Flowing to the Bounce

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Refs:

Chigusa, TM, Shoji, 1906.10829 [hep-ph]

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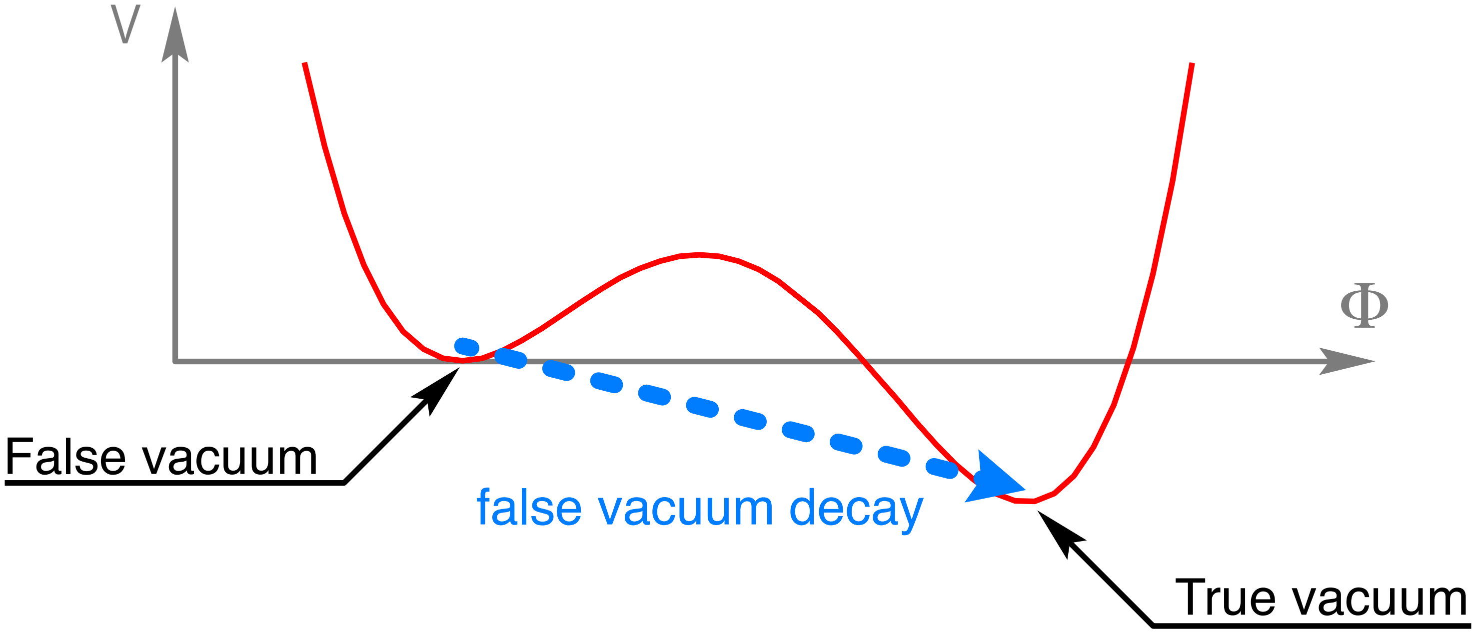
1. Introduction

The subject today: a new method to calculate the bounce



- False vacua show up in many particle-physics models
- Tunneling process is dominantly induced by the field configuration called "bounce"

Today, I try to explain

- Why is the calculation of the bounce difficult?
- What is our new idea?
- Why does it work?
- Does it really work?

Outline

1. Introduction
2. Bounce
3. Calculating Bounce with Flow Equation
4. Numerical Analysis
5. Summary

2. Bounce

Calculation of the decay rate *à la* Coleman

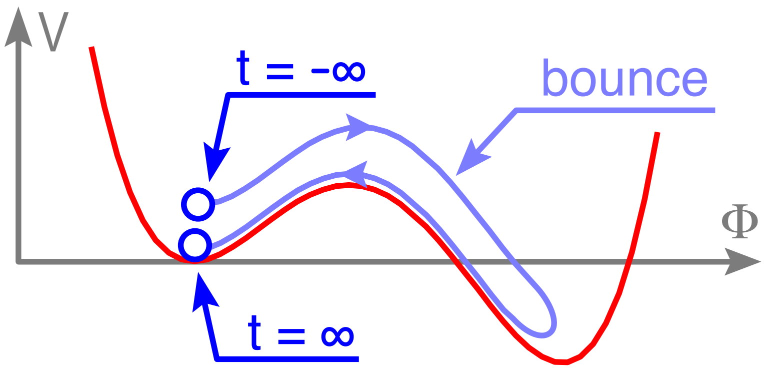
- The decay rate is related to Euclidean partition function

