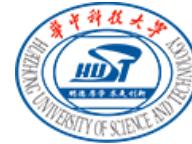


# $(g-2)_\mu$ versus Flavor Changing Neutral Current Induced by the Light $(B-L)_{\mu\tau}$ Boson

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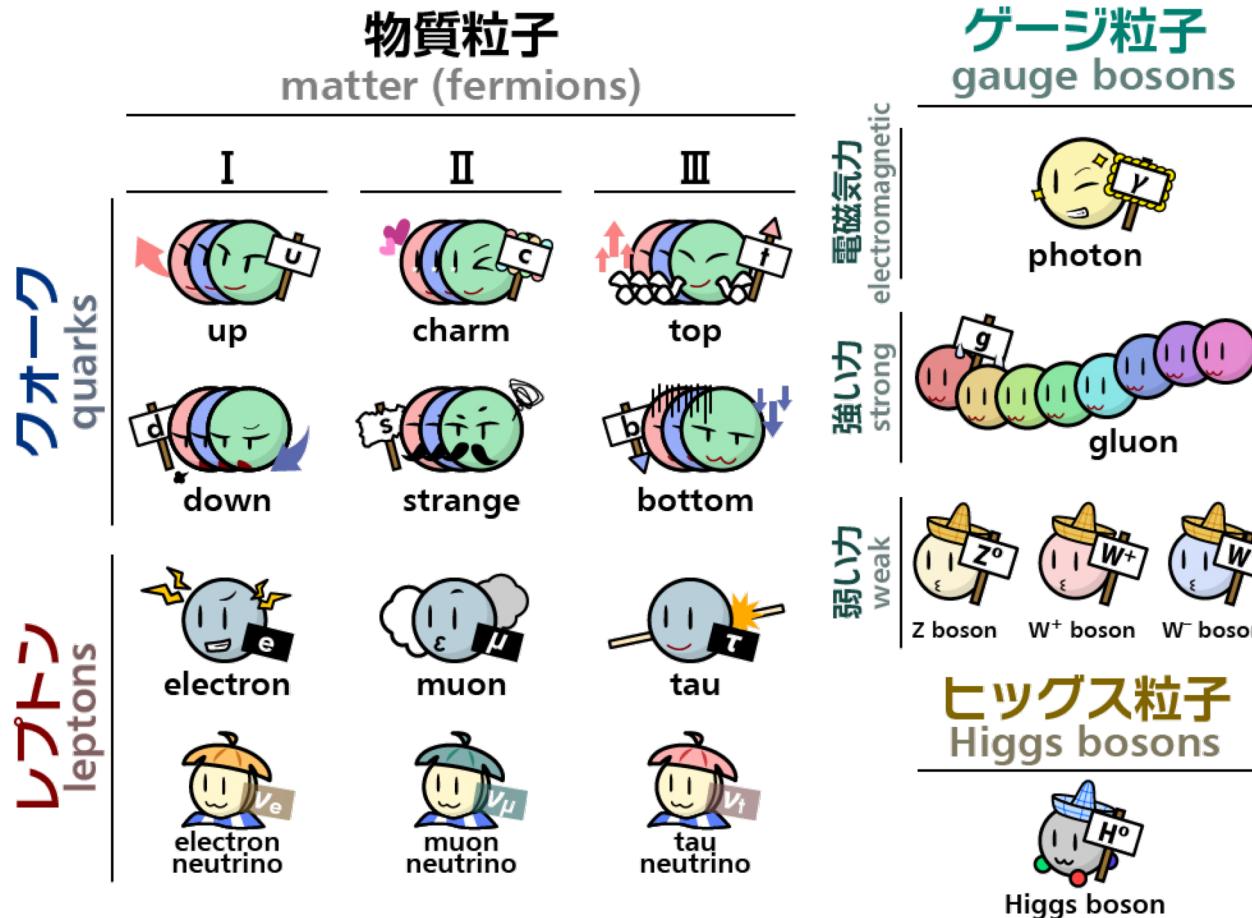


HUZHONG UNIVERSITY  
OF SCIENCE & TECHNOLOGY

Based on arXiv:1905.11018 [hep-ph]

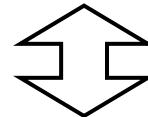
# Introduction

- Standard Model (SM): gauge  $G_{\text{SM}} = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$



# Introduction

- We should solve and explain some mysteries  
ex. neutrino masses: massless in SM (no right-handed neutrinos)

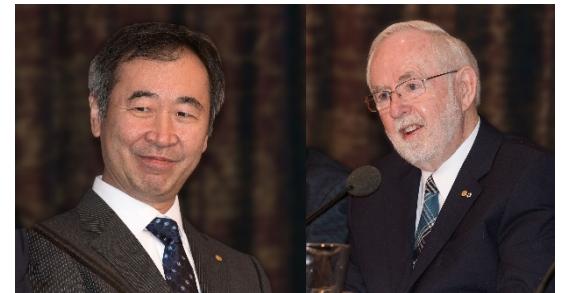


Extended model is needed

- Neutrino masses are confirmed in some experiments

Key: neutrino oscillation

$$P(\nu_e \rightarrow \nu_\mu) \propto \sin^2 \left( \frac{|m_{\nu_1}^2 - m_{\nu_2}^2|}{4E} L \right)$$



- Super-Kamiokande → Neutrinos are oscillated!

Nobel Prize (2015): T. Kajita, A. B. McDonald



- Tiny neutrino masses

$$\sum m_\nu < 0.12 \text{ eV}$$

Planck Collab., arXiv:1807.06209 [astro-ph.CO]

Parameter	best-fit	$3\sigma$
$\Delta m_{21}^2$ [ $10^{-5}$ eV $^2$ ]	7.37	6.93 – 7.96
$\Delta m_{31(23)}^2$ [ $10^{-3}$ eV $^2$ ]	2.56 (2.54)	2.45 – 2.69 (2.42 – 2.66)

# Introduction

- One of the interesting models → **B-L model**  
charges: +1 (+1/3) for Baryons (quarks), -1 for Leptons

- New  $U(1)$  gauge sym.

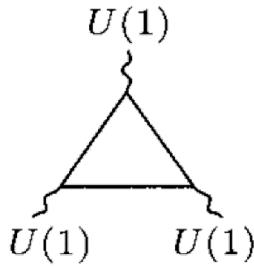


Figure from Peskin, Schroeder

	$Q_L$	$u_R$	$d_R$	$L_L$	$e_R$	$\nu_R$
$U(1)_{B-L}$	+1/3	+1/3	+1/3	-1	-1	-1
d.o.f	$3 \times 2$	3	3	2	1	1

$$-6 \times 1/3 + 3 \times 1/3 + 3 \times 1/3 - 2 \times (-1) + 1 \times (-1) = 1$$

$$-6 \times 1/3 + 3 \times 1/3 + 3 \times 1/3 - 2 \times (-1) + 1 \times (-1) + \textcolor{red}{1} \times (-1) = 0$$

Note:  
 $U(1)_{B-L}^3$  also cancels

- RHv is needed for gauge anomaly

- It appears from some high-energy theories

ex. Grand Unified Theory:  $SO(10) \rightarrow G_{SM} \times U(1)_{B-L}$

# Introduction

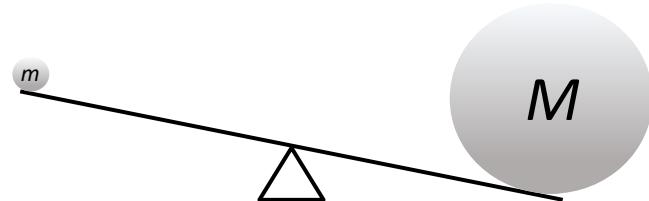
- New terms with  $RH\nu$

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \underbrace{Y \bar{L}_L \tilde{H} \nu_R}_{<\mathbf{H}^0> \neq 0 \rightarrow \text{Dirac mass term, } m} + \frac{\lambda_\nu}{2} \Phi \overline{\nu_R^c} \nu_R$$

$\Phi$ : new scalar (charge 2)

Seesaw mechanism:  $\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$   $<\Phi> \neq 0 \rightarrow \text{Majorana mass term, } M$

