

重力波宇宙論の展望

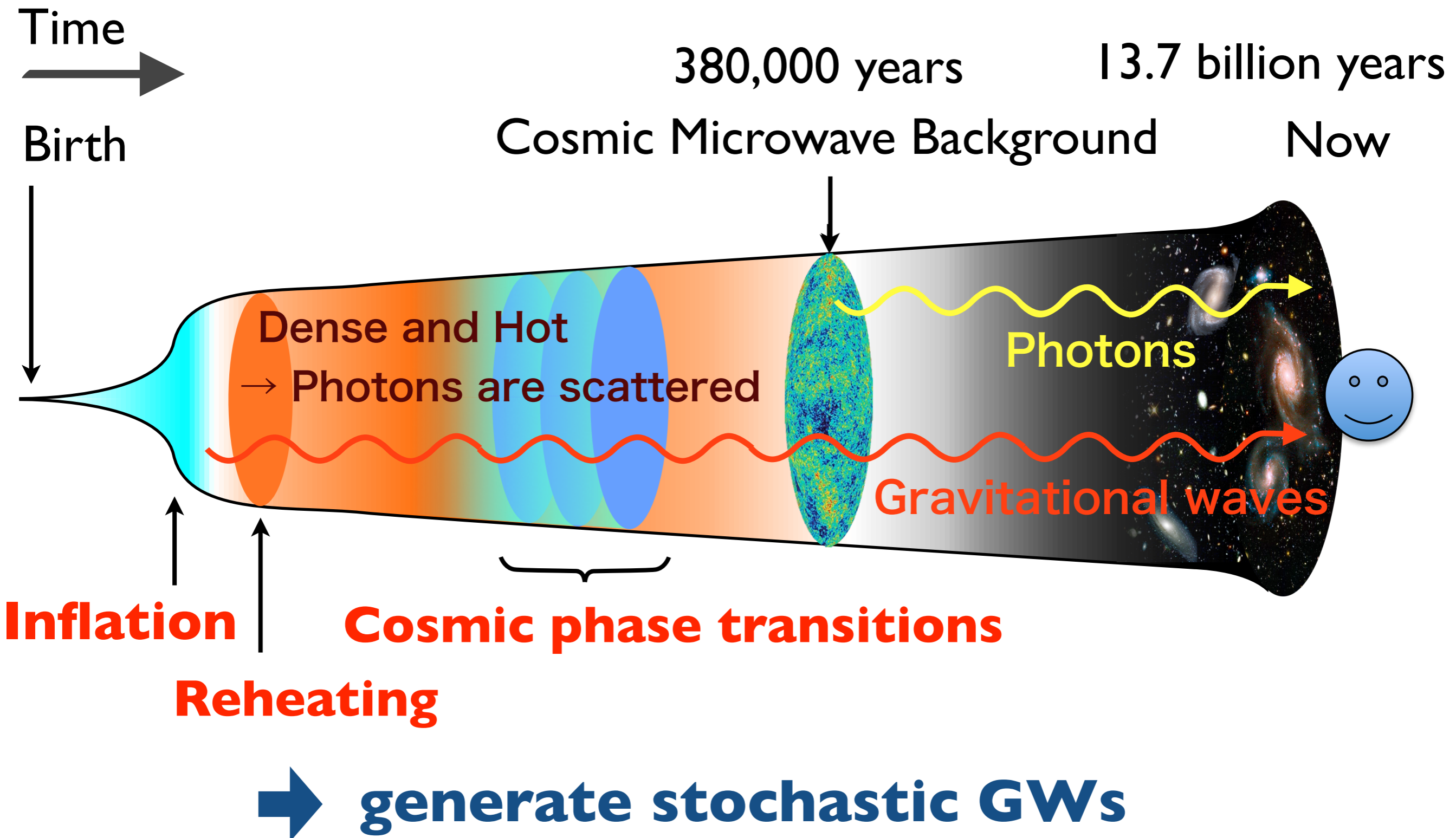
黒柳 幸子

(IFT UAM-CSIC / 名古屋大学)

2020年9月2日

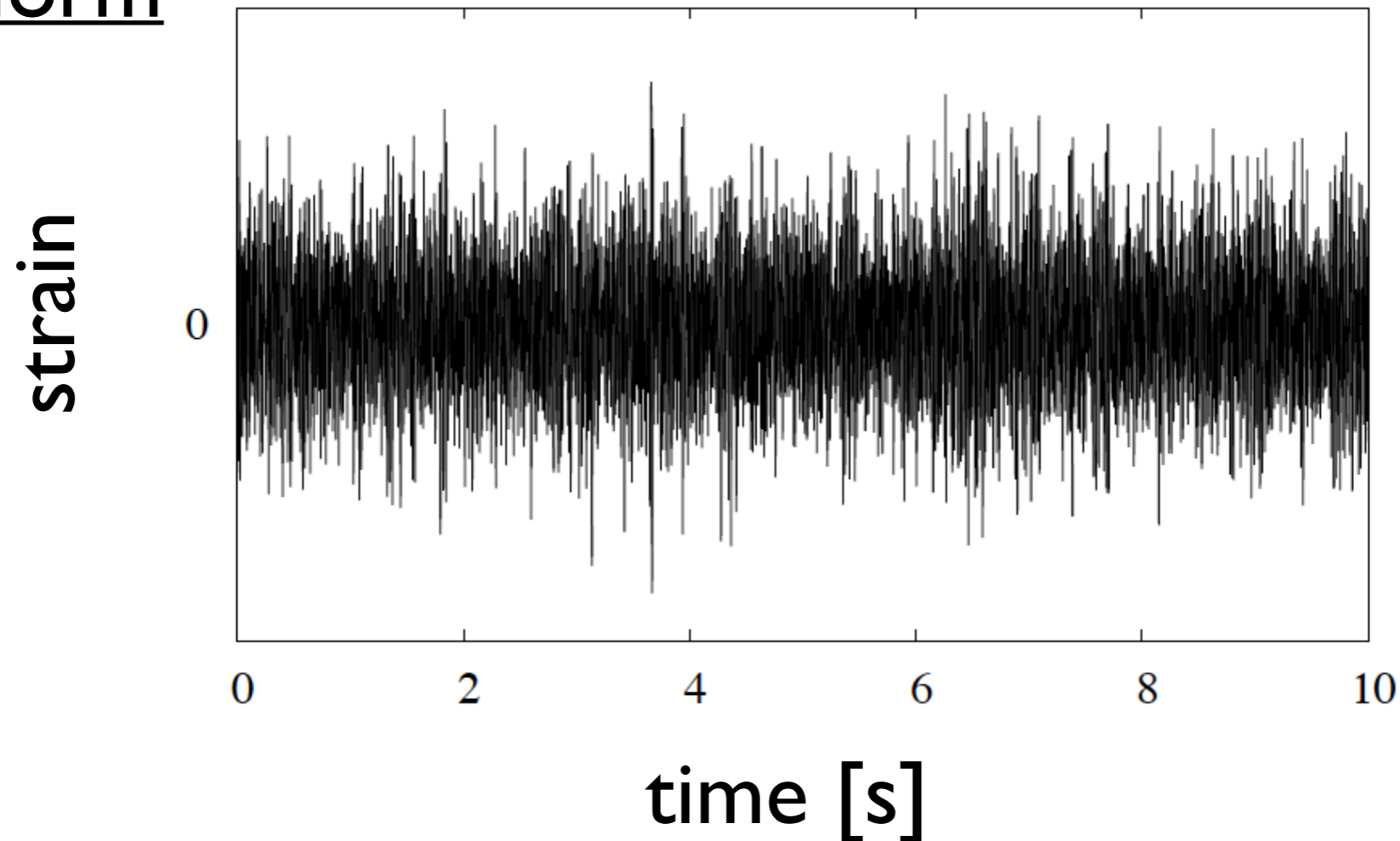
素粒子物理学の進展2020

GW is the only way to directly observe the early universe



Stochastic GW background

Waveform



Continuous and random gravitational wave (GW) signal coming from all directions → very similar to noise

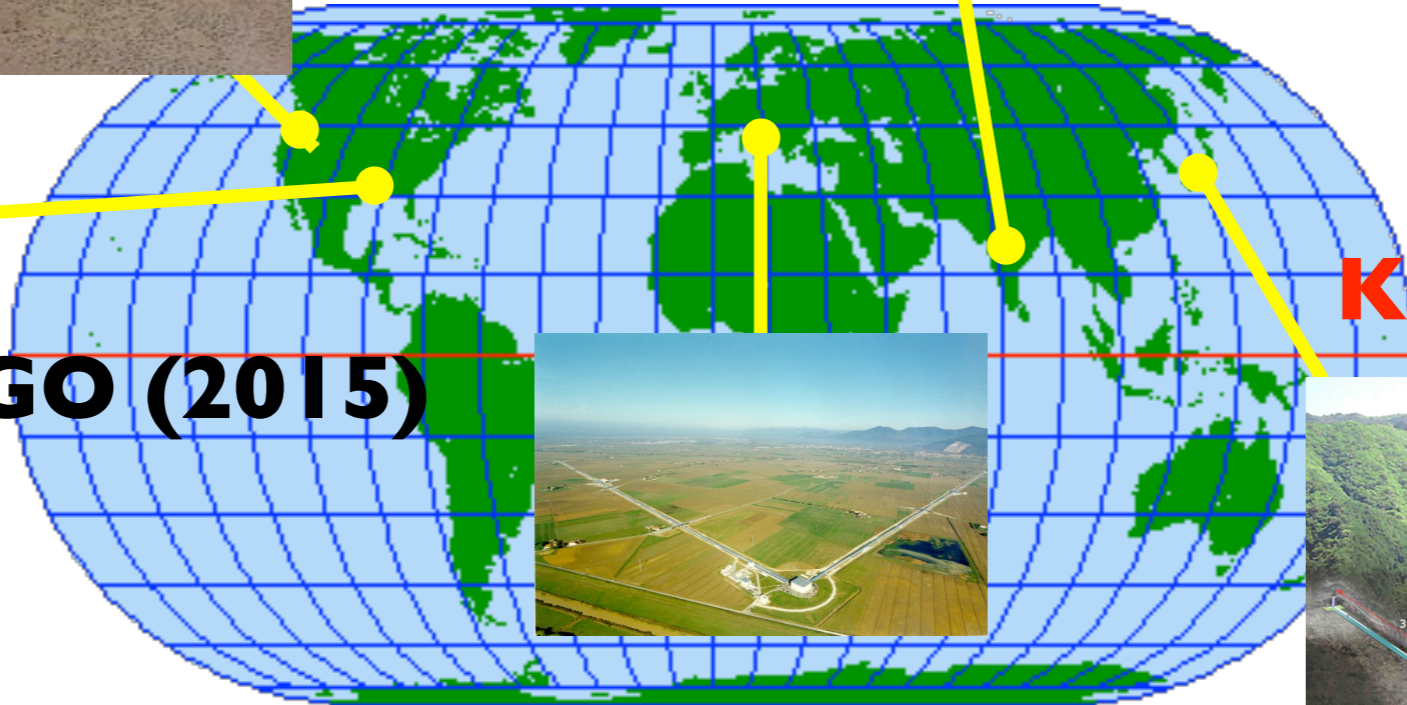
GW detectors in the world



LIGO-India (2025)



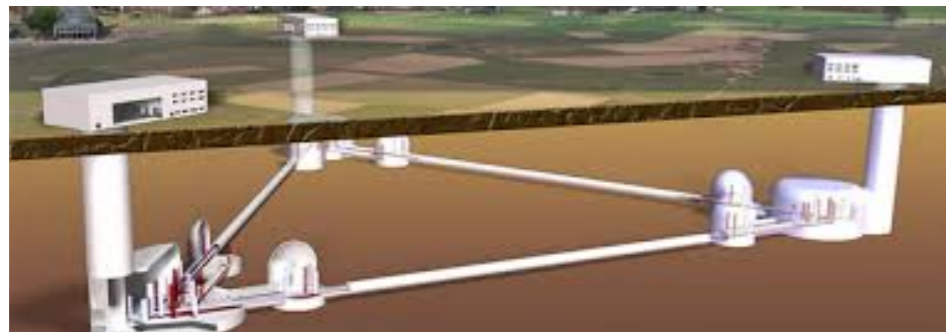
Advanced-LIGO (2015)



KAGRA (2020)



Advanced-VIRGO (2017)



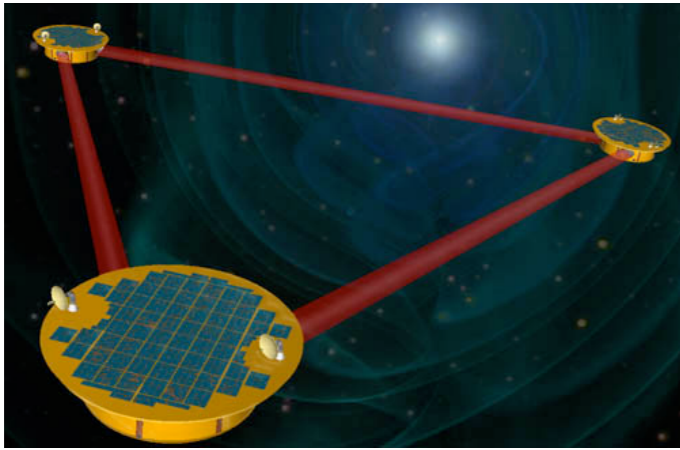
Future

LIGO A+, Voyager (2020')

Cosmic explorer (2030')

Einstein Telescope

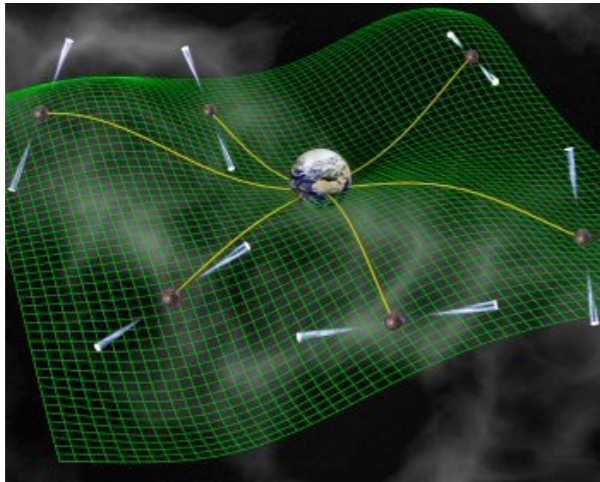
Other GW experiments



Space missions

LISA (2034)

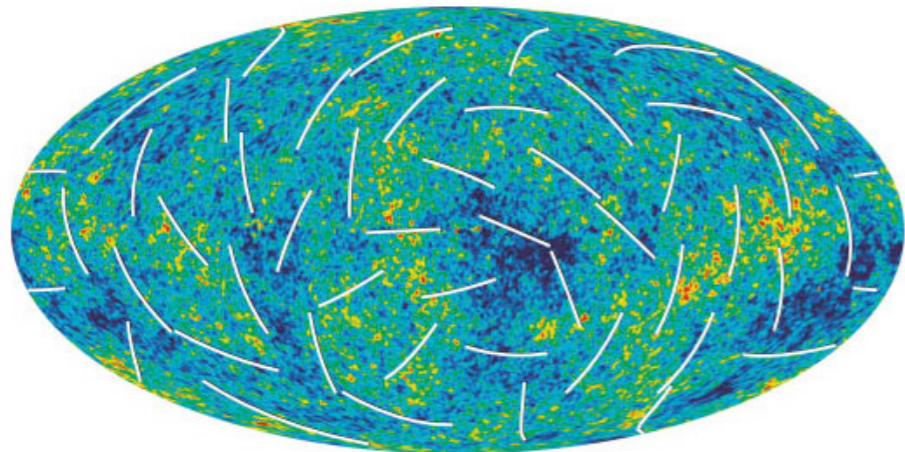
DECIGO, TianQin, Taiji



Pulsar timing array

ongoing: NANOGrav, EPTA, PPTA

SKA (2020-)



B-mode polarization in CMB

Ground-based experiments

LiteBIRD (2024-5?)

