Electroweak Vacuum Stability and Renormalized Vacuum Field Fluctuation

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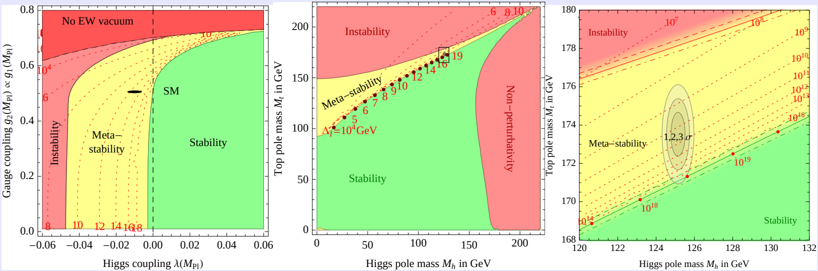
Based mainly on: **K, Kohri** and **H, Mastui**, Phys.Rev. D94 (2016) no.10, 103509, arXiv:1607.08133 (to appear in JCAP), arXiv:1704.06884, arXiv:1708.?????

Electroweak Vacuum Metastability

The recent LHC experiments of the Higgs boson mass $m_h = 125.09 \pm 0.21(\mathrm{stat}) \pm 0.11(\mathrm{syst})\ \mathrm{GeV}$ and the top quark mass $m_t = 172.44 \pm 0.13(\mathrm{stat}) \pm 0.47(\mathrm{syst})\ \mathrm{GeV}$ suggest that the electroweak vacuum is metastable and finally cause a catastrophic vacuum decay through quantum tunneling.



[D. Buttazzo, G. Degrassic, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, A. Strumia, JHEP 1312 (2013)]

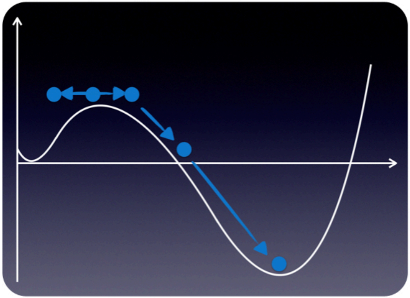
Electroweak Vacuum Stability during Large-scale Inflation

- In de Sitter space or inflationary Universe, the vacuum field fluctuation $\left\langle \delta\phi^2 \right\rangle$ enlarges in proportion to the Hubble scale H.

$$\left\langle \delta\phi^2 \right\rangle^{1/2} \approx \mathcal{O}(H) \gtrsim \Lambda_I \approx 10^{11}\ \mathrm{GeV} \implies \textit{CATASTROPHE !?}$$

If the inflationary vacuum fluctuation $\left\langle \delta\phi^2 \right\rangle$ overcomes the barrier of $V_{\mathrm{eff}}(\phi)$, it can trigger off a false vacuum decay. The preheating or reheating case are also trouble.

[J. R. Espinosa, G. F. Giudice, and A. Riotto, JCAP 0805, 002 (2008), M. Herranen, T. Markkanen, S. Nurmi, and A. Rajantie, Phys.Rev.Lett.113 211102 (2014), A. Hook, J. Kearney, B. Shakya, and K. M. Zurek, JHEP 01,061 (2015)]



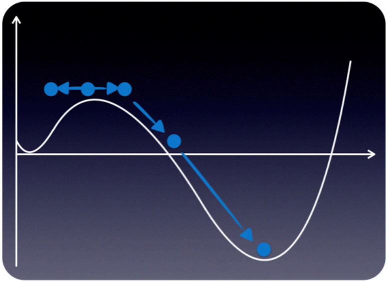
Electroweak Vacuum Stability around Evaporating Black Hole

- In Schwarzschild space, $\langle \delta\phi^2 \rangle$ proportional to inverse of the black-hole mass $M_{\rm BH}$ and approximately approach the Hawking thermal fluctuation near the black-hole event horizon.

$$\langle \delta\phi^2 \rangle^{1/2} \approx \mathcal{O}\left(1/M_{\rm BH}\right) \gtrsim \Lambda_I \approx 10^{11}\ \mathrm{GeV} \Longrightarrow \textit{CATASTROPHE !?}$$

Recent discussion about the vacuum stability around the evaporating (primordial) black hole which can be formulated by large density fluctuations in the early universe, has been growing.

