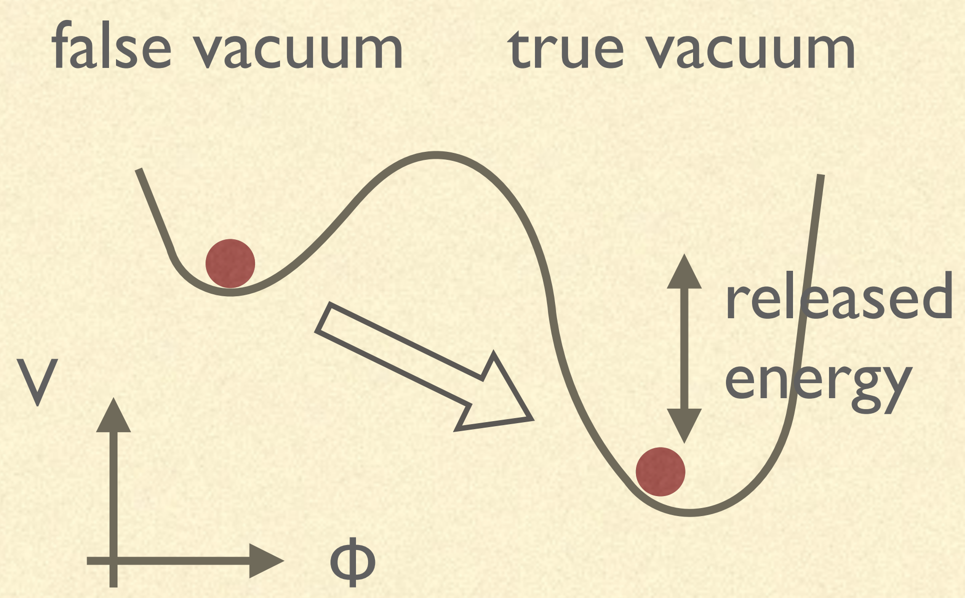
- Low temperature



GRAVITATIONAL WAVES AS A PROBE TO PHASE TRANSITION

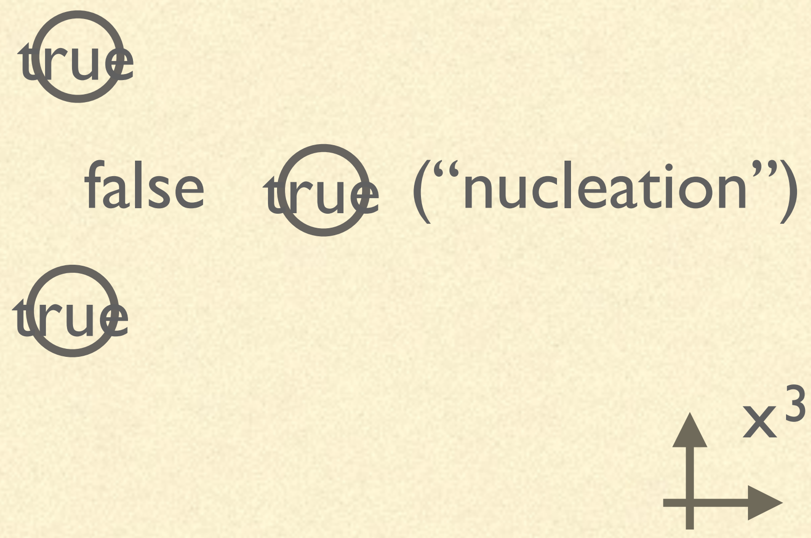
- How thermal first order phase transition produces GWs

- Field space



Quantum tunneling

- Position space



Bubble formation & GW production

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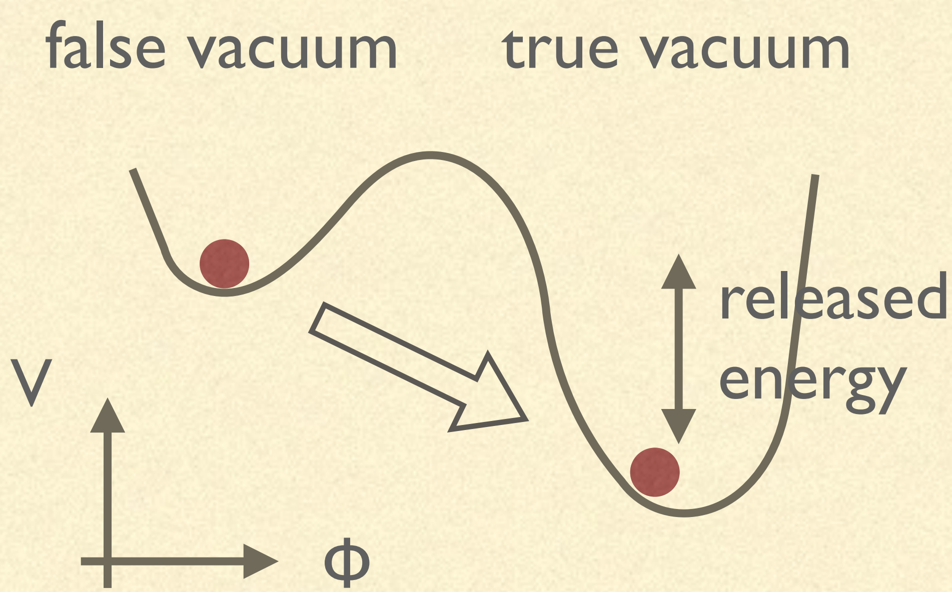

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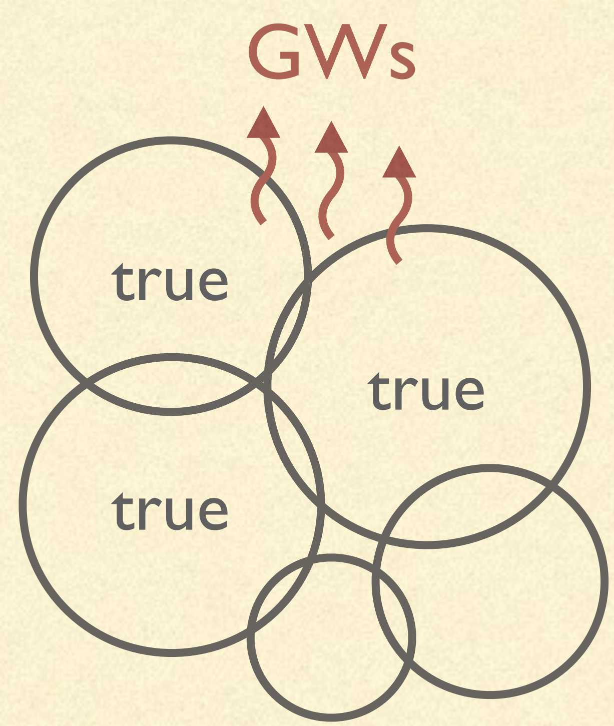
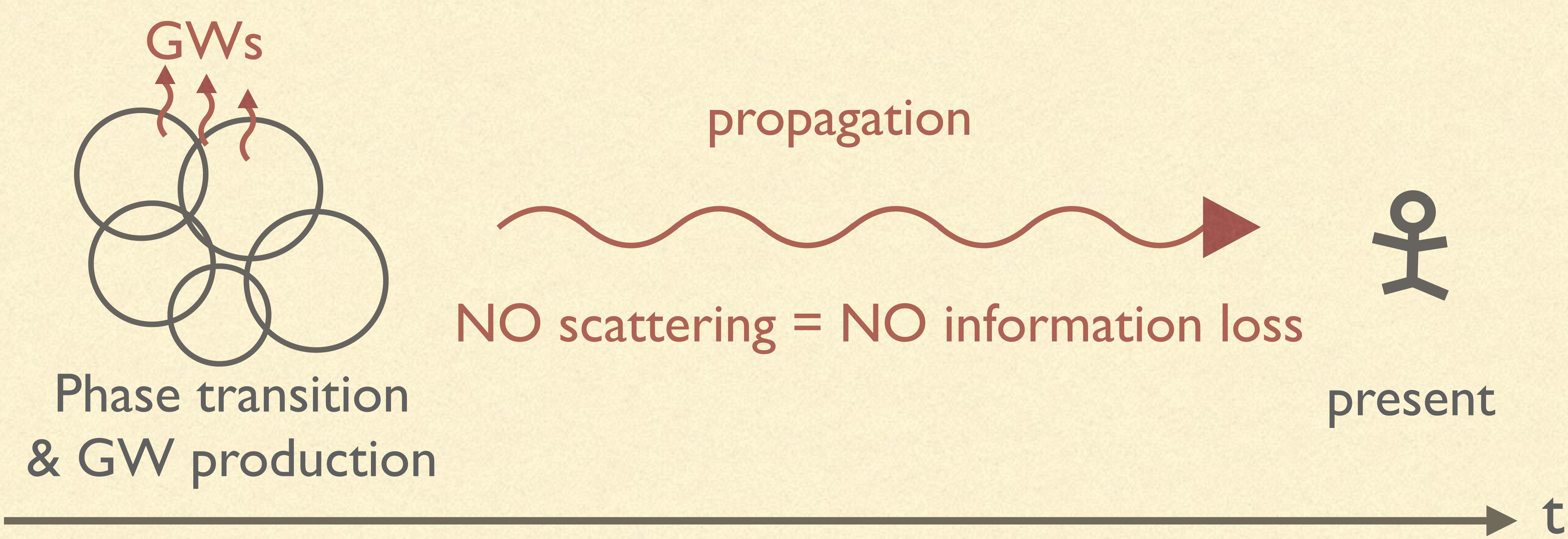


Bubble walls source GWs

Bubble formation & GW production

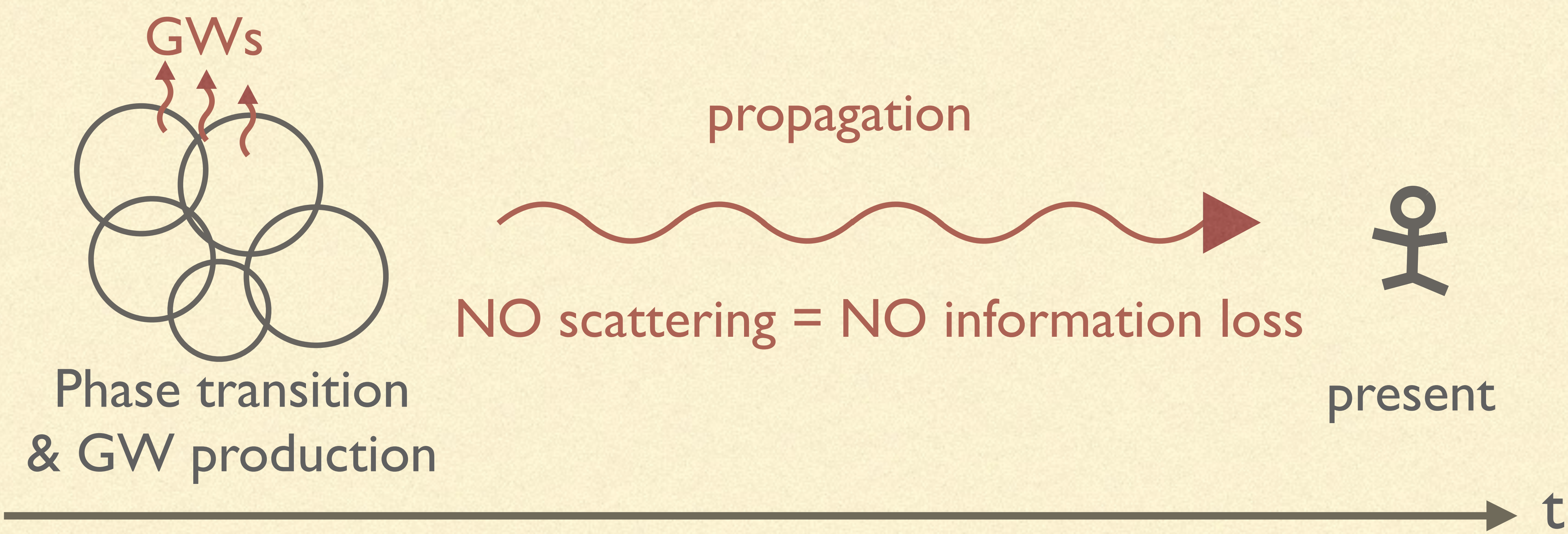
GRAVITATIONAL WAVES AS A PROBE TO PHASE TRANSITION

