Kobayashi-Maskawa Institute for the Origin of Particles and the Universe

PPP2018@YITP, 6-10 Aug 2018

False vacuum decay in gauge theory

~Standard model and beyond~

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(KMI, Nagoya U.)

Phys. Lett. B771(2017)281; M. Endo, T. Moroi, M. M. Nojiri, YS
JHEP11(2017)074; M. Endo, T. Moroi, M. M. Nojiri, YS
Phys. Rev. Lett. 119 (2017) no.21, 211801; S. Chigusa, T. Moroi, YS
Phys. Rev. D97 (2018) no.11, 116012; S. Chigusa, T. Moroi, YS

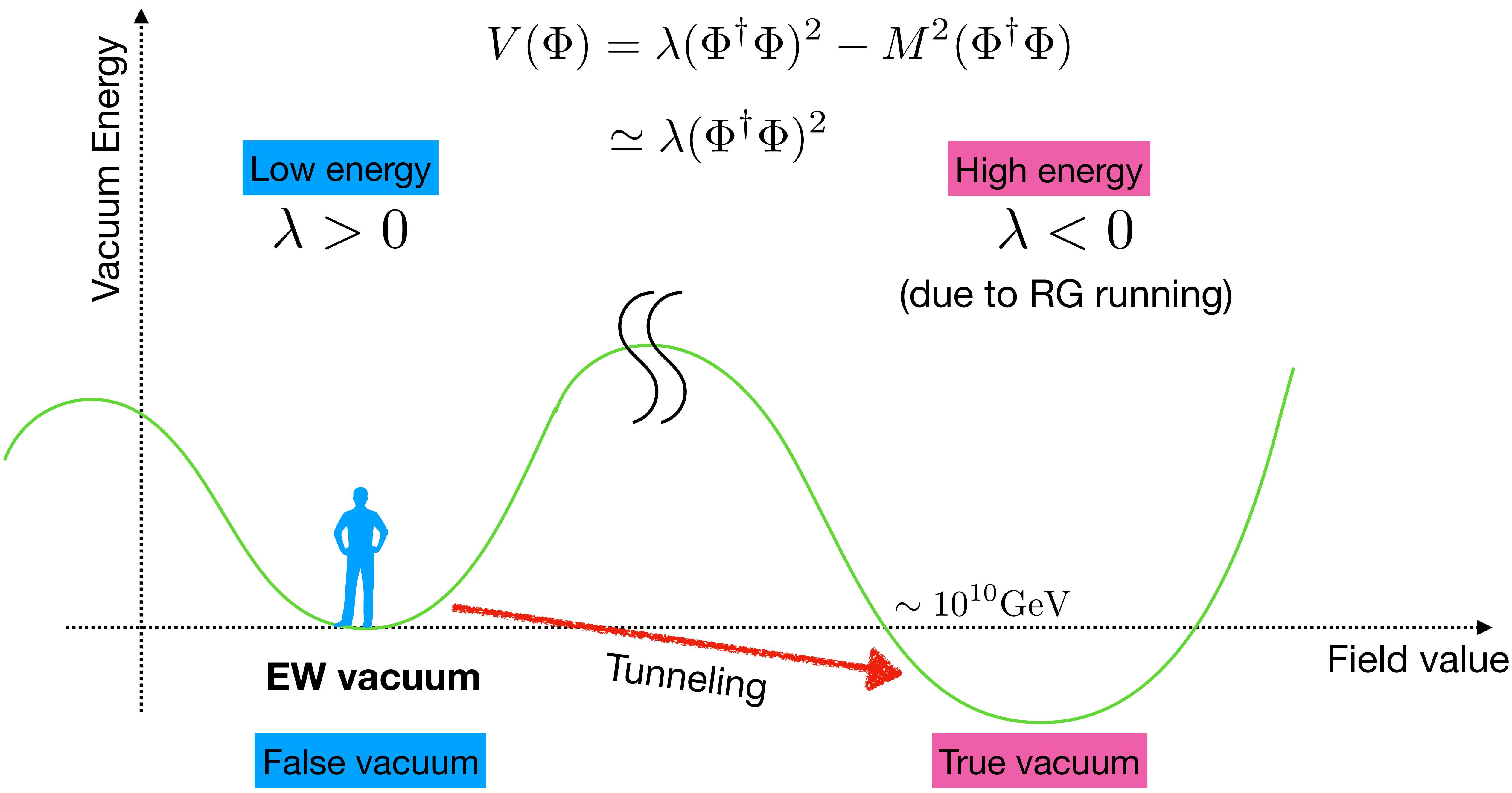
https://github.com/YShoji-HEP/ELVAS

Introduction

Vacuum decay in the standard model

$$V(\Phi) = \lambda(\Phi^\dagger\Phi)^2 - M^2(\Phi^\dagger\Phi)$$

$$\simeq \lambda(\Phi^\dagger\Phi)^2$$



Bubble nucleation rate

[C. G. Callan, S. R. Coleman, '77]

A rate to have a bubble in unit volume

$$\gamma = Ae^{-B}$$

$$B = S_E(\bar{\phi})$$

$$= \frac{8\pi^2}{3|\lambda|}$$

Bounce solution
(Fubini-Lipatov instanton)

$$\bar{\phi}(r) = \sqrt{\frac{8}{|\lambda|}} \frac{R}{R^2 + r^2}$$

for a negative λ

Independent of R
(classical scale invariance)

R : Size of the instanton

Bubble nucleation rate

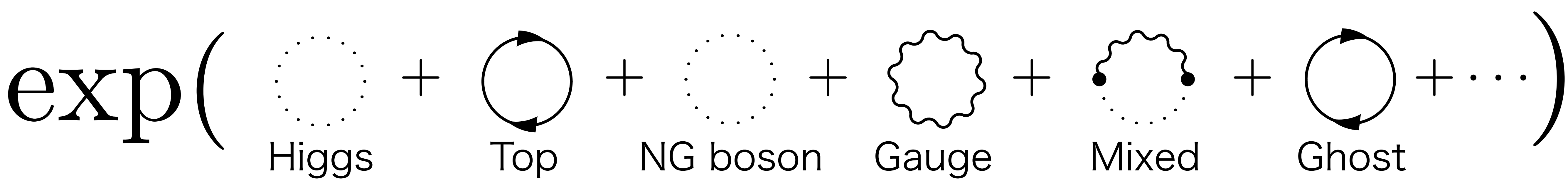
[C. G. Callan, S. R. Coleman, '77]

A rate to have a bubble in unit volume

$$\gamma = Ae^{-B}$$

Quantum correction to B

$$A \sim \left(\frac{\det S''_E|_{\mathrm{bounce}}}{\det S''_E|_{\mathrm{false}}} \right)^{-1/2}$$



Typical scale?

A rate to have a bubble in unit volume

$$\gamma = Ae^{-B}$$

$$A \sim \left(\frac{\det S_E''|_{\text{bounce}}}{\det S_E''|_{\text{false}}} \right)^{-1/2}$$

Dimensional analysis

$$A \sim M^4$$

Typical scale

What is the typical scale?

SM is classically scale invariant

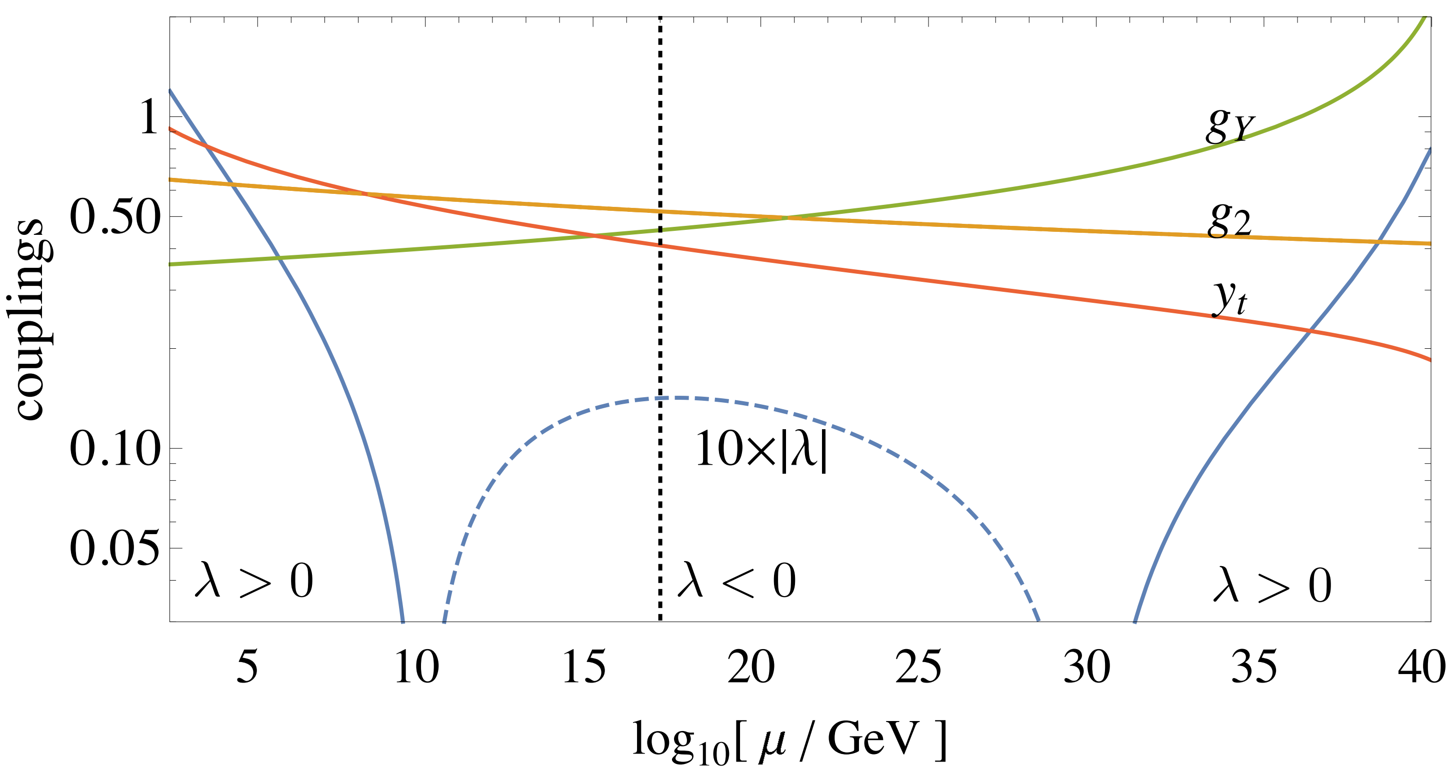
Any size of bounce is allowed

Renormalization scale?

A rate to have a bubble in unit volume

$$\gamma = Ae^{-B}$$

$$B = S_E(\bar{\phi}) = \frac{8\pi^2}{3|\lambda|}$$



What is the renormalization scale for λ?

Calculate A!