[P. Burda, R. Gregory, and I. Moss, Phys.Rev.Lett.115,071303 (2015), N. Tetradis, JCAP 1609,036 (2016), D. Gorbunov, D. Levkov, and A. Panin, arXiv:1704.05399, K. Mukaida and M. Yamada, arXiv:1706.04523]



Electroweak Vacuum Metastability in Curved Background

- We assume the bare Higgs potential in the curved spacetime.

$$V(\phi) \equiv \frac{1}{2}\left(m^2 + \xi R\right)\phi^2 + \frac{\lambda}{4}\phi^4 + \cdots$$

where ξ is the non-minimal curvature coupling.

- The Friedmann- Lemaitre-Robertson-Walker (FLRW) metric and the Schwarzschild metric can be written by

$$ds^2 = -dt^2 + a(t)^2\left[\frac{dr^2}{1-Kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]$$

$$ds^2 = -\left(1 - \frac{2M_{\mathrm{BH}}}{r}\right)dt^2 + \frac{dr^2}{1-2M_{\mathrm{BH}}/r} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

The Vacuum Field Fluctuation in Curved Spacetime

- In quantum field theory (QFT), the two-point correlation function $\left\langle \delta\phi^2 \right\rangle$ corresponding to the vacuum field fluctuation $\left\langle \delta\phi^2 \right\rangle \longrightarrow \infty$ becomes (quadratically or logarithmically) divergent.

$$\left\langle \delta\phi^2 \right\rangle = \int d^3k \left|\delta\phi_k\left(t,x\right)\right|^2 = \int_0^\infty \frac{dk}{k}\Delta^2_{\delta\phi}\left(k\right) \longrightarrow \infty.$$

- These UV divergences must be removed by using the regularization or the renormalization method, and we take out the dynamical field fluctuation being depend on the curved background, i.e. H, R, $R_{abcd}R^{abcd}$.
- In QFT, there are various regularization methods like cutoff regularization, dimensional regularization, ζ-function regularization, lattice regularization, point-splinting regularization (QFT in curved spacetime), adiabatic regularization (QFT in curved spacetime) etc.

The Vacuum Field Fluctuation in De-Sitter (Inflation) Spacetime

The vacuum field fluctuation $\langle \delta\phi^2 \rangle$ of the Higgs field can be written by

$$\langle \delta\phi^2 \rangle = \frac{1}{4\pi^2 a^2(\eta)} \int_0^\infty dk k^2 \Omega_k^{-1} \left\{ 1 + 2|\beta_k|^2 + \alpha_k \beta_k^* \delta\varphi_k^2 + \alpha_k^* \beta_k \delta\varphi_k^{*2} \right\}$$

where $\alpha_k(\eta)$ and $\beta_k(\eta)$ are the Bogoliubov coefficients.

$$\langle \delta\phi^2 \rangle = \langle \delta\phi^2 \rangle^{(\mathrm{s})} + \langle \delta\phi^2 \rangle^{(\mathrm{d})}$$
$$\langle \delta\phi^2 \rangle^{(\mathrm{s})} = \frac{1}{4\pi^2 a^2(\eta)} \int_0^\infty dk k^2 \Omega_k^{-1}$$
$$\langle \delta\phi^2 \rangle^{(\mathrm{d})} = \frac{1}{4\pi^2 a^2(\eta)} \int_0^\infty dk k^2 \Omega_k^{-1} \{2n_k + 2\mathrm{Re} z_k\}$$

where we introduce $n_k = |\beta_k|^2$ which can be interpreted as the particle number density and $z_k = \alpha_k \beta_k^* \delta\varphi_k^2$.

