Flavor Physics in the multi-Higgs doublet models induced by the left-right symmetry

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arXiv:1804:07478 [hep-ph]

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Introduction

- Standard Model (SM) is succeeded by the discovery of the Higgs boson in 2012

- The SM is almost consistent with experimental data

$\Updownarrow$

- There are still some problems and questions which cannot solve in the SM framework
    - fermion mass hierarchy
    - charge quantization
    - fine-tuning of the Higgs mass (hierarchy problem)
    - massless neutrino
    - no dark matter candidates
    - ...

Physics beyond the SM is needed!

Introduction

- One of the interesting extensions

  extra Higgs doublets → realistic Yukawa couplings

- If we introduce extra Higgs $SU(2)_L$ doublets, there are some Yukawa couplings

  e.g.
$$Y^1_{ij}\overline{Q^i_L}\widetilde{H}_u u^j_R + Y^2_{ij}\overline{Q^i_L}\widetilde{H}_d u^j_R + \cdots$$
$$Y^1_{ij}\overline{Q^i_L}H_d d^j_R + Y^2_{ij}\overline{Q^i_L}H_u d^j_R + \cdots$$

  Y_u and Y_d are obtained by the linear combinations of these Yukawas

- We consider the left-right symmetric model (LR model)

  can resolve the strong CP problem Babu, Mohapatra, PRD **41**, 1286 (1990)

Introduction

- LR model: gauge symmetry

$$SU(3)_C \times SU(2)_L \times \underline{SU(2)_R} \times U(1)_{B-L}$$

right-handed fermions are doublets under this symmetry

$$Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}, \quad L_R = \begin{pmatrix} n_R \\ e_R \end{pmatrix}$$

right-handed neutrino

- Higgs fields become bi-doublet fields : (**2**,**2**) under $SU(2)_L \times SU(2)_R$

$$\Phi = \begin{pmatrix} \widetilde{H}_u & H_d \end{pmatrix}$$

H_u, H_d: $SU(2)_L$ doublets

Yukawa coupling (e.g. quark sector)

$$\Rightarrow \quad Y^1_{ij} \overline{Q^i_L}\, \Phi\, Q^j_R + Y^2_{ij} \overline{Q^i_L}\, \widetilde{\Phi}\, Q^j_R + \mathrm{h.c.}$$

we can obtain the realistic Yukawa couplings

Introduction

- LR model predicts multi-Higgs doublets

  → many physical modes in Higgs doublets

  (heavy neutral Higgs, charged Higgs, ...)

- Interesting point: heavy Higgs coupling is flavor violating one

$$Y^u_{H\,ij}\overline{Q^i_L}\,\widetilde{H}\,u^j_R + Y^d_{H\,ij}\overline{Q^i_L}\,H\,d^j_R + \mathrm{h.c.}\ \Rightarrow$$ tree-level processes

- These couplings are related to fermion masses and mixings

$$Y^{u,d}:\ \text{linear comb. of } Y^1, Y^2, \ldots$$
$$Y^{u,d}_H:\ \text{linear comb. of } Y^1, Y^2, \ldots$$

- Then we can obtain explicit predictions from the flavor physics

Introduction

- We consider the following scenario:

based on SUSY LR model → at least 4 Higgs doublets are needed

$10^{12\text{-}13}$ GeV — LR symmetry is broken: $SU(2)_R \times U(1)_{B\text{-}L} \to U(1)_Y$

≈

Majorana neutrino mass is very heavy

type-I seesaw mechanism works, Y^ν would be large

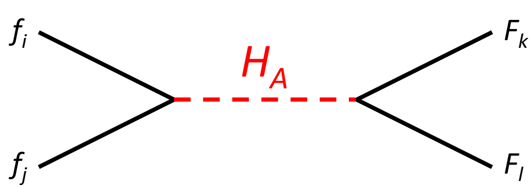
$O(100)$ TeV — SUSY breaking scale

heavy Higgs doublet masses are $O(100)$ TeV

EW scale

Some flavor processes will be enhanced

main topic of this work



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    - Lepton flavor violation
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SUSY LR model

S. Iguro, Y. Muramatsu, Y. Omura, YS, arXiv:1804:07478 [hep-ph]

LR model

- Right-handed fermions → SU(2)$_R$ doublet

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad L_R = \begin{pmatrix} n_R \\ e_R \end{pmatrix}$$

- Matter contents and charge assignments

Field	$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
Q_L^i	$(\mathbf{3},\ \mathbf{2},\ \mathbf{1},\ 1/3)$
$Q_R^{c\,i}$	$(\overline{\mathbf{3}},\ \mathbf{1},\ \mathbf{2},\ -1/3)$
L^i	$(\mathbf{1},\ \mathbf{2},\ \mathbf{1},\ -1)$
$L_R^{c\,i}$	$(\mathbf{1},\ \mathbf{1},\ \mathbf{2},\ 1)$
$\Phi_{1,2}$	$(\mathbf{1},\ \mathbf{2},\ \mathbf{2},\ 0)$
Δ_R	$(\mathbf{1},\ \mathbf{1},\ \mathbf{3},\ -2)$
$\overline{\Delta}_R$	$(\mathbf{1},\ \mathbf{1},\ \mathbf{3},\ 2)$
Δ_L	$(\mathbf{1},\ \mathbf{3},\ \mathbf{1},\ 2)$
$\overline{\Delta}_L$	$(\mathbf{1},\ \mathbf{3},\ \mathbf{1},\ -2)$
S	$(\mathbf{1},\ \mathbf{1},\ \mathbf{1},\ 0)$

Bi-doublet Higgs fields

to break SU(2)$_R$ × U(1)$_{B\text{-}L}$
to induce heavy RHν masses

LR model

- Superpotential: $W = W_{vis} + W_{SB} + W_{\Delta_L}$

$$W_{vis} = \underbrace{Y_{ji}^{a\,*} \hat{Q}_L^i \tau_2 \Phi_a \tau_2 \hat{Q}_R^{c\,j} + Y_{ji}^{l\,a\,*} \hat{L}^i \tau_2 \Phi_a \tau_2 \hat{L}_R^{c\,j}}_{\text{Yukawa int.}} + \underbrace{\lambda_{ij}^{\nu}\, \hat{L}_R^{c\,i} \tau_2 \Delta_R \hat{L}_R^{c\,j}}_{\substack{\text{Majorana mass} \\ \text{(effective)}}} + \underbrace{\mu^{ab}\, Tr\left(\tau_2 \Phi_a^T \tau_2 \Phi_b\right)}_{\mu\text{-term}}$$

$$W_{SB} = m(S)\, Tr\left(\Delta_R \overline{\Delta}_R\right) + w(S)$$

⇨ LR symmetry breaking: $SU(2)_R \times U(1)_{B\text{-}L} \rightarrow U(1)_Y$

$$W_{\Delta_L} = (\underline{m_L} + m(S))\, Tr\left(\Delta_L \overline{\Delta}_L\right) + \lambda_{ij}^{\nu\,*} L^i \tau_2 \Delta_L L^j$$

soft LR breaking term

⇨ Δ_L and $\overline{\Delta}_L$ terms (irrelevant to our work)

- *W* is invariant the following parity transformation:

$$Q_L \rightarrow Q_R^{c\,*},\ L_L \rightarrow L_R^{c\,*},\ \Delta_L \rightarrow \Delta_R^*,\ \overline{\Delta}_L \rightarrow \overline{\Delta}_R^*,\ \Phi_a \rightarrow \Phi_a^\dagger,\ S \rightarrow S^*$$

→ Yukawa matrices are Hermitian matrices

Yukawa couplings – SUSY LR model

- SUSY LR model: two bi-doublet chiral superfields are needed

we cannot use $\tilde{\Phi}$ in the Superpotential which is the holomorphic function of chiral superfields

$$Y_{ji}^{a*} Q_L^i \tau_2 \Phi_a \tau_2 Q_R^j \quad (a = 1, 2)$$

$\Phi_a = \begin{pmatrix} \widetilde{H_u^a} & H_d^a \end{pmatrix}$: bi-doublet field:

→ 4 Higgs doublet model! (H_u^1, H_u^2, H_d^1, H_d^2)

- Yukawa couplings

$$Y_{ij}^1 \overline{Q_L^i} \widetilde{H_u^1} u_R^j + Y_{ij}^2 \overline{Q_L^i} \widetilde{H_u^2} u_R^j$$

$$Y_{ij}^1 \overline{Q_L^i} H_d^1 d_R^j + Y_{ij}^2 \overline{Q_L^i} H_d^2 d_R^j$$

