Modular symmetry in magnetized orbifold models

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Introduction : Flavor Symmetry

4D Standard Model
Mysteries of Flavor Structure

- Origin of 3 generation
- Origin of mass hierarchy
- Origin of flavor mixing

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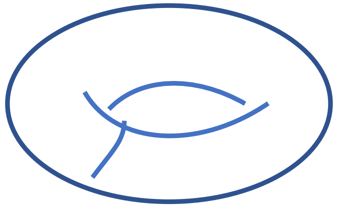
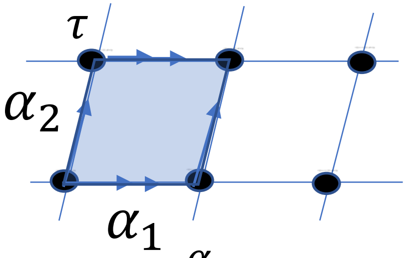
Flavor Symmetry
Non-Abelian Discrete Groups
(e.g.) $S_4(\simeq \Delta(24))$

Introduction : Modular Flavor Symmetry

+ Compact space
(e.g.) $T^2 \simeq \boldsymbol{C}/\Lambda$





Complex Structure Modulus: $\tau = \frac{\alpha_2}{\alpha_1}$

Geometrical Structure
(e.g.) Modular symmetry

$$\Psi(X_D) = \psi^i(x_4) \otimes \psi'_i(y_d)$$

$$\int d^D X\, g\Psi(X)\Psi(X)\Psi(X) = \int d^4x\, Y_{ijk}(\tau)\psi^i(x)\psi^j(x)\psi^k(x)$$

$$= \int d^4x \left(g \int d^d y\, \psi'_i(y)\psi'_j(y)\psi'_k(y)\right) \psi^i(x)\psi^j(x)\psi^k(x) =$$

4D Standard Model
Mysteries of Flavor Structure

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Modular Flavor Symmetry
Finite Modular Subgroups
Γ_N (e.g.) $\Gamma_4 \cong S_4 (\simeq \Delta(24))$

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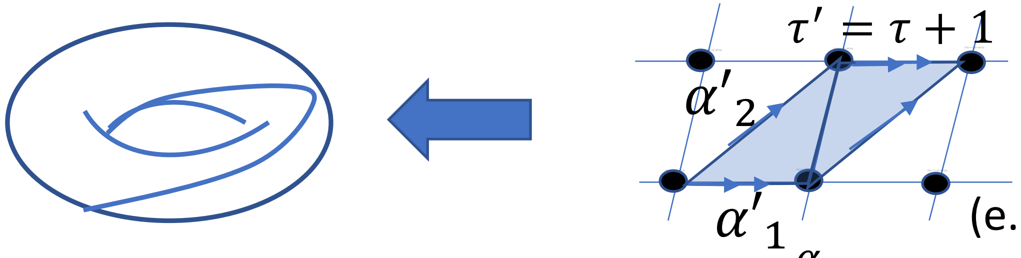

(e.g.) $\gamma: \tau \to \tau + 1$

Complex Structure Modulus: $\tau = \frac{\alpha_2}{\alpha_1}$

Modular Forms of Weight k ($\in 2Z$): $Y(\tau)$

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Modular Transformation: $\gamma: \tau \to \frac{a\tau+b}{c\tau+d}$ $\quad ad - bc = 1$ $(a, b, c, d \in Z)$

$$\gamma: Y(\tau) \to Y\left(\frac{a\tau+b}{c\tau+d}\right) = (c\tau+d)^k \rho(\gamma) Y(\tau)$$

$$\gamma: \psi_I(x_4) \to (c\tau+d)^{-k_I} \rho(\gamma) \psi_I(x_4) \qquad (k = \textstyle\sum_I k_I)$$

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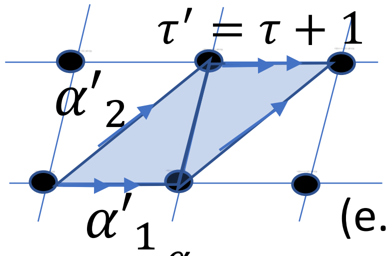

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(double covering group of Γ_N)