The Vacuum Field Fluctuation in De-Sitter (Inflation) Spacetime

The divergences of $\left\langle \delta\phi^2 \right\rangle^{(\mathrm{s})}$ correspond to the Minkowskian divergences

$$\left\langle \delta\phi^2 \right\rangle^{(\mathrm{s})} = \frac{M^2(\phi)}{16\pi^2} \times \left[\ln\left(\frac{M^2(\phi)}{\mu^2} \right) - \frac{1}{\epsilon} - \log 4\pi - \gamma - \frac{3}{2} \right]$$

$$M^2(\phi) = m^2 + 3\lambda\phi^2 + (\xi - 1/6) R$$

These divergences can be canceled by the counterterms δm^2, $\delta\xi$ and $\delta\lambda$.

$$\left\langle \delta\phi^2 \right\rangle^{(\mathrm{s})}_{\mathrm{ren}} = \frac{M^2(\phi)}{16\pi^2} \left[\ln\left(\frac{M^2(\phi)}{\mu^2} \right) - \frac{3}{2} \right]$$

From the above renormalized vacuum field fluctuations $\left\langle \delta\phi^2 \right\rangle^{(\mathrm{s})}_{\mathrm{ren}}$, we can construct the one-loop effective potential on the curved spacetime

$$V_{\mathrm{eff}}(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \xi R \phi^2 + \frac{\lambda}{4} \phi^4 + \frac{M^2(\phi)}{16\pi^2} \left[\ln\left(\frac{M^2(\phi)}{\mu^2} \right) - \frac{3}{2} \right]$$

The Vacuum Field Fluctuation in De-Sitter (Inflation) Spacetime

The adiabatic regularization is the powerful method to obtain the dynamical vacuum fluctuation.

$$\left\langle \delta\phi^2 \right\rangle^{(\mathrm{d})} = \left\langle \delta\phi^2 \right\rangle - \left\langle \delta\phi^2 \right\rangle^{(\mathrm{s})} = \frac{1}{4\pi^2 a^2(\eta)} \int_0^\infty dk k^2 \Omega_k^{-1} \left\{ 2n_k + 2\mathrm{Re} z_k \right\}$$
$$= \frac{1}{4\pi^2 a^2(\eta)} \left[\int_0^\infty dk 2k^2 |\delta\chi_k|^2 - \int_0^\infty dk k^2 \Omega_k^{-1} \right]$$

where we must determine exactly $\delta\chi(\eta)$. In de Sitter spacetime, the dynamical (renormalized) vacuum field fluctuation $\left\langle \delta\phi^2 \right\rangle^{(\mathrm{d})}$ can be summarized as

$$\left\langle \delta\phi^2 \right\rangle^{(\mathrm{d})} \simeq \begin{cases} H^3 t/4\pi^2 & (M(\phi) = 0) \\ 3H^4/8\pi^2 M^2(\phi) & (M(\phi) \ll H) \\ H^2/24\pi^2 & (M(\phi) \gtrsim H) \end{cases}$$

The Electroweak Vacuum Stability in Inflationary Universe

In curved spacetime, the effective Higgs potential must include the dynamical vacuum field fluctuation $\left\langle\delta\phi^2\right\rangle^{(\mathrm{d})}$

$$V_{\mathrm{eff}}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}\xi R\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\lambda}{2}\left\langle\delta\phi^2\right\rangle^{(\mathrm{d})}\phi^2 + \sum_{i=1}^{9}\frac{n_i}{64\pi^2}M_i^4(\phi)\left[\log\frac{M_i^2(\phi)}{\mu^2} - C_i\right]$$

$$M_i^2(\phi) = \kappa_i\phi^2 + \kappa_i\left\langle\delta\phi^2\right\rangle^{(\mathrm{d})} + \kappa_i' + \theta_i R$$

