Gauge hierarchy problem in asymptotically safe gravity –the resurgence mechanism

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Based on C. Wetterich and **M.Y.**, Phys. Lett. B 770, 268

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First of all…

The “resurgence” mechanism

differs from

a method to resume perturbative series.

(Misumi san’s talk)

What’s resurgence?

According to “Oxford Dictionary of English”,

an increase or revival after a period of
little activity, popularity

Plan

- Revisit gauge hierarchy problem
- Asymptotically safe gravity
- The resurgence mechanism

Gauge hierarchy problem

- Renormalized Higgs mass

$$\int^{\Lambda} d^4p \, \frac{1}{p^2} \sim \Lambda^2$$

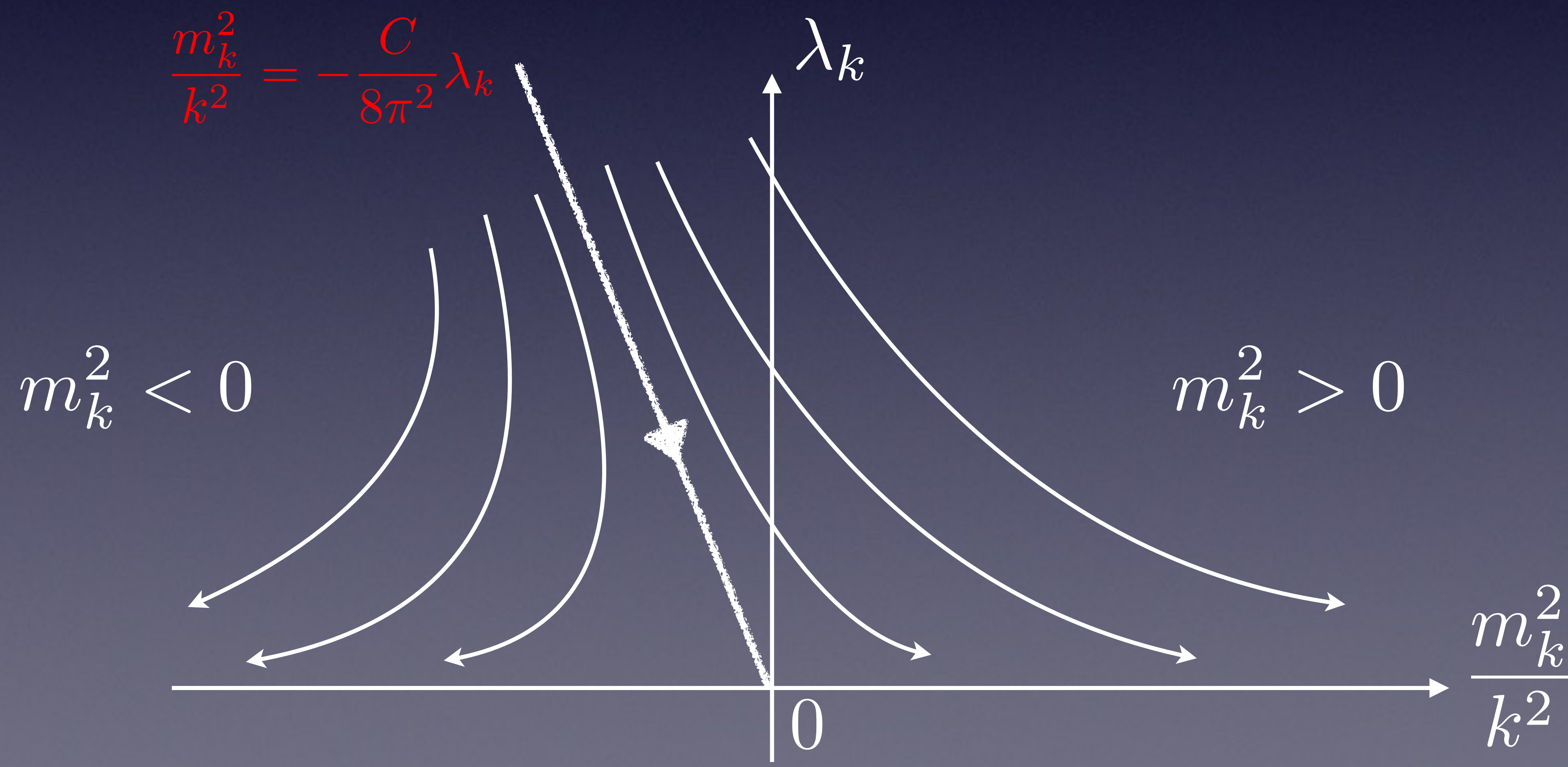
$$m_R^2 = m_\Lambda^2 + \frac{\Lambda^2}{16\pi^2}(\lambda + \cdots)$$

- $O(10^2\ \text{GeV})^2 = -O(10^{19}\ \text{GeV})^2 + O(10^{19}\ \text{GeV})^2$

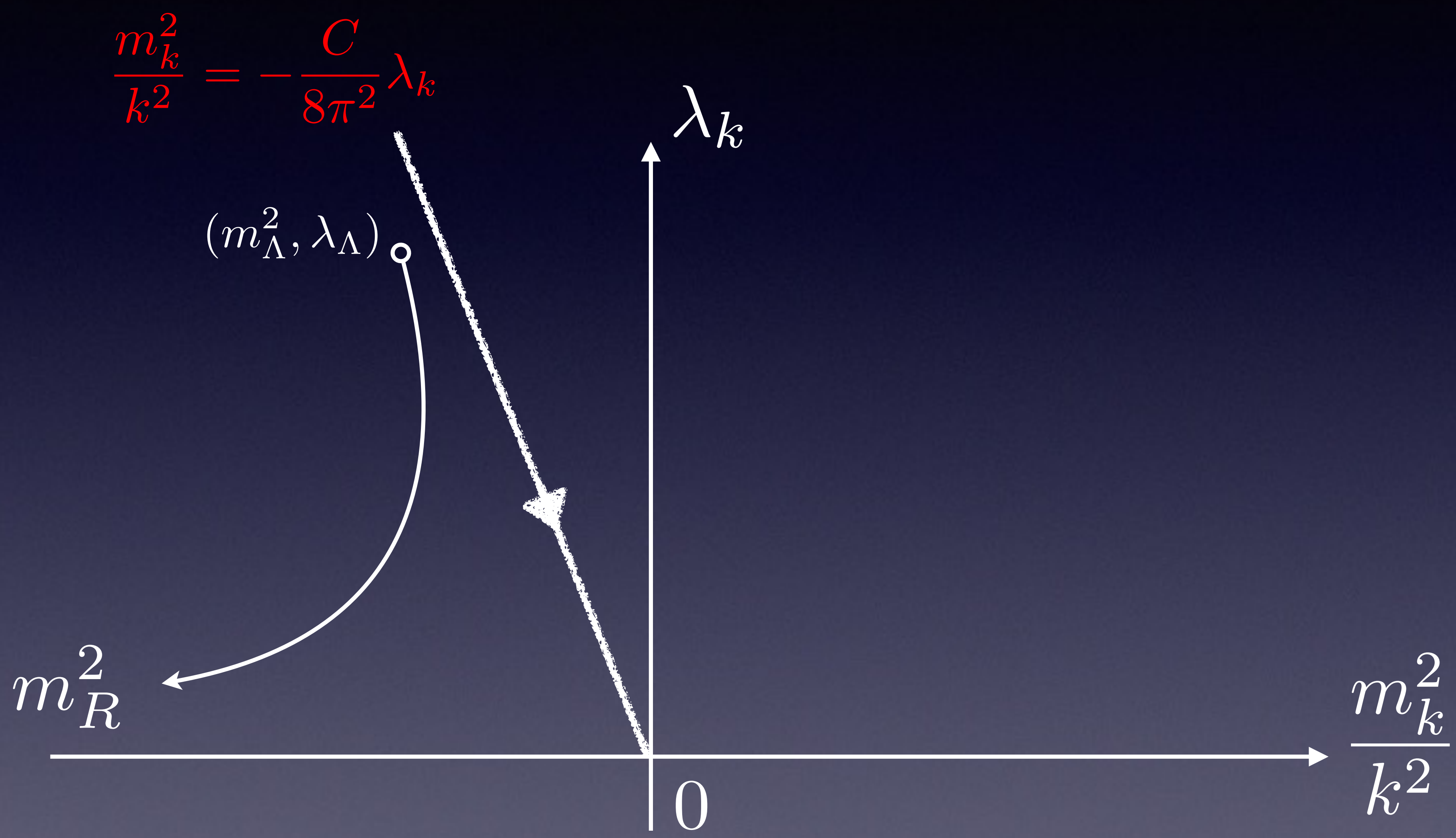
$$m_R^2 \ll m_\Lambda^2$$

In viewpoint of Wilson RG

$$\Gamma_k = \int d^4x \left[\frac{1}{2}(\partial_\mu \phi)^2 - \frac{m_k^2}{2}\phi^2 - \frac{\lambda_k}{4}\phi^4 \right]$$



