Phase Transitions in Twin Higgs Models

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Contents

- Naturalness of the Higgs mass
- Twin Higgs Models
- Phase Transitions in Twin Higgs Models (Cosmological impact)

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- Naturalness of the Higgs mass
- Twin Higgs Models
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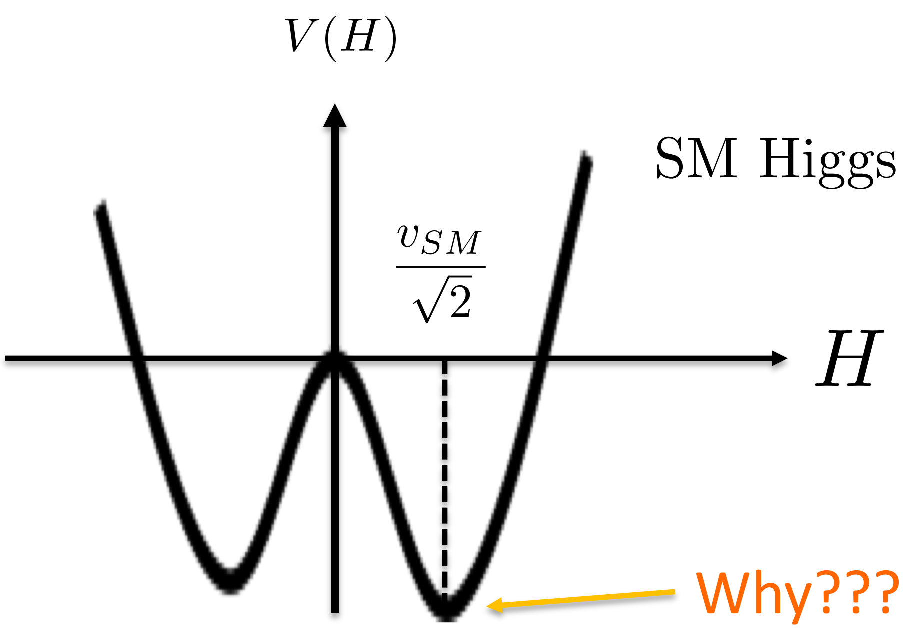
Standard Model is incomplete

SM describes phenomenology around the electroweak scale

However, there are problems…

Dynamics of Electroweak Symmetry Breaking

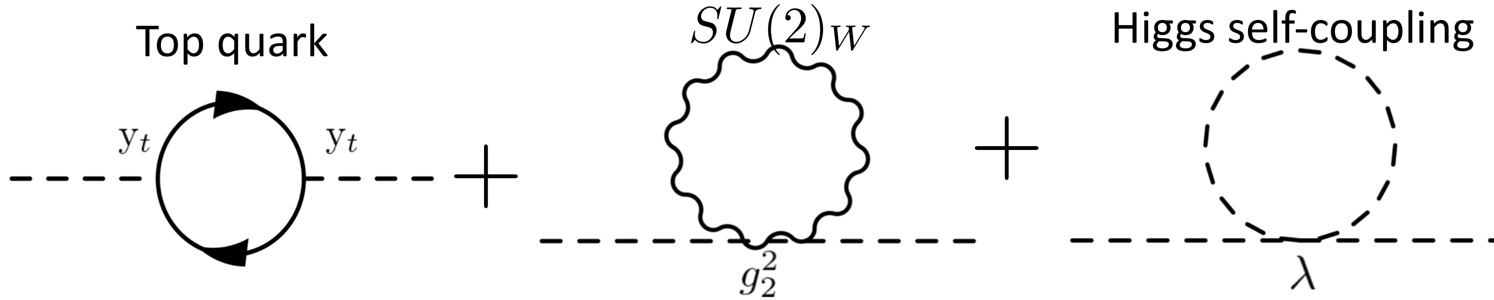


SM Higgs Potential: $V(\phi) = m^2|H|^2 + \lambda_{SM}|H|^4$

$$\boxed{m_R^2 = \underbrace{m_{bare}^2}_{\mathcal{O}(M_{pl}^2)} + \underbrace{\delta m^2}_{\mathcal{O}(M_{pl}^2)}}$$

Naturalness of the Higgs mass

$$\delta m_h^2 =$$



$$-\frac{3y_t^2}{4\pi^2}\Lambda^2 \qquad +\frac{9g_2^2}{32\pi^2}\Lambda^2 \qquad +\frac{\lambda}{4\pi^2}\Lambda^2$$

Λ : cut − off scale

The measure of Fine-Tuning: $\Delta \equiv \dfrac{(m_h^R)^2}{\delta m_h^2}$

For example, $\Delta < 10^{-2}$ $\qquad (m_h^R)^2 = (m_h^{bare})^2 + \delta m_h^2$

15625 = 98715625 - 98700000

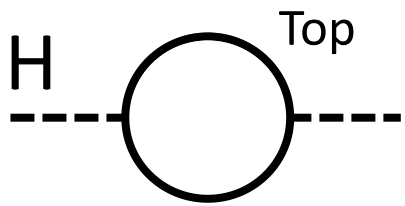
Large Hierarchy Problem

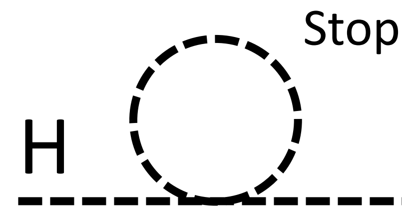
Unnatural Cancellation!