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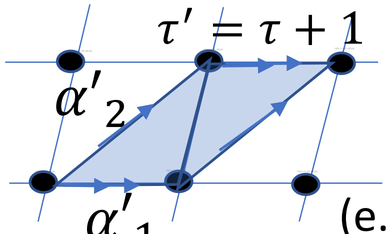

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Finite Modular Subgroups
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(double covering group of Γ_N)

ρ: representation of

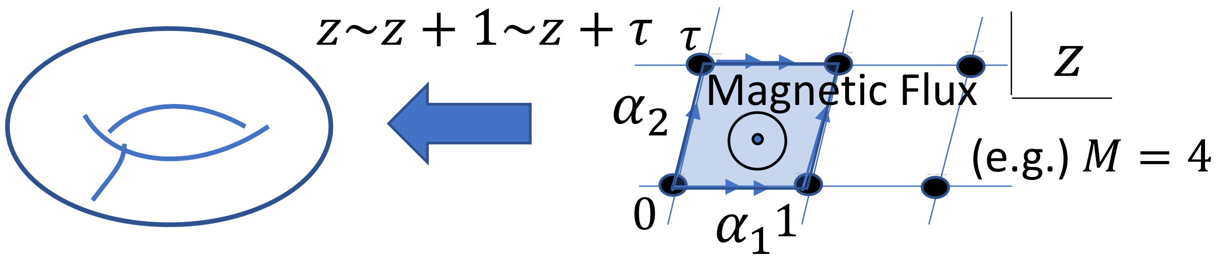
Assumption:
Flavor group, Representation, Weight
How can they be determined?

Introduction : Magnetized T^2 model

+ Compact space
(e.g.) $T^2 \simeq \boldsymbol{C}/\Lambda$

with Magnetic Flux: $(2\pi)^{-1}\int_{T^2} F = M \in Z$

Geometrical Structure
(e.g.) Modular symmetry



(Coordinate,Modulus) : (z, τ)

4D Standard Model
Mysteries of Flavor Structure

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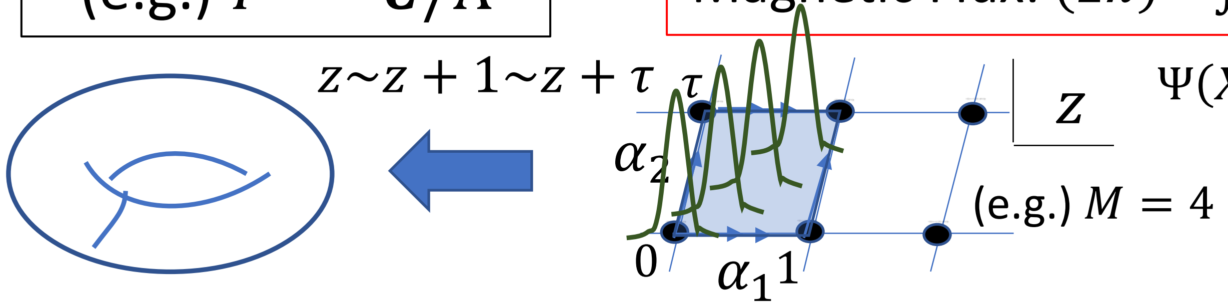


$\Psi(X_D) = \psi^i(x_4) \otimes \psi'_i(z,\tau)$

(Coordinate,Modulus) : (z,τ)

Wavefunctions : $\psi(z,\tau)$

4D Standard Model
Mysteries of Flavor Structure

- Origin of 3 generation

(e.g.)
M generation chiral fermions on magnetized T^2

Assumption:
Flavor group, Representation, Weight
How can they be determined?

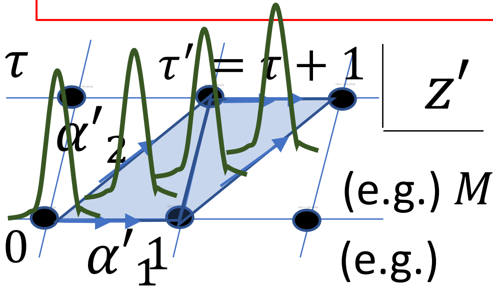
Modular Flavor Symmetry in Magnetized T^2 model

+ Compact space
(e.g.) $T^2 \simeq \boldsymbol{C}/\Lambda$

with Magnetic Flux: $(2\pi)^{-1}\int_{T^2} F = M \in 2Z$

Geometrical Structure
(e.g.) Modular symmetry

$z \sim z+1 \sim z+\tau$



(e.g.) $M = 4$

(e.g.)

