Modular 群によるニュートリノ振動の解析

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in progress

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Introduction

クォーク・レプトンの世代構造の起源を求めて

Neutrino oscillation parameters

FIT3.2 (2018)

3σ interval	Normal Hierarchy	Inverted Hierarchy
Δm^2_{21}	$[6.80\,,8.02]\times10^{-5}[\mathrm{eV}^2]$	$[6.80\,,8.02]\times10^{-5}[\mathrm{eV}^2]$
Δm^2_{31}	$[2.399\,,2.593]\times10^{-3}[\mathrm{eV}^2]$	$-[2.369\,,2.562]\times10^{-3}[\mathrm{eV}^2]$
$\sin^2\theta_{12}$	$[0.272\,,0.346]$	$[0.272\,,0.346]$
$\sin^2\theta_{23}$	$[0.418\,,0.613]$	$[0.435\,,0.616]$
$\sin^2\theta_{13}$	$[1.981\,,2.436]\times10^{-2}$	$[2.006\,,2.452]\times10^{-2}$

5 parameters are available now

- ニュートリノ質量の大きさ？
- Δm^2_{31} の符号？
- CPを破る位相の大きさ？

➡ 予言力のあるモデルを作ることは難しい

フレーバー対称性を考える

フレーバー model

S_3, A_4, S_4, A_5 ... などによる多くのモデルが提案されてきた

Modular 群 の部分群に含まれる

Recent achievement

$\Gamma(2) \cong S_3$: T. Kobayashi, K. Tanaka, T.H. Tatsuishi, [arXiv: 1803.10391]

$\Gamma(3) \cong A_4$: F. Feruglio, [arXiv: 1706.08749]

$\Gamma(4) \cong S_4$: J.T. Penedo, S.T. Petocov, [arXiv: 1806.11040]

modular 対称性をもつニュートリノのフレーバー model で現象論を展開

-- 実験と consistent な解を探し当てた

motivation

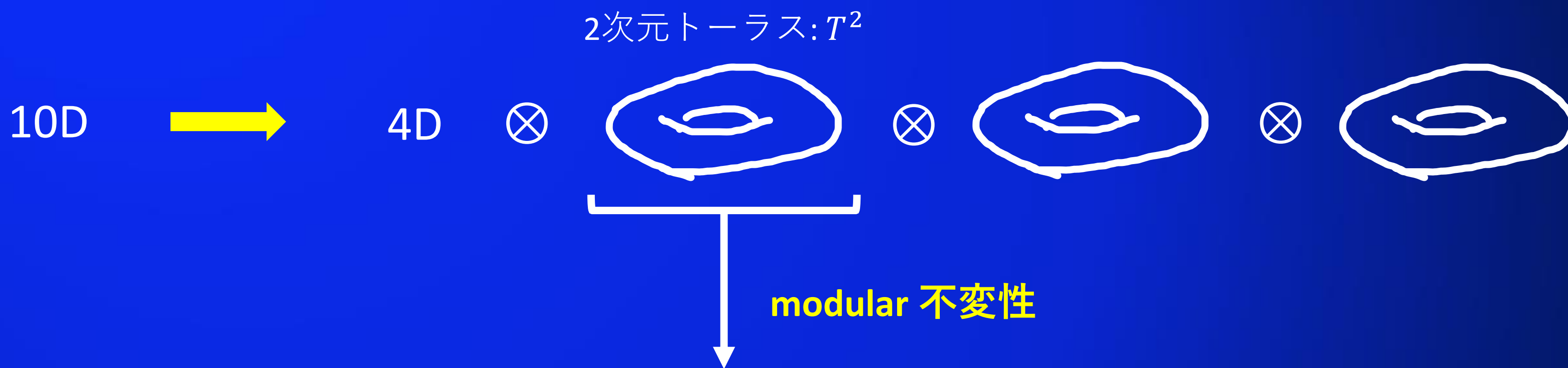
model parameter と 物理量の詳細な関係を解析したい ($\Gamma(3) \cong A_4$)

Outline

- **モジュラー群**
  - 群としての性質
  - モジュラー不変な量 (modular form)

