# Higgs inflation, unitarity, and scalaron

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Based mainly on 1907.00993, 2002.11739 and 2102.12501

with R. Jinno, K. Nakayama, K. Mukaida and J. van de Vis.

See also 1609.05209, 1701.07665 and 2008.01096.

# Higgs inflation

- Higgs inflation (HI): Standard Model Higgs = inflaton.

- Require non-minimal coupling to gravity $\xi$ :

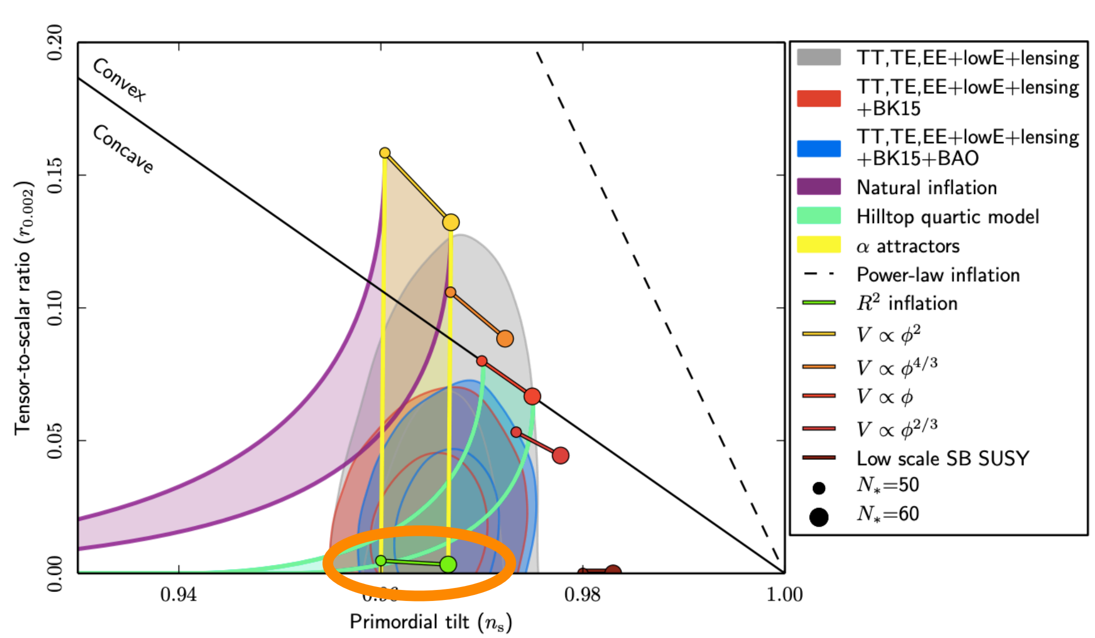
$$\mathscr{L} = \frac{M_P^2}{2}\left(1 + \frac{\xi \phi_i^2}{M_P^2}\right) R + \frac{1}{2}\left(\partial \phi_i\right)^2 - \frac{\lambda}{4}\phi_i^4,$$

where $\phi_i$ : Higgs with $i = 1,\ldots,N_s(=4)$, $R$ : Ricci scalar.

- Consistent with CMB observation for $\xi^2 \simeq 2 \times 10^9 \lambda$ .

[Bezrukov, Shaposhnikov 07; Planck 2018]

Assume $\xi \gg 1$ in this talk (true unless $\lambda$: tiny).

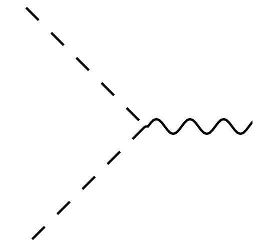


# Tree-level unitarity violation

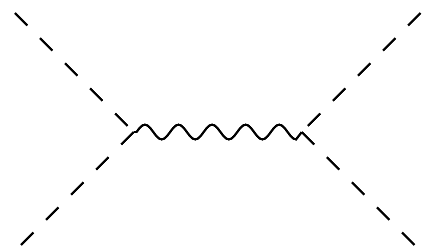
- Large $\xi$ induces tree-level unitarity violation.

[Burgess+ 09,10; Barbon+ 09; Hertzberg 10; ...]

  - Scalar-scalar-graviton vertex:

$$\xi R\phi_i^2 \ni \xi\phi_i^2\partial^2 h \sim$$  .

  - 4-point scattering amplitude $\phi_i\phi_i \to \phi_j\phi_j$ .

$$A_{\rm tree}^{(ii\to jj)} =$$  $$= \frac{1}{M_P^2}\left[\frac{(1+6\xi)^2}{6}s - \left(\frac{s}{6} - \frac{tu}{s}\right)\right].$$

Tree-level unitarity violation at $\sqrt{s} \sim \dfrac{M_P}{\xi} \ll M_P$ around $\phi_i = 0$.

- Could be fine during inflation, with finite VEV. [Bezrukov+ 10]

- Problematic after inflation, during (p)reheating epoch. [YE, Jinno, Mukaida, Nakayama 16]

Think more about unitarity of HI.

# Outline

1. Introduction
2. Unitarity violation during preheating
3. Unitarity and scalaron in large $N$
4. Summary

# Outline

1. Introduction

more cosmology-ish

2. Unitarity violation during preheating

3. Unitarity and scalaron in large $N$

more QFT-ish

4. Summary

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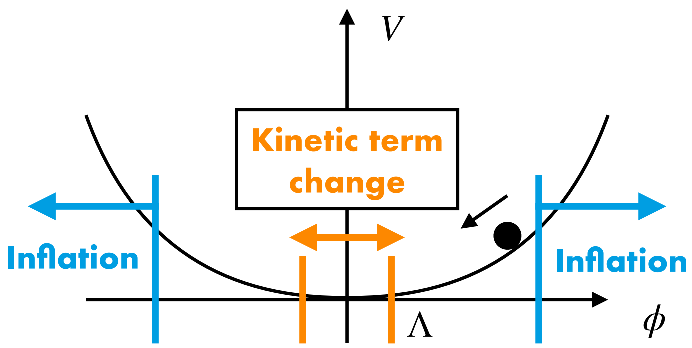
# Spiky oscillation after inflation

- Higgs fields have a non-tirivial target space in Einstein frame:

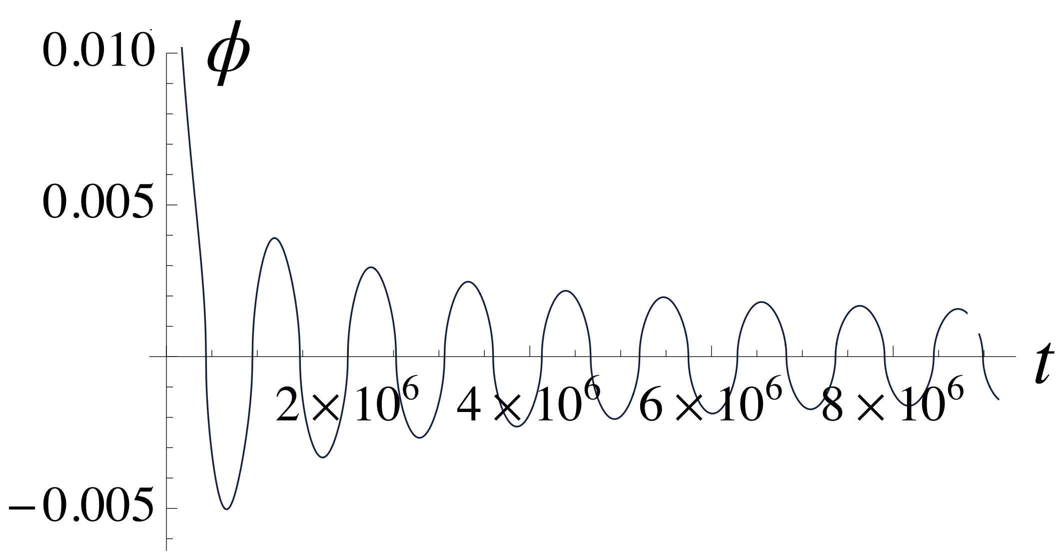
$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + \frac{1}{2}h_{ab}g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right]$$