(Coordinate,Modulus) : (z, τ) $\quad \gamma: (z,\tau) \to (z, \tau+1)$

Wavefunctions : $\psi(z,\tau)$

$\Psi(X_D) = \psi^i(x_4) \otimes \psi'_i(z,\tau)$ [2] K. Hoshiya, et al., PTEP 2021 (2021) 3, 033B05

$\int d^D X\, g \Psi(X)\Psi(X)\Psi(X) = \int d^4x\, Y_{ijk}(\tau)\psi^i(x)\psi^j(x)\psi^k(x)$

$= \int d^4x \left(g \int d^2z\, \psi'_i(z)\psi'_j(z)\psi'_k(z)\right)\psi^i(x)\psi^j(x)\psi^k(x) =$

Modular Transformation: $\gamma: (z,\tau) \to \left(\frac{z}{c\tau+d}, \frac{a\tau+b}{c\tau+d}\right)$

$\gamma: \psi(z,\tau) \to \psi\left(\frac{z}{c\tau+d}, \frac{a\tau+b}{c\tau+d}\right) = (c\tau+d)^{1/2}\rho(\gamma)\psi(z,\tau)$

$\gamma: \psi_I(x_4) \to (c\tau+d)^{-1/2}\rho(\gamma)^{-1}\psi_I(x_4)$

4D Standard Model
Mysteries of Flavor Structure

(e.g.) [1] S. Kikuchi, et al., Phys.Rev.D 102 (2020) 10, 105010

- Origin of 3 generation

M generation chiral fermions on magnetized T^2

Modular Flavor Symmetry
Finite Modular Subgroups
$\tilde{\Gamma}_{2M}$ (e.g.) $\tilde{\Gamma}_4 \cong \tilde{S}_4 (\simeq \widetilde{\Delta}(24))$
(quadruple covering group of Γ_{2M})

ρ: representation of

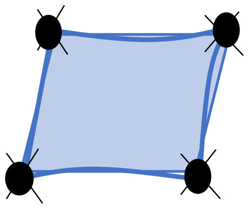
Assumption:
Flavor group, Representation, Weight
How can they be determined !

Modular Flavor Symmetry in Magnetized T^2/Z_2 model

+ Compact space
(e.g.) T^2/Z_2

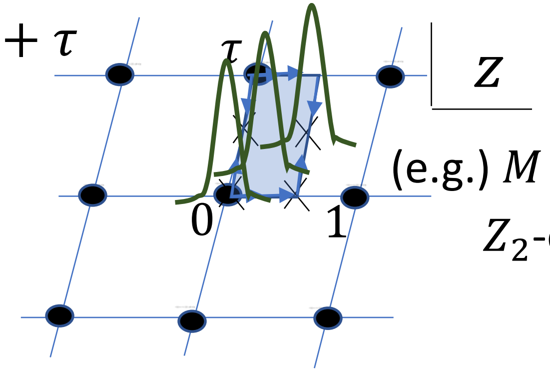
with Magnetic Flux: $(2\pi)^{-1}\int_{T^2} F = M \in 2Z$

Geometrical Structure
(e.g.) Modular symmetry



$z \sim z+1 \sim z+\tau$

$z \sim -z$



(e.g.) $M=4$

Z_2-even

Table.1: eigenmode $[(\alpha_1, \alpha_\tau) = (0,0)]$

M	2	4	6	8
T^2/Z_2 even	2	3	4	5
T^2/Z_2 odd	0	1	2	3

[3] S. Kikuchi, et al., arXiv:2101.00826

$\tilde{\Delta}(6M^2)$ $\quad \tilde{\Delta}(96)$ $\tilde{\Delta}(384)$

$\gamma: \psi_I(x_4) \rightarrow (c\tau+d)^{-1/2}\rho(\gamma)^{-1}\psi_I(x_4)$

4D Standard Model
Mysteries of Flavor Structure

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(e.g.)
3 generation chiral fermions on magnetized T^2/Z_2

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Modular Flavor Symmetry
Finite Modular Subgroups