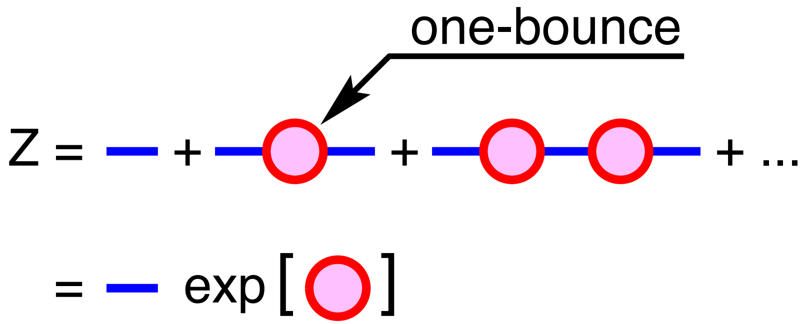
$$Z = \langle \mathsf{FV}|e^{-HT}|\mathsf{FV}\rangle \simeq \int \mathcal{D}\phi\, e^{-\mathcal{S}[\phi]} \propto \exp(i\gamma VT)$$

- Euclidean action

$$\mathcal{S}[\phi] = \int d^D x \left(\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + V\right)$$

- The false vacuum decay is dominated by the classical path



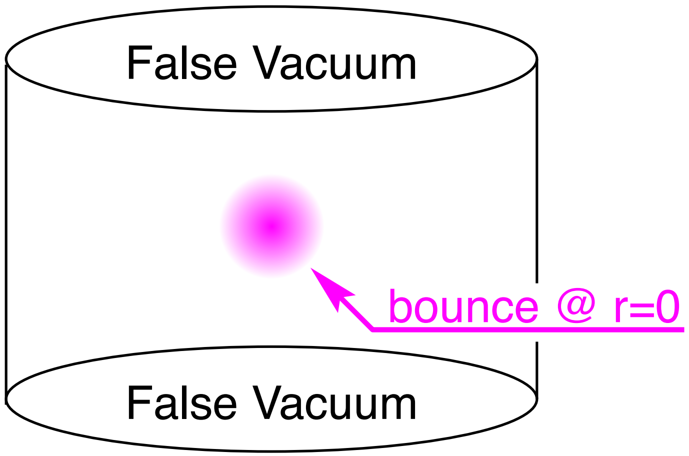


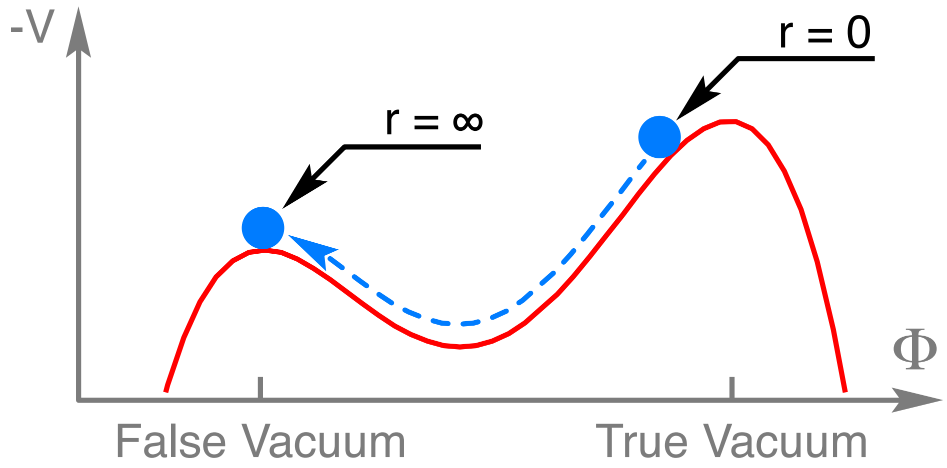
The bounce: spherical solution of Euclidean EoM

[Coleman; Callan & Coleman]

$$\left[\partial^2\phi - \frac{\partial V}{\partial\phi}\right]_{\phi\to\bar\phi} = \left[\partial_r^2\phi + \frac{D-1}{r}\partial_r\phi - \frac{\partial V}{\partial\phi}\right]_{\phi\to\bar\phi} = 0$$

with $\begin{cases}\bar\phi(r=\infty) = v: & \text{false vacuum}\\ \bar\phi'(0) = 0\end{cases}$





Bounce is important for the study of false vacuum decay

$$\gamma = \mathcal{A} e^{-\mathcal{S}[\bar{\phi}]}$$

Why is the calculation of $\bar{\phi}$ so difficult?

Bounce is a saddle-point solution of the EoM

Expansion of the action around the bounce: $\phi = \bar{\phi} + \Psi$

- $\mathcal{S}[\bar{\phi} + \Psi] = \mathcal{S}[\bar{\phi}] + \frac{1}{2} \int d^D x \Psi \mathcal{M} \Psi + O(\Psi^3)$

$$\mathcal{M} \equiv -\partial_r^2 - \frac{D-1}{r} \partial_r + \left. \frac{\partial^2 V}{\partial \phi^2} \right|_{\phi \to \bar{\phi}}$$: fluctuation operator

- $\mathcal{M}$ has one negative eigenvalue (which we call λ_-)