- GWs propagates until the present without losing information because of the Planck-suppressed interaction of gravitions



GRAVITATIONAL WAVES AS A PROBE TO PHASE TRANSITION

GWs can be a unique probe to unknown high-energy particle physics

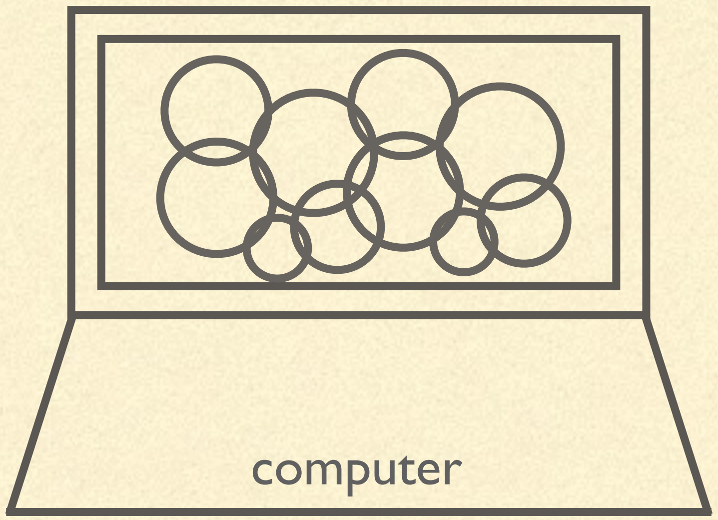


THEORETICAL PREDICTION FOR GW SPECTRUM

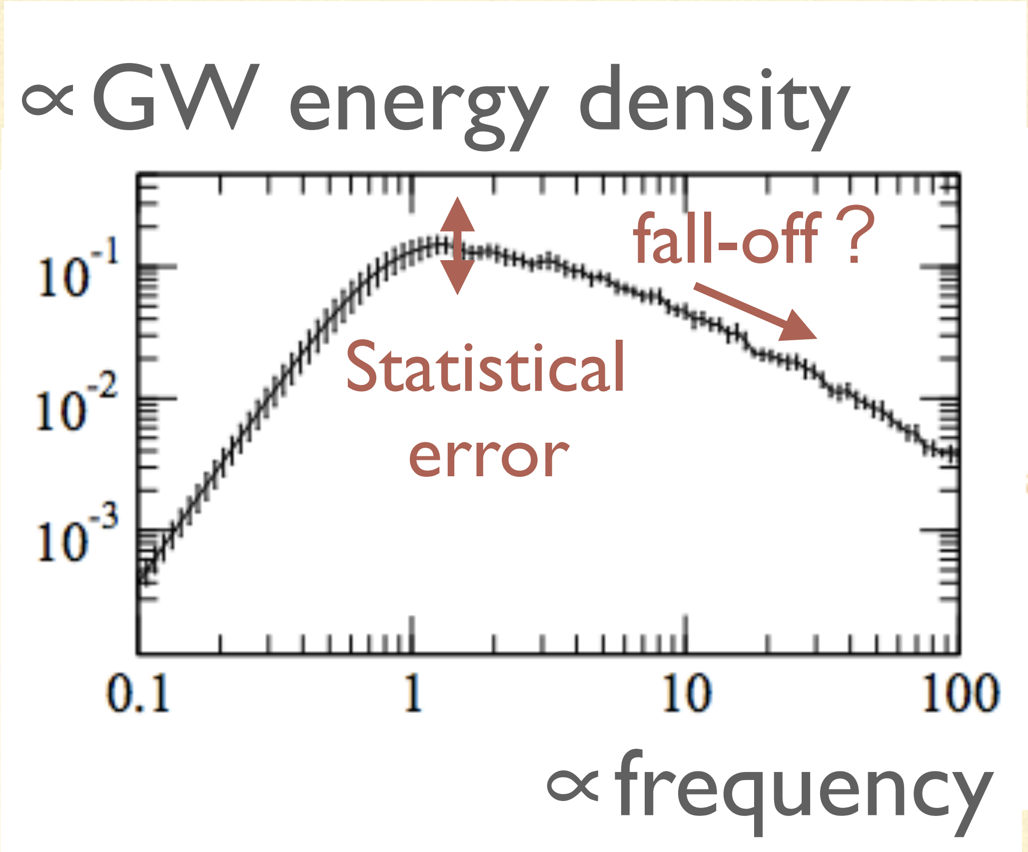
- We must fix theoretical prediction for GW spectrum
- GW spectrum from bubble collisions is usually calculated

  by NUMERICAL SIMULATION

[Huber et al., '08]



→



SUMMARY

- GW spectrum from bubble collisions is

Exactly

determined by analytic calculation

in the same setup as in numerical simulations

TALK PLAN

0. Introduction & Summary

1. GWs produced in first-order phase transition

   (We define the setup here)

2. Analytic derivation of the GW spectrum

3. Future applications & Summary

1. GWs produced in first-order phase transition

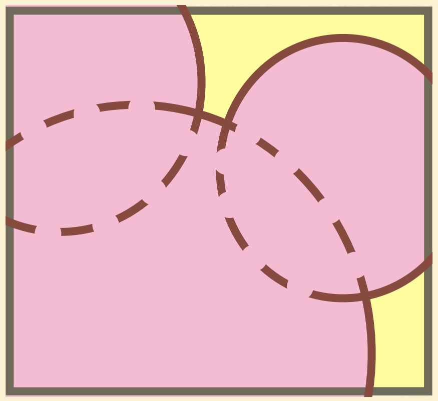
I. GWS PRODUCED IN PHASE TRANSITION

- What do we need to calculate GW production ?

- Definition of GWs : $ds^2 = -dt^2 + a^2(t)(\delta_{ij} + \underline{2h_{ij}})dx^i dx^j$

- Propagation of GWs : $\Box h_{ij} = 8\pi G \underline{K_{ij,kl}} \boxed{T_{kl}}$ ← We need this

projection to tensor mode — Energy-momentum tensor (from bubble walls)



T_{ij} is determined by

- Bubble distribution

- Energy-momentum profile around nucleated bubbles

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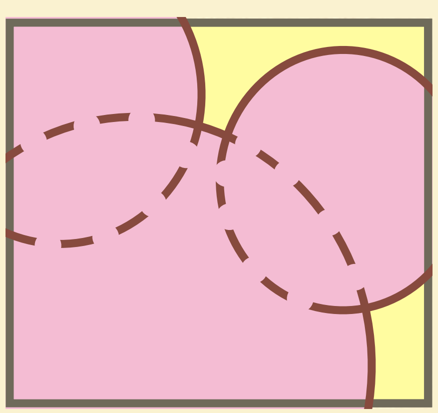


T_{ij} is determined by

- Bubble distribution
- Energy-momentum profile around nucleated bubbles

I. GWS PRODUCED IN PHASE TRANSITION

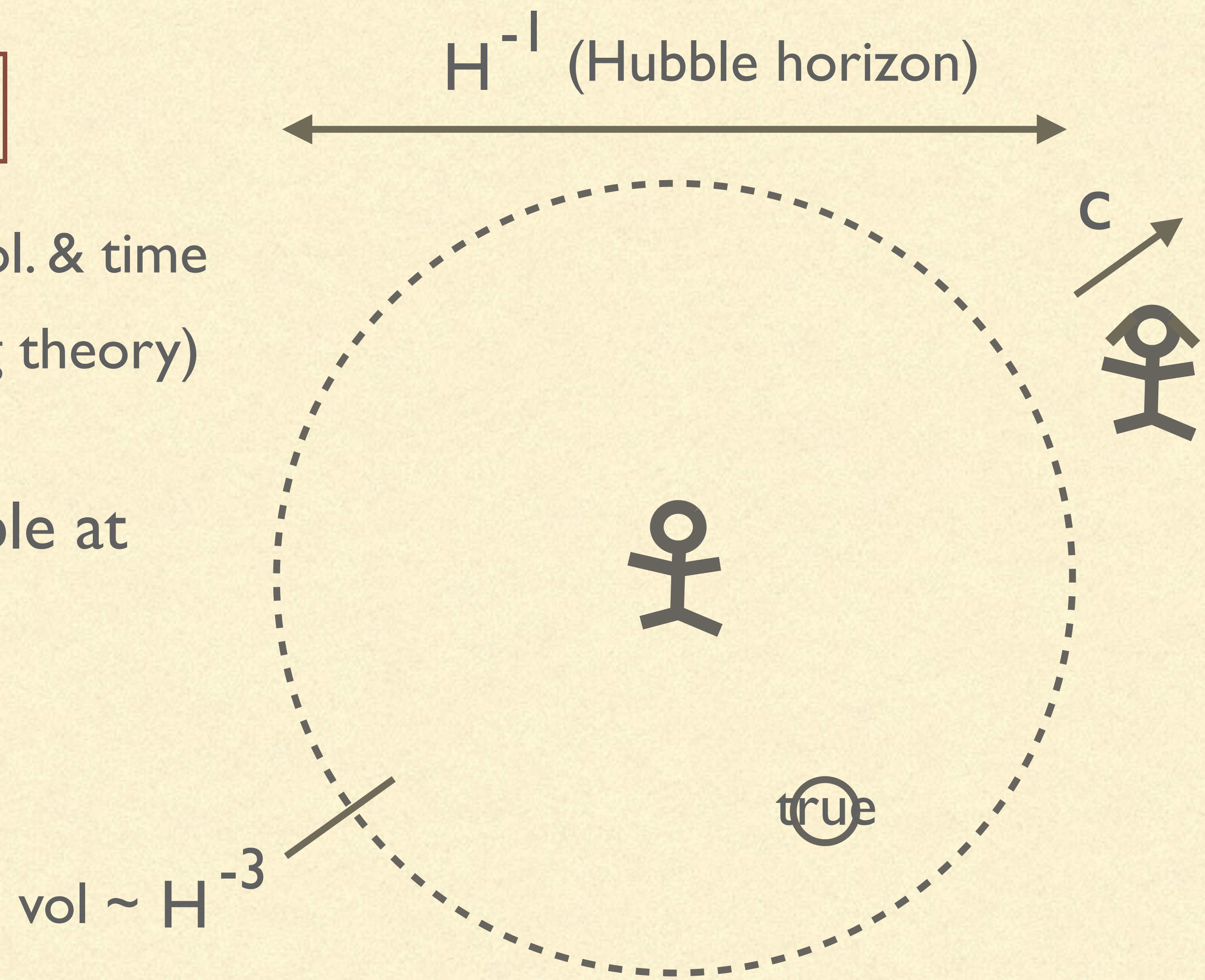
- When first order phase transition occurs