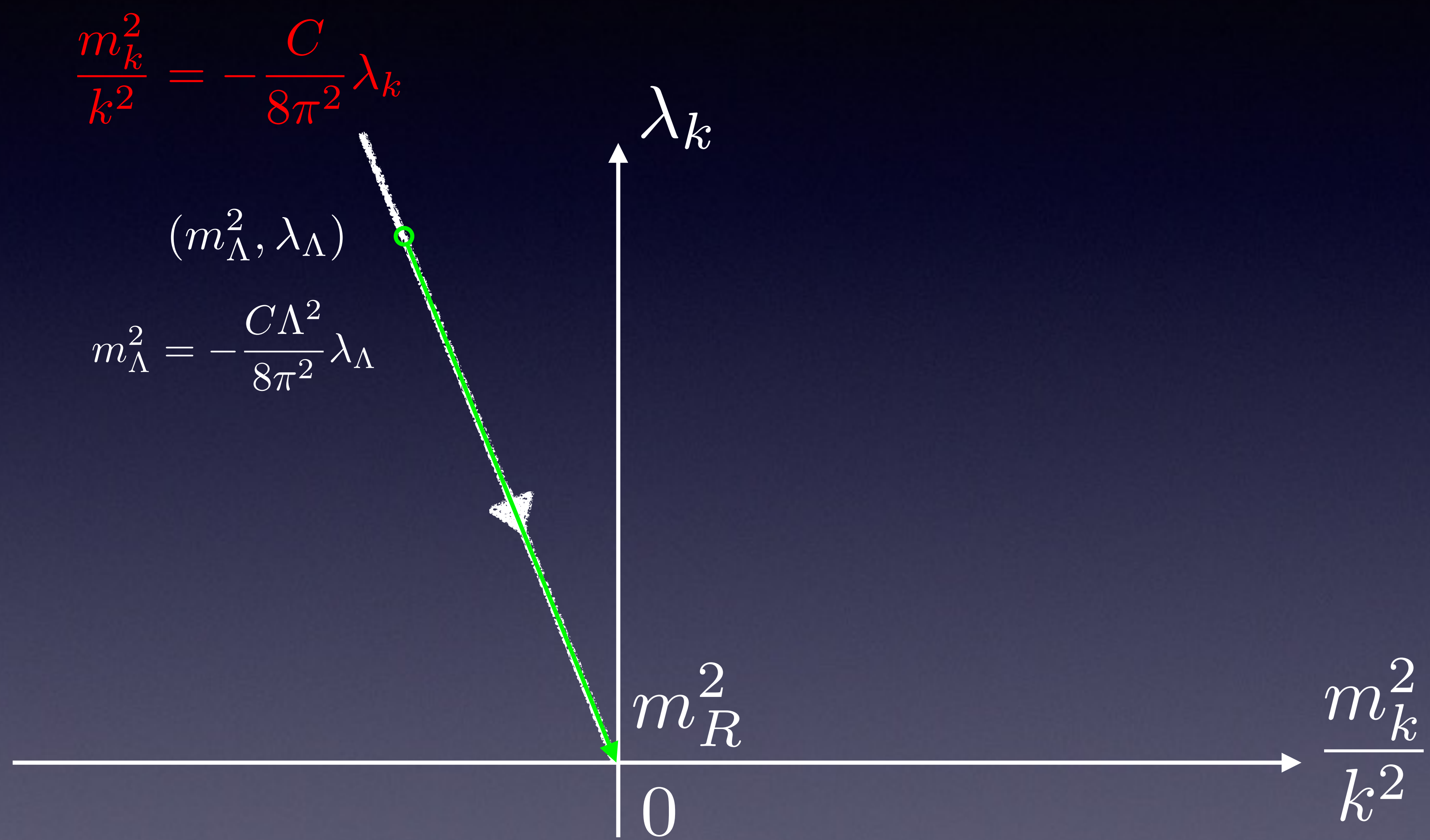
In viewpoint of Wilson RG



$$m_R^2 = m_\Lambda^2 + \frac{C\Lambda^2}{8\pi^2}\lambda_{k=0}$$

$$\lambda_\Lambda \simeq \lambda_{k=0}$$

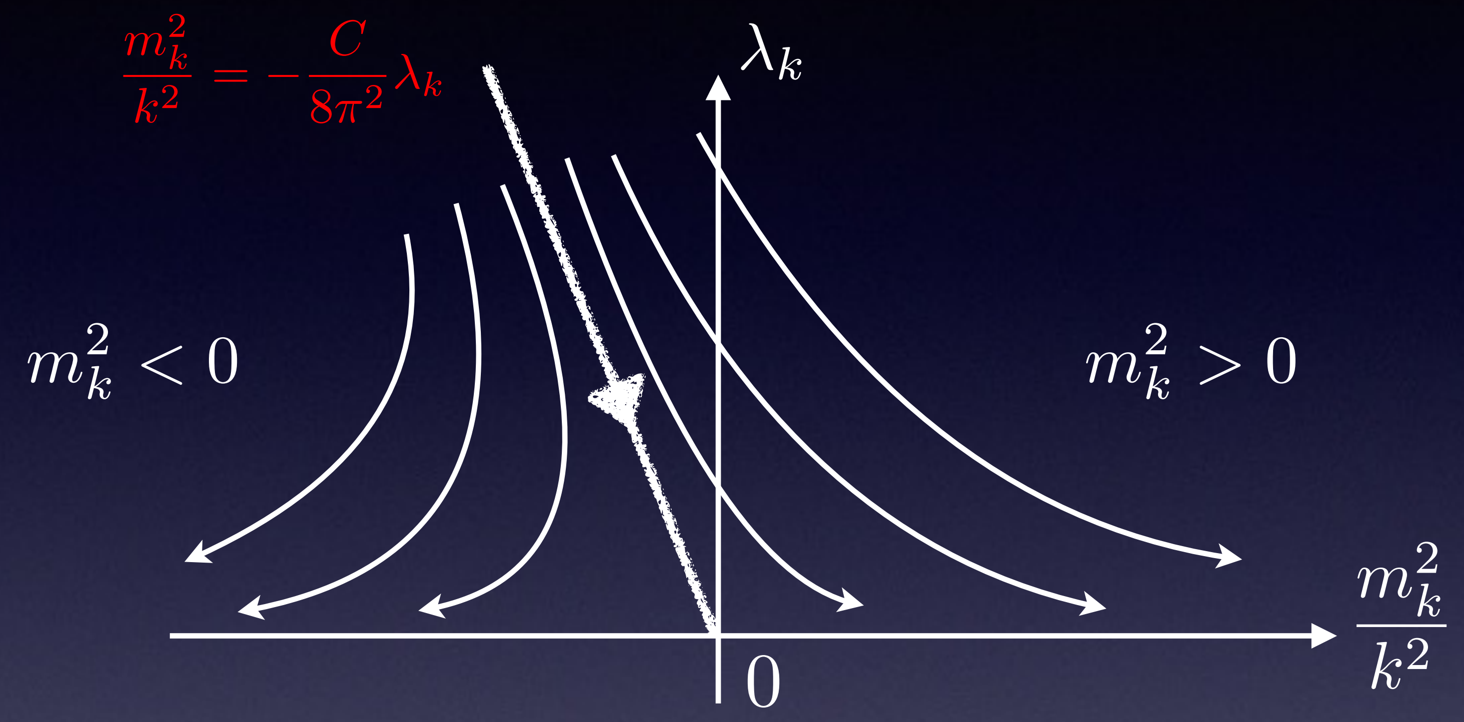
In viewpoint of Wilson RG



$$m_R^2 = m_\Lambda^2 + \frac{C\Lambda^2}{8\pi^2}\lambda_{k=0} = 0$$

$$\lambda_\Lambda \simeq \lambda_{k=0}$$

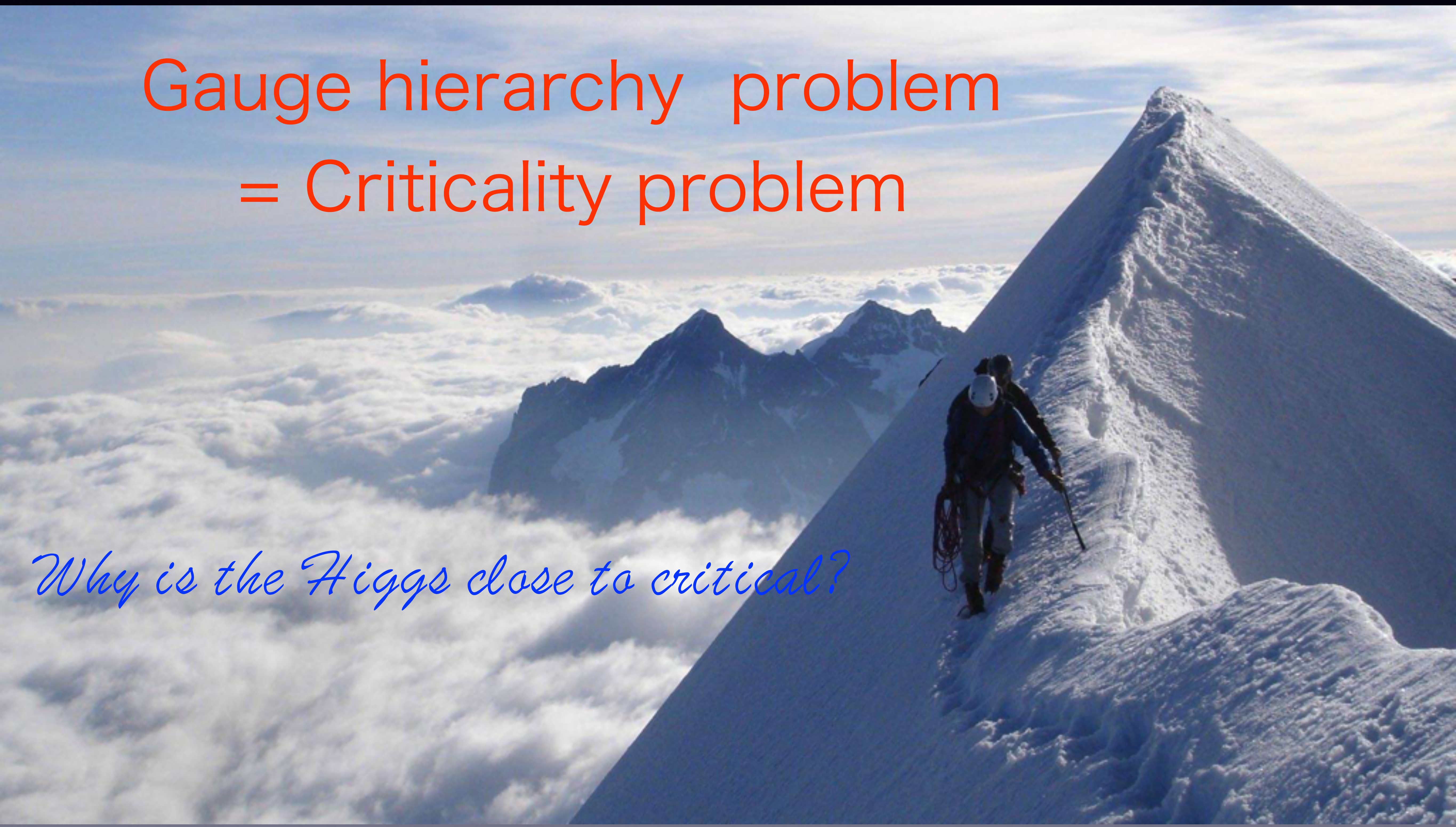
In viewpoint of Wilson RG

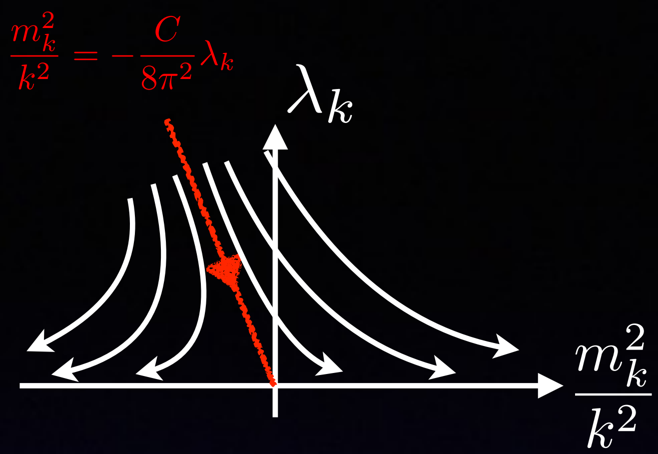


- Λ^2 determines the position of phase boundary (critical line).
- The phase boundary corresponds to the massless (critical) theory.
- To obtain small m_R, put the bare parameters close to the phase boundary.

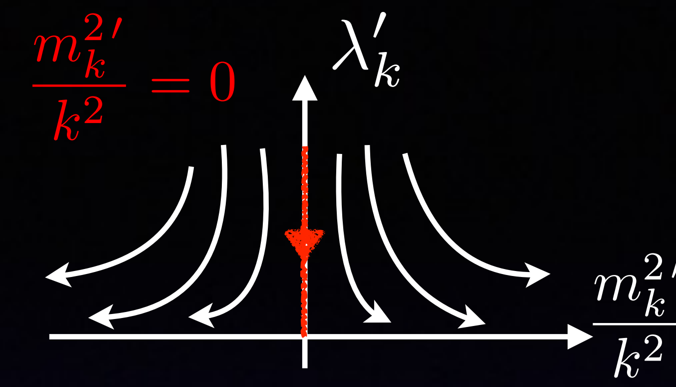
In viewpoint of Wilson RG

Picture from web





Discussion



W. A. Bardeen, FERMILAB-CONF-95-391-T
H. Aoki, S, Iso, Phys. Rev. D86, 013001

- Λ^2 is spurious?
  - The **position** of phase boundary is physically meaningless.
- The distance between the flow and the boundary is physically meaningful.
- In perturbation theory, Λ^2 is always subtracted by the counter term or dimensional regularization.

K-I Aoki et. al., hep-th/0002231

  - Rotation of coordinate. ➡ $C = 0$

$$\frac{m_k^2}{k^2} = -\frac{C}{8\pi^2}\lambda_k \quad \rightarrow \quad \frac{m_k^{2\prime}}{k^2} = 0$$

  - Choice of renormalization scheme ⇔ Choice of coordinate of theory space
- Gauge hierarchy problem ⇄ The bare theory of Higgs is almost scale (conformally) invariant.