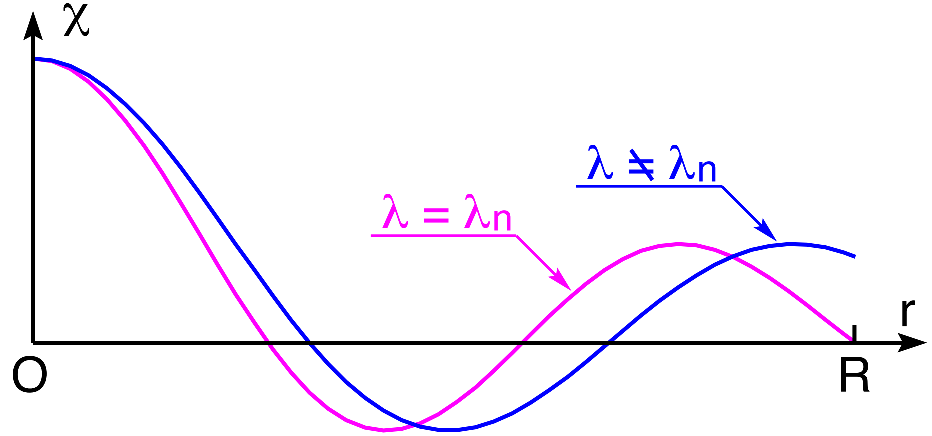
[Callan & Coleman]

Fluctuation around the bounce: $\phi = \bar{\phi} + \Psi$

- $\partial_r \Psi(r = 0) = 0$
- $\Psi(r = \infty) = 0$

We expand Ψ by using eigenfunctions of $\mathcal{M}$

$\Rightarrow \mathcal{M}\chi = \lambda\chi$



We need to impose relevant boundary conditions

- $\partial_r \chi_n(r = 0) = 0$
- $\chi_n(r = \infty) = 0$

An evidence of the existence of negative eigenvalue

- Functions are expanded by χ_n (eigenfunctions of $\mathcal{M}$)

  $\langle \chi_n | \chi_m \rangle = \delta_{nm}$, where $\langle f | f' \rangle \equiv \int_0^\infty dr r^{D-1} f(r) f'(r)$
- $f(r) = \sum_n \langle f | \chi_n \rangle \chi_n(r)$
- $\langle f | \mathcal{M} f \rangle = \sum_n \lambda_n \langle \chi_n | f \rangle^2$

Example: $f(r) = r\partial_r \bar{\phi}$

- $\langle (r\partial_r \bar{\phi}) | \mathcal{M} (r\partial_r \bar{\phi}) \rangle = -(D-2) \int_0^\infty dr r^{D-1} (\partial_r \bar{\phi})(\partial_r \bar{\phi}) < 0$
- $r\partial_r \bar{\phi}$: fluctuation w.r.t. the "scale transformation"

  $\bar{\phi}((1+\epsilon)r) \simeq \bar{\phi}(r) + \epsilon\, r\partial_r \bar{\phi} + \cdots$

Undershoot-overshoot method to calculate the bounce

$$\partial_r^2\phi + \frac{D-1}{r}\partial_r\phi - \frac{\partial V}{\partial\phi} = 0$$

2nd term is a "friction," which disappears as $r \to \infty$

There should exist bounce, satisfying $\bar{\phi}'(0) = 0$ and $\bar{\phi}(\infty) = v$