- **模型と解析結果**
  - いくつかの模型と結果を紹介

4 + 6 次元時空のトーラスコンパクト化



3 世代のレプトン L_1, L_2, L_3

2次元トーラスの modular 不変性 の破れからフレーバー構造を得る

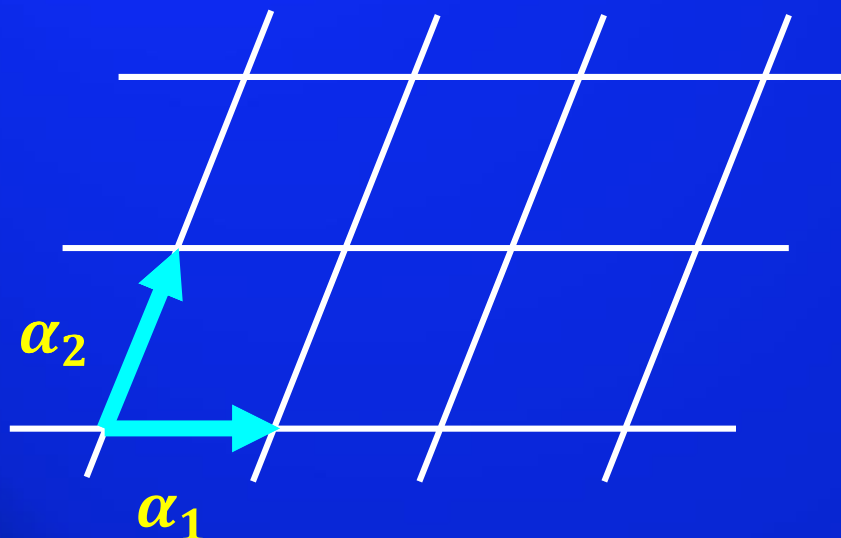
従来のフレーバーモデル

新粒子 (flavon) を導入

フレーバー対称性 ($S_3, A_4, \ldots$) を仮定し、理論に制限を与える

どんな群なのか？

2次元トーラス上に張られた格子



軸の変換：$SL(2,Z)$

$$\begin{pmatrix} \alpha_2' \\ \alpha_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix}$$

$ad - bc = 1$　　a, b, c, d: 整数

無数の変換が可能

→ **2種類の変換**で張られる

modulus τ による書き換え

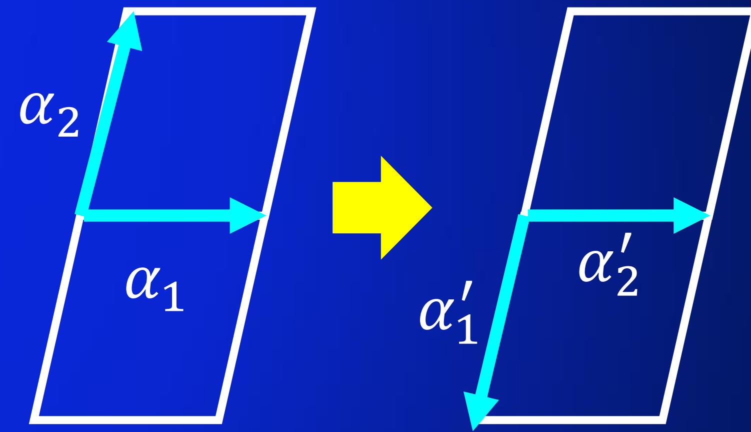
$$\tau \equiv \frac{\alpha_2}{\alpha_1} \longrightarrow \tau' \equiv \frac{\alpha'_2}{\alpha'_1} = \frac{a\tau + b}{c\tau + d}$$

$$ad - bc = 1$$

S 変換：

$$\begin{pmatrix} \alpha'_2 \\ \alpha'_1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ -\alpha_2 \end{pmatrix}$$

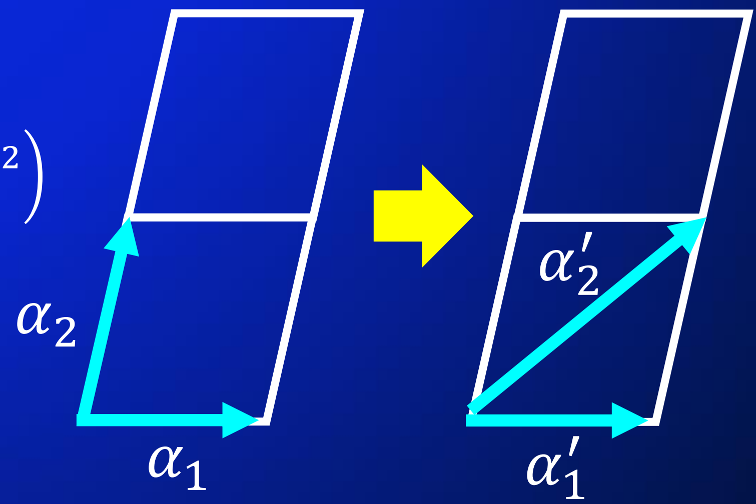
$$\boldsymbol{\tau' = -\frac{1}{\tau}}$$



T 変換：

$$\begin{pmatrix} \alpha'_2 \\ \alpha'_1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix} = \begin{pmatrix} \alpha_1 + \alpha_2 \\ \alpha_1 \end{pmatrix}$$