A rate to have a bubble in unit volume

$$\gamma = Ae^{-B}$$

$$A \sim \left(\frac{\det S''_E|_{\text{bounce}}}{\det S''_E|_{\text{false}}} \right)^{-1/2} \qquad B = S_E(\bar{\phi}) = \frac{8\pi^2}{3|\lambda|}$$

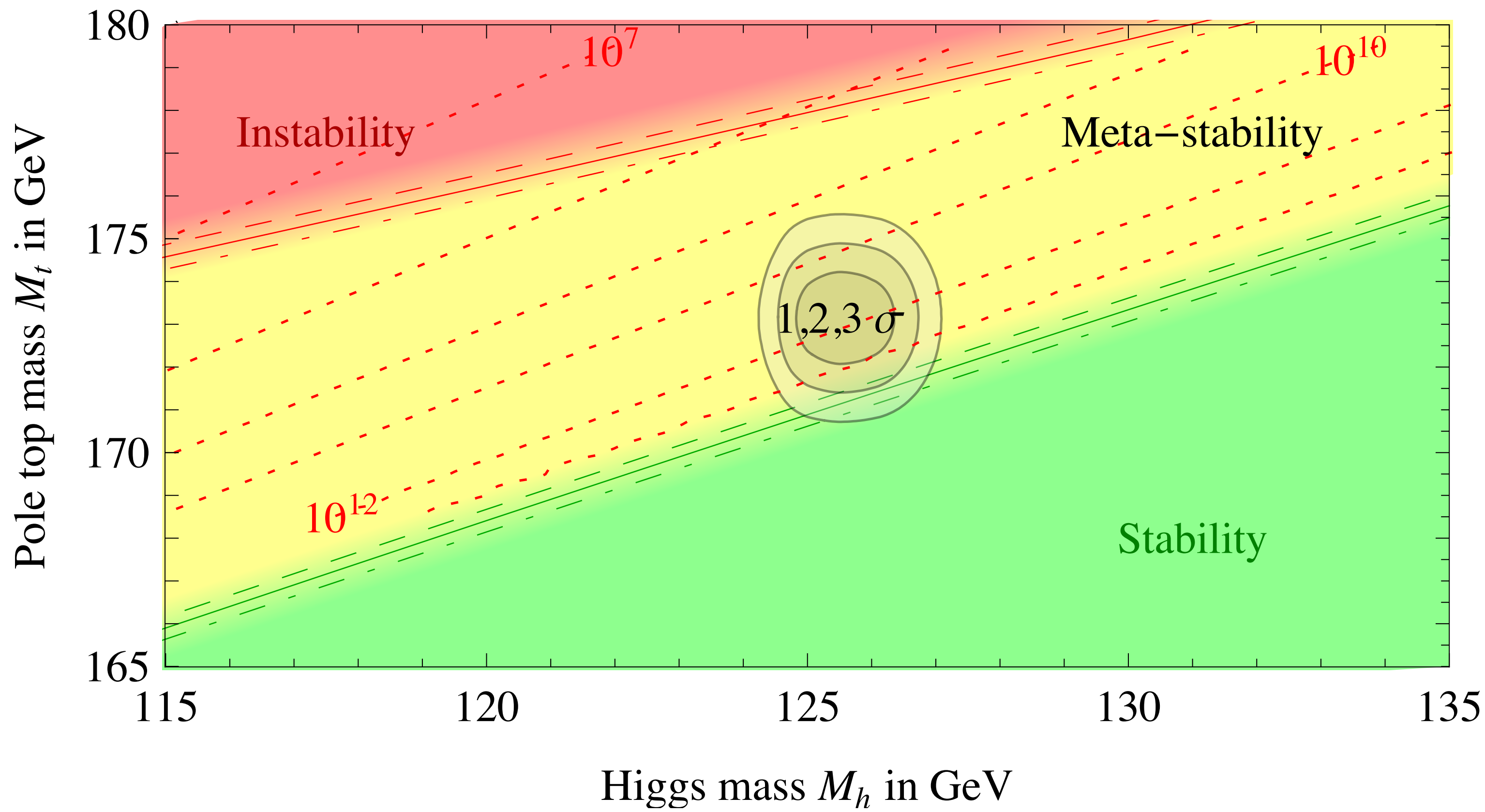
If we calculate A, we do not have such problems

We do not need "typical scale"

The renormalization scale uncertainty is canceled

Standard Model @ one-loop

G. Ishidori, G. Ridolfi, A. Strumia; '01



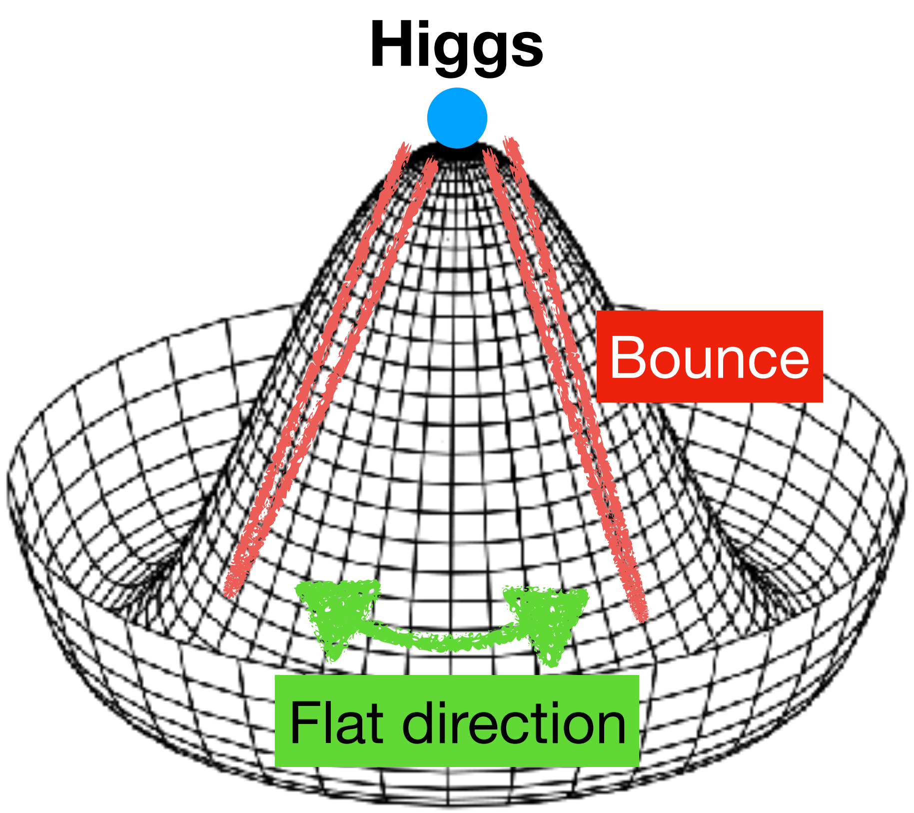
However,

There were no established ways to subtract dilatational and gauge zero modes

They use an approx. for the gauge zero mode subtraction, which is gauge dependent

Gauge zero mode

[A. Kusenko, K. M. Lee, E. J. Weinberg; '97]



$$\det S_E''\big|_{\mathrm{bounce}} = 0$$

However,

There were no established ways to subtract dilatational and gauge zero modes

They use an approx. for the gauge zero mode subtraction, which is gauge dependent

→ **We have found the correct treatment of the gauge zero mode**

JHEP 1711 (2017) 074, M. Endo, T. Moroi, M. M. Nojiri, YS

Contents

- Introduction
- Gauge invariant analytic results
  - Standard Model
- ELVAS (c++ package for ELectroweak VAcuum Stability)
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- Summary

Gauge invariant analytic result

Improvement

$$\gamma = \int dR \frac{d\gamma}{dR}$$

We improve the treatment of the R-integral

3

$$\frac{d\gamma}{dR} = e^{-\frac{8\pi^2}{3|\lambda|}} \times A'^{(h)} \times A^{(t)} \times A^{(W,Z)} \times \cdots$$

We get analytic results

2

We use a different gauge fixing so that we can treat gauge zero modes

1

Gauge fixing

NG boson

$$\mathcal{L}_{\rm GF} = (\partial_\mu A_\mu - g\bar{\phi}\chi)^2 \quad \rightarrow \quad \mathcal{L}_{\rm GF} = \frac{1}{\xi}(\partial_\mu A_\mu)^2$$

	$\mathcal{L}_{\rm GF} = (\partial_\mu A_\mu - g\bar{\phi}\chi)^2$	$\mathcal{L}_{\rm GF} = \frac{1}{\xi}(\partial_\mu A_\mu)^2$
Numerical Evaluation	✓	✗
Treatment of gauge zero modes	✗	✓
Semi-analytical Evaluation	✓	✓

Phys. Lett. B771(2017)281

JHEP11(2017)074

Gauge contribution

$$A^{(A_\mu,\varphi)} = \prod_{J=0}^{\infty} \left(\frac{\det \mathcal{M}_J^{(S,L,\varphi)}}{\det \widehat{\mathcal{M}}_J^{(S,L,\varphi)}} \right)^{-(2J+1)^2/2} \left(\frac{\det \mathcal{M}_J^{(T)}}{\det \widehat{\mathcal{M}}_J^{(T)}} \right)^{-(2J+1)^2}$$

$$\mathcal{M}_J^{(T)} \equiv -\Delta_J + g^2 \bar{\phi}^2$$

$$\mathcal{M}_J^{(S,L,\varphi)} \equiv \begin{pmatrix} -\Delta_J + \frac{3}{r^2} + g^2\bar{\phi}^2 & -\frac{2L}{r^2} & g\bar{\phi}' - g\bar{\phi}\partial_r \\ -\frac{2L}{r^2} & -\Delta_J - \frac{1}{r^2} + g^2\bar{\phi}^2 & -\frac{L}{r} g\bar{\phi} \\ 2g\bar{\phi}' + g\bar{\phi}\partial_r + \frac{3}{r} g\bar{\phi} & -\frac{L}{r} g\bar{\phi} & -\Delta_J + \frac{(\Delta_0 \bar{\phi})}{\bar{\phi}} \end{pmatrix}$$

$$+ \left(1 - \frac{1}{\xi}\right) \begin{pmatrix} \partial_r^2 + \frac{3}{r}\partial_r - \frac{3}{r^2} & -L\left(\frac{1}{r}\partial_r - \frac{1}{r^2}\right) & 0 \\ L\left(\frac{1}{r}\partial_r + \frac{3}{r^2}\right) & -\frac{L^2}{r^2} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\Delta_J = \partial_r^2 + \frac{3}{r}\partial_r - \frac{L^2}{r^2}$$

$$L = \sqrt{4J(J+1)}$$

hatted operator: $\bar{\phi} \to 0$

Theorem

[J. H. van Vleck, '28; R. H. Cameron and W. T. Martin, '45; …]

We give a proof for our case in JHEP11(2017)074

$$\frac{\det \mathcal{M}}{\det \widehat{\mathcal{M}}} = \left(\lim_{r\to\infty} \frac{\det[\psi_1(r)\cdots\psi_n(r)]}{\det[\hat{\psi}_1(r)\cdots\hat{\psi}_n(r)]} \right) \left(\lim_{r\to 0} \frac{\det[\psi_1(r)\cdots\psi_n(r)]}{\det[\hat{\psi}_1(r)\cdots\hat{\psi}_n(r)]} \right)^{-1}$$