- If $\phi(0) \lesssim v_c$

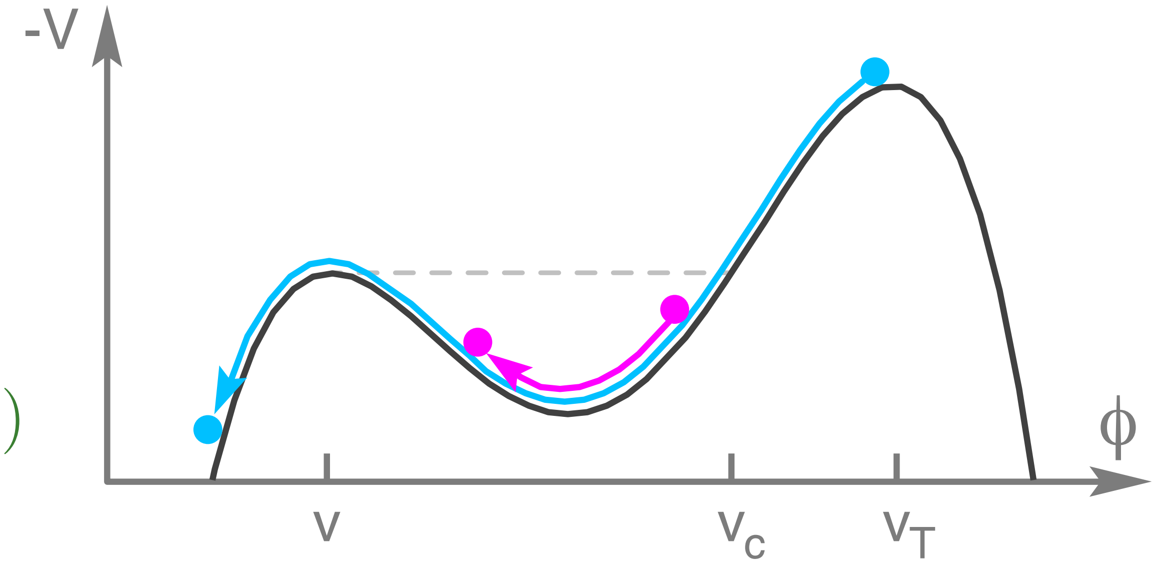
  $\Rightarrow$ Undershoot

- If $\phi(0) \simeq v_T$

  $\Rightarrow$ Overshoot

- There exists right $\phi(0)$

  $\Rightarrow \phi(\infty) = v$



It is not easy to obtain bounce in general

$\Rightarrow$ In particular, more difficulties with multi-fields

There has been various methods and attempts

- Undershoot-overshoot method
- Dilatation maximization

  [Claudson, Hall, Hinchliffe ('83)]
- Improved action

  [Kusenko ('95); Kusenko, Langacker, Segre ('96); Dasgupta ('96)]
- Squared EoM

  [Moreno, M. Quiros, M. Seco ('98); John ('98)]
- Backstep

  [Cline, Espinosa, Moore, Riotto ('98); Cline, Moore, Servant ('99)]

- Improved potential

  [Konstandin, Huber ('06); Park ('10)]

- Path deformation

  [Wainwright ('11)]

- Perturbative method

  [Akula, Balazs, White ('16); Athron et al. ('19)]

- Multiple shooting

  [Masoumi, Olum, Shlaer ('16)]

- Tunneling potential

  [Espinosa ('18); Espinosa, Konstandin ('18)]

- Polygon approximation

  [Guada, Maiezza, Nemevsek ('18)]

- Machine learning

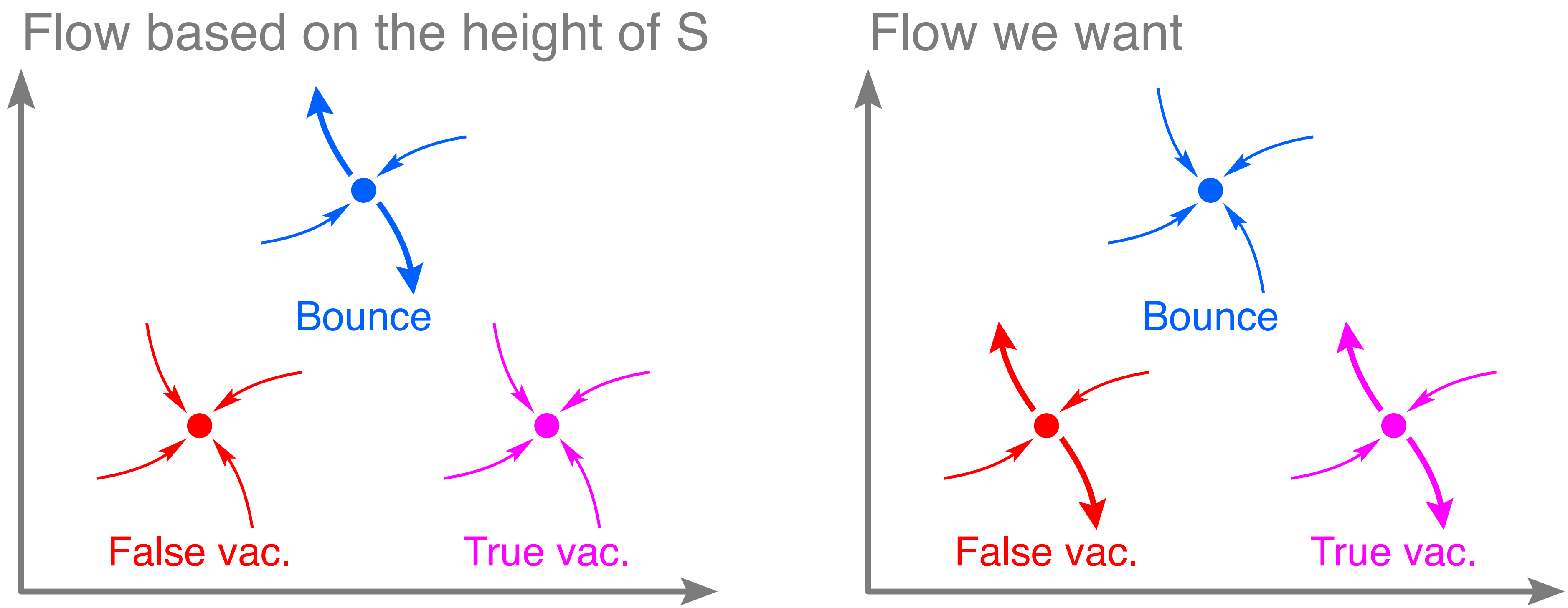
  [Jinno ('18); Piscopo, Spannowsky, Waite ('19)]

3. Bounce from Flow Equation

We want a flow eq. which has bounce as a stable fixed point

- $\partial_s \Phi(r, s) = \mathcal{G}[\Phi]$
- $\Phi(r, s \to \infty) = \bar{\phi}(r)$

Schematic view of the flow on the configuration space



Flow based on the height of $\mathcal{S}$

$$\partial_s \Phi(r,s) = F(r,s)$$

$$F \equiv -\frac{\delta \mathcal{S}[\Phi]}{\delta \Phi} = \partial_r^2 \Phi + \frac{D-1}{r}\partial_r \Phi - \frac{\partial V(\Phi)}{\partial \Phi}$$