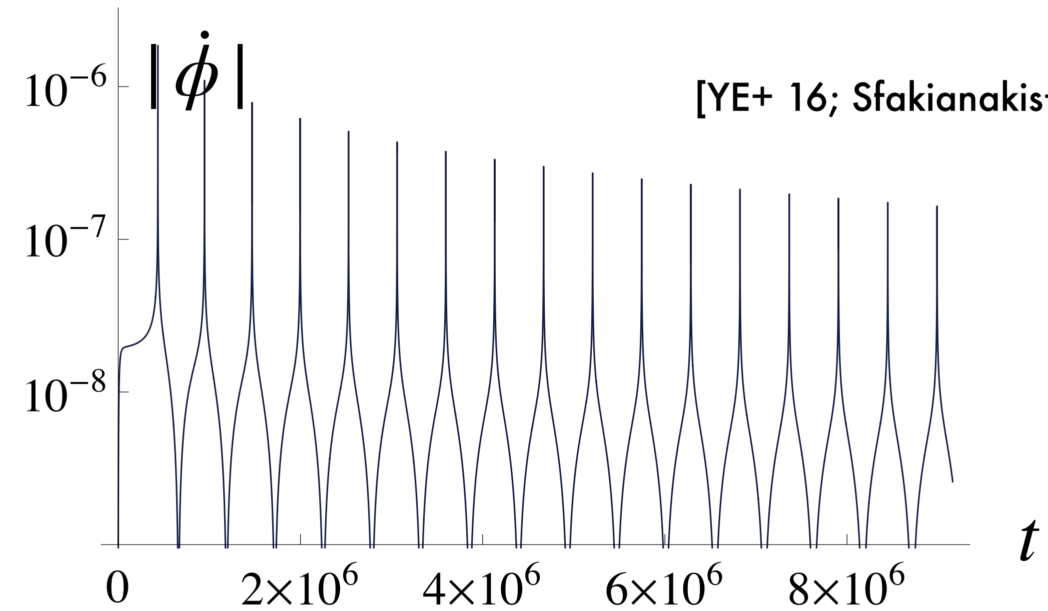
with $h_{ab} = \frac{1}{\Omega^4}\begin{pmatrix} \Omega^2 + \frac{6\xi^2\phi^2}{M_P^2} & \frac{6\xi^2\phi\chi}{M_P^2} \\ \frac{6\xi^2\phi\chi}{M_P^2} & \Omega^2 + \frac{6\xi^2\chi^2}{M_P^2} \end{pmatrix}$ with $\chi$ : NG mode(s) and $\Omega^2 = 1 + \xi\frac{\phi^2+\chi^2}{M_P^2}$ for HI.

- Kinetic term drastically changes for $|\phi| \lesssim M_P/\xi$,




➡ a "spiky" feature for $|\phi| \lesssim M_P/\xi$, causing unitarity violation.




[YE+ 16; Sfakianakis+ 18; Nguyen+ 19]

# Spiky oscillation after inflation

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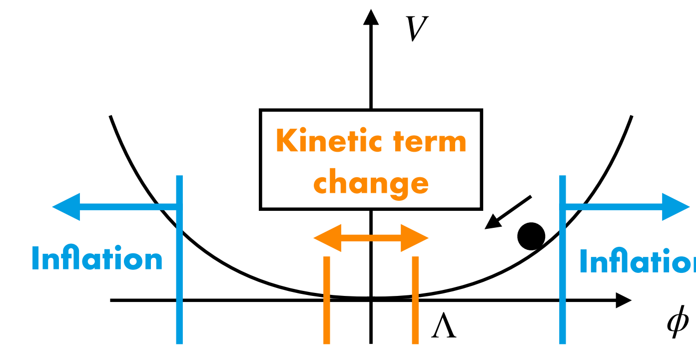


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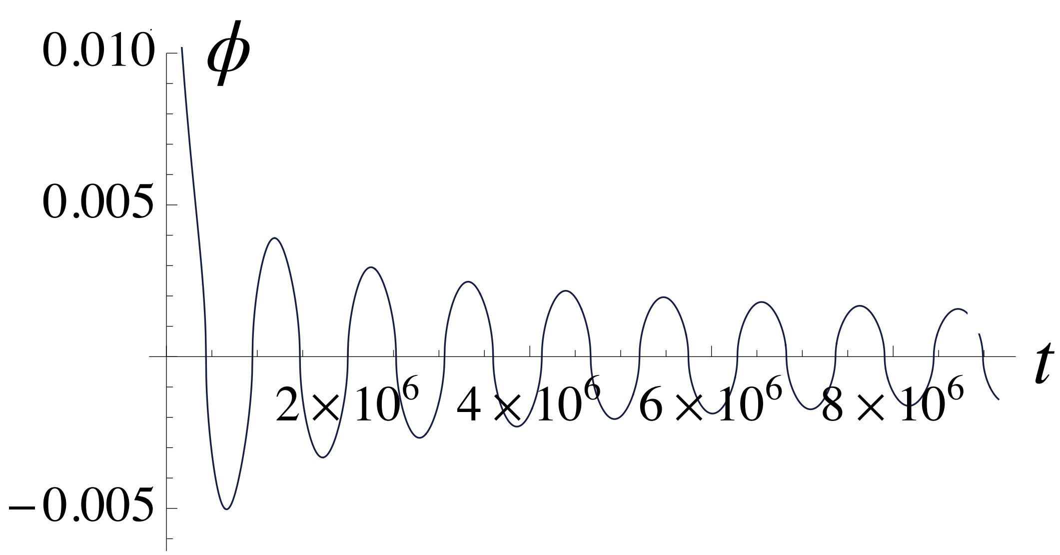
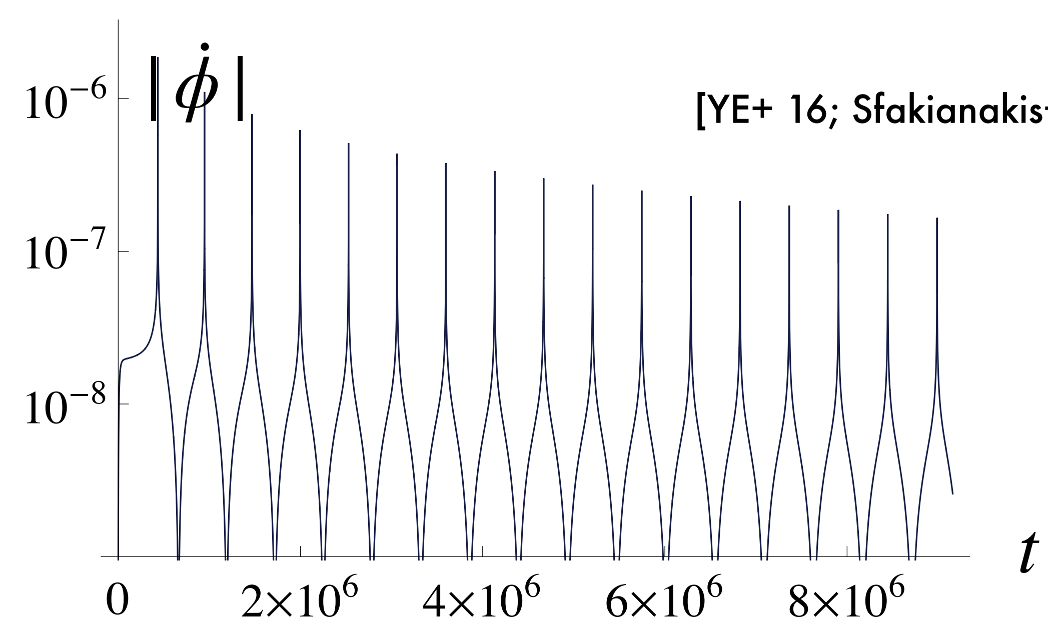



