R_K(*) anomaly comes from vector-like compositeness

Kenji Nishiwaki (KIAS)

니시와키, 켄지

based on collaboration with
Shinya Matsuzaki (Nagoya Univ.), Ryoutaro Watanabe (Montréal Univ.)

[arXiv:1706.01463]

Talk @ the workshop ‘Progress in Particle Physics 2017’, Kyoto, Japan;
1st August 2017
Points

1. Hidden “QCD” $\Rightarrow$ multiple vector candidates for B anomaly.

2. Various virtues in the vector-like compositeness

3. Large part of parameter space waits for being explored.
INTRODUCTION
B anomalies [review]

\[ R_K \equiv \frac{\mathcal{B}(B \to K\mu^+\mu^-)}{\mathcal{B}(B \to Ke^+e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036 \quad \text{for } 1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2 \]  

[\text{LHCb, arXiv:1406.6482}]

\[ R_{K^*} \equiv \frac{\mathcal{B}(B \to K^*\mu^+\mu^-)}{\mathcal{B}(B \to K^*e^+e^-)} = \begin{cases} 0.660^{+0.110}_{-0.070} \pm 0.024 & \text{for } (2m_\mu)^2 < q^2 < 1.1 \text{ GeV}^2 \\ 0.685^{+0.113}_{-0.069} \pm 0.047 & \text{for } 1.1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2 \end{cases} \]  

[LHCb (seminar in CERN on 18th April), arXiv:1705.05802]

suggested lepton flavor violation (2.2-2.6\sigma) [=1 in SM]
**B anomalies**

- **$R_K$**
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  \end{cases}
  \]

suggesting lepton flavor violation (2.2-2.6σ) [=1 in SM]

- **LHCb, arXiv:1506.08777**
  ![LHCb plot 1](image1.png)

- **Belle, arXiv:1612.05014**
  ![Belle plot 1](image2.png)

- **ATLAS, ATLAS-CONF-2017-023**
  ![ATLAS plot 1](image3.png)

deviations being observed in associated variables
Suggestions from global fit(s) [review]

\[ \mathcal{H}_{\text{eff}}^{\text{NP}} = -\frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_{i,\ell} \left( C_i^\ell O_i^\ell + C_{i}^\prime \ell O_{i}^\prime \ell \right) + \text{h.c.} \]

\[ O_9^\ell = (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma_\mu \ell), \quad O_9^{\prime \ell} = (\bar{s} \gamma_\mu P_R b)(\bar{\ell} \gamma_\mu \ell), \]

\[ O_{10}^\ell = (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma_\mu \gamma_5 \ell), \quad O_{10}^{\prime \ell} = (\bar{s} \gamma_\mu P_R b)(\bar{\ell} \gamma_\mu \gamma_5 \ell), \]

[global fit result for new physics]

[W.Altmannshofer et al., arXiv:1704.05435]

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>best fit</th>
<th>1σ</th>
<th>2σ</th>
<th>pull</th>
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<tbody>
<tr>
<td>( C_9^\mu )</td>
<td>-1.59</td>
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<td>( C_9^\mu )</td>
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<td>( C_9^c )</td>
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<td>[+0.79, +2.53]</td>
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✓ (effective) vector interaction

[in the SM]

[pictures from Web]

\( (C_9^{\text{SM}} = -C_{10}^{\text{SM}} \sim 4) \)

[see also e.g., arXiv:1704.15340, 1704.05435, 1704.05438, 1704.05444, 1704.05446, 1704.05447, 1704.05672, 1704.7347, 1704.07397, 1704.08168]
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- **(effective) vector interaction**
- **s and b should be left-handed** (right-handed is irrelevant).
- **Lepton part is ambiguous** (vector-like, left-handed,...).

\[ (C_9^{SM} = -C_{10}^{SM} \sim 4) \]

[see also e.g., arXiv:1704.15340, 1704.05435, 1704.05438, 1704.05444, 1704.05446, 1704.05447, 1704.05672, 1704.7347, 1704.07397, 1704.08168]
How about $Z'$? [review]

A straightforward candidate: $Z'$ vector boson → What is quantum number?

**Basic pheno. strategy:** $U(1)_{\mu-L_T}' +$ vector-like quarks

- $U(1)'$ breaking (by a new scalar)
- Yukawa mixing among SM quarks & vector-like ones

---

**Fundamental $Z'$ scenarios work** [(with additional fermion(s) and scalar(s))].
How about Z’? [review]

a straightforward candidate: Z’ vector boson  ➔ What is quantum number?

basic pheno. strategy: \( U(1)’_{L\mu-L\tau} + \text{vector-like quarks} \)

\[ b_L \quad \langle \phi \rangle \quad U(1)’ \text{ breaking (by a new scalar)} \]

\[ s_L \quad \langle \phi \rangle \quad \text{Yukawa mixing among SM quarks & vector-like ones} \]

Fundamental Z’ scenarios work [(with additional fermion(s) and scalar(s)].

Q: How about composite case?
0. Introduction (finished)

1. Hidden “QCD” $\Rightarrow$ multiple vector candidates for B anomaly.

2. Various virtues in the vector-like compositeness

3. Large part of parameter space waits for being explored.

Summary
When a coupling becomes strong, composite particles appear.

\[
q_{L/R} = \begin{pmatrix}
  u \\
  d \\
  s \\
  \vdots
\end{pmatrix}_{L/R}
\]

\(SU(3)_L \times SU(3)_R\) (approximated) chiral symmetry

\(p\) (momentum)

\(q\) \(g\) \(\bar{q}\) \(p'\)

weekly coupled (UV-fundamental) particles

\(\Lambda_{QCD}\)

[arXiv:hep-ex/0606035]
When a coupling becomes strong, composite particles appear.

\( \langle \bar{q}^A q^B \rangle \sim \Lambda_{QCD}^3 \delta^{AB} \) (confinement)

Spontaneous breakdown: 
\( SU(3)_L \times SU(3)_R \rightarrow SU(3)_V \)

\( q \bar{q} g \bar{q} \)

strongly coupled (theory of composite particles)

\( \Lambda_{QCD} \)

weekly coupled (UV-fundamental) particles

\( q_L / R = \begin{pmatrix} u \\ d \\ s \\ \vdots \end{pmatrix} \)

SU(3)_L \times SU(3)_R (approximated) chiral symmetry

\( p'' \)

\( p' \) (momentum)
When a coupling becomes strong, composite particles appear. The theory of composite particles is approximated by chiral symmetry. Weakly coupled (UV-fundamental) particles become strongly coupled (theory of composite particles).

Chiral symmetry governs low-energy composite (meson) spectrum.

