Hypercharge flux in SO(32) heterotic string theory

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based on

arXiv:1801.03684 [hep-th] JHEP **05** (2018) 045
arXiv:1808.XXXXX [hep-th] (with K. Takemoto)

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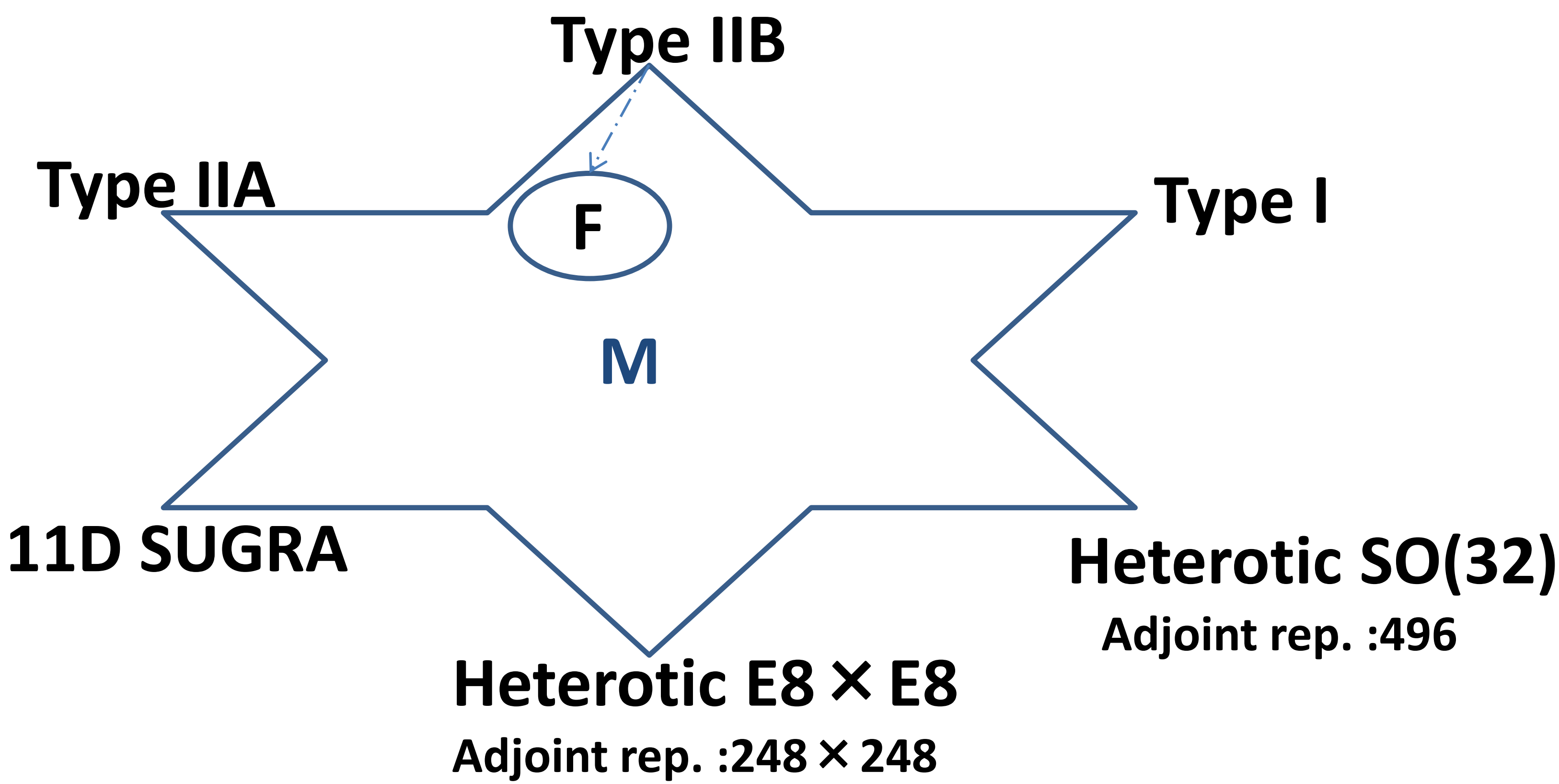
Why (super)string theory ?