Gauge hierarchy problem

- RG equation for dimensionless scalar mass (deviation from phase boundary)

$$\bar{m}^2 = \frac{m^2}{k^2}$$

$$-k\frac{d\bar{m}^2}{dk} = \left(2 - \frac{\lambda}{16\pi^2}\right)\bar{m}^2 =: \gamma_m \bar{m}^2$$

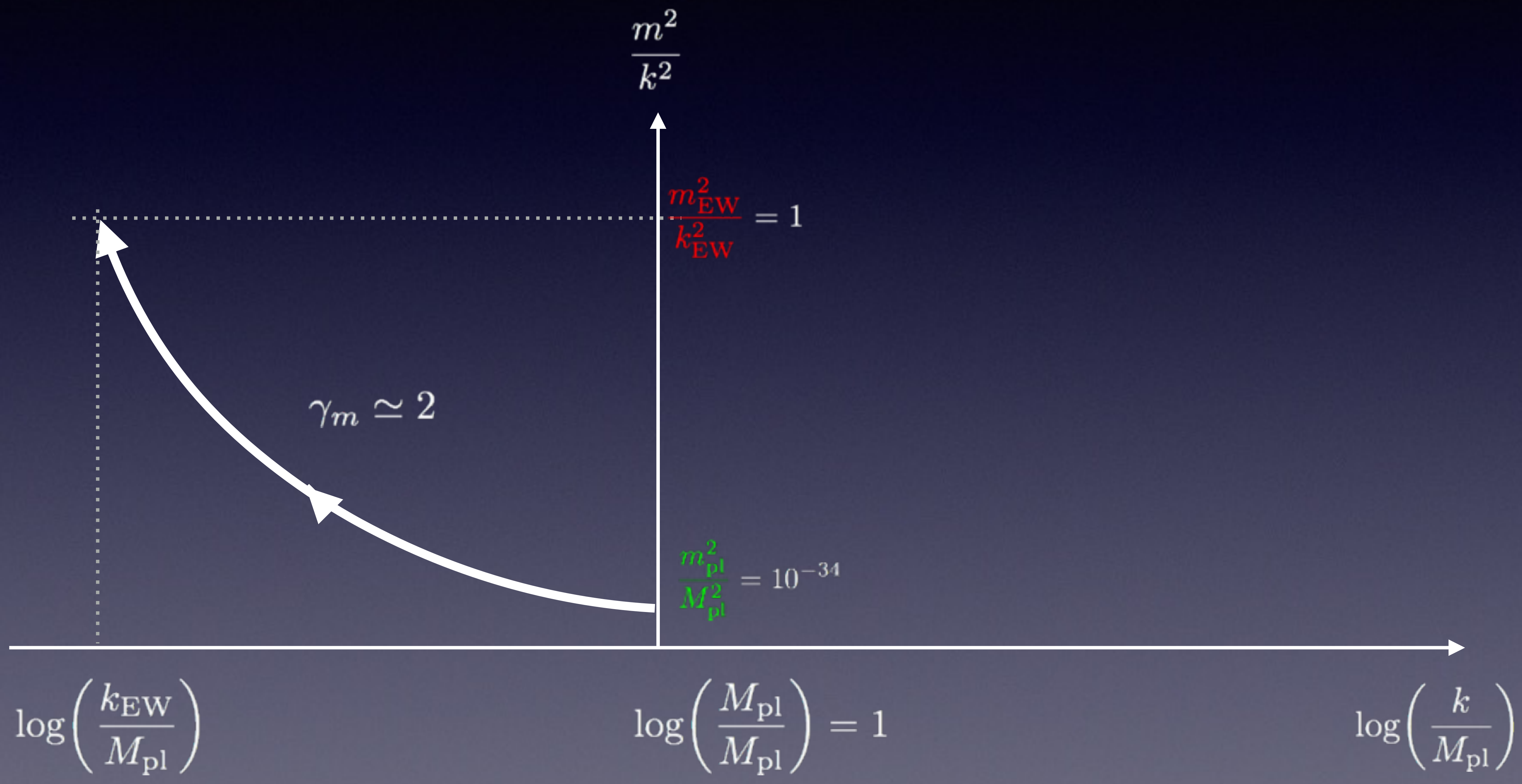
canonical dim. anomalous dim. effective dim.

$$\bar{m}^2 = \bar{m}_0^2 \left(\frac{k}{\Lambda}\right)^{\gamma_m}$$

From observations

$\gamma_m \simeq 2$ $\quad \bar{m}^2 = \dfrac{m_{\rm EW}^2}{k_{\rm EW}^2} \simeq 1$ → $\bar{m}_0^2 = \dfrac{m_{\rm pl}^2}{M_{\rm pl}^2} \simeq 10^{-34}$ $\quad \Lambda = M_{\rm pl}$

RG flow of scalar mass



Comment

- If m_0=0, m_R=0 is realized.
  - The theory exactly on the phase boundary.
- Idea of **classical** scale (conformal) invariance
  - How to generate the EW scale? (scalegenesis)
  - Dimensional transmutation or Dynamical symmetry breaking with TeV scale. (Next Matsuzaki san's talk)

Summary so far

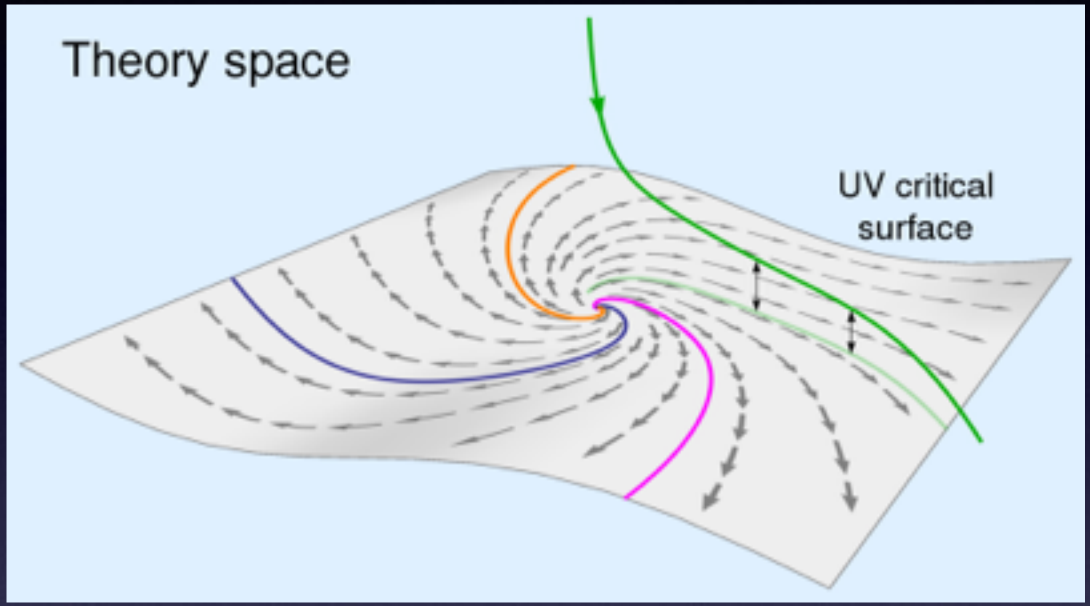
- Gauge hierarchy problem is criticality problem.
    - Why is Higgs close to the phase boundary?
- Λ^2 may be not physically meaningful.
    - The deviation from the phase boundary is physical.
- The positive canonical dimension 2 causes gauge hierarchy problem.

