## Probing phase transition keeping the symmetry

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[In preparation]

# Introduction

 $\star$  The shape of Higgs potential is still undetermined.

 $V_{SM}(\phi) = \mu^2 |\phi|^2 + \lambda |\phi|^4$  (The SM case)



It is important to explore the shape of the Higgs potential.

★ The dynamics of electroweak phase transition (EWPT) is governed by the shape of the Higgs potential.

If the model realize the first-order EWPT,

- Baryon asymmetry of the universe may be explained by electroweak baryogenesis scenario.
- The model may be tested by the gravitational wave (GW) observation experiment.

### How can we realize the first-order EWPT?

# **First-order EWPT**

- ★ First-order EWPT is that the electroweak symmetry breaking is produced by the bubble dynamics.
- $\star$  Effective potential for first-order EWPT

$$V_{\rm eff}(\varphi, T) = D(T^2 - T_0^2)\varphi^2 + (e - ET)\varphi^3 + \frac{\lambda(T)}{4}\varphi^4$$

(Under high temperature approximation)

e : Mixing effects at the tree level

*E* : Loop effects of bosons

The SM cannot realize the first-order EWPT.

[Y. Aoki, F. Csikor, Z. Fodor and A. Ukawa, Phys. Rev. D 60, 013001 (1999)]

The model beyond the SM, especially the extended Higgs model, can realize the first-order EWPT.



Maya Fishbach's slide



# **Motivation**

#### $\star$ Multi-step phase transition (PT) could be produced in some extended Higgs models.

[D. Land and E. D. Carlson, Phys. Lett. B 292 (1992), 107, A. Hammerschmitt, J. Kripfganz and M. G. Schmidt, Z. Phys. C 64 (1994), 105]



*h*: The SM Higgs boson *φ*: Additional scalar boson

[M. J. Ramsey-Musolf, JHEP 09 (2020), 179]

★ A symmetry in the Higgs potential at zero temperature can be assured, that depends on the model parameters.

$$V = \mu_h^2 h^2 + \mu_\phi^2 \phi^2 + \lambda_h h^4 + \lambda_\phi \phi^4 + \lambda_{h\phi} h^2 \phi^2 \qquad \qquad h \to h, \quad \phi \to -\phi$$

In this potential case, the potential keep  $Z_2$  symmetry when the PT realizes by (a) and (b).  $\rightarrow$  We could explore a symmetry in the potential by the sign of (a) and (B) PT...

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In our work, we focus on the extended Higgs model with  $Z_2$  symmetry and an isospin N-plet scalar field and discuss the possibility of the PT, especially (b).

#### model parameters.

$$V = \mu_h^2 h^2 + \mu_\phi^2 \phi^2 + \lambda_h h^4 + \lambda_\phi \phi^4 + \lambda_{h\phi} h^2 \phi^2 \qquad \qquad h \to h, \quad \phi \to -\phi$$

In this potential case, the potential keep  $Z_2$  symmetry when the PT realizes by (a) and (b).  $\rightarrow$  We could explore a symmetry in the potential by the sign of (a) and (B) PT...

## **Extended Higgs model**





 $V(\Phi_1, \Phi_2) = -\mu_1^2 |\Phi_1|^2 - \mu_2^2 |\Phi_2|^2 + \lambda_1 |\Phi_1|^4 + \lambda_2 |\Phi_2|^4 + \lambda_{12} |\Phi_1|^2 |\Phi_2|^2$ 

$$\Phi_1 = \left( \mathbf{G}^{\pm}, \frac{1}{\sqrt{2}} \left( \mathbf{h} + i\mathbf{G}^0 \right) \right)^T$$

$$\Phi_1 \to \Phi_1, \quad \Phi_2 \to -\Phi_2 \qquad \Phi_2 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \phi^1 + i\phi^{1,i} \\ \phi^2 + i\phi^{2,i} \\ \vdots \\ \phi^N + i\phi^{N,i} \end{array} \right)$$

 $\star$  Stationary points of the potential

$$\frac{\partial V(\langle \Phi_1 \rangle, \langle \Phi_2 \rangle)}{\partial \langle \Phi_1 \rangle} = \frac{\partial V(\langle \Phi_1 \rangle, \langle \Phi_2 \rangle)}{\partial \langle \Phi_2 \rangle} = 0$$



The condition of minimum points at red point could be obtained by Hesse matrix analysis.