[YE+ 16; Sfakianakis+ 18; Nguyen+ 19]

# Target space and unitarity

An easy-to-use condition of unitarity violation from target space

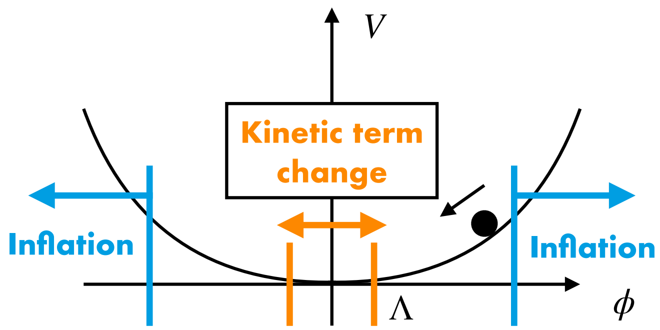
[YE, Jinno, Nakayama, van de Vis 21]

- NG boson has mass from target space curvature → feels spikes:

$$m_\chi^2 = \nabla^\chi V_\chi - \dot{\phi}^2 R^\chi{}_{\phi\phi\chi}\,, \qquad \text{e.g.} \left(1+\frac{\chi^2}{\Lambda^2}\right)(\partial\phi)^2 \to m_\chi^2 = -\frac{\dot{\phi}^2}{\Lambda^2}.$$

- Inflaton motion changes for $|\phi| \lesssim \Lambda$ with curvature $R[h] \sim \Lambda^{-2}$ ($\Lambda \sim M_P/\xi$ fro HI).

➡ typical momentum scale: $k_{\rm spike} \sim (\Lambda/\dot{\phi}_{\rm origin})^{-1}$ .




- Cut-off also $\sim \Lambda$ since the curvature affects e.g. scattering amplitudes.

- With energy cons. $\dot{\phi}^2_{\rm origin} \sim V_{\rm inf}$ , unitarity violation $k_{\rm spike} \gtrsim \Lambda$ translates to

$V_{\rm inf} \gtrsim \Lambda^4$: simply compare inflation energy scale and cut-off.

e.g. $V_{\rm inf}/\Lambda^4 \sim \lambda\xi^2 \sim 10^{-9}\xi^4$ for HI → unitarity violation for $\xi \gtrsim 10^2$ .

- Applicable to other inflation models (can see e.g. running kinetic inflation violates unitarity).

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# Main question (of 1907.00993)

- Tree-level unitarity of Higgs inflation is violated at $\sim M_P/\xi$.

**BUT**

- Renormalization group equations:

* Scalaron mass: $m_s^2 = \frac{M_P^2}{12\alpha}$.

$$\beta_\alpha \equiv \frac{d\alpha}{d\ln\mu} = -\frac{N_s}{1152\pi^2}\left(1+6\xi\right)^2, \quad \mathscr{L}_{\text{c.t.}} = \alpha R^2 .$$

➡ $\alpha = \mathcal{O}(\xi^2) \gg 1$ inevitably induced, implying light scalaron.

- Scalaron lifts the cut-off scale to $M_P$.

[YE 17; Gorbunov, Tkareva 18]

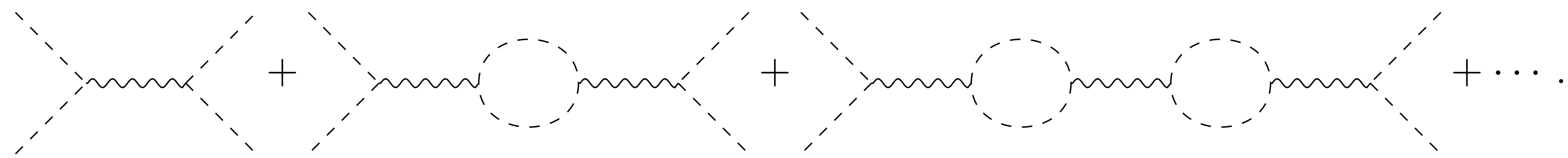
➡ **quantum correction to the rescue?**

Study quantum correction in a controllable way → **large $N_S$ limit.**

# Large $N_s$ analysis

- Take $N_s \to \infty$ with $N_s\xi^2/M_P^2$ fixed.

- Leading: vacuum polarization diagrams ("dressed amplitude")

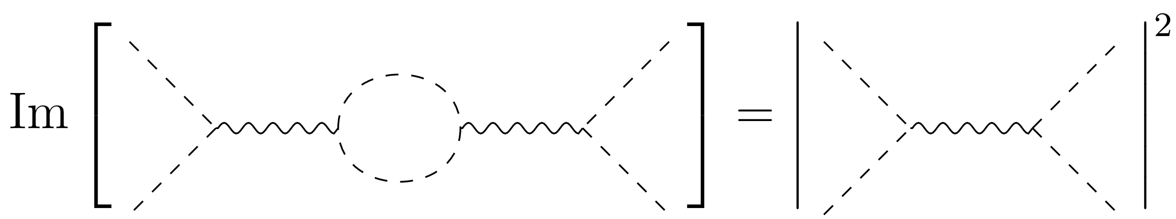
$A^{(ii\to jj)}_{\rm dressed} \equiv$ 

- Study forward scattering of flavor-singlet state: $|1\rangle \equiv \frac{1}{\sqrt{N_s}}\sum|\phi_i\phi_i\rangle\,.$

$$\Rightarrow \quad A^{(1\to 1)} = N_s A^{(ii\to jj)}_{\rm dressed} \quad \text{at the leading order.}$$

- Partial wave expansion: $A^{(1\to 1)} = 32\pi\sum a^{(l)}P_l(\cos\theta)\,.$

$$\Rightarrow \quad a^{(0)} = a^{(0)}_{\rm tree}\left[1 + a^{(0)}_{1-{\rm loop}}/a^{(0)}_{\rm tree} + \left(a^{(0)}_{1-{\rm loop}}/a^{(0)}_{\rm tree}\right)^2 + \cdots\right] = \frac{a^{(0)}_{\rm tree}}{1 - a^{(0)}_{1-{\rm loop}}/a^{(0)}_{\rm tree}}\,.$$

- 

$$\Rightarrow \quad {\rm Im}\left[a^{(0)}_{1-{\rm loop}}\right] = \left|a^{(0)}_{\rm tree}\right|^2.$$

$\Rightarrow$ ${\rm Im}\left[a^{(0)}\right] = \left|a^{(0)}\right|^2$: unitarity satisfied.

phrased "self-healing mechanicsm" in [Aydemir+12; Calmet, Casadio 13]

# New degree of freedom

Question: how to understand this "physically"?

- Dressed amplitude develops a pole.

$$A_{\text{dressed}} \sim \frac{A_{\text{tree}}}{1 - A_{1-\text{loop}}/A_{\text{tree}}}, \quad A_{1-\text{loop}}/A_{\text{tree}} \propto s \text{ (non-renormalizable).}$$

$$A_{\text{dressed}} \sim \frac{1}{1 - s/m^2}.$$

A new DOF emerges due to resummation, unitarizing the theory.