$\mathcal{M}, \widehat{\mathcal{M}}$: (n x n) radial fluctuation operators

$\mathcal{M}\psi_i = 0,\ \widehat{\mathcal{M}}\hat{\psi}_i = 0$: independent solutions (regular at r=0)

Decomposition of simultaneous differential equations

[JHEP11(2017)074; M. Endo, T. Moroi, YS]

$$\mathcal{M}_J^{(S,L,\varphi)}\Psi_i = 0$$

$$\Psi_i = \begin{pmatrix} \partial_r \chi \\ \frac{L}{r}\chi \\ g\bar{\phi}\chi \end{pmatrix} + \begin{pmatrix} \frac{1}{rg^2\bar{\phi}^2}\eta \\ \frac{1}{Lr^2g^2\bar{\phi}^2}\partial_r(r^2\eta) \\ 0 \end{pmatrix} + \begin{pmatrix} -2\frac{\bar{\phi}'}{g^2\bar{\phi}^3}\zeta \\ 0 \\ \frac{1}{g\bar{\phi}}\zeta \end{pmatrix},$$

$$\Delta_J \chi = \frac{2\bar{\phi}'}{rg^2\bar{\phi}^3}\eta + \frac{2}{r^3}\partial_r\left(\frac{r^3\bar{\phi}'}{g^2\bar{\phi}^3}\zeta\right) - \xi\zeta$$

$$\left(\Delta_J - g^2\bar{\phi}^2 - 2\frac{\bar{\phi}'}{\bar{\phi}}\frac{1}{r^2}\partial_r r^2\right)\eta = -\frac{2L^2\bar{\phi}'}{r\bar{\phi}}\zeta$$

$$\Delta_J \zeta = 0$$

Gauge invariant analytic formulae

We can calculate all of the determinants analytically

$$\gamma = \int d\ln R \frac{1}{R^4} \left[\mathcal{A}'^{(h)}\mathcal{A}^{(\sigma)}\mathcal{A}^{(\psi)}\mathcal{A}^{(A_\mu,\varphi)}e^{-\mathcal{B}}\right]_{\overline{\mathrm{MS}},\ \mu\sim 1/R},$$

where

$$\left[\ln \mathcal{A}'^{(h)}\right]_{\overline{\mathrm{MS}}} = -\frac{3}{4} - 6\ln A_G + \frac{5}{2}\ln\frac{\pi}{3} - \frac{5}{2}\ln\frac{|\lambda|}{8} + 3\ln\frac{\mu R}{2},$$

$$\left[\ln \mathcal{A}^{(\sigma)}\right]_{\overline{\mathrm{MS}}} = -\frac{1}{2}\mathcal{S}_\sigma(z_\kappa) + \frac{\kappa}{|\lambda|} + \frac{\kappa^2}{3|\lambda|^2}\left(1 + \gamma_E + \ln\frac{\mu R}{2}\right),$$

$$\left[\ln \mathcal{A}^{(\psi)}\right]_{\overline{\mathrm{MS}}} = -\frac{y^4}{3|\lambda|^2}\left(1 + \gamma_E + \ln\frac{\mu R}{2}\right) - \frac{2y^2}{3|\lambda|}\left(\frac{25}{4} + \gamma_E + \ln\frac{\mu R}{2}\right) + \mathcal{S}_\psi(z_y),$$

$$\left[\ln \mathcal{A}^{(A_\mu,\varphi)}\right]_{\overline{\mathrm{MS}}} = \ln\mathcal{V}_G + \left[\ln \mathcal{A}'^{(A_\mu,\varphi)}\right]_{\overline{\mathrm{MS}}},$$

$$\begin{aligned}\left[\ln \mathcal{A}'^{(A_\mu,\varphi)}\right]_{\overline{\mathrm{MS}}} = &\left(\frac{1}{3} + \frac{2g^2}{|\lambda|} + \frac{g^4}{|\lambda|^2}\right)\left(1 + \gamma_E + \ln\frac{\mu R}{2}\right) \\ &- \frac{g^4}{|\lambda|^2}\left(\frac{31}{3} - \pi^2\right) - \frac{1}{4} - \frac{1}{3}\gamma_E - 2\ln A_G - \frac{1}{2}\ln\frac{|\lambda|}{8} + \frac{1}{2}\ln\pi \\ &- \frac{3}{2}\mathcal{S}_\sigma(z_g) - \frac{3}{2}\ln\Gamma(1 - z_g)\Gamma(2 + z_g).\end{aligned}$$

[A. Andreassen, W. Frost, M. D. Schwartz; ’17, S. Chigusa, T. Moroi, YS; ’18]

Integral over the bounce size

R-dependence of the integrand **at the one-loop level**

$$\gamma = \int dR \frac{d\gamma}{dR} \propto \int d\ln R \; R^{-4-\frac{8\pi^2 \beta_\lambda^{(1)}}{3\lambda^2}}$$

$\beta_\lambda^{(1)}$: 1-loop beta function of λ

The integral apparently diverge!

However, the result can converge if one includes two or more loops

RG improvement

For each bounce size R, we take the renormalization scale as $\mu \simeq R^{-1}$

(Corresponding to resummation of higher loop logarithmic corrections)

RG improved integrand (SM)