- Quantum Gravity

- Unified theory

Good candidate for the unified theory of the gauge and gravitational interactions

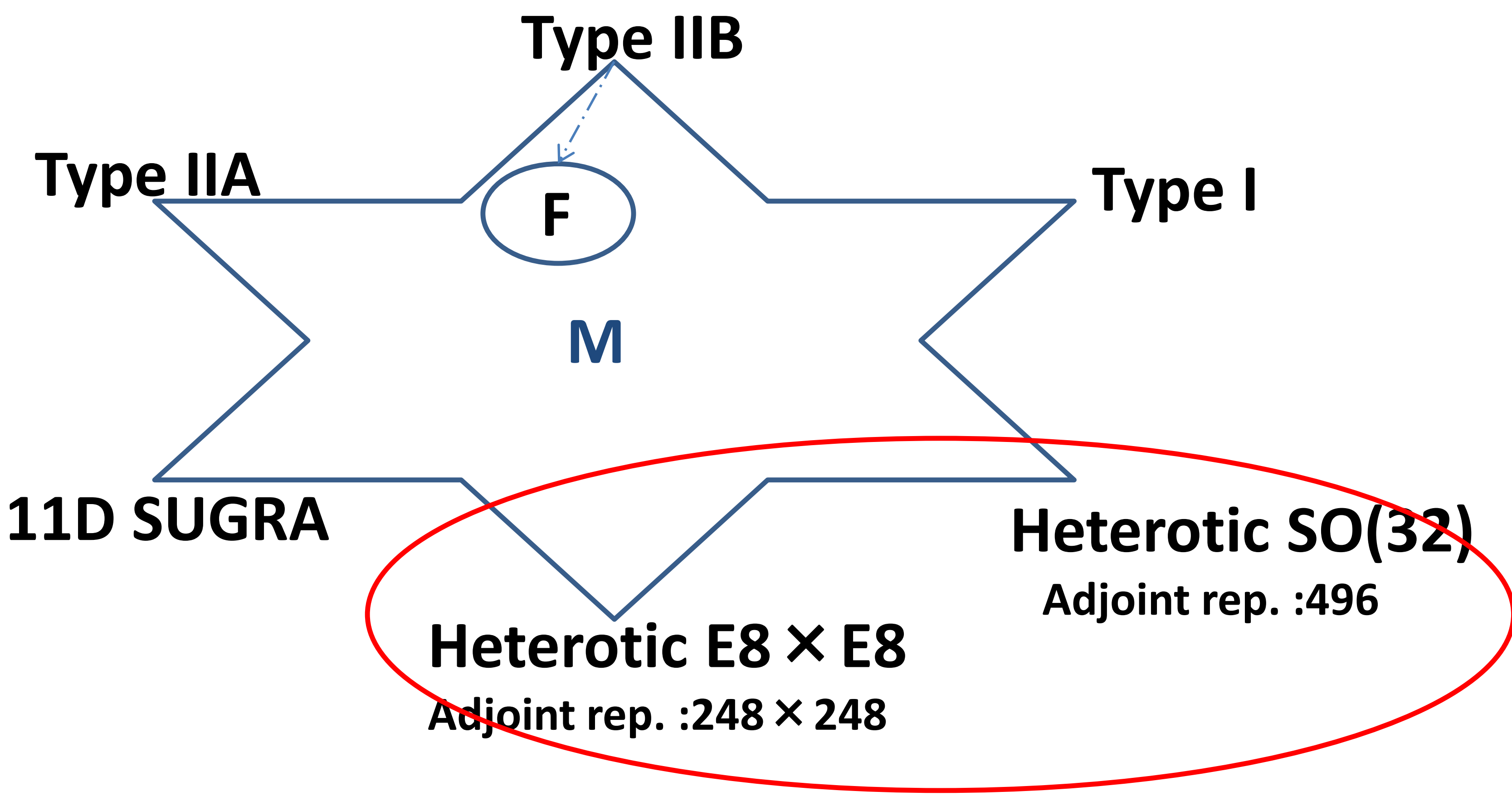
Superstring theory / M theory



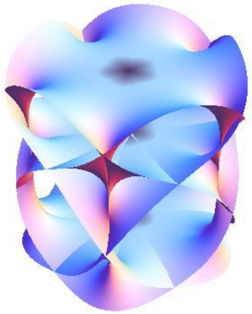
Where is the Standard Model ?

Why three generations ?

Superstring theory / M theory



10D Superstring theory ⟹ 4D Standard Model(SM)



SUSY-preserving
6D internal spaces:

1. Orbifolds
   classified by “orbifolder”
   Nilles, Ramos-Sanchez, Vaudrevange, Wingerter (‘11)
2. Calabi-Yau (CY)

Problem: in perturbative superstring, many 4D string vacua

<u>Can we derive conditions to derive the SM in general CY ?</u>

Outline

- Introduction

- **Heterotic Standard Models on smooth CY**

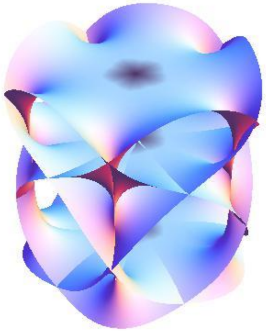
i) Model-building approach
ii) General formula
iii) Concrete model

- **Conclusion**

Heterotic Standard Models on smooth Calabi-Yau (CY)

“Standard embedding”

Candelas-Horowitz-Strominger-Witten (‘85)



6D Calabi-Yau (CY)Manifold

○Ricci-flat manifold

$$R_{ij} = 0$$

○SU(3) holonomy

- Gauge symmetry breaking:

$$E_8 \times E_8 \to E_6 \times \cancel{SU(3)} \times E_8 \qquad (A_i^{SU(3)} = w_i^{\text{spin}})$$