- Pseudo-scalars (pions) as pseudo NG bosons
- Vector mesons (rhos) as gauge bosons of hidden local symmetry ($SU(3)_V$, gauged)

[pictures from Web]

[reviewed by e.g., Harada,Yamawaki, arXiv:hep-ph/0302103]
Such a confining gauge theory is fascinating since:

- dynamically-realized symmetry breaking
- ‘predictive’ (limited # of parameters)
- Low-energy meson theory is managed by the chiral symmetry
- New vector particles are introduced in a consistent way!

FAQ: Can we obtain `composite Z` for \( R_{K(*)} \)?

Chiral symmetry governs low-energy composite (meson) spectrum.

- pseudo-scalars (pions) as pseudo NG bosons
- vector mesons (rhos) as gauge bosons of hidden local symmetry (\( SU(3)_V \), gauged)

[reviewed by e.g., Harada, Yamawaki, arXiv:hep-ph/0302103]
We consider an SU($N_{HC}$) confining gauge theory (fermion: $F$, gauge boson: $g'$)
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In a situation that $\rho_\mu$ “mix with” the SM gauge boson, $\rho_\mu$ may couple with the SM fermions in an effective way!

Gauge-invariant (effective) operator including it can be written down in terms of hidden local symmetry (with nonlinear basis).
We consider an SU($N_{HC}$) confining gauge theory (fermion: $F$, gauge boson: $g'$)

\[
F \rightarrow F' \quad F' \rightarrow g'
\]

\[
\rho_\mu \quad V_{\mu}^{SM}
\]

holding the Same (SM) quantum #

In a situation that $\rho_\mu$ “mix with” the SM gauge boson, $\rho_\mu$ may couple with the SM fermions in an effective way!

**Configuration:**

<table>
<thead>
<tr>
<th>new (HC) quarks</th>
<th>new (HC) leptons</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU(8)$_L \times$SU(8)$_R$ chiral symmetries</td>
<td></td>
</tr>
<tr>
<td>$F_{L/R} = \begin{pmatrix} Q \ L \end{pmatrix}_{L/R}$</td>
<td></td>
</tr>
<tr>
<td>$Q_{L/R} = \begin{pmatrix} U \ D \ N \end{pmatrix}_{L/R}$</td>
<td></td>
</tr>
<tr>
<td>$L_{L/R} = \begin{pmatrix} E \ N \ L \end{pmatrix}_{L/R}$</td>
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</table>

Effectively, formed mesons can couple to SM doublet fermions (left-handed) ← favored by the flavor results
### Configuration:

**SU(8)_L x SU(8)_R**

Chiral symmetries:

$$F_{L/R} = \begin{pmatrix} Q \\ L \end{pmatrix}_{L/R}$$

(one-family model)

<table>
<thead>
<tr>
<th><strong>QCD (quarks)</strong></th>
<th><strong>Hidden “QCD” (new fermions)</strong></th>
<th><strong>[N_f=8: 8^2-1 = 63 d.o.f.s]</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{L/R} = \begin{pmatrix} u \ d \ s \ \vdots \end{pmatrix}_{L/R}$</td>
<td>$F_{L/R} = \begin{pmatrix} U_R \ U_G \ U_B \ D_R \ D_G \ D_B \ N \ E \end{pmatrix}_{L/R}$</td>
<td></td>
</tr>
</tbody>
</table>

**SU(8)_L x SU(8)_R**

Chiral symmetries:

$$F_{L/R} = \begin{pmatrix} Q \\ L \end{pmatrix}_{L/R}$$

(one-family model)

**Configuration:**

**new (HC) quarks**

**SU(8)_L x SU(8)_R**

Chiral symmetries:

$$F_{L/R} = \begin{pmatrix} Q \\ L \end{pmatrix}_{L/R}$$

(one-family model)

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<tr>
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<th><strong>confined</strong></th>
<th><strong>external gauge interactions</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SU(N_{HC})</strong></td>
<td><strong>SU(3)_c</strong></td>
<td><strong>SU(2)_W</strong></td>
</tr>
<tr>
<td>$Q_{L/R} = \begin{pmatrix} U \ D \end{pmatrix}_{L/R}$</td>
<td>$N_{HC}$</td>
<td>3</td>
</tr>
<tr>
<td>$L_{L/R} = \begin{pmatrix} N \ E \end{pmatrix}_{L/R}$</td>
<td>$N_{HC}$</td>
<td>1</td>
</tr>
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Effectively, formed mesons can couple to SM doublet fermions (left-handed) ← favored by the flavor results.
[vector meson spectrum] NOT ONLY $Z'$ candidates!

<table>
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<tr>
<th>composite vector</th>
<th>constituent</th>
<th>color</th>
<th>isospin</th>
</tr>
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<tbody>
<tr>
<td>$\rho^\alpha_{(8)\alpha}$</td>
<td>$\frac{1}{\sqrt{2}} \bar{Q} \gamma_\mu \lambda^a \tau^\alpha Q$</td>
<td>octet</td>
<td>triplet</td>
</tr>
<tr>
<td>$\rho^0_{(8)\alpha}$</td>
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<td>octet</td>
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<td>$\rho^\alpha_{(3)c} \left( \bar{\rho}^\alpha_{(3)c} \right)$</td>
<td>$\frac{1}{\sqrt{2}} \bar{Q} c \gamma_\mu \tau^\alpha L$ (h.c.)</td>
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<td>$\rho^\alpha_{(1)'}$</td>
<td>$\frac{1}{2\sqrt{3}} (\bar{Q} \gamma_\mu \tau^\alpha Q - 3 \bar{L} \gamma_\mu \tau^\alpha L)$</td>
<td>singlet</td>
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Vector-like hidden “QCD” [HC] (cont’d)  

15 in total

- Massive gluons
- Vector leptoquarks
- $Z'$ (and $W'$) included
The forms which discriminate charges, involving the HC-quark and -lepton numbers. Then the nonlinear bases form. We may relate the charges of the HC fermions with those of the SM quark and lepton numbers attached in lower scripts (1, 2). The covariant derivatives for the HC fermions can also be written in terms of the 8\( \rho \) and \( \rho' \) form. We may relate the charges of the HC fermions with those of the SM quark and lepton fields. The SM-covariant derivatives, \( SU(8) \) gauge group in the new bases are also useful:

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\[ [L^{f}_\mu]_{8 \times 8} = \left( \begin{array}{c|c} 1_{2 \times 2} \otimes g_s G_{\mu}^\alpha \lambda^a_{\frac{1}{2}} + (g_W W_{\mu}^\tau \tau^\alpha + \frac{1}{6} g_Y B_{\mu}) \otimes 1_{3 \times 3} & 0_{6 \times 2} \\ \hline 0_{2 \times 6} & g_W W_{\mu}^\tau \tau^\alpha - \frac{1}{2} g_Y B_{\mu} \cdot 1_{2 \times 2} \end{array} \right) \]

\[ \rho = \left( \begin{array}{c} (\rho_{QQ})_{6 \times 6} (\rho_{QL})_{6 \times 2} \\ (\rho_{LQ})_{2 \times 6} (\rho_{LL})_{2 \times 2} \end{array} \right) \]
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<td>$\rho_{(1)}^\alpha$</td>
<td>$\frac{1}{2} (\bar{Q}^{\mu}<em>\gamma \tau^\alpha Q + \bar{L}^{\mu}</em>\gamma \tau^\alpha L)$</td>
<td>singlet</td>
<td>triplet</td>
</tr>
</tbody>
</table>