  - Transition occurs at $\Gamma \sim H^4$

    Γ : nucleation rate per unit vol. & time (determined by underlying theory)

  because  finds one bubble at

  $\Gamma \times \underset{H^{-3}}{\text{vol}} \times \underset{H^{-1}}{t} \sim 1$



I. GWS PRODUCED IN PHASE TRANSITION

- How long first order phase transition lasts

  - Duration of PT is determined by the changing rate of Γ (= β)

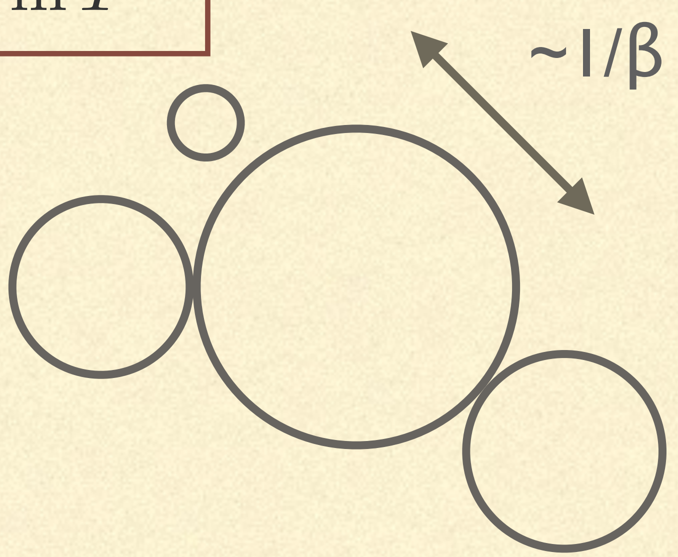
    $\Gamma = \Gamma_* e^{\beta(t - t_*)}$: Taylor exp. around transition time t_*

    because

    $$\beta \simeq \frac{d(S_3/T)}{dt} \simeq H \frac{d(S_3/T)}{d \ln T}$$

    $$\beta / H \gtrsim 1$$

    1. Γ significantly changes with time interval $\delta t \sim 1/\beta$
    2. So, the first bubble typically collides with others δt after nucleation



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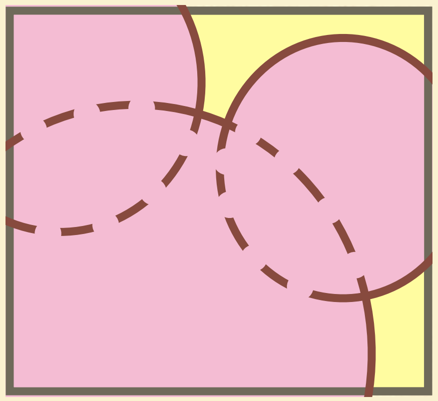


T_{ij} is determined by

- Bubble distribution

- Energy-momentum profile around nucleated bubbles

I. GWS PRODUCED IN PHASE TRANSITION

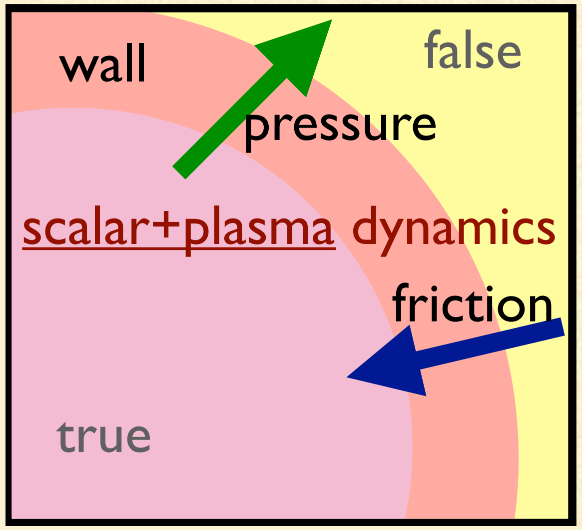
- Single bubble profile

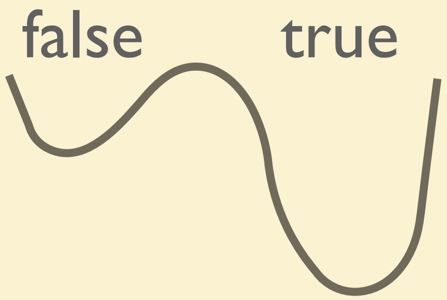
  - Main players : scalar field & plasma

  - Wall (where the scalar field value changes) wants to expand ("pressure")

  - Wall is pushed back by plasma ("friction")

  - These dynamics are generally complicated & hard to solve, so, let's try some qualitative classification





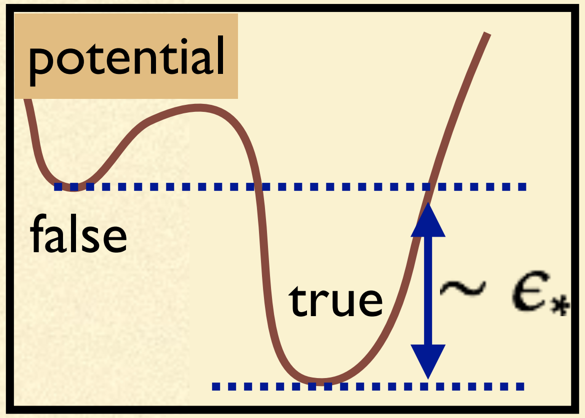
I. GWS PRODUCED IN PHASE TRANSITION

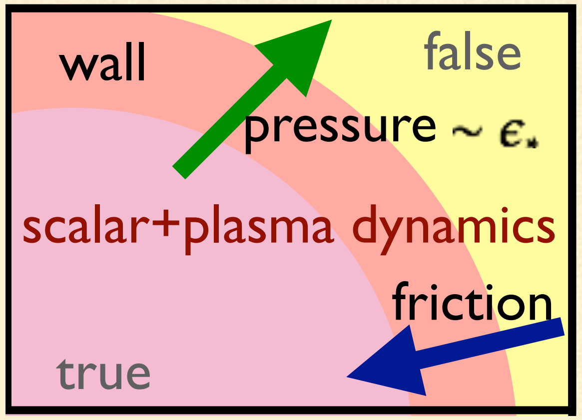
- Single bubble profile : Qualitative classification

  - Roughly speaking,

$$\alpha \equiv \epsilon_* / \rho_{\text{radiation}}$$

determines the late-time behavior of the bubble wall





(R) Runaway case : Pressure dominates friction

$\alpha \gtrsim O(1)$

→ Wall velocity approaches speed of light

Energy is dominated by scalar motion (wall itself)

(T) Terminal velocity case : Pressure & friction are in balance

$\alpha \lesssim O(1)$

→ Bubble walls reach a terminal velocity (<c)

Energy is dominated by plasma around walls

I. GWS PRODUCED IN PHASE TRANSITION

- Gravitational-wave sources

  - Usually categorized into 3 classes

    (1) Bubble wall collision : Scalar field dynamics
    (& also plasma)

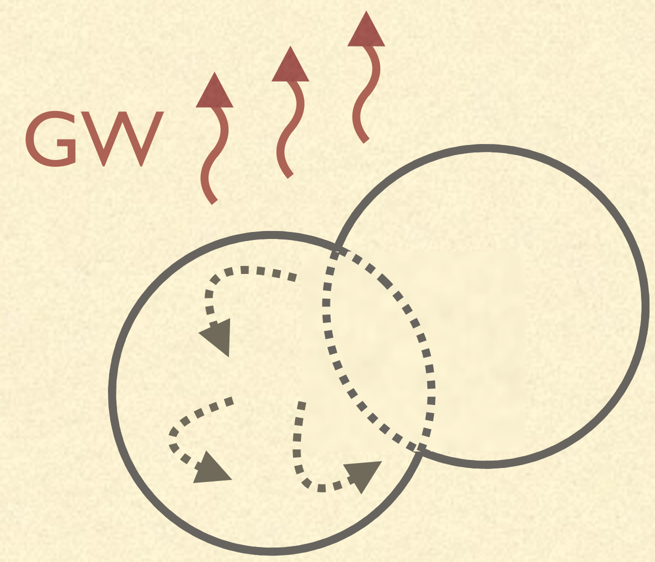
    (2) Sound wave : Plasma dynamics after collision

    (3) Turbulence : Plasma dynamics after collision

  - Which is important in (R)Runaway & (T)Terminal vel. cases?

    (R) → (1) Bubble wall collision

    (T) → (2,3) Sound wave (& Turbulence)



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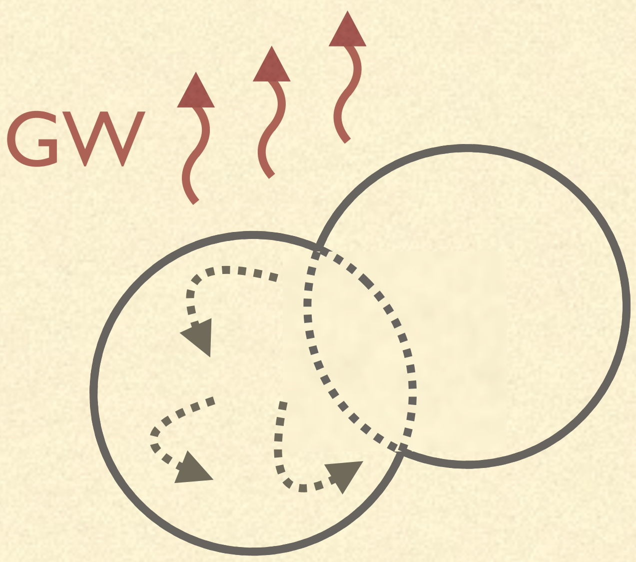


I. GWS PRODUCED IN PHASE TRANSITION

- EM profile of nucleated bubbles

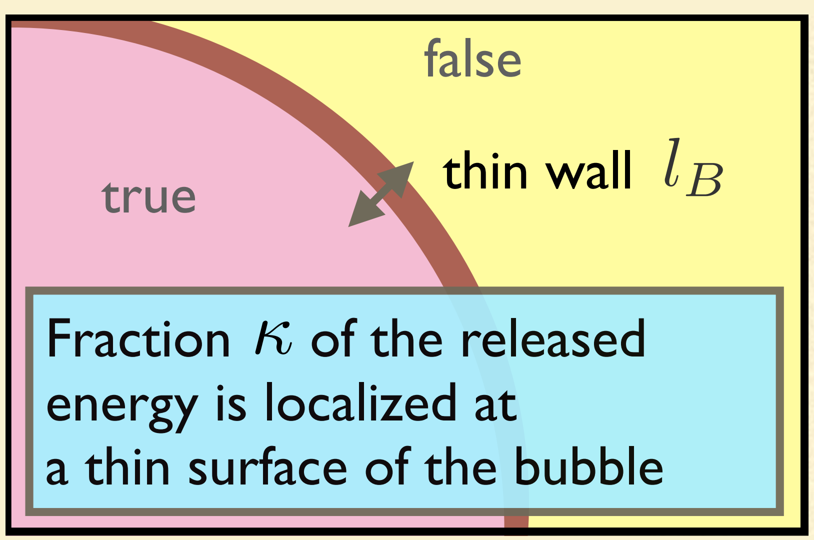
  - Thin-wall & envelope approximations

ϵ_* : released energy density

κ : fraction of ϵ_* localized at the wall

$r_B(t)$: bubble radius

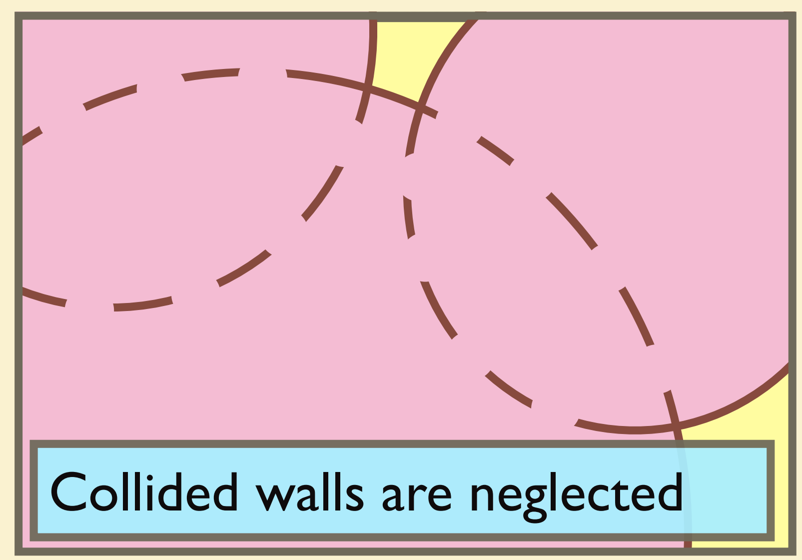
Thin-wall



$$T_{ij}(t,\mathbf{x}) = \kappa \cdot \frac{4\pi}{3} r_B(t)^3 \epsilon_* \cdot \frac{1}{4\pi r_B(t)^2 l_B} \cdot \hat{v}_i \hat{v}_j$$