$$\to SU(3)_C \times SU(2)_L \times U(1)_Y \quad \text{(Wilson lines)}$$

- Number of chiral generation = |Euler number of CY|/2

$$|\chi_{\text{CY}}| = 6$$

"Standard embedding"

Requirements:

① Wilson-line breaking (possible for restricted CYs)

- We require non-contractible one-cycles (non-simply-connected CY)

E.g.,
195 non-simply-connected CICYs among total 7890 CICYs

CICY=Complete Intersection Calabi-Yau

② Small Euler number of CY (3 generations of quarks)

$$|\chi_{\mathrm{CY}}| = 6$$

Two approaches in the heterotic model building on smooth CY

1. "Standard embedding"

$$A_i^{SU(3)} = w_i^{\mathrm{spin}}$$

$$E_8 \to E_6 \times SU(3) \to G_{\mathrm{SM}} \times G_{\mathrm{hid}}$$

2. "Non-standard embedding"

$$A_i^{SU(3)} \neq w_i^{\mathrm{spin}}$$

$$E_8 \to G_{\mathrm{SM}} \times G_{\mathrm{hid}}$$

- SM vacua directly with the SM gauge group

“Non-standard embedding”

○Internal $U(1)$ gauge fluxes F

$$\frac{1}{2\pi}\int_{\Sigma_i} F = \; m^{(i)} \in \mathbb{Z}$$

Σ_i : Two-cycles of CY

E.g., Hypercharge flux

$$SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$< F_{U(1)_Y} > \propto \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$$