Behavior of fluctuations around the bounce

$$\Phi(r,s) = \bar{\phi}(r) + \sum_n a_n(s)\chi_n(r)$$

$$\Rightarrow \sum_n \dot{a}_n \chi_n \simeq -\mathcal{M} \sum_n a_n \chi_n = -\sum_n \lambda_n a_n \chi_n$$

$$\Rightarrow \dot{a}_n \simeq -\lambda_n a_n$$

Because of χ_-, bounce cannot be a stable fixed point

$\Rightarrow$ This does not work

Flow equation of our proposal, which has a parameter β

$$\partial_s \Phi(r,s) = F(r,s) - \beta \langle F|g\rangle g(r)$$

$g(r)$: some function with $\langle g|g\rangle = 1$

$$g(r) \equiv \sum_n c_n \chi_n(r)$$

We will see:

With relevant choices of $g(r)$ and β, the bounce becomes a stable fixed point of our flow equation

For $\beta \neq 1$:

$\partial_s \Phi = 0 \;\; \Rightarrow \;\; F = 0$ (solution of EoM)

$\Leftrightarrow$ Fixed points do not depend on β

Behavior of the fluctuation: $\Phi(r,s) = \bar{\phi}(r) + \sum_n a_n(s)\chi_n(r)$

$$F(r,s) \simeq -\mathcal{M}(\Phi - \bar{\phi}) = -\sum_m \lambda_m a_m \chi_m$$

$$\langle F|g\rangle \simeq -\sum_m \lambda_m c_m a_m$$

$$\dot{a}_n \simeq -\lambda_n a_n + \beta \sum_m c_n c_m \lambda_m a_m \equiv -\sum_m \Gamma_{nm}(\beta) a_m$$

In the matrix form:

$$\dot{\vec{a}} \simeq -\Gamma(\beta)\vec{a}$$

$$\Gamma(\beta) = \left(\mathrm{I} - \beta\vec{c}\,\vec{c}^{\,T}\right)\mathsf{diag}(\lambda_-, \lambda_1, \lambda_2, \cdots)$$

Eigenvalues of Γ: γ_n (which are complex in general)

$$\Rightarrow \vec{a} \sim \sum_n \vec{v}_n e^{-\gamma_n s}$$

If $\mathrm{Re}\,\gamma_n > 0$ for $\forall n$, then $\vec{a}(s \to \infty) = 0$

We first study $\det\Gamma(\beta) = \prod_n \gamma_n$

Notice: $\det\left(\mathrm{I} - \beta\vec{c}\,\vec{c}^T\right) = 1 - \beta$

$$\left(\mathrm{I} - \beta\vec{c}\,\vec{c}^T\right)\vec{c} = (1-\beta)\vec{c}$$

$\left(\mathrm{I} - \beta\vec{c}\,\vec{c}^T\right)\vec{v}_\perp = \vec{v}_\perp$, if $\vec{c}^T\vec{v}_\perp = 0$

$\det\Gamma(\beta) = (1-\beta)\prod_n \lambda_n$

$\Rightarrow$ $\det\Gamma(\beta) > 0$, if $\beta > 1$

$\Rightarrow$ Taking $\beta > 1$, real parts of all the eigenvalues of Γ may become positive

Existence proof of $g(r)$ which realizes $\mathrm{Re}\,\gamma_n > 0$ for $\forall n$

$g(r) = \chi_-$, i.e., $\vec{c} = (1, 0, 0, \cdots)^T$

$\Rightarrow \Gamma(\beta) = \mathrm{diag}(1-\beta, 1, 1, \cdots)\,\mathrm{diag}(\lambda_-, \lambda_1, \lambda_2, \cdots)$

A guideline to choose $g(r)$

$\Rightarrow$ We should take $g(r)$ with sizable c_-

$g(r) \equiv \sum_n c_n \chi_n(r)$ with $\sum_n c_n^2 = 1$

Our choice: $g(r) \propto r\partial_r \Phi(r, s)$

- $\langle (r\partial_r \bar{\phi}) | \mathcal{M} (r\partial_r \bar{\phi}) \rangle = -(D-2) \int_0^\infty dr r^{D-1} (\partial_r \bar{\phi})(\partial_r \bar{\phi})$
- Empirically, it works well (see the numerical results)

If $\Phi(s \to \infty, r)$ goes to a stable fixed point with $\beta > 1$

1. $\Phi(s \to \infty, r)$ is a solution of EoM

2. $\Phi(s \to \infty, r)$ satisfies the BCs relevant for the bounce

3. $\Phi(s \to \infty, r)$ cannot be the false or true vacuum

$\Leftrightarrow$ Real parts of the eigenvalues of $\Gamma(\beta > 1)$ are all positive because $\Phi(s \to \infty, r)$ is stable against fluctuations

$\Leftrightarrow$ $\det\Gamma(\beta = 0) < 0$, so the fluctuation operator around $\Phi(s \to \infty, r)$ has a negative eigenvalue