Plan

- Revisit gauge hierarchy problem
- Asymptotically safe gravity
- The resurgence mechanism

Asymptotic safety

Picture from Wiki



- Suggested by S. Weinberg

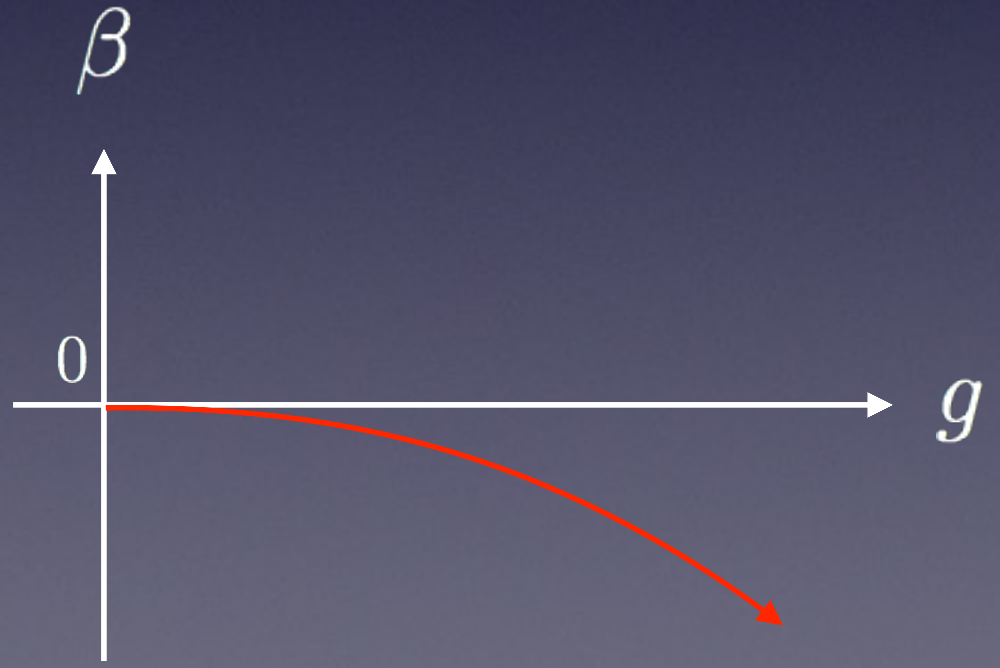
  S. Weinberg, Chap 16 in General Relativity

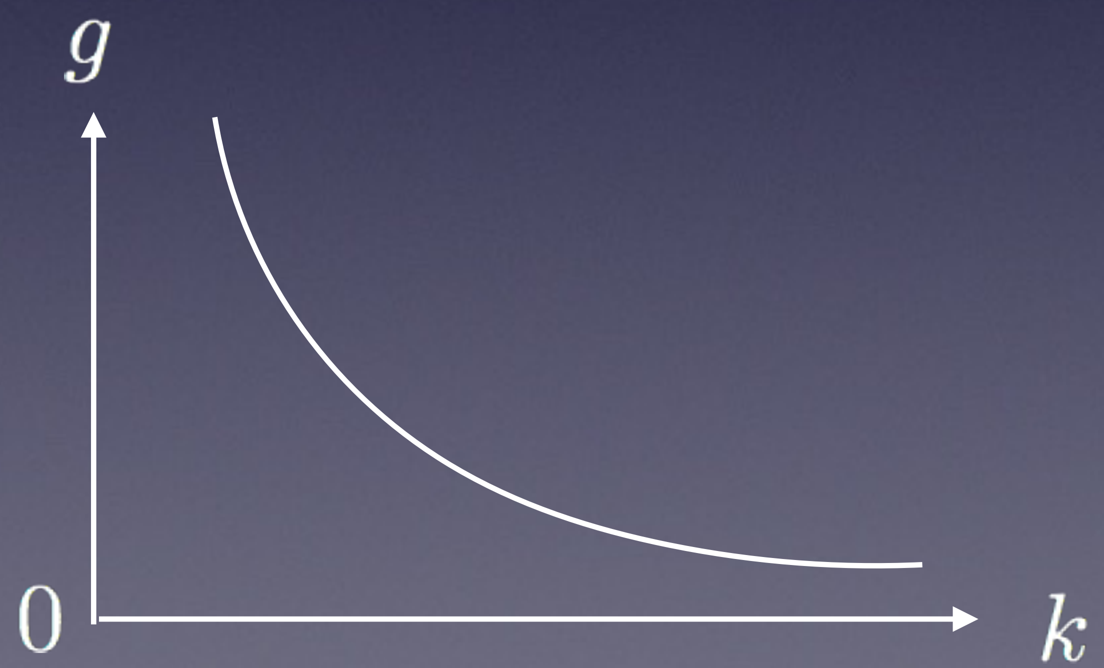
- Existence of UV fixed point
  - Continuum limit $k \to \infty$.
  - UV critical surface (UV complete theory) is defined by relevant operators.
  - Dimension of UV critical surface = number of free parameters.
- Generalization of asymptotic free