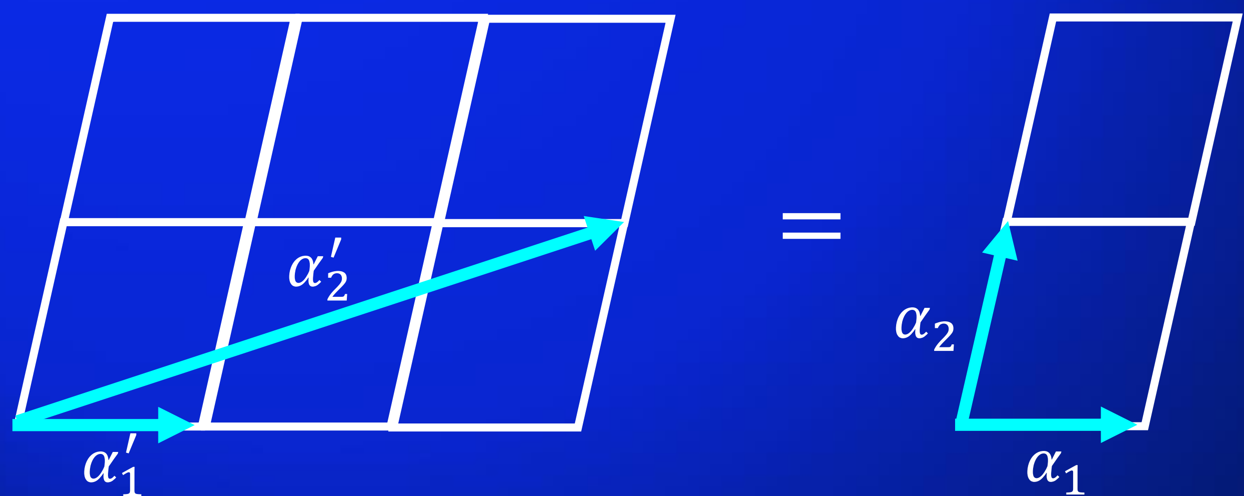
$$\boldsymbol{\tau' = \tau + 1}$$



Modular 群：Modular 変換で取りうる軸の基底の数々(**無限**個の要素)

$$S^2 = 1, \qquad (ST)^3 = 1$$

合同部分群 $\Gamma(N)$：$\boldsymbol{T^N = 1}$ の合同関係式を課す (**有限**個の要素)



$$\Gamma(2) \cong S_3,\ \Gamma(3) \cong A_4,\ \Gamma(4) \cong S_4\ ,\ \Gamma(5) \cong A_5$$

modular 不変な理論

$$\tau' = \frac{a\tau + b}{c\tau + d}, \quad ad - bc = 1$$

一般の chiral multiplet が受ける変換

$$\phi_i(\tau) \to (c\tau + d)^{-k_i}\rho_i(\gamma)\phi_i(\tau)$$

k_i : weight

S. Ferrara, D, Lust, A. Shapere, S. Theisen , Phys. Lett. B 225, 4 (1989)

$\rho_i(\gamma)$: $\gamma(= S, T)$ 変換に対応する$\Gamma(N)$ の表現行列

modular form (multiplet)

$$f(\tau) \to (c\tau + d)^{k}\rho(\gamma)f(\tau)$$

modular invariant superpotential

$$w = \sum_i f_{i1,i2\ldots in_i}(\tau)\phi_{(i1)}\phi_{(i2)} \ldots \phi_{(in_i)}$$

$k_1 + k_2 \ldots + k_{n_i} = k$ & $\rho f(\tau) \times \rho_i\phi_i(\tau) \times \cdots \times \rho_{n_i}\phi_{n_i}(\tau) \ni \mathbf{1}$ of $\Gamma(N)$

Modular form

Dedekind eta function を基に modular weight k = 2 の modular formを作る

Dedekind eta-function

$$\eta(\tau) \equiv q^{1/24} \prod_{n=1} (1 - q^n) \qquad q = e^{2\pi i \tau}$$

S 変換

$$\eta(-1/\tau) = \sqrt{-i\tau}\,\eta(\tau)$$

T 変換

$$\eta(\tau + 1) = e^{\pi i/12}\eta(\tau)$$

$\Gamma(2) \cong S_3$: T. Kobayashi, K. Tanaka, T.H. Tatsuishi, [arXiv: 1803.10391]

Main topic $\Gamma(3) \cong A_4$: F. Feruglio, [arXiv: 1706.08749]

$\Gamma(4) \cong S_4$: J.T. Penedo, S.T. Petocov, [arXiv: 1806.11040]

Modular weight 2 のmodular form (A_4 triplet) F. Feruglio, [arXiv: 1706.08749]

$$\begin{pmatrix} Y_1(-1/\tau) \\ Y_2(-1/\tau) \\ Y_3(-1/\tau) \end{pmatrix} = \tau^2 \rho(S) \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix}, \qquad \begin{pmatrix} Y_1(\tau+1) \\ Y_2(\tau+1) \\ Y_3(\tau+1) \end{pmatrix} = \rho(T) \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix}$$