- Similar phenomena observed for other models within large $N$.

e.g. 4-fermi → pions, $O(N)$ NLSM → $\sigma$-mesons, $CP^{N-1}$ NLSM → $\rho$-meson, EWChPT→ Higgs

[Nambu, Jona-Lasinio 61] [Bardeen+ 76; Brezin+ 76] [D'Adda+ 78,79; Bando+ 85,88; ...] [Dobado+ 90, 00]

- Identified as **scalaron** in the case of Higgs inflation. [YE 19]

# Emergence of scalaron

[YE 19]

- Dressed amplitude:

$$a^{(0)} = -\frac{N_s\left(1+6\xi\right)^2 s}{2304\pi\alpha}\left[s\left(1-\frac{i\pi}{\ln(s/\Lambda_\alpha^2)}\right)-\frac{M_P^2}{12\alpha}\right]^{-1}$$

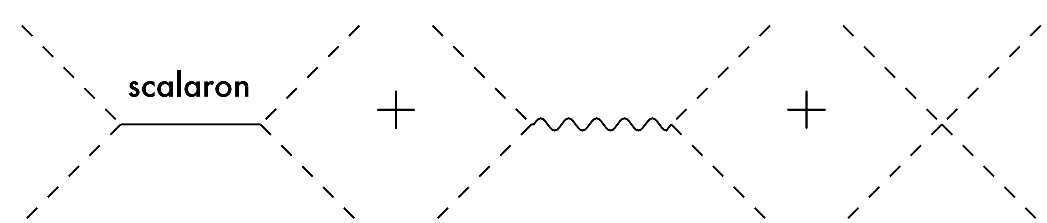
where $\alpha = -\frac{N_s\left(1+6\xi\right)^2}{2304\pi^2}\ln\left(\frac{s}{\Lambda_\alpha^2}\right)$ : coefficient of counter term,

$\Lambda_\alpha$: parameter choice of the theory, or "dimensional transmutation" (the same as $\Lambda_{\rm QCD}$).

- Consider a theory with scalaron: $\mathscr{L} = \bar{\alpha}R^2 + \xi R\phi_i^2/2$.

$$\Rightarrow \mathscr{L} = -\frac{1}{144\bar{\alpha}}\left[\frac{3M_P^2}{2}-\left(\sigma+\frac{\sqrt{6}M_P}{2}\right)-\frac{6\xi+1}{2}\phi_i^2\right]^2$$ in conformal frame with $\sigma$: scalaron.

- Compute 4-point amplitude $\phi_i\phi_i \to \phi_j\phi_j$ with the $R^2$ term.

$\bar{A}_{\rm tree} =$ 

.

- We obtain $A_{\rm dressed} = \bar{A}_{\rm tree}$ by identifying $\alpha = \bar{\alpha}$ → new DOF = scalaron.

# Correspondence

[YE, Mukaida, van de Vis 20]

| O($N$) NLSM | Higgs inflation |
| --- | --- |
| pions $\pi_i$ | Higgs fields $\phi_i$, conformal mode of metric $\Phi$ |
| target space: $\pi_i^2 + h^2 = v^2, \quad \left(\pi_i, h\right) \in \mathbb{R}^{(N+1)}$ | target space: $\frac{6\xi + 1}{2}\phi_i^2 + \left(h + \frac{\Phi}{2}\right)^2 = \frac{\Phi^2}{4}, \quad \left(\Phi, \phi_i, h\right) \in \mathbb{R}^{(1,N+1)}$ |
| sigma meson $\sigma$ | scalaron $\sigma$ |

* Higgs-$R^2$ model as LSM: useful also to compute RGE.

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- Scalaron lifts the cut-off scale to $M_P$.

[YE 17; Gorbunov, Tkareva 18]

**quantum correction to the rescue?**

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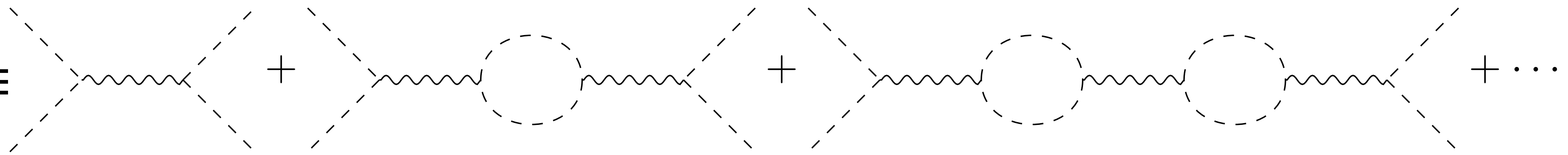
# Summary

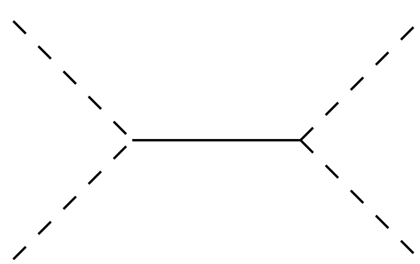
- Higgs inflation "self-heals" unitarity at large $N$.

$A_{\text{dressed}} \equiv$



- Interpreted as emergence of scalaron.

$A_{\text{dressed}} \sim$



$$= -\frac{\left(1+6\xi\right)^2}{6}\frac{m_s^2}{M_P^2}\frac{s}{s-m_s^2}, \quad m_s^2 = \frac{M_P^2}{12\alpha_1}.$$

1. Higgs inflation is actually a "Higgs-$R^2$" model (at least in large $N$).
2. (P)reheating can be studied without unitarity issue.

Scalaron smears spiky features, see [He+ 18; Bezrukov+ 19; ...].

# Back up

# Nonminimal coupling

- Interaction between Higgs and the metric around flat spacetime:

$$\mathscr{L}_{\rm int} = \frac{\delta g_{\mu\nu}}{2} T^{\mu\nu} + \mathcal{O}(\delta g^2),$$

$$T_{\mu\nu} = \partial_\mu \phi_i \partial_\nu \phi_i - \eta_{\mu\nu} \left( \frac{1}{2} (\partial \phi_i)^2 - \frac{\lambda}{4} \phi_i^4 \right) + \xi \left( \partial_\mu \partial_\nu - \eta_{\mu\nu} \Box \right) \phi_i^2 .$$

- Decompose the metric (after gauge fixing) as

$$\delta g_{\mu\nu} = 2\varphi \eta_{\mu\nu} + h^\perp_{\mu\nu}, \quad h^{\perp\mu}{}_\mu = 0, \ \ \partial^\mu h^\perp_{\mu\nu} = 0.$$

$$\Rightarrow \quad \mathscr{L}_{\rm int} = -3\xi\varphi \Box \phi_i^2 + \left( \xi \text{ indepenent terms} \right) .$$

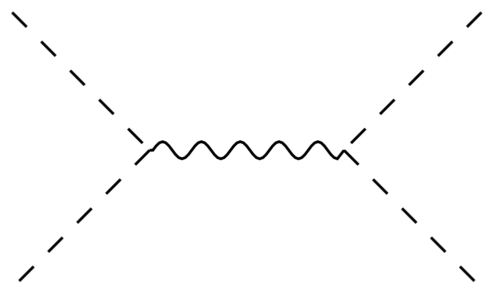
$\xi$ controls interaction between Higgs and $\varphi$ (conformal mode).

- As a result, $\xi$ affects the spin-0 part of the theory.

# Tree-level unitarity violation

- Four-point scattering amplitude $\phi_i\phi_i \to \phi_j\phi_j$ :

$$A_{\rm tree}^{(ii\to jj)} =$$



$$= \frac{1}{M_P^2}\left[\frac{\left(1+6\xi\right)^2}{6}s - \left(\frac{s}{6} - \frac{tu}{s}\right)\right].$$

$\delta g_{\mu\nu} = 2\varphi\eta_{\mu\nu} + h^{\perp}_{\mu\nu}$

- Amplitude exceeds unity at $\sqrt{s} \sim M_P/\xi$ .

  ➡ Tree-level unitarity violation at $M_P/\xi \ll M_P$ around $\phi_i = 0$.

[Burgess+ 09,10; Barbon+ 09; Hertzberg 10; ...]

- Could be fine during inflation (with finite VEV), [Bezrukov+ 10]

- But problematic after inflation, during preheating epoch.

[YE, Jinno, Mukaida, Nakayama 16]

  ➡ Think more about unitarity issue of HI.

# Violent particle production

- Spike mass scale: $m_{\rm sp} \equiv \Delta t_{\rm sp}^{-1} \sim \sqrt{\lambda} M_P$ : well above $M_P/\xi$ .