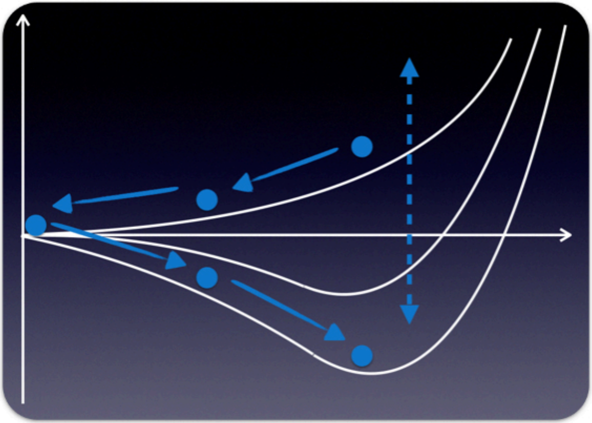
We can take the renormalization scale $\mu^2 \approx \phi^2 + R + \left\langle\delta\phi^2\right\rangle^{(\mathrm{d})}$.

$$\mu \approx \left(R + \left\langle\delta\phi^2\right\rangle^{(\mathrm{d})}\right)^{1/2} \gtrsim \Lambda_I \approx 10^{11}\ \mathrm{GeV} \implies \frac{\lambda(\mu)\phi^4}{4} < 0$$

[M. Herranen, T. Markkanen, S. Nurmi, and A. Rajantie, Phys.Rev.Lett. 113,211102 (2014), **K, Kohri** and **H, Mastui**, arXiv:1607.08133 (to appear in JCAP), arXiv:1704.06884]

The Electroweak Vacuum Stability in Inflationary Universe

The inflationary vacuum fluctuation of the Higgs field $\langle \delta\phi^2 \rangle$ destabilize the effective Higgs potential $V_{\rm eff}(\phi)$.



The Electroweak Vacuum Stability in Inflationary Universe

For $\mu \approx \left(R + \left\langle \delta\phi^2 \right\rangle^{(\mathrm{d})}\right)^{1/2} \gtrsim \Lambda_I$, the destabilization of $V_{\mathrm{eff}}(\phi)$ can be determined by

$$\left|\frac{1}{2}\xi(\mu)R\phi^2\right| \gtrless \left|\frac{\lambda(\mu)}{2}\left\langle \delta\phi^2 \right\rangle^{(\mathrm{d})}\phi^2\right| \Longrightarrow V_{\mathrm{eff}}(\phi) \gtrless 0$$

In de-Sitter spacetime, the destabilization condition of $V_{\mathrm{eff}}(\phi)$ can be given by

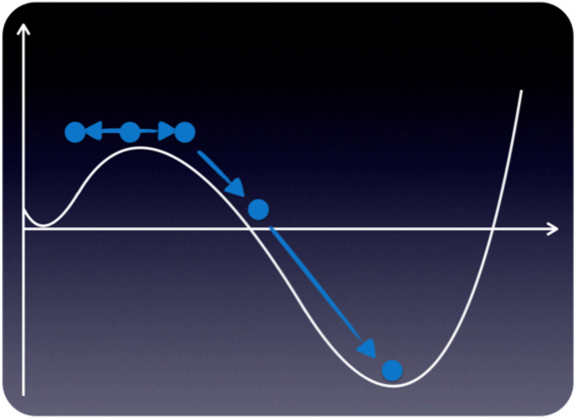
$$\left(R + \left\langle \delta\phi^2 \right\rangle^{(\mathrm{d})}\right)^{1/2} \gtrsim \Lambda_I, \quad \xi(\mu)R < |\lambda(\mu)| \left\langle \delta\phi^2 \right\rangle^{(\mathrm{d})} \Longrightarrow \mathit{CATASTROPHE}$$

Thus, we can obtain the constraint of the non-minimal coupling $\xi(\mu)$

$$H_{\mathrm{inf}} \gtrsim \Lambda_I \text{ and } \xi(\mu) \lesssim \mathcal{O}\left(10^{-3}\right) \Longrightarrow \text{Destabilized}$$

The Electroweak Vacuum Stability in Inflationary Universe

From the vacuum Higgs field fluctuations $\langle \delta\phi^2 \rangle$, the catastrophic Higgs Anti-de Sitter (AdS) domains or bubbles can be formed.