Coupling Y は 定数ではなく τ の関数

Y_1, Y_2, Y_3 は Dedekind eta function で与えられる (τ だけの関数) :

$$Y_1(\tau) = \frac{i}{2\pi}\left[\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - 27\frac{\eta'(3\tau)}{\eta(3\tau)}\right]$$

$$Y_2(\tau) = -\frac{i}{\pi}\left[\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega^2\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right]$$

$$Y_3(\tau) = -\frac{i}{\pi}\left[\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega^2\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right]$$

$$\omega = e^{2\pi i/3}$$

$$Y = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} = \begin{pmatrix} 1 + 12q + 36q^2 + 12q^3 + \dots \\ -6q^{1/3}(1 + 7q + 8q^2 + \dots) \\ -18q^{2/3}(1 + 2q + 5q^2 + \dots) \end{pmatrix}, \qquad q = e^{2\pi i\tau}$$

$$Y_2^2 + 2Y_1Y_3 = 0$$

Modular weight 2 のmodular form (A_4 triplet)

$$\begin{pmatrix} Y_1(-1/\tau) \\ Y_2(-1/\tau) \\ Y_3(-1/\tau) \end{pmatrix} = \tau^2 \boldsymbol{\rho}(\boldsymbol{S}) \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix}, \qquad \begin{pmatrix} Y_1(\tau+1) \\ Y_2(\tau+1) \\ Y_3(\tau+1) \end{pmatrix} = \boldsymbol{\rho}(\boldsymbol{T}) \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix}$$

$\boldsymbol{\rho}(\boldsymbol{\gamma})$: $\gamma(=S,T)$ 変換に対応する$\Gamma(N)$ の表現行列

$\boldsymbol{\Gamma(3) \cong A_4}$ を考える場合

$$\rho(S) = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad \rho(T) = \begin{pmatrix} 1 & & \\ & \omega & \\ & & \omega^2 \end{pmatrix} \qquad \omega = e^{2\pi i/3}$$

R.A. Toorop, F. Feruglio, C. Hagedorn [arXiv: 1112.1340]

$S^2 = (ST)^3 = T^3 = 1$ を反映

preliminary

モデルの解析 $\Gamma(3) \cong A_4$ モデル without flavon

Model I : Type I seesaw

Normal Hierarchy : **consistent**

Inverted Hierarchy : **inconsistent**

Model II : Weinberg operator

Normal Hierarchy : **inconsistent**

Inverted Hierarchy : **inconsistent**

Model III : Dirac neutrino

Normal Hierarchy : **inconsistent**

Inverted Hierarchy : **consistent**

Model

preliminary

	L	e_R, μ_R, τ_R	ν_R	H_u	H_d	Y
$SU(2)$	2	1	1	2	2	1
A_4	3	$1, 1'', 1'$	3	1	1	3
$-k_I$	-1 (1)	-1 (-3)	-1	0	0	$k=2$

$Y \equiv \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}$: modular forms of weight 2

$\Gamma(3) \cong A_4$ 不変 superpotential

$$w_e = \alpha e_R H_d (LY) + \beta \mu_R H_d (LY) + \gamma \tau_R H_d (LY)$$

$$w_D = g(\nu_R H_u L Y)_{\mathbf{1}}$$

$$w_N = \Lambda(\nu_R \nu_R Y)_{\mathbf{1}}$$

$m_\nu = -m_D^T m_N m_D$ or $m_\nu = m_D$

Model I : Type I seesaw

Model III : Dirac neutrino

preliminary

モデルに含まれるパラメータについて

	L	e_R, μ_R, τ_R	ν_R	H_u	H_d	Y
$SU(2)$	2	1	1	2	2	1
A_4	3	$1, 1'', 1'$	3	1	1	3
$-k_I$	$-1\ (1)$	$-1\ (-3)$	-1	0	0	$k=2$

$Y \equiv \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}$: modular forms of weight 2