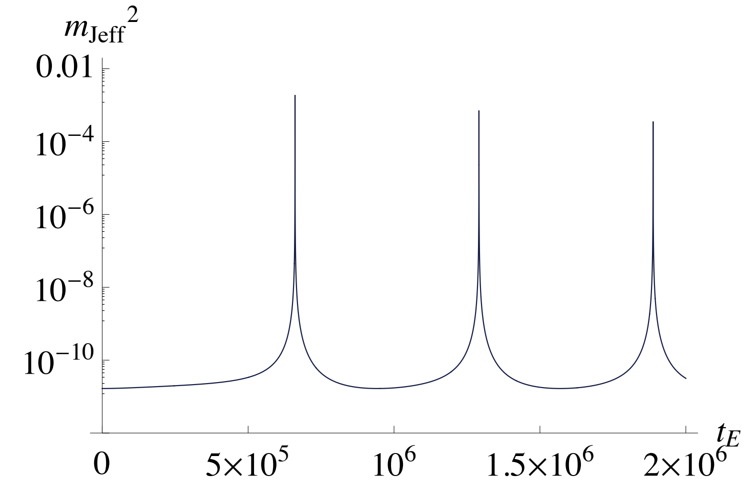
  Dangerous for unitarity.

- Goldstone boson (or longitudinal gauge boson) couples to spike.

  e.g. a complex scalar: $\phi = \phi_r e^{i\theta}/\sqrt{2}$.

  Action for $\theta$ after canonical normalization: $\mathscr{L}_{\rm NG} = \frac{1}{2}(\partial\theta)^2 - \frac{m_\theta^2}{2}\theta^2$,

$$m_\theta^2 = \frac{m_{\rm eff}^2}{\Omega^2} - \frac{\Omega}{d^2(1/\Omega)} dt^2, \quad \Omega^2 = 1 + \xi\phi_r^2/M_P^2 .$$



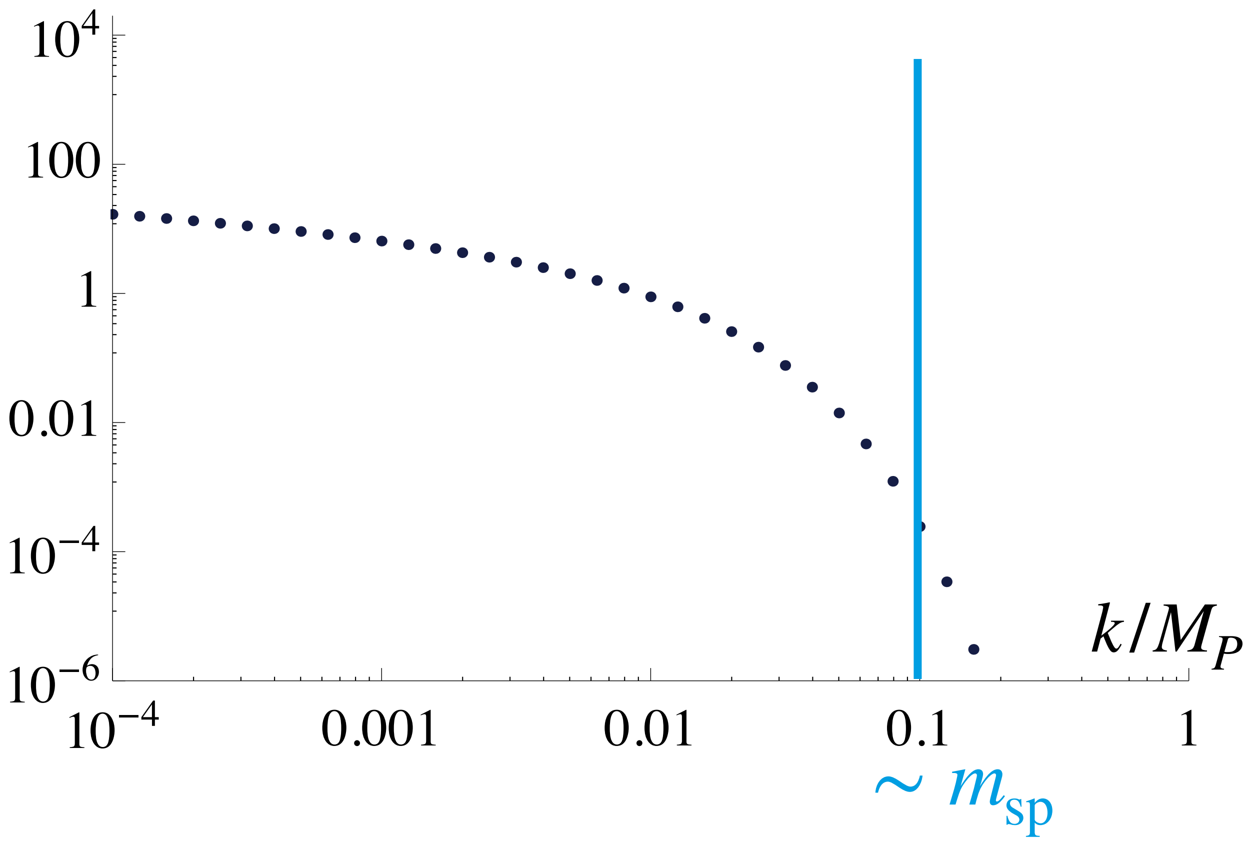

* $m_\theta^2 \neq 0$ due to $\dot{\phi}_r \neq 0$.

- Goldstone boson with $k \sim \sqrt{\lambda} M_P$ : efficiently produced.