Generation mechanisms

Overlapped astrophysical GWs

interval of events $\Delta T < f^{-1}$

- Black hole binaries
- Neutron star binaries
- White dwarf binaries
- Supernovae

etc...

Cosmological GWs

generated in the early Universe

- Inflation
- Preheating
- Phase transitions
- Gauge fields
- Cosmic strings

etc...

GW generation

Evolution equation of GWs

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = 16\pi G\Pi_{ij}$$

I. Non-negligible initial condition

- **Inflation**
→ quantum fluctuations in the spacetime

2. Sourced by matter component of the Universe

- **Preheating**
→ rapid particle productions
- **Phase transition**
→ bubble collisions
- **Cosmic strings**
→ heavy string objects generated at phase transition

GW generation

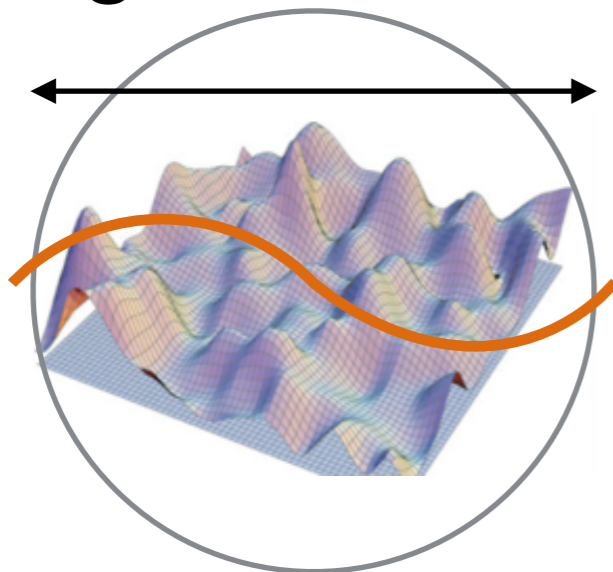
Evolution equation of GWs

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = 16\pi G\Pi_{ij}$$

$\lambda_{\text{GW}} < \text{Hubble horizon size}$

$$\rightarrow f/a < H$$

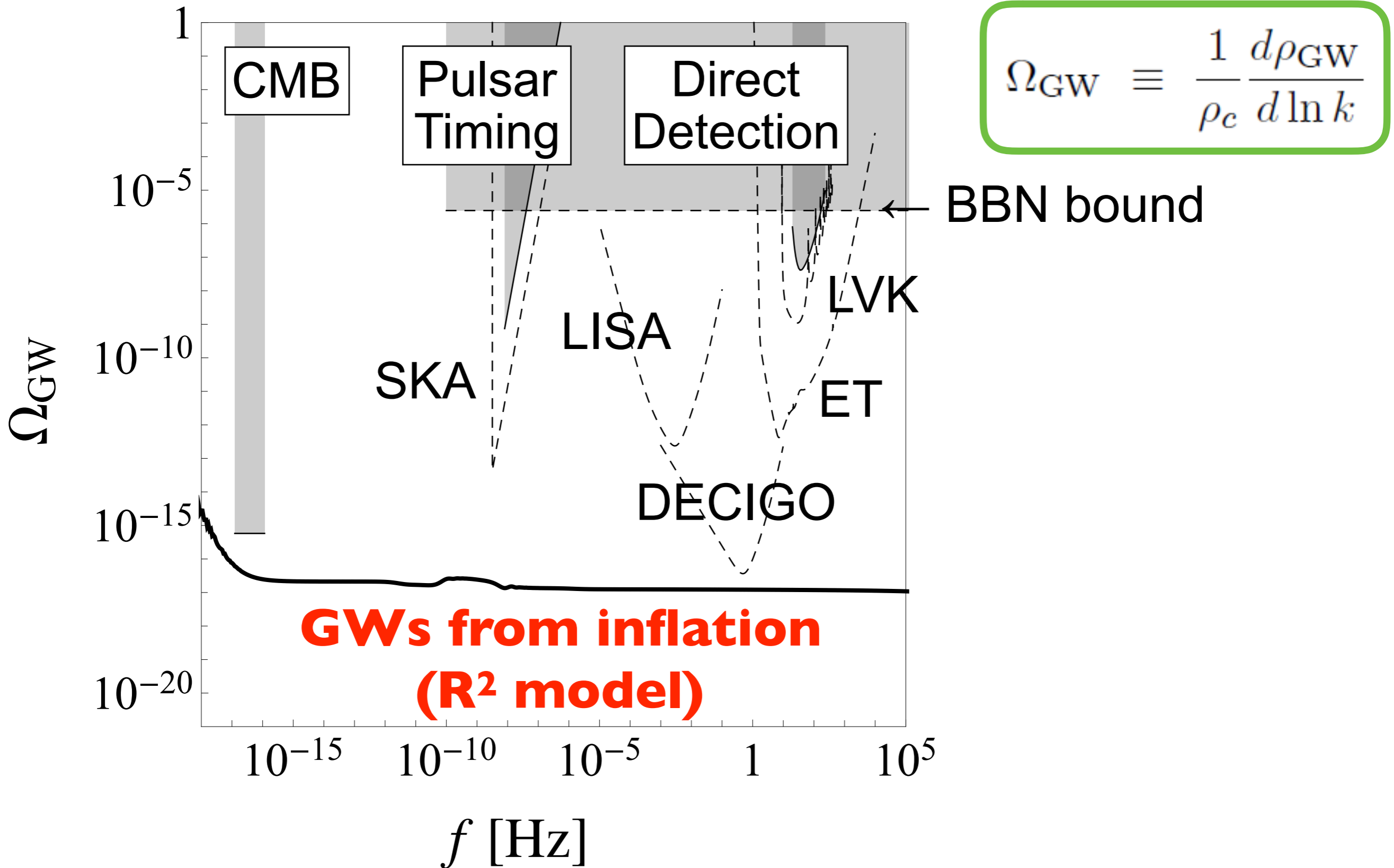
Hubble horizon
= region of causality



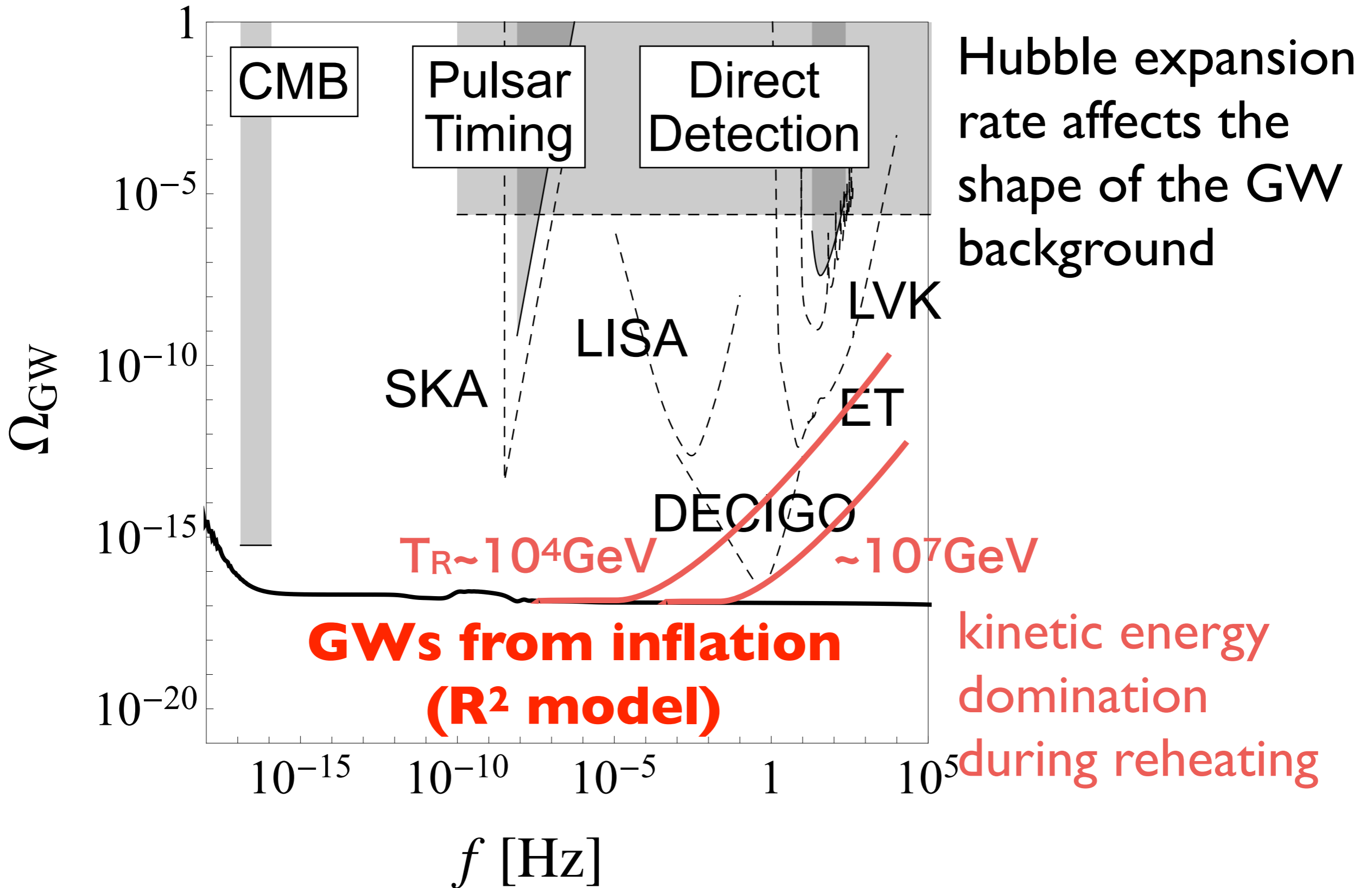
2. Sourced by matter component of the Universe

- **Preheating**
→ rapid particle productions
- **Phase transition**
→ bubble collisions
- **Cosmic strings**
→ heavy string objects generated in phase transition

GW spectrum and sensitivities



GW spectrum and sensitivities



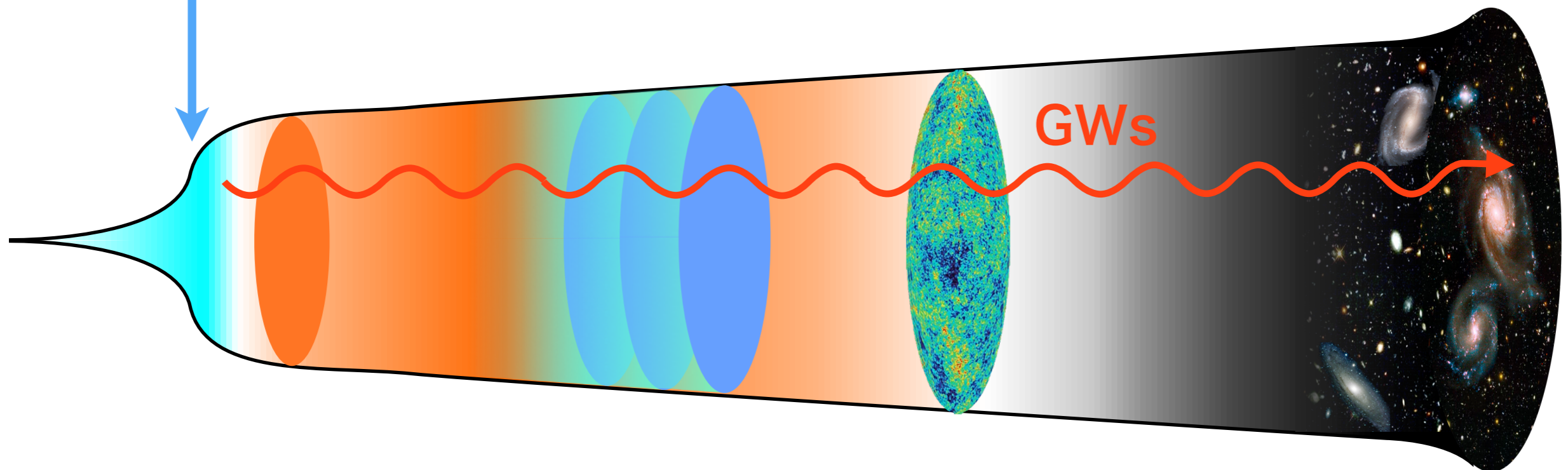
Gauge field production during inflation

Inflation: accelerated expansion in the early Universe driven by a scalar field (inflaton)

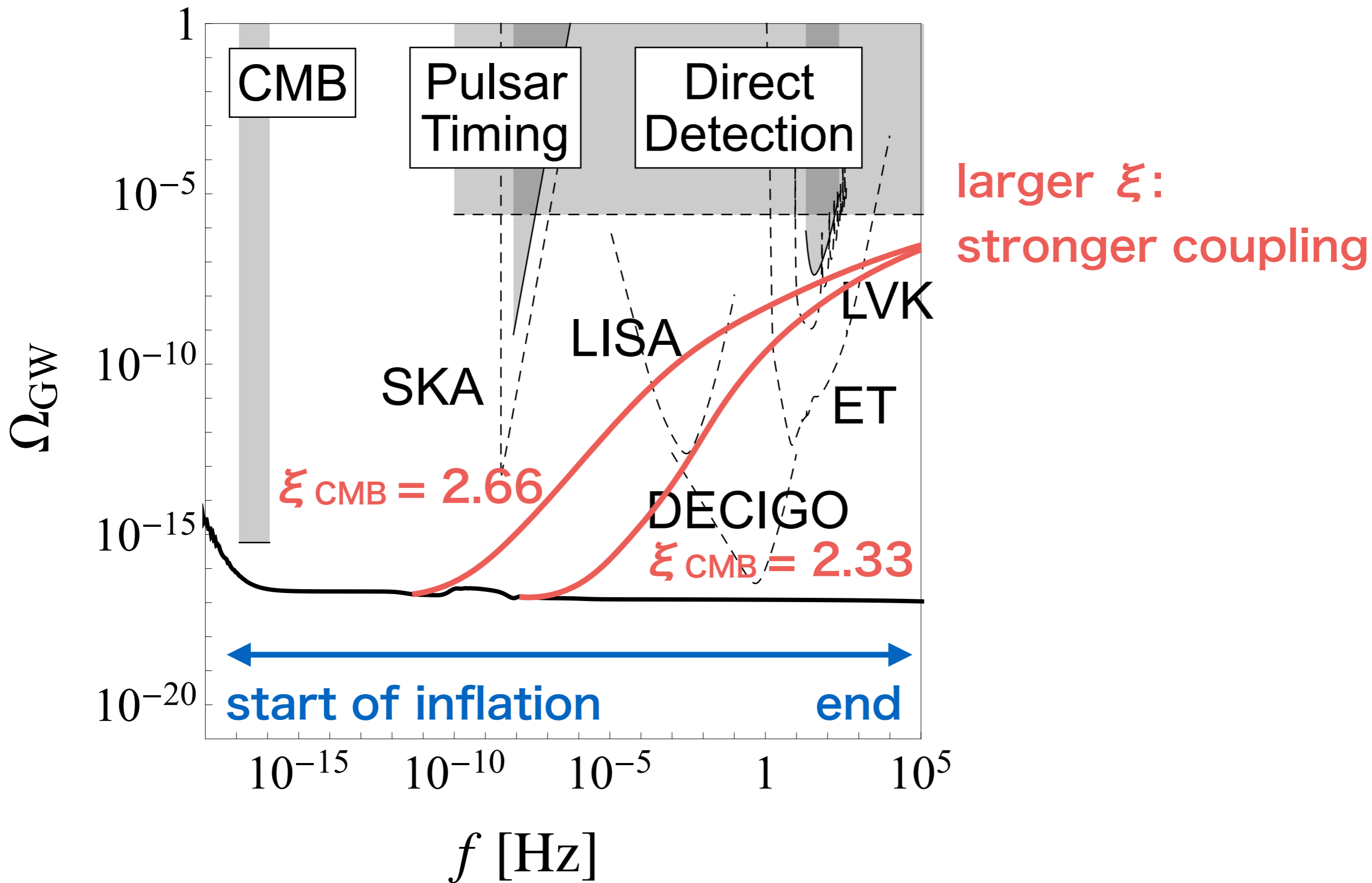
coupling between inflaton and a gauge field

$$\Delta\mathcal{L}_{\text{int}} = -\frac{1}{4f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