τ を与えれば決まる

$$w_e = \alpha e_R H_d (LY) + \beta \mu_R H_d (LY) + \gamma \tau_R H_d (LY)$$

$$w_D = g(\nu_R H_u L Y)_{\mathbf{1}}$$

$$w_N = \Lambda (\nu_R \nu_R Y)_{\mathbf{1}}$$

α, β, γ : real without loss of generality

charged lepton mass で決まる

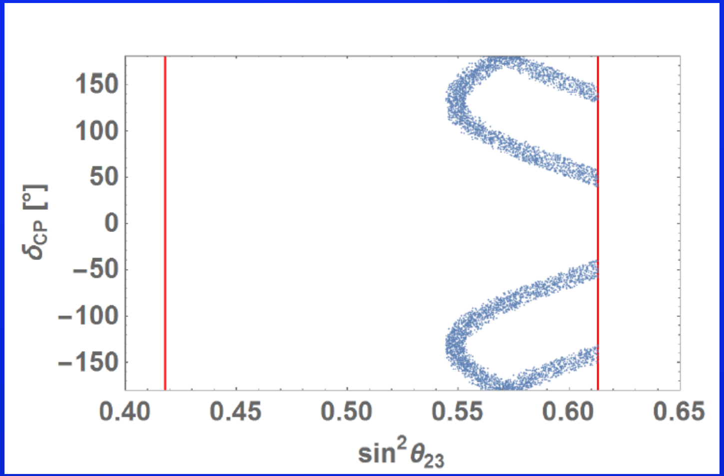
$$w_D = v_u \begin{pmatrix} \nu_{R1} \\ \nu_{R2} \\ \nu_{R3} \end{pmatrix} \otimes \left[g_1 \begin{pmatrix} 2\nu_e Y_1 - \nu_\mu Y_3 - \nu_\tau Y_2 \\ 2\nu_\tau Y_3 - \nu_e Y_2 - \mu Y_1 \\ 2\nu_\mu Y_2 - \nu_\tau Y_1 - \nu_e Y_3 \end{pmatrix} \oplus g_2 \begin{pmatrix} \nu_\mu Y_3 - \nu_\tau Y_2 \\ \nu_e Y_2 - \nu_\mu Y_1 \\ \nu_\tau Y_1 - \nu_e Y_3 \end{pmatrix} \right]$$

2 つの複素 parameters : τ , $g \left(\equiv \frac{g_2}{g_1} \right)$

Model I : Type I seesaw (NH)

preliminary

$\mathrm{Im}[\tau]$	0.90-1.12
$\mathrm{Re}[\tau]$	$\pm$(0.01-0.07) $\pm$(0.94-1.10)
g	1.43-2.12
ϕ_g	$\pm(76\text{-}104)^\circ$



FIT3.2 (2018)

3σ interval	Normal Hierarchy	
Δm^2_{21}	$[6.80\,, 8.02] \times 10^{-5}[\mathrm{eV}^2]$	
Δm^2_{31}	$[2.399\,, 2.593] \times 10^{-3}[\mathrm{eV}^2]$	
$\sin^2\theta_{12}$	$[0.272\,, 0.346]$	
$\sin^2\theta_{23}$	**$[0.418\,, 0.613]$**	
$\sin^2\theta_{13}$	$[1.981\,, 2.436] \times 10^{-2}$	

$\sin^2\theta_{23}$: $0.538^{+0.033}_{-0.069}$ (1σ range)

$\sin^2\theta_{12}$, $\sin^2\theta_{13}$ --- allowed in full range of 3σ

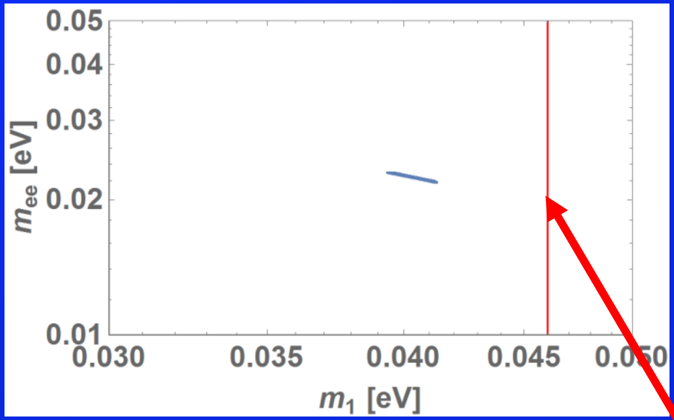
$\sin^2\theta_{23} > 0.54$ --- will be tested in near future

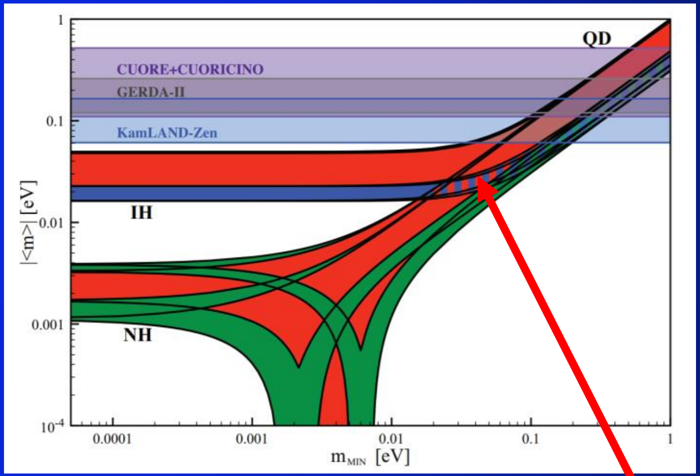
Maximum CP violation ($\delta_{CP} = \pm 90^\circ$) is realized around 1σ value