(1% tuning is needed)

SUPERSYMMETRY

SUSY provides an excellent solution to Hierarchy Problem

$\delta m^2 =$ 

$+$ 

$$\Lambda^2 \qquad\qquad -\Lambda^2$$

$$\simeq \frac{3y_t^2}{8\pi^2} m_{\mathrm{stop}}^2 \log\left(\frac{\Lambda^2}{m_{\mathrm{stop}}^2}\right)$$

Quadratic divergence is cancelled by Top partner (Stop).
(SUSY protects quadratic divergence mass corrections.)

Soft SUSY-breaking mass is important for fine-tuning.

Scalartop is a colored state ➡ Strong Bounds

Soft Mass must be Heavy

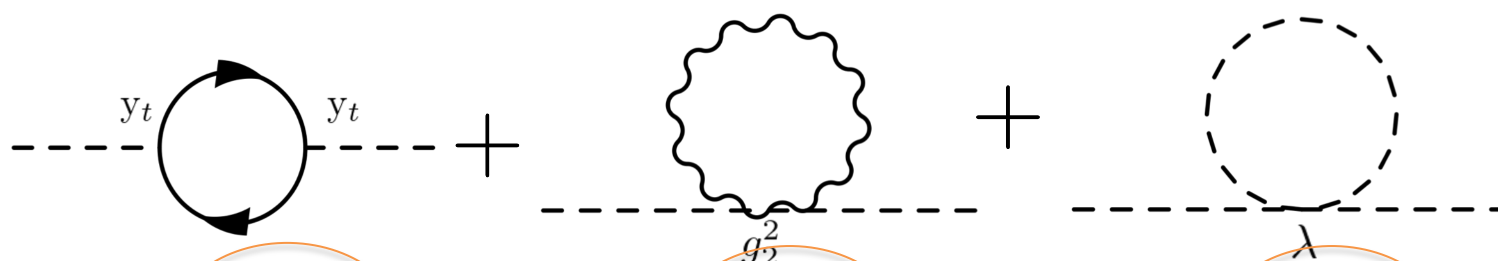
$M_{soft} >> \mathcal{O}(1)\mathrm{TeV}$

Little Hierarchy Problem!

$\Delta < 0.01$

Hierarchy Problem

$\delta m_h^2 =$



$$-\frac{3y_t^2}{4\pi^2}\Lambda^2 \qquad +\frac{9g_2^2}{32\pi^2}\Lambda^2 \qquad +\frac{\lambda}{4\pi^2}\Lambda^2$$

Problem is quadratic-divergence sensitive

$\Lambda:\ \mathrm{cut-off\ scale}$

Contents

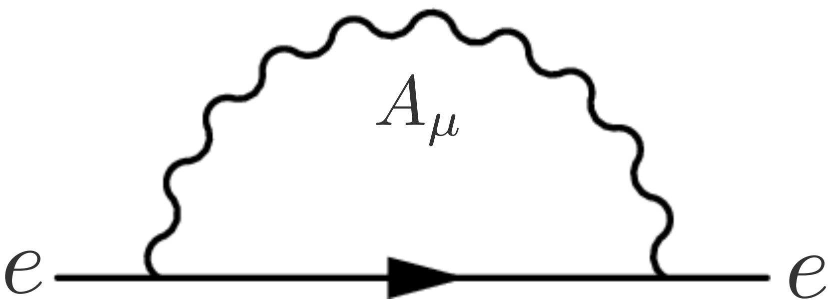
- ☐ Naturalness of the Higgs mass
- ☐ Twin Higgs Models
- ☐ Phase Transitions in Twin Higgs Models
  (Cosmological impact)

Electron mass is natural

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{e}_L\bar{\sigma}^\mu D_\mu e_L + \bar{e}_R\sigma^\mu D_\mu e_R - m_e(\bar{e}_L e_R + \bar{e}_R e_L)$$

$$D_\mu \equiv \partial_\mu - ieA_\mu$$

$U(1)_V \times U(1)_A$ invariant $U(1)_V$ invariant



Charge	e_L	e_R
$U(1)_V$	$+1$	$+1$
$U(1)_A$	$+1$	-1

Naive dimensional analysis $\delta m_e \sim \Lambda$

$$\Delta_{m_e} \sim 10^{-19}, (\Lambda \sim M_{pl})$$

However, $\delta m_e = m_e \frac{3\alpha}{2\pi} \log\left(\frac{\Lambda}{m_e}\right)$

Natural !!

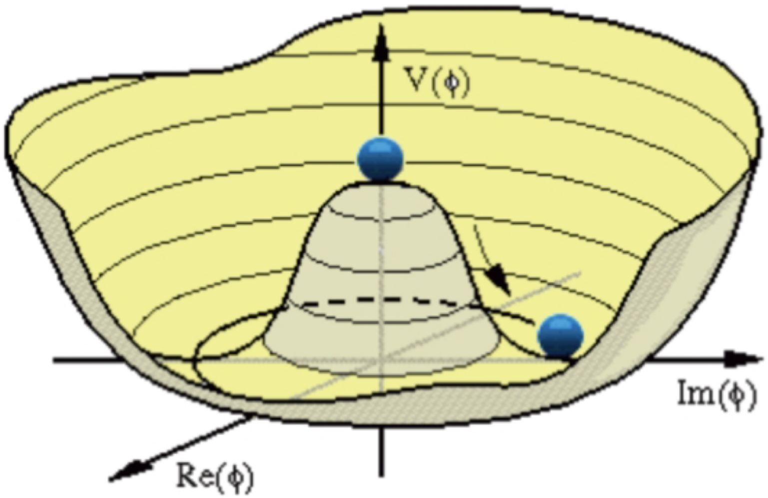
Why log sensitivity?

m_e is the only parameter which breaks the $U(1)_A$ symmetry.