- Popular in the F-theory $SU(5)$GUT *Beasley-Heckman-Vafa, Donagi-Wijnholt ('08)*
- Direct flux breaking scenario is applicable in the Heterotic context

Blumenhagen-Honecker-Weigand ('05)

○Internal $U(1)$ gauge fluxes F

- Gauge symmetry breaking

$$SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$$

- Chiral and net-number of zero-modes, given by

$$N_{\text{gen}} = \frac{1}{(2\pi)^3} \int_{\text{CY}} \left[\frac{1}{6} \text{tr}(F^3) + \frac{1}{12} \text{tr}(R^2) \wedge \text{tr}(F) \right]$$

Background curvatures F and R
give rise to the three-generation of quarks and leptons

Q, L, u^c, d^c, e^c : $N_{\text{gen}} = -3$

No chiral exotics : $N_{\text{gen}} = 0$

Internal $U(1)_a$ gauge fluxes F_a

$$\frac{1}{2\pi}\int_{\Sigma_i} F_a = m_a^{(i)} \in \mathbb{Z}$$

Σ_i : Two-cycles of CY

- Chiral index

$$N_{\rm gen} = \frac{1}{(2\pi)^3}\int_{\rm CY}\left[\frac{1}{6}{\rm tr}(F^3) + \frac{1}{12}{\rm tr}(R^2)\wedge {\rm tr}(F)\right]$$

$$\Downarrow$$

$$N_{\rm gen} = \frac{1}{6}\sum_{a,b,c} X_{abc} Y_a Y_b Y_c + \frac{1}{12}\sum_a Z_a Y_a$$

Y_a: $\mathrm{U}(1)_a$ charges of zero-modes

X_{abc}, Z_a depends on the topological data of CY and $m_a^{(i)}$.

Internal $U(1)_a$ gauge fluxes F_a

- Chiral index

$$N_{\text{gen}} = \frac{1}{6}\sum_{a,b,c} X_{abc} Y_a Y_b Y_c + \frac{1}{12}\sum_a Z_a Y_a$$

Y_a: $\mathrm{U}(1)_a$ charges of zero-modes

- Index is determined only by variables (X_{abc}, Z_a)
- Applicable in all CYs

It opens up a possibility of
searching for the three-generation SM in a background-independent way

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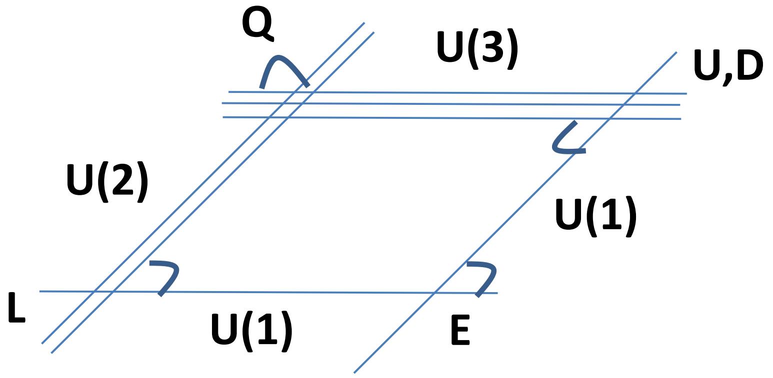
$E_8 \times E_8$ heterotic Standard Models are well studied by

Donagi-Ovrut-Pantev-Waldram ('00), Blumenhagen-Honecker-Weigand ('05)
Anderson-Gray-Lukas-Palti ('12)

SO(32) heterotic Standard Models

$\Updownarrow$ **S- and T-dualities**

Intersecting D6-brane models in type IIA string
(Several stacks of D-branes → MSSM or Pati-Salam model)



Our research:
<u>SO(32) heterotic SM (MSSM) vacua directly with the SM gauge group from smooth CYs</u>