[SM gauge boson structure of $(q,l)_L$ in SU(8)$_V$ form]

\[
\left[ L^f_\mu \right]_{8\times8} = \begin{pmatrix}
1_{2\times2} \otimes g_s G^a_\mu \frac{\lambda^a}{2} + \left( g_W W^\mu_\gamma \tau^\alpha + \frac{1}{6} g_Y B^\mu \right) \otimes 1_{3\times3} & 0_{6\times2} \\
0_{2\times6} & g_W W^\mu_\gamma \tau^\alpha \frac{1}{2} g_Y B^\mu \cdot 1_{2\times2}
\end{pmatrix}
\]

for SU(2)$_w$-doublet SM quarks

for SU(2)$_w$-doublet SM leptons

The following **flavor-changing interaction** can be added gauge-invariantly.

**undetermined coefficients**

\[
g_L^i \times (q^\text{SM}_i \tilde{q}^\text{SM}_i)_L \gamma^\mu \left( g^\text{SM}_V V^\text{SM}_\mu - g_\rho \rho_\mu + \cdots \right) (q^\text{SM}_j \tilde{q}^\text{SM}_j)_L
\]

**flavor indices (in gauge eigenbasis)**

\[
(\text{SM}^i_j) \text{ } L
\]
0. Introduction (finished)

1. Hidden “QCD” $\Rightarrow$ multiple vector candidates for B anomaly.

2. Various virtues in the vector-like compositeness

3. Large part of parameter space waits for being explored.

• Summary
We adopted the flavor texture: 
\[ g^i_j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & g^{33}_L \end{pmatrix} \]

The form of the electroweak sector must be concerned, which is rephrased in a way that the flavor anomalies in Eq. (IV.1) are the chiral projectors defined as

\[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_D & \sin \theta_D \\ 0 & -\sin \theta_D & \cos \theta_D \end{pmatrix} \]

\[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_L & \sin \theta_L \\ 0 & -\sin \theta_L & \cos \theta_L \end{pmatrix} \]

The final form of the above result, we keep the term up to \[ g^{33}_L \] -diagonal components of the \[ g \] couplings to the SM fermions. We also discuss a reasonable setup assuming (3,3) only in gauge eigenbasis.
Important points for current pheno. [B.Bhattacharya et al., arXiv:1609.09078]

We adopted the flavor texture:

\[ g_{ij}^L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & g_{33}^L \\ 0 & 0 & 0 \end{pmatrix} \]

\[ (u_L)^i = U^{iI}(u'_L)^I, \quad (d_L)^i = D^{iI}(d'_L)^I, \quad (e_L)^i = L^{iI}(e'_L)^I, \quad (\nu_L)^i = L^{iI}(\nu'_L)^I, \]

automatically determined (with CKM matrix)

assuming \((3,3)\) only in gauge eigenbasis

\[ L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_D & \sin \theta_D \\ 0 & -\sin \theta_D & \cos \theta_D \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_L & \sin \theta_L \\ 0 & -\sin \theta_L & \cos \theta_L \end{pmatrix} \]

assuming \(2\leftrightarrow 3\) matter generation mixings

\[ \mathcal{L}_{V_{fL\bar{f}L}}^{\text{direct}} = g_{ij}^L \bar{q}_L^i \gamma_\mu \left[ g_s G_a^{\mu} \left( 1_{2\times2} \otimes \frac{\lambda_a}{2} \right) + \left( g_W W^{\alpha \mu} \frac{\sigma^\alpha}{2} + \frac{g_Y}{6} B^\mu 1_{2\times2} \right) \otimes 1_{3\times3} + g_\rho \rho_{QQ}^\mu \right] q_L^j \]

overall factor

\[ -g_{L}^i g_\rho \left[ \bar{q}_L^i \gamma_\mu \rho_{QL}^\mu l_L^i + \text{h.c.} \right], \]

\[ \text{correction to } f\bar{f}\text{-V}_{SM} \text{ interaction} \]

\[ \text{f-f- } \rho \text{ interaction} \]

\[ g_\rho \gg g_{SM} \text{ is required via EW precisions.} \]

\[ \rightarrow g_\rho = 6 \text{ (vector dominance in QCD)} \]

[flavor-changing effective interaction]
We adopted the flavor texture:

\[ g_L^{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & g_{L33}^{33} \end{pmatrix} \]

\[ (u_L)^i = U^iI(u_L')^I, \quad (d_L)^i = D^iI(d_L')^I, \quad (e_L)^i = L^iI(e_L')^I, \quad (\nu_L)^i = L^iI(\nu_L')^I, \]

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**automatically determined (with CKM matrix)** assuming (3,3) only in gauge eigenbasis

**Important points for current pheno.**

**Direct**

\[ \mathcal{L}_{\text{direct}}^V_{fLfL} = \sum_{ij} \bar{q}_L^{i\gamma \mu} \left[ g_s G^\mu_a \left( \frac{1}{2} \frac{\lambda_a}{2} \right) \right] q_L^{j} \]

\[ + \bar{l}_L^{\gamma \mu} \left[ g_W W^{\alpha \mu} \frac{\sigma^\alpha}{2} + g_Y B^{\mu} \right] q_L^{j} \]

\[ - \bar{l}_L^{\gamma \mu} \left[ g_Y W^{\alpha \mu} \frac{\sigma^\alpha}{2} - g_Y B^{\mu} \right] q_L^{j} \]

Overall factor

\[ - g_L^{ij} g_\rho \left[ \bar{q}_L^{i\gamma \mu} \rho_{QL}^{\mu} l_L^{j} + \text{h.c.} \right] \]

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Important points for current pheno. (cont’d)

- Vector-like HC rho mesons ⇒ harmless (tree-level) oblique corrections
Important points for current pheno. (cont’d)

- **vector-like HC rho mesons** ⇒ **harmless (tree-level) oblique corrections**

4 (+1) couplings are relevant for (pure) HC vector-$\rho$ phenomena:

$$m_\rho, \ g_\rho^L \ [= [g_\Lambda]^{33} \ast g_\rho], \ \theta_D, \ \theta_L, \ \langle g_\rho \rangle$$
Important points for current pheno. (cont’d)

- Vector-like HC rho mesons ⇒ harmless (tree-level) oblique corrections

- 4 (+1) couplings are relevant for (pure) HC vector-\(\rho\) phenomena:

\[ m_\rho, g_{\rho L} [= [g_L]^{33} \ast g_\rho], \theta_D, \theta_L, (g_\rho) \]

- No dynamical EWSB (vector-like) ⇒ the fundamental Higgs doublet should be introduced (like the SM).