When we take $m_e \to 0$ limit, $U(1)_A$ symmetry is restored.

$$\delta m_e \to 0, \ (m_e \to 0) \Rightarrow \delta m_e \propto m_e$$

U(1) Toy Model (Symmetry Protection)



$$V(|\phi|) = \lambda\left(|\phi|^2 - \frac{f^2}{2}\right)^2$$

$$\phi(x) = \frac{1}{\sqrt{2}}(f + \sigma(x))e^{i\frac{a(x)}{f}}$$

$\sigma(x)$:Massive Mode $\quad m_\sigma = \sqrt{2\lambda}f$

$a(x)$: Massless Goldstone Mode

$$\mathcal{L}(\sigma, a) = -\frac{1}{2}\partial_\mu a(x)\partial^\mu a(x) + \mathcal{L}_\sigma(\sigma)$$

NG Boson has shift-symmetry: $\quad a(x) \rightarrow a(x) + const$

Add explicit breaking source: $\mathcal{L}_{U(1)breaking} = -\rho f^3(\phi + \phi^*)$

NG Boson acquires mass: $m_a = \sqrt{2\rho}f$

$\rho \rightarrow 0$ U(1) symmetry is restored $\quad \delta m_a^2 \propto m_a^2$

Twin Higgs

[Z. Chacko, H.-S. Goh, and R. Harnik,Phys. Rev. Lett.96, 231802 (2006)]

Twin Higgs provides an elegant solution to the Little Hierarchy Problem

SM Higgs is considered as pseudo-Nambu-Goldstone Boson

$$\mathcal{H} = \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \end{pmatrix}$$ U(4) Fundamental Representation

Spontaneous symmetry breaking $U(4) \to U(3)$

$$V = \lambda \left(|\mathcal{H}|^2 - \frac{f^2}{2} \right)^2 \qquad \langle \Phi_4 \rangle = f/\sqrt{2}$$

7 Goldstone Modes + one massive mode

4 of them are identified with Standard Model Higgs

SM-like Higgs: $H_A \equiv \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$ Another Higgs : $H_B = \begin{pmatrix} \Phi_3 \\ \Phi_4 \end{pmatrix}$

Mass corrections

In general, matter sector breaks the U(4) symmetry explicitly

$$-y_t \bar{Q}_L \tilde{H}_A u_R + h.c. \qquad H_A : SU(2)_W \times U(1)_Y$$

SM Higgs cannot be considered as pNGB

$\hat{u}$: Twin top quark

Introduce copy of SM

$$-\hat{y}_t \bar{\hat{Q}}_L \tilde{H}_B \hat{u}_R + h.c. \qquad H_B : SU(2)_{\hat{W}} \times U(1)_{\hat{Y}}$$

$$V_{\mathrm{eff}} \supset -\frac{3}{8\pi^2}\Lambda^2 (y_t^2 |H_A|^2 + \hat{y}_t^2 |H_B|^2) + \frac{9}{64\pi^2}\Lambda^2 (g_2^2 |H_A|^2 + \hat{g}_2^2 |H_B|^2)$$

$$y_t = \hat{y}_t, \;\; g_2 = \hat{g}_2$$

$$V_{\mathrm{eff}} \supset \left(-\frac{3y_t^2}{8\pi^2} + \frac{9g_2^2}{64\pi^2} \right) \Lambda^2 (|H_A|^2 + |H_B|^2)$$

Large mass corrections respect the U(4) symmetry

Twin Higgs

Let us introduce copy of SM

Twin Z_2 Symmetry

Standard Model

$$H_A$$

$$SU(3)_C \times SU(2)_W \times U(1)_Y$$

Quarks and Leptons

Twin Partner

$$H_B$$

$$SU(3)_{\hat{C}} \times SU(2)_{\hat{W}} \times U(1)_{\hat{Y}}$$

Twin Quarks and Leptons

Higgs Mixing

Every quadratic divergent mass corrections are cancelled by its Twin partner

Most important point is that the Twin partners do not have SM charge!
(Neutral Naturalness)

Higgs potential

Sub-leading correction does not respect the U(4) symmetry

$$V(\varphi)_{\rm CW} = n\frac{m^4(H_A,\ H_B)}{64\pi^2}\log\left[\frac{m^2(H_A,\ H_B)}{\Lambda^2}\right]$$

$$V_{\rm eff} = \lambda\left(|H_A|^2 + |H_B|^2 - \frac{f^2}{2}\right)^2 + \sigma f^2|H_A|^2 + \kappa(|H_A|^4 + |H_B|^4)$$

Spontaneously symmetry breaking $U(4) \to U(3)$

This term must be dominant compared to U(4) breaking term $\lambda >> \sigma,\ \kappa$

Twin Z_2 breaking term

$f > 2v_A$

Radiative Corrections

V_{CW}

from top quark Twin top quark

U(4) explicit breaking term controls pNGB mass!