Model I : Type I seesaw (a) NH

preliminary

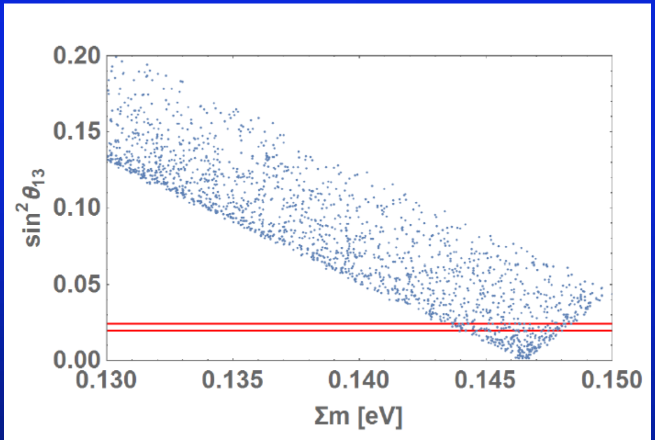
neutrinoless double beta decay





from PDG 2018

ニュートリノ質量の上限 : $\Sigma m_i \leq 0.16$



この近く

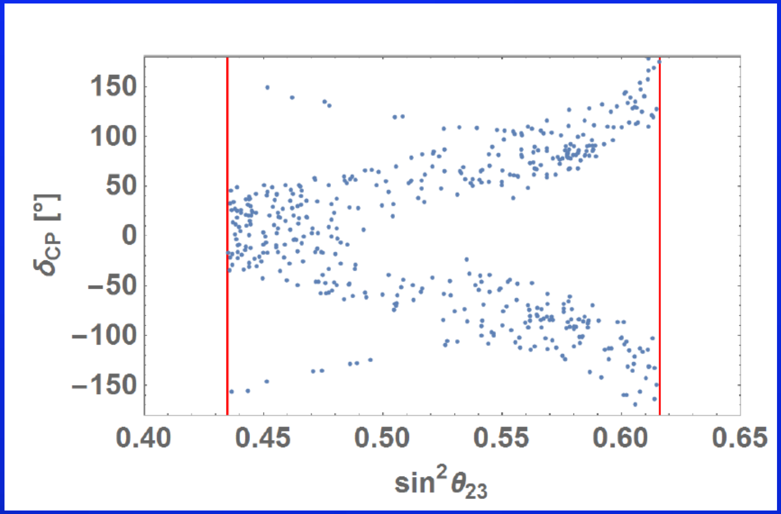
$m_1 \sim m_2 \sim 40$ [meV]
$m_3 \sim 60$ [meV]

近い将来の実験で検証可能

Model III : Dirac neutrino (IH)

preliminary

$\mathrm{Im}[\tau]$	1.17-1.32
$\mathrm{Re}[\tau]$	$\pm(0.46\text{-}0.54)$ $\pm(1.46\text{-}1.50)$
g	1.20-1.22
ϕ_g	$\pm(85\text{-}89)^\circ$ $\pm(91\text{-}95)^\circ$



FIT3.2 (2018)

3σ interval		Inverted Hierarchy
Δm_{12}^2		$[6.80\,,8.02]\times 10^{-5}[\mathrm{eV}^2]$
Δm_{13}^2		$-[2.369\,,2.562]\times 10^{-3}[\mathrm{eV}^2]$
$\sin^2\theta_{12}$		$[0.272\,,0.346]$
$\sin^2\theta_{23}$		$[0.435\,,0.616]$
$\sin^2\theta_{13}$		$[2.006\,,2.452]\times 10^{-2}$

$\sin^2\theta_{23}$: $0.554^{+0.023}_{-0.033}$ (1σ range)

$\sin^2\theta_{12}\,,\sin^2\theta_{23}\,,\sin^2\theta_{13}$ --- allowed in full range of 3σ