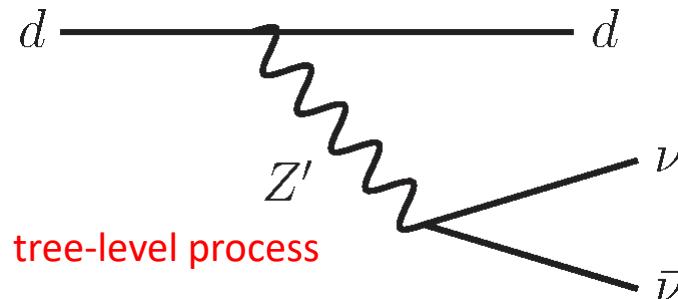
Minkowski, PLB **67**, 421 (1977);  
 Gell-Mann, Ramond, Slansky (proceedings) (1979);  
 Yanagida (proceedings) (1979);  
 Glashow, "Quarks and Leptons";  
 Mohapatra, Senjanovic, PRL **44**, 912 (1980)



- New gauge boson:  $Z'$

interactions with fermions:  $\mathcal{L}_{Z'} = g_{B-L} \left[ \left( \frac{1}{3} \right) \bar{q} \gamma^\mu q + (-1) \bar{\ell} \gamma^\mu \ell \right] Z'_\mu$

→ Contributes to some predictions



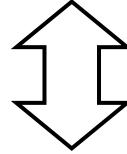
# Introduction

- $\mu$  couples to new gauge particles  $\rightarrow (g-2)_\mu$

Hamiltonian:  $H = -\mu \cdot \mathbf{B} - \mathbf{d} \cdot \mathbf{E}$

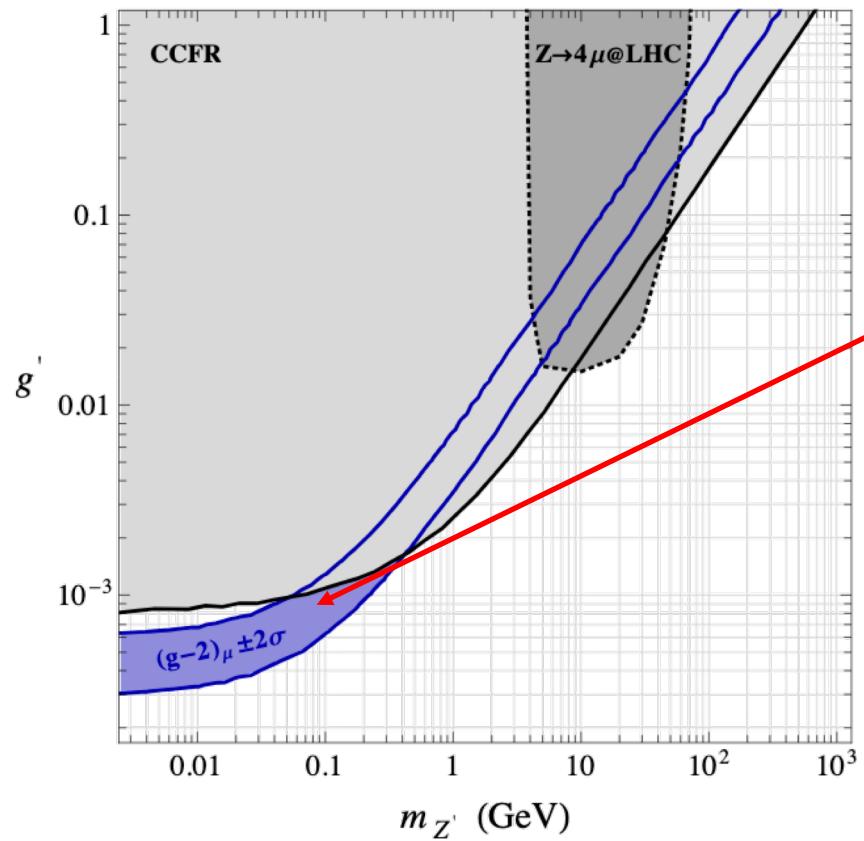
$q$ : charge,  $m$ : mass,  $\mathbf{s}$ : spin

$$\rightarrow \text{Magnetic moment: } \mu = g \left( \frac{q}{2m} \right) \mathbf{s}$$

- $a_\mu = \frac{g-2}{2} \rightarrow a_\mu = 0$  (tree level)
- 1-loop QED:  $a_\mu = \alpha/(2\pi)$  (Schwinger)
- SM prediction:  $a_\mu^{\text{SM}} = (11659182.04 \pm 3.56) \times 10^{-10}$   
 A. Keshavarzi et al., PRD 97, 114025 (2018)  
3.7 $\sigma$  deviation!
- Experimental result:  $a_\mu^{\text{exp}} = (11659208.9 \pm 6.3) \times 10^{-10}$   
PDG

# Introduction

- New physics explanation: **B-L model**,  $L_\mu$ - $L_\tau$  model, ...
- There is favored parameter space in light  $Z'$  mass region



$M_{Z'} \sim 10\text{-}400 \text{ MeV} \text{ & } g' \sim (3\text{-}15) \times 10^{-4}$

main focus of this work

# Introduction

K. Zhao Feng and Y.S, arXiv:1905.11018 [hep-ph]

- Our setup: 2<sup>nd</sup> and 3<sup>rd</sup> generations have U(1)<sub>B-L</sub> charges

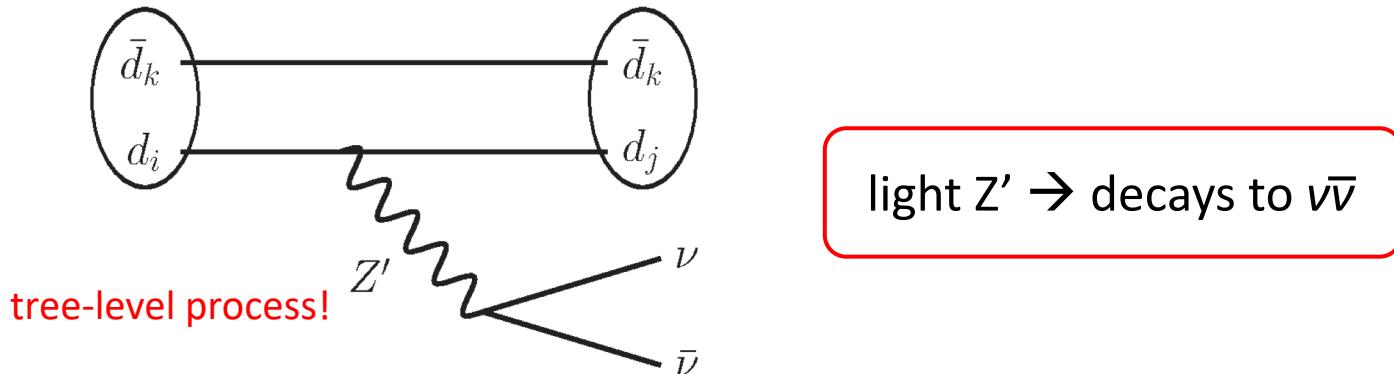
$$Z' \text{ interactions: } \mathcal{L}_{Z'} = \frac{g_{B-L}}{3} \sum_{i=2,3} \bar{q}_i \gamma^\mu q_i Z'_\mu$$

- In mass basis,

$$\mathcal{L}_{Z'} = \frac{g_{B-L}}{3} \sum_{a,b=1,2,3} C_{ab}^q \bar{q}_a \gamma^\mu q_b Z'_\mu$$