$\sigma,\ \kappa$: Technically Natural

Low-Energy Effective Theory

Integrating Out Massive Mode

$$|H_B|^2 = \frac{f^2}{2} - |H_A|^2$$

$$V_{\text{eff}}^{\text{low-energy}} = \left(\sigma f^2 - \kappa f^2\right)|H_A|^2 + 2\kappa |H_A|^4$$

It should match with SM Higgs potential!

$$2\kappa = \lambda_{\text{SM}},\ (\sigma - \kappa) f^2 = \lambda_{\text{SM}} v_{\text{SM}}^2$$

$V_{\text{eff}}^{\text{low-energy}}$

is valid up to $\Lambda \sim 4\pi f$

$\Lambda \sim 5\text{TeV} \leftrightarrow f \sim 400\text{GeV}$

How is the tuning?

$$\Delta_\sigma = \frac{2\frac{v_{SM}^2}{f^2}}{1 - 2\frac{2v_{SM}^2}{f^2}} \sim 2\frac{v_{SM}^2}{f^2}$$

$$\Delta_\sigma > \frac{1}{10} \Rightarrow \frac{v_{SM}}{f} > 0.23$$

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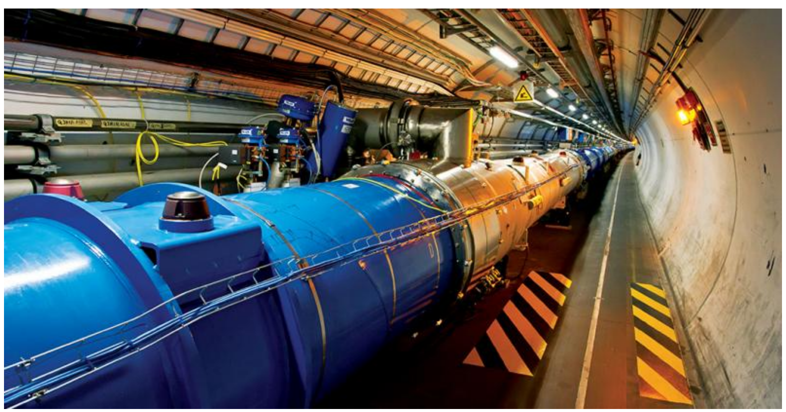
Cosmology!