- The 125GeV Higgs signal strengths are good.
Important points for current pheno. (cont’d)

- vector-like HC rho mesons ⇒ harmless (tree-level) oblique corrections

- 4 (+1) couplings are relevant for (pure) HC vector-$\rho$ phenomena:

\[ m_\rho, g_\rho L \left[= [g_L]^{33} \right. * g_\rho \left. \right], \theta_D, \theta_L, (g_\rho) \]

- No dynamical EWSB (vector-like) ⇒ the fundamental Higgs doublet should be introduced (like the SM).

  ✔ The 125GeV Higgs signal strengths are good.

Fascinating aspects:

✔ The candidate of Zs and their mass scale are dynamically generated.
✔ The $C_9 = -C_{10}$ texture (for $b \to sll$) is naturally realized.
✔ Apparently gauge-anomaly free.
✔ Lots of new particles (EW-safe) are ‘derived’.

( ✯ The lightest baryon may be stable (⇒ a dark matter candidate) 
  e.g., [T.Hur & P.Ko, arXiv:1103.2571])
( ✯ Scale-invariant extension (⇒ hierarchy problem))
0. Introduction (finished)

1. Hidden “QCD” $\Rightarrow$ multiple vector candidates for B anomaly.

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• Summary
Flavor results

HC $\rho : |g_\rho g_{L}^{33}| = 1$, $m_\rho = 1$ TeV

[Image of contour plot with annotations]

an example of amplitude:

\[ b \rightarrow s\mu\mu \]

\[ \tau \rightarrow 3\mu \]

\[ \tau \rightarrow \phi\mu \]

\[ R_{K}(\ast) \]

\[ \Delta M_s \]

\[ \text{‘minimal’ constraint} \]

adopted $C_9 (= -C_{10})$ favored range: $C_9^{\mu\mu}|_{\text{best}} = -0.61$ & $C_9^{\mu\mu}|_{+3\sigma} = -0.23$

All of vector $\rho$s (massive gluons, vector LQ, W’s, Z’s) are taken into account.
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[B.Capdevila et al., arXiv:1704.05340]
For $\mu$, the best-fit point, Fig. 35 we also found that the two allowed spots (which were divided by the anomaly) are presented with dotted, dashed, and solid curves, respectively. For the close-up plot, we have taken the parameter range favored up to the figure. Note that the left-side spot is barely viable when the $g\rho g_{L}^{33}$ are shown whereas the others are simply omitted. The best-fit point, $B_{\mu\mu}$ from the best-fit point ($\theta_{L}$, $\theta_{D}$), where $\theta_{L} \simeq 0$ ("left-side") and $\theta_{L} \simeq \pi/4$ ("right-side") are only allowed. In this case, the lepton sector only has the connection to the HC mass basis. In this case, the lepton sector only has the connection to the HC mass basis.

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[B. Capdevila et al., arXiv:1704.05340]
We can enlarge \(|g_\rho g^L_{33}|\) as much as possible (if perturbation is valid).

- adopted \(C_9 (= -C_{10})\) favored range: \(C_9^{\mu\mu}|_{\text{best}} = -0.61\) & \(C_9^{\mu\mu}|_{+3\sigma} = -0.23\)
- All of vector \(\rho\)s (massive gluons, vector LQ, W’s, Z’s) are taken into account.
Situation after fixing the mixing angles ($\theta_D \sim 0$)

**in scenario I**

(EW + Flavor) measurements: $g_\rho = 6$

**in scenario II**

(EW + Flavor) measurements: $g_\rho = 6$

relevant EW constraints: $A_{FB}^{(0,\tau)}$, $A_{FB}^{(0,\mu)}$, $R_b$

[Flavor-favored region] is inside [EW allowed region (2\sigma)].
**for scenario I & II**

For $pp \to jj$ [acceptance = 0.69]

- No bound for 'flavored' region

(PDF set: CTEQ6L1)

**for scenario I**

For $pp \to \tau^+ \tau^-$

- Typically, $m_\rho > 1$ TeV

**for scenario II**

For $pp \to \mu^+ \mu^-$

- Typically, $m_\rho > a$ few TeV
Virtues of (vector-like) composite model are (e.g.,)

- The candidate of Zs and their mass scale are dynamically generated.
- The $C_9 = -C_{10}$ texture (for $b \to sll$) is naturally realized.
- Apparently gauge-anomaly free.
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- Well-defined TeV-scale vector leptoquarks

The $R_K(\ast)$ anomalies are addressed consistently.

Discovering lots of new particles is expected at the LHC, distinguishable from other scenarios.
Virtues of (vector-like) composite model are (e.g.,)

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The $R_{K^{(*)}}$ anomalies are addressed consistently.

Discovering lots of new particles is expected at the LHC, distinguishable from other scenarios.

thank you:-)
\[
R_D = \frac{\mathcal{B}(\bar{B} \to D\tau\bar{\nu})}{\mathcal{B}(\bar{B} \to D\ell\bar{\nu})}, \quad R_D^* = \frac{\mathcal{B}(\bar{B} \to D^*\tau\bar{\nu})}{\mathcal{B}(\bar{B} \to D^*\ell\bar{\nu})}
\]

(\ell = e \text{ or } \mu)
In our scenario, nonzero contributions to $R_{D(*)}$ are found (via $W$’s and vector LQ). However, they are cancelled out in the degenerated $\rho$ mass limit. ⇒ Only negligible effect remains.
**$R_D(*)$ anomaly**

$$R_D = \frac{\mathcal{B}(\bar{B} \to D \tau \bar{\nu})}{\mathcal{B}(\bar{B} \to D \ell \bar{\nu})} \quad \text{,} \quad R_{D*} = \frac{\mathcal{B}(\bar{B} \to D^* \tau \bar{\nu})}{\mathcal{B}(\bar{B} \to D^* \ell \bar{\nu})}$$

($\ell = e$ or $\mu$)

In our scenario, nonzero contributions to $R_D(*)$ are found (via $W$’s and vector LQ). However, they are cancelled out in the degenerated $\rho$ mass limit.

$\Rightarrow$ Only negligible effect remains.