$$\gamma = \int d\ln R \frac{d\gamma}{d\ln R}$$

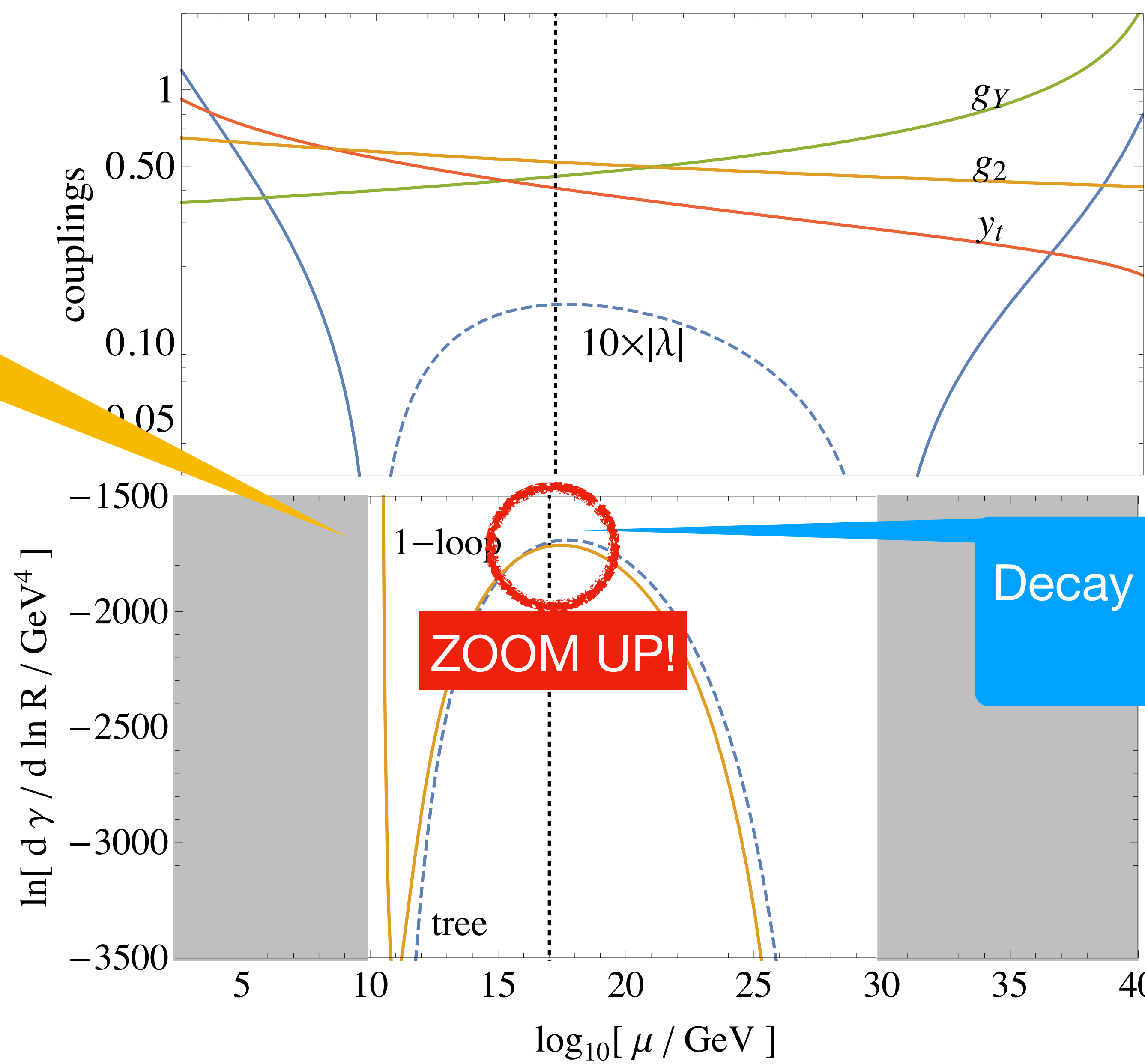
break down of perturbation

Remove by hand

$$\ln \frac{d\gamma}{d\ln R}$$

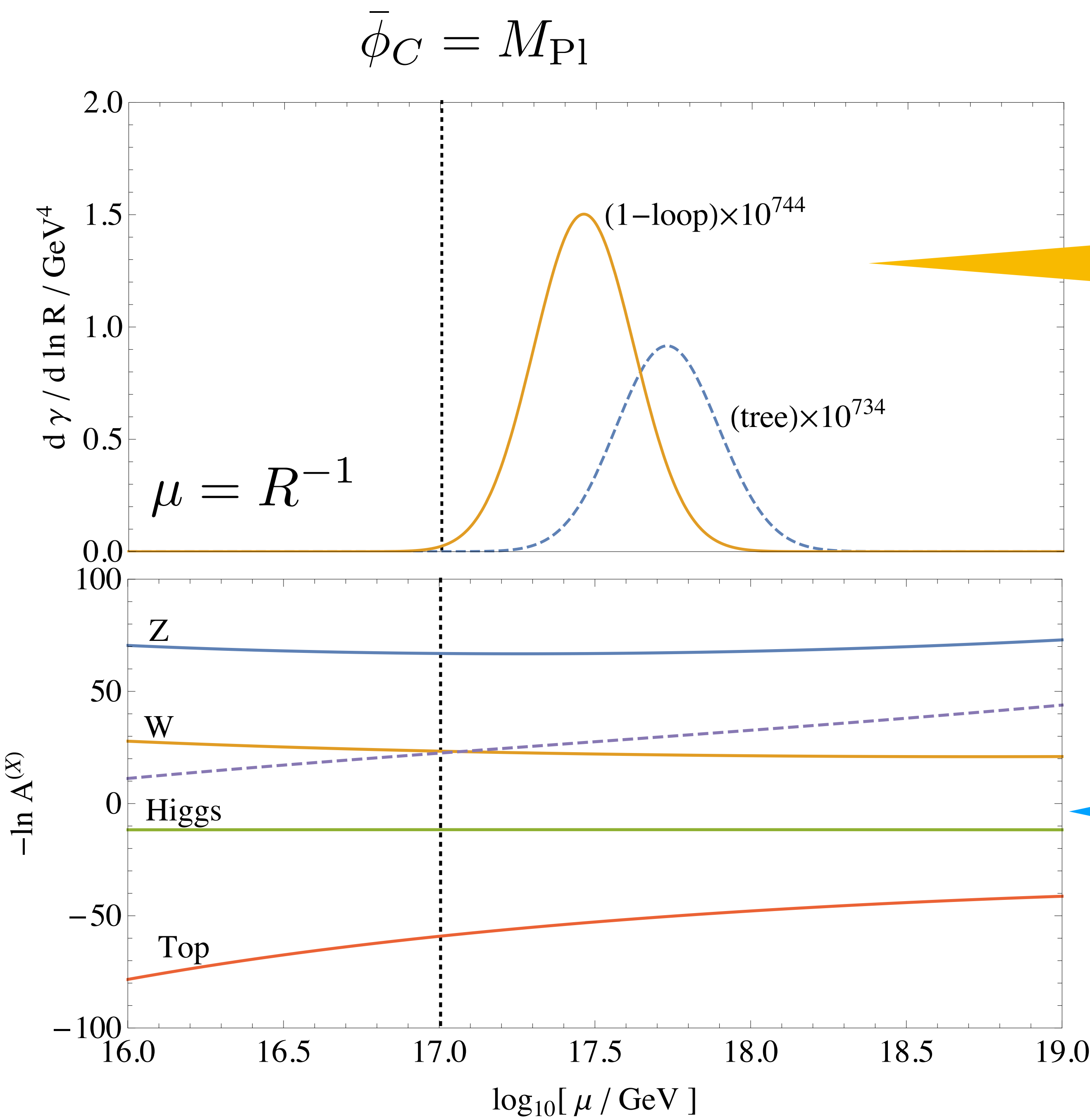
with

$$\mu = R^{-1}$$



Decay rate is dominated by this small region

Standard Model



The center of the bounce exceeds the Planck scale

(1) Ignore effects of gravity

$$\log_{10}\left[\gamma_\infty \times \mathrm{Gyr\ Gpc}^3\right] = -580^{+40}_{-44}{}^{+183}_{-328}{}^{+145}_{-218}{}^{+2}_{-1},$$

$$m_h \quad m_t \quad \alpha_s \quad \mu$$

(2) Impose a cutoff of R

$$\log_{10}\left[\gamma_{\rm Pl} \times \mathrm{Gyr\ Gpc}^3\right] = -582^{+40}_{-45}{}^{+184}_{-329}{}^{+144}_{-218}{}^{+2}_{-1},$$

There are cancellation among quantum corrections

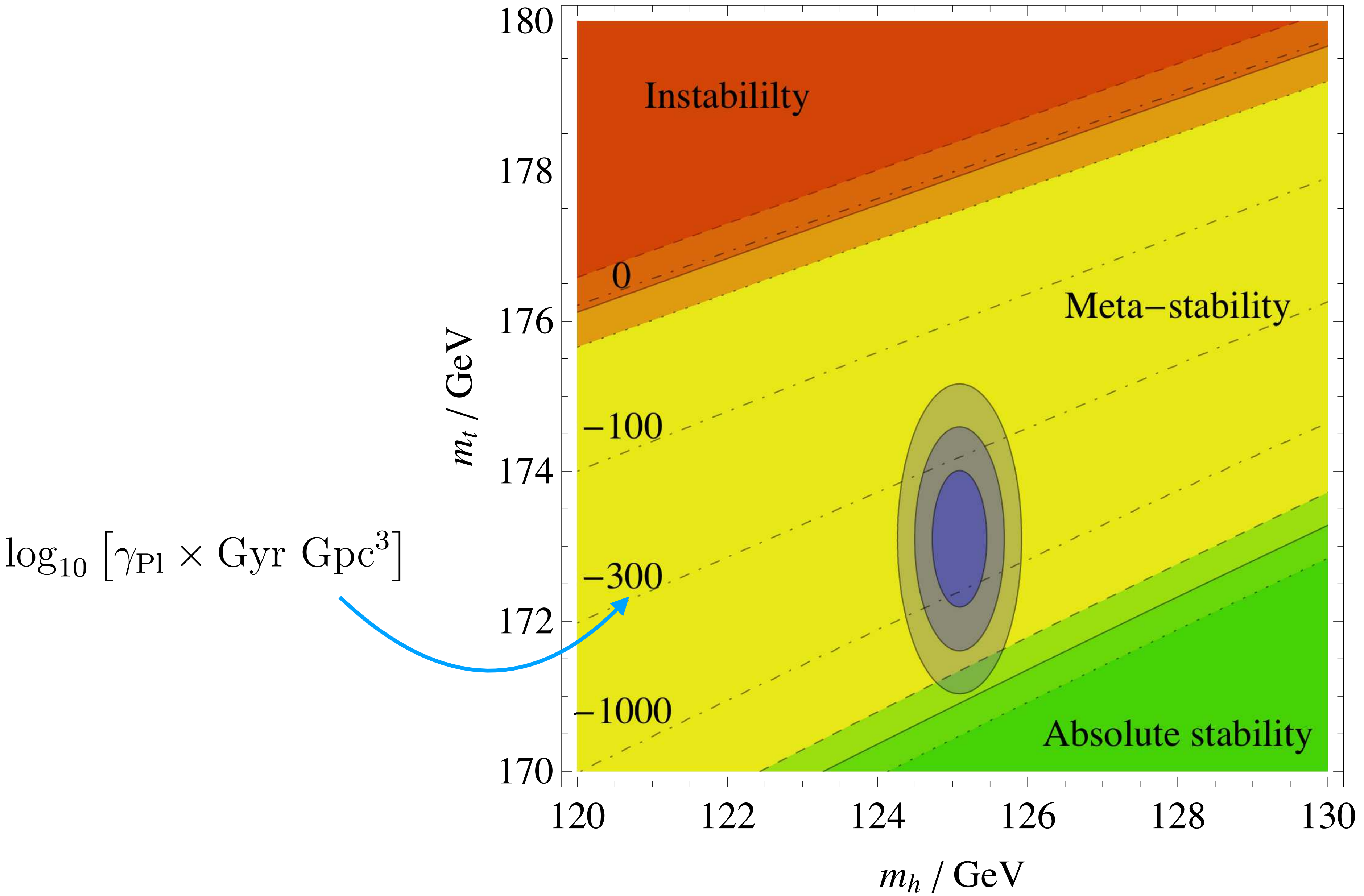
Ignoring QC, we have

$$\log_{10}\left[\gamma_{\rm Pl}^{(\rm tree)} \times \mathrm{Gyr\ Gpc}^3\right] = -575$$

($\bar{\phi}_C$: field value at the center of the bounce)

Standard Model

S. Chigusa, T. Moroi, YS; ‘ 18



cf.) A. Andreassen, W. Frost, M. D. Schwartz; ‘17
S. Chigusa, T. Moroi, YS; ‘ 17

ELVAS

(c++ package for ELectroweak VAcuum Stability)

＊実在する都市、背景とは一切関係ありません。

Who don't even want to write a code by theirselves

should go to https://github.com/YShoji-HEP/ELVAS

ELVAS

C++ Package for ELectroweak VAcuum Stability

Introduction

ELVAS is a C++ package for the calculation of the decay rate of a false vacuum at the one-loop level, based on the formulae developed in [1, 2]. ELVAS is applicable to models with the following features:

- Only one scalar boson is responsible for the vacuum decay.
- Classical scale invariance (approximately) holds. In particular, the potential of the scalar field responsible for the vacuum decay should be well approximated by the quartic form for the calculation of the bounce solution. (Thus, the bounce is nothing but the so-called Fubini instanton.)
- The instability of the scalar potential occurs due to RG effects; thus, the quartic coupling constant becomes negative at a high scale.

If you use ELVAS in scholarly work, please cite [1] and [2].

Assumptions

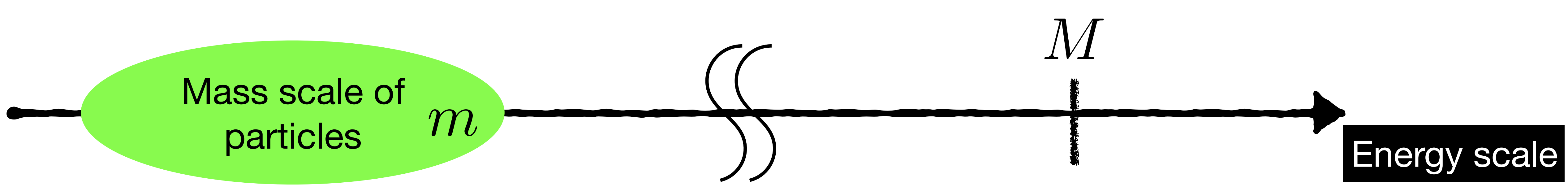
We consider a more general case where the scalar potential is roughly given by

$$V(\phi) = \frac{\lambda}{4}\phi^4$$

Other terms are neglected since

$$m \ll M$$

Lambda becomes smallest
at a high energy scale



No instability at a low energy scale

The bounce field can have global/local symmetry

Laurent expansions

The package uses expanded expressions (< 0.05% accuracy)

F Numerical Recipe

In this Appendix, we give fitting formulae of the prefactors at the one-loop level. Contrary to the analytic formulae including various special functions with complex arguments, which may be inconvenient for numerical calculations, the fitting formulae give a simple procedure to perform a numerical calculation of the decay rate with saving computational time. Compared to the analytic expressions, the errors of the fitting formulae are 0.05% or smaller.