- Denote 4 Higgs doublets as $\hat{H}_I = (H_u^1, H_u^2, H_d^1, H_d^2)$,

$$\hat{H}_I = U_{Ih}\, h + U_{IA}\, H_A$$

I = 1, 2, 3, 4; A = 1, 2, 3

SM Higgs

Heavy Higgs doublets

Yukawa couplings – SUSY LR model

$$Y^1_{ij}\overline{Q^i_L}\widetilde{H^1_u}u^j_R + Y^2_{ij}\overline{Q^i_L}\widetilde{H^2_u}u^j_R$$
$$Y^1_{ij}\overline{Q^i_L}H^1_d d^j_R + Y^2_{ij}\overline{Q^i_L}H^2_d d^j_R$$

& $\hat{H}_I = U_{Ih}\,h + U_{IA}\,H_A$

- Yukawa couplings with
  - SM Higgs $Y^u_{ij}\overline{Q^i_L}\,\widetilde{h}\,u^j_R + Y^d_{ij}\overline{Q^i_L}\,h\,d^j_R$
  - Heavy Higgs $Y^u_{A\,ij}\overline{Q^i_L}\widetilde{H_A}\,u^j_R + Y^d_{A\,ij}\overline{Q^i_L}H_A d^j_R$
- Relation between Y_{ij} and Y_{Aij}

$$\boxed{\begin{pmatrix} Y^u_{A\,ij} \\ Y^d_{A\,ij} \end{pmatrix} = \frac{1}{\Delta_h}\begin{pmatrix} U^*_{1A} & U^*_{2A} \\ U_{3A} & U_{4A} \end{pmatrix}\begin{pmatrix} U_{4h} & -U^*_{2h} \\ -U_{3h} & U^*_{1h} \end{pmatrix}\begin{pmatrix} Y^u_{ij} \\ Y^d_{ij} \end{pmatrix}}$$

$$\Delta_h = U^*_{1h}U_{4h} - U_{3h}U^*_{2h}$$

Note: we can obtain Y_A^{ν} and Y_A^{e} by replacing $u \to \nu$ and $d \to e$

Yukawa couplings – SUSY LR model

- Neutrino sector: there are right-handed neutrinos

  → we have the Majorana mass terms: $(M_\nu)_{ij}\overline{\nu_R^{c\,i}}\nu_R^j + \mathrm{h.c.}$

- Tiny neutrino masses are obtained by seesaw mechanism (type-I)

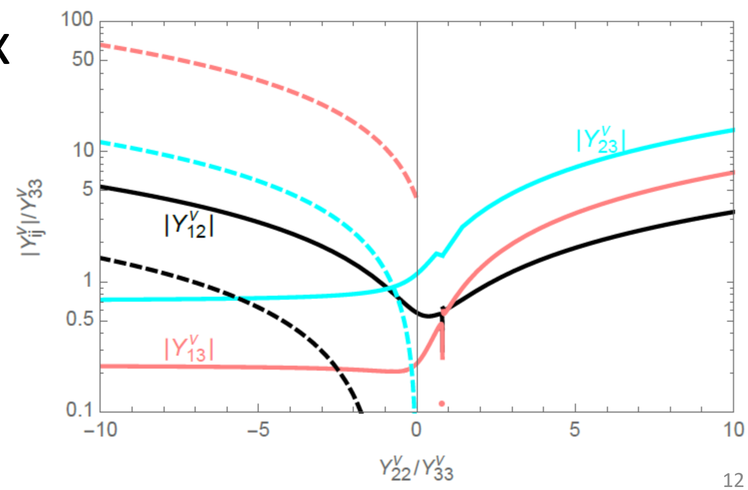
$$\boxed{(V_{\mathrm{PMNS}})_{ik} m_k^\nu (V_{\mathrm{PMNS}}^T)_{kj} = v^2 Y_{ik}^\nu \left(M_\nu^{-1}\right)_{kl} Y_{jl}^\nu}$$

- Assume: (i) M_ν is the diagonal form $M_\nu = \mathrm{diag}(M_{\nu 1}, M_{\nu 2}, M_{\nu 3})$

  (ii) Y^ν is a hermitian matrix

- We can calculate the elements of Y^ν from observables

  $\Delta m_{12}^2, \Delta m_{13}^2, \vartheta_{12}, \ldots$



4-fermi couplings

- Integrating out the heavy scalars, we obtain

$$\mathcal{L}_{eff} = \frac{1}{M_{H_A}^2} Y^{f\,\dagger}_{A\,ij} Y^{F}_{A\,kl} \left(\overline{f^i_R} f^j_L\right)\left(\overline{F^k_L} F^l_R\right) \qquad f, F = u, d, e, \nu$$

$$\hat{U}_H^\dagger M_H^2 \hat{U}_H = \mathrm{diag}(-\mu_h^2, M_{H_1}^2, M_{H_2}^2, M_{H_3}^2) \qquad \hat{H}_I = U_{Ih}\, h + U_{IA}\, H_A$$

M_H^2: mass matrix for Higgs doublets, $\hat{U}_H = (U_{Ih}, U_{I1}, U_{I2}, U_{I3})$

$$(C_4^d)^{ij}_{kl} = \begin{pmatrix} Y^{u\,\dagger}_{ij} & y^d_i \delta_{ij} \end{pmatrix} \begin{pmatrix} \Lambda^{-2}_{uu} & \Lambda^{-2}_{ud} \\ \Lambda^{-2}_{du} & \Lambda^{-2}_{dd} \end{pmatrix} \begin{pmatrix} Y^u_{kl} \\ y^d_k \delta_{kl} \end{pmatrix} \quad \text{for} \quad (\overline{d^i_R} d^j_L)(\overline{d^k_L} d^l_R)$$

$$(C_4^e)^{ij}_{kl} = \begin{pmatrix} Y^{\nu\,\dagger}_{ij} & y^e_i \delta_{ij} \end{pmatrix} \begin{pmatrix} \Lambda^{-2}_{uu} & \Lambda^{-2}_{ud} \\ \Lambda^{-2}_{du} & \Lambda^{-2}_{dd} \end{pmatrix} \begin{pmatrix} Y^\nu_{kl} \\ y^e_k \delta_{kl} \end{pmatrix} \quad \text{for} \quad (\overline{e^i_R} e^j_L)(\overline{e^k_L} e^l_R)$$

$$\vdots$$

$$|\Delta_h|^2 \Lambda^{-2}_{uu} = (M_H^{-2})_{33} |U_{4h}|^2 + (M_H^{-2})_{44} |U_{3h}|^2 - (M_H^{-2})_{34} U^*_{3h} U_{4h} - (M_H^{-2})_{43} U^*_{4h} U_{3h}$$
$$|\Delta_h|^2 \Lambda^{-2}_{ud} = -(M_H^{-2})_{33} U^*_{2h} U^*_{4h} - (M_H^{-2})_{44} U^*_{3h} U^*_{1h} + (M_H^{-2})_{34} U^*_{3h} U^*_{2h} + (M_H^{-2})_{43} U^*_{4h} U^*_{1h}$$
$$|\Delta_h|^2 \Lambda^{-2}_{du} = \Lambda^{-2\,*}_{ud}$$
$$|\Delta_h|^2 \Lambda^{-2}_{dd} = (M_H^{-2})_{33} |U_{2h}|^2 + (M_H^{-2})_{44} |U_{1h}|^2 - (M_H^{-2})_{34} U^*_{2h} U_{1h} - (M_H^{-2})_{43} U^*_{1h} U_{2h} + |\Delta_h|^2 \mu_h^{-2}$$

We treat Λ_{ab} (a, b = u, d) as the parameters of this model

Flavor Physics

K meson mixing

- Effective Hamiltonian

$$\mathcal{H}_{eff}^{\Delta F=2} = -(C_4^d)_{ij}^{ij}(\overline{d_R^i} d_L^j)(\overline{d_L^i} d_R^j) + \text{h.c.}$$

- Meson mixing → $i \neq j$, then $(C_4^d)_{ij}^{ij} = Y_{ji}^{u\,*} Y_{ij}^u \Lambda_{uu}^{-2}$

- Observables: $\epsilon_K = \dfrac{\kappa_\epsilon e^{i\varphi_\epsilon}}{\sqrt{2}(\Delta M_K)_{\text{exp}}} \text{Im}(M_{12}^K), \quad \Delta M_K = 2\text{Re}(M_{12}^K)$