ρ: representation of $(\tilde{\Gamma}_{2M} \supset)\tilde{\Delta}(6M^2)$ $(M=4,8)$
(quadruple covering group of $\Delta(6M^2)$)

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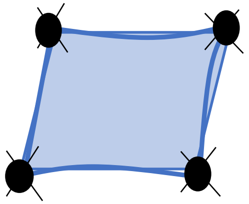

$z \sim z+1 \sim z+\tau$

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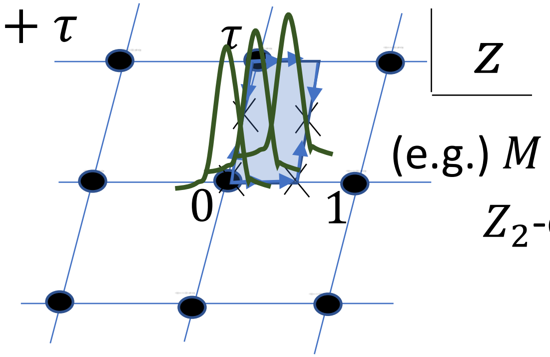

(e.g.) $M=5$
Z_2-even

Table.2: eigenmode $[(\alpha_1, \alpha_\tau) = (1/2, 1/2)]$

M	1	3	5	7
T^2/Z_2 even	0	1	2	3
T^2/Z_2 odd	1	2	3	4

[3] S. Kikuchi, et al., arXiv:2101.00826

$\Gamma_M \times Z_8$ $\quad A_5 \times Z_8$ $\quad PSL(2, Z_7) \times Z_8$

$\gamma: \psi_I(x_4) \rightarrow (c\tau+d)^{-1/2}\rho(\gamma)^{-1}\psi_I(x_4)$

4D Standard Model
Mysteries of Flavor Structure

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(e.g.)
3 generation chiral fermions on magnetized T^2/Z_2

Assumption:
Flavor group, Representation, Weight
How can they be determined !

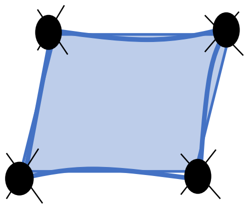
Modular Flavor Symmetry
Finite Modular Subgroups
ρ: representation of $\Gamma_M \times Z_8$ $(M=5,7)$

Modular Flavor Symmetry in Magnetized T^2/Z_2 model

+ Compact space
(e.g.) T^2/Z_2

with Magnetic Flux: $(2\pi)^{-1}\int_{T^2} F = M \in 2Z+1$

Geometrical Structure
(e.g.) Modular symmetry



$z \sim z+1 \sim z+\tau$

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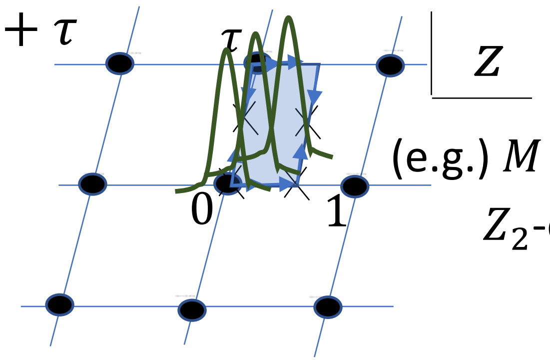

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$\Gamma_M \times Z_8$ $A_5 \times Z_8$ $PSL(2, Z_7) \times Z_8$

$\gamma: \psi_I(x_4) \to (c\tau+d)^{-1/2} \rho(\gamma)^{-1} \psi_I(x_4)$

4D Standard Model
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(e.g.)
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Anomalous?

Modular Flavor Symmetry
Finite Modular Subgroups
$\Gamma_M \times Z_8$ $(M=5,7)$

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Identification of Anomalous/Anomaly Free Symmetry

in Path Integral $Z = \int D\bar{\psi} D\psi \, e^{-S[\psi,\bar{\psi}]}$

$$\psi \to U_A \psi, \ \ \bar{\psi} \to U_A \bar{\psi} \ \Rightarrow \ D\bar{\psi} D\psi \to (\det U_A)^{-2} D\bar{\psi} D\psi$$