- Higgs

$$-\left[\ln \mathcal{A}'^{(h)}\right]_{\overline{\rm MS}} = -0.99192944327027 + 2.5\ln|\lambda| - 3\ln\mu R. \tag{F.1}$$

- Scalar

Let $x = \kappa/|\lambda|$. For $x < 0.7$,

$$\begin{aligned}-\left[\ln \mathcal{A}^{(\sigma)}\right]_{\overline{\rm MS}} = &-0.239133939224974x^2 + 0.222222222222222x^3\\ &- 0.134704602106396x^4 + 0.102278606592866x^5\\ &- 0.0839329261179402x^6 + 0.0715956882048009x^7\\ &- 0.0625481711576628x^8 + 0.0555697470602515x^9\\ &- 0.0500042455037409x^{10} - 0.333333333333333x^2\ln\mu R.\end{aligned} \tag{F.2}$$

For $x > 0.7$,

$$\begin{aligned}-\left[\ln \mathcal{A}^{(\sigma)}\right]_{\overline{\rm MS}} = &-0.0261559272783723 + 0.0000886704923163256/x^4\\ &+ 0.0000962000962000962/x^3 + 0.000198412698412698/x^2\\ &+ 0.00105820105820106/x + 0.111111111111111x\\ &- 0.181204187497805x^2 + (-0.00555555555555556\\ &+ 0.166666666666667x^2)\ln x - 0.333333333333333x^2\ln\mu R.\end{aligned} \tag{F.3}$$

- Fermion

Let $x = y^2/|\lambda|$. For $x < 1.3$,

$$\begin{aligned}-\left[\ln \mathcal{A}^{(\psi)}\right]_{\overline{\rm MS}} = &\,0.644934545116661x + 0.00511497150510 9x^2\\ &- 0.0366953662258276x^3 + 0.00476307962690785x^4\\ &- 0.000845451274112082x^5 + 0.000168244913551417x^6\\ &- 0.0000353785958610453x^7 + 7.67709260595572\times10^{-6}x^8\\ &+ (0.666666666666667x + 0.333333333333333x^2)\ln\mu R.\end{aligned} \tag{F.4}$$

For $x > 1.3$,

$$\begin{aligned}-\left[\ln \mathcal{A}^{(\psi)}\right]_{\overline{\rm MS}} = &-0.227732960077634 + 0.00260942760942761/x^3\\ &+ 0.00271164021164021/x^2 + 0.00820105820105820/x\\ &+ 0.537901879626 70x + 0.296728717591129x^2\\ &+ (-0.0611111111111111 - 0.333333333333333x\\ &- 0.166666666666666x^2)\ln x\\ &+ (0.666666666666667x + 0.333333333333333x^2)\ln\mu R.\end{aligned} \tag{F.5}$$

- Gauge

Let $x = g^2/|\lambda|$. For $x < 1.4$,

$$\begin{aligned}-\left[\ln \mathcal{A}'^{(A_\mu,\varphi)}\right]_{\overline{\rm MS}} = &- 0.966861032843 73 - 1.76813696868318x\\ &+ 0.615931515658 41x^2 + 0.145084271024101x^3\\ &- 0.0241469799983579x^4 + 0.00555917805602827x^5\\ &- 0.00145020891759152x^6 + 0.000402580447036276x^7\\ &- 0.000115821925959136x^8\\ &+ 0.5\ln|\lambda| + (-0.333333333333333 - 2x - x^2)\ln\mu R.\end{aligned} \tag{F.6}$$

For $x > 1.4$,

$$\begin{aligned}-\left[\ln \mathcal{A}'^{(A_\mu,\varphi)}\right]_{\overline{\rm MS}} = &- 27.0091748854198 + 0.000266011476948977/x^4\\ &+ 0.000288600288600289/x^3 + 0.000595238095238095/x^2\\ &+ 0.00317460317460317/x + 1.56519636465016x\\ &- 0.0798836302494 4x^2 + (-3.54033527491510\times10^{-6}/x^5\\ &- 0.0000404609745704583/x^4 - 0.000517900474501 87/x^3\\ &- 0.00828640759202 99/x^2 - 0.265165042944955/x\\ &+ 4.24264068711929)\sqrt{x}\arcsin\left[\frac{s}{\sqrt{s^2 + 79164837199872x^9}}\right]\\ &+ (-6.01666666666667 + 0.5x^2)\ln x\\ &+ 1.5\ln[3.14159265358979(-98796.7402597403\\ &+ 136316.571428571x - 136594.285714286x^2\\ &+ 92160x^3 + 7372800x^4 + 6553600x^5)]\\ &+ 0.5\ln|\lambda| + (-0.333333333333333 - 2x - x^2)\ln\mu R,\end{aligned} \tag{F.7}$$

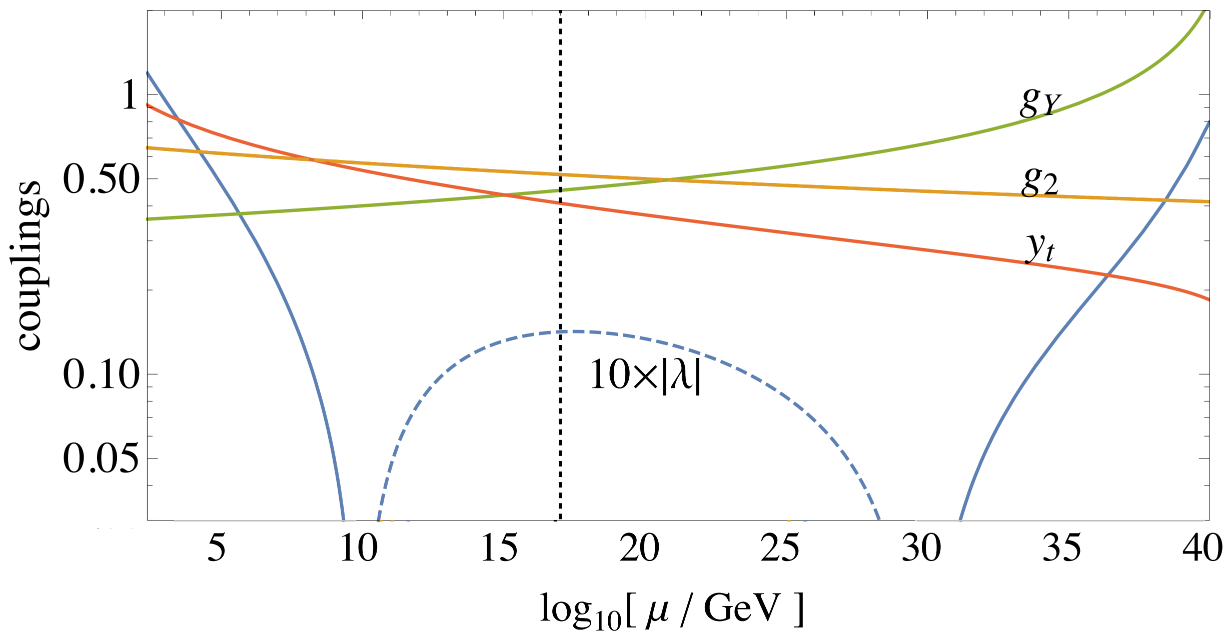
where

$$s = 7 + 80x + 1024x^2 + 16384x^3 + 524288x^4 - 8388608x^5. \tag{F.8}$$

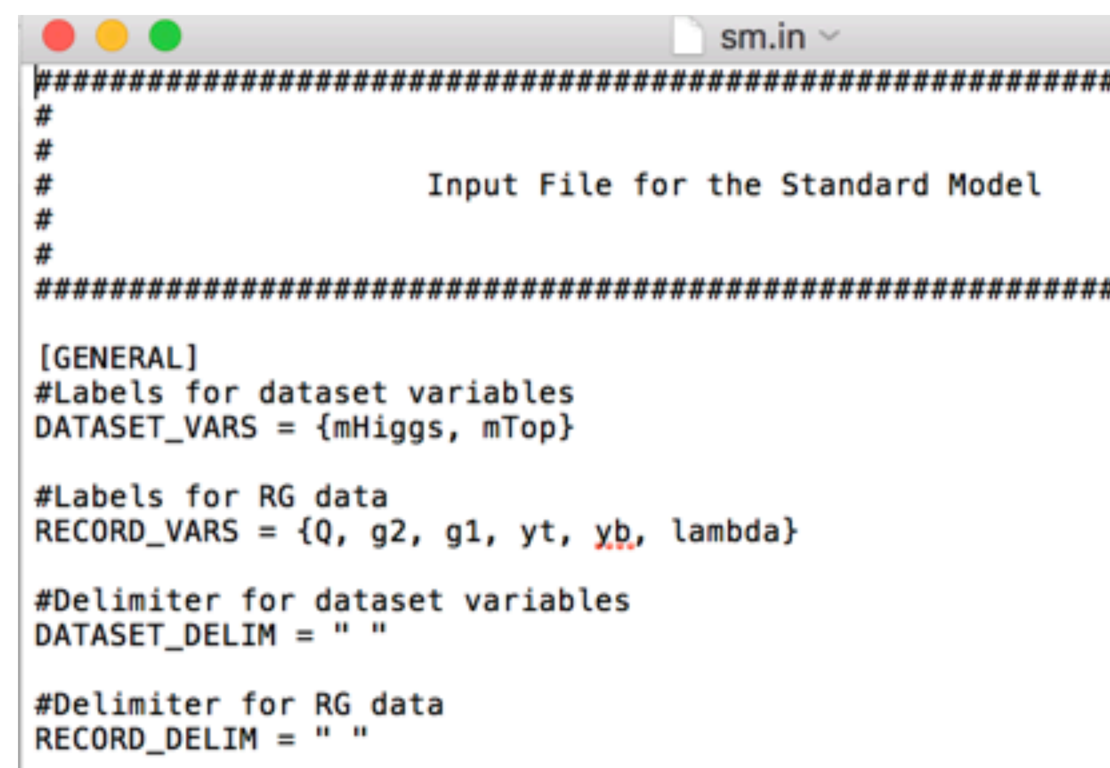
[S. Chigusa, T. Moroi, YS; ’18]

What you need

Renormalization group evolution of couplings



Model file



```
sm.in
###########################################################
#
#
#                    Input File for the Standard Model
#
#
###########################################################

[GENERAL]
#Labels for dataset variables
DATASET_VARS = {mHiggs, mTop}

#Labels for RG data
RECORD_VARS = {Q, g2, g1, yt, yb, lambda}

#Delimiter for dataset variables
DATASET_DELIM = " "

#Delimiter for RG data
RECORD_DELIM = " "
```