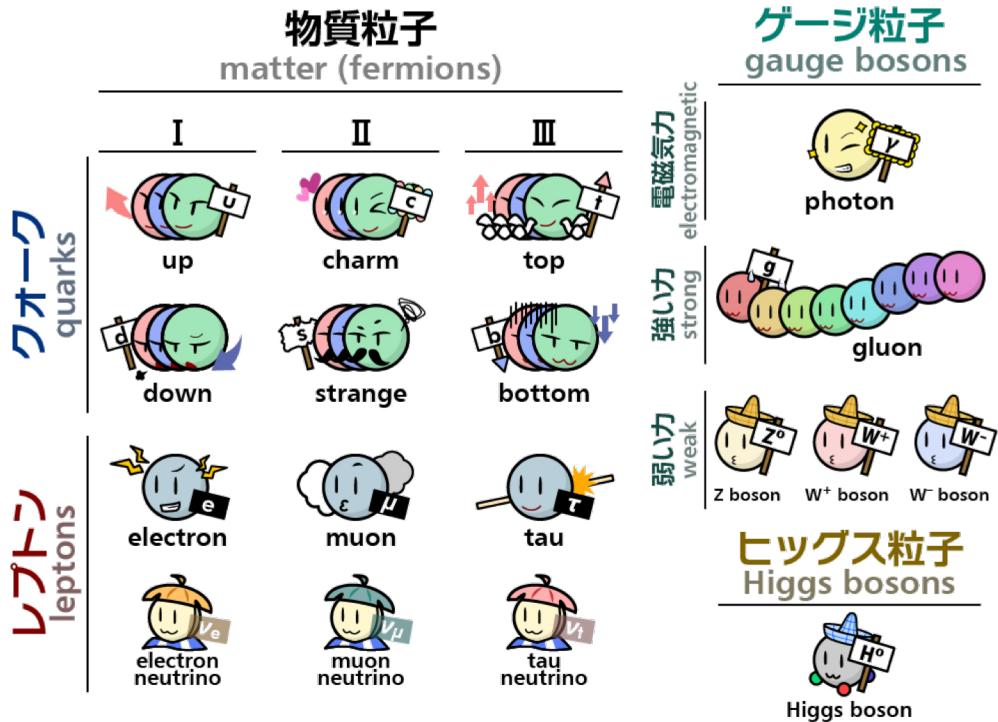
elements of diagonalizing matrix for Yukawas  
→ Flavor Violating Couplings (FVCs)

- New contributions:  $t \rightarrow q Z'$ ,  $P_1 \rightarrow P_2 Z'$  ( $Z' \rightarrow \nu\bar{\nu}$ )



# Contents

- Introduction (7) → ✓ done!
- Model details (4)
- $(g-2)_\mu$  (3)
- Quark FCNCs (6)
- Summary (1)



+ which New Physics? → Light Z'!

# Model details

K. Zhaofeng and YS, arXiv:1905.11018 [hep-ph]

# Model details

- We consider  $G_{\text{SM}} \times U(1)_{B-L}$
- 2<sup>nd</sup> and 3<sup>rd</sup> generations are charged under  $U(1)_{B-L}$
- Contents ( $i = 2, 3$ ):

	spin	$SU(2)_L$	$U(1)_Y$	$(B - L)_{\mu\tau}$
$Q_i$	1/2	2	1/6	1/3
$u_{R,i}$	1/2	1	2/3	1/3
$d_{R,i}$	1/2	1	-1/3	1/3
$U_L$	1/2	1	2/3	1/3
$U_R$	1/2	1	2/3	1/3
$\mathcal{F}$	0	1	0	1/3
$L_i$	1/2	2	-1/2	-1
$e_{R,i}$	1/2	1	-1	-1
$N_{R,i}$	1/2	1	0	-1
$H$	0	2	1/2	0
$\Phi$	0	1	0	2

need for realization of CKM

right-handed neutrinos

tiny  $v$  mass via seesaw mechanism

←  $\langle \Phi \rangle$  breaks  $U(1)_{B-L}$

# Model details

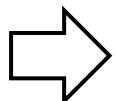
- Yukawa couplings for quarks

$$-\mathcal{L} \supset Y_{11}^u \bar{Q}_1 \tilde{H} u_{R,1} + Y_{ij}^u \bar{Q}_i \tilde{H} u_{R,j} + Y_{11}^d \bar{Q}_1 H d_{R,1} + Y_{ij}^d \bar{Q}_i H d_{R,j}$$

→ No Yukawas between 1<sup>st</sup> and the other generations: ~~CKM~~

- Vector-like quarks: Integrate out

$$-\mathcal{L}_U = M_U \bar{U}_L U_R + M_{Ui} \bar{U}_{LuR,i} + \lambda_1 \bar{U}_L u_{R,1} \mathcal{F} + \lambda_i \bar{Q}_i \tilde{H} U_R + h.c.$$



$$-\mathcal{L} \supset Y_{ab}^u \bar{Q}_a \tilde{H} u_{R,b} + Y_{11}^d \bar{Q}_1 H d_{R,1} + Y_{ij}^d \bar{Q}_i H d_{R,j}$$

$a = 1, 2, 3; i = 2, 3$

- If we introduce doublet flavons with U(1)<sub>B-L</sub> charge +1/3,

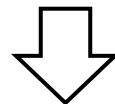
$$-\mathcal{L} \supset \tilde{Y}_{1i}^u \bar{Q}_1 \tilde{H}_{\mu\tau} u_{R,i} + \tilde{Y}_{i1}^d \bar{Q}_i H_{\mu\tau} d_{R,1} + h.c.$$

→ vector-like quarks are not needed

# Model details

- $Z'$  couplings of quarks

$$-\mathcal{L}_{Z'} \supset \frac{g_{B-L}}{3} [\overline{Q'}_i \gamma^\mu Q'_i + \overline{u'}_{R,i} \gamma^\mu u'_{R,i} + \overline{d'}_{R,i} \gamma^\mu d'_{R,i}] Z'_\mu$$



mass basis from flavor one:  $q_L^i = U_q^{ij} q_L^j$  and  $q_R^i = W_q^{ij} q_R^j$

$U_q, W_q$ : diagonalizing matrices for Yukawa

$$-\mathcal{L}_{Z'}^q = \frac{g_{B-L}}{3} \bar{q}_i \gamma^\mu (V_{ij}^q - \gamma_5 A_{ij}^q) q_j Z'_\mu : \text{Flavor violating couplings (FVCs)}$$

$$V_{ij}^q = \frac{1}{2} \sum_{k=2,3} [(U_q^\dagger)_{ik} (U_q)_{kj} + (W_q^\dagger)_{ik} (W_q)_{kj}] = \delta_{ij} - \frac{(U_q^\dagger)_{i1} (U_q)_{1j} + (W_q^\dagger)_{i1} (W_q)_{1j}}{2},$$

$$A_{ij}^q = \frac{1}{2} \sum_{k=2,3} [(U_q^\dagger)_{ik} (U_q)_{kj} - (W_q^\dagger)_{ik} (W_q)_{kj}] = -\frac{(U_q^\dagger)_{i1} (U_q)_{1j} - (W_q^\dagger)_{i1} (W_q)_{1j}}{2}.$$

Note: our FVCs are related to  
**(1, i)-element of  $U_q$  and  $W_q$**

- Different type of quark FCNC:

➤ Singlet flavon case → only **up** sector

$$Y_u = \begin{pmatrix} y_{11}^u & y_{12}^u & y_{13}^u \\ y_{21}^u & y_{22}^u & y_{23}^u \\ y_{31}^u & y_{32}^u & y_{33}^u \end{pmatrix}, \quad Y_d = \begin{pmatrix} y_{11}^d & 0 & 0 \\ 0 & y_{22}^d & y_{23}^d \\ 0 & y_{32}^d & y_{33}^d \end{pmatrix}$$

➤ Doublet flavon case → **up** and **down** sectors

$$Y_u = \begin{pmatrix} y_{11}^u & y_{12}^u & y_{13}^u \\ 0 & y_{22}^u & y_{23}^u \\ 0 & y_{32}^u & y_{33}^u \end{pmatrix}, \quad Y_d = \begin{pmatrix} y_{11}^d & 0 & 0 \\ y_{21}^d & y_{22}^d & y_{23}^d \\ y_{31}^d & y_{32}^d & y_{33}^d \end{pmatrix}$$

# Model details

- The size of FVCs: depend on  $g_{\text{B-L}}$  and  $U_q, W_q$
- $g_{\text{B-L}} \rightarrow$  determine from  $(g-2)_\mu$
- $U_q \rightarrow$  size from CKM matrix:

$$V_{\text{CKM}} = U_u^\dagger U_d = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

- No cancellation means

$$(U_u)_{c'u,u'c}, (U_d)_{s'd,d's} \lesssim \lambda, \quad (U_u)_{c't,t'c}, (U_d)_{s'b,b's} \lesssim \lambda^2, \quad (U_u)_{t'u,u't}, (U_d)_{b'd,d'b} \lesssim \lambda^3$$

and diagonal element  $\sim 1$

- For  $W_q$ , there are no concrete bound, but if  $U_q \sim W_q$ , similar inequalities are applied

$$(g\!-\!2)_\mu$$

# $(g-2)_\mu$

- Our scenario has the possibility to solve  $(g-2)_\mu$  deviation  
current result:

$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (27.06 \pm 7.26) \times 10^{-10}$$

K. Hagiwara *et al.*, J. Phys. G **38**, 085003 (2011)  
A. Keshavarzi *et al.*, PRD **97**, 114025 (2018)  
G. W. Bennet *et al.*, PRD **73**, 072003 (2006)  
B. L. Roberts, Chin. Phys. C **34**, 741 (2010)

- $Z'$  coupling with leptons:

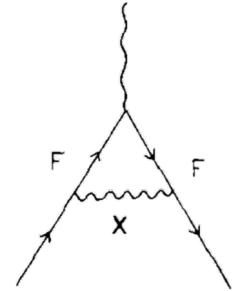
$$-\mathcal{L}_{Z'}^l = -g_{B-L}(\bar{\mu}\gamma^\mu\mu + \bar{\tau}\gamma^\mu\tau + \bar{\nu}_\mu\gamma^\mu\nu_\mu + \bar{\nu}_\tau\gamma^\mu\nu_\tau)Z'_\mu$$

- When  $M_{Z'}$  is light,  $\Delta a_\mu$  can be calculated by

J. P. Leveille, NPB **137**, 63 (1978)

$$\Delta a_\mu = \frac{g_{B-L}^2}{8\pi^2} \int_0^1 \frac{2x^2(1-x)}{x^2 + (M_{Z'}^2/m_\mu^2)(1-x)} dx$$

- In light mass region there are other constraints  
neutrino trident production,  $e^+e^- \rightarrow 4\mu$ , BBN, ...



# $(g-2)_\mu$

- Neutrino trident production  
ruled out the mass range  $M_{Z'} \gtrsim 400$  MeV

CCFR Collab., PRL 66, 3117 (1991)

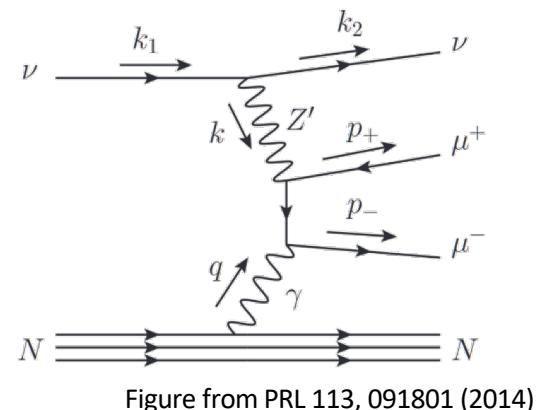


Figure from PRL 113, 091801 (2014)

- $e^+e^- \rightarrow 4\mu$  ( $e^+e^- \rightarrow Z'\mu^+\mu^-, Z' \rightarrow \mu^+\mu^-$ )  
ruled out the mass range  $M_{Z'} \gtrsim 2m_\mu \simeq 210$  MeV with  $g' \sim O(10^{-3})$

BaBar Collab., PRD 94, 011102 (2016)

- BBN (Big Bang Nucleosynthesis): light  $Z' \rightarrow$  effective relativistic d.o.f  
ruled out the mass range  $M_{Z'} \lesssim O(1)$  MeV

B. Ahlgren et al., PRL 111, 199001 (2013)  
A. Kamada et al., PRD 92, 113004 (2015)  
M. Escudero et al., JHEP 1903, 071 (2019)

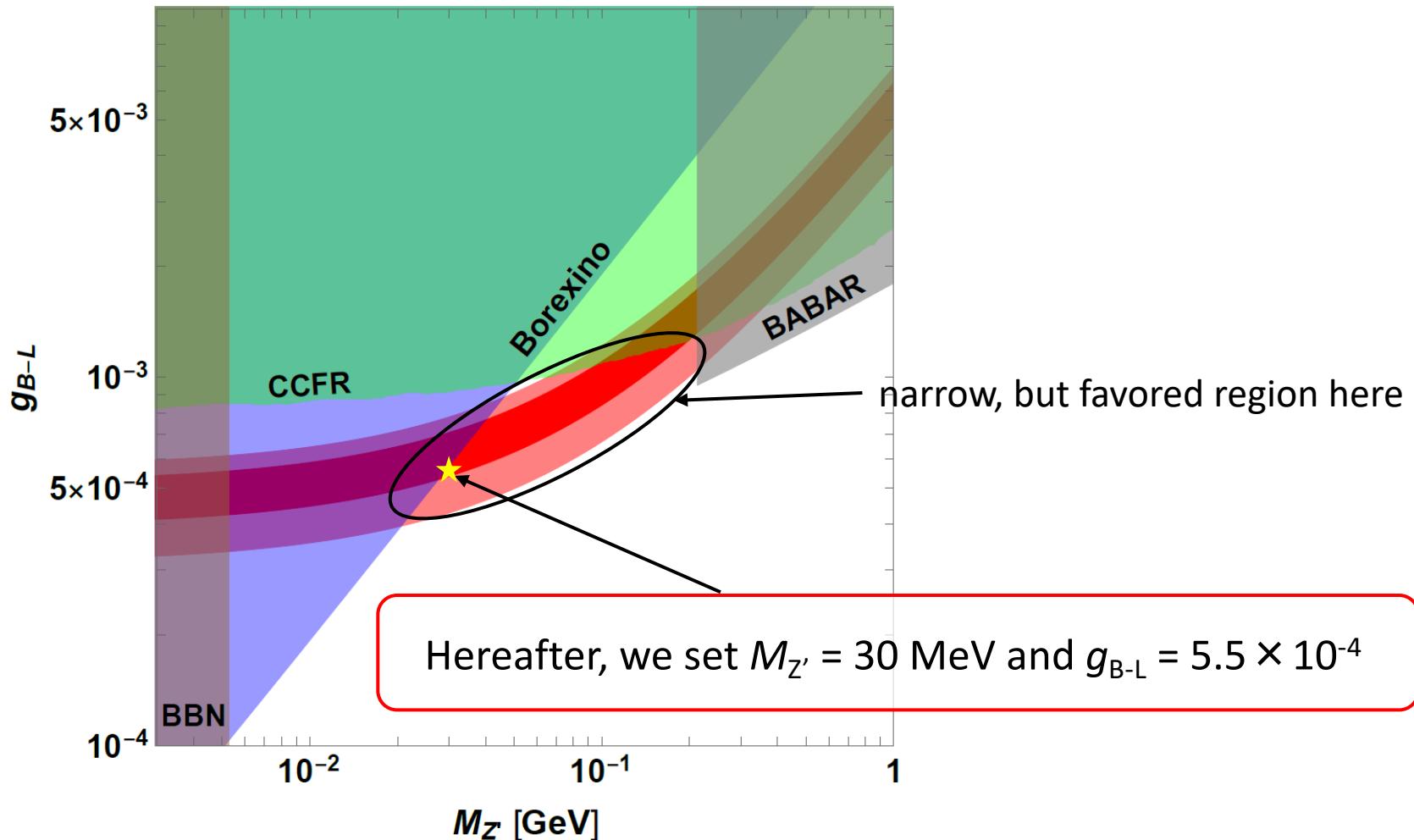
- $e-\nu$  scattering (Borexino)

G. Bellini et al., PRL 107, 141302 (2011); R. Harnik et al., JCAP 1207, 026 (2012); Borexino Collab., arXiv:1707.09279 [hep-ex]

even when  $e$  doesn't couple to  $Z'$  at tree level, it does at loop level  
the bound is depend on model, especially kinetic mixing  $\chi$

# $(g-2)_\mu$

- Bounds on  $Z'$  mass and coupling



# Quark FCNCs

# Singlet and doublet flavon models

- $t \rightarrow q Z'$  decay

M. D. Goodsell *et al*, EPJC **77**, 758 (2017)

$$\begin{aligned} \Gamma(t \rightarrow qZ') &= \frac{m_t}{32\pi} \lambda(1, x_q, x')^{1/2} \left[ \left( 1 + x_q - 2x' + \frac{(1-x_q)^2}{x'} \right) (|(g_L^u)_{qt}|^2 + |(g_R^u)_{qt}|^2) \right. \\ &\quad \left. - 12\sqrt{x_q} \text{Re} \left( (g_L^u)_{qt} (g_R^u)_{qt}^* \right) \right] \\ &\approx \frac{m_t^3}{32\pi} \lambda(1, x_q, x')^{1/2} \frac{|(g_L^u)_{qt}|^2 + |(g_R^u)_{qt}|^2}{M_{Z'}^2} \quad \text{when } x_q \ll 1, x' \ll 1 \end{aligned}$$

$$x_q \equiv m_q^2/m_t^2, \quad x' \equiv M_{Z'}^2/m_t^2 \quad \text{and} \quad \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$$