Modular Transformation: $\gamma: \tau \to \frac{a\tau+b}{c\tau+d}$

$\gamma: \psi_I(x_4) \to (c\tau + d)^{-1/2} \rho(\gamma)^{-1} \psi_I(x_4)$ $\quad$ $\rho(\gamma) \in$ Flavor Group G

$\det\rho(\gamma) = 1 \Rightarrow$ Anomaly Free

$\det\rho(\gamma) \neq 1 \Rightarrow$ Anomalous

[4] Y. Kariyazono, et al., Phys.Rev.D 100 (2019) 4, 045014

Identification of Anomalous/Anomaly Free Symmetry

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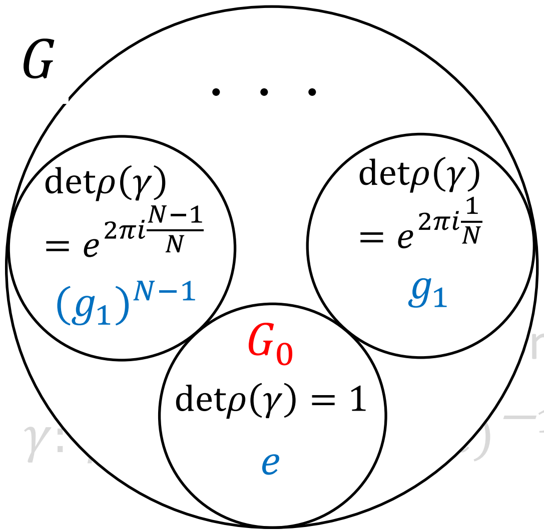
n: $\gamma : \tau \to \frac{a\tau+b}{c\tau+d}$

γ: $)^{-1/2} \rho(\gamma)^{-1} \psi_I(x_4)$



$\det\rho(a) = 1 \quad \det\rho(b) = e^{2\pi i \frac{k}{8}} \quad Z_8$

$a \quad b$

$\Cap \quad \Cap$

(e.g.)

$M = 5,7 \qquad \Gamma_M \times Z_8$ [3] S. Kikuchi, et al., arXiv:2101.00826

$M = 4,8 \qquad \tilde{\Delta}(6M^2) \simeq \Delta(3M^2) \rtimes Z_8$ (cf. $\Delta(6M^2) \simeq \Delta(3M^2) \rtimes Z_2$)

$\rho(\gamma) \in$ Flavor Group $G \ \simeq \ \underline{G_0} \rtimes \underline{Z_N}$

Anomaly Free Anomalous [5] in progress

$\det\rho(\gamma) = 1 \Rightarrow$ Anomaly Free $\Rightarrow$ $G_0 = \{g_0 \in G \,|\, \det g_0 = 1\}$ is Normal Subgroup of G $\Rightarrow G/G_0 \simeq Z_N$

$\det\rho(\gamma) \neq 1 \Rightarrow$ Anomalous

If $\exists g_1$ which satisfies $\det g_1 = e^{2\pi i \frac{1}{N}}$ is $g_1 \in Z_N$

Broken Symmetry by Non-Perturbative Effect

(non-perturbative effect)
D-brane instanton effect
↓
Majorana mass term of ν_R

$$e^{-S_{cl}} \int d^2\alpha d^2\beta \, e^{-d(\tau)_a^{ij}\alpha_i\beta_j\nu_R^a}$$
$$= \left(e^{-S_{cl}}\varepsilon_{ik}\varepsilon_{jl}d(\tau)_a^{ij}d(\tau)_b^{kl}\right)\nu_R^a\nu_R^b$$
$$= m(\tau)_{ab}\nu_R^a\nu_R^b$$
$$(\tilde{\Delta}(96) \supset) \; S'_4 \quad \tilde{\Delta}(96) \; \tilde{\Delta}(96)$$

[6] K. Hoshiya, arXiv:2103.07147

ν_R^a: 3-generation $(a = 1,2,3)$ Right-Handed Neutrinos
(Assume) from $M = 4$ Z_2-even modes
⇒ triplet of $\tilde{\Delta}(96)$