To concrete our analysis, we focus on the branching:

$$SO(32) \supset SO(16) \supset SU(3)_C \times SU(2)_L \times \Pi_{a=1}^{5} U(1)_a$$

$496 \supset 120 \supset$ MSSM particles ($\sim 7 \times 10^7$ possibilities)

$16 \supset$ Exotics

We introduce internal $U(1)_a$ gauge fluxes F_a ($a = 1,2,3,4,5$)

- Chiral index only depends on variables X_{abc}, Z_a

$$N_{\rm gen} = \frac{1}{6} \sum_{a,b,c} X_{abc} Y_a Y_b Y_c + \frac{1}{12} \sum_a Z_a Y_a$$

Can we constrain 35 variables X_{abc} and 5 variables Z_a ?

Phenomenological requirements:

- Chiral index

$$N_{\text{gen}} = \frac{1}{6}\sum_{a,b,c} X_{abc} Y_a Y_b Y_c + \frac{1}{12}\sum_a Z_a Y_a$$

Y_a: $\mathrm{U}(1)_a$ charges of zero-modes

5 conditions : Q, L, u^c, d^c, e^c : $N_{\text{gen}} = -3$

6 conditions : No chiral exotics : $N_{\text{gen}} = 0$

Theoretical conditions:

① Masslessness conditions for $U(1)_Y = \sum_{a=1}^{5} f_a U(1)_a$

10D Green-Schwarz terms

$$\int_{10\mathrm{D}} B_6 \wedge \mathrm{tr}(F^2) \qquad \int_{10\mathrm{D}} B_2 \wedge X_8$$

4D Green-Schwarz terms $\Downarrow$ Gauge fluxes

$$\int_{4\mathrm{D}} b \wedge F_{\mathrm{U(1)_Y}}$$

b: string axions

To ensure the masslessness of $U(1)_Y$ gauge boson,

$$\sum_a \mathrm{tr}(T_a^2) f_a m_a^{(i)} = 0 \qquad \sum_{a,b,c,d} \mathrm{tr}(T_a T_b T_c T_d) f_a X_{bcd} = 0$$

Theoretical conditions:

② To admit the spinorial rep. in the first excited mode

$$\sum_a \mathrm{tr}(T_a) m_a^{(i)} = 2\kappa^{(i)} \in 2\mathbb{Z}$$

①,②より、$m_\alpha^{(i)}$ $(\alpha = 1,2)$は$m_A^{(i)}(A = 3,4,5)$と$\kappa^{(i)}$で表すことが可能

40 variables $\{X_{abc}, Z_a\}$

$\Downarrow$ Theoretical conditions ①, ②

23 variables

Against several branching of $SO(16) \to SU(3) \times SU(2) \times \Pi_a U(1)_a$

three-generation models are possible, e.g.,

$$SO(16) \to SO(6) \times SO(4) \times SO(2)^3$$
$$\to SU(3) \times SU(2) \times \Pi_{a=1}^{5} U(1)_a$$

$$X_{333} = p_1, X_{334} = p_2, X_{335} = p_3, X_{344} = p_4, X_{345} = 3, X_{355} = p_5, X_{444} = p_6,$$
$$X_{445} = p_7, X_{455} = -6 - p_2 + p_3 + p_4 + p_5 + p_7, X_{555} = -p_1 + p_6;$$
$$Z_3 = -2p_1, Z_4 = -2p_6, Z_5 = 2p_1 - 2p_6 \quad \cdots$$

p_m : integers ($m = 1,2,\cdots,16$)

- Other U(1)s become massive through the GS mechanism in general.

- Supersymmetric and stability conditions are required to be checked for each CYs.