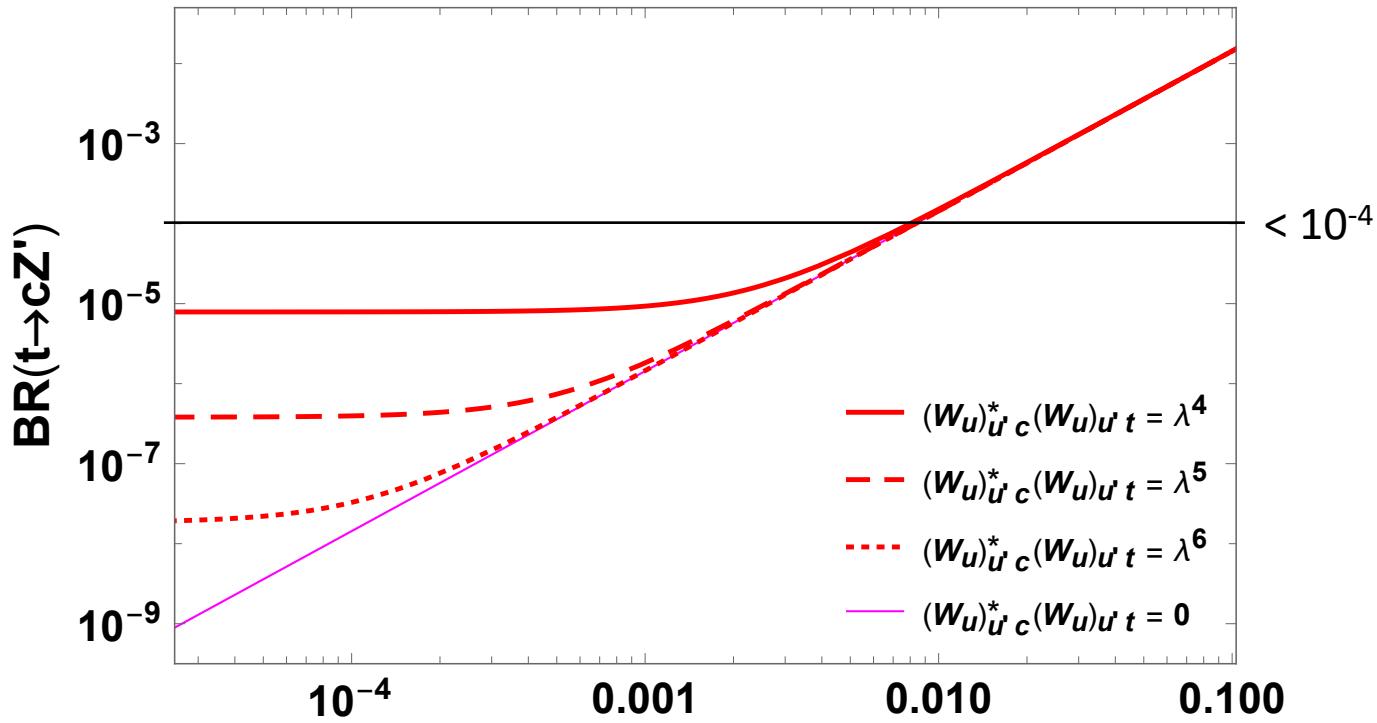
- **FVCs:**  $(g_L^u)_{ij} = \frac{g_{B-L}}{3} (V_{ij}^u + A_{ij}^u) = -\frac{g_{B-L}}{3} (U_u^\dagger)_{i1} (U_u)_{1j},$   
 $(g_R^u)_{ij} = \frac{g_{B-L}}{3} (V_{ij}^u - A_{ij}^u) = -\frac{g_{B-L}}{3} (W_u^\dagger)_{i1} (W_u)_{1j}$
- The results are proportional to  $|g_{B-L}|^2/M_{Z'}^2$  when  $M_{Z'} < \mathcal{O}(10)$  GeV

# Singlet and doublet flavon models

- Result of  $t \rightarrow c Z'$  decay

$t \rightarrow Wb$  is dominant mode in top quark decay

$$M_{Z'} = 30 \text{ MeV}, g_{B-L} = 5.5 \times 10^{-4}$$



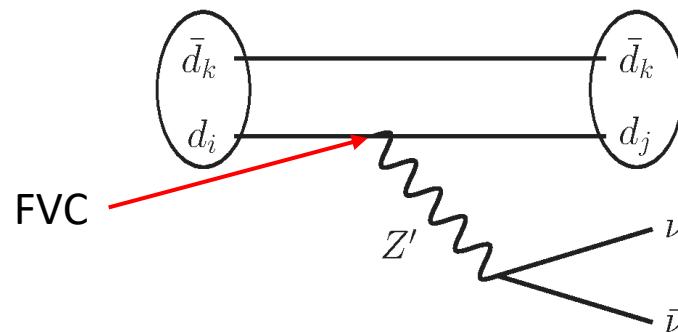
Note: our  $Z'$  decays mainly to  $\nu$ -pair  $(U_u)_{u'}^* c (U_u)_{u'} t < 8 \times 10^{-3} \sim \lambda^{3.2}$

→ no concrete bounds...

→ Consistent with the CKM matrix!

# Doublet flavon model

- Up sector: predictions are not changed
- FCNC in down sector, especially focus on  $\nu$ -pair in final state  
 $Z' \rightarrow \nu \bar{\nu}$
- Meson decay such as  $B \rightarrow K\nu\bar{\nu}$  and  $K \rightarrow \pi\nu\bar{\nu}$   
these processes are **tree level** ones



- Since  $Z'$  is light, it is produced through meson decay directly  
 $\rightarrow \text{BR}(B \rightarrow KZ') \times \frac{\text{BR}(Z' \rightarrow \nu\bar{\nu})}{\approxeq 1}$  and  $\text{BR}(K \rightarrow \pi Z') \times \frac{\text{BR}(Z' \rightarrow \nu\bar{\nu})}{\approxeq 1}$

# Doublet flavon model

- Branching ratios

$$\text{BR}(B^+ \rightarrow K^+ Z') = \frac{|(g_L^d)_{sb} + (g_R^d)_{sb}|^2}{64\pi} \frac{\lambda(m_{B^+}, m_{K^+}, M_{Z'})^{3/2}}{M_{Z'}^2 m_{B^+}^3 \Gamma_{B^+}} \left[ f_+^{B^+ K^+}(M_{Z'}^2) \right]^2,$$

$$\text{BR}(K^+ \rightarrow \pi^+ Z') = \frac{|(g_L^d)_{ds} + (g_R^d)_{ds}|^2}{64\pi} \frac{\lambda(m_{K^+}, m_{\pi^+}, M_{Z'})^{3/2}}{M_{Z'}^2 m_{K^+}^3 \Gamma_{K^+}} \left[ f_+^{K^+ \pi^+}(M_{Z'}^2) \right]^2,$$

$f_+^{M_1 M_2}(M_{Z'}^2)$  : M1 → M2 form factor at  $M_{Z'}$

P. Ball and R. Zwicky, PRD **71**, 014015 (2005)  
F. Mescia and C. Smith, PRD **76**, 034017 (2007)

- FVCs:  $(g_L^d)_{ij} = \frac{g_{B-L}}{3}(V_{ij}^d + A_{ij}^d) = -\frac{g_{B-L}}{3}(U_d^\dagger)_{i1}(U_d)_{1j}$ ,  
 $(g_R^d)_{ij} = \frac{g_{B-L}}{3}(V_{ij}^d - A_{ij}^d) = -\frac{g_{B-L}}{3}(W_d^\dagger)_{i1}(W_d)_{1j}$
- Masses and decay widths:

$m_{K^+}$	493.677(16) MeV	$\tau_{K^+}$	$1.2380(20) \times 10^{-8}$ s
$m_{K_L}$	497.611(13) MeV	$\tau_{K_L}$	$5.116(21) \times 10^{-8}$ s
$m_{B^+}$	5279.32(14) MeV	$\tau_{B^+}$	$1.638(4) \times 10^{-12}$ s
$m_{\pi^+}$	139.57061(24) MeV		
$m_{\pi^0}$	134.9770(5) MeV		