The Electroweak Vacuum Stability in Inflationary Universe

The probability function $P\left(\phi, \left\langle\delta\phi^2\right\rangle^{(\mathrm{d})}\right)$ with $\left\langle\delta\phi^2\right\rangle^{(\mathrm{d})}$ as follows:

$$P\left(\phi, \left\langle\delta\phi^2\right\rangle^{(\mathrm{d})}\right) = \frac{1}{\sqrt{2\pi\left\langle\delta\phi^2\right\rangle^{(\mathrm{d})}}} \exp\left(-\frac{\phi^2}{2\left\langle\delta\phi^2\right\rangle^{(\mathrm{d})}}\right)$$

$$P\left(\phi < \phi_{\mathrm{max}}\right) \equiv \int_{-\phi_{\mathrm{max}}}^{\phi_{\mathrm{max}}} P\left(\phi, \left\langle\delta\phi^2\right\rangle^{(\mathrm{d})}\right) d\phi = \mathrm{erf}\left(\frac{\phi_{\mathrm{max}}}{\sqrt{2\left\langle\delta\phi^2\right\rangle^{(\mathrm{d})}}}\right)$$

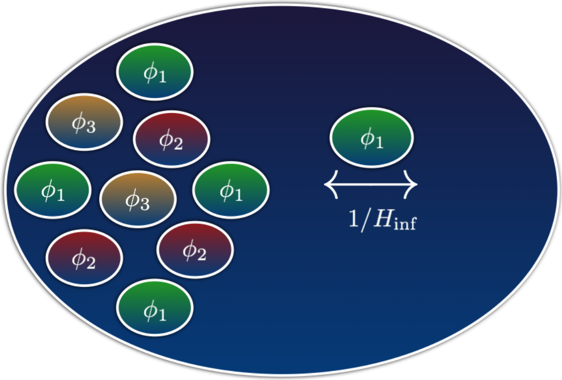
The probability to generate Higgs AdS domains or bubbles can be given by

$$P\left(\phi > \phi_{\mathrm{max}}\right) = 1 - \mathrm{erf}\left(\frac{\phi_{\mathrm{max}}}{\sqrt{2\left\langle\delta\phi^2\right\rangle^{(\mathrm{d})}}}\right) \simeq \frac{\sqrt{2\left\langle\delta\phi^2\right\rangle^{(\mathrm{d})}}}{\pi\phi_{\mathrm{max}}} \exp\left(-\frac{\phi_{\mathrm{max}}^2}{2\left\langle\delta\phi^2\right\rangle^{(\mathrm{d})}}\right)$$

[A. D. Linde, Nucl.Phys.B 372,421 (1992), J. R. Espinosa, G. F. Giudice, E. Morgante, A. Riotto, L. Senatore, A. Strumia, and N. Tetradis, JHEP 09,174 (2015)]

The Electroweak Vacuum Stability in Inflationary Universe

At the end of the inflation, there are huge no correlation patches being equivalent to $e^{3N_{\mathrm{hor}}}$



The Electroweak Vacuum Stability in Inflationary Universe

The vacuum decay probability of the Universe can be expressed as

$$e^{3N_{\rm hor}} P\left(\phi > \phi_{\rm max}\right) < 1 \implies \frac{\left\langle \delta\phi^2 \right\rangle^{(\rm d)}}{\phi_{\rm max}^2} < \frac{1}{6N_{\rm hor}}$$

where we can take the e-folding number $N_{\rm hor} \simeq N_{\rm CMB} \simeq 60$. During inflation, we can obtain the restriction of the non-minimal coupling $\xi(\mu) \gtrsim O\left(10^{-2}\right)$ not to generate the unwanted AdS domains or bubbles.