$\kappa_\epsilon = 0.94 \pm 0.02, \quad \varphi_\epsilon = 0.2417 \times \pi$

$(\Delta M_K)_{\text{exp}} = 3.484(6) \times 10^{-12}$ MeV

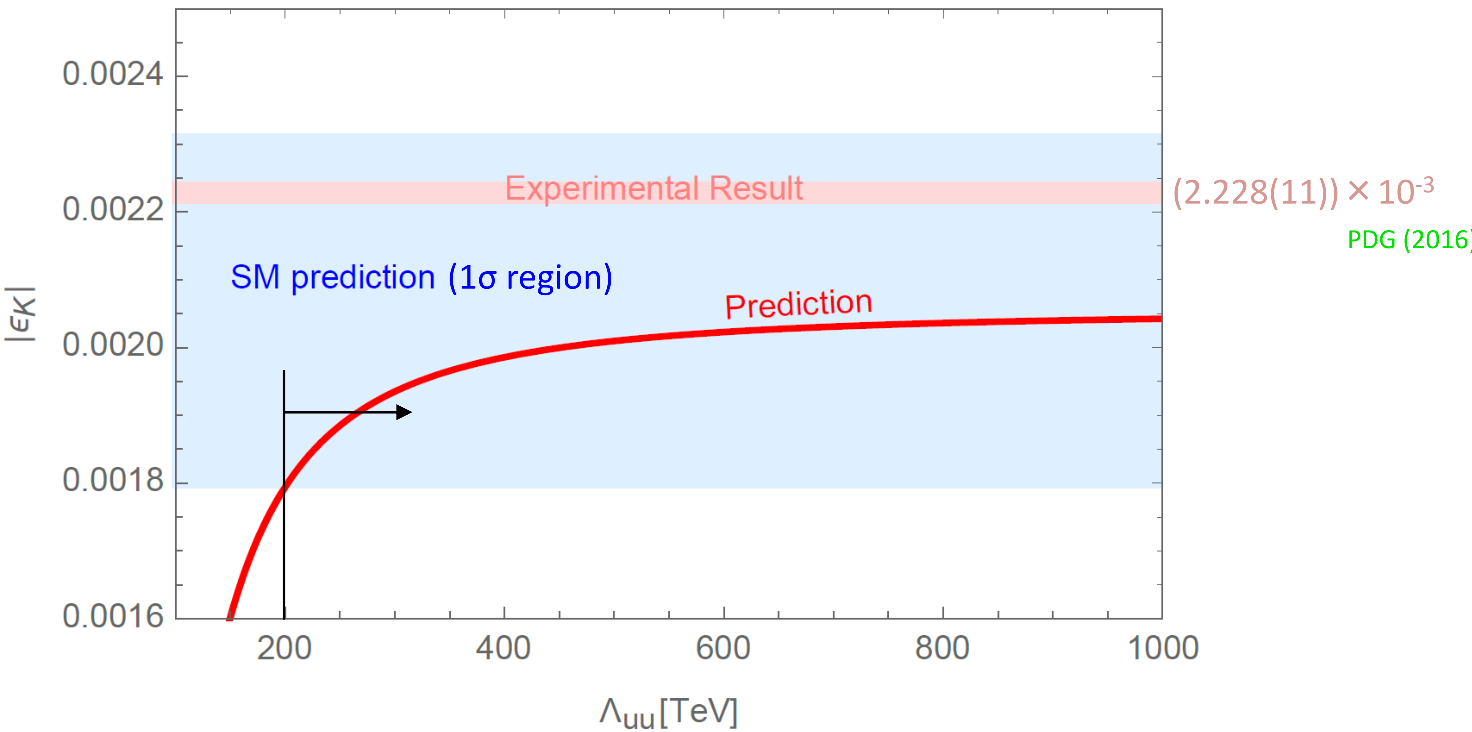
$$M_{12}^{K\,*} = (M_{12}^K)_{\text{SM}}^* \underbrace{- (C_4^d)_{sd}^{sd} \times \frac{1}{4}\left(\frac{m_K}{m_s + m_d}\right)^2 m_K F_K^2 B_4}_{\text{NP contribution from this model}}$$

$F_K = 156.1(11)$ MeV, $\quad B_4 = 0.9(2)$

$$(M_{12}^K)_{\text{SM}}^* = \frac{G_F^2}{12\pi^2} F_K^2 \hat{B}_K m_K M_W^2 \left\{ \lambda_c^2 \eta_1 S_0(x_c) + \lambda_t^2 \eta_2 S_0(x_t) + 2\lambda_c \lambda_t \eta_3 S(x_c, x_t) \right\}$$

K meson mixing

- Result



$$\boxed{\Lambda_{uu} \gtrsim 200 \text{ TeV}}$$

leptonic B meson decay

- Effective Hamiltonian

$$\mathcal{H}_{eff}^{\Delta F=1} = -(C_4^{de})_{ij}^{kl}(\overline{d_R^i}d_L^j)(\overline{e_L^k}e_R^l) - C_{\rm SM}^{ij}(\overline{d_L^i}\gamma^\mu d_L^j)(\overline{e^k}\gamma_\mu\gamma_5 e^k) + \text{h.c.}$$

$$C_{\rm SM}^{bq} = -\frac{G_F^2 M_W^2}{\pi^2}\sum_{n=c,t} V_{nb}^* V_{nq}\,\eta_Y\,\frac{x_n}{4}\left\{\frac{4-x_n}{1-x_n} + \frac{3x_n}{(1-x_n)^2}\ln x_n\right\}$$

- Wilson coefficients ($i \neq j$): $(C_4^{de})_{ij}^{kl} = Y_{ji}^{u\,*}Y_{kl}^{\nu}\Lambda_{uu}^{-2} + Y_{ji}^{u\,*}y_k^e\delta_{kl}\Lambda_{ud}^{-2}$

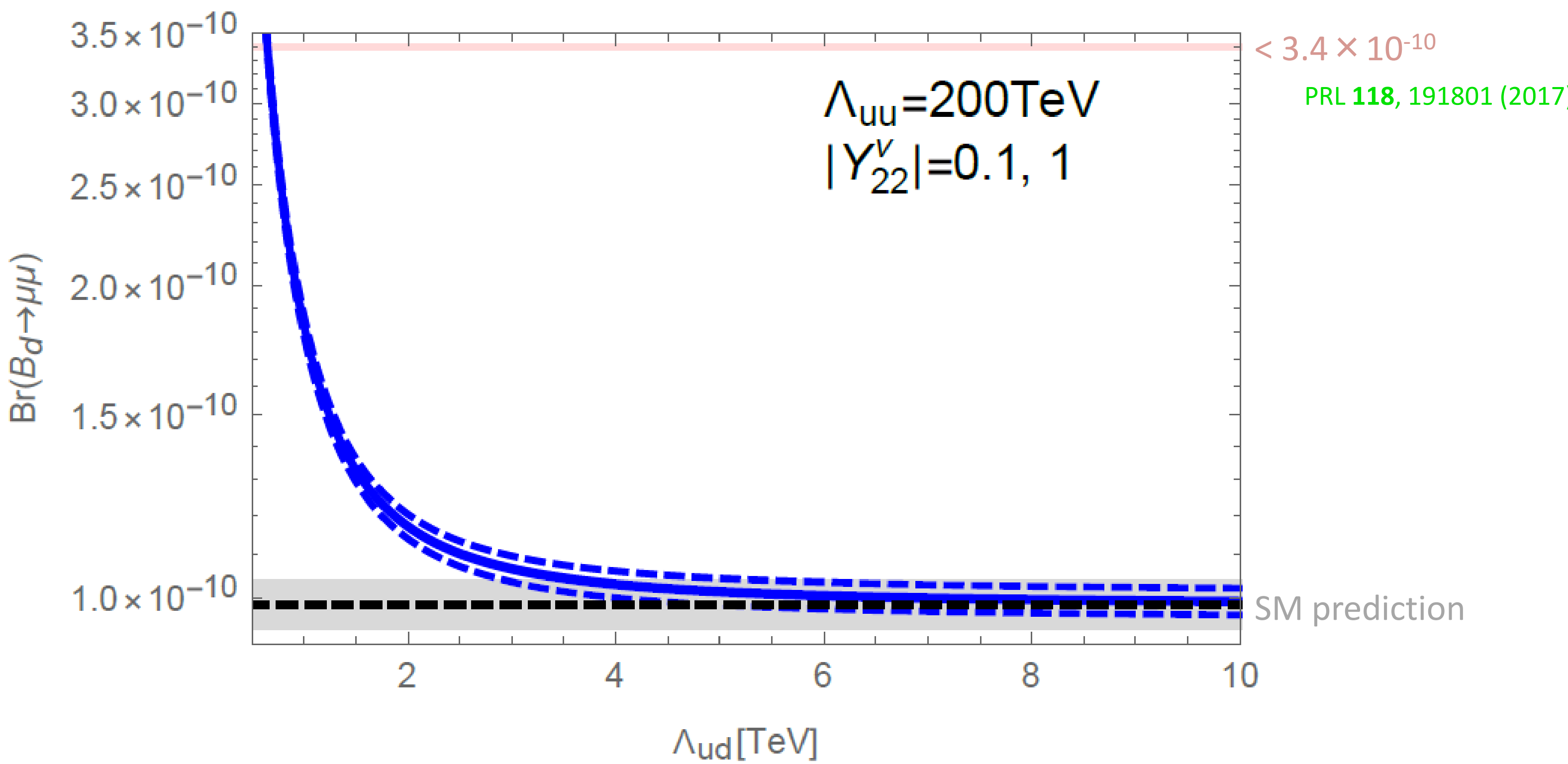
- Branching Ratio:

$$Br(B_q \to e_k\overline{e_l}) = \frac{\tau_{B_q}}{128\pi}(m_{e_k}+m_{e_l})^2 m_{B_q}F_{B_q}^2\sqrt{\left(1-\frac{(m_{e_k}+m_{e_l})^2}{m_{B_q}^2}\right)\left(1-\frac{(m_{e_k}-m_{e_l})^2}{m_{B_q}^2}\right)}$$
$$\times\left\{\left|\frac{R_{B_q}}{m_{e_k}+m_{e_l}}\{(C_4^{de})_{bq}^{kl} + (C_4^{de})_{qb}^{lk\,*}\} - \delta_{kl}\,C_{\rm SM}^{bq}\right|^2\left(1-\frac{(m_{e_k}-m_{e_l})^2}{m_{B_q}^2}\right)\right.$$
$$\left.+\left|\frac{R_{B_q}}{m_{e_k}+m_{e_l}}\{(C_4^{de})_{bq}^{kl} - (C_4^{de})_{qb}^{lk\,*}\}\right|^2\left(1-\frac{(m_{e_k}+m_{e_l})^2}{m_{B_q}^2}\right)\right\}$$

where $R_{B_q} = \dfrac{m_{B_q}^2}{m_b + m_q}$

leptonic B_d meson decay

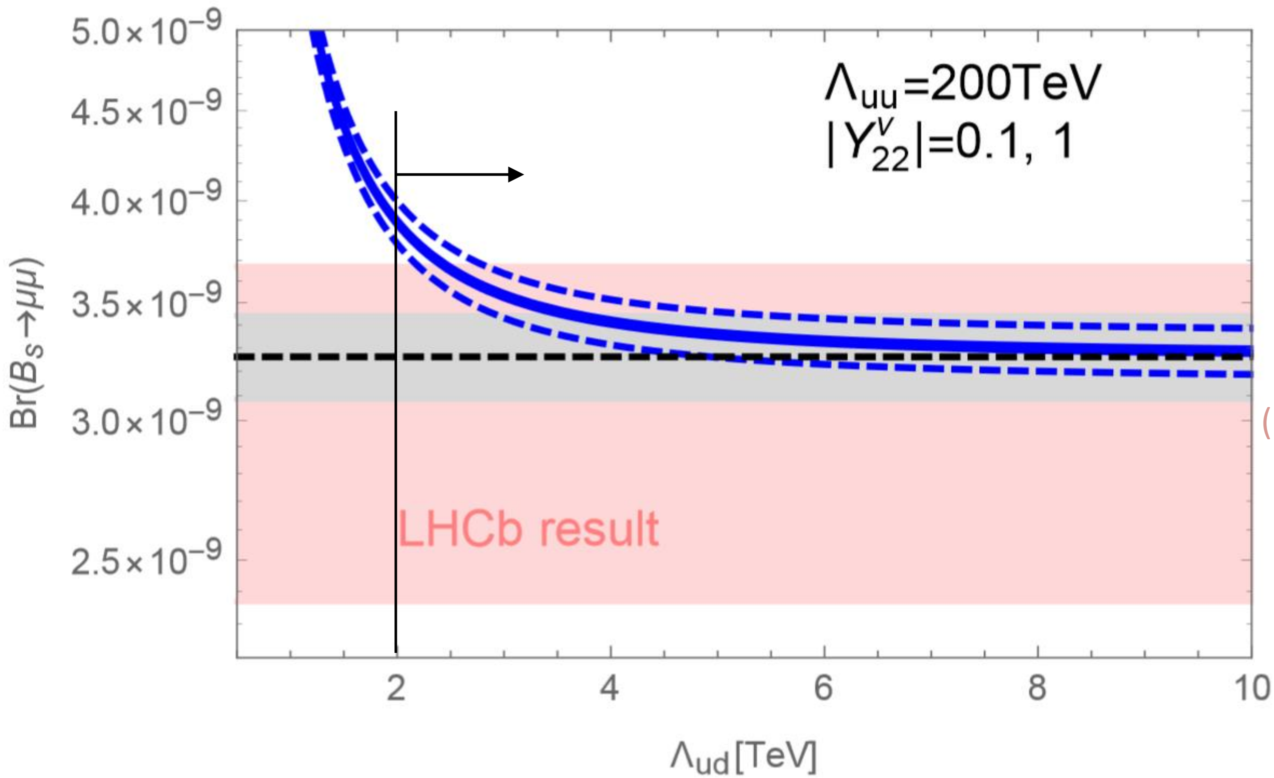
- Result



The prediction is safe even when Λ_{ud} is $O(1)$ TeV

leptonic B_s meson decay

- Result



$(3.0 \pm 0.6^{+0.3}_{-0.2}) \times 10^{-3}$

PRL **118**, 191801 (2017)

$$\boxed{|\Lambda_{ud}| \gtrsim 2\ \mathrm{TeV}}$$

lepton flavor violation (LFV) – $\ell_l \to \ell_i \bar{\ell}_j \ell_k$

- Effective Hamiltonian

$$\mathcal{H}_{eff}^{\mathrm{LFV}} = -(C_4^e)_{kl}^{ij}(\overline{e_R^i} e_L^j)(\overline{e_L^k} e_R^l)$$

- LFV → $(C_4^e)_{kl}^{ij} = Y_{ji}^{\nu\,*} Y_{kl}^{\nu} \Lambda_{uu}^{-2} + y_i^e \delta_{ij} Y_{kl}^{\nu} \Lambda_{du}^{-2} + y_k^e \delta_{kl} Y_{ji}^{\nu\,*} \Lambda_{ud}^{-2}$

- Branching Ratio for $\mu \to 3e$

$$\mathrm{BR}(\mu \to 3e) = \frac{m_\mu^5 \tau_\mu}{6144\pi^3}\left(|(C_4^e)_{12}^{11}|^2 + |(C_4^e)_{11}^{12}|^2\right)$$

- Current bound (SINDRUM exp.): $\mathrm{BR}(\mu \to 3e) < 1.0 \times 10^{-12}$

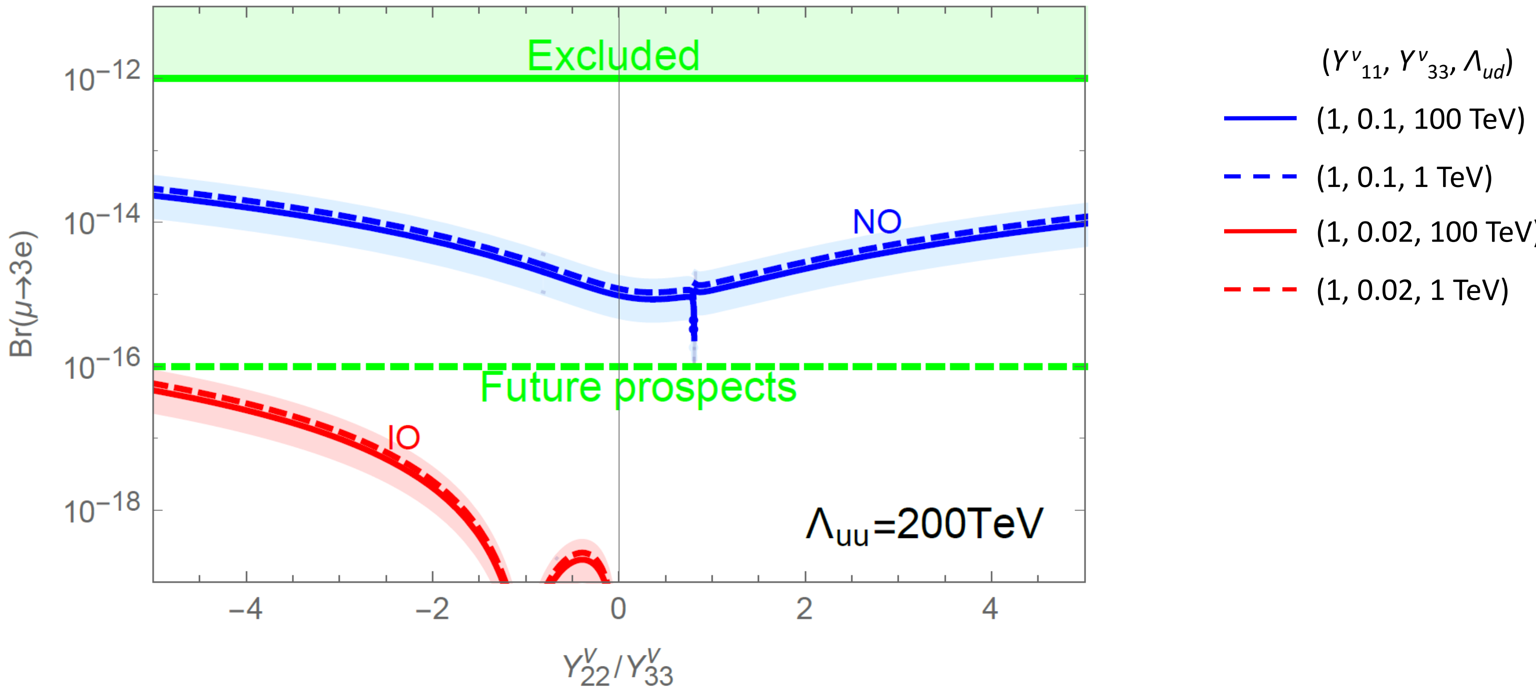
SINDRUM Collab., NPB **299**, 1 (1988)