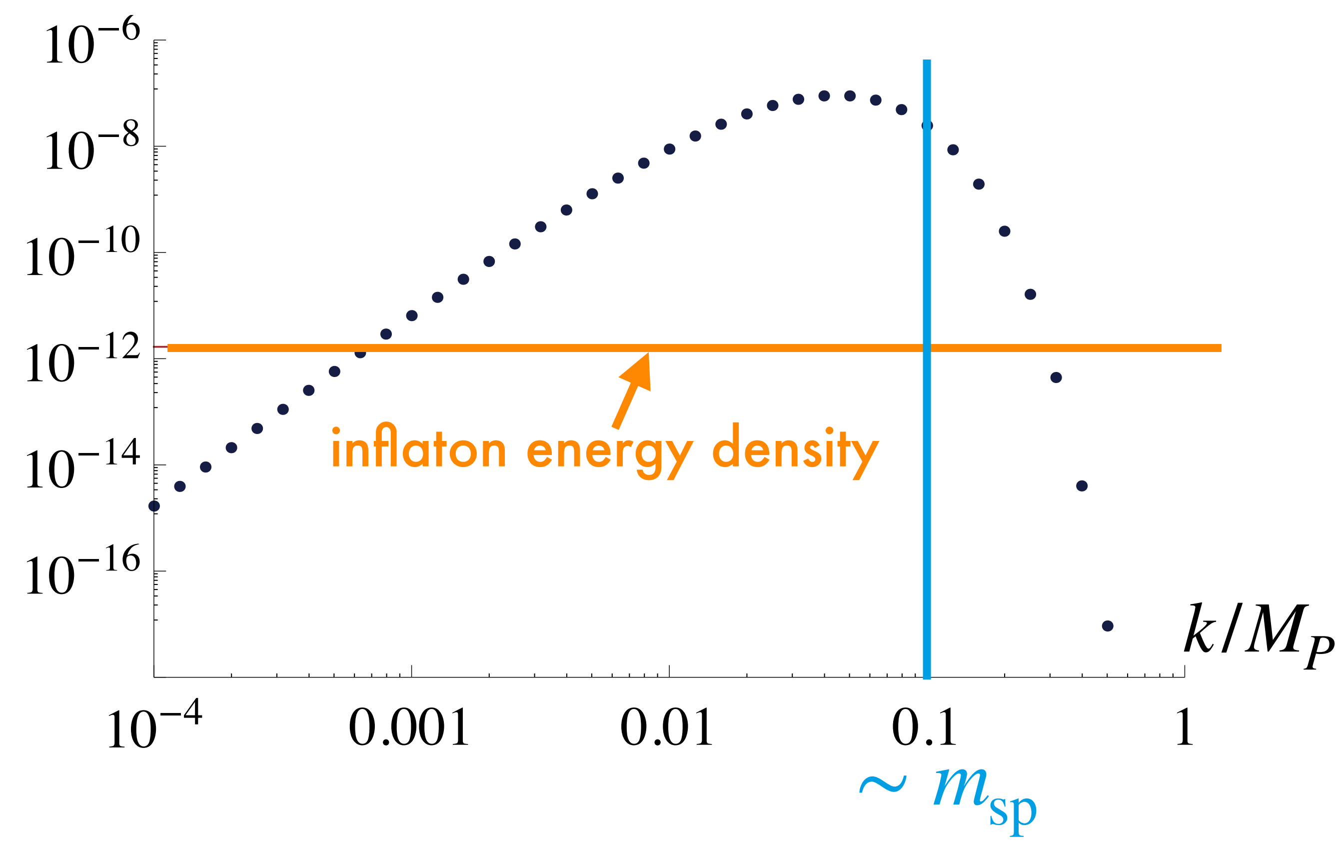
# Violent particle production

Numerical results for $\lambda = 10^{-2}$:

$f_\theta$ (occupation number)




$k^3\omega_k f_\theta$ (energy density)




[YE, Jinno, Mukaida, Nakayama 16]

- Particles with $k \sim m_{\rm sp}$ are efficiently produced.
- Longitudinal mode of gauge boson plays the same role for the gauged case.

* Transverse/longitudinal modes obey different EoMs for $\dot{\phi} \neq 0$.

# Action for vector boson

- Consider the following gauge boson action:

$$S_A = \int d\tau d^3x \left[ -\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{m_A^2}{2}\eta^{\mu\nu}A_\mu A_\nu \right].$$

- Decomposed in the unitary gauge (no Goldstone bosons) as

$$S_A = \int \frac{d\tau d^3k}{2\,(2\pi)^3} \left[ \left(k^2 + m_A^2\right) \left| A_0 + \frac{i\vec{k}\cdot\vec{A}'}{k^2+m_A^2} \right|^2 + \left|\vec{A}'\right|^2 - \left|\vec{k}\times\vec{A}\right|^2 - \frac{\left|\vec{k}\cdot\vec{A}'\right|^2}{k^2+m_A^2} - m_A^2\left|\vec{A}\right|^2 \right].$$

- After integrate out the auxiliary field $A_0$, it becomes

$$S_A = \frac{1}{2}\int \frac{d\tau d^3k}{(2\pi)^3} \left[ \left|\vec{A}'\right|^2 - \left|\vec{k}\times\vec{A}\right|^2 - \frac{\left|\vec{k}\cdot\vec{A}'\right|^2}{k^2+m_A^2} - m_A^2\left|\vec{A}\right|^2 \right].$$

# Mode decomposition

- In the previous slide, we obtain

$$S_A = \frac{1}{2}\int \frac{d\tau d^3k}{(2\pi)^3}\left[\left|\vec{A}'\right|^2 - \left|\vec{k}\times\vec{A}\right|^2 - \frac{\left|\vec{k}\cdot\vec{A}'\right|^2}{k^2+m_A^2} - m_A^2\left|\vec{A}\right|^2\right].$$

- Decompose into longitudinal/transverse modes:

$$\vec{A} = \vec{A}_T + \frac{\vec{k}}{k}\tilde{A}_L, \quad \vec{k}\cdot\vec{A}_T = 0, \quad A_L \equiv \frac{m_A}{\sqrt{k^2+m_A^2}}\tilde{A}_L\,.$$

- In terms of them, the action becomes

$$S_A = S_{A_T} + S_{A_L}, \quad S_{A_T} = \frac{1}{2}\int \frac{d\tau d^3k}{(2\pi)^3}\left[\left|\vec{A}'_T\right|^2 - \left(k^2+m_A^2\right)\left|\vec{A}_T\right|^2\right],$$

$$S_{A_L} = \frac{1}{2}\int \frac{d\tau d^3k}{(2\pi)^3}\left[\left|A'_L\right|^2 - \left(k^2+m_A^2 - \frac{k^2}{k^2+m_A^2}\left(\frac{m''_A}{m_A} - \frac{3m_A'^2}{k^2+m_A^2}\right)\right)\left|A_L\right|^2\right].$$

EoMs for longitudinal/transverse modes different
for time-dependent symmetry breaking field.

# Spin-2 sector

- Vacuum pol. diagrams contain divergences.

  ➡ Renormalized by $\mathscr{L}_{\text{c.t.}} = \alpha R^2 + \alpha_2 \left( R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right)$.

  * We have no choice but including these terms.

- Renormalization group equations:

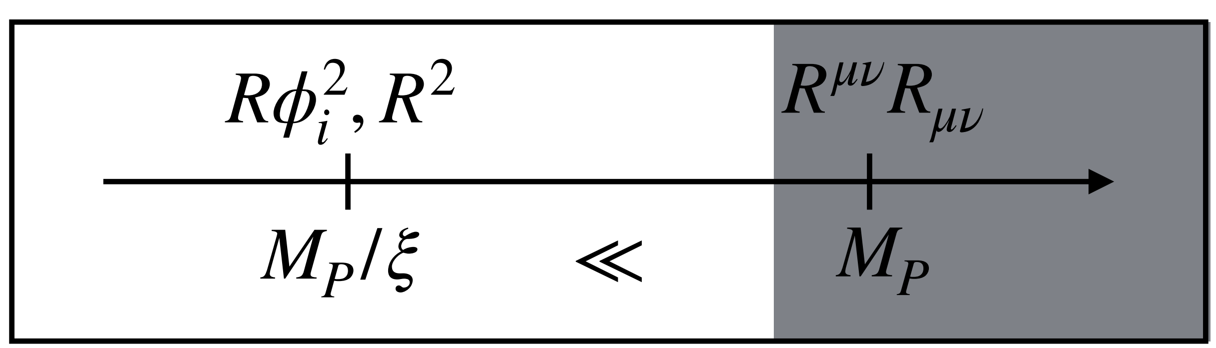
$$\beta_\alpha \equiv \frac{d\alpha}{d \ln \mu} = - \frac{N_s}{1152\pi^2} \left(1 + 6\xi\right)^2, \quad \beta_{\alpha_2} \equiv \frac{d\alpha_2}{d \ln \mu} = - \frac{N_s}{960\pi^2} .$$

The hierarchy $\alpha \sim \mathcal{O}(\xi^2) \gg \alpha_2 \sim \mathcal{O}(1)$ naturally exits.

- Alternatively, the coupling for the spin-2 is suppressed:

$$T_{\mu\nu} \ni \xi \left( \partial_\mu \partial_\nu - \eta_{\mu\nu} \Box \right) \phi_i^2$$

➡ $h^{\perp}_{\mu\nu} T^{\mu\nu}$: independent of $\xi$.



# Motivations (of 2002.11739)

- A frame-independent way to see emergence of scalaron?

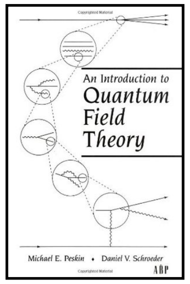
$\beta_\alpha$ depends on the nonminimal coupling $\xi$.

$\rightarrow$ No large enhancement of $R^2$, e.g., in the Einstein frame.

How to understand emergence of scalaron in other frames?

- Correspondence to the ordinary $\mathrm{O}(N)$ NLSM analysis?

The analysis so far is quite similar to the $\mathrm{O}(N)$ NLSM.

e.g. Large $N$ analysis on phase diagram in Ch.13 of 


* Symmetric phase = appearance of $\sigma$-meson = target space flattened = UV completion

Can elaborate this correspondence more?