**A ‘null’ result from LHCb?**

![Diagram of B → D* π⁺ π⁻ (+N) transitions](image)

- $B \to D^* \pi^+ \pi^- (+N)$
- $B^0 \to D^* \tau^+ \nu_\tau$
- $\Delta z > 4\sigma_{\Delta z}$

---

**[HFLAV, arXiv:1612.07233v2]**

- BuBar, PRL109,101802(2012)
- Belle, PRD92,072014(2015)
- LHCb, PRL115,111803(2015)
- Belle, PRD94,072007(2016)
- Belle, PRL118,211801(2017)

**Average**

- $\Delta \chi^2 = 1.0$ contours

**[CERN LHC seminar, 06/06/2017, A.R.Vidal (LHCb)]**

- LHCb muonic
  - PRL 115 (2015) 111803
  - $0.336 \pm 0.027 \pm 0.030$

- LHCb 3-prong
  - LHCb-PAPER-2017-017
  - $0.285 \pm 0.019 \pm 0.029$

- LHCb average
  - $0.306 \pm 0.016 \pm 0.022$

- Fajfer et al. (SM)
  - PRD 85 (2012) 094025
  - $0.252 \pm 0.003$
**typical spectrum**
\( \Lambda_{\text{HC}} \approx 1 \text{ TeV}, \ \Lambda_{\text{UV}} \approx 10^{16} \text{ GeV} \)

\[
M_{\pi^0}^{(1)} \sim \mathcal{O}(f_\pi) = \mathcal{O}(100) \text{ GeV},
\]

\[
M_{\pi^0}^{(1)'} \sim 2 \text{ TeV},
\]

\[
M_{\pi^0}^{(1)} \sim 2 \text{ TeV},
\]

\[
M_{\pi^0}^{(3)} \sim 3 \text{ TeV},
\]

\[
M_{\pi^0}^{(8)} \sim 4 \text{ TeV},
\]

\[M^2_{\pi^{(3),(8)}} \sim C_2 \alpha_s(M_\pi) \Lambda_{\text{HC}}^2 \ln \frac{\Lambda_{\text{UV}}^2}{\Lambda_{\text{HC}}^2}, \text{ with } C_2 = \frac{4}{3} (3) \text{ for color-triplet (octet)}\]

[Matsuzaki & Yamawaki, arXiv:1508.07688]

large mass correction via near-conformal (walking) gauge theory
Misc on pions (skippable)

**typical spectrum**
\( \Lambda_{\text{HC}} \sim 1 \text{ TeV}, \ \Lambda_{\text{UV}} \sim 10^{16} \text{GeV} \)

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\]
\[
M_{\pi^{\pm,3}}^{(1')} \sim 2 \text{ TeV},
\]
\[
M_{\pi^{\pm,3}}^{(1)} \sim 2 \text{ TeV},
\]
\[
M_{\pi^{\pm,3,0}}^{(3)} \sim 3 \text{ TeV},
\]
\[
M_{\pi^{\pm,3,0}}^{(8)} \sim 4 \text{ TeV},
\]

**\((\rho, \pi)\)-interactions**
\( a \equiv m_\rho^2/(g_\rho^2 f_\pi^2) \leftarrow \sim 2 \text{ in vector dominance} \)

\[
\mathcal{L}_{\rho-\pi-\pi} = a g_\rho i \text{tr} \left[[\partial_\mu \pi, \pi] \rho^\mu\right], \quad \leftarrow \text{decay channel of } \rho
\]
\[
\mathcal{L}_{\nu-\pi-\pi} = 2i \left(1 - \frac{a}{2}\right) \text{tr} \left[[\partial_\mu \pi, \pi] \nu^\mu\right], \quad \leftarrow \sim 0
\]
\[
\mathcal{L}_{\nu-\nu-\pi-\pi} = -\text{tr} \left[[\nu_\mu, \pi] [\nu^\mu, \pi]\right], \quad \leftarrow \text{`gg\rightarrow \pi\pi' pair production (evaded)}
\]
\[
\mathcal{L}_{\pi-\pi-\pi-\pi} = -\frac{3}{f_\pi} \text{tr} \left\{[\partial_\mu \pi] [\pi, [\pi, \partial^\mu \pi]]\right\},
\]
Misc on pions (skippable)

Typical spectrum
($\Lambda_{\text{HC}} \sim 1\text{TeV}$, $\Lambda_{\text{UV}} \sim 10^{16}\text{GeV}$)

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M_{\pi^0(1)'} \sim \mathcal{O}(f_\pi) = \mathcal{O}(100) \text{GeV},
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\]

(\(\rho, \pi\))-interactions

\[
a \equiv m_\rho^2 / (g_\rho f_\pi^2)
\]

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\]

Typical pionic decays

- $\rho_{(3)}^0 \to \bar{\pi}_{(3)}^0 \pi_{(1)'}^0 : m_{\pi\pi} \sim (3 + \mathcal{O}(0.1)) \text{TeV},$
- $\rho_{(3)}^\alpha \to \bar{\pi}_{(3)}^\alpha \pi_{(1)'}^0 : m_{\pi\pi} \sim (3 + \mathcal{O}(0.1)) \text{TeV},$
- $\rho_{(8)}^0 \to \bar{\pi}_{(3)}^0 \pi_{(3)}^0 : m_{\pi\pi} \sim (3 + 3) \text{TeV} = 6 \text{TeV},$
- $\rho_{(8)}^\alpha \to \bar{\pi}_{(3)}^\alpha \pi_{(3)}^\alpha : m_{\pi\pi} \sim (3 + 3) \text{TeV} = 6 \text{TeV},$
- $\rho_{(1)'}^0 \to \bar{\pi}_{(3)}^0 \pi_{(3)}^0 : m_{\pi\pi} \sim (3 + 3) \text{TeV} = 6 \text{TeV},$
- $\rho_{(1)'}^\alpha \to \bar{\pi}_{(3)}^\beta \pi_{(3)}^\gamma : m_{\pi\pi} \sim (1 + 2) \text{TeV} = 3 \text{TeV},$
- $\rho_{(1)}^\alpha \to \bar{\pi}_{(1)}^\beta \pi_{(1)}^\gamma : m_{\pi\pi} \sim (1 + 2) \text{TeV} = 3 \text{TeV}.$

For $m_\rho \lesssim 3 \text{TeV}$, $\rho$ decay width is narrow.
Misc on pions (skippable)

**typical spectrum**

(\(\Lambda_{\text{HC}} \approx 1 \, \text{TeV}, \ \Lambda_{\text{UV}} \approx 10^{16} \, \text{GeV}\))

\[
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M_{\pi^{\pm,3}_{(1)}} & \sim 2 \, \text{TeV}, \\
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\end{align*}
\]