for bubble wall region with width l_B

Envelope



[Kosowsky, Turner, Watkins, PRD45 ('92)]

I. GWS PRODUCED IN PHASE TRANSITION

- To summarize, GW production of bubble collisions has been calculated

  in the literature in the following setup

  - Propagation of GWs : $\Box h_{ij} = 8\pi G K_{ij,kl} \underline{T_{kl}}$

    EM tensor of bubble walls

  - Nucleation rate : $\Gamma = \Gamma_* e^{\beta(t - t_*)}$

  - Thin-wall and envelope approximations

I. GWS PRODUCED IN PHASE TRANSITION

参考までに、、

★Rough estimation of GW amplitude

Present GW amplitude & frequency are obtained just by redshifting

~quadrupole factor ~radiation fraction today

$$h^2\Omega_{\rm GW,peak} \sim \mathcal{O}(10^{-2})\mathcal{O}(10^{-5})\left(\frac{\beta}{H_*}\right)^{-2}\left(\frac{\alpha}{1+\alpha}\right)^2$$

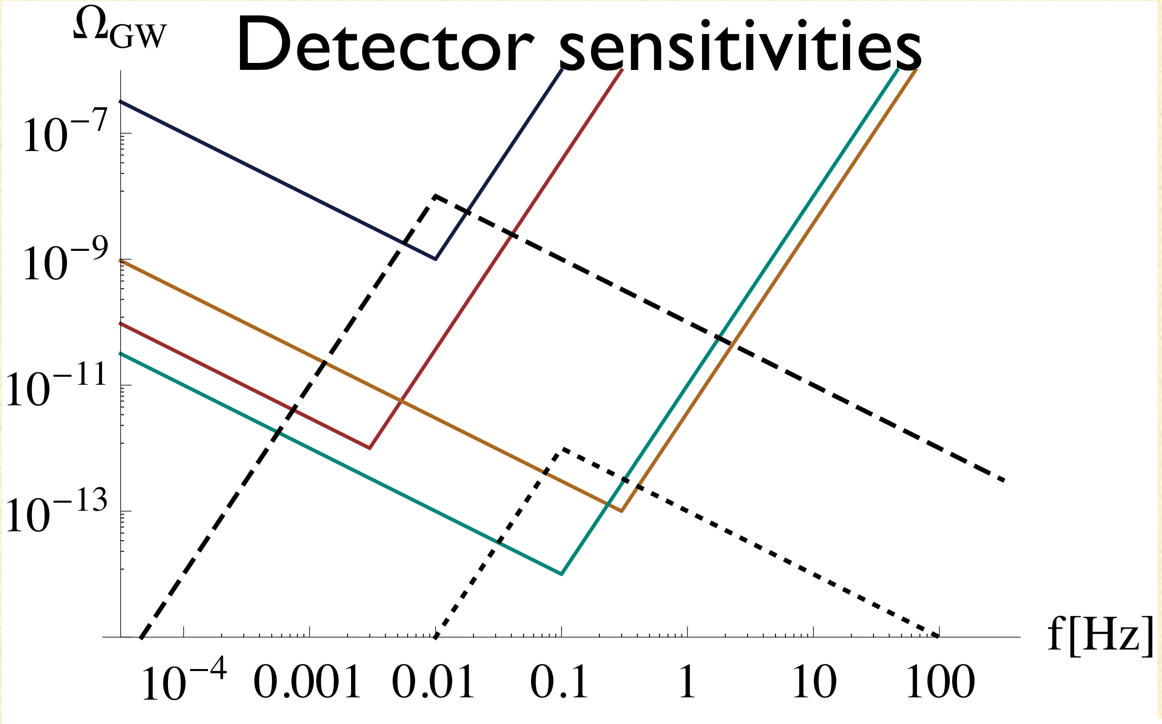
duration time

$$f_{\rm peak} \sim \frac{\beta}{H_*}\frac{T_*}{10^8{\rm GeV}}[{\rm Hz}]$$

$\Omega_{\rm GW} = \rho_{\rm GW}/\rho_{\rm tot}$

T_* : temp. just after transition

H_* : H just after transition



To have large GW, small β/H_* and large α are preferred!!

2. Analytic derivation of the GW spectrum

2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- The essence : GW spectrum is determined by

$$\langle T_{ij}(t_x, \mathbf{x}) T_{kl}(t_y, \mathbf{y}) \rangle_{\rm ens}$$

II
Ensemble average

[Caprini et al. '08]

2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- The essence : GW spectrum is determined by $\langle T_{ij}(t_x, \mathbf{x}) T_{kl}(t_y, \mathbf{y}) \rangle_{\rm ens}$

  - Why ?   Note : indices omitted below

    Formal solution of EOM : $\Box h \sim T \quad \rightarrow \quad h \sim \int^t dt' \; {\rm Green}(t, t') T(t')$

    Energy density of GWs (~ GW spectrum) :

$$\rho_{\rm GW}(t) \sim \frac{\langle \dot{h}^2 \rangle_{\rm ens}}{8\pi G} \sim \int^t dt_x \int^t dt_y \cos(k(t_x - t_y)) \boxed{\langle TT \rangle_{\rm ens}}$$

same as massless scalar field

substitute the formal solution

Note : ensemble average because of the stochasticity of the bubbles

2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- Estimation of the ensemble average $\langle T(t_x, \mathbf{x}) T(t_y, \mathbf{y}) \rangle_{\rm ens}$

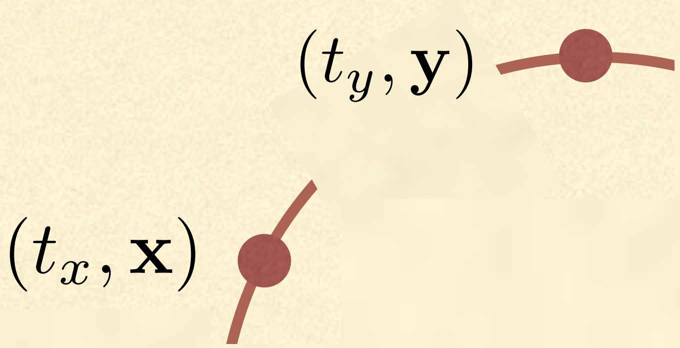
  - Trivial from the definition of ensemble average

(P) Probability part — (V) Value part

$$\langle T(t_x, \mathbf{x}) T(t_y, \mathbf{y}) \rangle_{\rm ens} = \sum \left(\begin{array}{c} \text{Probability for} \\ T(t_x, \mathbf{x}) T(t_y, \mathbf{y}) \neq 0 \end{array} \right) \times \left(\begin{array}{c} \text{Value of} \\ T(t_x, \mathbf{x}) T(t_y, \mathbf{y}) \\ \text{in that case} \end{array} \right)$$

||

$$\left(\begin{array}{c} \text{Probability that} \\ \text{bubble walls are} \\ \text{passing through} \\ (t_x, \mathbf{x}) \& (t_y, \mathbf{y}) \end{array} \right)$$



3. GWS FROM BUBBLE COLLISION

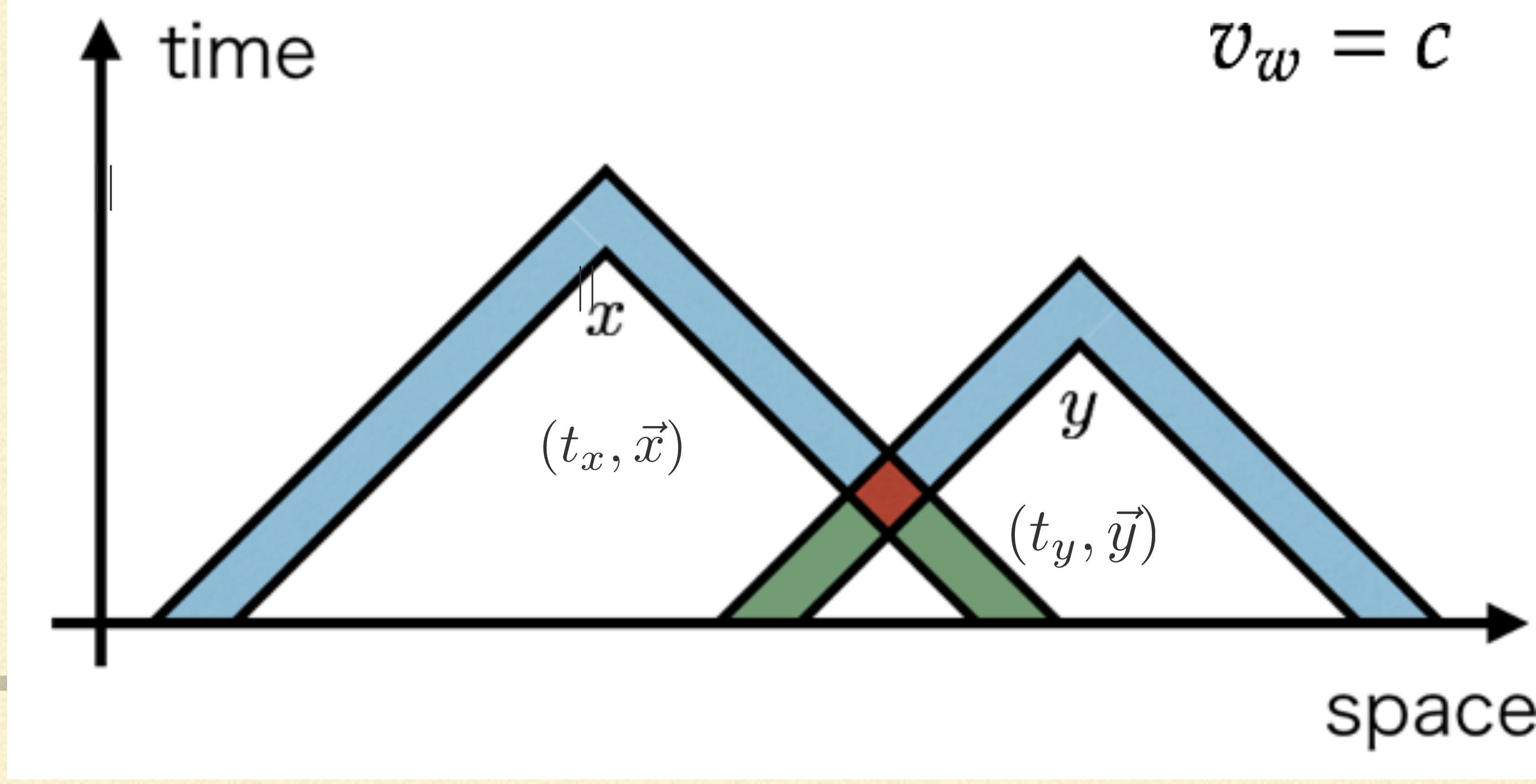
- Let consider the conditions where T(x)T(y) is non-zero.