PDG

# Doublet flavon model

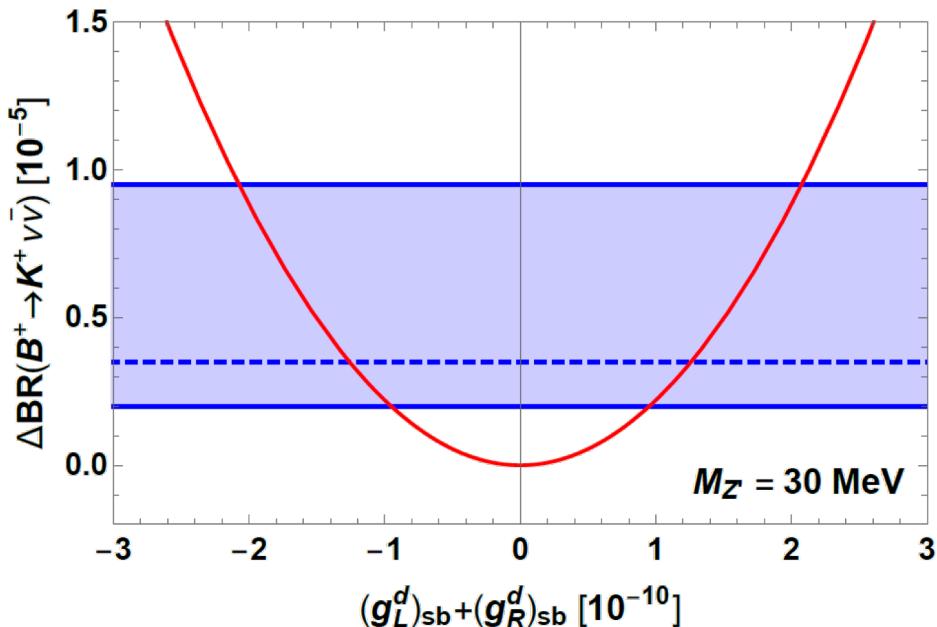
- Constraint:  $\Delta \text{BR}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{exp}} = (0.35^{+0.60}_{-0.15}) \times 10^{-5}$

J. P. Lees *et al.* [BaBar Collab.], PRD **87**, 112005 (2013)

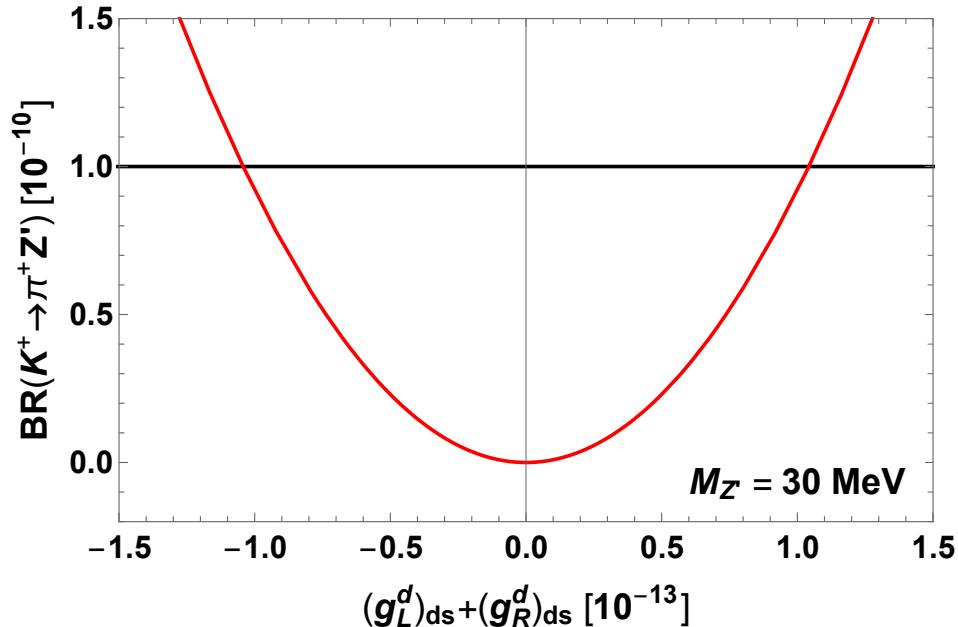
$$\text{BR}(K^+ \rightarrow \pi^+ Z') < 1.0 \times 10^{-10} \text{ for } M_{Z'} = 30 \text{ MeV}$$

A. V. Artamonov *et al.* [BNL-E949 Collab.], PRD **79**, 092004 (2009)

- Results



$$0.95 \times 10^{-10} \leq |(g_L^d)_{sb} + (g_R^d)_{sb}| \leq 2.1 \times 10^{-10}$$



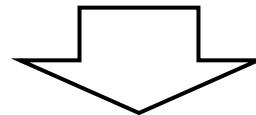
$$|(g_L^d)_{ds} + (g_R^d)_{ds}| < 0.11 \times 10^{-12}$$

# Doublet flavon model

- Summary of bounds:

$$0.95 \times 10^{-10} \leq |(g_L^d)_{sb} + (g_R^d)_{sb}| \leq 2.1 \times 10^{-10}$$

$$|(g_L^d)_{ds} + (g_R^d)_{ds}| < 0.11 \times 10^{-12}$$



$$\times 3/(5.5 \times 10^{-4}) (= 3/g_{B-L})$$

$$5.2 \times 10^{-7} \leq |(U_d)_{d's}^* (U_d)_{d'b}| \leq 1.1 \times 10^{-6}, \quad |(U_d)_{d'd}^* (U_d)_{d's}| < 6.2 \times 10^{-10}$$

- Taking unitary condition into account,  $|(U_d)_{d'd}|^2 + |(U_d)_{d's}|^2 + |(U_d)_{d'b}|^2 = 1$  allowed patterns are

$ (U_d)_{d'd} $	$ (U_d)_{d's} $	$ (U_d)_{d'b} $
$< 10^{-9}$	$\sim 1$	$\simeq \mathcal{O}(10^{-6})$
$< 10^{-3}$	$\simeq \mathcal{O}(10^{-6})$	$\sim 1$

Cannot be  $O(1)$

⇒ Inconsistent with CKM structure!

# Summary

# Summary

- We consider B-L extended model  
 $2^{\text{nd}}$  and  $3^{\text{rd}}$  generations are charged under  $U(1)_{\text{B-L}}$
- We should introduce some flavon (singlet, double)
- In singlet flavon case, only up sector has FVCs of  $Z'$ , and FCNC top decay is interesting:  
 $\text{BR}(t \rightarrow c Z') \sim O(10^{-4})$ , which consistent with CKM bounds
- In doublet flavon case, down sector also have FVCs of  $Z'$ , so strong bound from meson decay with  $\nu$ -pair:  
excluded unless highly tuned cancellation between  $g_L^d$  and  $g_R^d$
- We can expect that our scenario (especially singlet case) can be tested by future experiments  
 $(g-2)_\mu$ ,  $\nu$  physics: NA64, DUNE, ...; top FCNC: CLIC, FCC, ...