$\Leftrightarrow$ For the fluctuation operator around the false or true vacuum, $\det\Gamma(\beta = 0) > 0$

$\Rightarrow$ Thus, $\Phi(s \to \infty, r)$ is a bounce

4. Numerical Analysis

We considered single- and double scalar cases:

- Single-scalar case:

$$V^{(1)} = \frac{1}{4}\phi^4 - \frac{k_1+1}{3}\phi^3 + \frac{k_1}{2}\phi^2$$

  - False vacuum: $\phi = 0$
  - True vacuum: $\phi = 1$

- Double-scalar case:

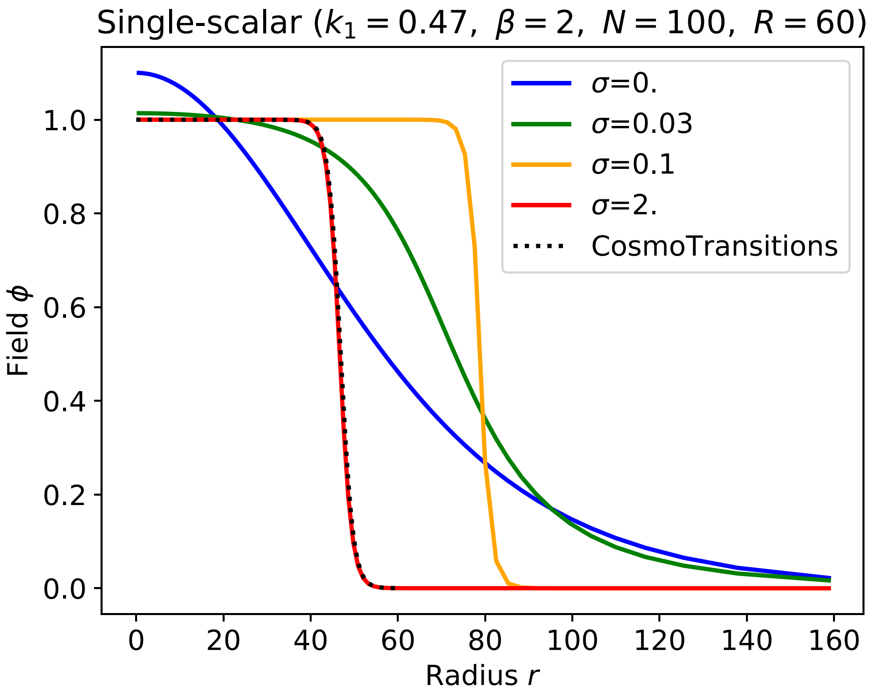
$$V^{(2)} = \left(\phi_x^2 + 5\phi_y^2\right)\left[5(\phi_x - 1)^2 + (\phi_y - 1)^2\right] + k_2\left(\frac{1}{4}\phi_y^4 - \frac{1}{3}\phi_y^3\right)$$

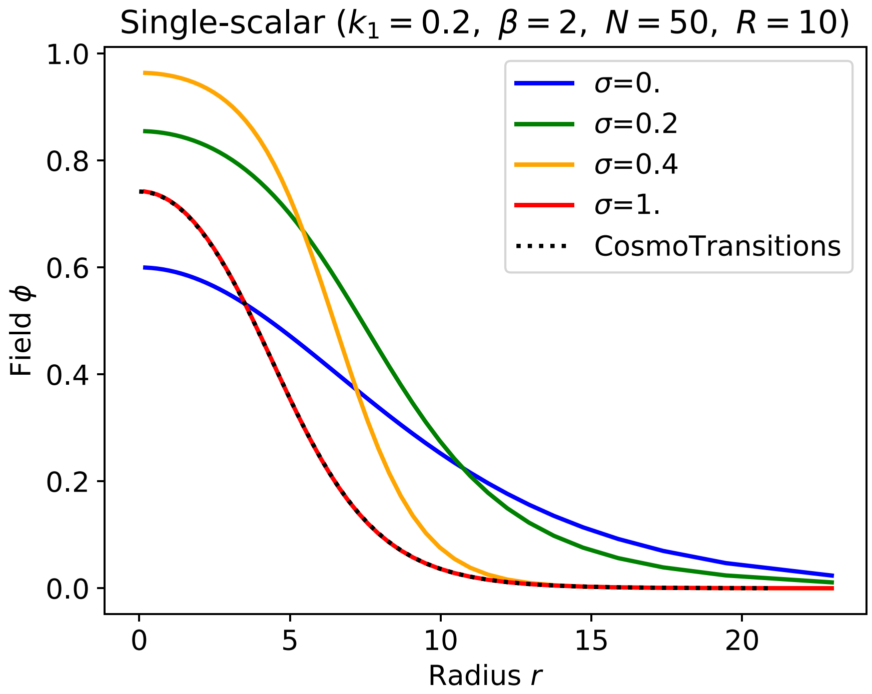
  - False vacuum: $(\phi_x, \phi_y) = (0, 0)$
  - True vacuum: $(\phi_x, \phi_y) = (1, 1)$

- We compare our results with those of `CosmoTransitions` [Wainwright]

Single-scalar case (with $D = 3$)