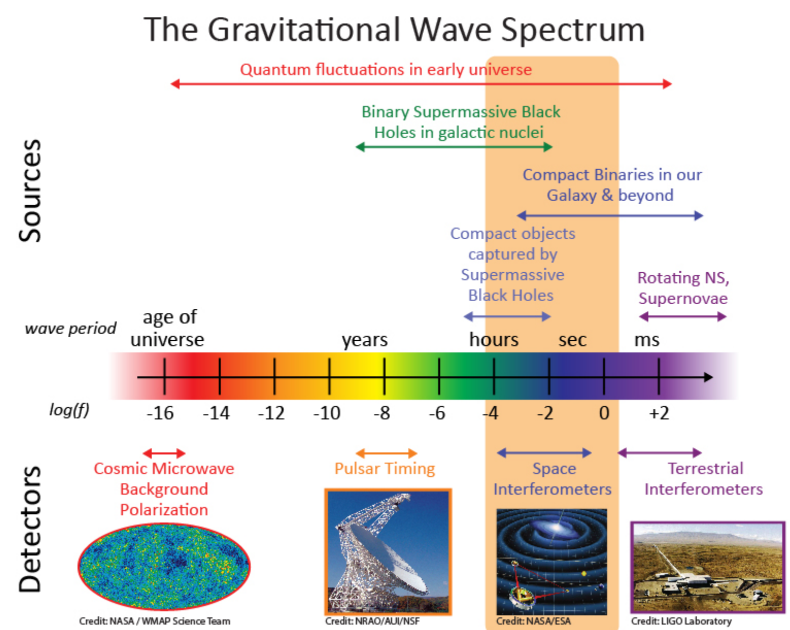
Twin Higgs

Collider searches



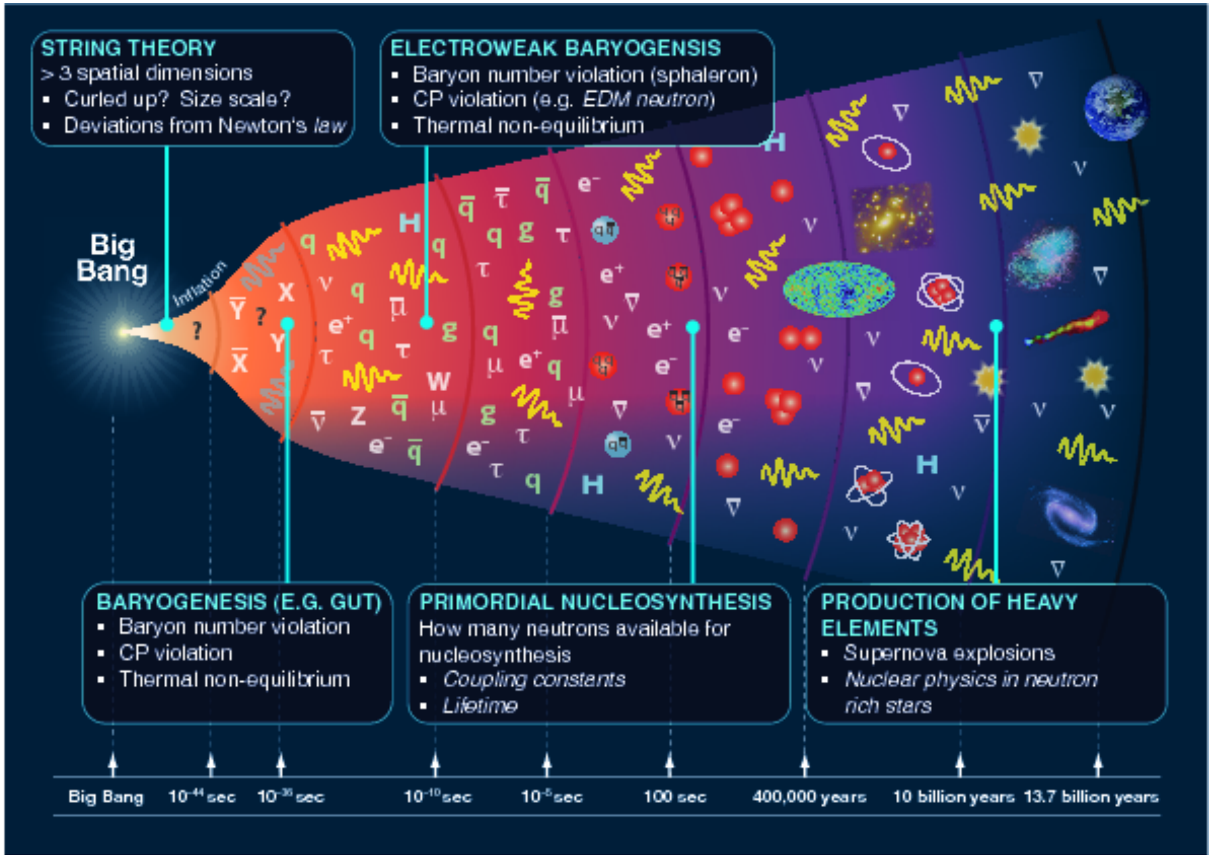
Large Hadron Collider

Gravitational wave



LISA, DECIGO and BBO

Thermal History of the early Universe

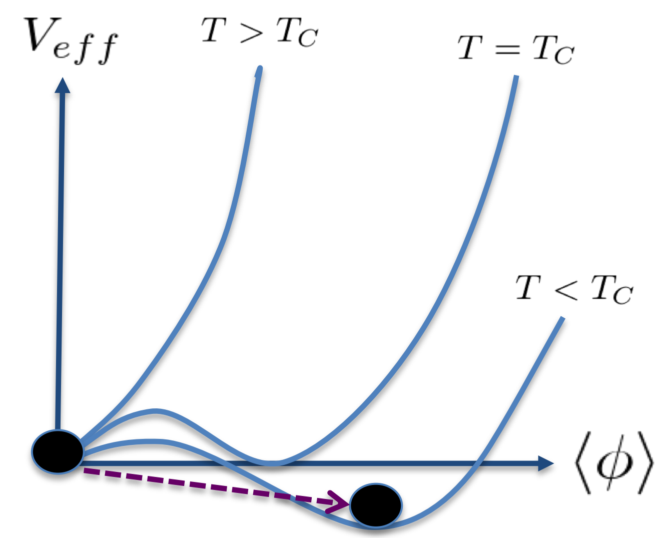


Spontaneous symmetry breaking in the early Universe

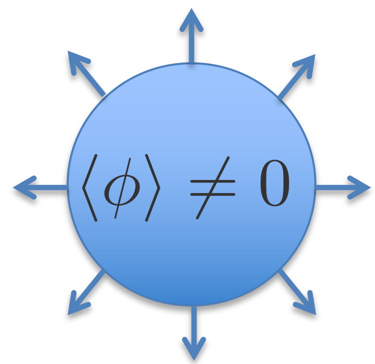
→ Phase transition

First-order phase transition and GW

First order phase transition

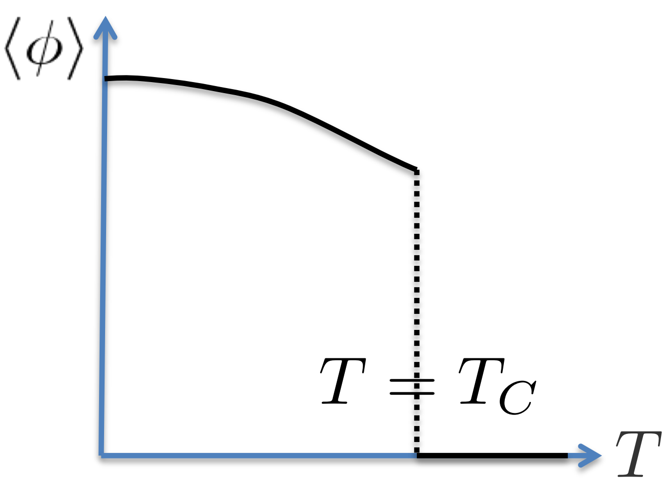


Tunneling



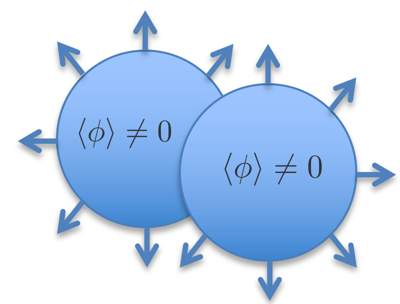
First order phase transition proceeds through bubble nucleation

Order parameter is Higgs VEV.