Asymptotic safety

- Asymptotic free

$$\partial_t g = -\beta_0 g^3, \quad \beta_0 > 0$$

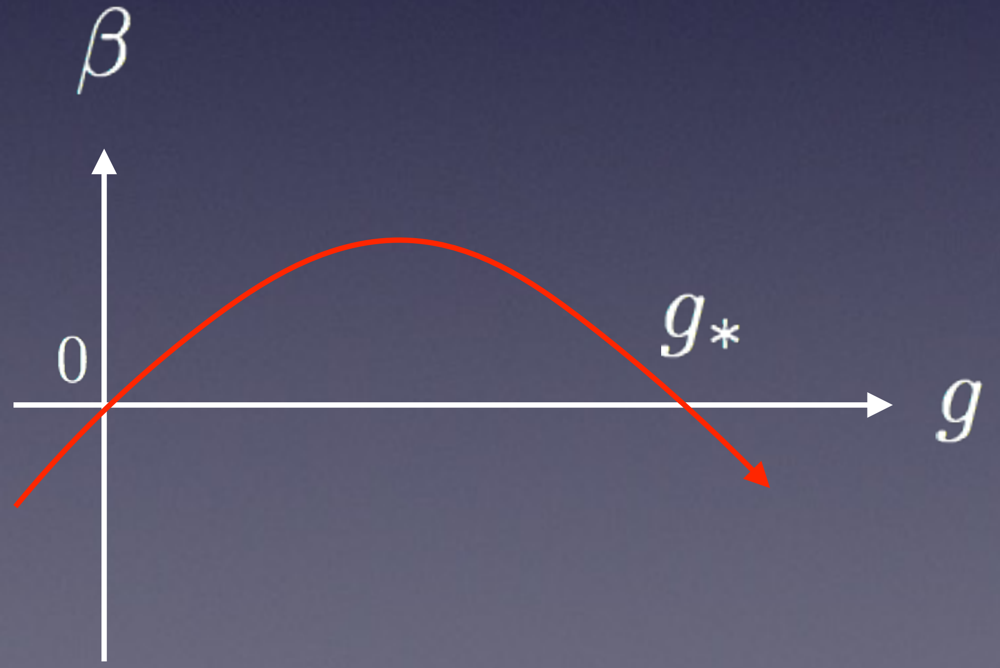


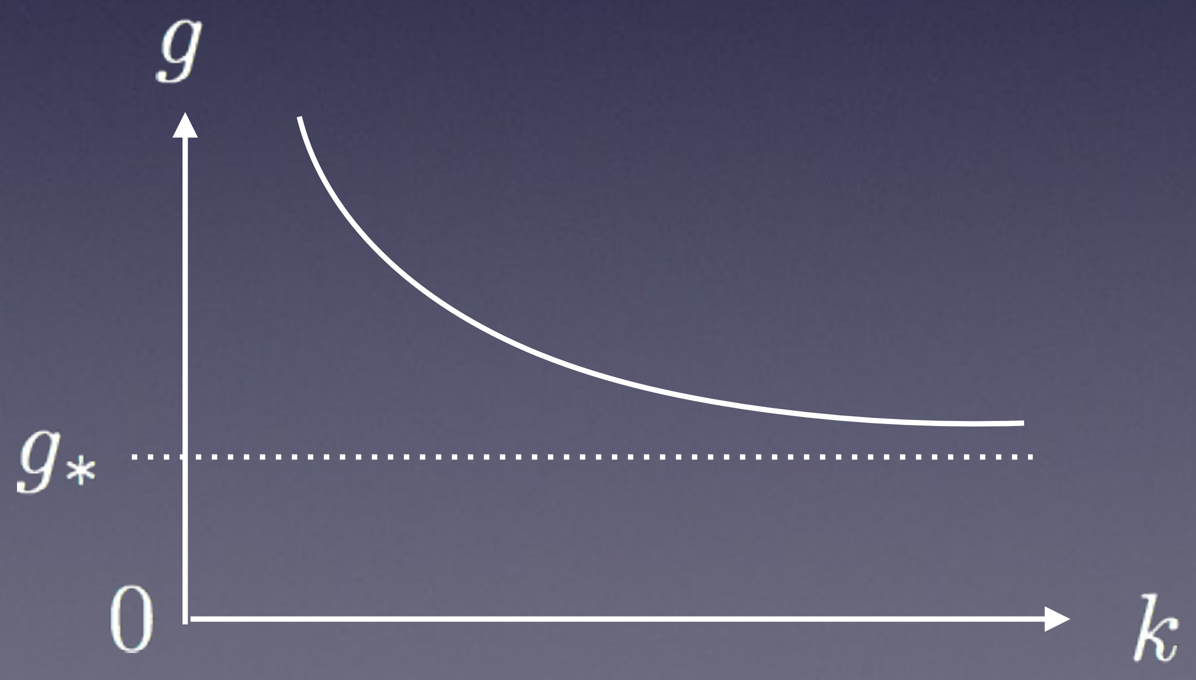


Asymptotic safety

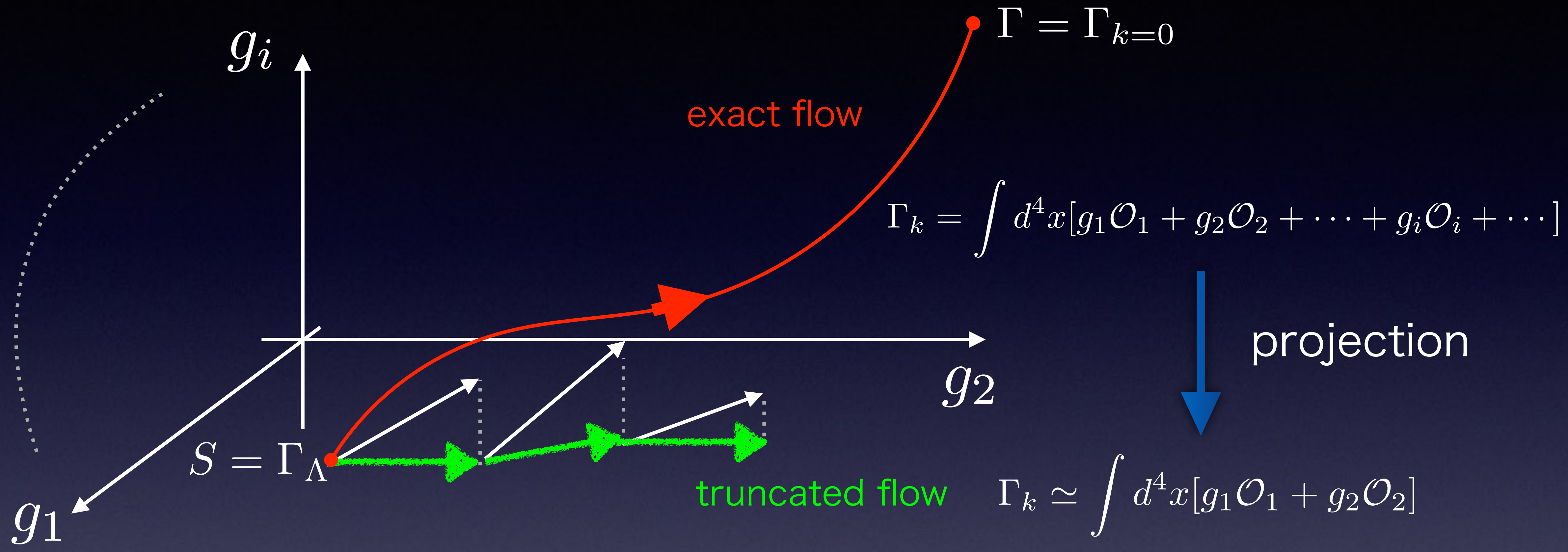
- Asymptotic safety $\quad g = G_N k^2$

$$\partial_t g = 2g - \beta_0 g^2, \quad \beta_0 > 0$$





Functional renormalization group



Wetterich equation

$$k\partial_k\Gamma_k = \frac{1}{2}\mathrm{Str}[(\Gamma_k^{(2)} + R_k)^{-1}k\partial_k R_k]$$

Critical exponent

- Classification of flow around FP

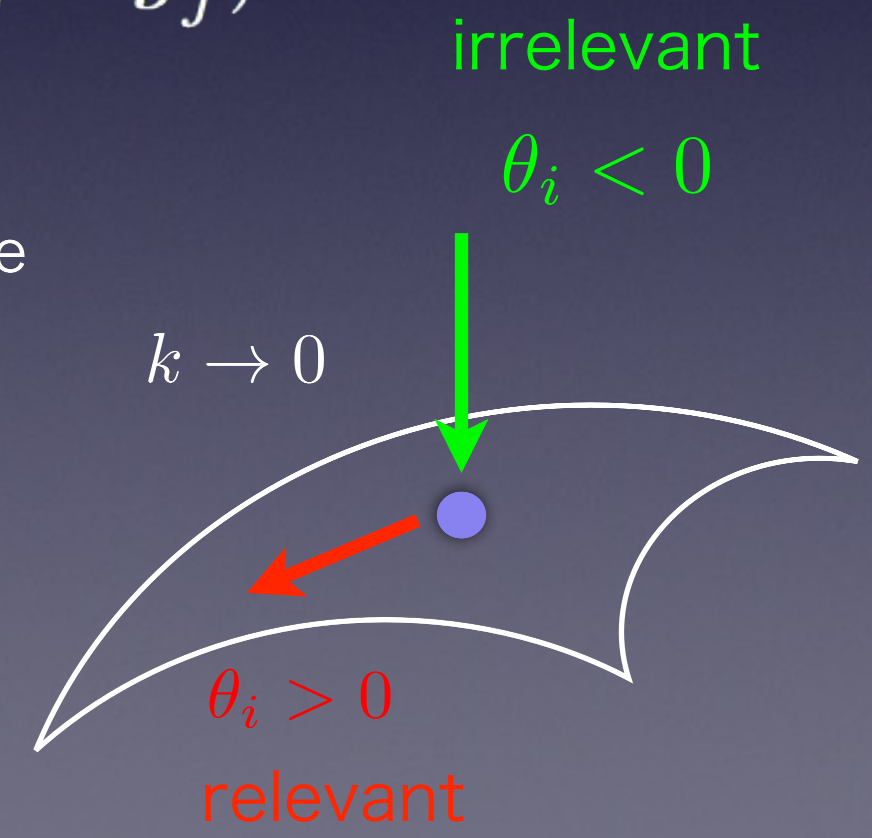
- RG eq. around FP g^*

$$-k\frac{dg_i}{dk} = \cancel{\beta_i(g^*)} + \left.\frac{\partial \beta_i}{\partial g_j}\right|_{g=g^*}(g_j - g_j^*) + \cdots$$

eigenvalue

- Solution of RG eq.

$$g_i(k) = g_i^* + \sum_j^N \zeta_j^i \left(\frac{k}{\Lambda}\right)^{-\boldsymbol{\theta_j}}$$



Earlier studies

Cf.
O. Lauscher, M. Reuter, Phys. Rev. D66, 025016
K. Falls, et. al., arXiv: 1301.4191
D. Benedeti, et. al,. Mod. Phys. Lett. A24, 2233

- Pure gravity
  - Truncation: $f(R),\ \alpha R + \beta R^2 + \gamma R_{\mu\nu} R^{\mu\nu},\ \text{etc.}$
  - There exists a UV fixed point in gravity sector
  - Number of relevant operators: 3 ⇔ 3 free parameters
- With matters
  - stability of FP
  - scalar-gravity system
  - Higgs-Yukawa system
  - gauge field system
  - Fermionic system

Cf.
R. Percacci, D. Perini, Phys. Rev. D67, 081503
P. Dona, et. al., Phys. Rev. D89, 084035
J. Meibohm, et. al., Phys.Rev. D93, 084035
R. Percacci, D. Perini, Phys. Rev. D68, 044018
G. Narain, R. Percacci, CQG 27, 075001
R. Percalli et. al, Phys. Lett. B689, 90
G. P. Vacca, O. Zanusso, Rhys. Rev. Lett. 105, 231601
J. Daum et. al., JHEP 01, 084
U. Harst, M. Reuter, JHEP 05, 119
A. Eichhorn, H. Gies, New Phys. J., 113, 125012
A. Eichhorn, Phys. Rev. D86, 105021
etc.