- $H_{\rm inf} \gtrsim \Lambda_I$ and $\xi(\mu) \lesssim \mathcal{O}\left(10^{-3}\right) \implies$ Destabilized
- $H_{\rm inf} \gtrsim \Lambda_I$ and $\mathcal{O}\left(10^{-3}\right) \lesssim \xi(\mu) \lesssim \mathcal{O}\left(10^{-2}\right) \implies$ AdS domains
- $H_{\rm inf} \lesssim \Lambda_I$ or $\mathcal{O}\left(10^{-2}\right) \lesssim \xi(\mu) \implies$ Stable

The Electroweak Vacuum Stability in Inflationary Universe

So far we consider the vacuum field fluctuation of the Higgs field, but the vacuum field fluctuation of the W and Z bosons and top quark enlarge in proportional to the Hubble scale $H_{\rm inf}$ and contribute to the effective Higgs potential as the effective masses

$$V_{\rm eff}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}\xi R\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\lambda}{2}\left\langle \delta\phi^2 \right\rangle^{(\rm d)}\phi^2 + \frac{1}{4}g^2\left\langle \delta W^2 \right\rangle^{(\rm d)}$$
$$+ \frac{1}{4}\left[g^2 + g'^2\right]\left\langle \delta Z^2 \right\rangle^{(\rm d)} + \frac{1}{2}y_t^2\left\langle \delta t^2 \right\rangle^{(\rm d)} + \sum_{i=1}^{9}\frac{n_i}{64\pi^2}M_i^4(\phi)\left[\log\frac{M_i^2(\phi)}{\mu^2} - C_i\right].$$

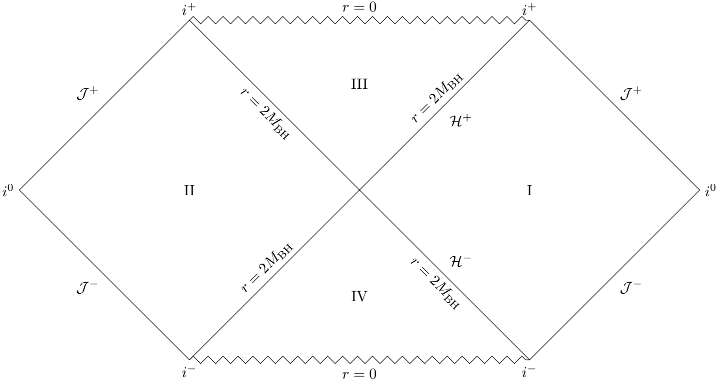
If the vacuum field fluctuations of W and Z bosons and the top quark are larger than the Higgs one, the effective Higgs potential can be stabilized

$$\left\langle \delta\phi^2 \right\rangle^{(\rm d)} \lesssim \left\langle \delta W^2 \right\rangle^{(\rm d)}, \left\langle \delta Z^2 \right\rangle^{(\rm d)}, \left\langle \delta t^2 \right\rangle^{(\rm d)} \Longrightarrow \text{Stabilized}$$
$$\xi(\mu) \gtrsim \xi_W(\mu),\ \xi_Z(\mu),\ \xi_t(\mu) \qquad \text{non} - \text{catastrophe}$$

Since these fluctuations of W and Z bosons and top quark depend on their mass as well as Higgs field, they can be suppressed by their non-minimal coupling $\xi_{W,Z,t}(\mu)$.

The Vacuum Field Fluctuation in Schwarzschild Spacetime

$$ds^2 = -\left(1 - \frac{2M_{\rm BH}}{r}\right) dt^2 + \frac{dr^2}{1 - 2M_{\rm BH}/r} + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$



The Vacuum Field Fluctuation in Schwarzschild Spacetime

The renormalized vacuum field fluctuation in the Boulware vacuum $|0_{\rm B}\rangle$ (vacuum polarization around a static star) and the Unruh vacuum $|0_{\rm U}\rangle$ (evaporating black hole) via the point-splitting regularization can be give by

$$\langle 0_{\rm B}|\delta\phi^2(x)|0_{\rm B}\rangle_{\rm ren} = \frac{1}{16\pi^2}\int_0^\infty \frac{d\omega}{\omega}\left[\sum_{l=0}^{\infty}(2l+1)\left[\left|R_l^{in}(r;\omega)\right|^2 + \left|R_l^{out}(r;\omega)\right|^2\right] - \frac{4\omega^2}{1-2M_{\rm BH}/r}\right] - \frac{M_{\rm BH}^2}{48\pi^2 r^4(1-2M_{\rm BH}/r)}$$