・condition(1): No bubble is nucleated inside

the light cones of x and y.← envelope approximation

・condition(2): Bubbles are nucleated on the surface of

the light cones of x and y.
↑thin wall approximation



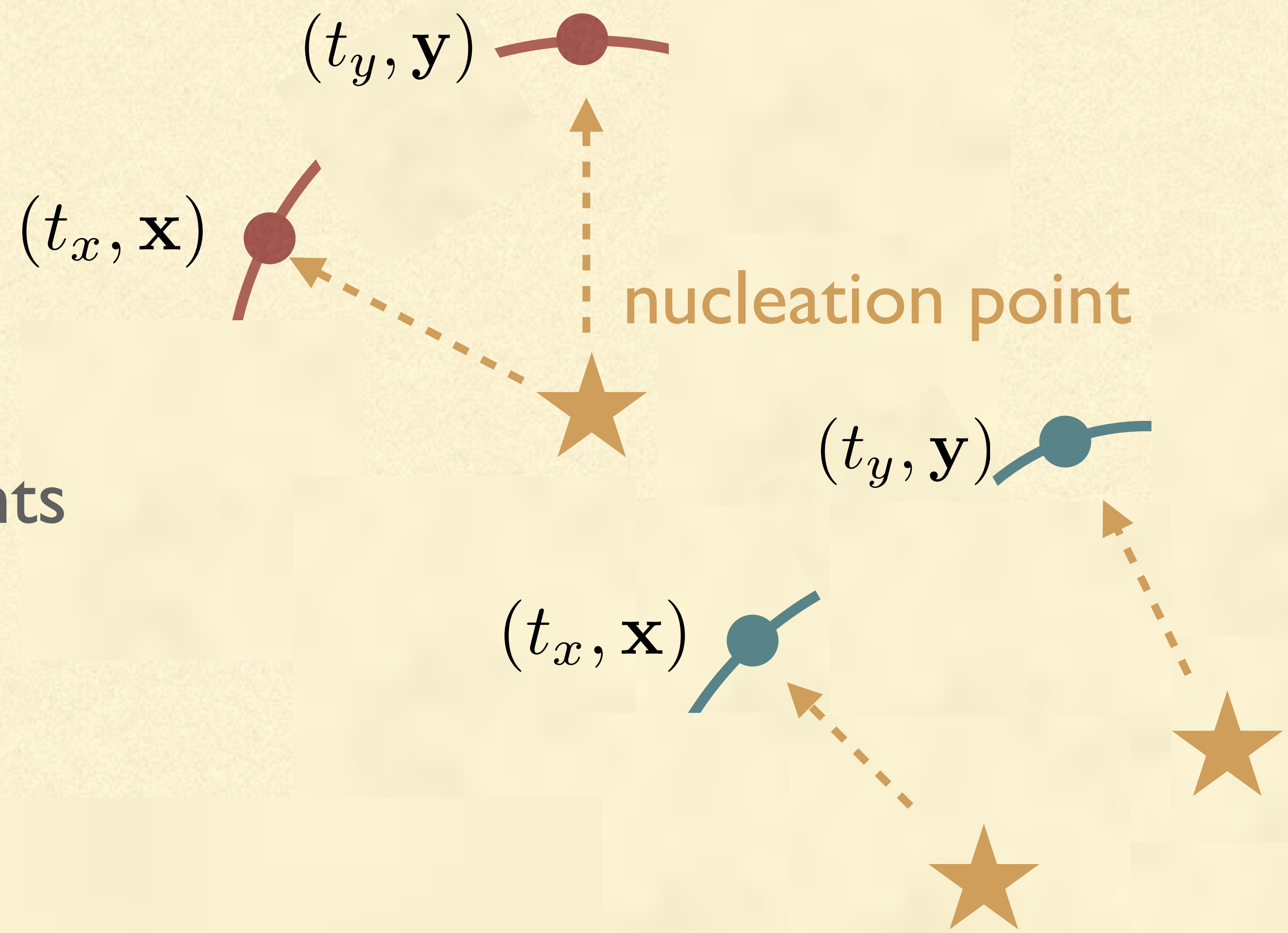
2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- Estimation of the ensemble average $\langle T(t_x, \mathbf{x}) T(t_y, \mathbf{y}) \rangle_{\text{ens}}$

  - 2 exclusive possibilities for $T(t_x, \mathbf{x}) T(t_y, \mathbf{y})$ to be nonzero

    1. single nucleation point

    2. double nucleation points



2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- Final expression $\Delta(k/\beta) \equiv \frac{3}{8\pi G}\frac{\beta^2\rho_{\rm tot}}{\kappa^2\epsilon_*^2}\Omega_{\rm GW}(k)$ $\Omega_{\rm GW}(k) \equiv \frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm GW}}{d\ln k}$

contribution to $\rho_{\rm GW}$ from each ln k

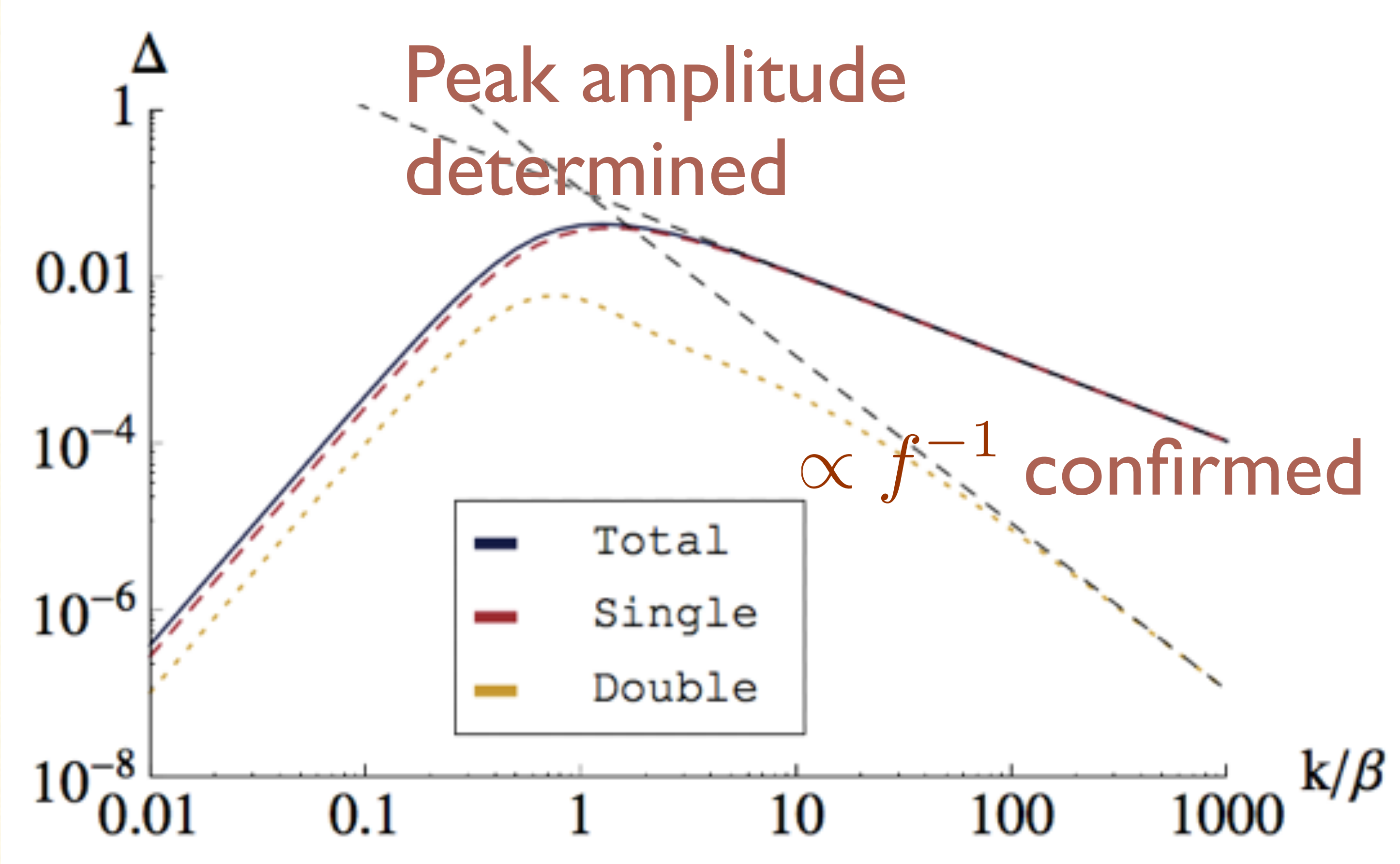
single	$$\Delta^{(s)} = \frac{k^3}{12\pi}\int_0^\infty dt_d \int_{t_d}^\infty dr\, \frac{e^{-r/2}\cos(kt_d)}{r^3\mathcal{I}(t_d,r)} \times \left[j_0(kr)F_0 + \frac{j_1(kr)}{kr}F_1 + \frac{j_2(kr)}{k^2r^2}F_2\right]$$	$$\begin{aligned} F_0 &= 2(r^2-t_d^2)^2(r^2+6r+12), \\ F_1 &= 2(r^2-t_d^2)\left[-r^2(r^3+4r^2+12r+24) + t_d^2(r^3+12r^2+60r+120)\right], \\ F_2 &= \frac{1}{2}\left[r^4(r^4+4r^3+20r^2+72r+144) - 2t_d^2r^2(r^4+12r^3+84r^2+360r+720) + t_d^4(r^4+20r^3+180r^2+840r+1680)\right] \end{aligned}$$
double	$$\Delta^{(d)} = \frac{k^3}{96\pi}\int_0^\infty dt_d \int_{t_d}^\infty dr\, \frac{e^{-r}\cos(kt_d)}{r^4\mathcal{I}(t_d,r)^2} \times \frac{j_2(kr)}{k^2r^2}G(t_d,r)G(-t_d,r)$$	$$G(t_d,r) = (r^2-t_d^2)\left[(r^3+2r^2)+t_d(r^2+6r+12)\right]$$

contains many polynomials, exponentials, and Bessel functions, but just that

2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- Result

  - Consistent with numerical simulation within factor ~2



$$\Delta(k/\beta) \equiv \frac{3}{8\pi G}\frac{\beta^2 \rho_{\rm tot}}{\kappa^2 \epsilon_*^2}\Omega_{\rm GW}(k)$$

$$\Omega_{\rm GW}(k) \equiv \frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm GW}}{d\ln k}$$

3. Future application & Summary

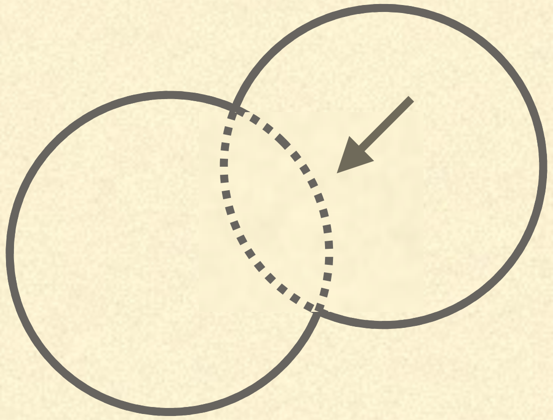
3. FUTURE APPLICATION & SUMMARY

- Future application

  Extend analytic method to more general setups.