- Future prospect (Mu3e exp.): $\mathrm{BR}(\mu \to 3e) < 1 \times 10^{-16}$

Mu3e Collab., EPJ Web Conf. **118**, 01028 (2016)

lepton flavor violation (LFV) – $\ell_l \to \ell_i \bar{\ell}_j \ell_k$

- Result



Our predictions can be covered by the future exp.
if the active neutrino is in the NO

Summary

Summary

- We discussed SUSY LR model - the multi-Higgs doublet model

  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group

  there are 4 Higgs doublets H_u^1, H_u^2, H_d^1, H_d^2

  ✓ Energy scales: TeV-scale << SUSY breaking scale << LR breaking scale

- Heavy Higgs doublets have flavor violating couplings

  relation between $Y_{Aij} \leftrightarrow Y_{ij}$: explicit predictions

- Because of the size of Y^ν and M_{HA}, NP contributions are enhanced

  even if M_{HA} is $O(100)$ TeV

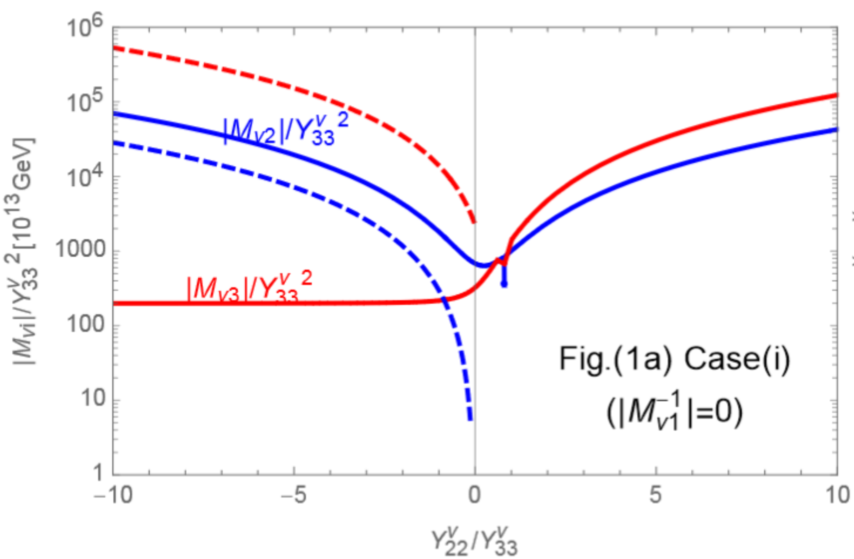
- Obtained bounds:

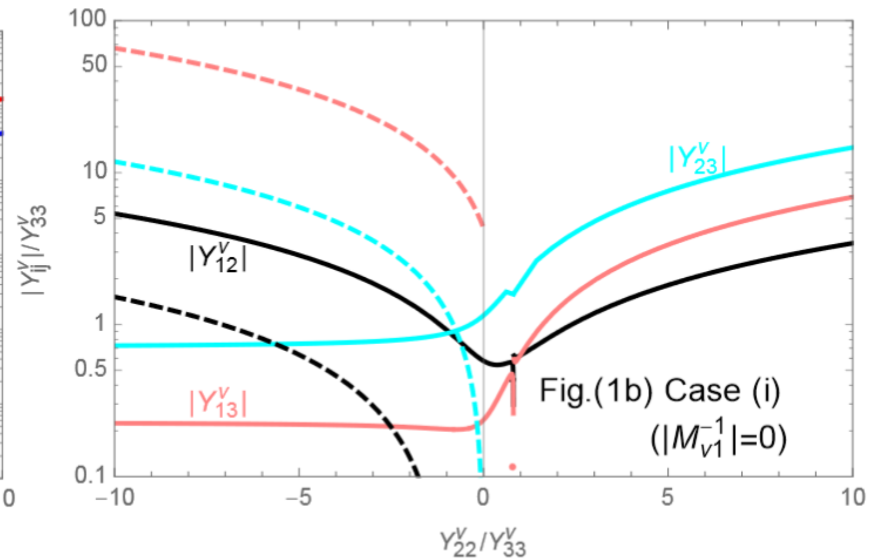
Observables	bounds
ϵ_K	$\Lambda_{uu} \gtrsim 200\,\mathrm{TeV}$
$Br(B_s \to \mu\mu)$	$\lvert\Lambda_{ud}\rvert \gtrsim 2\,\mathrm{TeV}$
$\sigma(pp \to e^+e^-)\rvert_{\sqrt{s}=13\mathrm{TeV}}$	$\left\lvert\Lambda_{du}^{(ue)}\right\rvert \gtrsim \sqrt{\lvert Y_{11}^\nu\rvert} \times 115\,\mathrm{GeV},\ \left\lvert\Lambda_{uu}^{(ue)}\right\rvert \gtrsim \sqrt{\lvert Y_{11}^\nu\rvert} \times 384\,\mathrm{GeV},$ $\lvert\Lambda_{du}\rvert \gtrsim \sqrt{\lvert Y_{11}^\nu\rvert} \times 533\,\mathrm{GeV}$
$\mu - e$ conversion	$\Lambda_{uu}^{(ue)} \gtrsim 1\,\mathrm{TeV},\ \Lambda_{du}^{(ue)} \gtrsim 2\,\mathrm{TeV}$
$Br(K \to e\nu)$	$\sqrt{\lvert\Lambda_{du}\rvert^2 + \left\lvert\Lambda_{du}^{(ue)}\right\rvert^2} \gtrsim \sqrt{\lvert Y_{11}^\nu\rvert} \times 6\,\mathrm{TeV}$