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- \(\rho_{(3)}^0 \rightarrow \bar{\pi}_{(3)}^0 \pi_{(1)'}^0: m_{\pi\pi} \sim (3 + \mathcal{O}(0.1)) \, \text{TeV},\)
- \(\rho_{(3)}^\alpha \rightarrow \bar{\pi}_{(3)}^\alpha \pi_{(1)'}^0: m_{\pi\pi} \sim (3 + \mathcal{O}(0.1)) \, \text{TeV},\)
- \(\rho_{(8)}^0 \rightarrow \bar{\pi}_{(3)}^0 \pi_{(3)}^0: m_{\pi\pi} \sim (3 + 3) \, \text{TeV} = 6 \, \text{TeV},\)
- \(\rho_{(8)}^\alpha \rightarrow \bar{\pi}_{(3)}^0 \pi_{(3)}^\alpha: m_{\pi\pi} \sim (3 + 3) \, \text{TeV} = 6 \, \text{TeV},\)
- \(\rho_{(1)'}^0 \rightarrow \bar{\pi}_{(3)}^0 \pi_{(3)}^0: m_{\pi\pi} \sim (3 + 3) \, \text{TeV} = 6 \, \text{TeV},\)
- \(\rho_{(1)'}^\alpha \rightarrow \bar{\pi}_{(1)}^\alpha \pi_{(1)'}^0: m_{\pi\pi} \sim (1 + 2) \, \text{TeV} = 3 \, \text{TeV},\)
- \(\rho_{(1)}^\alpha \rightarrow \bar{\pi}_{(1)}^\beta \pi_{(1)}^\gamma: m_{\pi\pi} \sim (1 + 2) \, \text{TeV} = 3 \, \text{TeV}.\)

If this factor is less than a few, no problem.

**Typical cross section of resonant \(\pi\) production (through WZW anomaly term)**

\[
\sigma(GG \rightarrow \pi^0_{(1)'}, \gamma\gamma) \sim 0.1 \, \text{fb} \times \left( \frac{N_{\text{HC}}}{3} \right)^2 \left[ \frac{\alpha_s}{0.1} \right]^2 \left[ \frac{B(\pi^0_{(1)'}, \gamma\gamma)}{10^{-3}} \right] \left( \frac{M_{\pi^0_{(1)'}}}{f_\pi} \right)^2
\]

\[\checkmark\] typical cross section of resonant \(\pi\) production (through WZW anomaly term)
Composite scenario: QCD as showing example

If a gauge theory is strongly-coupled, composite mesons (and other types) are observed (like QCD below ~1 GeV).

**QCD Lagrangian**

- $\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^0 + \bar{q}_L \gamma^\mu L_{\mu} q_L + \bar{q}_R \gamma^\mu R_{\mu} q_R + \bar{q}_L [S + i\mathcal{P}] q_R + \bar{q}_R [S - i\mathcal{P}] q_L$

  **above $\Lambda_{QCD}$**
  - couplings to external gauge fields ($W^\pm, Z, \gamma$)
  - current mass terms (via the Higgs mechanism)
  - [explicit breaking: $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_{\nu}$]

  **$\mathcal{L}_{QCD}^0 = \bar{q} i \not{\partial} q - \frac{1}{2} \text{tr} [G_{\mu\nu} G^{\mu\nu}]$**

  - pure QCD part

  **$q_{L/R} = \begin{pmatrix} u \\ d \\ s \\ \vdots \end{pmatrix}_{L/R}$**

  - $SU(N_f)_L \times SU(N_f)_R$ global flavor (chiral) symmetry, realized
Composite scenario: QCD as showing example

If a gauge theory is strongly-coupled, composite mesons (and other types) are observed (like QCD below ~1GeV).

**[QCD Lagrangian]**

- \[ \mathcal{L}_{QCD} = \mathcal{L}_{QCD}^0 + \bar{q}_L \gamma^\mu L \mu q_L + \bar{q}_R \gamma^\mu R \mu q_R + \bar{q}_L [S + i\mathcal{P}] q_R + \bar{q}_R [S - i\mathcal{P}] q_L \]

Confined around below

- \[ \mathcal{L}_{QCD}^0 = \frac{1}{2} \text{tr} \left[ G_{\mu\nu} G^{\mu\nu} \right] \] pure QCD part

- SU(\(N_f\))L × SU(\(N_f\))R global flavor (chiral) symmetry, realized

&lt;\(\bar{q}^A q^B\rangle \sim \Lambda_{QCD}^3 \delta^{AB} \text{(confinement)} \rightarrow SU(\(N_f\))L × SU(\(N_f\))R \rightarrow SU(\(N_f\))V spontaneously \rightarrow (\(N_f\))^2-1 #s of (pseudo) NG bosons emerge.\[

[Chiral perturbation theory ⇒ effective description]

Spin-one vector mesons can be described by hidden local symmetry (HLS).

SU(\(N_f\))L × SU(\(N_f\))R ⇒ [SU(\(N_f\))L × SU(\(N_f\))R]_\text{global} × [SU(\(N_f\))V]_\text{gauged} \rightarrow (\(N_f\))^2-1 #s of vector mesons are introduced.
Form of effective Lagrangian

Basic ingredients of chiral perturbation theory (with HLS):

1. \( \xi_R = \left( e^{iP/f_P} \cdot e^{\pm i\pi/f_\pi} \right) \) (non-linear basis of chiral symmetries)
   - would-be NGs
   - pions (NG bosons)
   - for rho mesons
   - (longitudinal d.o.f.s)

2. \( \rho_\mu = \rho_\mu^a T^a \) \( (T^a : SU(8) \text{ generators}) \) (HC rho meson fields)
Form of effective Lagrangian

Basic ingredients of chiral perturbation theory (with HLS):

\[ \xi^R_L = \left( e^{iP_\perp / f_\perp} \cdot e^{\pm i \pi / f_\pi} \right) \] (non-linear basis of chiral symmetries)

\[ \rho_\mu = \rho^a_\mu T^a \] (HC rho meson fields)

Materials for constructing effective Lagrangian:

\[ \rho_{\mu \nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - ig_\rho [\rho_\mu, \rho_\nu], \] [HC rho’s field strength]

\[ \hat{\alpha}_{\perp \mu} = \frac{D_\mu \xi_R \cdot \xi^\dagger_R - D_\mu \xi_L \cdot \xi^\dagger_L}{2i}, \quad \hat{\alpha}_{|| \mu} = \frac{D_\mu \xi_R \cdot \xi^\dagger_R + D_\mu \xi_L \cdot \xi^\dagger_L}{2i} \]

\[ D_\mu \xi_{R(L)} = \partial_\mu \xi_{R(L)} - ig_\rho \rho_\mu \xi_{R(L)} + i \xi_{R(L)} \mathcal{R}_\mu (\mathcal{L}_\mu), \] [(covariantized) Maurer-Cartan one-forms]