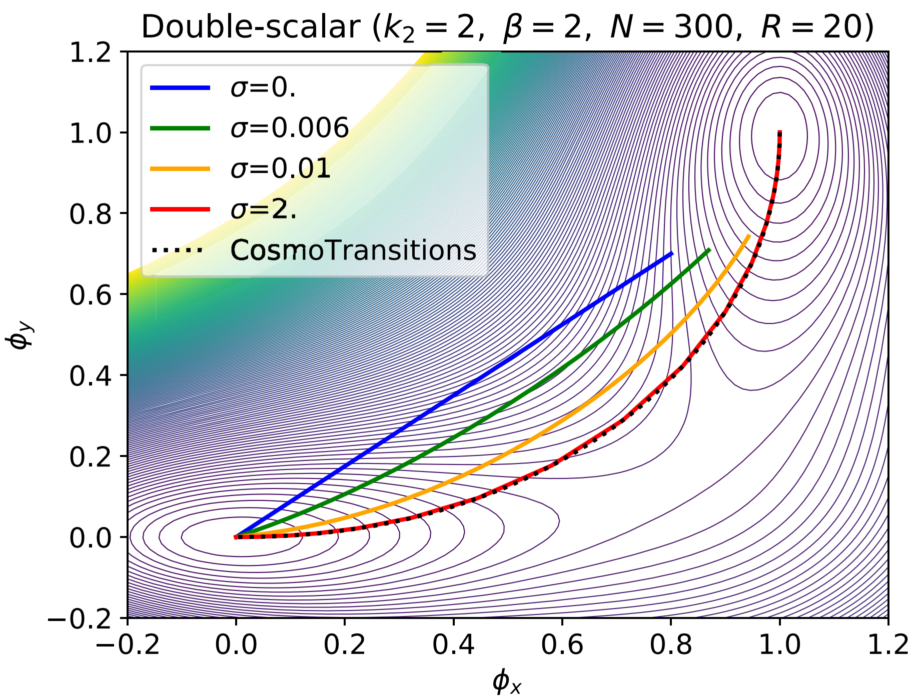
- Left: thin-wall (model 1a)
- Right: thick-wall (model 1b)

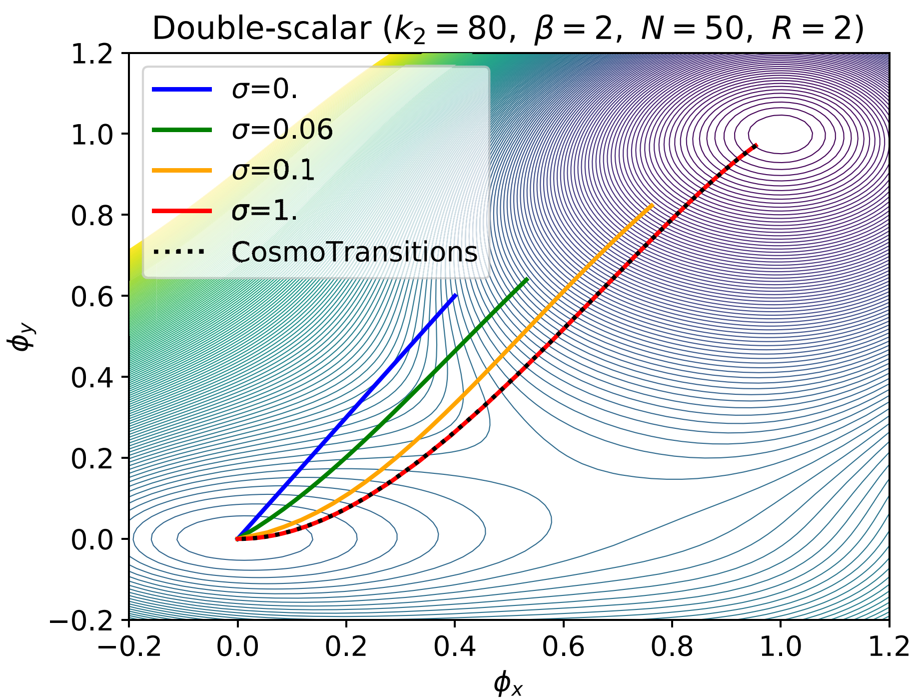




Double-scalar case (with $D = 3$)

- Left: thin-wall (model 2a)
- Right: thick-wall (model 2b)





Bounce action $\mathcal{S}[\bar{\phi}]$

Model	Our Result	`CosmoTransitions`
1a	1092.5	1092.8
1b	6.6298	6.6490
2a	1769.1	1767.7
2b	4.4567	4.4661

- Our results well agree with those of `CosmoTransitions`
- Bounce configuration (and its action) can be precisely calculated by using flow equation
- Compared to `CosmoTransitions`, our method gives better accuracy for the behavior of $\bar{\phi}(r \to \infty)$

Another approach

[Coleman, Glaser, Martin ('78); Sato ('19)]