## **Extended Higgs model**

 $\star$  Tree-level results for Hesse matrix

 $\star$ 

$$det[\mathcal{H}_{red}] = \mu_1^2 \left( \frac{\lambda_{12} \mu_1^2}{\lambda_1} - 2\mu_2^2 \right), \quad det[\mathcal{H}_{mag}] = \mu_2^2 \left( \frac{\lambda_{12} \mu_2^2}{\lambda_2} - 2\mu_1^2 \right)$$
$$det[\mathcal{H}_{blue}] = -4 \frac{\left( \lambda_{12} \mu_2^2 - 2\lambda_2 \mu_1^2 \right) \left( \lambda_{12} \mu_1^2 - 2\lambda_1 \mu_2^2 \right)}{\lambda_{12}^2 - 4\lambda_1 \lambda_2},$$

$$\frac{\lambda_{12}\mu_1^2}{\lambda_1} > 2\mu_2^2, \quad \frac{\lambda_{12}\mu_2^2}{\lambda_2} > 2\mu_1^2. \quad \text{(B)}$$

(Blue point is a saddle point)

Pink and red points are minimum values. ← These are the conditions of stability of a potential.



The condition of height of potential at tree-level

$$V(0, \sqrt{\mu_2^2/\lambda_2}) = -\mu_2^4/4\lambda_2, \quad V(\sqrt{\mu_1^2/\lambda_1}, 0) = -\mu_1^4/4\lambda_1$$

$$\mu_2^4/\lambda_2 < \mu_1^4/\lambda$$

$$-\mu_2^4/4\lambda_2 \text{ (magenta)} < \frac{\lambda_1\mu_2^4 + \lambda_2\mu_1^4 - \lambda_{12}\mu_1^2\mu_2^2}{\lambda_{12}^2 - 4\lambda_1\lambda_2} \text{ (blue)}, \quad \rightarrow \quad 0 < \frac{1}{4\lambda_2}(\lambda_{12}\mu_2^2 - 2\lambda_2\mu_1^2) + \lambda_1\lambda_2 + \lambda_2\lambda_2 + \lambda_2\lambda$$

### **Extended Higgs model**



### The conditions

 $\star$  Model parameters

 $(\mu_1^2,\mu_2^2,\lambda_1,\lambda_2,\lambda_{12})\to (v,v_s,m_h,\lambda_2,m_{\Phi_2}^2).$ 

#### $\star$ Three conditions at tree-level



 $\langle \Phi_{\hat{2}} \rangle$ 



### The conditions at one-loop level

 $\star$  The results with one-loop level

### In the cyan parameter region,

•The height of potential at red point is the lowest.

$$V_{eff}(v, 0) < V_{eff}(0, v_{S})$$
 ( $V_{eff} = V_{0} + V_{1-loop}$ )

•Red and magenta points are minimum values.

Numerical conditions

$$V_{eff}(v, 0) < V_{eff}(v, \pm 10^{-1}), V_{eff}(v \pm 10^{-1}, 0)$$
$$V_{eff}(0, v_S) < V_{eff}(\pm 10^{-1}, v_S), V_{eff}(0, v_S \pm 10^{-1})$$

•Condition of perturbative expansion

$$\lambda_i < (4\pi)^{1/2}$$



800 700  $v_s = 100 \text{ GeV}, \lambda_i < \sqrt{4 \pi}, T = 0$ 600  $Y_{\Phi_2} = 1$  and  $I_{\Phi_2} = 1/2$ [GeV] 500 **ਜੂ 400** 300 200 0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 λz

## The conditions at one-loop level

 $\star$  The results with one-loop level

### In the cyan parameter region,

E[]

• The height of potential at red point is the lowest.

 $V_{eff}(v, 0) < V_{eff}(0, v_S)$  ( $V_{eff} = V_0 + V_{1-loop}$ )

### We cannot clarify the possibility of the pattern of PT by the zero temperature analysis.

$$V_{eff}(0, v_{S}) < V_{eff}(\pm 10^{-1}, v_{S}), V_{eff}(0, v_{S} \pm 10^{-1})$$
•Condition of perturbative expansion
$$\lambda_{i} < (4\pi)^{1/2}$$
300
200
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5
$$\lambda_{2}$$

800

700



 $v_s = 100 \text{ GeV}, \lambda_i < \sqrt{4 \pi}, T = 0$ 

### The condition of PT



The PT occurs through Path I, when  $T_{\Phi} > T_{EW}$ .

[D. Land and E. D. Carlson, Phys. Lett. B 292 (1992), 107, M. J. Ramsey-Musolf, JHEP 09 (2020), 179]

Especially, the multi-step PT for Path I could be produced by large  $\lambda_2$  and  $v_{s}$ .

We will show the results without high temperature approximation.

### **Numerical results**

#### $\star$ Without high temperature approximation



Parameter region where PT for Path II can occur.

We compared the critical temperatures between (Green and red) and (Green and Magenta).

### Numerical results





There is a barrier between magenta and red points at zero temperature, therefore we need to make sure that the PT has been completed at the current university.

We used "Anybubble" to analyze multi-step PT.

[A. Masoumi, K. D. Olum and B. Shlaer, JCAP 01 (2017), 051]

## Gravitational wave from PT

- $\star$  The gravitational wave could be produced from first-order PT.
- ★ We estimate the S/N ratio for measurements of GW spectrum.