For example - Inclusion of non-envelope part is (probably) possible

→ Check for the enhancement of GWs from sound-waves



(recent hot topic)

3. FUTURE APPLICATION & SUMMARY

- Summary

  - GWs can be a probe to high-energy particle physics, especially to high-energy first-order PT

  - To extract particle-physics information, theoretical prediction for the spectrum must be fixed

  - We derived GW spectrum from bubble collisions analytically in the same setup as in numerical simulation literature

    → Spectrum in this setup has been completely determined

Back up

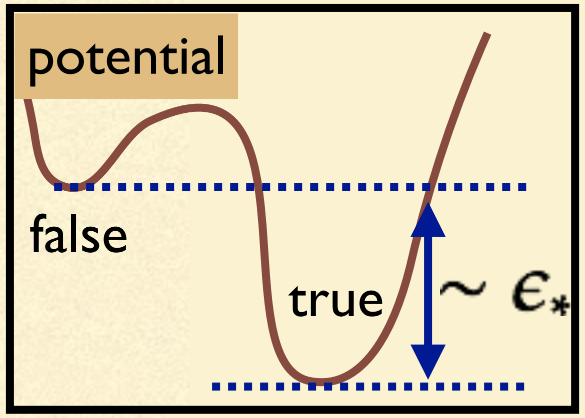
I. GWS PRODUCED IN PHASE TRANSITION

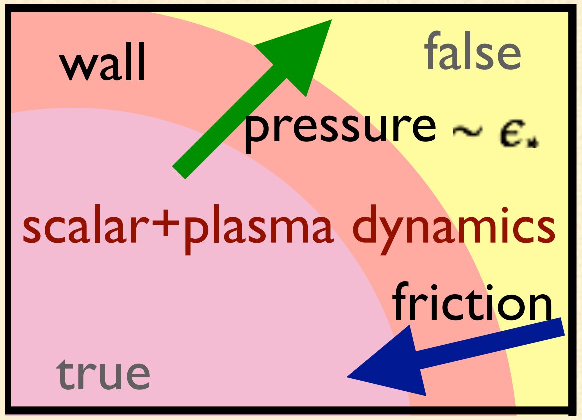
- How about ?

  - Roughly speaking,

$$\alpha \equiv \epsilon_* / \rho_{\rm radiation}$$

determines bubble-wall behavior



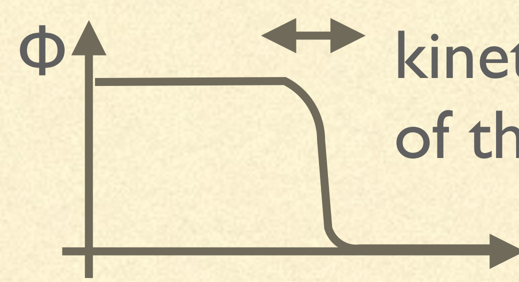


	Wall velocity approaches	Energy dominated by
(R) Runaway case $\alpha \gtrsim \mathcal{O}(1)$	speed of light (c)	scalar motion (wall itself)
(T) Terminal velocity case $\alpha \lesssim \mathcal{O}(1)$	terminal velocity (< c)	plasma around walls

I. GWS PRODUCED IN PHASE TRANSITION

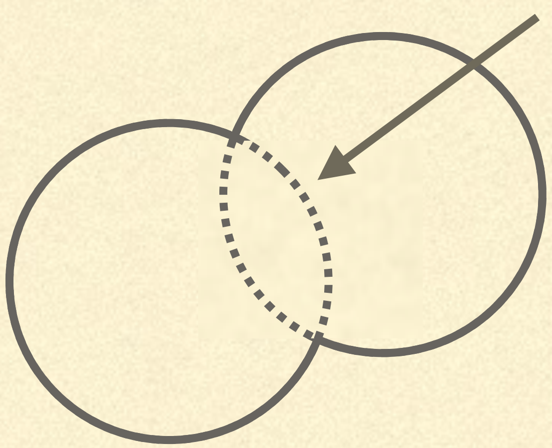
- How good are thin-wall & envelope approximations？

  - In (R) Runaway case,



{ thin-wall approx. → justified
{ envelope approx. → justified(?)

[Kosowski et al. '92]



  - (R) Runaway case is the one where large GW amplitude is expected

(N.B. sound wave-enhancement of (T) Terminal velocity case)

1. GWS PRODUCED IN PHASE TRANSITION

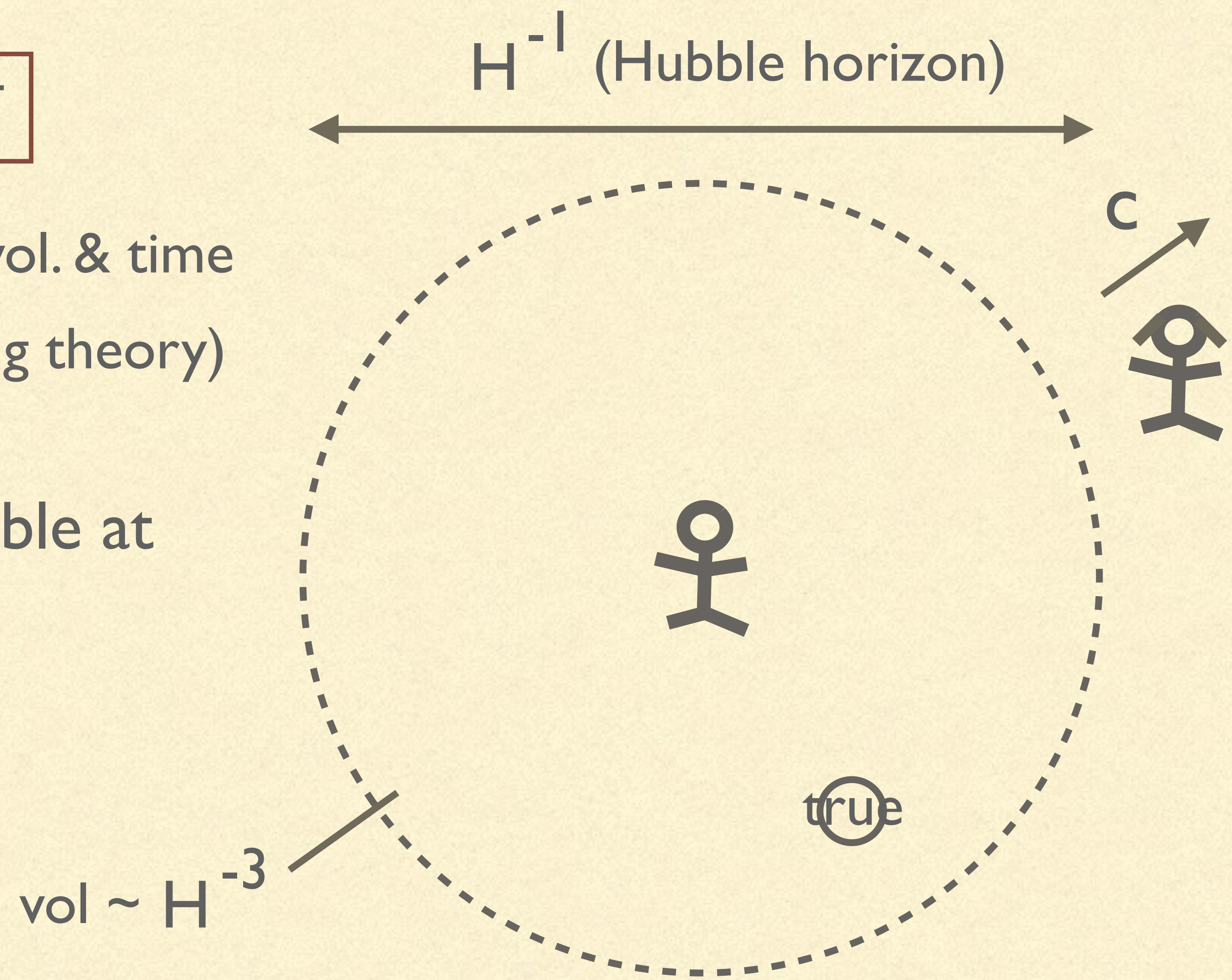
- When first order phase transition occurs

  - Transition occurs at $\Gamma \sim H^4$

Γ : nucleation rate per unit vol. & time
(determined by underlying theory)

because  finds one bubble at

$$\Gamma \times \underbrace{vol}_{H^{-3}} \times \underbrace{t}_{H^{-1}} \sim 1$$



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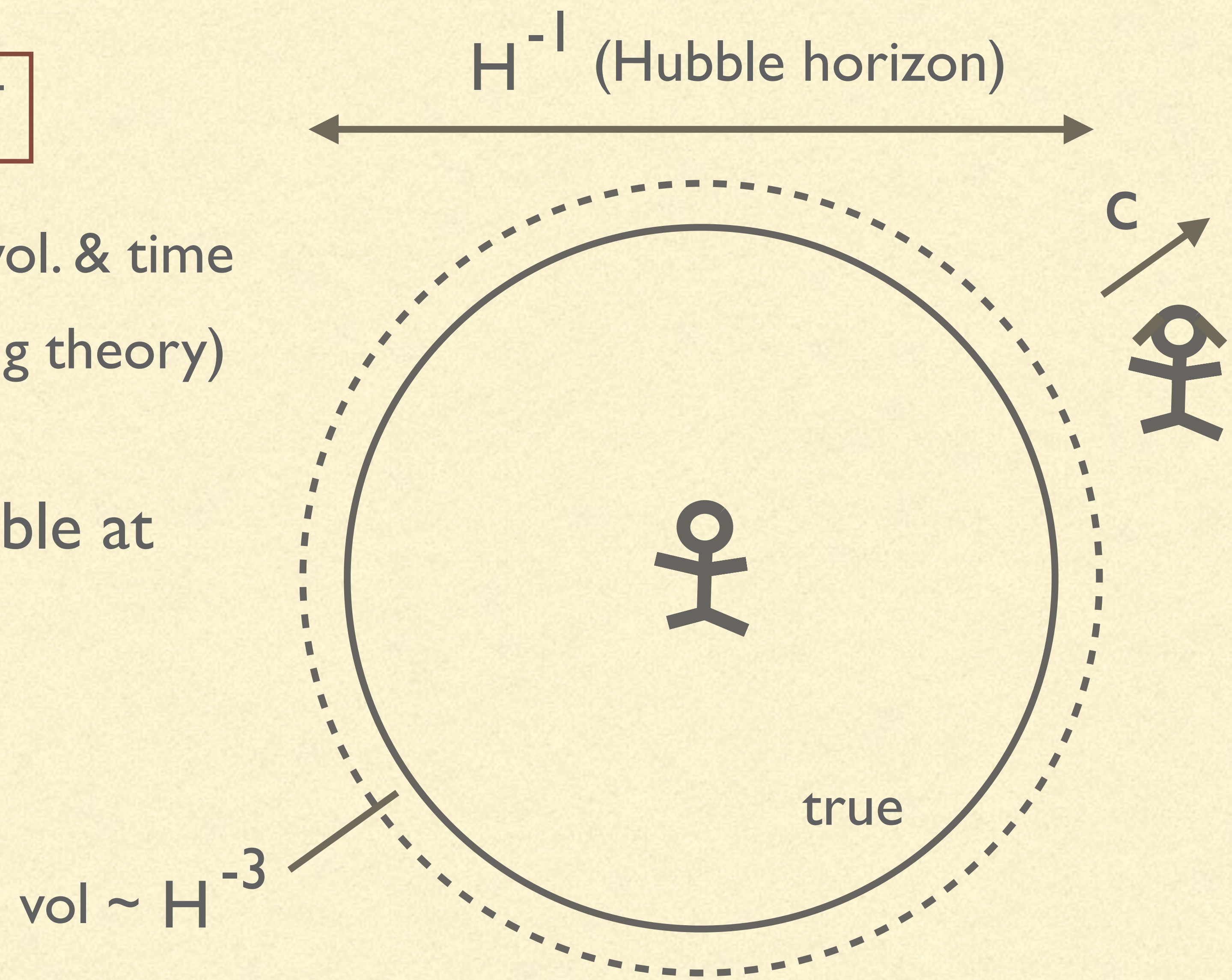