Back up

LR model – neutrino Yukawa

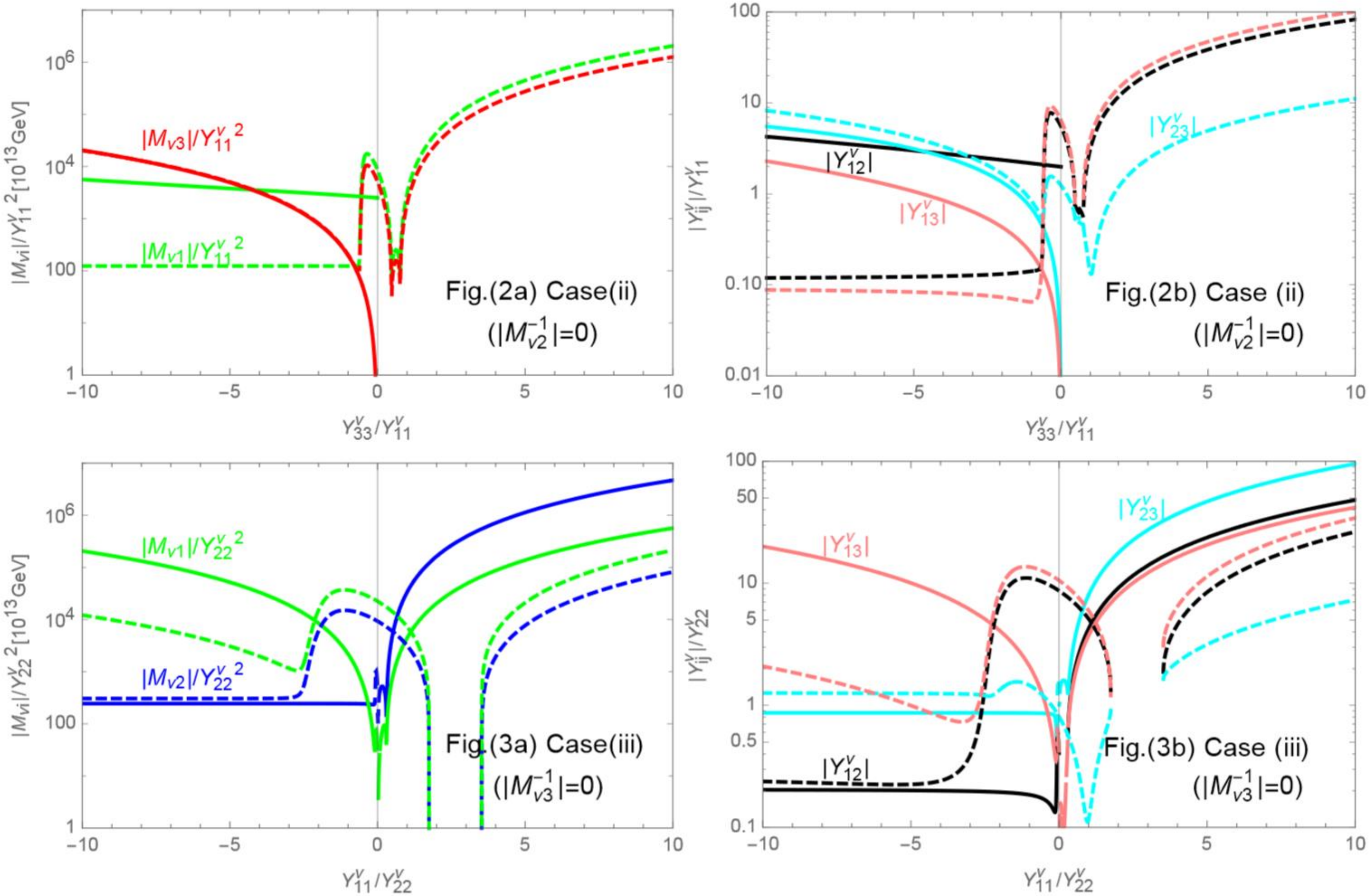
- Useful relation: $\boxed{(V_{\mathrm{PMNS}})_{ik} m_k^\nu (V_{\mathrm{PMNS}}^T)_{kj} = v^2 Y_{ik}^\nu \left(M_\nu^{-1}\right)_{kl} Y_{jl}^\nu}$
- Normalize the RHS with $Y^\nu{}_{33}$ → $v^2 \frac{Y_{ik}^\nu}{Y_{33}^\nu} \frac{Y_{33}^{\nu\,2}}{M_{\nu k}} \frac{Y_{jk}^\nu}{Y_{33}^\nu}$
- We can solve above equations with variables, $\frac{Y_{ij}^\nu}{Y_{33}^\nu}$ and $\frac{Y_{33}^{\nu\,2}}{M_{\nu i}}$





LR model – neutrino Yukawa

- Different Majorana mass case



lepton flavor violation (LFV) – μ-e conversion

- Effective Lagrangian

$$\mathcal{L}_{eff}^{\mu-\mathrm{e}} = \sum_{q=d,s} (C_4^{de})_{qq}^{e\mu} (\overline{q_R} q_L)(\overline{e_L}\mu_R) + (C_4^{de})_{qq}^{\mu e\,*} (\overline{q_L} q_R)(\overline{e_R}\mu_L)$$

$$+(C_4^{ue})_{uu}^{e\mu} (\overline{u_L} u_R)(\overline{e_L}\mu_R) + (C_4^{ue})_{uu}^{\mu e\,*} (\overline{u_R} u_L)(\overline{e_R}\mu_L)$$

- 4-fermi couplings $(C_4^{ue})_{ij}^{kl} = \begin{pmatrix} y_i^u \delta_{ij} & (V y^d V_R^\dagger)_{ij} \end{pmatrix} \begin{pmatrix} \left(\Lambda_{uu}^{(ue)}\right)^{-2} & \left(\Lambda_{ud}^{(ue)}\right)^{-2} \\ \left(\Lambda_{du}^{(ue)}\right)^{-2} & \left(\Lambda_{dd}^{(ue)}\right)^{-2} \end{pmatrix} \begin{pmatrix} (Y^\nu)_{kl} \\ y_k^e \delta_{kl} \end{pmatrix}$

$$|\Delta_h|^2 \left(\Lambda_{uu}^{(ue)}\right)^{-2} = (M_H^{-2})_{31} U_{4h}^2 + (M_H^{-2})_{42} U_{3h}^2 - (M_H^{-2})_{32} U_{3h} U_{4h} - (M_H^{-2})_{41} U_{4h} U_{3h}$$

$$|\Delta_h|^2 \left(\Lambda_{ud}^{(ue)}\right)^{-2} = -(M_H^{-2})_{31} U_{4h} U_{2h}^* - (M_H^{-2})_{42} U_{3h} U_{1h}^* + (M_H^{-2})_{32} U_{3h} U_{2h}^* + (M_H^{-2})_{41} U_{4h} U_{1h}^* + |\Delta_h|^2 \mu_h^{-2}$$

$$|\Delta_h|^2 \left(\Lambda_{du}^{(ue)}\right)^{-2} = -(M_H^{-2})_{31} U_{4h} U_{2h}^* - (M_H^{-2})_{42} U_{3h} U_{1h}^* + (M_H^{-2})_{32} U_{4h} U_{1h}^* + (M_H^{-2})_{41} U_{3h} U_{2h}^*$$

$$|\Delta_h|^2 \left(\Lambda_{dd}^{(ue)}\right)^{-2} = (M_H^{-2})_{31} U_{2h}^{*2} + (M_H^{-2})_{42} U_{1h}^{*2} - (M_H^{-2})_{32} U_{1h}^* U_{2h}^* - (M_H^{-2})_{41} U_{1h}^* U_{2h}^*$$

- SINDRUM II bound: $\Gamma(\mu\,\mathrm{Au} \rightarrow e\,\mathrm{Au}) / \Gamma_{cap}(\mu\,\mathrm{Au}) < 7 \times 10^{-13}$

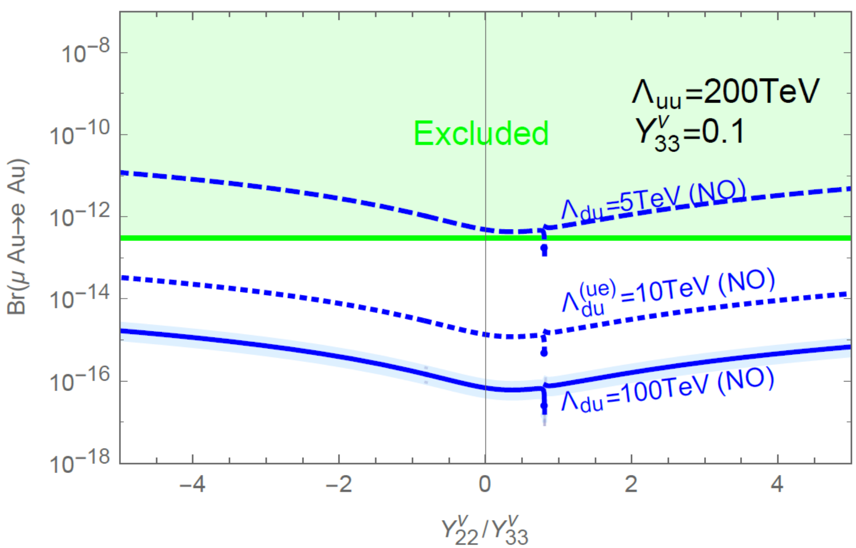
SINDRUM II Collab., EPJC **47**, 337 (2006)