# Frame-independent target space

[YE, Mukaida, van de Vis 20]

- Naive definition solely by scalar fields is frame-dependent.

$$\begin{cases} \mathscr{L}_J = \frac{M_P^2}{2}\Omega^2 R + \frac{1}{2}\left(\partial\phi_i\right)^2 + \cdots, \quad \Omega^2 = 1 + \frac{\xi\phi_i^2}{M_P^2}, \\ \mathscr{L}_E = \frac{M_P^2}{2} R + \frac{1}{2\Omega^4}\left(\Omega^2\delta_{ij} + \frac{6\xi^2\phi_i\phi_j}{M_P^2}\right)\partial\phi_i\partial\phi_j + \cdots. \end{cases}$$

Physics is frame-independent → a frame-independent definition is desirable.

- Frame-independent definition by including the conformal mode.

Metric decomposition: $g_{\mu\nu} = e^{2\varphi}\tilde{g}_{\mu\nu}, \quad \mathrm{Det}\left[\tilde{g}_{\mu\nu}\right] = -1.$

$\Phi = \sqrt{6}M_P e^{\varphi}$: conformal mode.

➡ Target space defined by $\left(\phi_i, \Phi\right)$: frame-independent!

∵ Weyl transformation = redefinition of $\Phi$ = coordinate transf. of target space.

# Higgs inflation as NLSM

[YE, Mukaida, van de Vis 20]

- Focus on the conformal mode of the metric as $g_{\mu\nu} = e^{2\varphi}\eta_{\mu\nu}$ .

$$S = \int d^4x \left[ -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{6\xi+1}{2}\left(\frac{\Box\Phi}{\Phi}\right)\phi_i^2 - \frac{\lambda}{4}\phi_i^4 \right].$$

- Can be simplified by field redefinitions as

$$S = \int d^4x \left[ -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{1}{2}\left(\partial h\right)^2 - \frac{\lambda}{4}\phi_i^4 \right],$$

where $h\left(\Phi, \phi_i\right) = \frac{1}{2}\left[\sqrt{\Phi^2 - 2\left(6\xi+1\right)\phi_i^2} - \Phi\right].$

Interpreted frame-independently as NLSM.

- $\Phi$ is ghost-like but harmless.

* Similar to $A_0$ of $\mathrm{U}(1)$ gauge boson in the Lorentz gauge $\partial_\mu A^\mu = 0$:

$$\mathscr{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} = -\frac{1}{2}\eta^{\alpha\beta}\partial^\mu A_\alpha \partial_\mu A_\beta = -\frac{1}{2}\left(\partial A_0\right)^2 + \frac{1}{2}\left(\partial A_i\right)^2.$$

# Scalaron as $\sigma$-meson

- Higgs inflation as NLSM: [YE, Mukaida, van de Vis 20]

$$\mathscr{L}_{\rm NLSM} = -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{1}{2}\left(\partial h\right)^2 - \frac{\lambda}{4}\phi_i^4, \quad h = \frac{1}{2}\left[\sqrt{\Phi^2 - 2\left(6\xi+1\right)\phi_i^2} - \Phi\right].$$

Naturally imply $\sigma$-meson that linearizes the NLSM:

$$\mathscr{L}_{\rm LSM} = -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{1}{2}(\partial\sigma)^2 - \frac{1}{144\alpha}\left[\frac{\Phi^2}{4} - \left(\sigma + \frac{\Phi}{2}\right)^2 - \frac{6\xi+1}{2}\phi_i^2\right]^2 - \frac{\lambda}{4}\phi_i^4.$$

- It is identified as the scalaron:

$$\mathscr{L} = \frac{M_P^2}{2}\left(1 + \frac{\xi\phi_i^2}{M_P^2}\right)R + \alpha R^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 - \frac{\lambda}{4}\phi_i^4.$$

Auxiliary field introduction + Weyl transformation

$$\mathscr{L} = \frac{M_P^2}{2}\left(1 - \frac{\sigma^2}{6M_P^2} - \frac{\phi_i^2}{6M_P^2}\right)R + \frac{1}{2}\left[\left(\partial\phi_i\right)^2 + (\partial\sigma)^2\right] - \frac{\lambda}{4}\phi_i^4 - \frac{1}{144\alpha}\left[\frac{3M_P^2}{2} - \left(\sigma + \sqrt{\frac{3}{2}}M_P\right)^2 - \frac{6\xi+1}{2}\phi_i^2\right]^2.$$

Flat kinetic term in the conformal frame!

$g_{\mu\nu} = e^{2\varphi}\eta_{\mu\nu}$ + rescaling fields

$$\mathscr{L} = -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{1}{2}(\partial\sigma)^2 - \frac{1}{144\alpha}\left[\frac{\Phi^2}{4} - \left(\sigma + \frac{\Phi}{2}\right)^2 - \frac{6\xi+1}{2}\phi_i^2\right]^2 - \frac{\lambda}{4}\phi_i^4.$$

* Remember this identification is frame-independent.

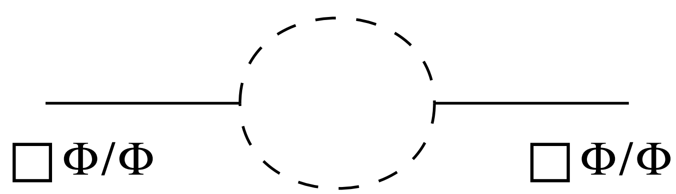
# Large $N_s$ analysis

[YE, Mukaida, van de Vis 20]

- Large $N_s$ can also be done with $\Phi$ and $\phi_i$.

$$\mathscr{L}_{\rm cl} = -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{6\xi+1}{2}\left(\frac{\Box\Phi}{\Phi}\right)\phi_i^2 - \frac{\lambda}{4}\phi_i^4 .$$

➡ leading order term:




$$: \mathscr{L}_{\rm c.t.} = 36\alpha\left(\frac{\Box\Phi}{\Phi}\right)^2 .$$

- Higher derivative term = an additional degree of freedom.

- This additional degree of freedom linearizes the target space = scalaron.

- Of course not a unique UV completion, but large $N_s$ limit picks up one among others.

# Renormalizability of LSM

[YE, Mukaida, van de Vis 20]

- The LSM with the Higgs mass and the cosmological constant is renormalizable.

(= renormalizability of (spin-0 part of) quadratic gravity)

➡ One can compute the RGEs without any ambiguity!

1-loop:

$$\beta_{g_1}^{(1)} = \frac{41}{10} g_1^3, \quad \beta_{g_2}^{(1)} = -\frac{19}{6} g_2^3, \quad \beta_{g_3}^{(1)} = -7 g_3^3,$$

$$\beta_{y_t}^{(1)} = y_t \left[ \frac{9 y_t^2}{2} - \frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2 \right],$$

$$\beta_{\lambda}^{(1)} = \left(8\bar{\xi}^2 - 8\bar{\xi} + 2\right)\bar{\xi}^2 \lambda_\alpha^2 + 24\bar{\xi}^2 \lambda \lambda_\alpha + 24\lambda^2 - 6 y_t^4 + \frac{27 g_1^4}{200} + \frac{9 g_2^4}{8} + \frac{9}{20} g_1^2 g_2^2 + \left[ 12 y_t^2 - \frac{9 g_1^2}{5} - 9 g_2^2 \right] \lambda,$$

$$\beta_{\lambda_m}^{(1)} = 2\bar{\xi}\left(2\bar{\xi} - 1\right)\lambda_\alpha^2 - 8\bar{\xi}\lambda_m^2 + \lambda_m \left[ 4\bar{\xi}^2 \lambda_\alpha + 8\bar{\xi}\lambda_\alpha - 3\lambda_\alpha + 12\lambda + 6 y_t^2 - \frac{9 g_1^2}{10} - \frac{9 g_2^2}{2} \right],$$

$$\beta_{\bar{\xi}}^{(1)} = \bar{\xi} \left[ \left(4\bar{\xi}^2 + 4\bar{\xi} - 3\right)\lambda_\alpha + 12\lambda + 6 y_t^2 - \frac{9}{10} g_1^2 - \frac{9}{2} g_2^2 \right],$$

$$\beta_{\lambda_\alpha}^{(1)} = \left(8\bar{\xi}^2 + 5\right)\lambda_\alpha^2,$$

$$\beta_{\lambda_\Lambda}^{(1)} = \frac{\lambda_\alpha^2}{2} - 2\lambda_\alpha \lambda_\Lambda - 16\bar{\xi}\lambda_\Lambda \lambda_m + 2\lambda_m^2.$$

* See 2008.01096 for an explicit form up to 2-loop.