→ the gauge field mode is amplified by $\sim e^{\pi\xi}$ $\xi \equiv \frac{\dot{\phi}}{2fH}$
The anisotropic stress generates GWs



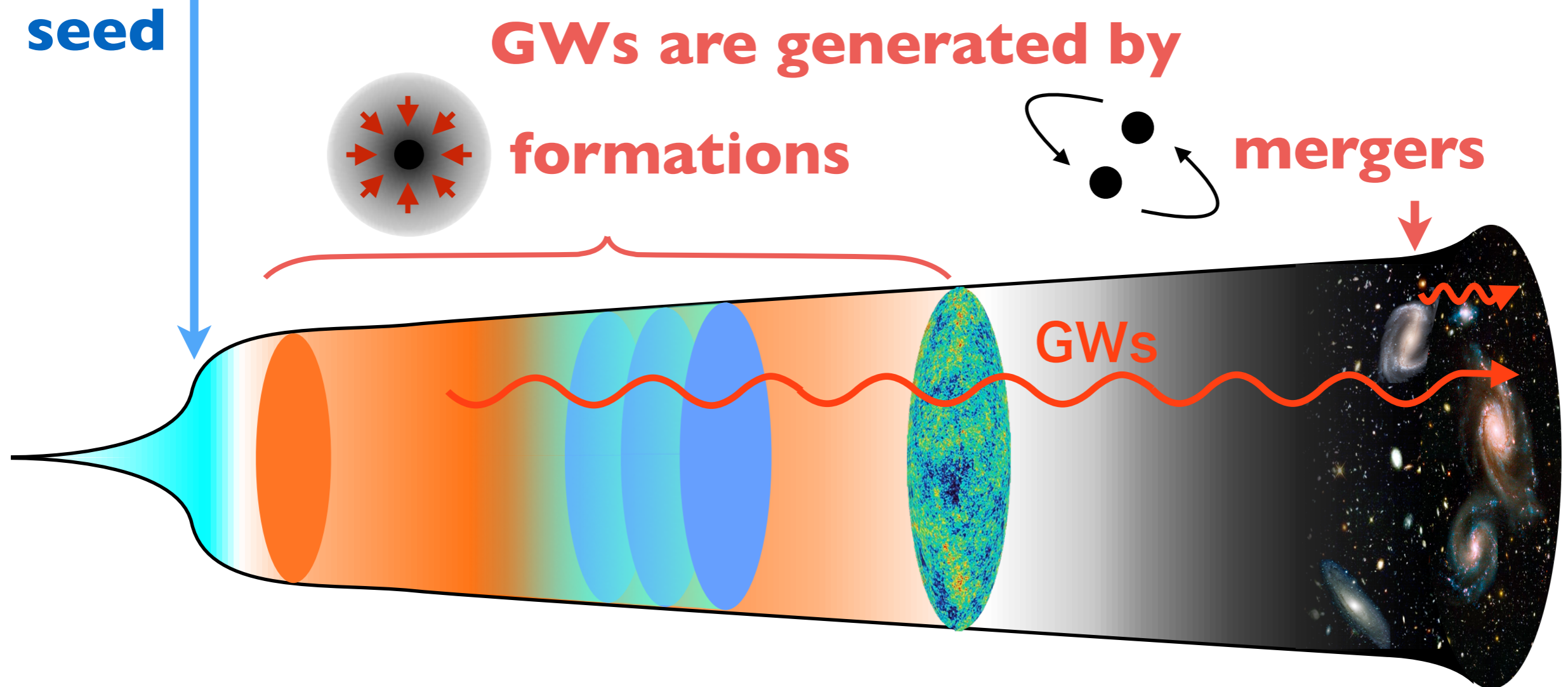
Gauge field production during inflation



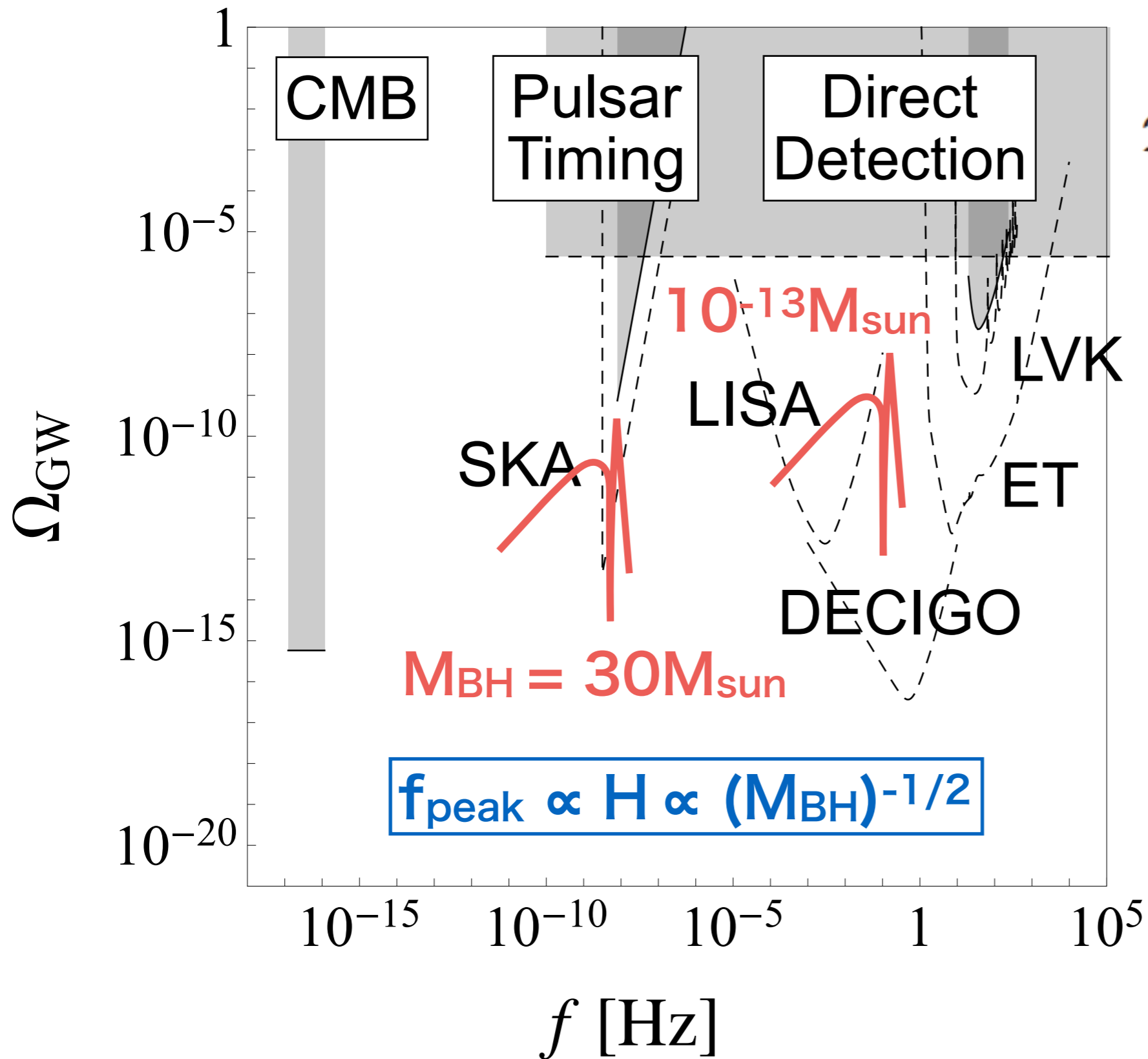
Primordial black holes

Prediction of CMB: $\mathcal{P}_{\mathcal{R}}(k) \sim 2 \times 10^{-9}$ at $k \sim 0.05 \text{Mpc}^{-1}$
 $\rightarrow f \sim 10^{-16} \text{Hz}$

large primordial curvature perturbations at small scale
can be produced by changing the dynamics of the inflaton



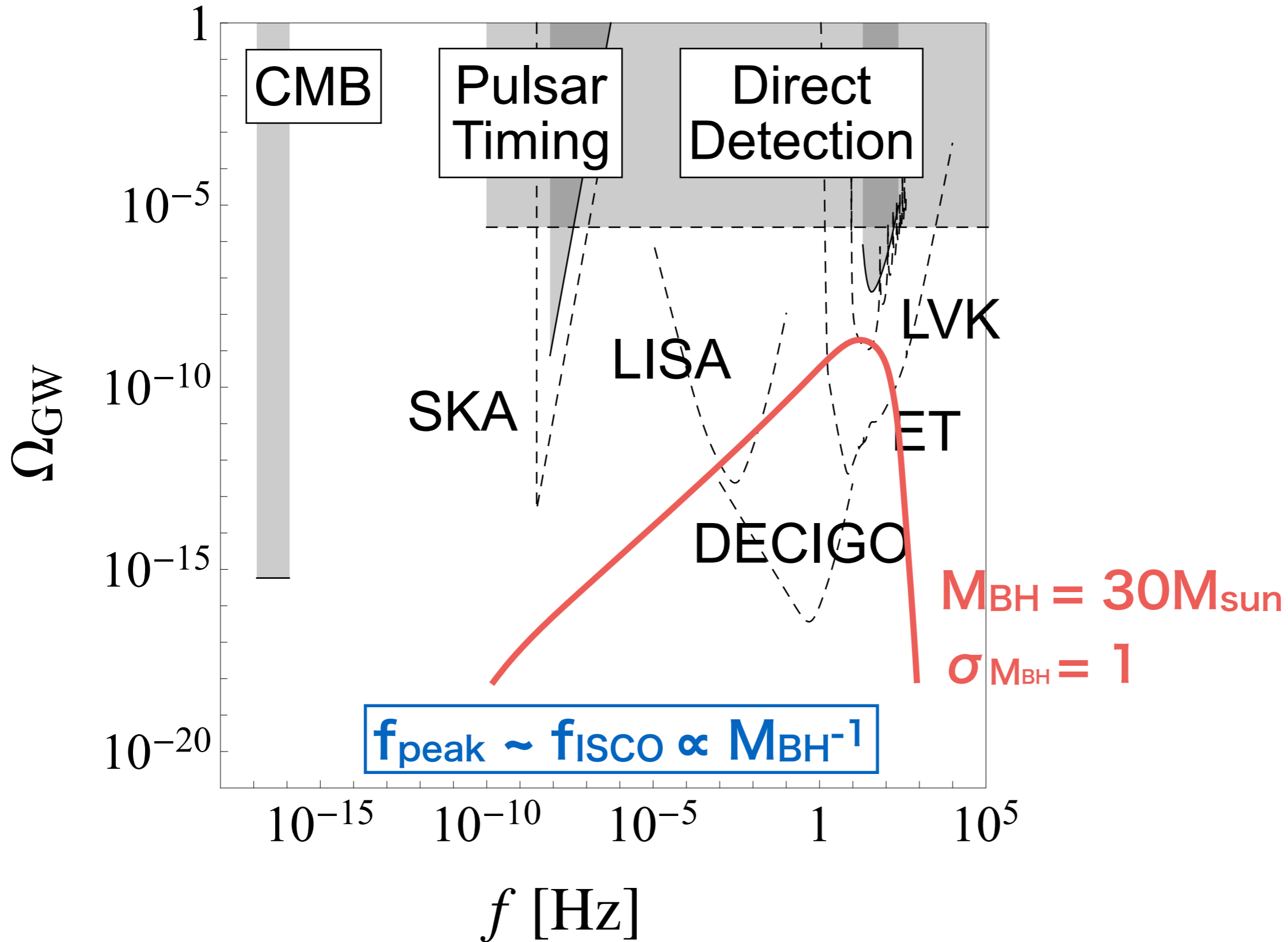
Primordial black holes: formation



$$\mathcal{P}_{\Psi}(k) = \mathcal{A}^2 \delta_D(\ln(k/k_p))$$

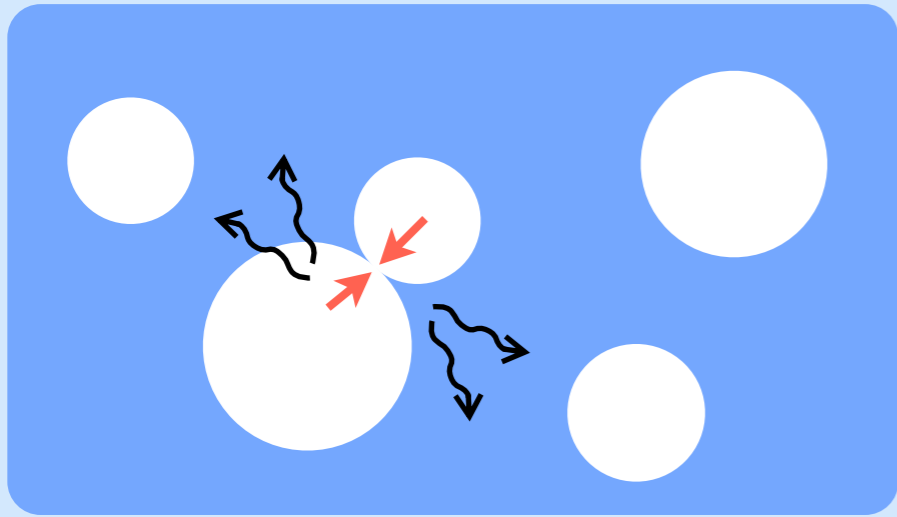
δ function peak is assumed for curvature perturbations