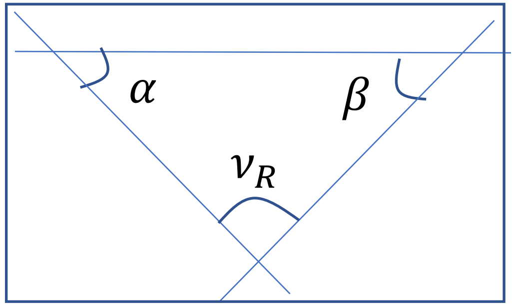
α_i, β_j: zero-modes $(i, j = 1,2)$ appeared by D-brane instanton effect

Modular Transformation: $\gamma: \tau \to \frac{a\tau+b}{c\tau+d}$

$\gamma: \psi_I(x_4) \to (c\tau+d)^{-1/2}\rho(\gamma)^{-1}\psi_I(x_4)$

[5] in progress

(T-dual) Intersecting D-brane model



D-brane instanton

Compact space

$$\tilde{\Delta}(96) \simeq \Delta(48) \rtimes Z_8$$
$$\cup \qquad \cup \qquad \cup$$
$$S'_4 \simeq \Delta'(24) \simeq \Delta(12) \rtimes Z_4 \simeq A_4 \rtimes Z_4$$

Broken Symmetry by Non-Perturbative Effect

(non-perturbative effect)
D-brane instanton effect
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Majorana mass term of ν_R

$$e^{-S_{cl}} \int d^2\alpha d^2\beta \; e^{-d(\tau)_a^{ij}\alpha_i\beta_j\nu_R^a}$$

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Modular Transformation: $\gamma: \tau \to \frac{a\tau+b}{c\tau+d}$

$$\rho_{inst}(\gamma) = \rho_\alpha(\gamma)\rho_\beta(\gamma)$$

$$\gamma: \psi_I(x_4) \to (c\tau + d)^{-1/2}\rho(\gamma)^{-1}\psi_I(x_4) \quad \Rightarrow \quad d^2\alpha d^2\beta \to (c\tau + d)^2 \det\rho_{inst}(\gamma) d^2\alpha d^2\beta$$

$$\gamma: m(\tau)_{ab}\nu_R^a\nu_R^b \to m(\tau')_{ab}\nu'^a_R\nu'^b_R = (c\tau + d)^2 \det\rho_{inst}(\gamma) m(\tau)_{a'b'}\nu_R^{a'}\nu_R^{b'}$$

[5] in progress

$$\tilde{\Delta}(96) \simeq \Delta(48) \rtimes Z_8$$

$$\cup \qquad \cup \qquad \cup$$

$$S'_4 \simeq \Delta'(24) \simeq \Delta(12) \rtimes Z_4 \simeq A_4 \rtimes Z_4$$

Broken Symmetry by Non-Perturbative Effect

(non-perturbative effect)
D-brane instanton effect

↓

Majorana mass term of ν_R

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$$= \left(e^{-S_{cl}}\varepsilon_{ik}\varepsilon_{jl}d(\tau)_a^{ij}d(\tau)_b^{kl}\right)\nu_R^a\nu_R^b$$

$$= m(\tau)_{ab}\nu_R^a\nu_R^b$$

$(\tilde{\Delta}(96) \supset)\ S'_4 \ \tilde{\Delta}(96)\ \tilde{\Delta}(96)$

ν_R^a: 3-generation $(a = 1,2,3)$ Right-Handed Neutrinos

(Assume) from $M = 4$ Z_2-even modes

$\Rightarrow$ triplet of $\tilde{\Delta}(96)$

α_i, β_j: zero-modes $(i, j = 1,2)$ appeared by D-brane instanton effect

Modular Transformation: $\gamma: \tau \to \frac{a\tau+b}{c\tau+d}$

$$\rho_{inst}(\gamma) = \rho_\alpha(\gamma)\rho_\beta(\gamma)$$

$$\gamma: \psi_I(x_4) \to (c\tau+d)^{-1/2}\rho(\gamma)^{-1}\psi_I(x_4) \quad \Rightarrow \quad d^2\alpha d^2\beta \to (c\tau+d)^2 \det\rho_{inst}(\gamma) d^2\alpha d^2\beta$$

$$\gamma: m(\tau)_{ab}\nu_R^a\nu_R^b \to m(\tau')_{ab}\nu'^a_R\nu'^b_R = (c\tau+d)^2\det\rho_{inst}(\gamma) m(\tau)_{a'b'}\nu_R^{a'}\nu_R^{b'}$$