Possible gauge branching satisfying all the requirements:

$$
SO(16) \to \begin{cases}
SO(6) \times SO(4) \times SO(2)^3 \\
SO(10) \times SO(6) \to SU(5) \times SU(3) \times U(1)^2 \\
SO(8) \times SO(4) \times SO(4) \to SU(4)_C \times SU(2)_L \times SU(2) \times U(1)^3 \\
SU(4)_C \times SU(4) \times U(1)^2 \to SU(4)_C \times SU(2)_L \times SU(2) \times U(1)^3 \\
SO(6) \times SO(6) \times SO(4) \to SU(3)_C \times SU(3) \times SU(2) \times U(1)^3 \\
SO(8) \times SO(6) \times SO(2) \to SU(4)_C \times SU(3) \times U(1)^3 \\
SO(8) \times SO(4) \times SO(2)^2 \to SU(4)_C \times SU(2)_L \times U(1)^3 \\
SO(6) \times SO(10) \to SU(3)_C \times SU(5) \times U(1)^2 \\
SO(6) \times SO(4) \times SO(4) \times SO(2) \to SU(3)_C \times SU(2)_L \times SU(2) \times U(1)^3 \\
SO(6) \times SO(6) \times SO(2)^2 \to SU(3)_C \times SU(3) \times U(1)^4
\end{cases}
$$

$$
\to G_{\mathrm{SM}} \times U(1)^4,
$$

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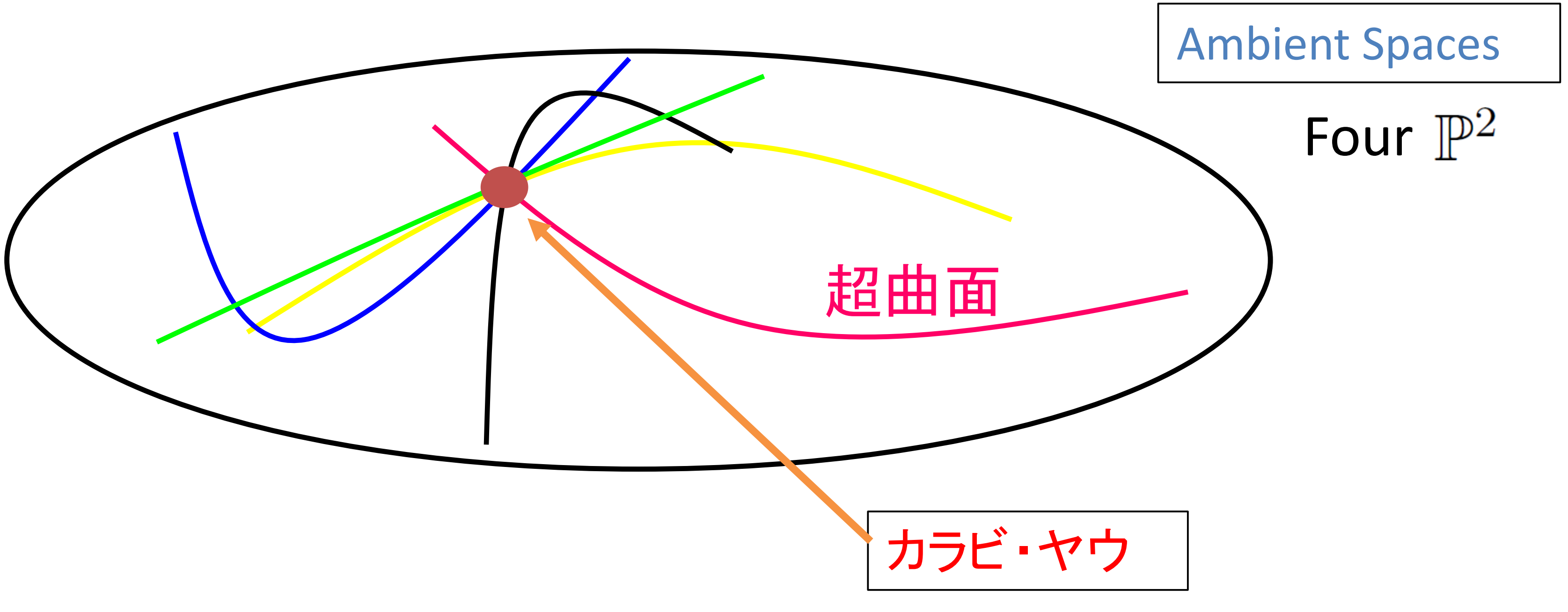
Concrete model