What you need

Renormalization group evolution of couplings

sm.dat

```
[DATASET] (1.2509000e+02 1.7310000e+02)
2.4000000e[illegible] 6.4591604e-01 4.6301691[illegible] 9.1368515e-01 1.5217632e-02 1.1956213e-01
3.8037437e+02 6.4352899e-01 4.6423116[illegible]01 8.9526450e-01 1.4739161e-02 1.1126386e-01
6.0285274e+02 6.4116422e-01 4[illegible]45398e-0[illegible] 8.7551393e-01 1.4299383e-02 1.0365348e-01
9.5545721e+02 6.3882178e-01 4.6[illegible]8555e-01 [illegible].5622280e-01 1.3893282e-02 9.6642915e-02
1.5142976e+03 6.3650165e-01 4.6792[illegible]09e-01 8.[illegible]321643e-01 1.3516710e-02 9.0159826e-02
2.4000000e+03 6.3420376e-01 4.69175[illegible]e-01 8.21[illegible]850e-01 1.3166205e-02 8.4144097e-02
3.8037437e+03 6.3192799e-01 4.7043482[illegible]-01 8.0549[illegible]40e-01 1.2838850e-02 7.8545287e-02
6.0285274e+03 6.2967421e-01 4.7170338e-[illegible]1 7.905518[illegible]-01 1.2532170e-02 7.3320684e-02
9.5545721e+03 6.2744223e-01 4.7298163e-01 7.7542741e-[illegible]1 1.2244044e-02 6.8433821e-02
1.5142976e+04 6.2523186e-01 4.7426976e-01 7.[illegible]6304415e-0[illegible] 1.1972648e-02 6.3853320e-02
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3.8037437e+04 6.2087512e-01 4.7587633e-01 7.38[illegible]833e-01 1.[illegible]73923e-02 5.5506151e-02
6.0285274e+04 6.1872829e-01 4.7819511e-01 7.2570[illegible]4e-01 1.12[illegible]06e-02 5.1694977e-02
9.5545721e+04 6.1660217e-01 4.7952445e-01 7.156865[illegible]-01 1.10255[illegible]e-02 4.8100126e-02
1.5142976e+05 6.1449651e-01 4.8086452e-01 7.0514[illegible] [illegible] [illegible]26e-02
2.4000000e+05 6.1241106e-01 4.8221550e-01 6.9504[illegible] [illegible] [illegible]70e-02
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6.0285274e+05 6.0829978e-01 4.8495086e-01 6.7502368e-01 1.[illegible]249568e-02 3.[illegible]583559e-02
9.5545721e+05 6.0627344e-01 4.8633561e-01 6.6705531e-01 1.0[illegible]6423e-02 3.2[illegible]6085e-02
1.5142976e+06 6.0426629e-01 4.8773196e-01 6.5841453e-01 9.910[illegible]490e-03 3.027[illegible]01e-02
2.4000000e+06 6.0227808e-01 4.8914011e-01 6.5007990e-01 9.75115[illegible]8e-03 2.782081[illegible]e-02
3.8037437e+06 6.0030054e-01 4.9056024e-01 6.4203195e-01 9.5981044[illegible]-03 2.5492412[illegible]02
6.0285274e+06 5.9835743e-01 4.9199253e-01 6.3425295e-01 9.4508899e-[illegible]3 2.3280928e-[illegible]
9.5545721e+06 5.9642450e-01 4.9343717e-01 6.2672670e-01 9.3091466e-03 2.1179706e-02
1.5142976e+07 5.9450950e-01 4.9489437e-01 6.1943837e-01 9.1725399e-03 1[illegible]9182617e-02
2.4000000e+07 5.9261217e-01 4.9636430e-01 6.1237436e-01 9.0407633e-03 1.[illegible]284003e-02
3.8037437e+07 5.9073229e-01 4.9784718e-01 6.0552216e-01 8.9135354e-03 1.54[illegible]8629e-02
6.0285274e+07 5.8886960e-01 4.9934321e-01 5.9887023e-01 8.7905975e-03 1.3761[illegible]42e-02
9.5545721e+07 5.8702387e-01 5.0085259e-01 5.9240793e-01 8.6717111e-03 1.212853[illegible]e-02
```

Dataset variables

+

Model file

sm.in

```
#################################################################
#
#
#                   Input File for the Standard Model
#
#
#################################################################

[GENERAL]
#Labels for dataset variables
DATASET_VARS = {mHiggs, mTop}

#Labels for RG data
RECORD_VARS = {Q, g2, g1, yt, yb, lambda}

#Delimiter for dataset variables
DATASET_DELIM = " "

#Delimiter for RG data
RECORD_DELIM = " "
```

What you need

Renormalization group evolution of couplings

sm.dat

```
[DATASET] (1.2509000e+02 1.7310000e+02)
2.4000000e+02 6.4591604e-01 4.6301691e-01 9.1368515e-01 1.5217632e-02 1.1956213e-01
3.8037437e+02 6.4352899e-01 4.6423116e-01 8.9526450e-01 1.4739161e-02 1.1126386e-01
6.0285274e+02 6.4116422e-01 4.6545398e-01 8.7551393e-01 1.4299383e-02 1.0365348e-01
[illegible]5545721e+02 6.3882178e-01 4.6668555e-01 8.5622280e-01 1.3893282e-02 9.5642915e-02
1.5142976e+03 6.3550165e-01 4.6792609e-[illegible] 8.3[illegible]43e-01 1.3516710e-02 9.0159826e-02
2.4000000e+03 6.3420376e-01 4.6917578e-[illegible] [illegible]8.2[illegible]850e-01 1.3166205e-02 8.4144097e-02
3.8037437e+03 6.3192799e-01 4.7043482e-01 [illegible].0349[illegible]40e-01 1.2838850e-02 7.8545287e-02
6.0285274e+03 6.2967421e-01 4.7170338e-01 7.[illegible]05518[illegible]e-01 1.2532170e-02 7.3320684e-02
9.5545721e+03 6.2744223e-01 4.7298163e-01 7.7[illegible]42741e-01 1.2244044e-02 6.8433821e-02
1.5142976e+04 6.2523186e-01 4.7426976e-01 7.630[illegible]415e-0[illegible] 1.1972648e-02 6.3853320e-02
2.4000000e+04 6.2304790e-01 4.7556794e-01 7.5033[illegible]21e-01 [illegible].1716401e-02 5.9551999e-02
3.8037437e+04 6.2087512e-01 4.7687633e-01 7.382383[illegible]e-01 1.[illegible]473923e-02 5.5506151e-02
6.0285274e+04 6.1872829e-01 4.7819511e-01 7.2570444e-01 1.1[illegible]4006e-02 5.1694977e-02
9.5545721e+04 6.1660217e-01 4.7952445e-01 7.1568657e-0[illegible] 1.102[illegible]588e-02 4.8100126e-02
1.5142976e+05 6.1449651e-01 4.8086452e-01 7.0514398e-01 [illegible] 4.4705326e-02
2.4000000e+05 6.1241106e-01 4.8221550e-01 6.9504037e-01 [illegible] 4.1496070e-02
3.8037437e+05 6.1034557e-01 4.8357755e-01 6.8534330e-01 [illegible] 3.8459373e-02
6.0285274e+05 6.0829978e-01 4.8495086e-01 6.7602368e-01 1.02[illegible]9568e-02 3.5583559e-02
9.5545721e+05 6.0627344e-01 4.8633561e-01 6.6705531e-01 1.007[illegible]423e-02 [illegible].2858085e-02
1.5142976e+06 6.0426629e-01 4.8773196e-01 6.5841453e-01 9.9104[illegible]0e-03 3.[illegible]273401e-02
2.4000000e+06 6.0227808e-01 4.8914011e-01 6.5007990e-01 9.751156[illegible]-03 2.78[illegible]0818e-02
3.8037437e+06 6.0030854e-01 4.9056024e-01 6.4203195e-01 9.5981044e-[illegible]3 2.5492[illegible]12e-02
6.0285274e+06 5.9835743e-01 4.9199253e-01 6.3425295e-01 9.4508899e-0[illegible] 2.328092[illegible]e-02
9.5545721e+06 5.9642450e-01 4.9343717e-01 6.2672670e-01 9.3091466e-03 [illegible].1179706e-02
1.5142976e+07 5.9450950e-01 4.9489437e-01 6.1943837e-01 9.1725399e-03 1.[illegible]182617e-0[illegible]
2.4000000e+07 5.9261217e-01 4.9636430e-01 6.1237436e-01 9.0407633e-03 1.7[illegible]84003e-02
3.8037437e+07 5.9073229e-01 4.9784718e-01 6.0552216e-01 8.9135354e-03 1.547[illegible]629e-02
6.0285274e+07 5.8886960e-01 4.9934321e-01 5.9887023e-01 8.7905975e-03 1.3761[illegible]42e-02
9.5545721e+07 5.8702387e-01 5.0085259e-01 5.9240793e-01 8.6717111e-03 1.212853[illegible]e-02
```

Records

+

Model file

sm.in

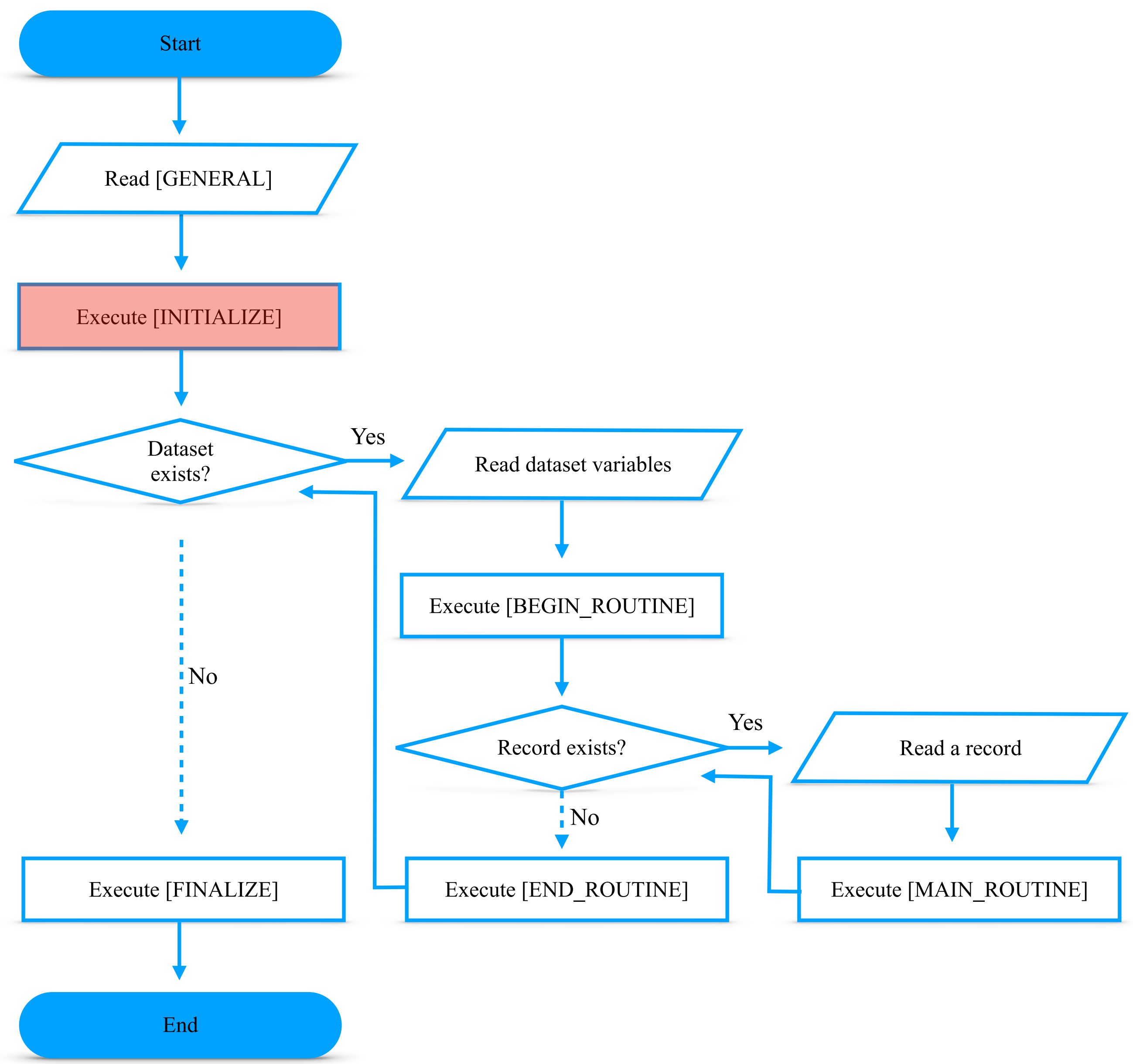
```
##########################################################
#
#
#                 Input File for the Standard Model
#
#
##########################################################

[GENERAL]
#Labels for dataset variables
DATASET_VARS = {mHiggs, mTop}

#Labels for RG data
RECORD_VARS = {Q, g2, g1, yt, yb, lambda}

#Delimiter for dataset variables
DATASET_DELIM = " "

#Delimiter for RG data
RECORD_DELIM = " "
```