There are three sources of the Gravitational Wave

Bubble collisions

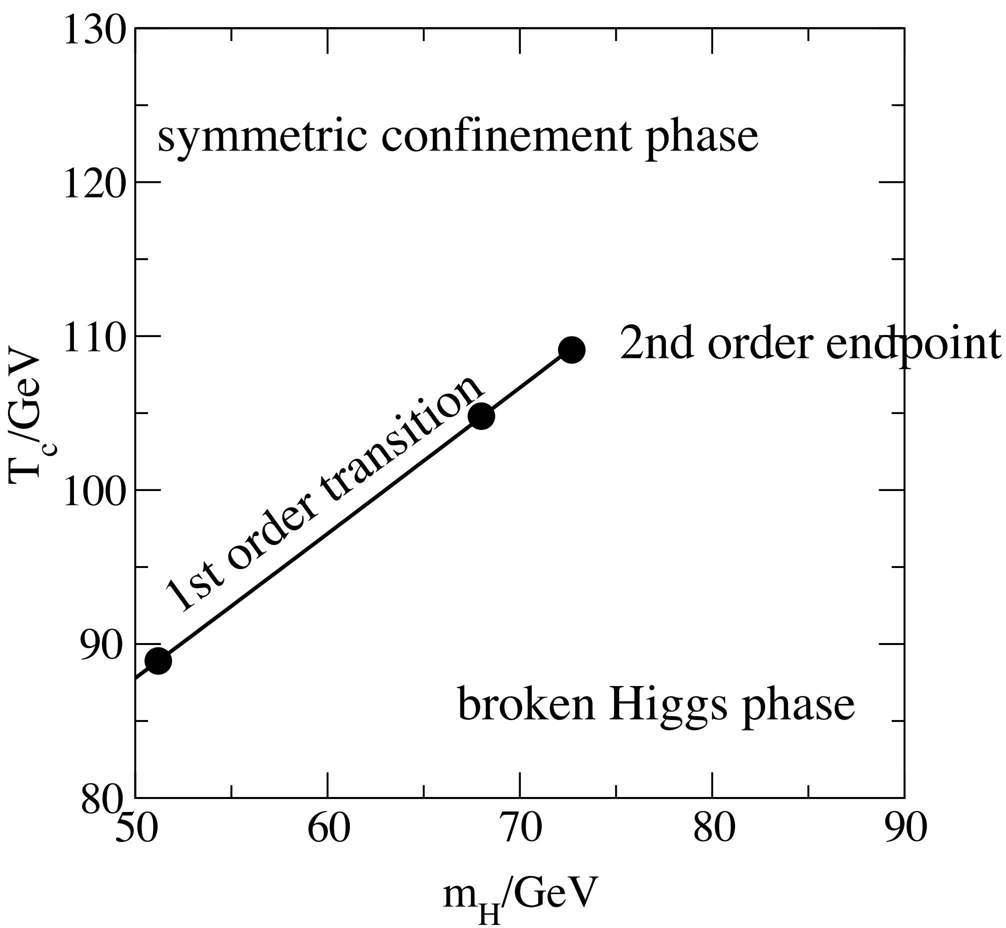


Sound Wave of the cosmic plasma

Turbulence of the plasma

Electroweak Phase Transition in SM

M. LAINE (2000)



First-order phase transition

Electroweak phase transition is not first order in SM.

BSM can change the situation!

Is it possible to realize first-order phase transitions in Twin Higgs Models?

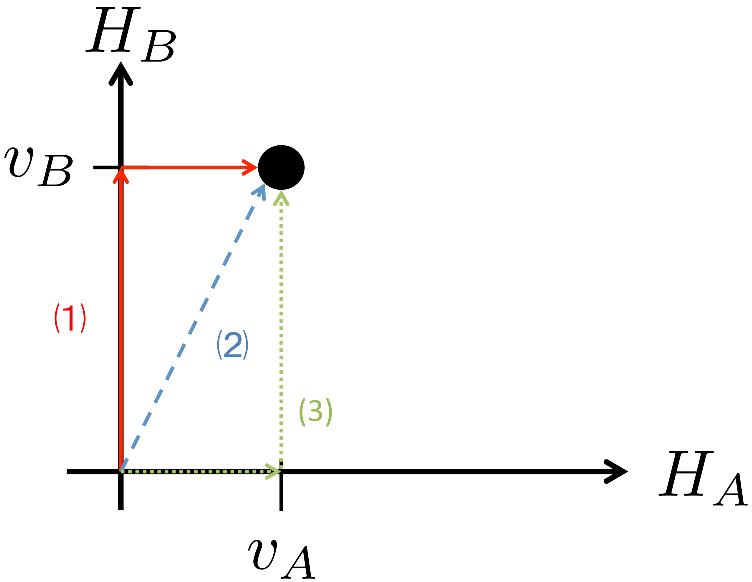
and

If the phase transition is first order, can we detect the GW?

Phase Transition(s) in Twin Higgs Models

There are two spontaneous symmetry breakings
(Global U(4) symmetry and EW symmetry)

$$V_{\text{eff}} = \lambda \left(|H_A|^2 + |H_B|^2 - \frac{f^2}{2} \right)^2 + \sigma f^2 |H_A|^2 + \kappa (|H_A|^4 + |H_B|^4)$$



$$(1)\ (0,\ 0) \to (0,\ v_B) \to (v_A,\ v_B)$$

U(4) Breaking Phase Transition ▸ Electroweak Phase Transition

$$(2)\ (0,\ 0) \to (v_A,\ v_B)$$

U(4) Breaking Phase Transition and Electroweak Phase Transition occur simultaneously

$$(3)\ (0,\ 0) \to (v_A,\ 0) \to (v_A,\ v_B)$$

Electroweak Phase Transition ▸ U(4) Breaking phase Transition

We consider the case (1)

Electroweak Phase Transition

Background field $H_A = \begin{pmatrix} 0 \\ \frac{\phi_A}{\sqrt{2}} \end{pmatrix}, \; H_B = \begin{pmatrix} 0 \\ \frac{\phi_B}{\sqrt{2}} \end{pmatrix}$

Take account of	d.o.f	Field dependent mass
$SU(2)_W$	$n_W = 6,$	$: \; m_W^2 = \frac{g_2^2 \phi_A^2}{4},$
$SU(2)_{\hat{W}}$	$n_{\hat{W}} = 9,$	$: \; m_{\hat{W}}^2 = \frac{\hat{g}_2^2 \phi_B^2}{4}$
$U(1)_Y$	$n_Z = 3,$	$: \; m_Z = (g_1^2 + g_2^2)\frac{\phi_A^2}{4}$
Top quark	$n_t = -12,$	$: \; m_t^2 = \frac{y_t^2 \phi_A^2}{2}$
Twin Top quark	$n_{\hat{t}} = -12,$	$: \; m_{\hat{t}}^2 = \frac{y_t^2 \phi_B^2}{2}$

$$V(\phi_A, \; \phi_B \; T) = V_0 + V_{\rm CW} + V_{\rm Thermal} + V_{\rm ring}$$