I. GWS PRODUCED IN PHASE TRANSITION

- How long first order phase transition lasts

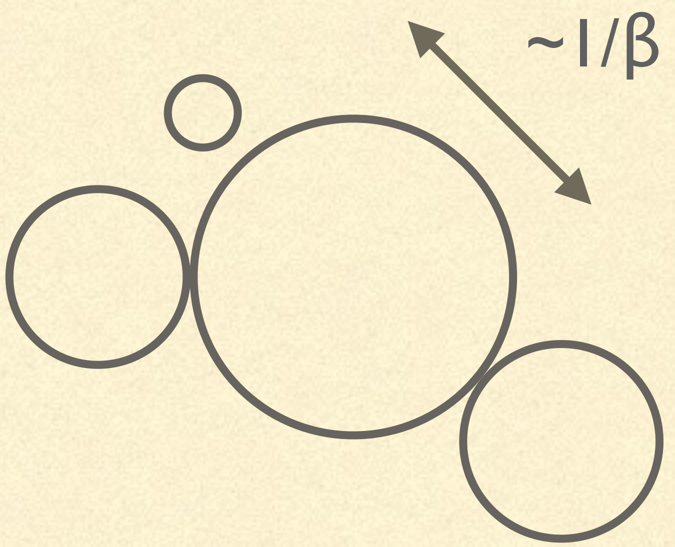
  - Duration of PT is determined by the changing rate of Γ

    $\Gamma = \Gamma_* e^{\beta(t - t_*)}$: Taylor exp. around transition time t_*

    as δt (duration time) ~ $1/\beta$ because

    1. Γ significantly changes with time interval $\delta t \sim 1/\beta$

    2. So, the first bubble typically collides with others δt after nucleation



2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- The essence : GW spectrum is determined by $\langle T_{ij}(t_x, \mathbf{x}) T_{kl}(t_y, \mathbf{y}) \rangle_{\rm ens}$

  - Ensemble average ?

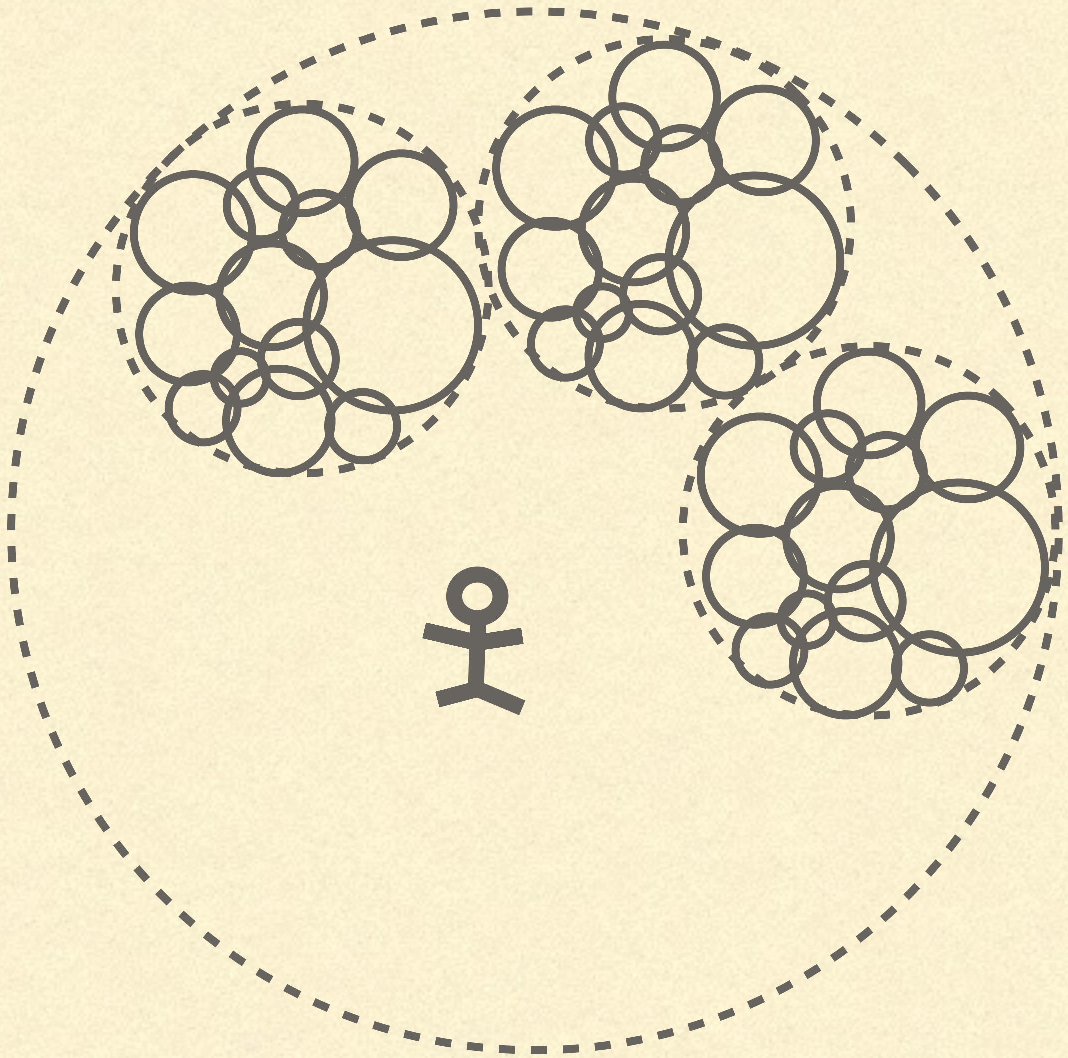
    Justified because bubble collisions are stochastic sources for GWs

    Horizon @ present

    ⊃ many horizons @ PT

    Horizon @ PT

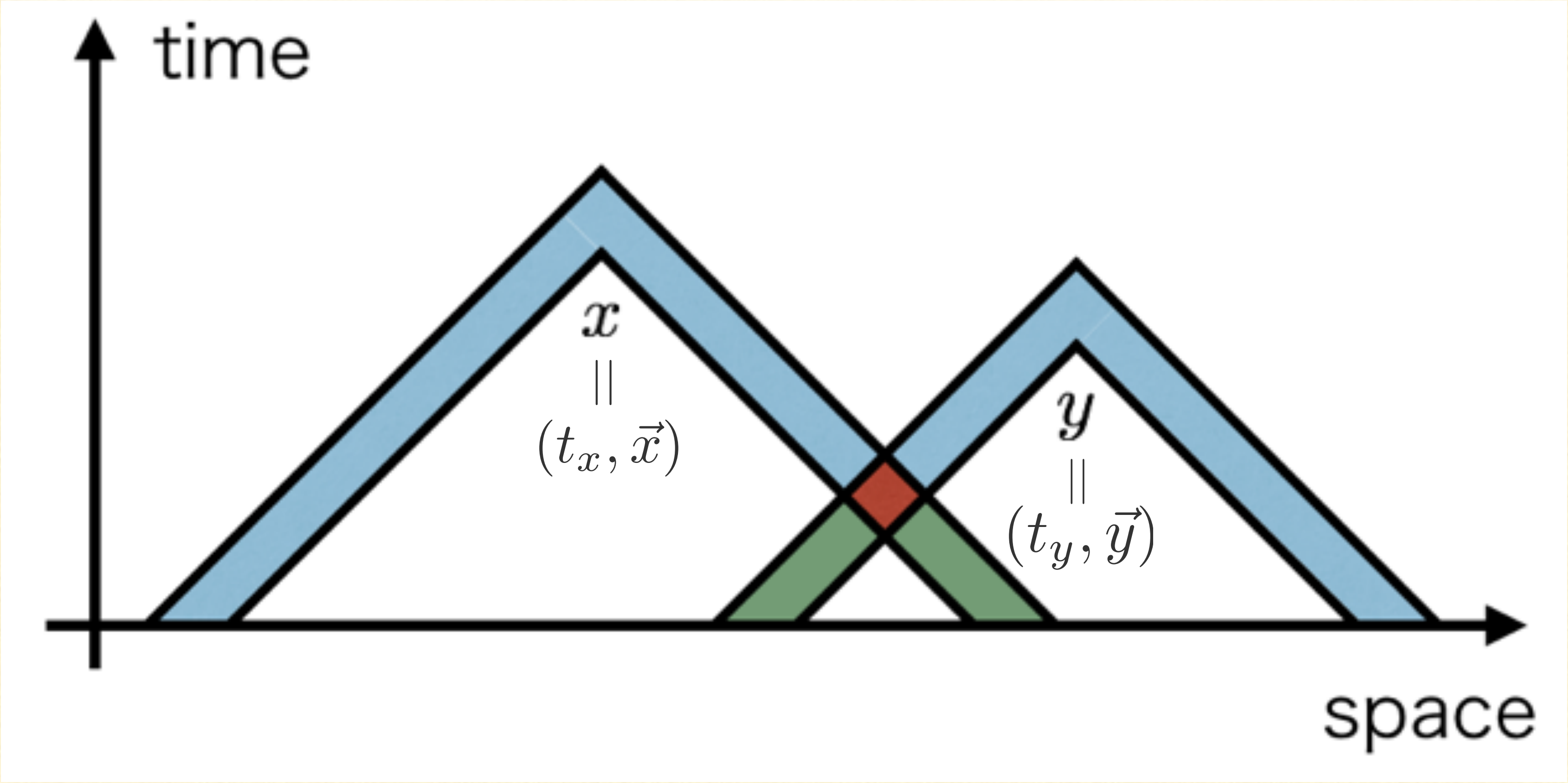
    ⊃ many bubbles (~ $(\beta/H)^3$)



2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- Estimation of the ensemble average $\langle T(t_x, \mathbf{x}) T(t_y, \mathbf{y}) \rangle_{\text{ens}}$

When does $T(x)T(y)$ has nonzero value ?