- The Higgs mass and the CC are naturally at the scalaron mass scale = hierarchy problem.

➡ They do not affect inflationary dynamics, but (p)reheating??

- EW scale parameters can be related to inflationary scale parameters (with $\xi$ and $\alpha$).

# Higgs-$R^2$ inflation

- Higgs-$R^2$ inflation: $\mathscr{L} = \xi R \left|H\right|^2 + \alpha_1 R^2 - \lambda \left|H\right|^4$.

$$U(\phi) = \frac{M_P^4}{4}\frac{1}{\xi^2/\lambda + 4\alpha_1}\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}\right)\right]^2$$

for $\xi, \alpha_1 \gg 1,\ \lambda > 0$, where $\phi$: canonical inflaton.

[YE 17; He+ 18; ...]

- It is consistent with CMB normalization for

$$\frac{\xi^2}{\lambda} + 4\alpha_1 \simeq 2 \times 10^9\,.$$

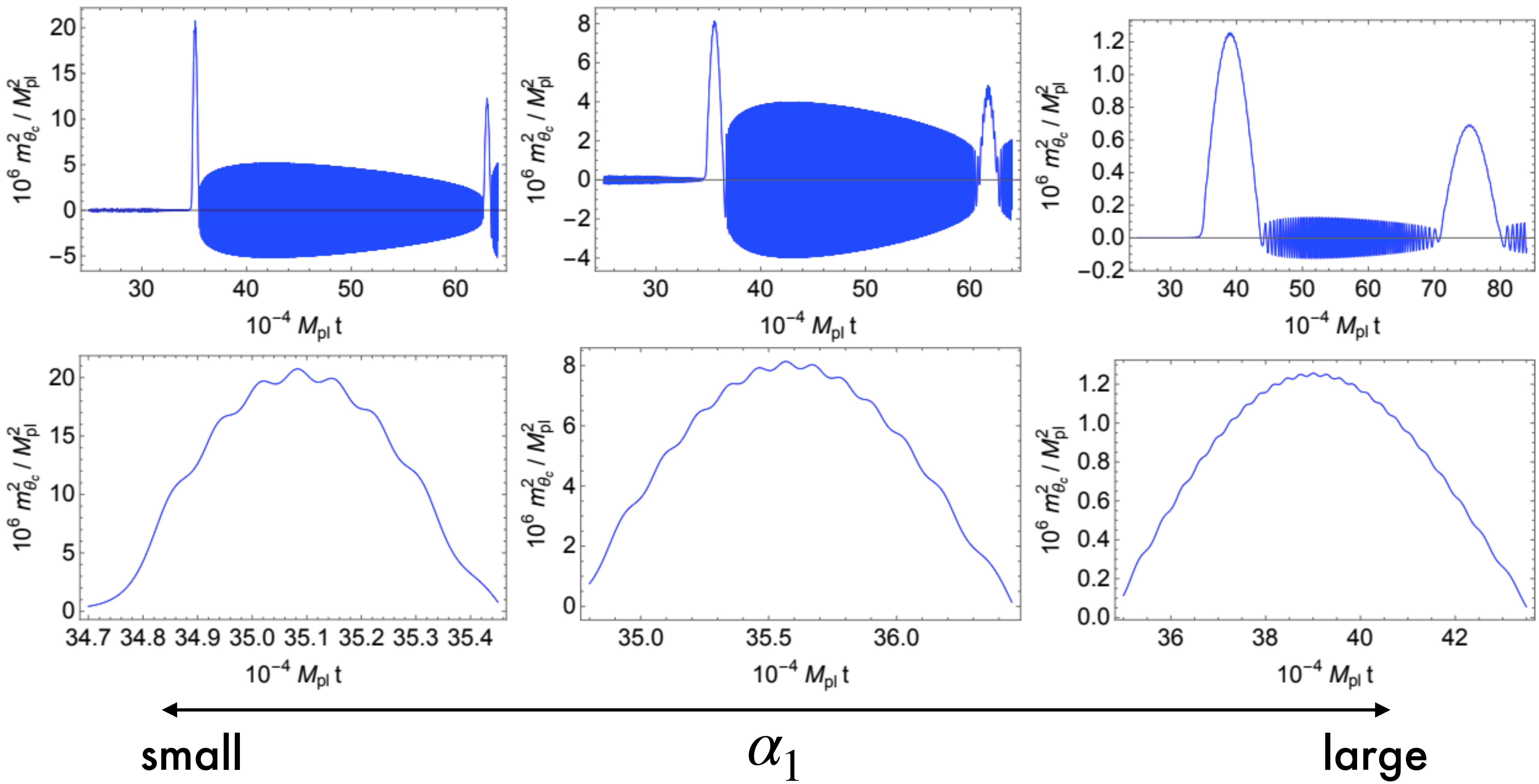
- Inflationary parameters are

$$n_s \simeq 1 - \frac{2}{N_e},\quad r \simeq \frac{12}{N_e^2}\quad \text{with}\quad N_e = [50,\ 60]\,.$$

# Scalaron heals spikes

- Scalaron smears spiky features:

[He+ 18; Bezrukov+ 19]



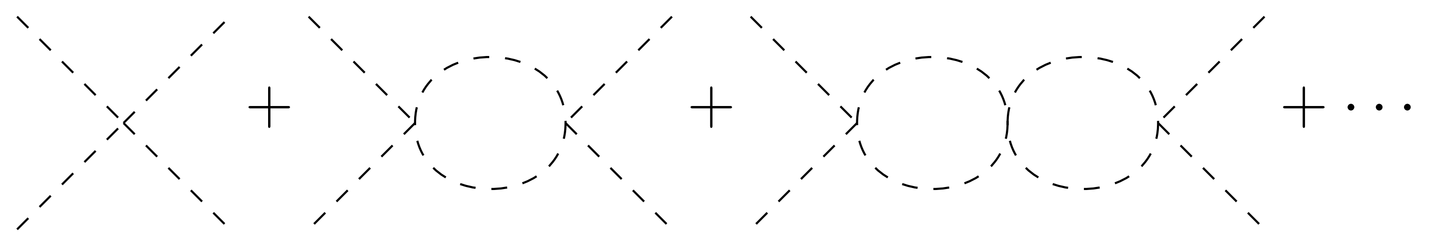
# Other inflation models

- In the Einstein frame, the non-minimal coupling is

$$\mathscr{L}_{\rm kin} = \frac{1}{2\left(1+\xi\phi_i^2/M_P^2\right)^2}\left[1+\frac{\xi\left(1+6\xi\right)\phi_j^2}{M_P^2}\right]\left(\partial\phi_k\right)^2 = \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{\xi\left(-1+6\xi\right)\phi_j^2}{M_P^2}\left(\partial\phi_i\right)^2 + \cdots .$$

Self-healing in the Einstein frame:

$A_{\rm dressed} =$




- Can be generalized to multi-field inflation with non-trivial kinetic terms.

e.g. running kinetic inflation: $\mathscr{L}_{\rm kin} = \left(1+\frac{\phi_j^2}{M^2}\right)\left(\partial\phi_i\right)^2,$ [Nakayama, Takahashi 10]

$\alpha$-attractor inflation: $\mathscr{L}_{\rm kin} = \frac{\left(\partial\phi_i\right)^2}{\left(1-\phi_j^2/6\alpha\right)^2}.$ [Kallosh, Linde 15; ...]