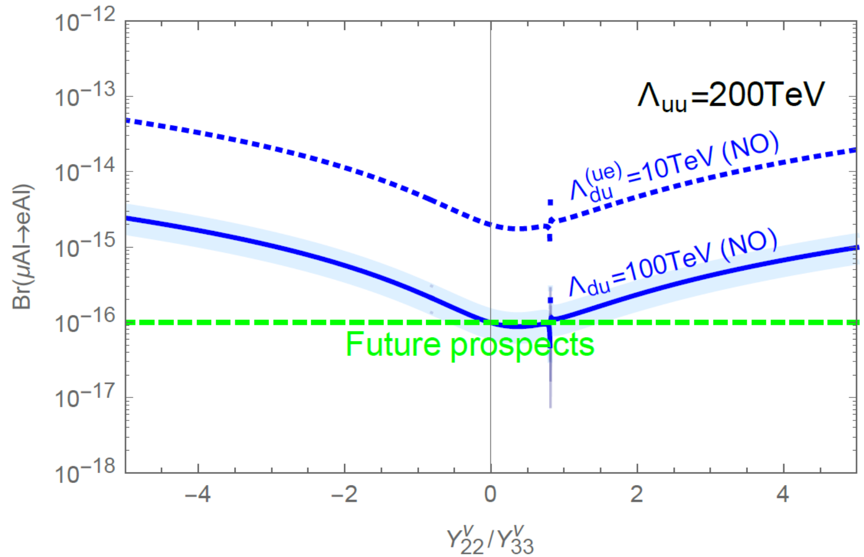
- Future prospect (COMET): $\sim 10^{-16}$ Level (in Al)

COMET Collab., PTEP **2013**, 022C01 (2013)

lepton flavor violation (LFV) – μ-e conversion

- Result





More severe constraint on Λ_{du}
depending on Y^{ν}_{ij}

Future sensitivity covers
the $O(0.1)$ Yukawa couplings region

From these analyses, we obtain the lower bounds

$$\Lambda^{(ue)}_{uu} \gtrsim 1\,\mathrm{TeV},\ \Lambda^{(ue)}_{du} \gtrsim 2\,\mathrm{TeV}$$

when Y^{ν}_{33} is $O(0.1)$

Leptonic meson decay with active ν

- Effective Hamiltonian

$$\mathcal{H}^C_{eff} = -(\widetilde{C}^{de}_4)^{kl}_{ij}(\overline{d^i_R}u^j_L)(\overline{\nu^k_L}e^l_R) - (\widetilde{C}^{ue}_4)^{kl}_{ij}\left(\overline{d^i_L}u^j_R\right)\left(\overline{\nu^k_L}e^l_R\right) + h.c.$$

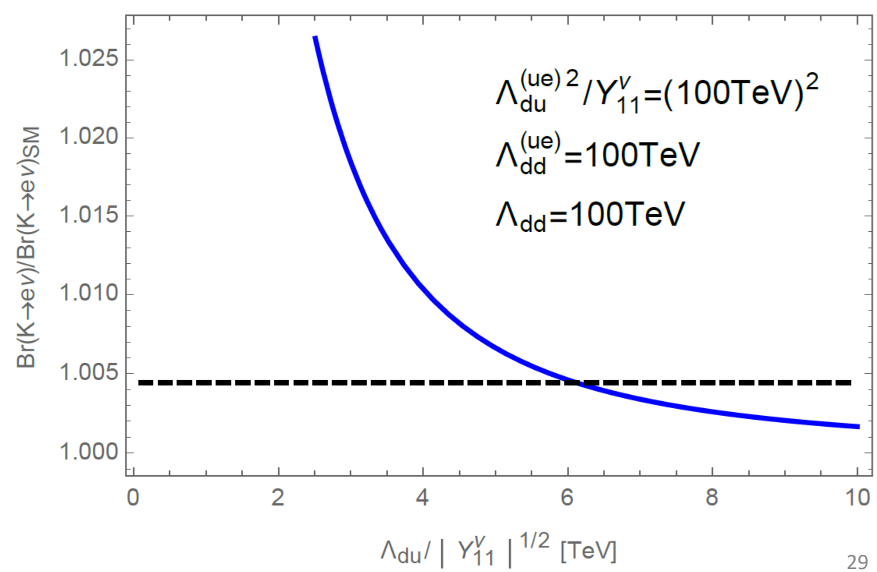
4-fermi couplings: $(\widetilde{C}^{de}_4)^{kl}_{ij} = (C^{de}_4)^{k'l}_{ij'} V^*_{jj'} (V_{PMNS})^*_{k'k}, \; (\widetilde{C}^{ue}_4)^{kl}_{ij} = -(C^{ue}_4)^{k'l}_{i'j} V^*_{i'i} (V_{PMNS})^*_{k'k}$

- Branching ratio

$$\frac{Br(B_q \to e_l \nu)}{Br_{\mathrm{SM}}(B_q)} = \left|\delta_{kl} + \frac{R_{B_q} y^b_d}{m_{e_l}(4G_F/\sqrt{2})}(\Lambda^{-2}_{du} + \Lambda^{(ue)-2}_{du})Y^\nu_{kl} + (\Lambda^{-2}_{dd} + \Lambda^{(ue)-2}_{dd})y^e_l \delta_{kl}\right|^2$$

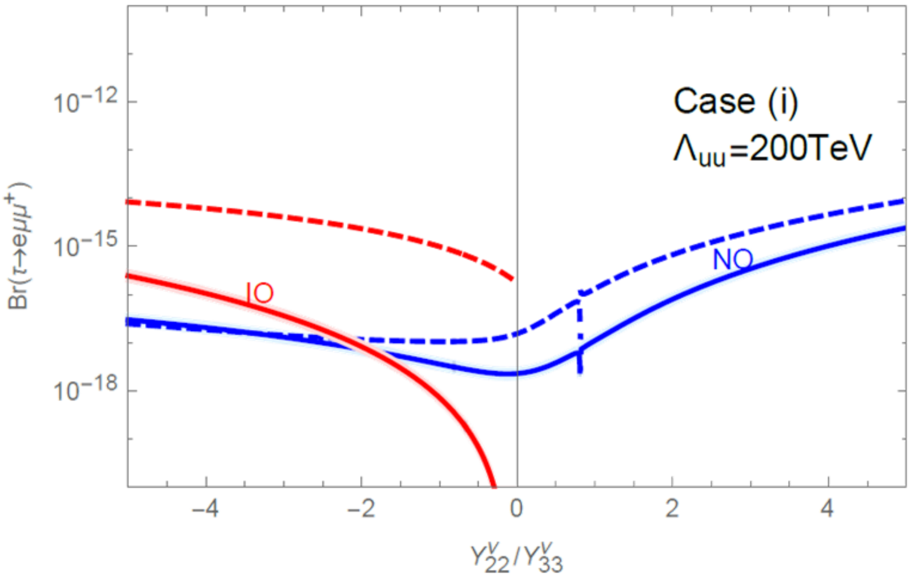
- BR of *K*→*eν* can be obtained by replacing $R_{B_q} y^b_d$ with $R_K y^s_d$

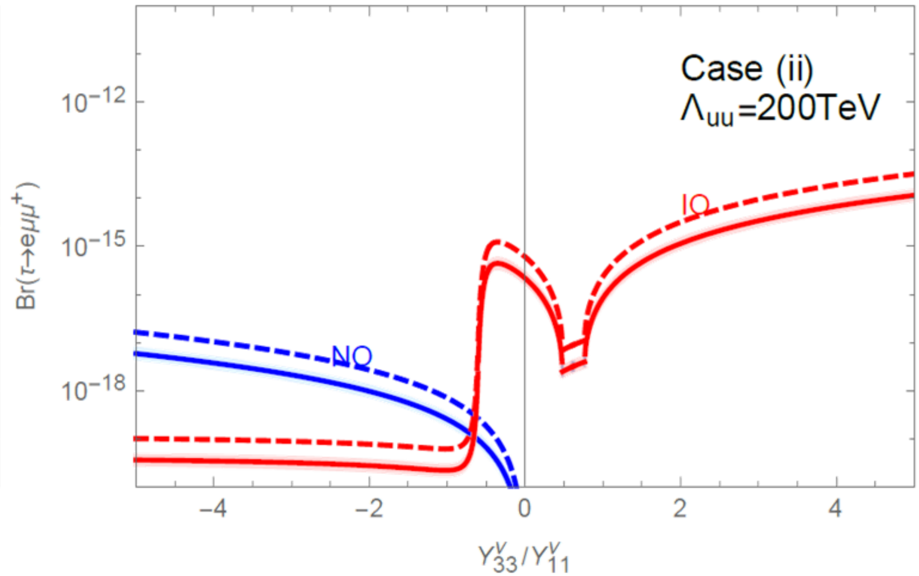
$$\boxed{\sqrt{|\Lambda_{du}|^2 + \left|\Lambda^{(ue)}_{du}\right|^2} / \sqrt{|Y^\nu_{11}|} \gtrsim 6\,\mathrm{TeV}}$$