[external] SM gauge bosons

[gauge transformations]

\[ \xi_L \rightarrow h(x) \cdot \xi_L \cdot g^\dagger_L (x), \quad \xi_R \rightarrow h(x) \cdot \xi_R \cdot g^\dagger_R (x), \]

\[ \rho_\mu \rightarrow h(x) \cdot \rho_\mu \cdot h^\dagger(x) + \frac{i}{g_\rho} h(x) \cdot \partial_\mu h^\dagger(x), \quad \rho_{\mu \nu} \rightarrow h(x) \cdot \rho_{\mu \nu} \cdot h^\dagger(x), \]

\[ \hat{\alpha}_{\perp \mu} \rightarrow h(x) \cdot \hat{\alpha}_{\perp \mu} \cdot h^\dagger(x), \quad \hat{\alpha}_{|| \mu} \rightarrow h(x) \cdot \hat{\alpha}_{|| \mu} \cdot h^\dagger(x). \]
Form of effective Lagrangian (cont’d)

Effective Lagrangian (lowest terms):

\[ \mathcal{L} = -\frac{1}{2} \text{tr}[\rho_{\mu\nu}^2] + f_\pi^2 \text{tr}[\hat{\alpha}_{\perp\mu}] + \frac{m_\rho^2}{g_\rho^2} \text{tr}[\hat{\alpha}_{||\mu}] + \cdots \]

rhos (‘kinetic’)  pions (‘kinetic’)  rhos (‘mass’)

HC pion decay constant  (typical) HC rho-meson mass scale
Form of effective Lagrangian (cont’d)

Effective Lagrangian (lowest terms):

HC pion decay constant

\[ \mathcal{L} = -\frac{1}{2} \text{tr}[\rho_{\mu \nu}^2] + f_\pi^2 \text{tr}[\hat{\alpha}_{\perp \mu}^2] + \frac{m_\rho^2}{g_\rho^2} \text{tr}[\hat{\alpha}_{|| \mu}^2] + \cdots \]

(Typical) HC rho-meson mass scale

SM gauge bosons  |  HC rho mesons

- \( \hat{\alpha}_{|| \mu} = \gamma_{\mu} - g_\mu \rho_{\mu} - \frac{i}{2 f_\pi^2} [\partial_\mu \pi, \pi] - \frac{i}{f_\pi} [A_\mu, \pi] + \cdots \)

- \( \hat{\alpha}_{\perp \mu} = \frac{\partial_\mu \pi}{f_\pi} + A_\mu - \frac{i}{f_\pi} [\gamma_{\mu}, \pi] - \frac{1}{6 f_\pi^3} [\pi, [\pi, \partial_\mu \pi]] + \cdots \)

\[ \mathcal{L}_\mu = \frac{R_\mu + L_\mu}{2} = \mathcal{L}_\mu^f, \quad A_\mu = \frac{R_\mu - L_\mu}{2} = 0 \]

\( f_L = \begin{pmatrix} q \\ l \end{pmatrix}_L, \quad f_R = \begin{pmatrix} q \\ l \end{pmatrix}_R \)

For SU(2)_W-doublet quarks

\[ [\mathcal{L}_\mu^f]_{8 \times 8} = \begin{pmatrix} 1_{2 \times 2} \otimes g_s G_\mu^a \frac{\lambda^a}{2} + (g_W W_\mu^a + \frac{1}{6} g_Y B_\mu) \otimes 1_{3 \times 3} \\ 0_{2 \times 6} \\ 0_{2 \times 6} \\ g_W W_\mu^a \tau^\alpha - \frac{1}{2} g_Y B_\mu \cdot 1_{2 \times 2} \end{pmatrix} \]

For SU(2)_W-doublet leptons
Form of effective Lagrangian (cont’d)

**Effective Lagrangian (lowest terms):**

- HC pion decay constant

\[ \mathcal{L} = -\frac{1}{2} \text{tr}[\rho^2_{\mu\nu}] + f^2_{\pi} \text{tr}[\hat{\alpha}^2_{\perp\mu}] + \frac{m^2_{\rho}}{g^2_{\rho}} \text{tr}[\hat{\alpha}^4_{||\mu}] + \cdots \]

- (typical) HC rho-meson mass scale

\[ \Psi_L \equiv \xi_L \cdot f_L, \quad \psi_L \equiv \xi_R \cdot f_L \]

\[ \Psi_L \rightarrow h(x) \cdot \Psi_L, \quad \psi_L \rightarrow h(x) \cdot \psi_L \]

- RHSM gauge bosons

- HC rho mesons

- SM gauge bosons

\[ \hat{\alpha}_{||\mu} = \mathcal{V}_\mu - g_{\rho} \rho_{\mu} - \frac{i}{2 f^2_{\pi}} [\partial_\mu \pi, \pi] - \frac{i}{f_{\pi}} [A_\mu, \pi] + \cdots \]

\[ \hat{\alpha}_{\perp\mu} = \frac{\partial_\mu \pi}{f_{\pi}} + A_\mu - \frac{i}{f_{\pi}} [\mathcal{V}_\mu, \pi] - \frac{1}{6 f^3_{\pi}} [\pi, [\pi, \partial_\mu \pi]] + \cdots \]

\[ \mathcal{V}_\mu = \frac{R_\mu + \mathcal{L}_\mu}{2} = \mathcal{L}_\mu^f, \quad A_\mu = \frac{R_\mu - \mathcal{L}_\mu}{2} = 0 \]

\[ [\mathcal{L}_\mu^f]_{8 \times 8} = \begin{pmatrix} 1_{2 \times 2} \otimes g_s G^a_{\mu} \frac{\lambda^a}{2} + (g_W W^a_{\mu} \tau^a + \frac{1}{6} g_Y B_{\mu}) \otimes 1_{3 \times 3} \\ 0_{2 \times 6} \end{pmatrix} \]

\[ g_W W^a_{\mu} \tau^a - \frac{1}{2} g_Y B_{\mu} \cdot 1_{2 \times 2} \]

- for SU(2)\(^W\)-doublet quarks

- for SU(2)\(^W\)-doublet leptons
Form of effective Lagrangian (cont’d)

**Effective Lagrangian (lowest terms):**

- **HC pion decay constant**
  \[ \mathcal{L} = -\frac{1}{2} \text{tr}[\rho_{\mu\nu}^2] + f_\pi^2 \text{tr}[\hat{\alpha}_{\mu\nu}] + \frac{m_\rho^2}{g_\rho^2} \text{tr}[\hat{\alpha}_{||\mu}] \]