Maximum CP violation ($\delta_{CP}=\pm 90^\circ$) is realized around 1σ value

Model III : Dirac neutrino (IH)

preliminary

$\mathrm{Im}[\tau]$	0.90-1.12
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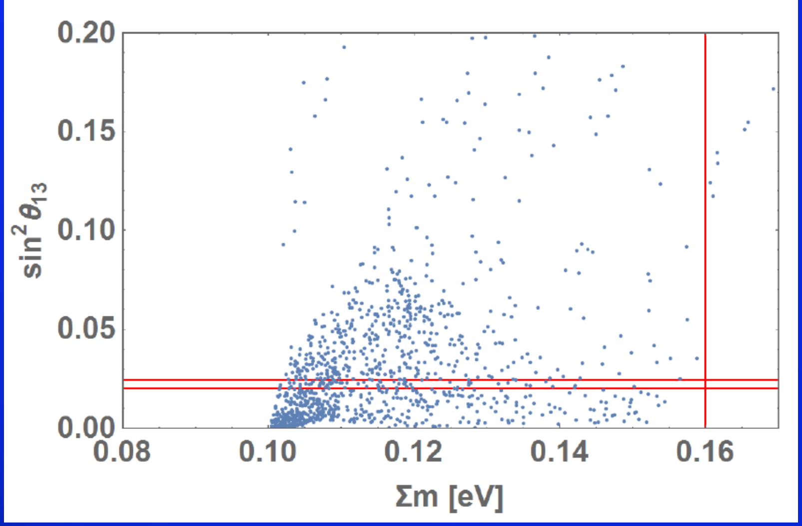

FIT3.2 (2018)

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Lightest neutrino mass >5.7 meV

Model I (NH) の結果とは対照的

- modular symmetry
  - 2次元トーラス上で定義 (トーラスコンパクト化から実現可能)
  - S_3, A_4, S_4, A_5 と同形の合同部分群を持つ
  - フレーバー対称性の枠組みとして応用可能

- flavon を導入しない A_4 模型を解析
  - Normal Hierarchy : Type I seesaw で consistent
  - Inverted Hierarchy : neutrino が Dirac particle とした場合に consistent

- 今後の展望
  - クォークセクター
  - 他の群: $\Gamma(2) \cong S_3, \Gamma(4) \cong S_4, \Gamma(5) \cong A_5$

Thank you

Modular weight 2 のmodular form

まず仮定として

modular form (component)

$$f_i(\tau) \to (c\tau + d)^{k_i} f_i(\tau)$$

次が証明可能

$$\frac{d}{d\tau}\sum_i \log f_i(\tau) \to (c\tau + d)^2 \left[\sum_i \log f_i(\tau) + (c\tau + d)\sum_i k_i\right]$$

$\boldsymbol{\sum_i k_i = 0}$ **ならば、** $\frac{d}{d\tau}\sum_i \log f_i(\tau)$ **の weight は 2**

$\boldsymbol{f_i(\tau)}$ **: Dedekind eta function を用いる**

$f_i(\tau) = \eta(c_i\tau)^{\alpha_i}$ **の weight は** $\alpha_i/2$ $\qquad$ $\boldsymbol{\sum_i \alpha_i = 0}$ **で** $\boldsymbol{\sum_i k_i = 0}$

Modular weight 2 のmodular form : $\frac{d}{d\tau}\sum_i \alpha_i \log\eta(c_i\tau)$ $\quad \boldsymbol{\sum_i \alpha_i = 0}$

Γ(N) のmultiplet を作る (A_4 の例)

$$Y_1 = 3c\frac{d}{d\tau}\left[\log\eta\left(\frac{\tau}{3}\right) + \log\eta\left(\frac{\tau+1}{3}\right) + \log\eta\left(\frac{\tau+2}{3}\right) - 3\log\eta(3\tau)\right]$$

$$Y_2 = -6c\frac{d}{d\tau}\left[\log\eta\left(\frac{\tau}{3}\right) + \omega^2\log\eta\left(\frac{\tau+1}{3}\right) + \omega\log\eta\left(\frac{\tau+2}{3}\right)\right]$$

$$Y_3 = -6c\frac{d}{d\tau}\left[\log\eta\left(\frac{\tau}{3}\right) + \omega\log\eta\left(\frac{\tau+1}{3}\right) + \omega^2\log\eta\left(\frac{\tau+2}{3}\right)\right]$$

$\boldsymbol{c}$ **: 任意係数**

modular form (A_4 triplet)

$$Y(\tau) \to (c\tau + d)^2\rho(\gamma)Y(\tau)$$

Modular weight 2 のmodular form (S_3 doublet)

$$\begin{pmatrix} Y_1(-1/\tau) \\ Y_2(-1/\tau) \end{pmatrix}_{\mathbf{2}} = \tau^2 \rho(S) \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}}, \qquad \begin{pmatrix} Y_1(\tau+1) \\ Y_2(\tau+1) \end{pmatrix}_{\mathbf{2}} = \rho(T) \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}}$$

$$Y_1(\tau) = \frac{i}{4\pi}\left[\frac{\eta'(\tau/2)}{\eta(\tau/2)} + \frac{\eta'((\tau+1)/2)}{\eta((\tau+1)/2)} - 8\frac{\eta'(2\tau)}{\eta(2\tau)}\right]$$

$$Y_2(\tau) = -\frac{\sqrt{3}i}{4\pi}\left[\frac{\eta'(\tau/2)}{\eta(\tau/2)} - \frac{\eta'((\tau+1)/2)}{\eta((\tau+1)/2)}\right]$$

$$\rho(S) = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \qquad \rho(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Modular weight 2 のmodular form (S_4 doublet & triplet)