Integrating out massive mode $\phi_B^2 = f^2 - \phi_A^2$

$$V(\phi_A, \; T)$$

Validity of the perturbation in finite temperature

$$f_B = \frac{1}{e^{\beta E} - 1}, \quad E \equiv k^2 + m_W^2(\phi) \qquad m_W(\phi) \sim g_2 \phi$$

$$f_B >> 1 \text{ when } m_W(\phi)\beta << 1$$

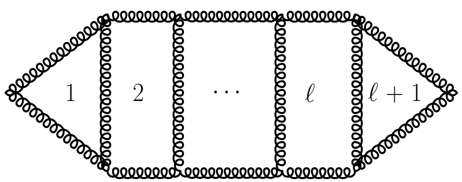
IR divergence comes from large occupation number

Perturbation theory breakdown when

Expansion parameter

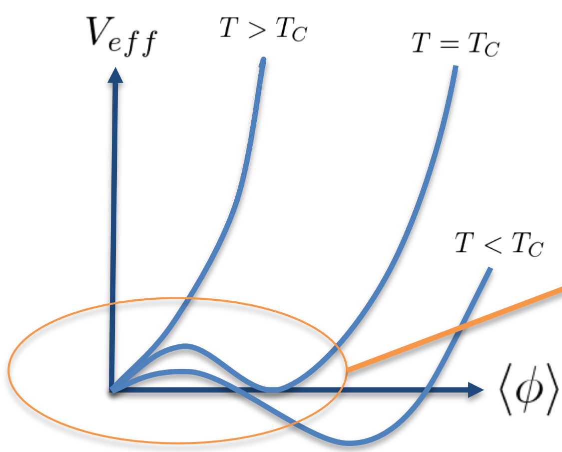
$$\gamma = g_2^2 \frac{T}{m_W(\phi)} \sim \frac{g_2 T}{\phi} > 1$$

Linde.(1980)



Numerical Simulation

$$\begin{array}{ll} g^{2l}T^4 & \text{for } l = 1,2 \\ g^6 T^4 \ln(T/m) & \text{for } l = 3 \\ g^6 T^4 (g^2 T/m)^{l-3} & \text{for } l > 3 \end{array}$$



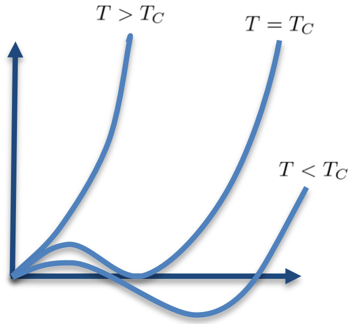
If $\gamma(T_C) > 1$, we cannot believe perturbation!

Peter Arnold(1994)

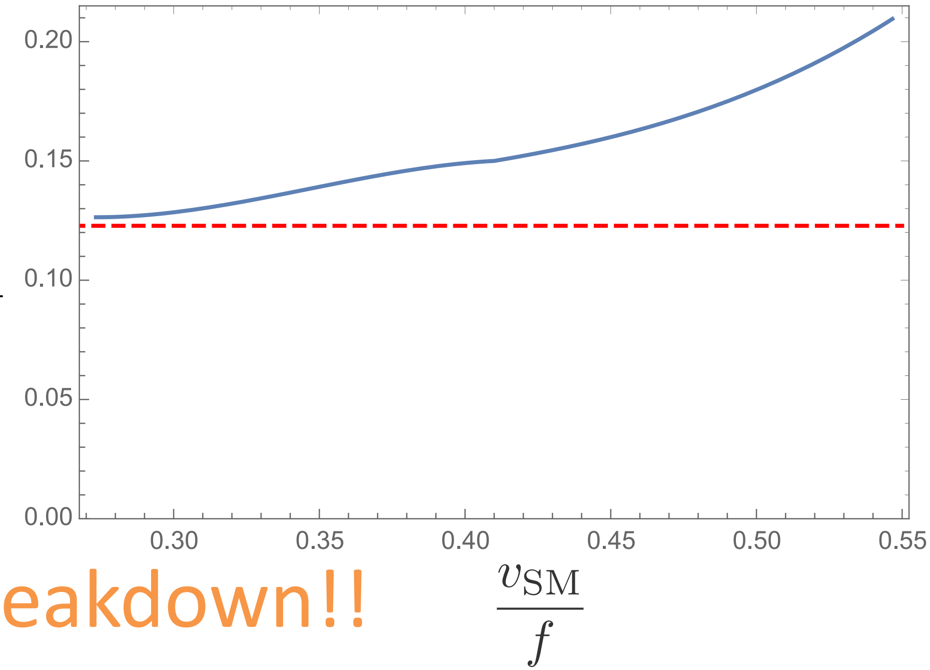
Order of the phase transition should be analyzed by the non-perturbative method

Result of the electroweak phase transition

$V(\phi_A,\ T)$



Red line represent the only SM contribution



Allowed region $\frac{v_{SM}}{f} < 0.5$ $\frac{\phi_A(T_C)}{T_C}$

cannot satisfy $\frac{\phi(T_C)}{T_C} > g_2 \sim 0.65$

Perturbative description breakdown!!