- **RH pion mass scale**
  
- **RH rho-meson mass scale**
  
- **SM gauge bosons**
- **HC rho mesons**

\[ \hat{\alpha}_{||\mu} = \mathcal{V}_\mu - g_\mu \rho_\mu - \frac{i}{2} \frac{f_\pi^2}{2} \left[ \partial_\mu \pi, \pi \right] - \frac{i}{f_\pi} \left[ \mathcal{A}_\mu, \pi \right] + \cdots \]

\[ \hat{\alpha}_{\mu\nu} = \frac{\partial_\mu \pi}{f_\pi} + \mathcal{A}_\mu - \frac{i}{f_\pi} \left[ \mathcal{V}_\mu, \pi \right] - \frac{1}{6} \frac{f_\pi^3}{f_\pi} \left[ \pi, \left[ \pi, \partial_\mu \pi \right] \right] + \cdots \]

\[ \mathcal{V}_\mu = \frac{\mathcal{R}_\mu + \mathcal{L}_\mu}{2} = \mathcal{L}_\mu^f, \quad \mathcal{A}_\mu = \frac{\mathcal{R}_\mu - \mathcal{L}_\mu}{2} = 0 \]

\[ \left[ \mathcal{L}_\mu^f \right]_{8 \times 8} = \begin{pmatrix} 1_{2 \times 2} \otimes g_s G_{\mu}^{a} \frac{\lambda_{a}}{2} + \left( g_{W} W_{\mu}^{a} \tau^{a} + \frac{1}{6} g_{Y} B_{\mu} \right) \otimes 1_{3 \times 3} \end{pmatrix} \]

\[ 0_{2 \times 6} \]

\[ g_{L}^{ij} \equiv \left( g_{1L} + 2 g_{2L} + g_{3L} \right)^{ij} \]

**Effective couplings of f_L-f_L-\rho (being gauge-invariant):**

\[ \mathcal{L}_{\rho f f} = g_{1L}^{ij} \left( \bar{\psi}_{L}^{i} \gamma^{\mu} \hat{\alpha}_{||\mu} \psi_{L}^{j} \right) + g_{2L}^{ij} \left( \bar{\psi}_{L}^{i} \gamma^{\mu} \hat{\alpha}_{||\mu} \psi_{L}^{j} + \text{h.c.} \right) + g_{3L}^{ij} \left( \bar{\psi}_{L}^{i} \gamma^{\mu} \hat{\alpha}_{||\mu} \psi_{L}^{j} \right) \]

(undetermined) 3\times3 matrices

(No additional fermion/scalar is required.)
How about $Z'$? [review]

**a straightforward candidate: $Z'$ vector boson**

<table>
<thead>
<tr>
<th><strong>basic pheno. strategy: $U(1)\prime_{L\mu-L_T}$</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>✔ $b_L$ breaking (by a new scalar)</td>
</tr>
<tr>
<td>✔ $s_L$ Yukawa mixing among SM quarks &amp; vector-like ones</td>
</tr>
</tbody>
</table>

**other (anomaly-free) $U(1)\prime$**

e.g., [L.Bian et al., arXiv:1707.04811]

- ✔ $U(1; y(L_\mu-L_T) + x(B_3-L_3))$
  
  $(x,y$: arbitrary values $)$

\[ L_Y = -\bar{u}M_u u - \bar{d}M_d d - \bar{l}M_l l - \bar{\nu}_R M_{\nu_R} \nu_R + \text{h.c.} \]

\[
\begin{align*}
M_u &= \begin{pmatrix}
y_{11}^u & y_{12}^u & 0 \\
y_{21}^u & y_{22}^u & 0 \\
h_{31}^u & h_{32}^u & y_{33}^u
\end{pmatrix}, \\
M_d &= \begin{pmatrix}
y_{11}^d & y_{12}^d & h_{13}^d \\
y_{21}^d & y_{22}^d & h_{23}^d \\
0 & 0 & y_{33}^d
\end{pmatrix}, \\
M_l &= \begin{pmatrix}
y_{11}^l & 0 & 0 \\
0 & y_{22}^l & 0 \\
0 & 0 & y_{33}^l
\end{pmatrix}, \\
M_D &= \begin{pmatrix}
y_{11}^\nu & 0 & 0 \\
0 & y_{22}^\nu & 0 \\
0 & 0 & y_{33}^\nu
\end{pmatrix}, \\
M_R &= \begin{pmatrix}
M_{11} & z_{12}^{(1)} & z_{13}^{(2)} \\
z_{21}^{(1)} & z_{22}^{(2)} & z_{23}^{(3)} \\
z_{31}^{(2)} & z_{32}^{(3)} & 0
\end{pmatrix}.
\]

**Fundamental $Z'$ scenarios work [(with additional fermion(s) and scalar(s)].**
How about Z’? [review]

**a straightforward candidate: Z’ vector boson**

- **basic pheno. strategy:** $U(1)_L'$
  - $b_L$ breaking (by a new scalar)
  - Yukawa mixing among SM quarks & vector-like ones

- **other (anomaly-free) $U(1)_L'$**
  - e.g., $[L. Bian et al., arXiv:1707.04811]$ in $U(1; y(L_\mu-L_T) + x(B_3-L_3))$
  - $(x,y$: arbitrary values $)$

$$
\mathcal{L}_Y = -\bar{u}M_u u - \bar{d}M_dd - \bar{l}M_ll - \bar{\nu}M_D\nu_R - (\nu^c R) eM_R\nu_R + \text{h.c.}
$$

- $M_u = \begin{pmatrix}
y_{11}^u(H_1) & y_{12}^u(H_1) & 0 \\
y_{21}^u(H_1) & y_{22}^u(H_1) & 0 \\
h_{31}^u(H_2) & h_{32}^u(H_2) & y_{33}^u(H_1)
\end{pmatrix}$
- $M_d = \begin{pmatrix}
y_{11}^d(\tilde{H}_1) & y_{12}^d(\tilde{H}_1) & h_{13}^d(\tilde{H}_2) \\
y_{21}^d(\tilde{H}_1) & y_{22}^d(\tilde{H}_1) & h_{23}^d(\tilde{H}_2) \\
0 & 0 & y_{33}^d(\tilde{H}_1)
\end{pmatrix}$
- $M_l = \begin{pmatrix}
y_{11}^l(\tilde{H}_1) & 0 & 0 \\
0 & y_{22}^l(\tilde{H}_1) & 0 \\
0 & 0 & y_{33}^l(\tilde{H}_1)
\end{pmatrix}$
- $M_D = \begin{pmatrix}
y_{11}^\nu(H_1) & 0 & 0 \\
0 & y_{22}^\nu(H_1) & 0 \\
0 & 0 & y_{33}^\nu(H_1)
\end{pmatrix}$
- $M_R = \begin{pmatrix}
M_{11} & z_{12}^{(1)}(\Phi_1) & z_{13}^{(2)}(\Phi_2) \\
z_{21}^{(1)}(\Phi_1) & 0 & z_{23}^{(3)}(\Phi_3) \\
z_{31}^{(2)}(\Phi_2) & z_{32}^{(3)}(\Phi_3) & 0
\end{pmatrix}$

**Fundamental Z’ scenarios work [(with additional fermion(s) and scalar(s)].**

**Q: How about composite case?**