J.T. Penedo, S.T. Petocov, [arXiv: 1806.11040]

$$\begin{pmatrix} Y_1(-1/\tau) \\ Y_2(-1/\tau) \end{pmatrix}_{\mathbf{2}} = \tau^2 \rho(S) \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}}, \qquad \begin{pmatrix} Y_1(\tau+1) \\ Y_2(\tau+1) \end{pmatrix}_{\mathbf{2}} = \rho(T) \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}}$$

$$\begin{pmatrix} Y_3(-1/\tau) \\ Y_4(-1/\tau) \\ Y_5(-1/\tau) \end{pmatrix}_{\mathbf{3}'} = \tau^2 \rho(S) \begin{pmatrix} Y_3(\tau) \\ Y_4(\tau) \\ Y_5(\tau) \end{pmatrix}_{\mathbf{3}'}, \qquad \begin{pmatrix} Y_3(\tau+1) \\ Y_4(\tau+1) \\ Y_5(\tau+1) \end{pmatrix}_{\mathbf{3}'} = \rho(T) \begin{pmatrix} Y_3(\tau) \\ Y_4(\tau) \\ Y_5(\tau) \end{pmatrix}_{\mathbf{3}'}$$

$$Y_1(\tau) = \frac{\eta'(\tau+1/2)}{\eta(\tau+1/2)} + \frac{\eta'(4\tau)}{\eta(4\tau)} + \omega\frac{\eta'(\tau/4)}{\eta(\tau/4)} + \omega^2\frac{\eta'((\tau+1)/4)}{\eta((\tau+1)/4)} + \omega\frac{\eta'((\tau+1)/4)}{\eta((\tau+1)/4)} + \omega^2\frac{\eta'((\tau+1)/4)}{\eta((\tau+1)/4)}$$

各係数のみ異なる

$$Y_2(\tau) = [1,1,\omega^2,\omega,\omega^2,\omega] \qquad Y_3(\tau) = [1,-1,-1,-1,1,1]$$

$$Y_4(\tau) = [1,-1,-\omega^2,-\omega,\omega^2,\omega] \qquad Y_5(\tau) = [1,-1,-\omega,-\omega^2,\omega,\omega^2]$$

$$\rho(S) = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} \quad \rho(S) = \frac{1}{3}\begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega & 2 & -\omega^2 \\ 2\omega^2 & -\omega & 2 \end{pmatrix} \qquad \rho(T) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \rho(T) = \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}$$

Modular invariant kinetic term

$$\frac{\left|\boldsymbol{\partial_\mu \phi_i}\right|^2}{\langle \boldsymbol{\tau} - \boldsymbol{\tau}^* \rangle^{\boldsymbol{k_i}}}$$

S. Ferrara, D, Lust, A. Shapere, S. Theisen , Phys. Lett. B 225, 4 (1989)

scalar component の modular 変換

$$\boxed{\phi_i(\tau) \rightarrow (c\tau + d)^{-k_i} \phi_i(\tau)}$$

Model II : Weinberg operator

preliminary

	L	e_R, μ_R, τ_R	ν_R	H_u	H_d	Y
$SU(2)$	2	1	1	2	2	1
A_4	3	$1, 1'', 1'$	3	1	1	3
$-k_I$	-1	-1	-1	0	0	$k=2$

$$Y \equiv \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}$$: modular forms of weight 2

$$w_e = \alpha e_R H_d(LY) + \beta \mu_R H_d(LY) + \gamma \tau_R H_d(LY)$$: charged lepton mass term

$$w_\nu = -\frac{1}{\Lambda}(H_u H_u LLY)_1$$: neutrino mass term

Normal Hierarchy : **inconsistent**

Inverted Hierarchy : **inconsistent**

Model III : Dirac neutrino

preliminary

	L	e_R, μ_R, τ_R	ν_R	H_u	H_d	Y
$SU(2)$	2	1	1	2	2	1
A_4	3	$1, 1'', 1'$	3	1	1	3
$-k_I$	-1	-1	-1	0	0	$k=2$

$Y \equiv \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}$: modular forms of weight 2

$$w_e = \alpha e_R H_d(LY) + \beta \mu_R H_d(LY) + \gamma \tau_R H_d(LY)$$

: charged lepton mass term

$$w_D = g(\nu_R H_u L Y)_{\mathbf{1}}$$

: neutrino mass term

Normal Hierarchy : **inconsistent**

Inverted Hierarchy : **consistent**