Primordial black holes: merger



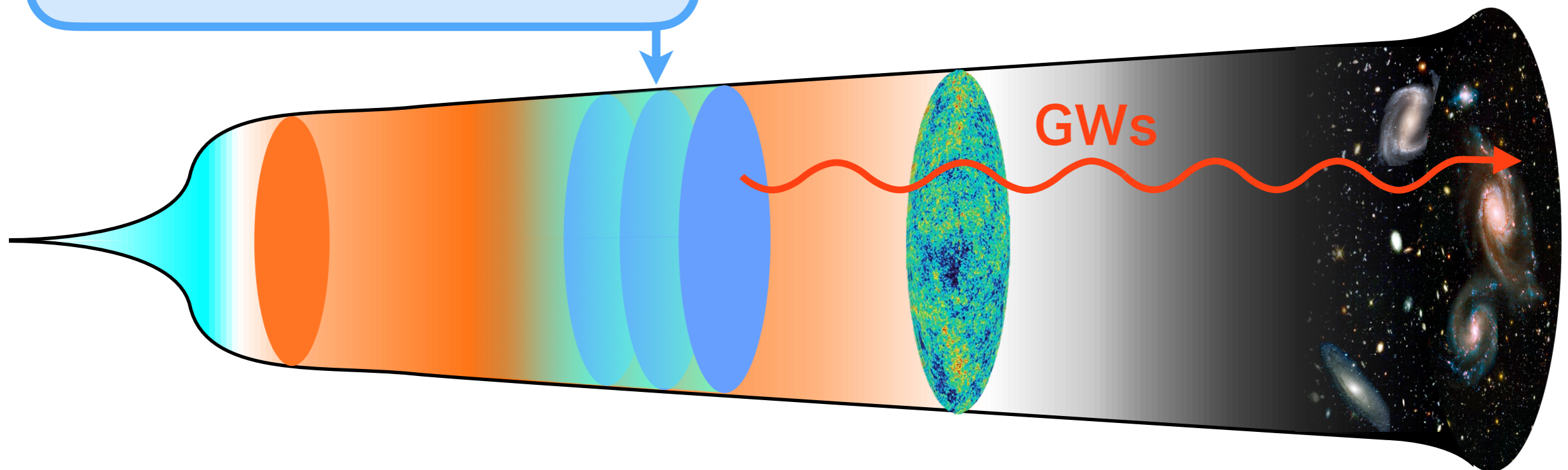
Cosmic phase transitions

**Bubbles appear
in a first order
phase transition**

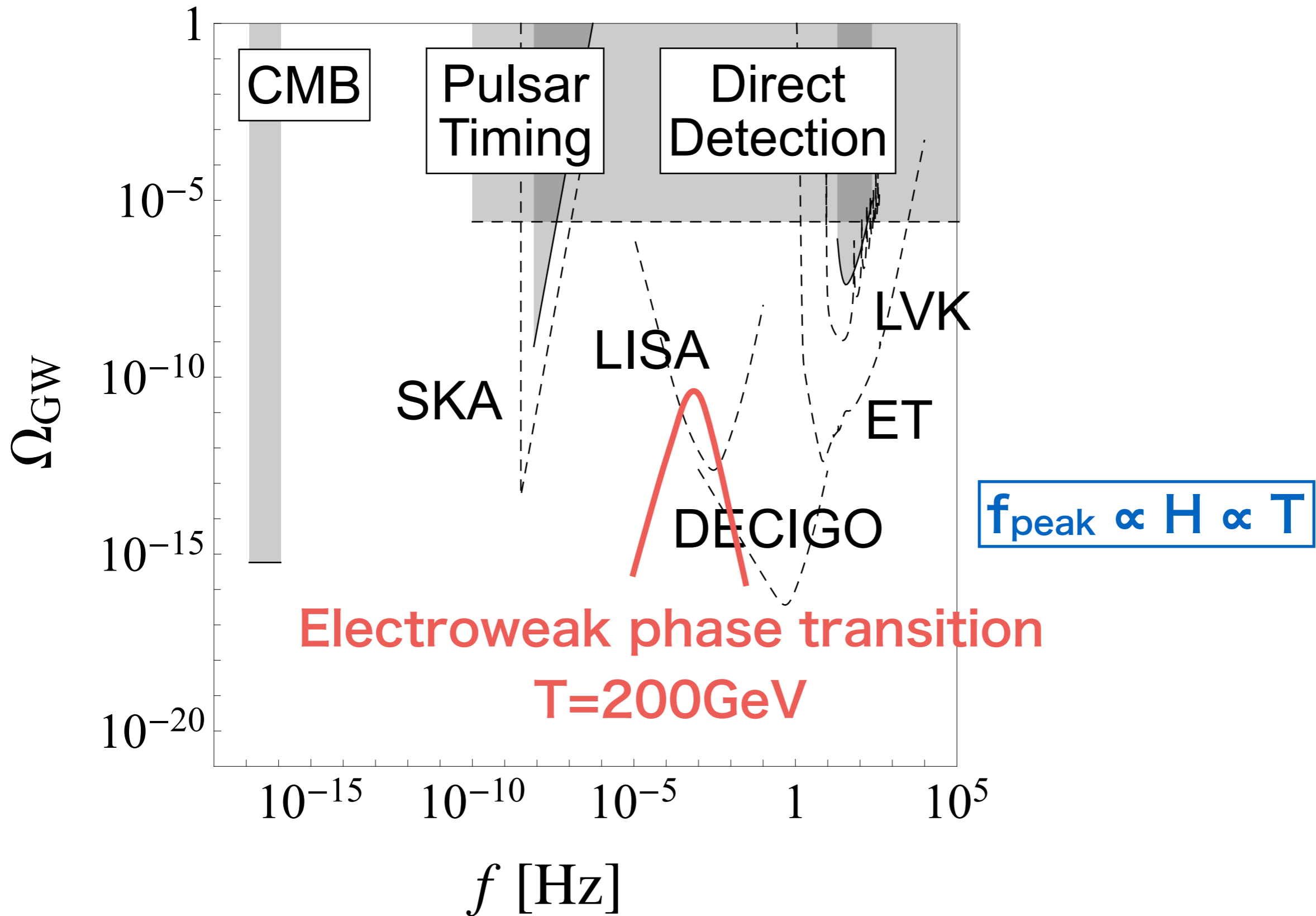


GWs are generated by

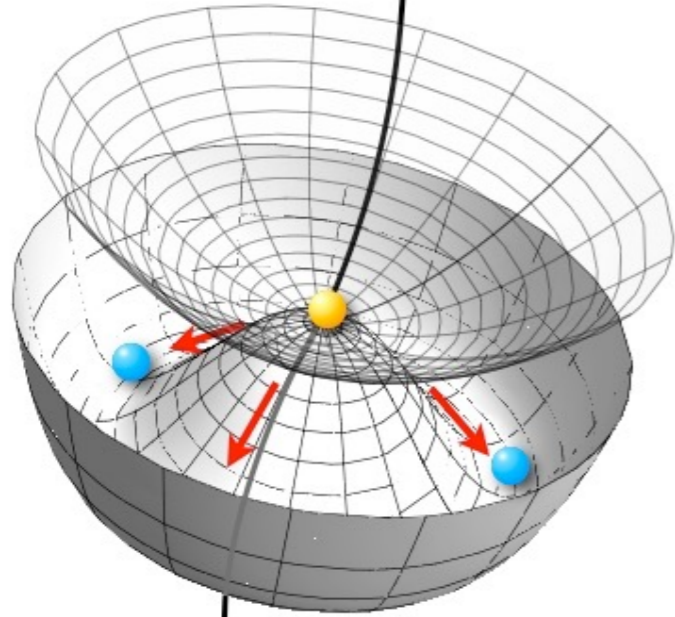
- 1. collision of bubbles**
- 2. sound waves ← dominant**
(sourced by fluid kinetic energy)
- 3. turbulence**
(nonlinear process)



Cosmic phase transitions



Cosmic strings



cosmic superstrings

inflation

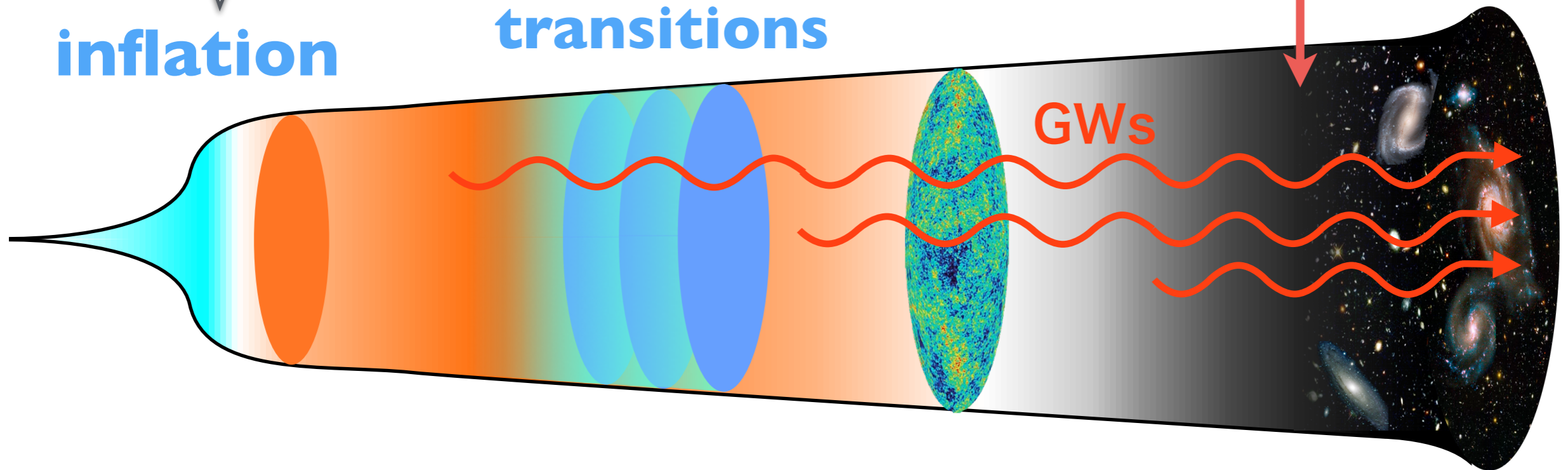
cosmic strings

Cosmic phase transitions

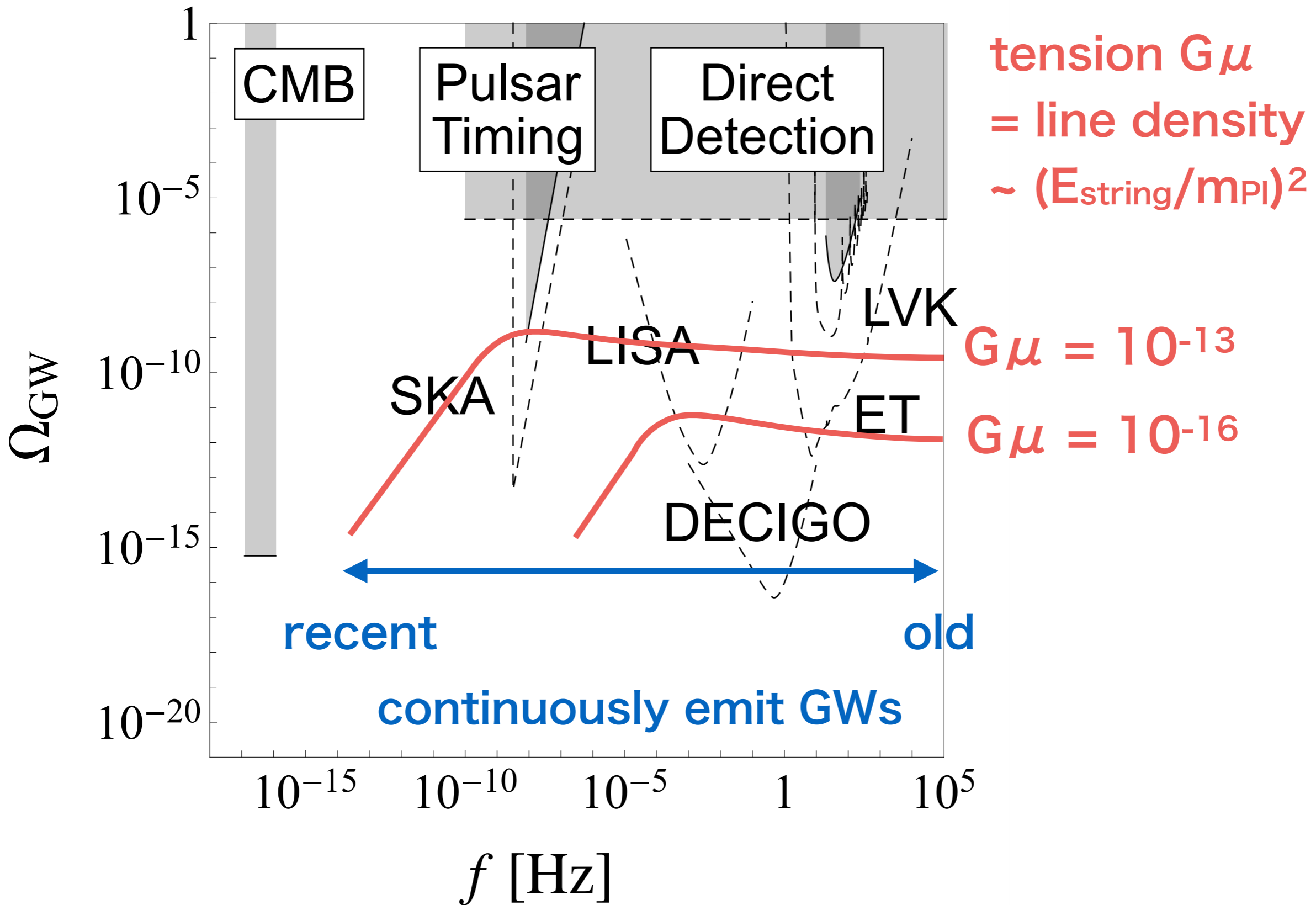
continuously emit GWs until today

kinks

cusps

A red-bordered box containing text and diagrams. The text 'continuously emit GWs until today' is at the top. Below it, the word 'kinks' is followed by a diagram of two intersecting lines forming an X-shape, with an arrow pointing to a similar shape where the lines are bent. Below that, the word 'cusps' is followed by a diagram of a closed loop with two sharp points, with arrows pointing towards these points.

Cosmic strings (cusps on loops)



How can we determine the origin of the GW background?

→ go beyond the spectral amplitude and use other types of information such as

- 1. Spectral shape**
- 2. Polarization**
- 3. Anisotropy**
- 4. Popcorn noise**
- (5. Non-Gaussianity)**

How to detect a stochastic background



Cross Correlation

detector 1

$$s_1(t) = h(t) + n_1(t)$$

detector 2

$$s_2(t) = h(t) + n_2(t)$$

$$\langle S \rangle = \int_{-T/2}^{T/2} dt \langle s_1(t) s_2(t) \rangle$$

$$= \int_{-T/2}^{T/2} dt \langle h^2(t) + \underbrace{h(t)n_2(t) + n_1(t)h(t) + n_1(t)n_2(t)}_{\text{no correlations} \rightarrow 0} \rangle$$

$$= \int_{-T/2}^{T/2} dt \langle \underline{h^2(t)} \rangle \text{ GW signal}$$

(for detectors at the same location)

s: observed signal
h: gravitational waves
n: noise

Optimal filtering

B. Allen & J. Romano, PRD 59, 102001 (1999)

$$S := \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' s_1(t) s_2(t') \underline{Q}(t, t')$$

filter function

With the approximation $h \ll n$, the signal-to-noise ratio is

$$\text{SNR}^2 = \frac{\mu^2}{\sigma^2} \approx \left(\frac{3H_0^2}{10\pi^2} \right)^2 T \frac{\left(\underline{\tilde{Q}}, \frac{\gamma(|f|)\Omega_{\text{gw}}(|f|)}{|f|^3 P_1(|f|)P_2(|f|)} \right)^2}{(\underline{\tilde{Q}}, \underline{\tilde{Q}})}$$

$(A, B) := \int_{-\infty}^{\infty} df A^*(f) B(f) P_1(|f|) P_2(|f|)$

**filter function
in Fourier space**

SNR is maximized when

$$\tilde{Q}(f) = \lambda \frac{\gamma(|f|)\Omega_{\text{gw}}(|f|)}{|f|^3 P_1(|f|)P_2(|f|)}$$

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$n(t) \rightarrow P_i(|f|)$: noise spectrum

observable:

$$s(t) = h(t) + n(t) \sim n(t)$$

↑
 $h \ll n$

→ obtained from data

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$h(t) \rightarrow \Omega_{\text{gw}}(|f|)$: GW spectrum