The rest is script routines

sm.in

```
###########################################################################
#
#
#                     Input File for the Standard Model
#
#
###########################################################################

[GENERAL]
#Labels for dataset variables
DATASET_VARS = {mHiggs, mTop}

#Labels for RG data
RECORD_VARS = {Q, g2, g1, yt, yb, lambda}

#Delimiter for dataset variables
DATASET_DELIM = " "

#Delimiter for RG data
RECORD_DELIM = " "

#Delimiter for output
OUTPUT_DELIM = " "

[INITIALIZE]
#Set ln(Q x R)
LN_QR = 0.

#Volume of the group space generated by the broken generators
lnVg = log(2. * pi^2)

#Upper bound on lnPhiC and lnR^(-1)
upper_bound = log(2.435e18)

#Print a header
print("mHiggs       mTop         log10(gamma x Gyr Gpc^3)")

[BEGIN_ROUTINE]
#Clear phiC and dlngamma/dR^(-1)
initialize()

#Lower bound on lnPhiC and lnR^(-1)
lower_bound = log(mTop * 10)

[MAIN_ROUTINE]
#The Higgs quartic coupling.
HIGGS_QUARTIC_COUPLING = lambda

#If Higgs quartic coupling is positive, skip this record
if(HIGGS_QUARTIC_COUPLING > 0, continue())
```

Executed at the beginning

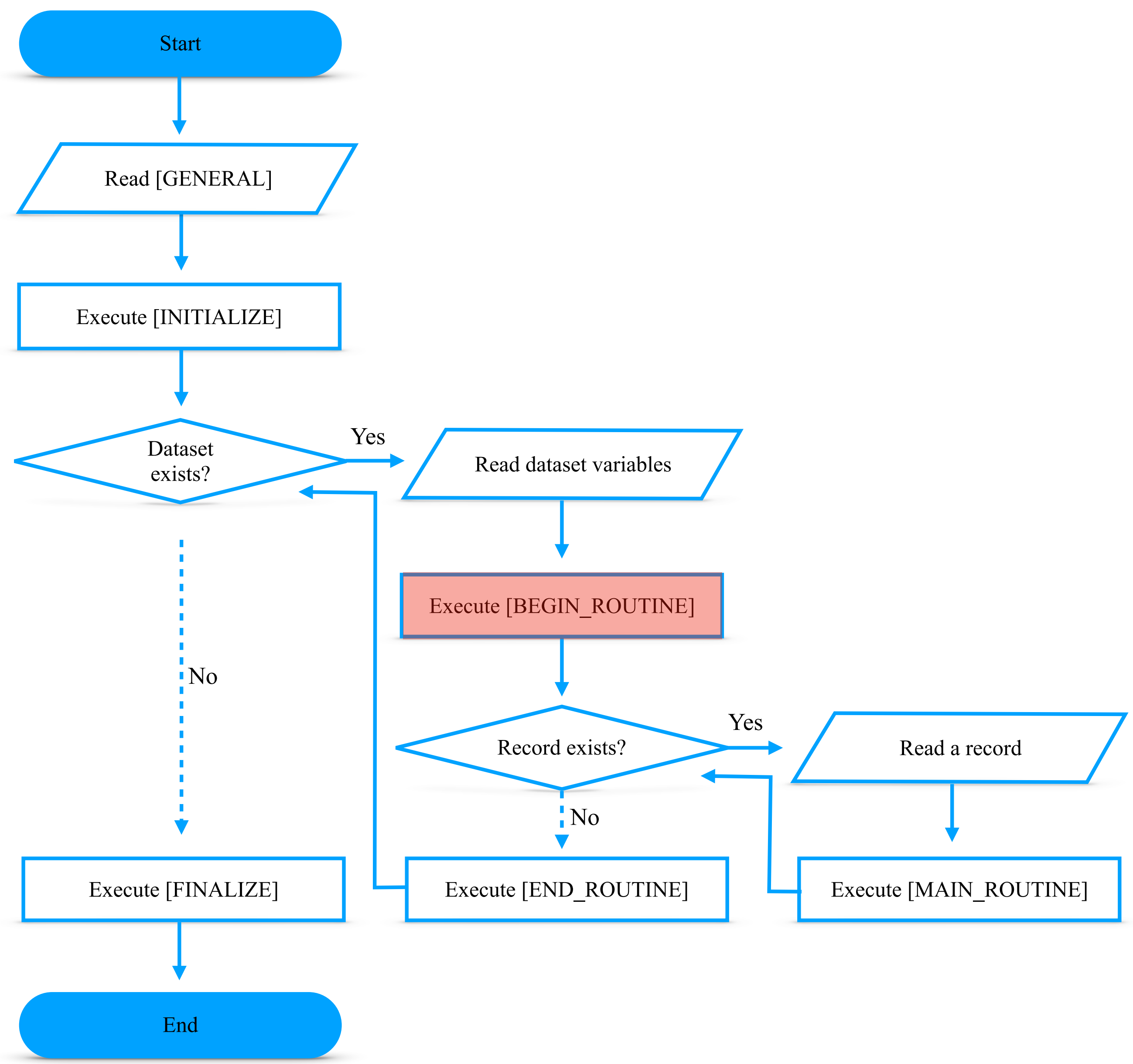


The rest is script routines

sm.in

```
###########################################################################
#
#
#                    Input File for the Standard Model
#
#
###########################################################################

[GENERAL]
#Labels for dataset variables
DATASET_VARS = {mHiggs, mTop}

#Labels for RG data
RECORD_VARS = {Q, g2, g1, yt, yb, lambda}

#Delimiter for dataset variables
DATASET_DELIM = " "

#Delimiter for RG data
RECORD_DELIM = " "

#Delimiter for output
OUTPUT_DELIM = " "

[INITIALIZE]
#Set ln(Q x R)
LN_QR = 0.

#Volume of the group space generated by the broken generators
lnVg = log(2. * pi^2)

#Upper bound on lnPhiC and lnR^(-1)
upper_bound = log(2.435e18)

#Print a header
print("mHiggs       mTop         log10(gamma x Gyr Gpc^3)")

[BEGIN_ROUTINE]
#Clear phiC and dlngamma/dR^(-1)
initialize()

#Lower bound on lnPhiC and lnR^(-1)
lower_bound = log(mTop * 10)

[MAIN_ROUTINE]
#The Higgs quartic coupling.
HIGGS_QUARTIC_COUPLING = lambda

#If Higgs quartic coupling is positive, skip this record
if(HIGGS_QUARTIC_COUPLING > 0, continue())
```

Executed for each dataset

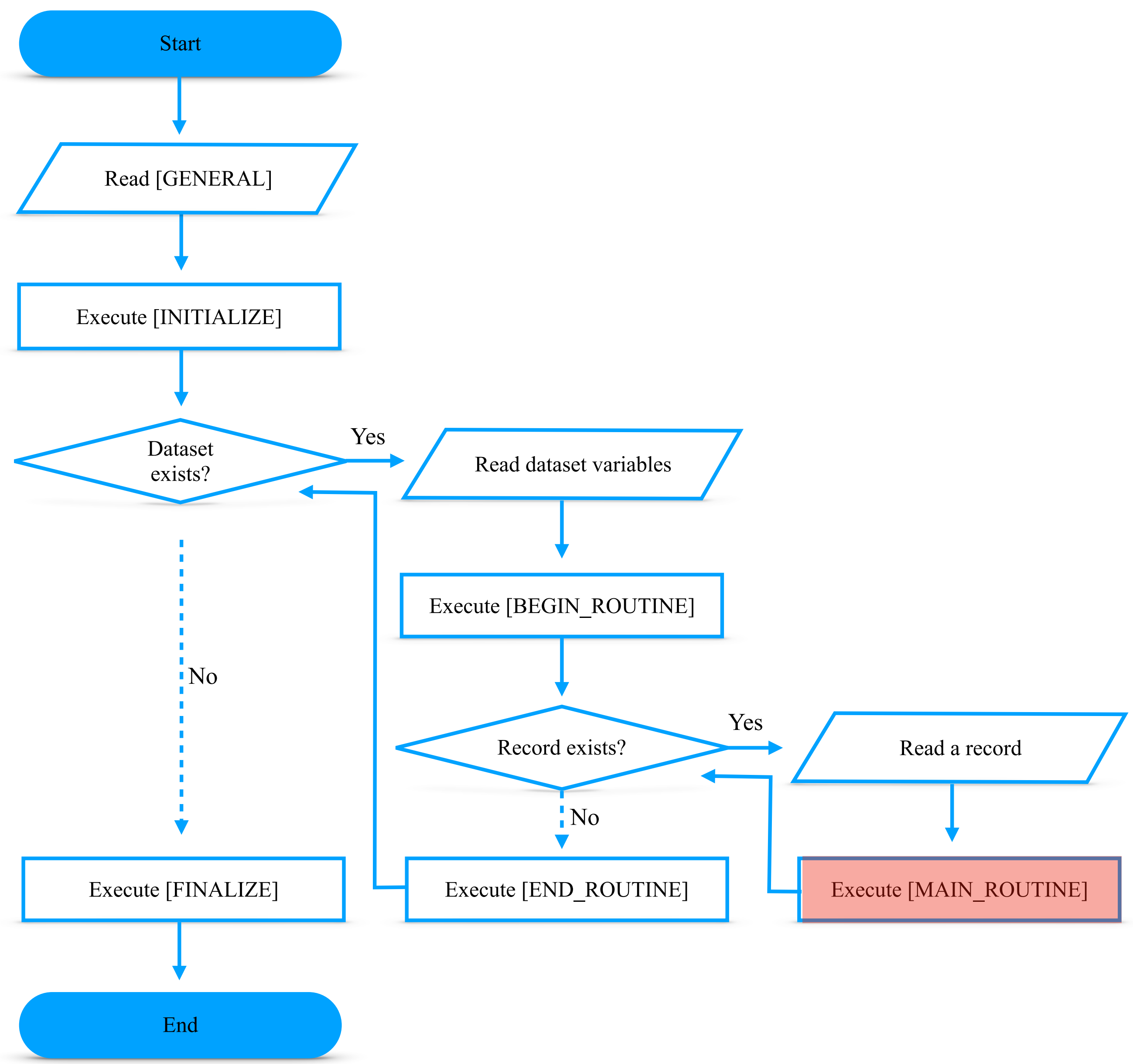


The rest is script routines

sm.in

```
##########################################################################
#
#
#                    Input File for the Standard Model
#
#
##########################################################################

[GENERAL]
#Labels for dataset variables
DATASET_VARS = {mHiggs, mTop}

#Labels for RG data
RECORD_VARS = {Q, g2, g1, yt, yb, lambda}

#Delimiter for dataset variables
DATASET_DELIM = " "

#Delimiter for RG data
RECORD_DELIM = " "

#Delimiter for output
OUTPUT_DELIM = " "

[INITIALIZE]
#Set ln(Q x R)
LN_QR = 0.

#Volume of the group space generated by the broken generators
lnVg = log(2. * pi^2)

#Upper bound on lnPhiC and lnR^(-1)
upper_bound = log(2.435e18)

#Print a header
print("mHiggs       mTop          log10(gamma x Gyr Gpc^3)")

[BEGIN_ROUTINE]
#Clear phiC and dlngamma/dR^(-1)
initialize()

#Lower bound on lnPhiC and lnR^(-1)
lower_bound = log(mTop * 10)

[MAIN_ROUTINE]
#The Higgs quartic coupling.
HIGGS_QUARTIC_COUPLING = lambda

#If Higgs quartic coupling is positive, skip this record
if(HIGGS_QUARTIC_COUPLING > 0, continue())
```