Tau decays

- $\tau \to 3l$ [bounds: $O(10^{-8})$, future: $(3\text{-}5)\times 10^{-10}$]





- Hadronic τ decay

$$Br(\tau \to l M_{qq'}) = \frac{3}{32\pi}\left|(C_4^{de})_{qq'}^{l\tau}\right|^2 m_\tau \tau_\tau R_{M_{qq'}}^2 F_{M_{qq'}}^2 \left(1 - \frac{m_{M_{qq'}}^2}{m_\tau^2}\right)^2$$

→ sufficiently suppress when Λ_{uu} and Λ_{ud} are large

SM predictions for meson mixing

$$(M_{12}^K)_{\rm SM} = \frac{G_F^2}{12\pi^2} F_K^2 \hat{B}_K m_K M_W^2 \left\{\lambda_c^2 \eta_1 S_0(x_c) + \lambda_t^2 \eta_2 S_0(x_t) + 2\lambda_c \lambda_t \eta_3 S(x_c, x_t)\right\}$$

x_i and λ_i denote m_i^2/M_W^2 and $(V_{\rm CKM})_{is}^*(V_{\rm CKM})_{id}$

$$(M_{12}^{B_q})_{\rm SM} = \frac{G_F^2}{12\pi^2} F_{B_q}^2 \hat{B}_{B_q} m_{B_q} M_W^2 \lambda_{B_q}^2 \eta_B S_0(x_t)$$

$\lambda_{B_q} = (V_{\rm CKM})_{tb}^*(V_{\rm CKM})_{tq}$

loop func.

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \log x}{2(1-x)^3}$$

$$S(x,y) = \frac{-3xy}{4(y-1)(x-1)} - \frac{xy(4-8y+y^2)\log y}{4(y-1)^2(x-y)} + \frac{xy(4-8x+x^2)\log x}{4(x-1)^2(x-y)}$$

Input parameters

m_e	0.511 MeV [39]	$\sin^2\theta_{12}$	$0.321^{+0.018}_{-0.016}$ [40]
m_μ	105.658 MeV [39]	$\sin^2\theta_{23}$ (NO)	$0.430^{+0.020}_{-0.018}$ [40]
m_τ	1776.86 ± 0.12 MeV [39]	$\sin^2\theta_{23}$ (IO)	$0.596^{+0.017}_{-0.018}$ [40]
τ_μ	3.33781×10^{15} MeV^{-1} [39]	$\sin^2\theta_{13}$ (NO)	$0.02155^{+0.00090}_{-0.00075}$ [40]
τ_τ	$(441.0\pm0.8)\times10^{6}$ MeV^{-1} [39]	$\sin^2\theta_{13}$ (IO)	$0.02140^{+0.00082}_{-0.00085}$ [40]
Δm^2_{12}	$(7.56\pm0.19)\times10^{-5}$ eV^2 [40]	δ/π (NO)	$1.40^{+0.31}_{-0.20}$ [40]
$\lvert\Delta m^2_{13}\rvert$ (NO)	$(2.55\pm0.04)\times10^{-3}$ eV^2 [40]	δ/π (IO)	$1.44^{+0.26}_{-0.23}$ [40]
$\lvert\Delta m^2_{13}\rvert$ (IO)	$(2.49\pm0.04)\times10^{-3}$ eV^2 [40]		

$m_d(2\text{ GeV})$	$4.8^{+0.5}_{-0.3}$ MeV [39]	λ	$0.22509^{+0.00029}_{-0.00028}$ [41]
$m_s(2\text{ GeV})$	95 ± 5 MeV [39]	A	$0.8250^{+0.0071}_{-0.0111}$ [41]
$m_b(m_b)$	4.18 ± 0.03 GeV [39]	$\overline{\rho}$	$0.1598^{+0.0076}_{-0.0072}$ [41]
$\frac{2m_s}{(m_u+m_d)}(2\text{ GeV})$	27.5 ± 1.0 [39]	$\overline{\eta}$	$0.3499^{+0.0063}_{-0.0061}$ [41]
$m_c(m_c)$	1.275 ± 0.025 GeV [39]	M_Z	$91.1876(21)$ GeV [39]
$m_t(m_t)$	160^{+5}_{-4} GeV [39]	M_W	$80.385(15)$ GeV [39]
α	$1/137.036$ [39]	G_F	$1.1663787(6)\times10^{-5}$ GeV^{-2} [39]
$\alpha_s(M_Z)$	$0.1182(12)$ [39]		

References: [39] PDG
[40] P. F. de Salas, et. al., arXiv:1708.01186
[41] CKMfitter

Input parameters

m_K	$497.611(13)$ MeV [39]	m_{B_s}	$5.3663(6)$ GeV [39]
τ_{K_L}	7.78×10^{13} MeV^{-1} [39]	τ_B	2.31×10^{12} GeV^{-1} [39]
τ_{K_S}	1.36×10^{11} MeV^{-1} [39]	τ_{B_s}	2.30×10^{12} GeV^{-1} [39]
F_K	$156.1(11)$ MeV [59]	m_B	$5.2795(3)$ GeV [39]
$\hat{B}_K$	$0.764(10)$ [59]	F_{B_s}	227.7 ± 6.2 MeV [59]
$(\Delta M_K)_{\rm exp}$	$3.484(6)\times10^{-12}$ MeV [39]	F_B	190.6 ± 4.6 MeV [59]
$\lvert\epsilon_K\rvert$	$(2.228(11)) \times 10^{-3}$ [39]	$\hat{B}_{B_s}$	$1.33(6)$ [59]
η_1	$1.87(76)$ [60]	$\hat{B}_B$	$1.26(11)$ [59]
η_2	$0.5765(65)$ [61]	η_B	$0.55(1)$ [61]
η_3	$0.496(47)$ [62]	B_4^d	$1.15(13)$ [63]
B_4	$0.9(2)$ [63]	B_4^s	$1.15(13)$ [63]

References:
[39] PDG
[59] J. Laiho et al., PRD **81** (2010) 034503
[60] J. Brod and M. Gorbahn, PRL **108** (2012) 121801
[61] A. J. Buras et al., NPB **347** (1990) 491
[62] J. Brod and M. Gorbahn, PRD **82** (2012) 094026
[63] V. Lubicz and C. Tarantino, Nuovo Cim. B **123** (2008) 674

Introduction

- SO(10) SUSY GUT is interesting

16 rep.

10 rep.

$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$

$u_R^c \quad e_R^c$

5bar rep.

$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

d_R^c

SU(5) singlet

ν_R^c

SU(5)

SO(10)

→ one generation of fermion is unified into 16 rep.

right-handed neutrino is naturally introduced

- Yukawa couplings: only one coupling, h_{ij}

$$W_Y = h_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H$$

$\mathbf{10}_H \supset H_u, H_d$

$$\Rightarrow \quad Y_u = Y_d = Y_e^T$$

@ GUT scale, M_{GUT}

⚠ conflict with the fermion masses and mixings

Introduction

- How avoid the relation?
  - ✓ Include additional Higgs fields
  - ✓ Include additional matter fields
  - ✓ Include higher dimensional operators
  - ✓ …

SO(10) → SU(5)

$$\mathbf{10} = \mathbf{5} + \bar{\mathbf{5}}$$

- 5bar fields mix with each other

$$\begin{pmatrix} f^{\rm SM} \\ f^h \end{pmatrix} = \begin{pmatrix} \hat{U}_{16}^f & \Delta U_f \\ \Delta U_f' & \hat{U}_{10}^f \end{pmatrix} \begin{pmatrix} f^{(16)} \\ f^{(10)} \end{pmatrix}$$

crucial point

- Because of this mixing, we can obtain realistic Yukawa couplings

e.g. down-type Yukawa couplings

$$(Y_d)_{ij} = (\hat{U}_{16}^d)_{ik} \left[(Y_u)_{kj} + \epsilon \, c_{kj}^d \right]$$

SO(10) GUT

- Higher dimensional operators

$$\frac{h'_{ij}}{\Lambda}\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{16}}_H\overline{\mathbf{16}}_H + \frac{h''_{ij}}{\Lambda}\mathbf{16}_i\mathbf{45}_A\mathbf{16}_j\mathbf{10}_H + \cdots$$

- Additional Higgs

$$Y^{(126)}_{ij}\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{126}}_H + Y^{(120)}_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{120}_H$$

$$\mathbf{16}\otimes\mathbf{16} = \mathbf{10}\oplus\mathbf{120}\oplus\mathbf{126}$$

$\Downarrow$

can be obtained realistic fermion masses and mixings