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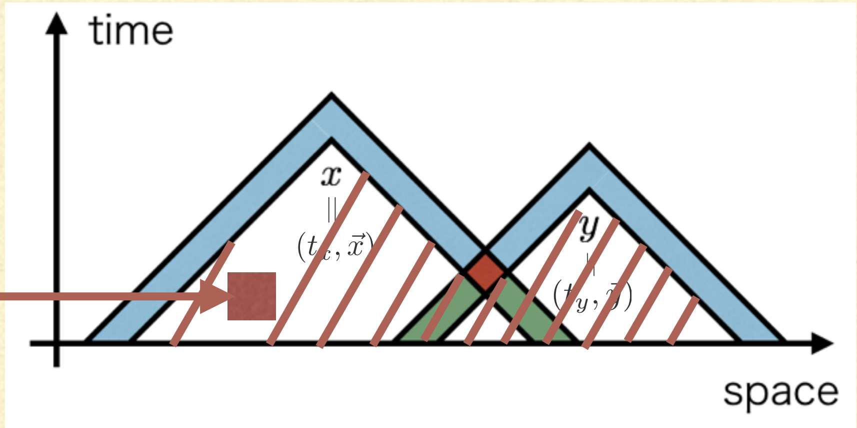
When does $T(x)T(y)$ has nonzero value ?

x & y must be in false vac. before t_x, t_y , respectively

Probability for x & y to be in false vacuum

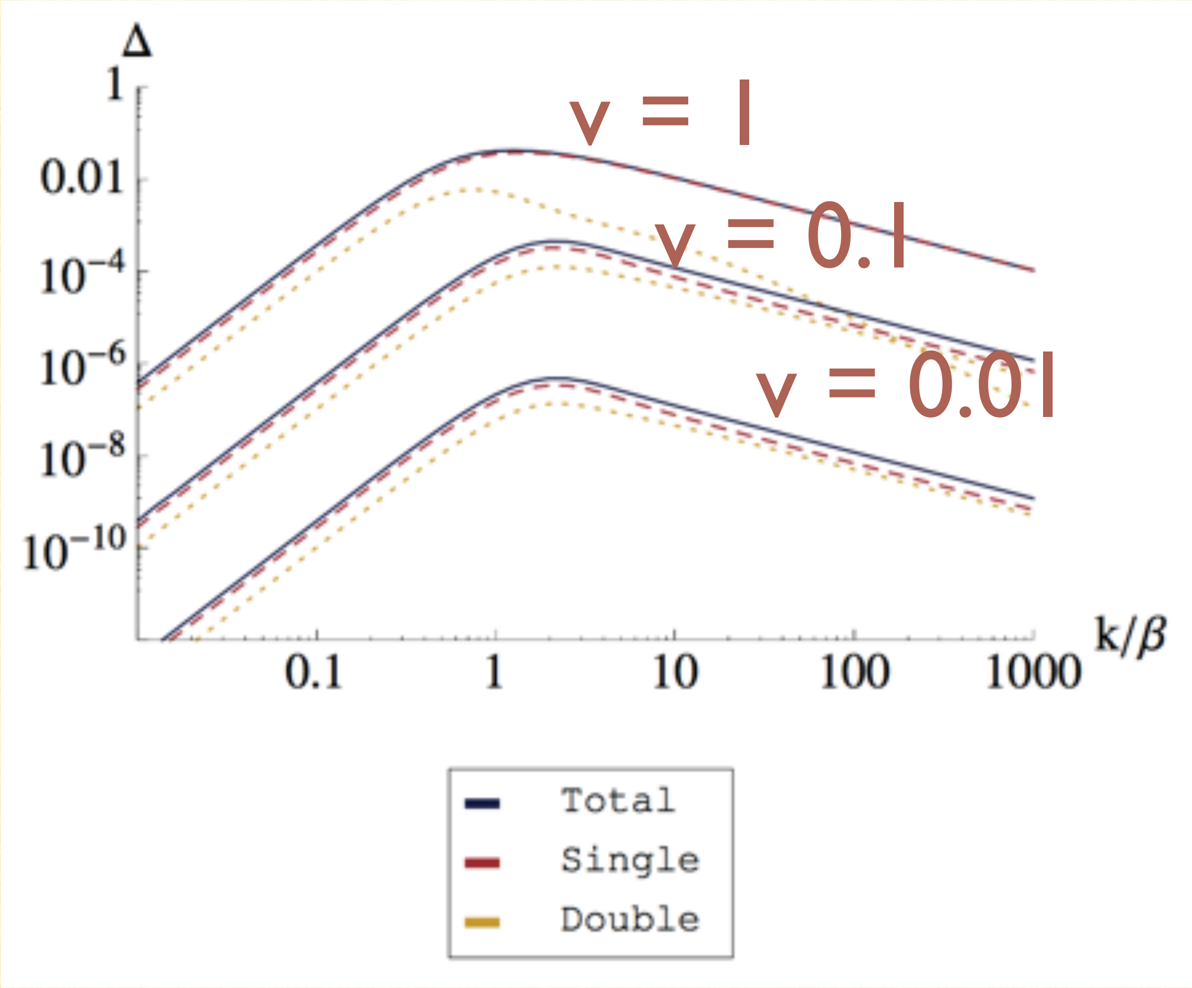
$$P(x,y) = \prod_i (1 - \Gamma dV_4^i) = e^{-\int d^4 z\, \Gamma(z)}$$

dV_4^i



2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- Wall velocity dependence



Wall velocity dependence determined

2. GWS PRODUCED IN PHASE TRANSITION

★Rough estimation of GW amplitude

Present GW amplitude & frequency are obtained just by redshifting

~quadrupole factor ~radiation fraction today

$$h^2\Omega_{\rm GW,peak} \sim \mathcal{O}(10^{-2})\mathcal{O}(10^{-5}) \left(\frac{\beta}{H_*}\right)^{-2} \left(\frac{\alpha}{1+\alpha}\right)^2$$

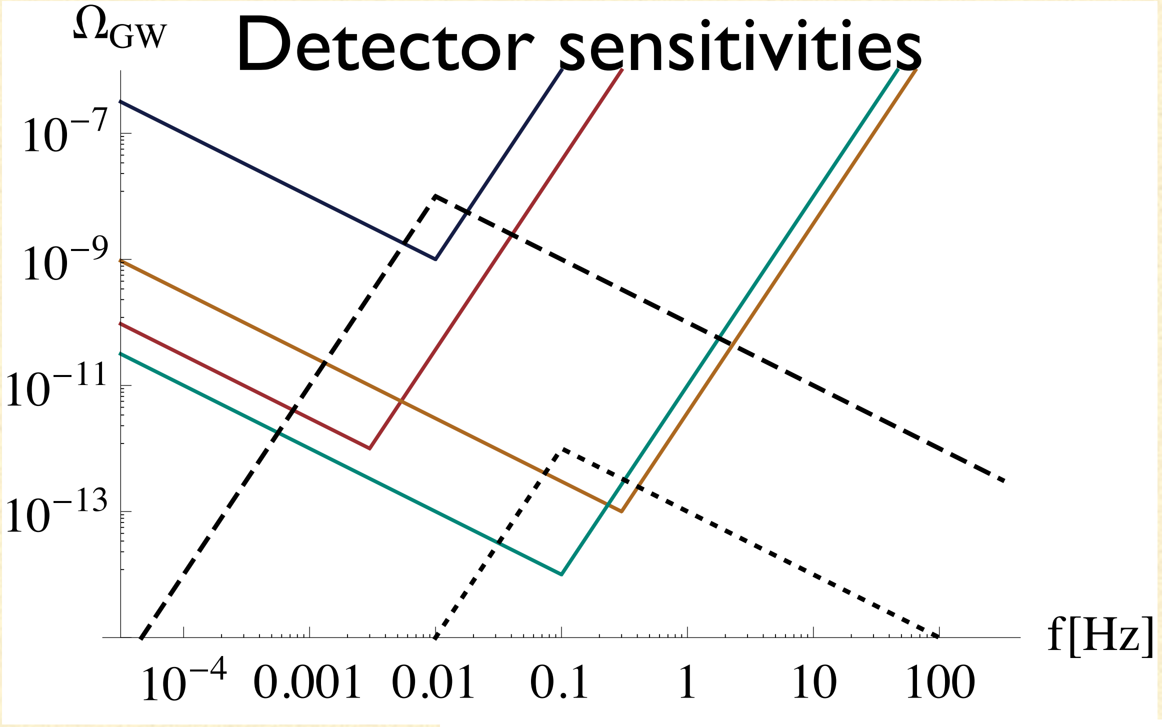
duration time

$$f_{\rm peak} \sim \frac{\beta}{H_*}\frac{T_*}{10^8 {\rm GeV}}[{\rm Hz}]$$

$\Omega_{\rm GW} = \rho_{\rm GW}/\rho_{\rm tot}$

T_* : temp. just after transition

H_* : H just after transition



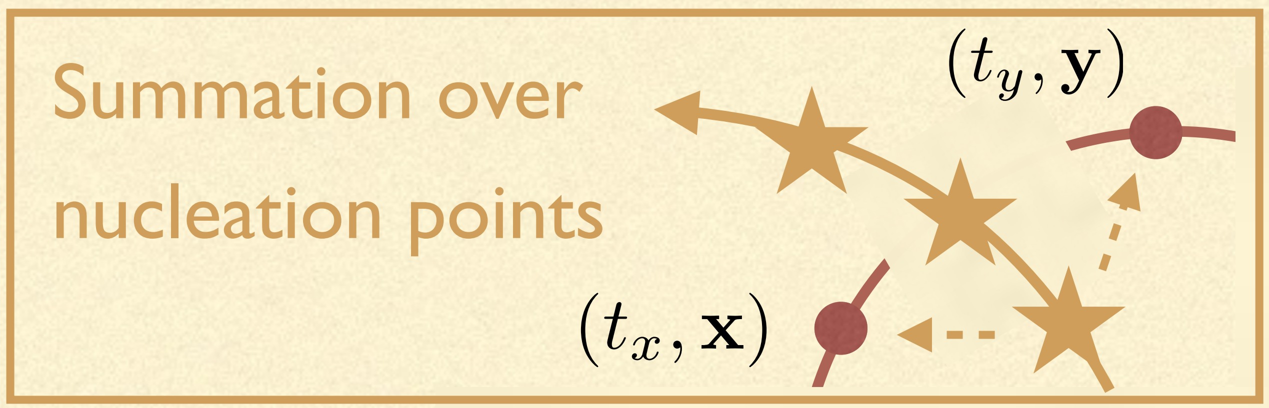
To have large GW, small β/H_* and large α are preferred!!

cf. SM with $m_H \sim 10$ GeV → $\beta/H \sim \mathcal{O}(10^5)$, $\alpha \sim \mathcal{O}(0.001)$

2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- Estimation of the ensemble average $\langle T(t_x, \mathbf{x}) T(t_y, \mathbf{y}) \rangle_{\text{ens}}$

  - Single nucleation-point contribution



$$\langle T_{ij}(t_x, \mathbf{x}) T_{kl}(t_y, \mathbf{y}) \rangle^{(s)} = \int \underline{dt_{\text{nucl}}} \; \underline{P(t_x, t_y, |\mathbf{x} - \mathbf{y}|) \Gamma(t_{\text{nucl}})}$$

$$\times \underline{\mathcal{T}(t_{\text{nucl}}, t_x, t_y, \Omega_{\text{nucl}}) (n_x)_i (n_x)_j (n_y)_k (n_y)_l}$$

Summation over nucleation points

(P) Probability part

(V) Value part