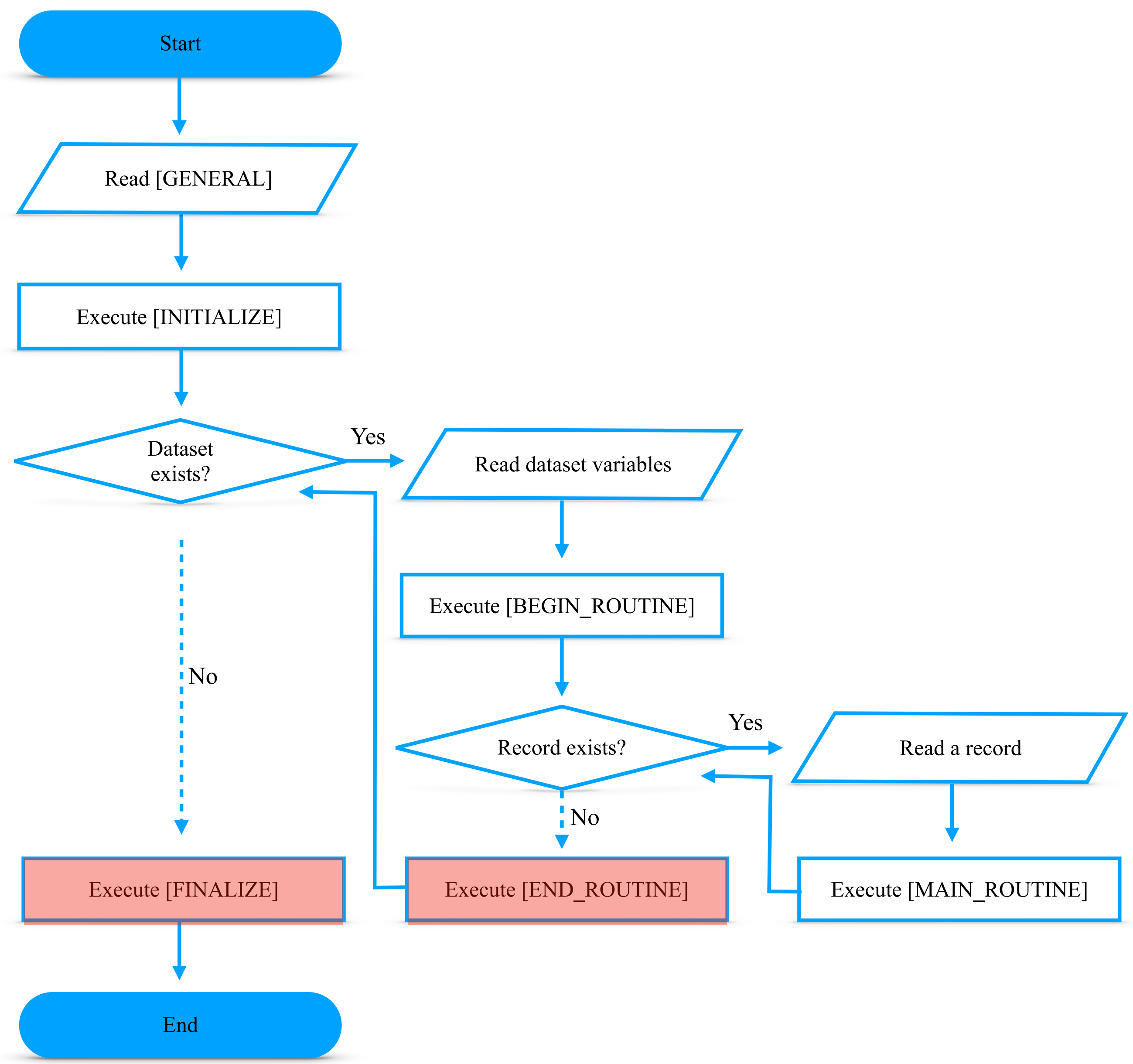
Executed for each record



The rest is script routines

sm.in

```
################################################################################
#
#
#                    Input File for the Standard Model
#
#
################################################################################

[GENERAL]
#Labels for dataset variables
DATASET_VARS = {mHiggs, mTop}

#Labels for RG data
RECORD_VARS = {Q, g2, g1, yt, yb, lambda}

#Delimiter for dataset variables
DATASET_DELIM = " "

#Delimiter for RG data
RECORD_DELIM = " "

#Delimiter for output
OUTPUT_DELIM = " "

[INITIALIZE]
#Set ln(Q x R)
LN_QR = 0.

#Volume of the group space generated by the broken generators
lnVg = log(2. * pi^2)

#Upper bound on lnPhiC and lnR^(-1)
upper_bound = log(2.435e18)

#Print a header
print("mHiggs       mTop         log10(gamma x Gyr Gpc^3)")

[BEGIN_ROUTINE]
#Clear phiC and dlngamma/dR^(-1)
initialize()

#Lower bound on lnPhiC and lnR^(-1)
lower_bound = log(mTop * 10)

[MAIN_ROUTINE]
#The Higgs quartic coupling.
HIGGS_QUARTIC_COUPLING = lambda

#If Higgs quartic coupling is positive, skip this record
if(HIGGS_QUARTIC_COUPLING > 0, continue())
```



And similarly for [END_ROUTINE] and [FINALIZE]

Important functions

The routines can be easily constructed

$$\gamma = \int d\ln R \frac{d\gamma}{d\ln R}$$

$$\boxed{e^{-\frac{8\pi^2}{3|\lambda|}} \times A^{(h)} \times A^{(t)} \times A^{(W,Z)}}$$

`InstantonB()`

`HiggsQC()`

`FermionQC(y)`

`GaugeQC(g_squared)`

`get_lngamma(lnRinv_min,lnRinv_max)`

(fitting & integration)

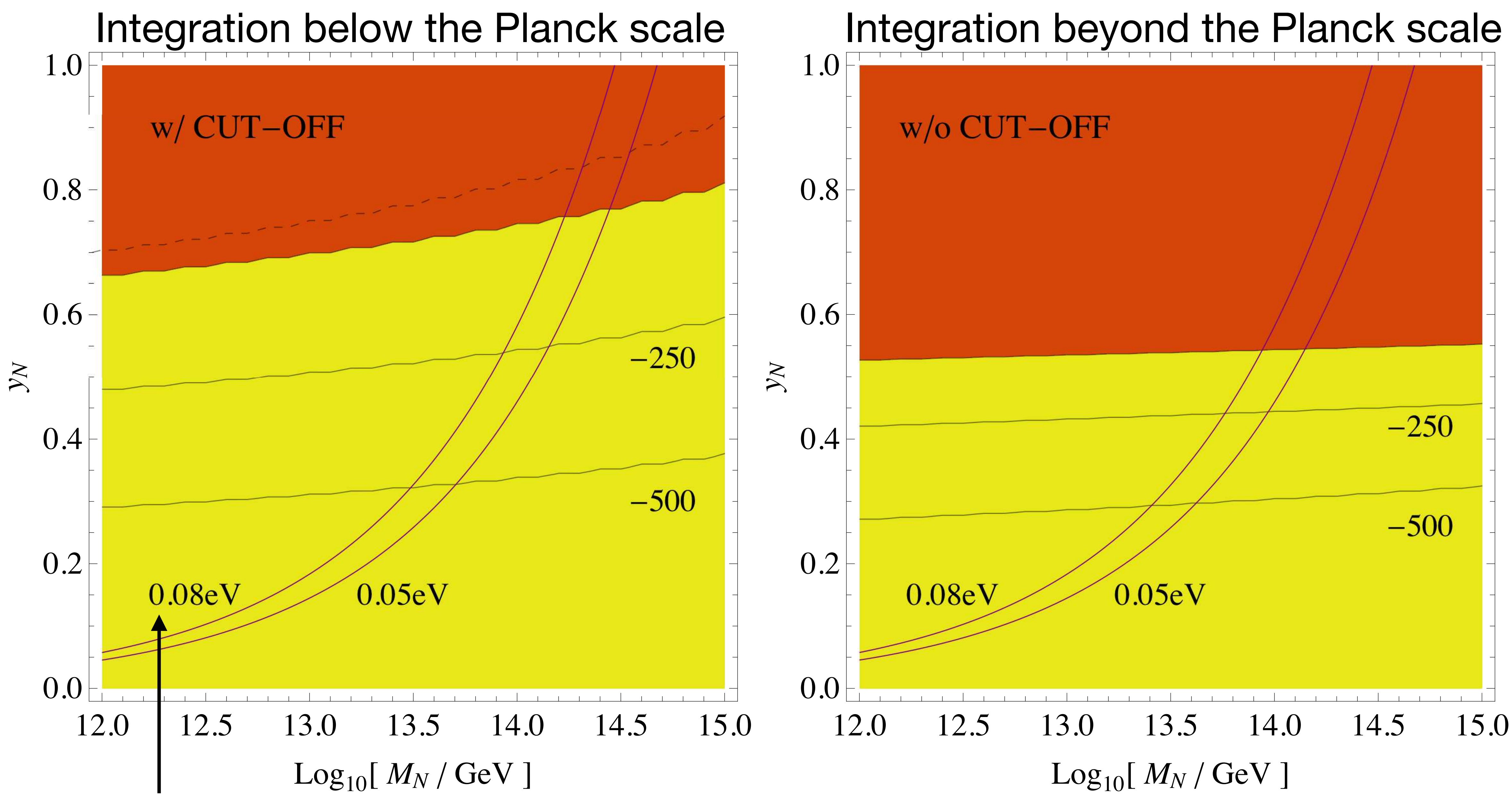
$+$

`ScalarQC(kappa)`

(for additional scalars)

Right handed neutrino

For simplicity, consider one RH neutrino



Neutrino mass

Summary

- We obtained analytic formulas for bubble nucleation rates, which are manifestly gauge invariant.
- We proposed a way to treat the integral over the bounce size, which gives a convergent result.
- We have confirmed that the SM vacuum is meta-stable, i.e. the lifetime is longer than the age of the Universe.
- We provide a c++ package that can calculate decay rates for generic models with approximate scale invariance.