1. Determine the configuration $\bar{\varphi}(r;\mathcal{P})$ which minimizes $\mathcal{S}$ on the hypersurface with constant $\mathcal{P}$

   $$\mathcal{P} \equiv \int d^D x V$$

   Flow equation:

   $$\partial_s \Phi(r,s) = F - \xi[\Phi] \frac{\partial V}{\partial \Phi}$$

   At the fixed point: $\bar{\varphi}(r;\mathcal{P}) = \Phi(r, s \to \infty)$

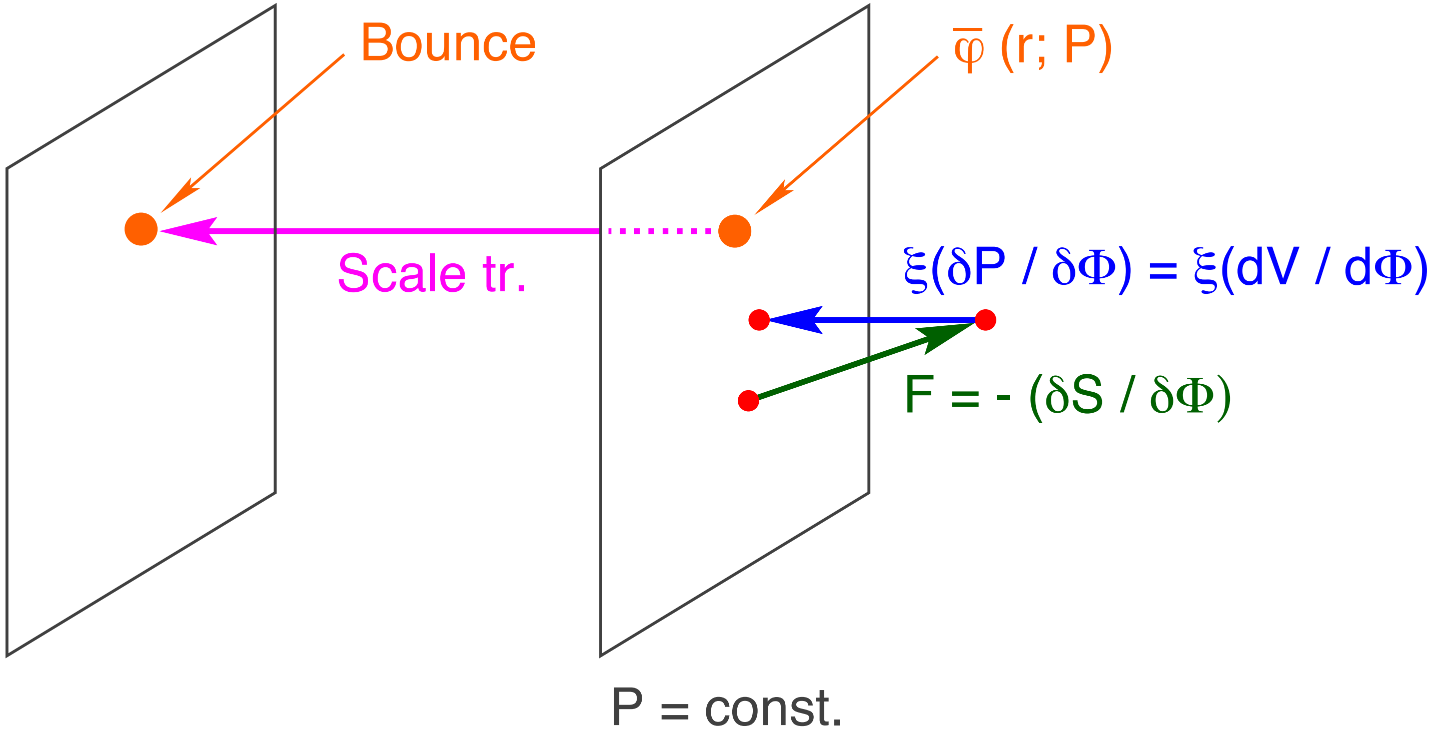
   $$\partial_r^2 \bar{\varphi} + \frac{D-1}{r} \partial_r \bar{\varphi} - \lambda^2 \frac{\partial V}{\partial \bar{\varphi}} = 0$$

   $$\lambda^2 = \xi[\Phi(s \to \infty)] + 1$$

2. Use scale transformation:

$$\partial^2_{r'}\,\bar{\varphi}(r;\mathcal{P}) + \frac{D-1}{r'}\partial_{r'}\,\bar{\varphi}(r;\mathcal{P}) - \frac{\partial V}{\partial \bar{\varphi}} = 0 \qquad r' = \lambda r$$

$$\Rightarrow\ \bar{\phi}(r) = \bar{\varphi}(\lambda^{-1} r, \mathcal{P})$$



5. Summary

We proposed a new method to calculate the bounce

- Our method is based on the gradient flow
- The bounce is obtained by solving the flow equation
- It can be easily implemented into numerical code

To-do list:

- Application to BSM models (in particular, SUSY)
  [Gunion, Haber, Sher; Casas, Lleyda, Munoz; Kusenko, Langacker, Segre; Camargo-Molina et al.; Chowdhury et al.; Blinov and Morrissey; Endo, Moroi, Nojiri; Endo, Moroi, Nojiri, Shoji; $\cdots$]
- Making a public code (?)

Please use our method, if you find any good application