Complete Intersection Calabi-Yau = 完全交叉カラビ・ヤウ

$$\begin{array}{c} \mathbb{P}^2 \\ \mathbb{P}^2 \\ \mathbb{P}^2 \\ \mathbb{P}^2 \end{array} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$



Concrete model

Complete Intersection Calabi-Yau = 完全交叉カラビ・ヤウ

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Topological data of CY:

- $h^{1,1} = 4$ (Number of Kähler moduli)
- Intersection number

$$d_{123} = 6, \quad d_{124} = d_{134} = d_{234} = 5, \quad d_{112} = d_{113} = d_{122} = d_{133} = d_{223} = d_{233} = 3,$$
$$d_{114} = d_{144} = d_{224} = d_{244} = d_{334} = d_{344} = 2, \quad d_{111} = d_{222} = d_{333} = d_{444} = 0,$$

- Second Chern number

$$c_2(T\mathcal{M}) = (36, 36, 36, 36)$$

Concrete model

$U(1)_a$ fluxes ($a = 1,2,3,4,5$)

$$m_1 = (1,0,0,-1), \qquad m_2 = (1,0,-1,3), \qquad m_3 = (0,1,0,-1),$$
$$m_4 = (0,0,1,-1), \qquad m_5 = (-1,-1,1,1),$$

- Supersymmetric and stability conditions are also satisfied at

Kähler moduli

Dilaton

$$t_1 = t_2 = t_3 = \frac{3+2\sqrt{3}}{3} t_4, \quad s = \frac{2+\sqrt{3}}{2\pi} t_4$$

Concrete model

✔ Gauge symmetry : $SO(32) \to SU(3)_C \times SU(2)_L \times U(1)_Y \times SO(16)'$

✔ Other $U(1)$s become massive through the GS mechanism

✔ Chiral spectrum:

MSSM particles + Extra vector-like Higgs + Singlets

✔ Allow for perturbative Yukawa couplings !

✔ No proton decay operators (constrained by massive $U(1)_{B-L}$)

Gauge coupling unification

- Against all branching of $SO(16) \to SU(3) \times SU(2) \times \Pi_{a=1}^{5} U(1)_a$

  Tree-level gauge couplings at the string scale,

$$g^2_{SU(3)_C} = g^2_{SU(2)_L} = \frac{5}{6} g^2_{U(1)_Y} = g_0^2$$

- Gauge fluxes induce the threshold corrections to the gauge couplings

$$g^{-2}_{SU(3)_C} = g_0^{-2} + \Delta_{\mathrm{th},3}$$

$$g^{-2}_{SU(2)_L} = g_0^{-2} + \Delta_{\mathrm{th},2}$$

$$g^{-2}_{U(1)_Y} = 5 g_0^{-2}/6$$

$\Delta_{\mathrm{th},3} \neq \Delta_{\mathrm{th},2}$

- Nonuniversal gauge kinetic functions
  (in contrast to $E_8 \times E_8$ heterotic string $\Delta_{\mathrm{th},3} = \Delta_{\mathrm{th},2}$)

Conclusion

- We have searched for $SO(32)$ heterotic SM vacua directly with the SM gauge group from smooth CYs

- Direct flux breaking (Hypercharge flux in F-theory)

$$SO(32) \to SU(3)_C \times SU(2)_L \times U(1)_Y \times SO(16)$$

  is applicable in general CY compactification

- General formula leading to
  (i) Three-generation of quarks and leptons
  (ii) No chiral exotics

Discussion

- General formula in the dual global F-theory context