→ **unknown**

→ **We prepare**

**a template bank
of spectral shape**

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Optimal filtering

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SNR is maximized when

$$\tilde{Q}(f) = \lambda \frac{\gamma(|f|) \Omega_{\text{gw}}(|f|)}{|f|^3 P_1(|f|) P_2(|f|)}$$

overlap reduction function
determined by detector response

$h(t) \rightarrow \Omega_{\text{gw}}(|f|)$: GW spectrum

→ **unknown**

→ **We prepare**

**a template bank
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$n(t) \rightarrow P_i(|f|)$: noise spectrum
observable:

$$s(t) = h(t) + n(t) \sim n(t)$$

↑
 $h \ll n$

→ **obtained from data**

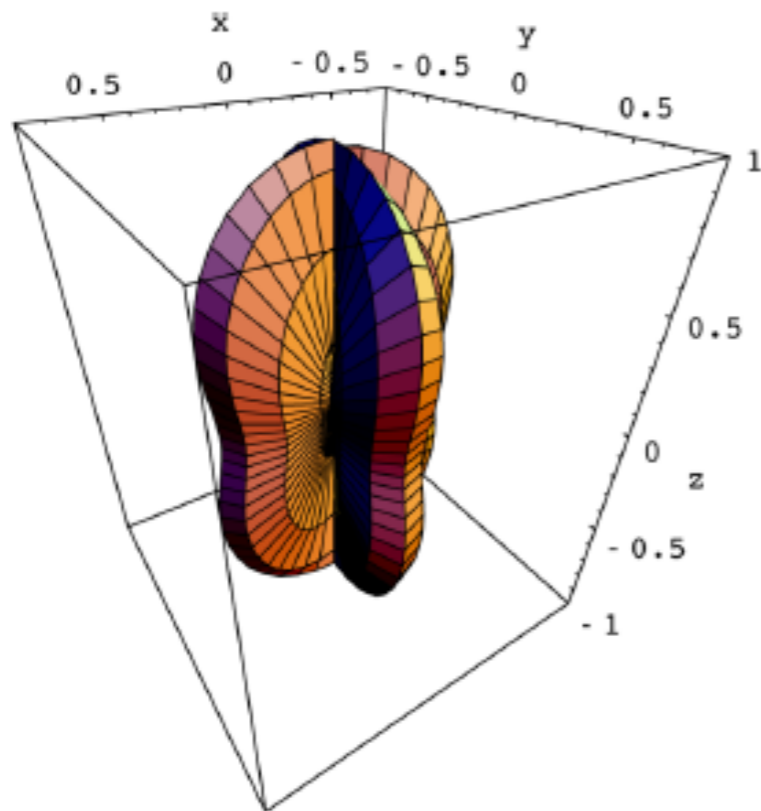
Detector response

Sensitivity of an interferometer depends on **source location** and **polarization**

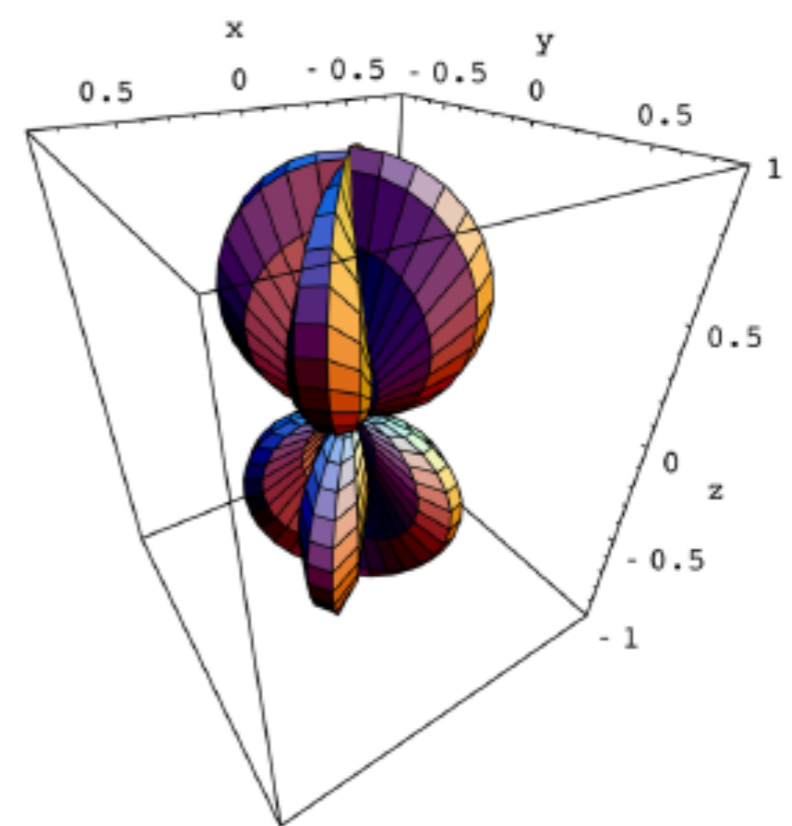
observable: $s(t) = \underline{F(\hat{\Omega})}h(t) + n(t)$
response function

$\hat{\Omega}$: direction of GW propagation

+ mode



x mode



detector arms

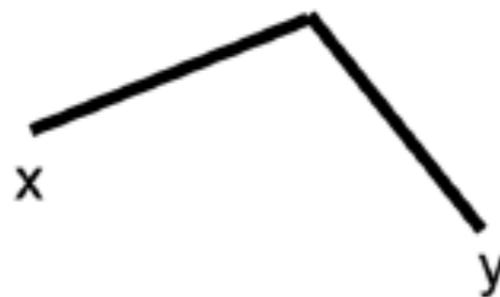


Figure from A. Nishizawa et al. PRD 79, 082002 (2009)

Overlap reduction function

For a stochastic GW background, we construct

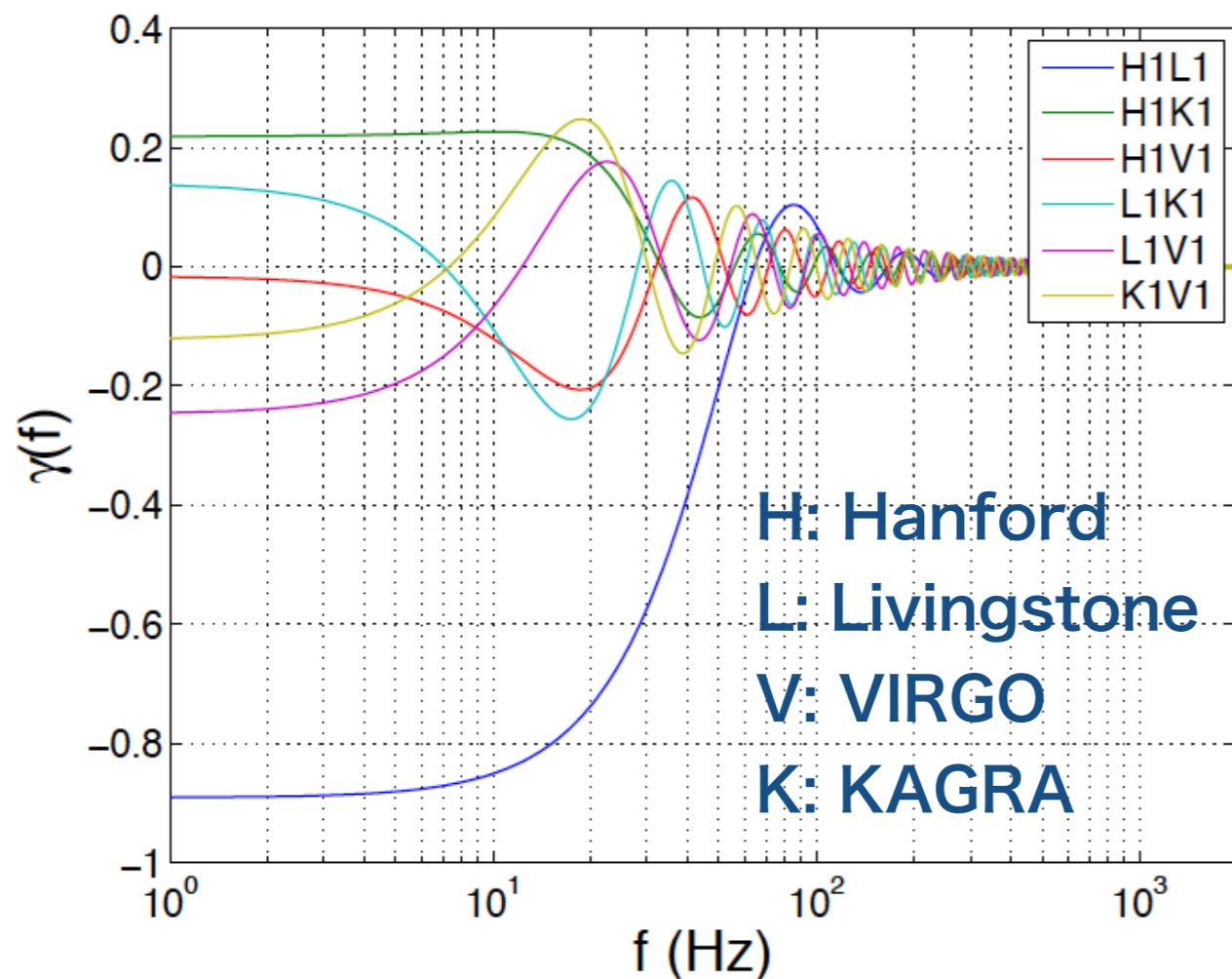
$$\gamma_{IJ}^T(f) \equiv \frac{5}{2} \int_{S^2} \frac{d\hat{\Omega}}{4\pi} e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{X} / c} (F_I^+ F_J^+ + F_I^\times F_J^\times)$$

integration over the whole sky

separation of detectors

response function

I and J denote different detectors



→ represents the amount of preserved correlation between two detectors

E. Thrane & J. D. Romano,
PRD 88, 124032 (2013)

I. Spectral shape

GW background is searched by changing
spectral shape template

filter function

$$\tilde{Q}(f) = \lambda \frac{\gamma(|f|) \Omega_{\text{gw}}(|f|)}{|f|^3 P_1(|f|) P_2(|f|)}$$

usually parametrized by
a single power law

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}$$

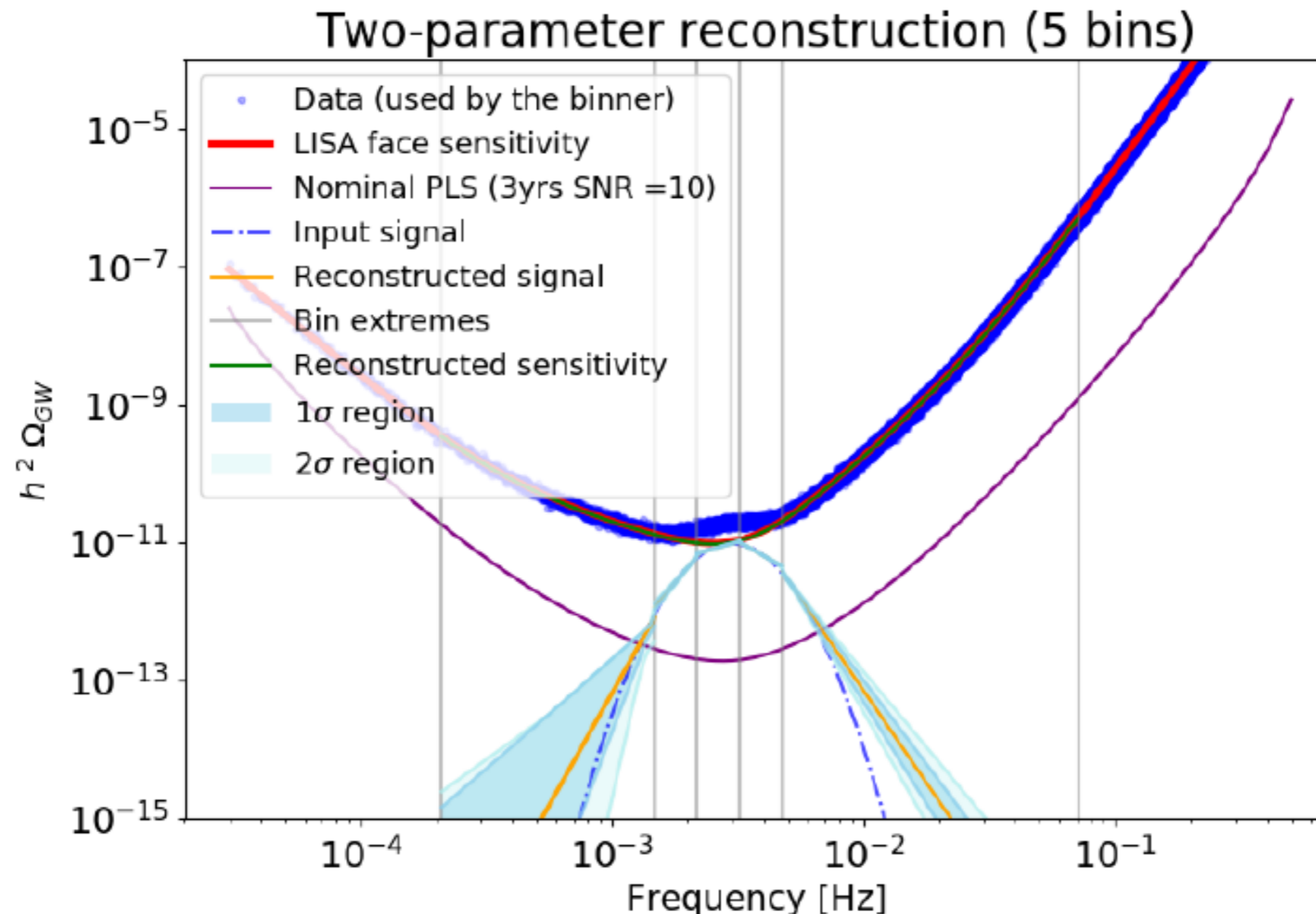
$$f_{\text{ref}} = 25\text{Hz}$$

LIGO & Virgo Collaboration, PRL 118, 121101 (2017)

→ but many models of GW background
predict a peaked shape

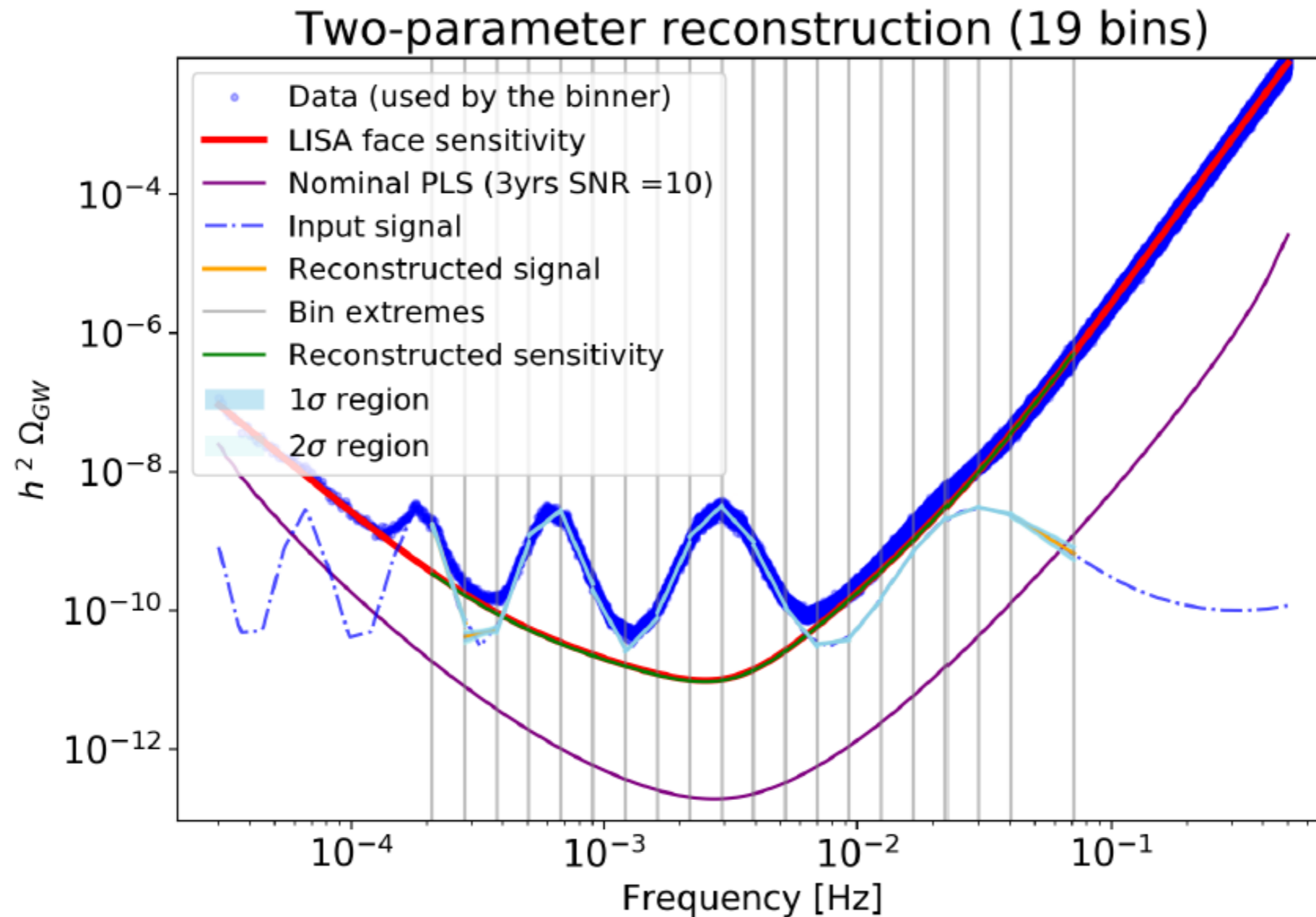
Multi-binning reconstruction

- Spectral tilt is fitted in each bin
- Binning is optimized by calculating the AIC