Back up slides

# $(g-2)_\mu$

- Experimental results so far:

BNL-E821 final report, PRD 73, 072003 (2006)

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	$\mu^+$	11 450 000(220 000)	4300
CERN II	1962-1968	$\mu^+$	11 661 600(3100)	270
CERN III	1974-1976	$\mu^+$	11 659 100(110)	10
CERN III	1975-1976	$\mu^-$	11 659 360(120)	10
BNL	1997	$\mu^+$	11 659 251(150)	13
BNL	1998	$\mu^+$	11 659 191(59)	5
BNL	1999	$\mu^+$	11 659 202(15)	1.3
BNL	2000	$\mu^+$	11 659 204(9)	0.73
BNL	2001	$\mu^-$	11 659 214(9)	0.72
Average			11 659 208.0(6.3)	0.54

# Model details

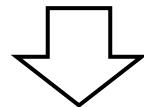
- Yukawa couplings for quarks

$$-\mathcal{L} \supset Y_{11}^u \bar{Q}_1 \tilde{H} u_{R,1} + Y_{ij}^u \bar{Q}_i \tilde{H} u_{R,j} + Y_{11}^d \bar{Q}_1 H d_{R,1} + Y_{ij}^d \bar{Q}_i H d_{R,j}$$

→ No Yukawas between 1<sup>st</sup> and the other generations: ~~CKM~~

- Vector-like quarks: Integrate out

$$-\mathcal{L}_U = M_U \bar{U}_L U_R + M_{Ui} \bar{U}_L u_{R,i} + \lambda_1 \bar{U}_L u_{R,1} \mathcal{F} + \lambda_i \bar{Q}_i \tilde{H} U_R + h.c.$$



$$\mathcal{L}_{eff} \supset \underline{c_{ab} \bar{u}_a i \gamma_\mu \partial^\mu P_R u_b} + Y_{ib} \bar{Q}_i \tilde{H} P_R u_b + h.c. \quad c_{ab} = \frac{M_{Ua} M_{Ub}^*}{M_U^2}, \quad Y_{ib} = \lambda_i \frac{M_{Ub}}{M_U}.$$

choose canonical basis by  $u_R \rightarrow \widetilde{W}_u u_R$        $M_{U1} = \lambda_1 v_f / \sqrt{2}$  :  $a = 1, 2, 3; i = 2, 3$

$$-\mathcal{L} \supset Y_{ab}^u \bar{Q}_a \tilde{H} u_{R,b} + Y_{11}^d \bar{Q}_1 H d_{R,1} + Y_{ij}^d \bar{Q}_i H d_{R,j}$$

# Model details

- Comparison between singlet and doublet flavons
  - Yukawas in singlet case

$$Y_u = \begin{pmatrix} y_{11}^u & y_{12}^u & y_{13}^u \\ y_{21}^u & y_{22}^u & y_{23}^u \\ y_{31}^u & y_{32}^u & y_{33}^u \end{pmatrix}, \quad Y_d = \begin{pmatrix} y_{11}^d & 0 & 0 \\ 0 & y_{22}^d & y_{23}^d \\ 0 & y_{32}^d & y_{33}^d \end{pmatrix}$$

diagonalizing matrices for  $Y_d$ :  $U_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_u^d & -\sin \theta_u^d \\ 0 & \sin \theta_u^d & \cos \theta_u^d \end{pmatrix}$ ,  $W_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_w^d & -\sin \theta_w^d \\ 0 & \sin \theta_w^d & \cos \theta_w^d \end{pmatrix}$

$$\begin{aligned} \rightarrow -\mathcal{L}_{Z'} &\supset \frac{g_{B-L}}{3} [\bar{d'}_{L,i} \gamma^\mu d'_{L,i} + \bar{d'}_{R,i} \gamma^\mu d'_{R,i}] Z'_\mu \\ &\rightarrow \frac{g_{B-L}}{3} [\bar{d}_{L,i} \gamma^\mu d_{L,i} + \bar{d}_{R,i} \gamma^\mu d_{R,i}] Z'_\mu \quad \text{No FVCs} \end{aligned}$$

- Only up sector has FVCs of  $Z'$

# Model details

- Comparison between singlet and doublet flavons
  - Yukawas in doublet case

$$Y_u = \begin{pmatrix} y_{11}^u & y_{12}^u & y_{13}^u \\ 0 & y_{22}^u & y_{23}^u \\ 0 & y_{32}^u & y_{33}^u \end{pmatrix}, \quad Y_d = \begin{pmatrix} y_{11}^d & 0 & 0 \\ y_{21}^d & y_{22}^d & y_{23}^d \\ y_{31}^d & y_{32}^d & y_{33}^d \end{pmatrix}$$

diagonalizing matrices for  $Y_d$ : arbitrary  $3 \times 3$  unitary matrices

$$\begin{aligned} \rightarrow -\mathcal{L}_{Z'} &\supset \frac{g_{B-L}}{3} \left[ \overline{d'}_{L,i} \gamma^\mu d'_{L,i} + \overline{d'}_{R,i} \gamma^\mu d'_{R,i} \right] Z'_\mu \\ &\rightarrow \frac{g_{B-L}}{3} \left[ \overline{d}_{L,a} (U_d^\dagger U_d)_{ab} \gamma^\mu d_{L,b} + \overline{d}_{R,a} (W_d^\dagger W_d)_{ab} \gamma^\mu d_{R,b} \right] Z'_\mu \end{aligned}$$

- Both up and down sectors have FVCs of  $Z'$

Note: in both cases, there are no FVCs in charged leptons sector

# Model details

- Comments on scalar sector

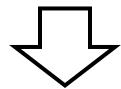
- singlet flavon case

$$|D_\mu \mathcal{F}|^2 + |D_\mu \Phi|^2 \supset \frac{1}{2} M_{Z'}^2 Z'_\mu Z'^\mu$$

$$\Rightarrow M_{Z'}^2 = g_{B-L}^2 (1/9 v_f^2 + 4 v_\phi^2) \sim \mathcal{O}(0.01 \text{GeV})^2 - \mathcal{O}(0.1 \text{GeV})^2$$

- $g_{B-L}$  should be  $\mathcal{O}(10^{-3})$  for  $(g-2)_\mu$ , then  $v_f < 30 - 300 \text{ GeV}$

$$\rightarrow M_{U1} = \lambda_1 v_f / \sqrt{2} \sim 100 \text{ GeV}$$



$Y_{ib} = \lambda_i \frac{M_{Ub}}{M_U}$  can be large enough to accommodate CKM when  $M_U \sim \mathcal{O}(10^{3-4}) \text{ GeV}$

- doublet flavon case  $\rightarrow$  enough for CKM:

$$(m_u^0)_{1i} = \tilde{Y}_{1i}^u v_{\mu\tau} / \sqrt{2} \quad \Rightarrow \quad v_{\mu\tau} / v_h \lesssim 1$$

# Model details

- Comments on gauge sector

- singlet flavon case  $\rightarrow$  no mass mixing

since there is no scalar which have both SM and  $U(1)_{B-L}$  charges

- doublet flavon case  $\rightarrow H_{\mu\tau}$  contributes to  $Z$  and  $Z'$  mass

$$M_G^2 = \frac{v_h^2}{4} \begin{pmatrix} g_Y^2(1+r_{\mu\tau}) & -g_Y g_2(1+r_{\mu\tau}) & \frac{2}{3}g_Y g_{B-L} r_{\mu\tau} \\ -g_Y g_2(1+r_{\mu\tau}) & g_2^2(1+r_{\mu\tau}) & -\frac{2}{3}g_2 g_{B-L} r_{\mu\tau} \\ \frac{2}{3}g_Y g_{B-L} r_{\mu\tau} & -\frac{2}{3}g_2 g_{B-L} r_{\mu\tau} & g_{B-L}^2 \left(\frac{4}{9}r_{\mu\tau} + 16r_\phi\right) \end{pmatrix}$$

$r_{\mu\tau} \equiv \frac{v_{\mu\tau}^2}{v_h^2}$  and  $r_\phi \equiv \frac{v_\phi^2}{v_h^2}$       in  $(B, W^3, Z')$  basis

- Because of the size of  $g_{B-L}$  and  $M_{Z'}$ , we ignore mass mixing  
we also choose  $r_{\mu\tau} \ll 1$  without loss of realization of  $M_{Z'} \sim O(10)$  MeV
- In addition, there is kinetic mixing:  $\mathcal{L} \supset -\frac{\chi}{2} F^{\mu\nu} Z'_{\mu\nu}$   
 $\chi \sim O(10^{-4}-10^{-5})$  in our scenario

# Kinetic mixing

- Kinetic mixing term can be written as  $\mathcal{L} \supset -\frac{\chi}{2} F^{\mu\nu} Z'_{\mu\nu}$
- $\chi$  is estimated by calculation of vacuum polarization diagram

$$\chi = -\frac{eg_{B-L}}{12\pi^2} \sum_f Q_f Q_f^{B-L} \left[ 6 \int_0^1 dx x(1-x) \ln \left( \frac{m_f^2 - k^2 x(1-x)}{\mu^2} \right) \right]$$

- In doublet flavon model,  $\chi$  is

$$\chi = -\frac{eg_{B-L}}{18\pi^2} \ln \left( \frac{m_c^2 m_t^2 m_\mu^3 m_\tau^3}{m_s m_b M_U^8} \right) \quad \text{we assume } \chi = 0 @ M_U$$

for singlet flavon case, simply set  $M_U = m_t$

- If  $M_U = 1 \text{ TeV}$ ,  $\chi = 4.6 \times 10^{-5}$  with  $g_{B-L} = 5.5 \times 10^{-4}$