Asymptotic safety of gravity and the Higgs boson mass

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Top mass from asymptotic safety

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An asymptotically safe solution to the U(1) triviality problem

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Graviton fluctuations erase the cosmological constant

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Graviton fluctuations induce strong non-perturbative infrared renormalization effects for the cosmological constant. The functional renormalization flow drives a positive cosmological constant towards zero, solving the cosmological constant problem without the need to tune parameters. We propose a simple computation of the graviton contribution to the flow of the effective potential for scalar fields. Within variable gravity we find that the potential increases asymptotically at most quadratically with the scalar field. With effective Planck mass proportional to the scalar field, the solutions of the derived cosmological equations lead to an asymptotically vanishing cosmological "constant" in the infinite future, providing for dynamical dark energy in the present cosmological epoch. Beyond a solution of the cosmological constant problem, our simplified computation also entails a sizeable positive graviton-induced anomalous dimension for the quartic Higgs coupling in the ultraviolet regime, substantiating the successful prediction of the Higgs boson mass within the asymptotic safety scenario for quantum gravity.

Plan

- Revisit gauge hierarchy problem
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Corrections to scalar mass

- RG equation for scalar mass

$$-k\frac{d\bar{m}^2}{dk} = \gamma_m \bar{m}^2$$

- Contributions from graviton fluctuation

$$\gamma_m = 2 - \frac{\lambda}{16\pi^2} - \frac{3g}{\pi(1-2\Lambda_{cc})^2} + \cdots$$

Quantum Scaling of scalar mass

- Critical exponent (effective dimension)

$$\theta_m = \frac{\partial \beta_m}{\partial m^2}\Big|_{g=g^*} = \gamma_m\Big|_{g=g^*} = 2 - \frac{\lambda^*}{16\pi^2} - \frac{3g^*}{\pi(1-2\Lambda_{\rm cc}^*)^2}$$

- At Gaussian-matter fixed point:

$$m^{*2} = 0,\ \lambda^* = 0,\ g^* \neq 0,\ \Lambda_{\rm cc}^* \neq 0$$

$$\theta_m = 2 - \frac{3g^*}{\pi(1-2\Lambda_{\rm cc}^*)^2}$$

Quantum Scaling of scalar mass

- Critical exponent

$$\theta_m = 2 - \frac{3g^*}{\pi(1 - 2\Lambda^*_{\rm cc})^2}$$

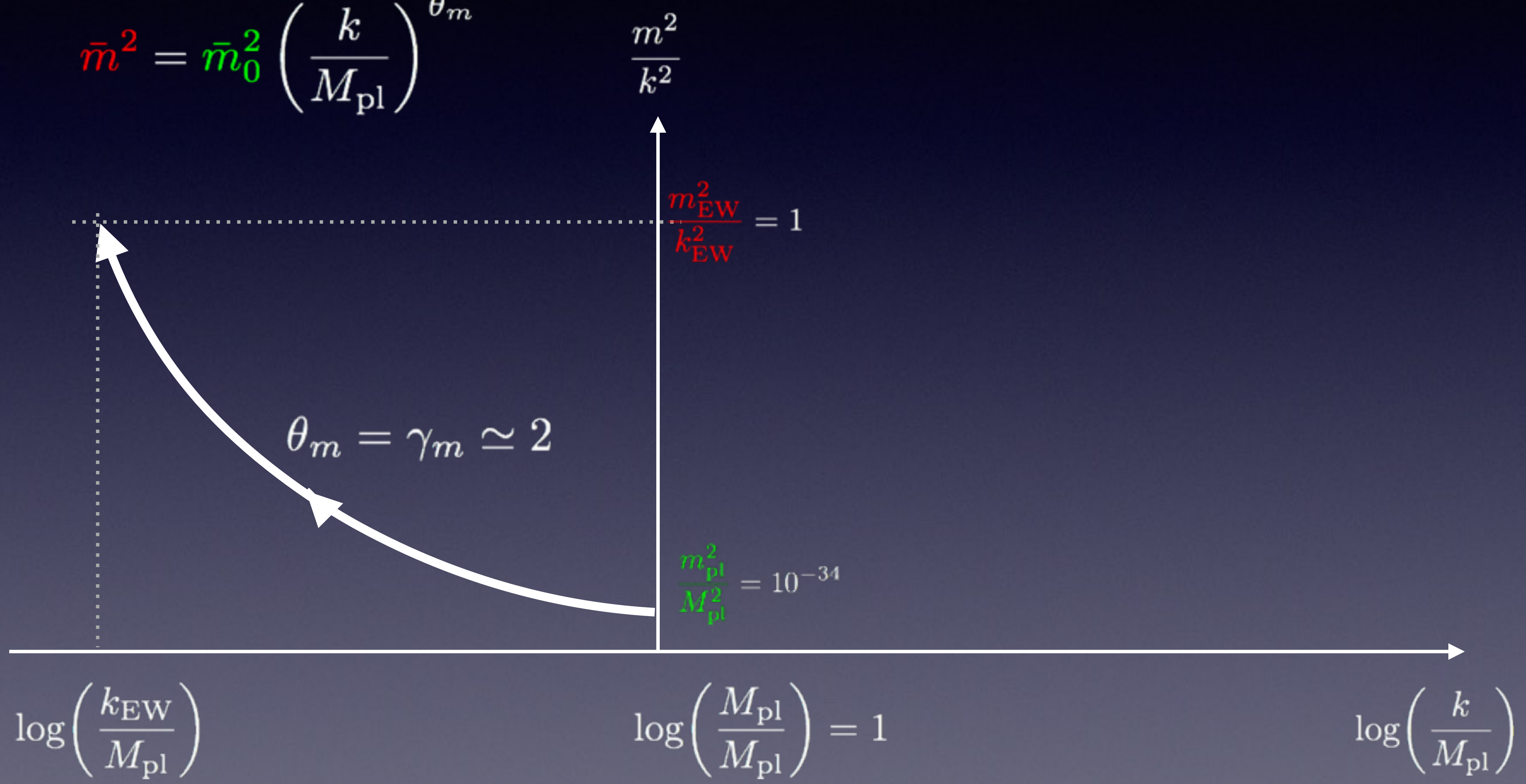
- If we have a large anomalous dimension