[5] in progress

↓

$\det\rho_{inst}(a) = 1$ $\quad \det\rho_{inst}(b) = -1$ $\quad Z_2$ is broken

$$\begin{array}{ccccc} & & a & & b \\ & & \in & & \in \\ \tilde{\Delta}(96) & \simeq & \Delta(48) & \rtimes & Z_8 \\ \cup & & \cup & & \cup \\ S'_4 \simeq \Delta'(24) & \simeq & \Delta(12) & \rtimes & Z_4 \simeq A_4 \rtimes Z_4 \end{array}$$

Broken Symmetry by Non-Perturbative Effect

(non-perturbative effect)
D-brane instanton effect

$e^{-S_{cl}} \int d^2\alpha d^2\beta \, e^{-d(\tau)_a^{ij}\alpha_i\beta_j\nu_R^a}$

$= \left(e^{-S_{cl}}\varepsilon_{ik}\varepsilon_{jl}d(\tau)_a^{ij}d(\tau)_b^{kl}\right)\nu_R^a\nu_R^b$

Majorana mass term of ν_R $= m(\tau)_{ab}\nu_R^a\nu_R^b$

ν_R^a: 3-generation $(a = 1,2,3)$ Right-Handed Neutrinos

(Assume) from $M = 4$ Z_2-even modes

$\Rightarrow$ triplet of $\widetilde{\Delta}(96)$

α_i, β_j: zero-modes $(i,j = 1,2)$ appeared by D-brane instanton effect

Z_2 is broken by anomaly

$(\widetilde{\Delta}(96) \supset) \, S'_4 \;\; \widetilde{\Delta}(96) \;\; \widetilde{\Delta}(96)$

$(\Delta'(96) \supset) \, S_4 \;\; \Delta'(96) \;\; \Delta'(96)$ $\Delta'(96)$ invariant

Modular Transformation: $\gamma: \tau \to \frac{a\tau+b}{c\tau+d}$

$\rho_{inst}(\gamma) = \rho_\alpha(\gamma)\rho_\beta(\gamma)$

$\gamma: \psi_I(x_4) \to (c\tau+d)^{-1/2}\rho(\gamma)^{-1}\psi_I(x_4) \;\; \Rightarrow \;\; d^2\alpha d^2\beta \to (c\tau+d)^2 \det\rho_{inst}(\gamma) d^2\alpha d^2\beta$

$\gamma: m(\tau)_{ab}\nu_R^a\nu_R^b \to m(\tau')_{ab}\nu'^a_R\nu'^b_R = (c\tau+d)^2 \det\rho_{inst}(\gamma) m(\tau)_{a'b'}\nu_R^{a'}\nu_R^{b'}$

[5] in progress

$\det\rho_{inst}(a) = 1 \quad \det\rho_{inst}(b^2) = 1$

$$
\begin{array}{ccccc}
 & & a & & b^2 \\
 & & \Cap & & \Cap \\
\Delta'(96) & \simeq & \Delta(48) & \rtimes & Z_4 \\
\cup & & \cup & & \cup \\
S_4 \simeq \Delta(24) & \simeq & \Delta(12) & \rtimes & Z_2 \simeq A_4 \rtimes Z_2
\end{array}
$$

Conclusion

We have studied modular symmetry in magnetized T^2/Z_2 orbifold models.

- 3 generation chiral fermions from magnetized T^2/Z_2 orbifold transform as modular $\tilde{\Delta}(6M^2) \simeq \Delta(3M^2) \rtimes Z_8$ $(M = 4,8)$ triplets or $\Gamma_M \times Z_8$ $(M = 5,7)$ triplets with modular weight $-k_l=-1/2$.

Conclusion

We have studied modular symmetry in magnetized T^2/Z_2 orbifold models.

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Modular symmetry can be anomalous, in general.

- Anomaly free transformation generate normal subgroup G_0 of modular flavor group G and the residue class group G/G_0 is isomorphic to Z_N.

⇒ There are cases when G can be decomposed as semidirect product of anomaly free G_0 and anomalous Z_N (i.e.) $G \simeq G_0 \rtimes Z_N$

Anomaly Free Anomalous

- A part of anomalous Z_N symmetry is actually broken by non-perturbative effect (e.g.) Majorana mass term by D-brane instanton