Large breaking scale f

- thermal decoupling (Boltzmann suppression)
- Twin sector corrections do not give a contribution

Sphaleron decoupling condition cannot be satisfied $\frac{\phi_A(T_C)}{T_C} > 1$

U(4) Breaking Phase Transition without UV completion

Thanks to the Twin $\mathbf{Z_2}$ symmetry, the situation is similar to the electroweak phase transition in SM.

U(4) breaking Phase Transition	Twin Z_2 Symmetry $y_t \simeq \widehat{y}_t, \ \ g_2 \simeq \widehat{g}_2$	Electroweak phase transition in SM

Well known

$$V = \frac{1}{2}M^2(T)\phi_B^2 - \frac{T}{2\pi}\left(\frac{\hat{g}_2^2\phi_B^2}{4}\right)^{3/2} - \frac{T}{4\pi}\left(\left(\frac{\hat{g}_2^2\phi_B^2}{4} + \Pi_W^{(i)}\right)^{\frac{3}{2}} - \left(\frac{\hat{g}_2^2\phi^2}{4}\right)^{\frac{3}{2}}\right) + \frac{\lambda + \kappa_1(T)}{4}\phi_B^4$$

$$M^2(T) = -\lambda f^2 + \frac{\hat{y}_t^2}{4}T^2 + \frac{3\hat{g}_2^2}{16}T^2 \qquad \kappa(T) = \kappa - \frac{3\hat{y}_t^4}{16\pi^2}\log\left(\frac{a_F T^2}{\mu^2}\right) + \frac{9\hat{g}_2^4}{256\pi^2}\log\left(\frac{a_B T^2}{\mu^2}\right)$$

However…

$$v_A \neq f, \ \ \lambda_{\rm SM} \neq \lambda + \kappa$$

Situation is similar to electroweak phase transition in SM

$$SU(2)_{\widehat{W}} : \ \widehat{g}_2 \leftrightarrow SU(2)_W : \ g_2$$

$$\left|\frac{\hat{g}_2(\Lambda)-g_2(\Lambda)}{g_2(\Lambda)}\right| \lesssim 0.1$$

$$\text{Breaking Scale} : \ f \leftrightarrow v_{SM}$$

(Critical Temperature is different)

$$\text{Twin Top Quark} : \ \widehat{y}_t \leftrightarrow \text{Top Quark} : \ y_t$$

$$\text{Higg self}-\text{coupling} : \ \lambda+\kappa \leftrightarrow \lambda_{SM}$$

In SM, order of electroweak phase transition depends on λ_{SM}/g_2^2

[K. Rummukainena, M. Tsypinb, K. Kajantiec, M. Laine, and M. Shaposhnikov] (1998)

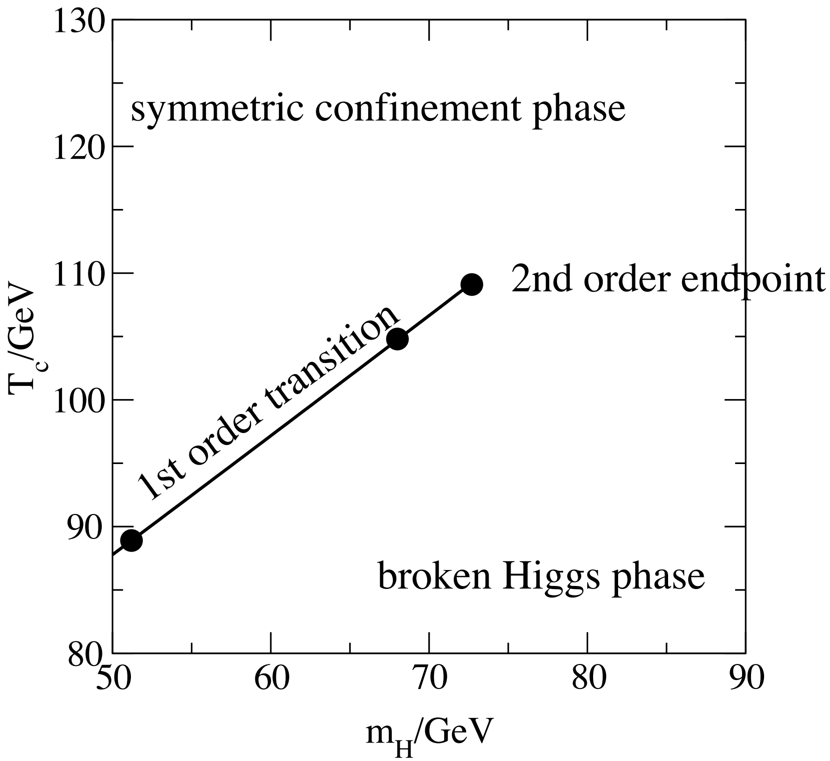
U(4) breaking phase transition depends on $(\lambda+\kappa)/\hat{g}_2^2$

We can use the result of electroweak phase transition in SM !

Result of the U(4) Phase Transition

M. LAINE (2000)

SM Rusult



$$\lambda_{SM} < 0.04$$

$$\lambda + \kappa < 0.04$$

First order phase transition!!

Matching condition with SM

$$\kappa = \frac{\lambda_{\rm SM}}{2} \sim 0.06$$

U(4) breaking phase transition cannot be first order without any UV completion

Summary

- Twin Higgs provides excellent solution to the Little Hierarchy problem
- Electroweak phase transition cannot be analyzed perturbatively in Twin Higgs Models
- It is difficult to realize the first-order U(4) breaking phase transition without any UV completion
- We also analyze the U(4) breaking phase transition with light twin stops in SUSY completion and calculate a typical GW amplitude