$$\langle 0_{\rm U}|\delta\phi^2(x)|0_{\rm U}\rangle_{\rm ren} = \frac{1}{16\pi^2}\int_0^\infty \frac{d\omega}{\omega}\left[\sum_{l=0}^{\infty}(2l+1)\left[\left|R_l^{in}(r;\omega)\right|^2 + \coth\left(\frac{\pi\omega}{\kappa}\right)\left|R_l^{out}(r;\omega)\right|^2\right] - \frac{4\omega^2}{1-2M_{\rm BH}/r}\right] - \frac{M_{\rm BH}^2}{48\pi^2 r^4(1-2M_{\rm BH}/r)}$$

[P. Candelas, Phys. Rev. D21, 2185 (1980)]

where we introduce $\kappa = (4M_{\rm BH})^{-1}$ which is the surface gravity of the black hole and the factor of $\coth\left(\frac{\pi\omega}{\kappa}\right)$ originates from the thermal features of the outgoing modes.

The Vacuum Field Fluctuation in Evaporating Black Hole

The renormalized vacuum fluctuation $\langle \delta\phi^2 \rangle_{\rm ren}$ around the evaporating black hole can be written as follows:

$$\langle \delta\phi^2 \rangle_{\rm ren} \simeq \begin{cases} \mathcal{O}\left(T_{\rm H}^2\right) & (r \rightarrow 2M_{\rm BH}) \\ 0 & (r \rightarrow \infty) \end{cases}$$

where the Hawking temperature $T_{\rm H} = 1/8\pi M_{\rm BH}$. The vacuum field fluctuation of the the Higgs, W and Z bosons and the top quark approximately approach the Hawking thermal fluctuations near the event horizon

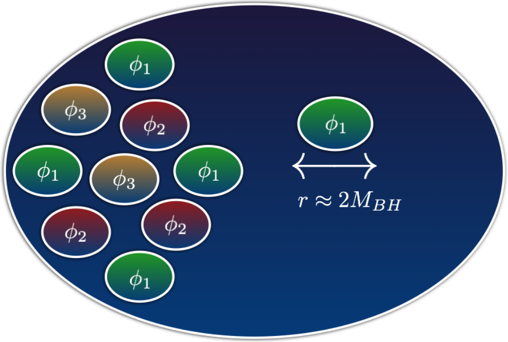
$$\langle \delta\phi^2 \rangle_{\rm ren} \simeq \langle \delta W^2 \rangle_{\rm ren} \simeq \langle \delta Z^2 \rangle_{\rm ren} \simeq \langle \delta t^2 \rangle_{\rm ren} \simeq \mathcal{O}\left(T_{\rm H}^2\right)$$

Thus, the maximal field value $\phi_{\rm max}$ being consistent with the hill of the effective Higgs potential can be given by

$$\phi_{\rm max}^2 \simeq \mathcal{O}\left(10^2\right) \cdot \left(\frac{1}{4} g^2(\mu) \langle \delta W^2 \rangle_{\rm ren} + \frac{1}{4}\left[g^2(\mu) + g'^2(\mu)\right] \langle \delta Z^2 \rangle_{\rm ren} + \frac{1}{2} y_t^2(\mu) \langle \delta t^2 \rangle_{\rm ren} + \frac{1}{2}\lambda(\mu) \langle \delta\phi^2 \rangle_{\rm ren}\right)$$

The Electroweak Vacuum Stability in Evaporating Black Hole

Let us consider a huge number of the evaporating or evaporated black holes $\mathcal{N}_{\mathrm{EBH}}$ where the large vacuum fluctuation exists only around the event horizon.