$$2 < \frac{3g^*}{\pi(1 - 2\Lambda^*_{\rm cc})^2}$$

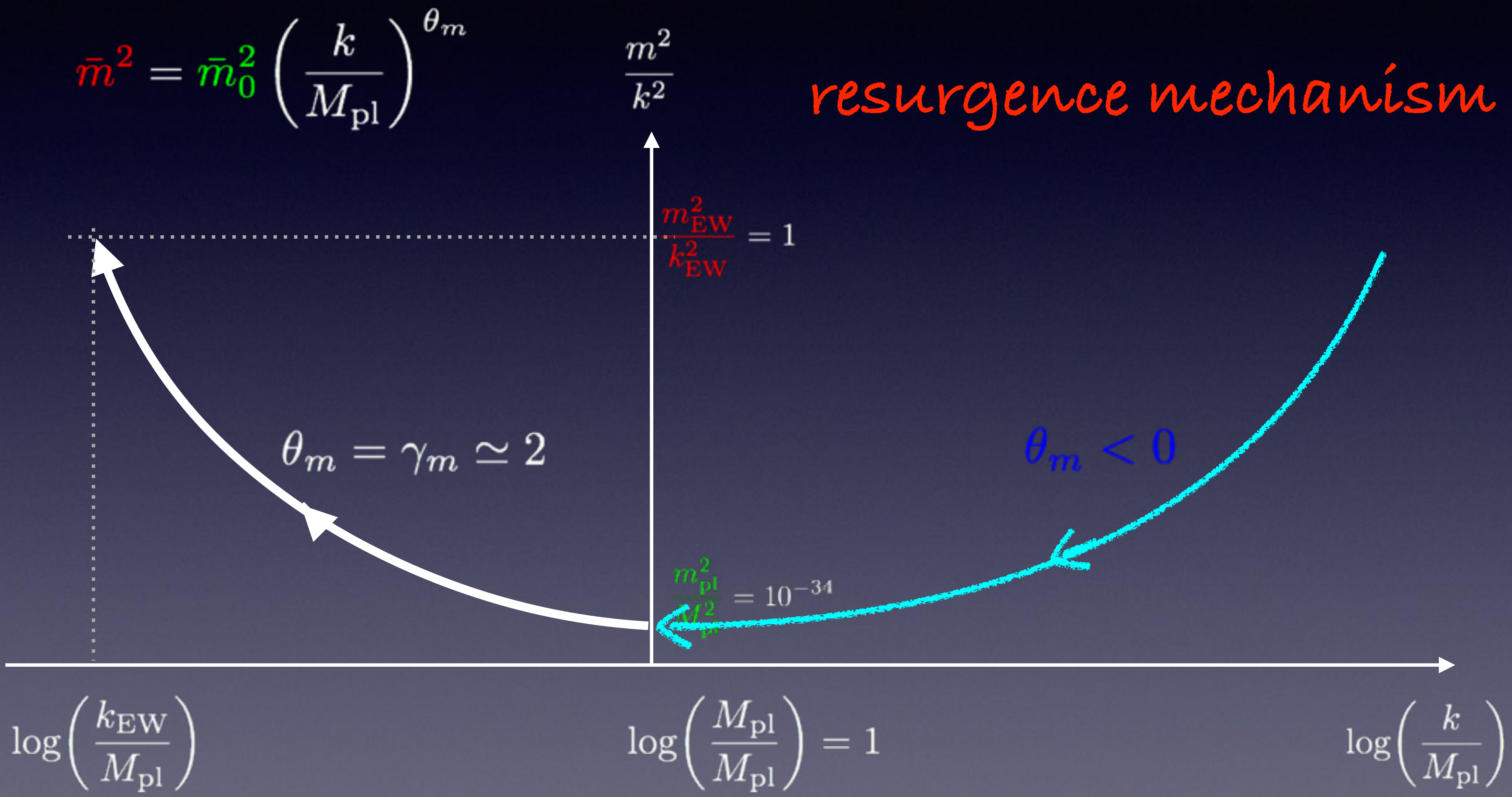
$$\theta_m = \gamma_m < 0 \qquad \bar{m}^2 = \bar{m}_0^2 \left(\frac{k}{\Lambda}\right)^{\gamma_m}$$

RG flow of scalar mass

$$\bar{m}^2 = \bar{m}_0^2 \left(\frac{k}{M_{\rm pl}}\right)^{\theta_m}$$



RG flow of scalar mass



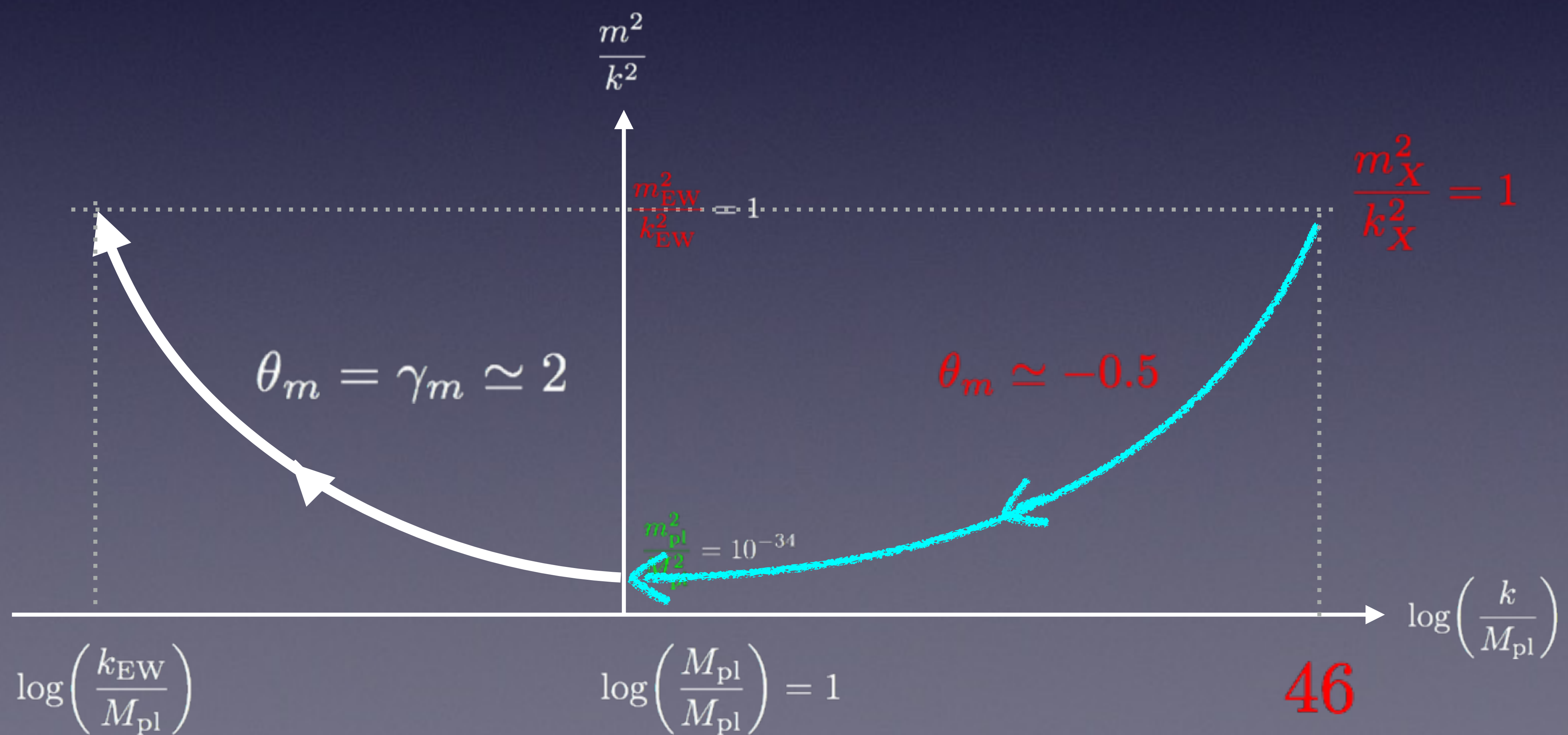
self-tune criticality

Possibility of $\theta < 0$

- Higgs-Yukawa model

K. oda and **M.Y.**, Class.Quant.Grav. 33 (2016) no.12, 125011

- We find $\theta_m \simeq -0.5$ which yields $k_X \simeq 10^{46} M_{\rm pl}$.



Comment on “Classical” scale invariance

- Gaussian-matter fixed point: Matter’s FP is trivial.
- In continuum limit k→∞, matter coupling constants converge to the trivial fixed point.
- The scalar couplings keep vanishing up to below the Planck scale.
- The “classical” scale invariance is realized at the Planck scale. In particular, the flatland is realized.
- What is the origin of the EW scale?

Summary

- Gauge hierarchy problem is criticality problem.
- Asymptotically safe gravity is one of strong candidates of quantum gravity.
- Graviton fluctuation could induce large anomalous dimension.
- Gauge hierarchy problem could be solved by the resurgence mechanism.
- We need new physics.

Prospects

- Dependence of critical exponent of scalar mass on number of particles. Working with J. M. Pawlowski, M. Reichert, C. Wetterich
- Higgs portal dark matter Working with A. Eichhorn, Y. Hamada, J. Lumma
- Neutrino mass (Majorana mass) Working with A. Eichhorn, A. Held. S. Lippold, Y. Hamada
- Strong CP problem?
- Weak gravity conjecture Working with T. Denz, A. Eichhorn, S. Lippold, J. M. Pawlowski, M. Reichert, F. Versteegen
- Relation with string theory