Multi-binning reconstruction

- Spectral tilt is fitted in each bin
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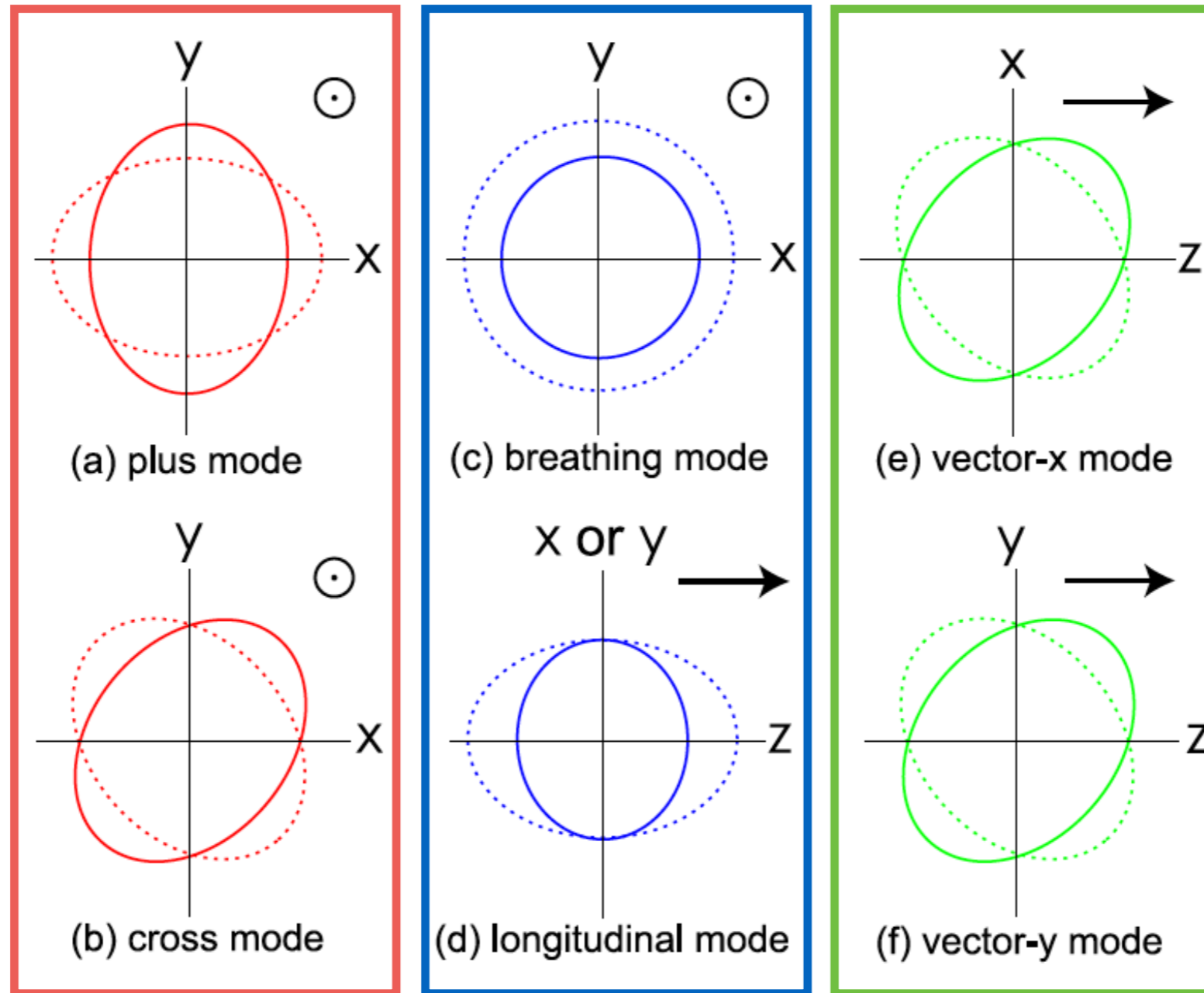


2. Polarization

Tensor

Scalar

Vector

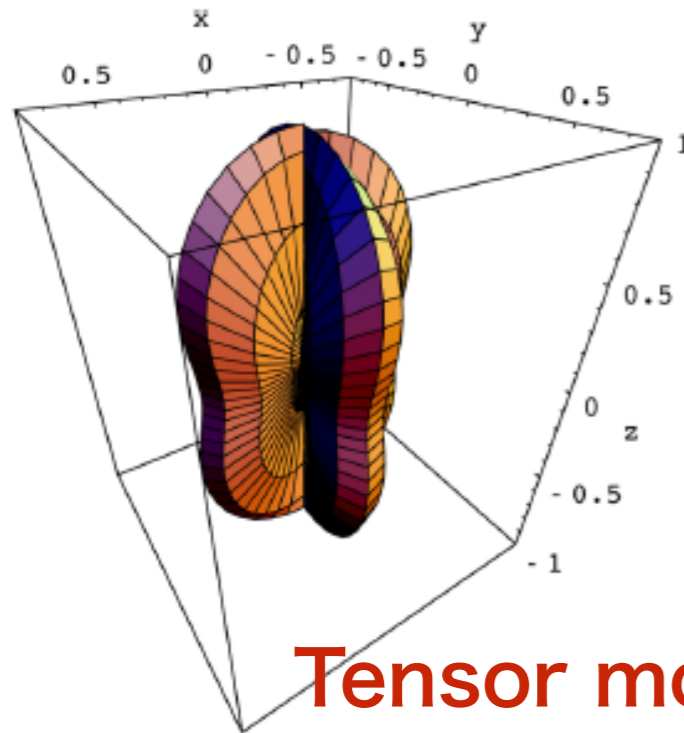


Linear + Circular modes

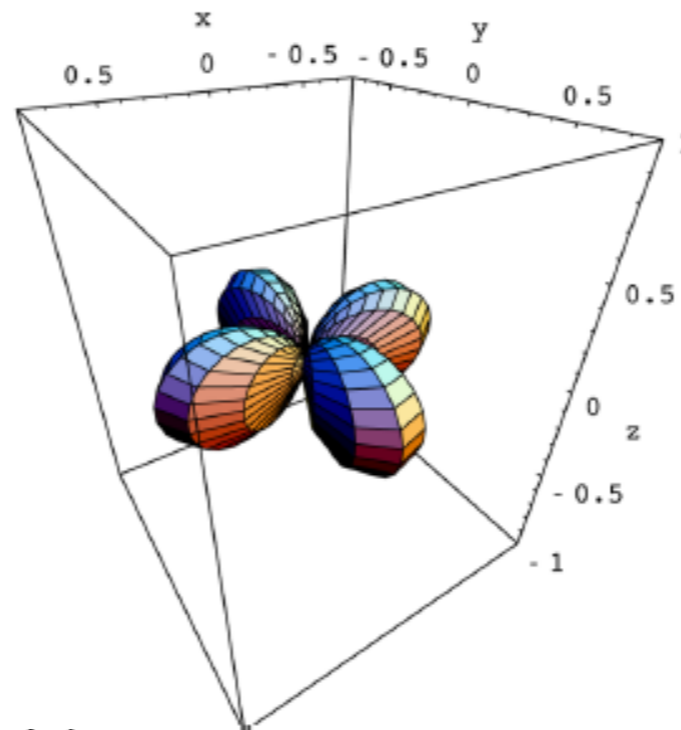
→ probe of non-GR

How can we detect polarization?

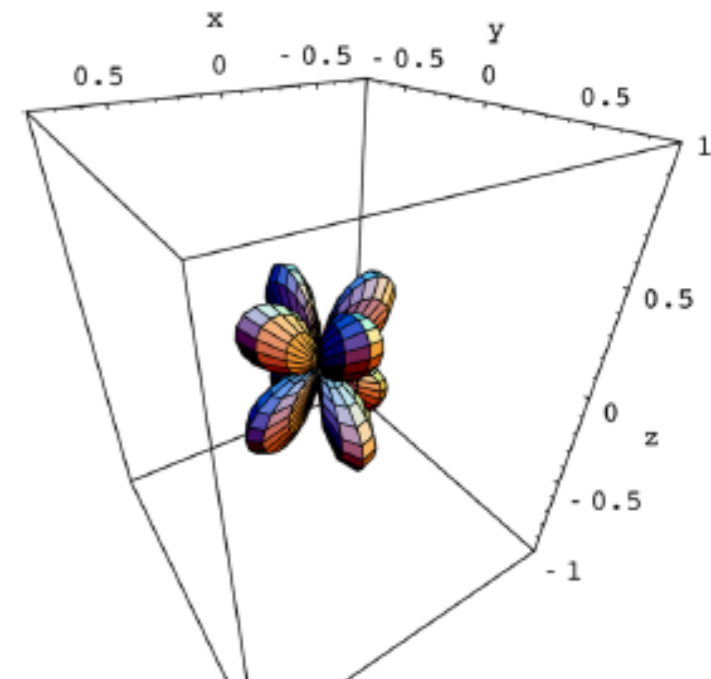
Detector response changes



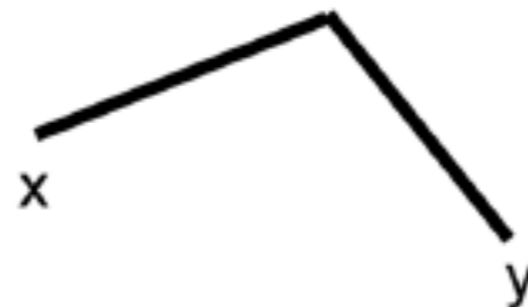
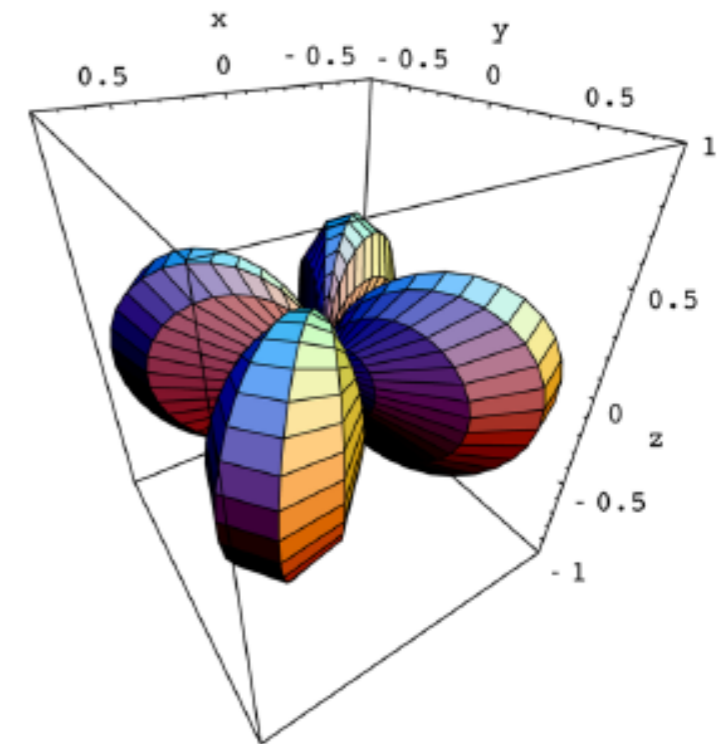
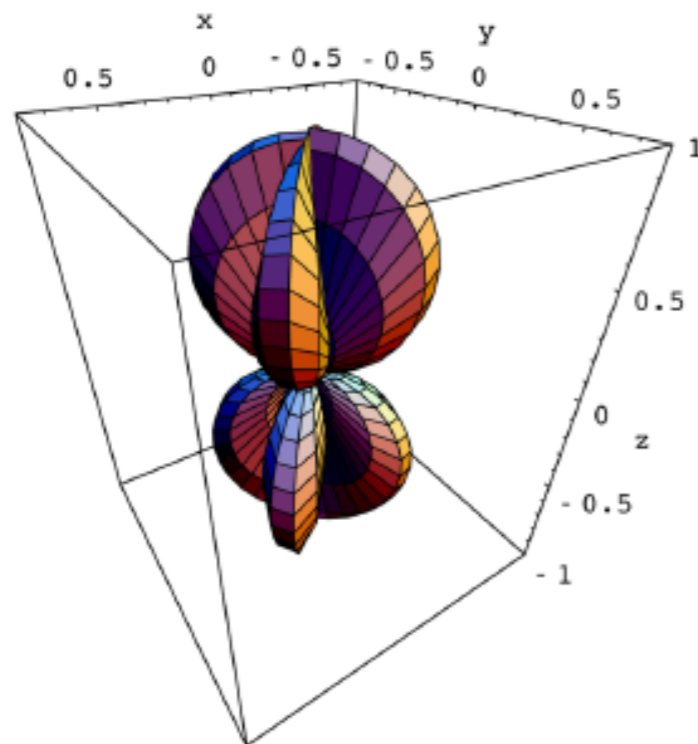
Tensor modes