The Electroweak Vacuum Stability in Evaporating Black Hole

By applying the probability function $P\left(\phi > \phi_{\rm max}\right)$

$$\mathcal{N}_{\rm EBH} \cdot P\left(\phi > \phi_{\rm max}\right) \simeq \frac{\mathcal{N}_{\rm EBH}\sqrt{2\left\langle\delta\phi^2\right\rangle_{\rm ren}}}{\pi\phi_{\rm max}} \exp\left(-\frac{\phi_{\rm max}^2}{2\left\langle\delta\phi^2\right\rangle_{\rm ren}}\right)$$
$$\approx \mathcal{N}_{\rm EBH} \cdot e^{-\mathcal{O}(100)} \lesssim 1$$

The constraint of the number of the evaporating black holes can be given by

$$\mathcal{N}_{\rm EBH} \lesssim \mathcal{O}\left(10^{43}\right), \quad \beta \equiv \left.\frac{\rho_{\rm PBH}}{\rho_{\rm tot}}\right|_{\rm formation} \lesssim \mathcal{O}\left(10^{-21}\right)\left(\frac{m_{\rm PBH}}{10^9 {\rm g}}\right)^{3/2}$$

where $\rho_{\rm PBH}$ and $\rho_{\rm tot}$ are the energy density of the PBHs and the total energy density of the Universe. This bound can be stronger than the known one for $m_{\rm PBH} \lesssim 10^9{\rm g}$ [B. J. Carr, **K, Kohri**, Y. Sendouda, and J. Yokoyama, Phys. Rev. D81, 104019 (2010)]

The Electroweak Vacuum Stability in Evaporating Black Hole

However, at the final stage of the evaporation of the black hole, the black-hole mass M_{BH} becomes extremely small and the Hawking temperature reaches the Planck scale where $T_{\rm H} \to \mathcal{O}\left(M_{\rm Pl}\right)$.

$$V_{\rm eff}\left(\phi\right) = \frac{\lambda_{\rm eff}\left(\phi\right)}{4}\phi^4 + \frac{\delta\lambda_{\rm bsm}}{4}\phi^4 + \frac{\lambda_6}{6}\frac{\phi^6}{M_{\rm Pl}^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_{\rm Pl}^4} + \cdots$$

$$\Longrightarrow V_{\rm eff}\left(\phi\right) = \frac{1}{2}\left(\lambda_{\rm eff}T_{\rm H}^2 + \kappa^2 T_{\rm H}^2 + \frac{\lambda_6 T_{\rm H}^4}{M_{\rm Pl}^2} + \frac{\lambda_8 T_{\rm H}^6}{M_{\rm Pl}^4} + \cdots\right)\phi^2$$

$$+ \frac{1}{4}\left(\lambda_{\rm eff} + \delta\lambda_{\rm bsm} + \frac{\lambda_6 T_{\rm H}^2}{M_{\rm Pl}^2} + \frac{\lambda_8 T_{\rm H}^4}{M_{\rm Pl}^4} + \cdots\right)\phi^4 + \cdots$$

where $\delta\lambda_{\rm bsm}$ express the running corrections from the BSM, λ_6 and λ_8 dimensionless coupling constants. Therefore, even a single evaporating black hole can be catastrophic for the vacuum stability through $T_{\rm H} \to \mathcal{O}\left(M_{\rm Pl}\right)$ although this possibility strongly depends on the Planck scale physics.

Conclusion and Outlook

- In the curved background, the vacuum field fluctuations $\langle \delta\phi^2 \rangle$ grow in proportional to the curvature scale.
- The large vacuum fluctuation of the Higgs field can destabilize the effective Higgs potential, or generate the Higgs AdS domains or bubbles.These unwanted phenomena could cause a catastrophic collapse of the Universe.
- In the de-Sitter spacetime, the non-minimal coupling $\xi(\mu)$ sway the cosmological destiny of the Higgs vacuum.
- In the Schwarzschild black-hole spacetime, the vacuum field fluctuation $\langle \delta\phi^2 \rangle$ approaches approximately the Hawking thermal fluctuation near the black-hole horizon. By incorporating the back-reaction effects of $\langle \delta\phi^2 \rangle$ and analyzing the stability of the vacuum, we show that one evaporating black hole does not cause serious problems in standard model vacuum and obtain an upper bound on the evaporating PBH abundance not to induce any catastrophe.