# Singlet flavon model

- Comment on  $t \rightarrow u Z'$  decay

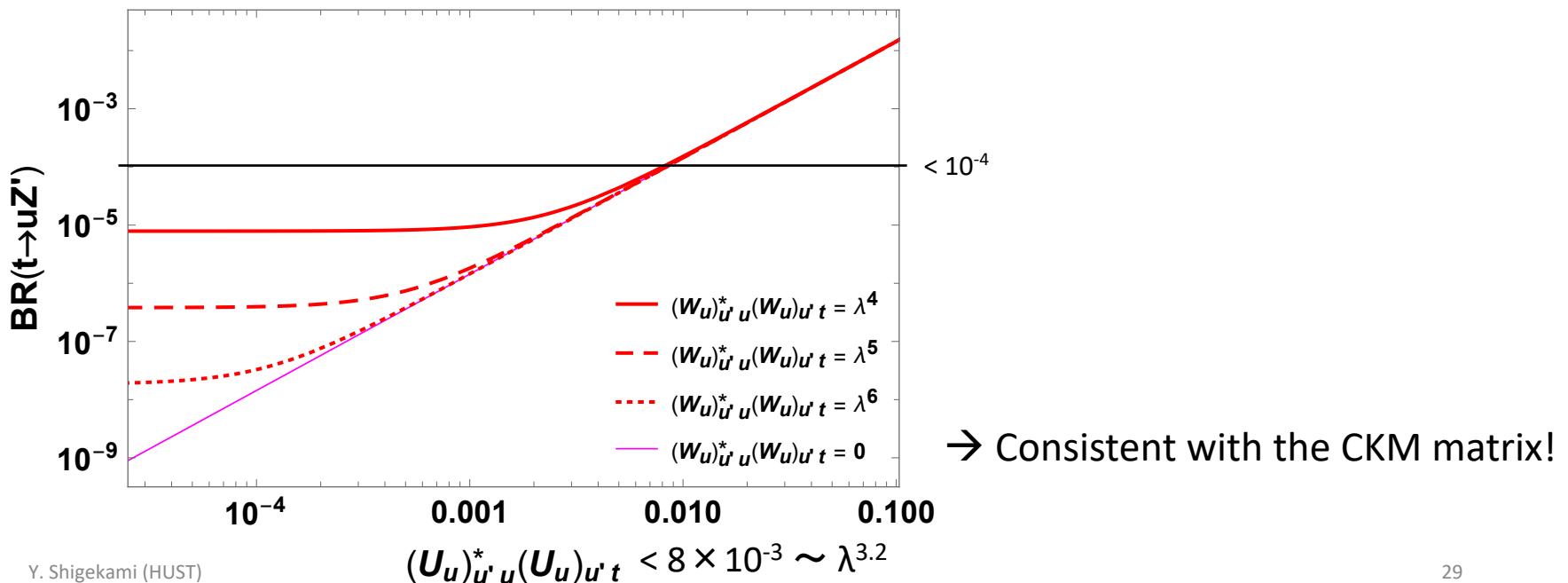
the difference only comes from  $x_q$ :

$$\Gamma(t \rightarrow qZ') \approx \frac{m_t^3}{32\pi} \lambda(1, x_q, x')^{1/2} \frac{|(g_L^u)_{qt}|^2 + |(g_R^u)_{qt}|^2}{M_{Z'}^2}$$

$\doteq 1$

- We obtain (almost) same result for  $t \rightarrow u Z'$  decay

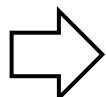
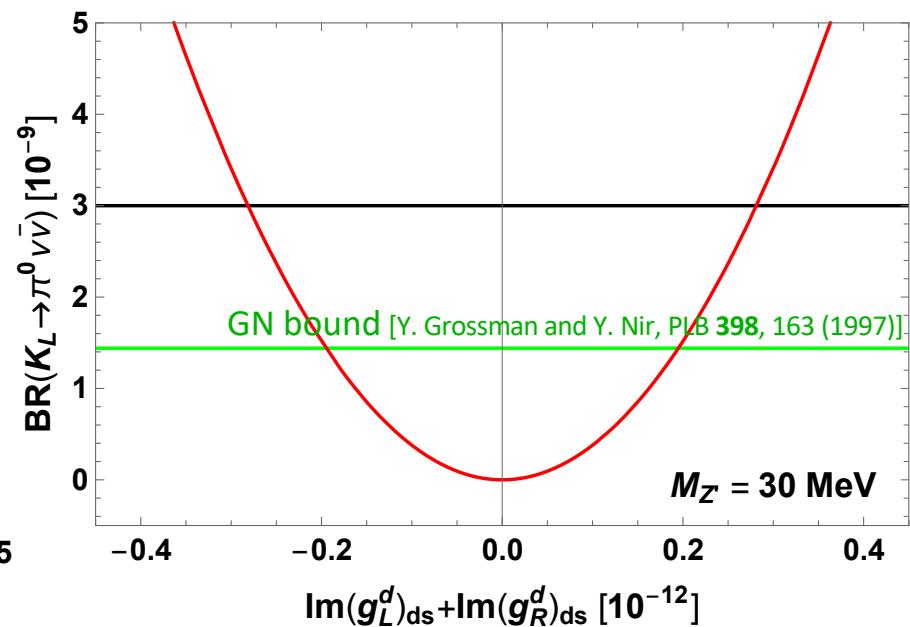
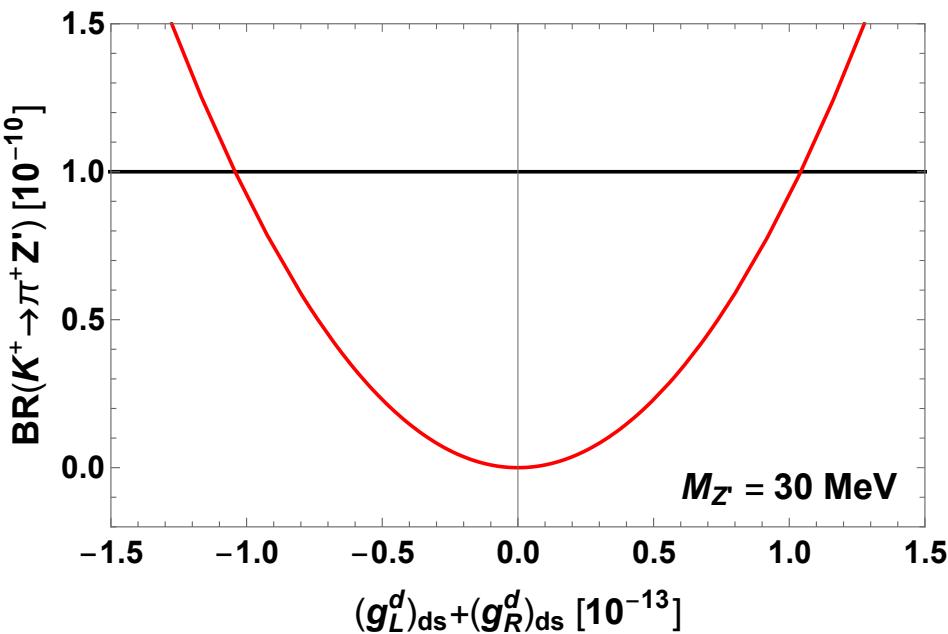
$$M_{Z'} = 30 \text{ MeV}, g_{B-L} = 5.5 \times 10^{-4}$$



# Doublet flavon model

A. V. Artamonov *et al.* [BNL-E949 Collab.], PRD **79**, 092004 (2009)  
 J. K. Ahn *et al.* [KOTO Collab.], PRL **122**, 021802 (2019)

- Constraints:  $\text{BR}(K^+ \rightarrow \pi^+ Z') < 1.0 \times 10^{-10}$  for  $M_{Z'} = 30 \text{ MeV}$   
 $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{exp}} < 3.0 \times 10^{-9}$
- Results



$$|(g_L^d)_{ds} + (g_R^d)_{ds}| < 0.11 \times 10^{-12},$$

$$|\text{Im}(g_L^d)_{ds} + \text{Im}(g_R^d)_{ds}| < 0.28 \times 10^{-12} \quad (0.20 \times 10^{-12} \text{ (GN bound)})$$