Scalar modes

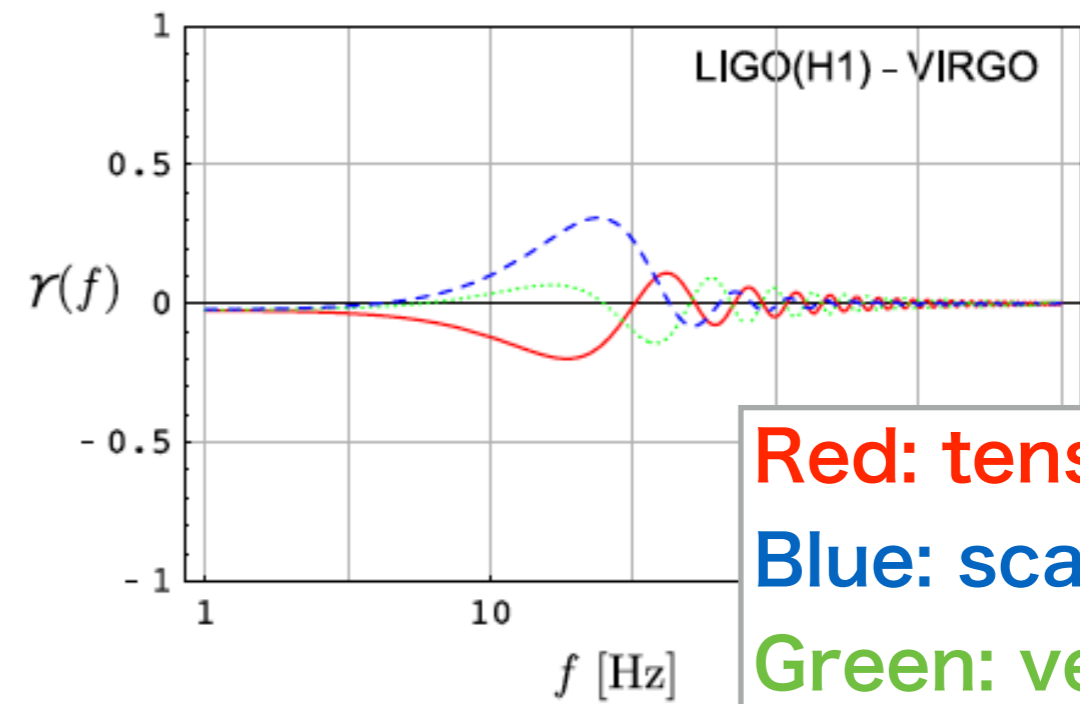
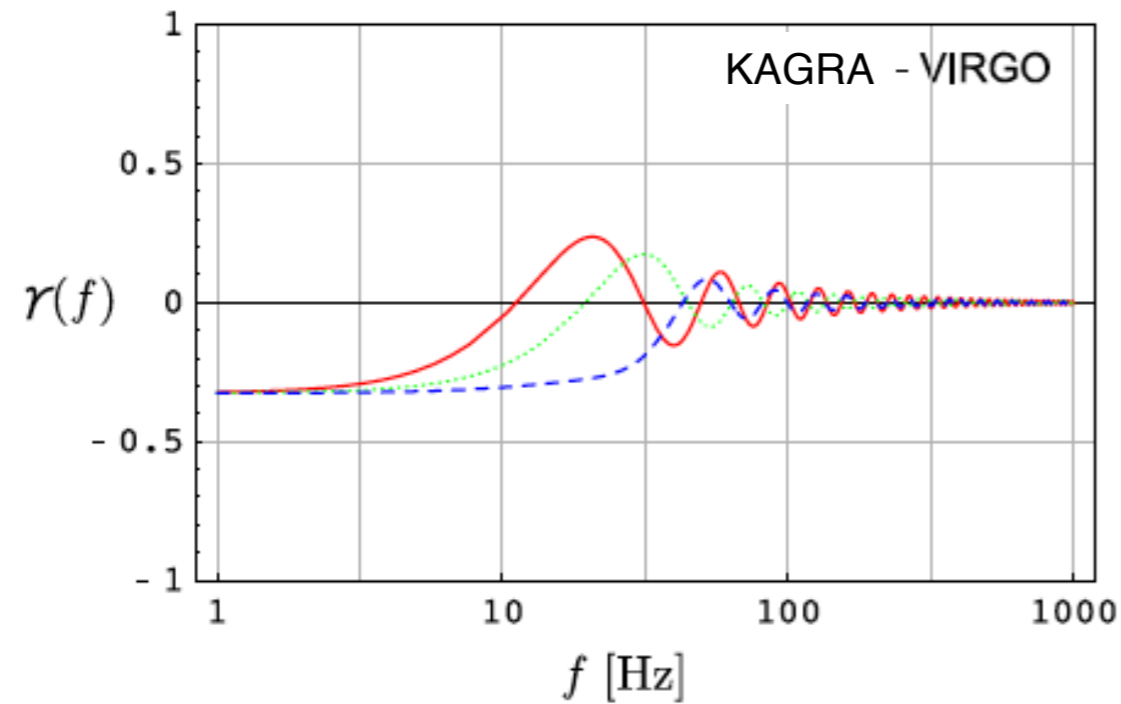


Vector modes

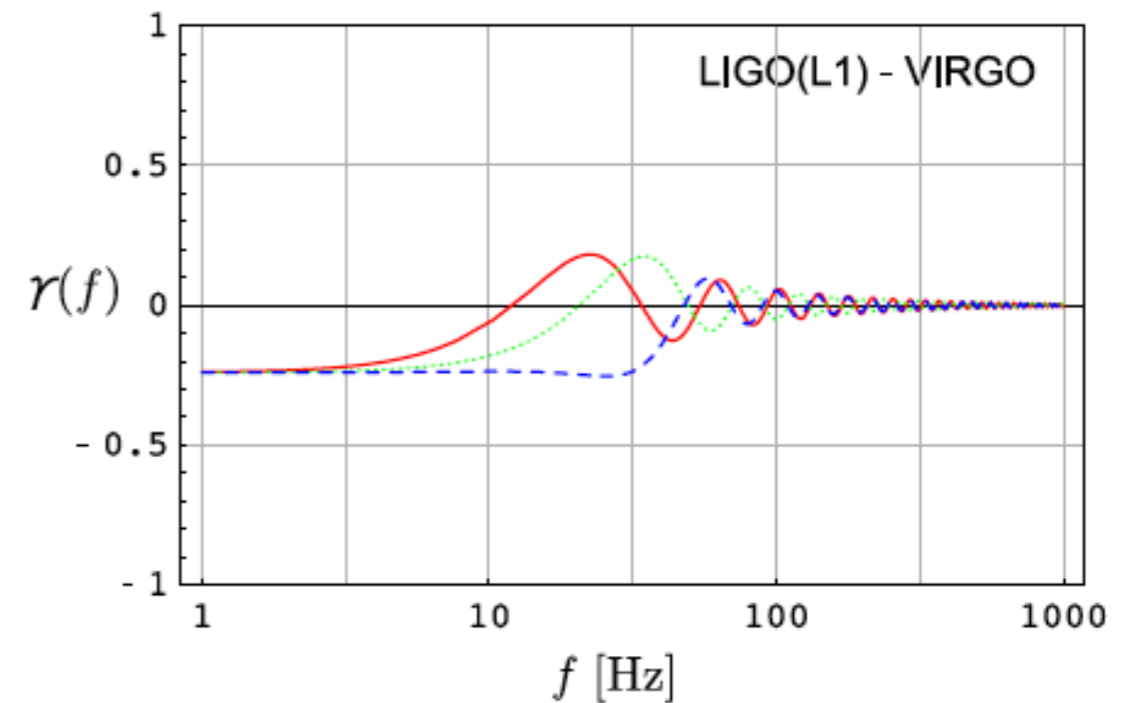
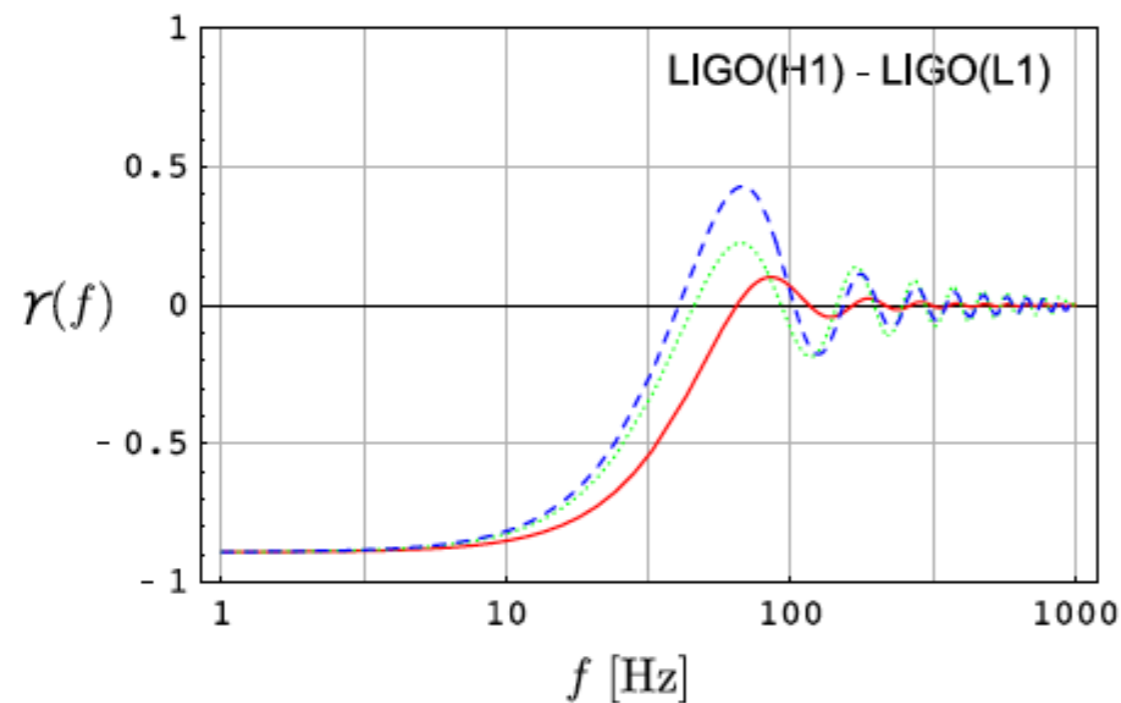


How can we detect polarization?

Overlap reduction functions



Red: tensor
Blue: scalar
Green: vector



How can we detect polarization?

Filter function

overlap reduction function spectral shape template

$$\tilde{Q}(f) = \lambda \frac{\gamma(|f|) \Omega_{\text{gw}}(|f|)}{|f|^3 P_1(|f|) P_2(|f|)}$$

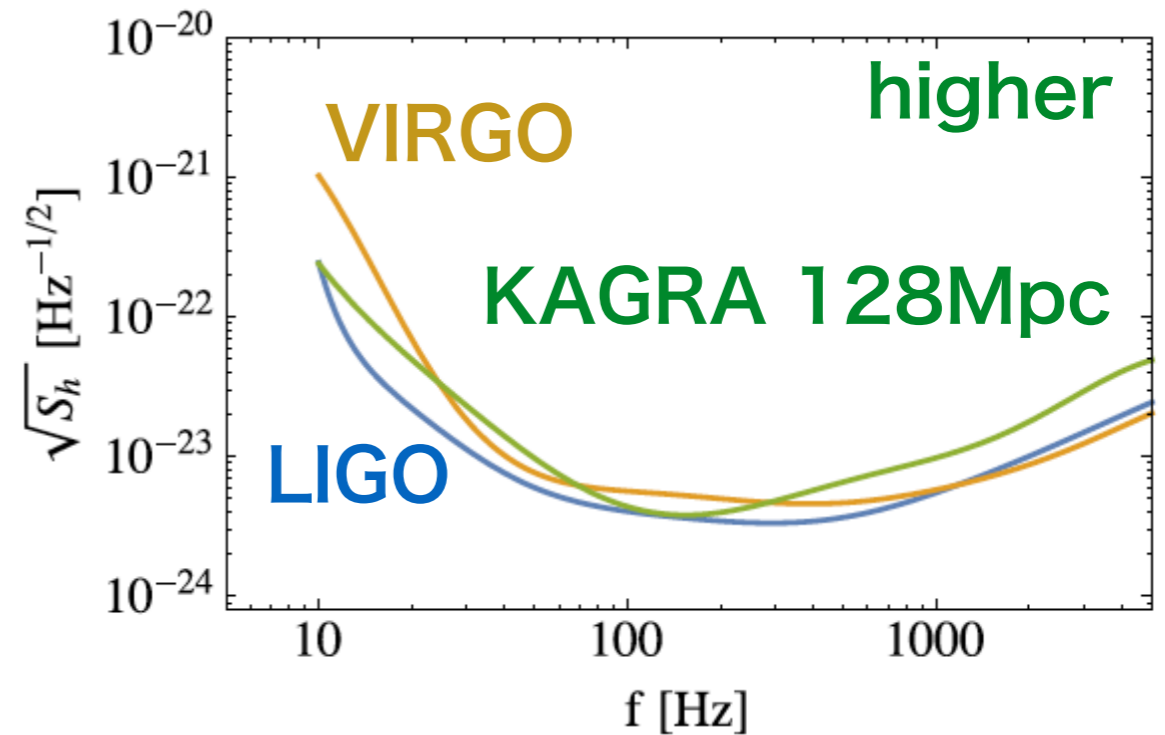
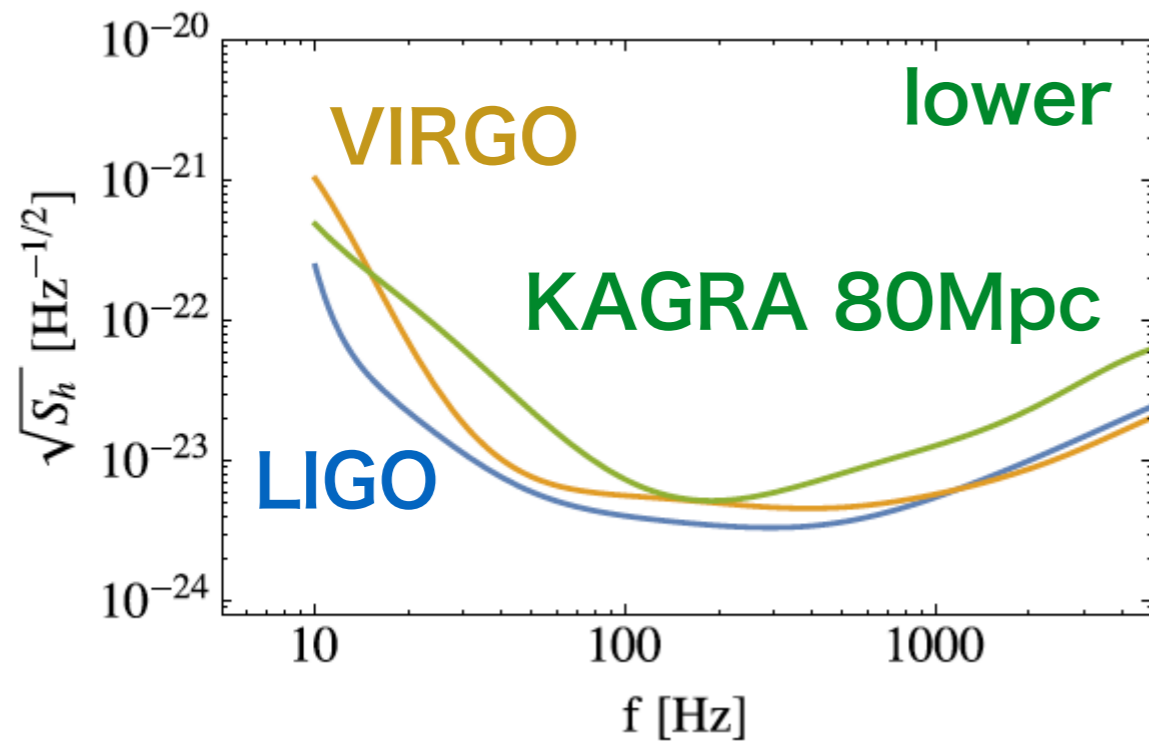
noise spectra of detectors

→ We can extract additional polarization modes by changing $\gamma(f)$

KAGRA will help



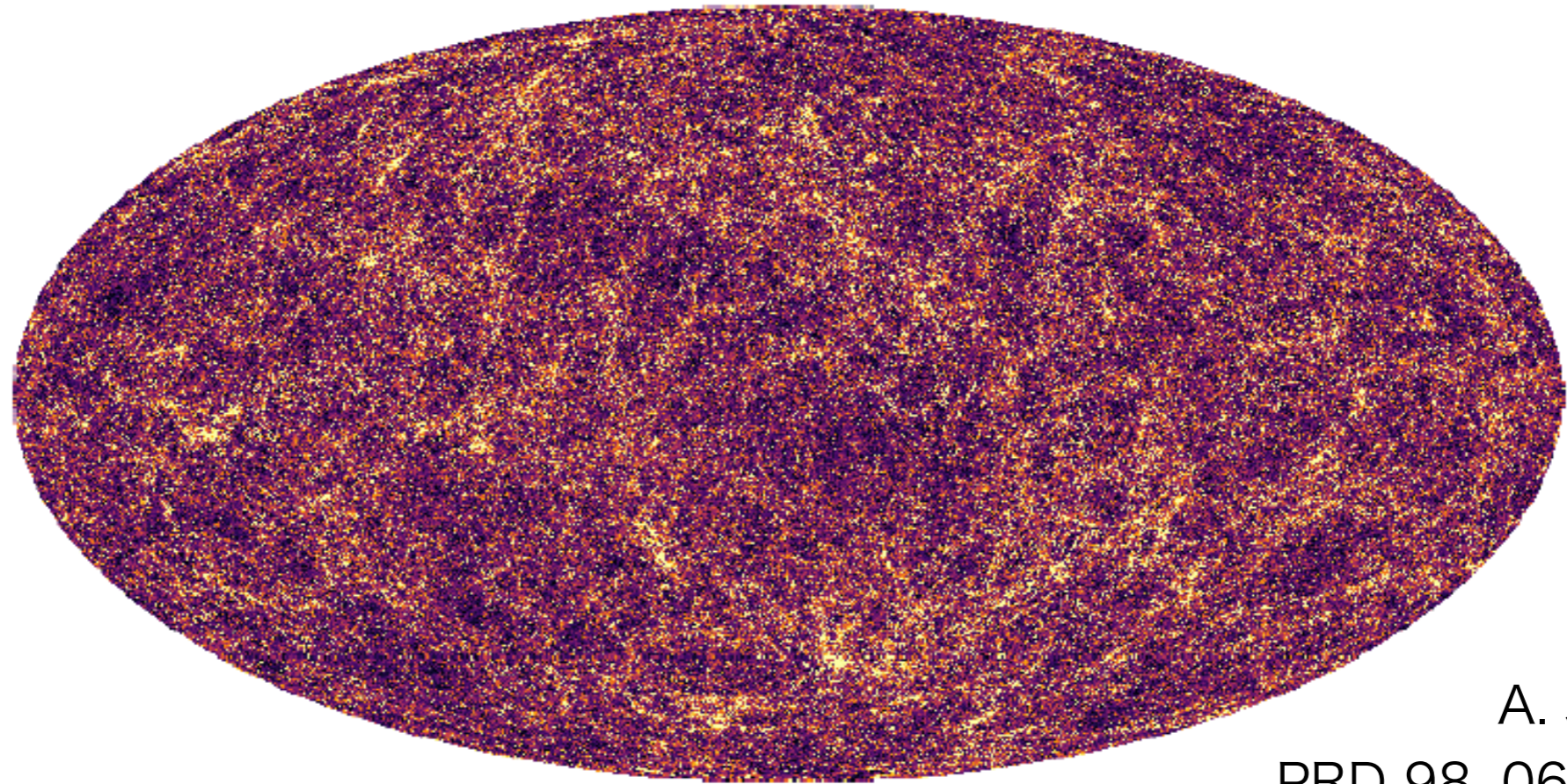
O4 sensitivity curves (2021-)



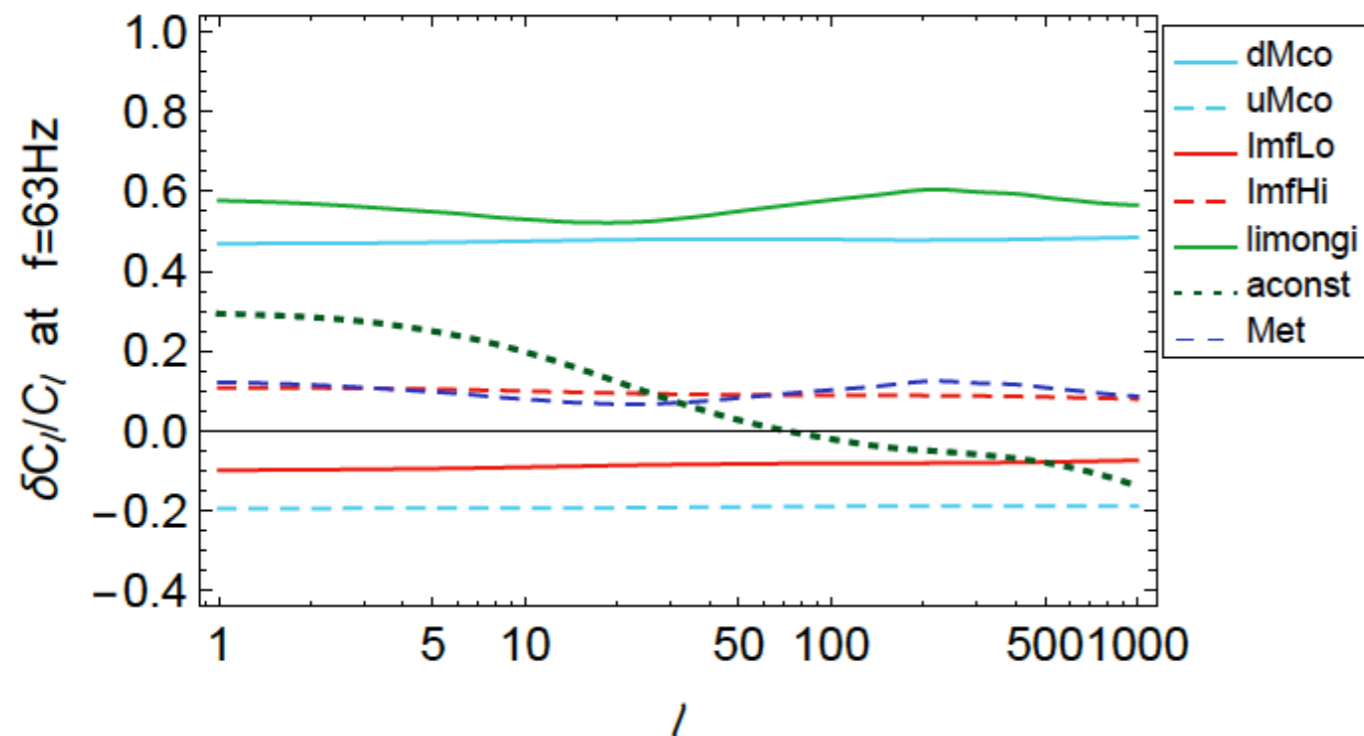
	Ω_{GW}^T	Ω_{GW}^V	$\xi \Omega_{\text{GW}}^S$
LIGO×2 + VIRGO	6.6×10^{-9}	1.7×10^{-8}	8.3×10^{-9}
+ KAGRA (80pc)	1.3×10^{-8}	2.1×10^{-8}	1.4×10^{-8}
+ KAGRA (128pc)	5.8×10^{-9}	9.5×10^{-9}	6.6×10^{-9}

ongoing work with A. Nishizawa & G. Liu (KAGRA collaboration)

3. Anisotropy



A. Jenkins et al.
PRD 98, 063501 (2018)



Anisotropy of BBH GW background

→ helpful to distinguish
between astrophysical and
cosmological origin

How can we detect anisotropy?

Overlap reduction function changes

$$\gamma_{IJ}^T(f) \equiv \frac{5}{2} \int_{S^2} \frac{d\hat{\Omega}}{4\pi} e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{X} / c} (F_I^+ F_J^+ + F_I^\times F_J^\times)$$

isotropic GWs integrated over the whole sky

1. Radiometry

S. W. Ballmer, CQG 23, S179 (2006)

$$\gamma_{IJ}^{\hat{\Omega}} \equiv \frac{1}{2} e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{X} / c} (F_I^+ F_J^+ + F_I^\times F_J^\times)$$

→ filter for a point source in direction $\hat{\Omega}$

2. Spherical harmonics

B. Allen & A. C. Ottewell, PRD 56, 545 (1997)

E. Thrane, PRD 80, 122002 (2009)

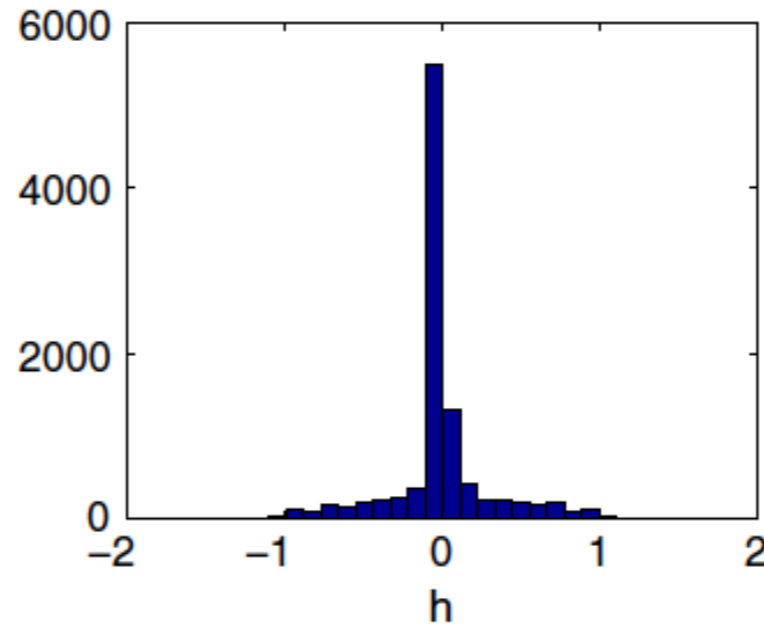
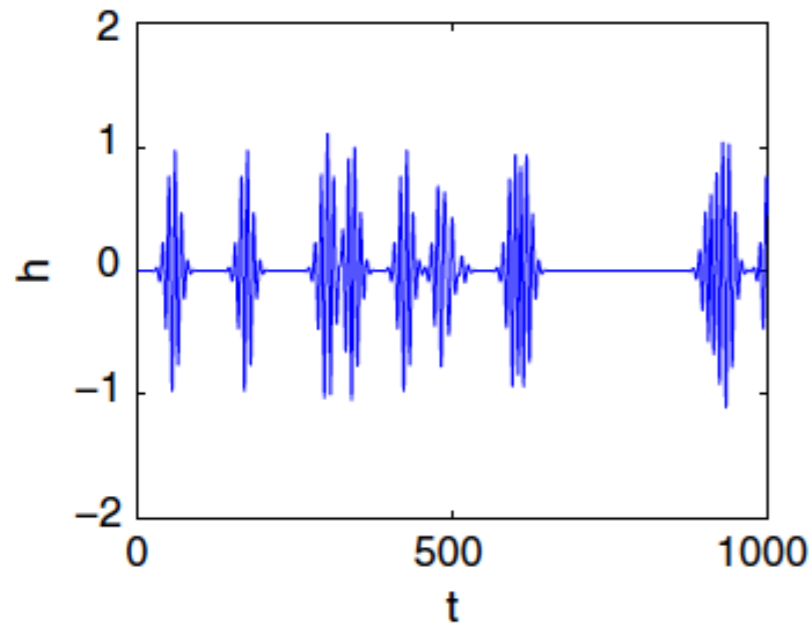
$$\gamma_{IJ,lm} \equiv \frac{1}{2} \int d\hat{\Omega} \underline{Y_{lm}(\hat{\Omega})} e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{X} / c} (F_I^+ F_J^+ + F_I^\times F_J^\times)$$

→ optimized for extended sources

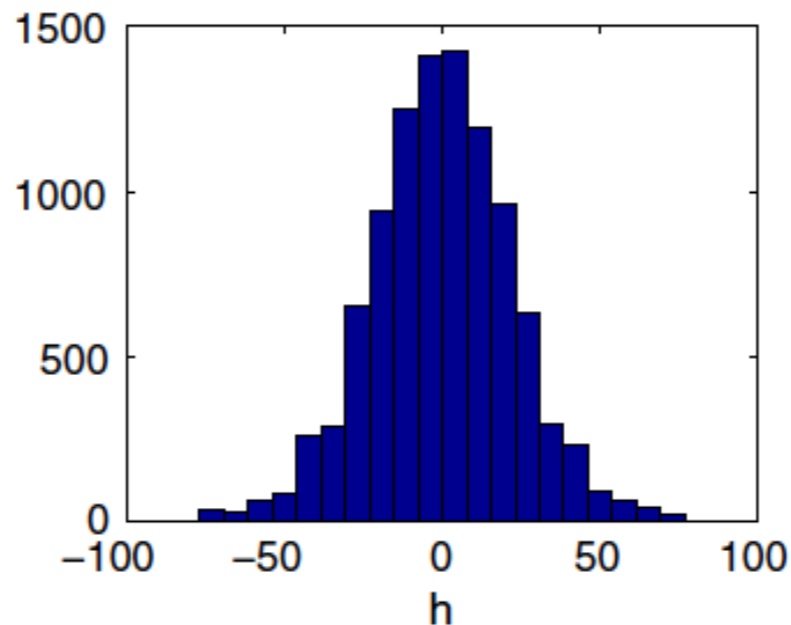
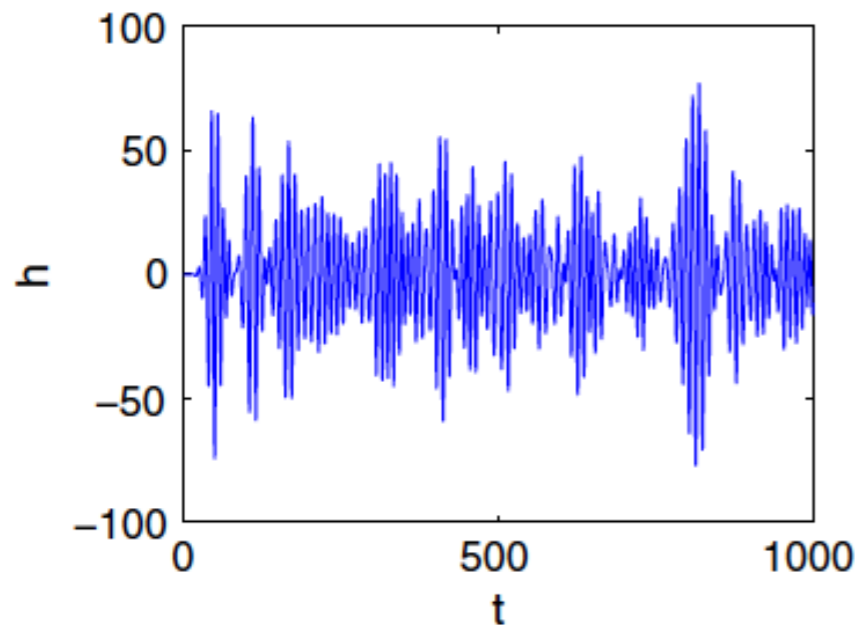
4. Popcorn noise

Non-overlapping GW bursts

E. Thrane, PRD 87, 043009 (2013)



non-Gaussian
distribution



Gaussian
distribution

→ helpful to distinguish between
astrophysical and cosmological origin

5. Non-Gaussianity

N. Bartolo et al. (LISA CosWG), JCAP 11, 034 (2018)

$$\langle h_{\lambda_1}(t, \vec{k}_1) h_{\lambda_2}(t, \vec{k}_2) h_{\lambda_3}(t, \vec{k}_3) \rangle = \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \mathcal{B}_{\lambda_1, \lambda_2, \lambda_3}(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

- Overlapped astrophysical sources
- Sub-horizon-produced cosmological GWs
 - Gaussian (the central limit theorem)
- Inflation
- Topological defects (long strings)
 - could generate non-Gaussianity

But...

Propagation through large scale structure suppresses the non-Gaussian signal due to Shapiro time delay

N. Bartolo et al. PRL 122, 211301 (2019); PRD 99, 103521 (2019)

Summary

- There are various mechanisms to generate a **stochastic GW background in the early universe**.
- Once it is detected, it will bring a breakthrough in the understanding of our Universe
- Information beyond the amplitude of a GW background

1. Spectral shape → applicable to all origins

2. Polarization → probe of non-GR

3. Anisotropy

4. Popcorn noise

) → typically of astrophysical origin

(5. Non-Gaussianity) → inflationary models

→ **Hints for discriminating origins**