### **Two first-order PTs**

[GeV]

Hu

 $\star$  Both PTs for (Green to magenta) and (magenta to red) may be first-order.

Potential along  $<\Phi_{2}>$  under high temperature approximation.

 $V_{eff}\left(0,\left<\Phi_{2}\right>,T\right)=\Sigma_{\Phi_{2}}T^{2}\left<\Phi_{2}\right>^{2}-E_{2}T\left<\Phi_{2}\right>^{3}+\lambda_{2}\left<\Phi_{2}\right>^{4}$ 



First-order PT at first step can be realized by Small  $\lambda_2$ .

#### But...

$$\begin{split} T_{EW}^2 &\sim \frac{12m_h^2}{3g'^2/2 + 9g^2/2 + 6y_t^2 + 2(3+n_1)\lambda_1 + (1+n_2)\lambda_{12}} \\ T_{\phi}^2 &\sim \frac{24\lambda_2 v_s^2}{3Y_{\Phi_2}^2 g'^2/2 + 3(Y_{\Phi_2}^2 + 2I_W^2)g^2/2 + 2(3+n_2)\lambda_2 + (1+n_1)\lambda_{12}}, \end{split}$$

The PT for Path I could be realized by large  $\lambda_2$  value or large  $v_s$ . At a blue star mark, two first-order PTs can be realized, however

the spectra couldn't be tested at future experiments.

GW spectrum: C. Caprini, et al., J.Cosmol. Astropart. Phys. 1604(04)(2016) 01.



 $\langle \Phi_2 \rangle$ 

[Qing-Hong Cao, Katsuya Hashino, Xu-Xiang Li<sup>,</sup> Jiang-Hao Yu]

### **Two first-order PTs**

 $\star$  Both PTs for (Green to magenta) and (magenta to red) may be first-order.

Potential along  $<\Phi_{2}>$  under high temperature approximation.

 $\frac{\langle \Phi_2 \rangle_C}{T_C} = \frac{E_2}{2\lambda_2}$  First-order PT at first step can be realized by Small  $\lambda_2$ .

But...

 $T_{EW}^{2} \sim \frac{12m_{h}^{2}}{3g'^{2}/2 + 9g^{2}/2 + 6y_{t}^{2} + 2(3 + n_{1})\lambda_{1} + (1 + n_{2})\lambda_{12}}$   $T_{\phi}^{2} \sim \frac{24\lambda_{2}v_{s}^{2}}{3Y_{\phi_{2}}^{2}g'^{2}/2 + 3(Y_{\phi_{2}}^{2} + 2I_{W}^{2})g^{2}/2 + 2(3 + n_{2})\lambda_{2} + (1 + n_{1})\lambda_{12}} \lambda_{2}v_{s}^{4} < m_{h}^{2}v^{2}$ 

The PT for Path I could be realized by large  $\lambda_2$  value or large  $v_s$ .

At least, one peak of Gw spectrum can be detected by future experiments: S/N (LISA) ~ 58.0, S/N (BBO) ~ 238.

GW spectrum: C. Caprini, et al., J.Cosmol. Astropart. Phys. 1604(04)(2016) 01.





[Qing-Hong Cao, Katsuya Hashino, Xu-Xiang Li<sup>,</sup> Jiang-Hao Yu]

## Summary

- ★ The shape of Higgs potential is still undetermined.
   The Higgs potential may have the symmetry.
- ★ In this time, we discussed the possibility of phase transitions keeping the symmetry at zero temperature.
- ★ As an example, we focused on the extended Higgs model with  $Z_2$  symmetry and an isospin N-plet scalar field and discuss the possibility of the phase transitions.
- ★ Especially, the multi-step phase transition could be realized by large  $\lambda_2$  value.
- ★ The potential being able to realize such phase transitions could be tested by the gravitational wave observastion.





### Backup

### **Isospin dependence**

- $\star$  We examine the isospin dependence for the possibility of the path of PT.
- ★ For large isospin of  $\Phi_2$ , the ratio of  $T_2$  and  $T_{EW}$  is

$$\frac{T_2^2}{T_{EW}^2} \sim \frac{2\lambda_2 (m_H^2 + \lambda_2 v_2^2) v_2^2 \left(1 + \frac{3I_{\Phi_2}\lambda_2}{\pi^2} \left(\ln\frac{T_2^2 \alpha_B}{Q^2} - \frac{3}{2}\right)\right)}{3I_{\Phi_2} m_W^2 m_h^2}$$

When the value of  $\lambda_2 I_{\phi_2}$  is large, multi-step PT could be generated.



[Qing-Hong Cao, Katsuya Hashino, Xu-Xiang Li Jiang-Hao Yu]



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When the value of  $\lambda_2 I_{\phi_2}$  is large, multi-step PT could be generated.

**★** For heavy  $m_H$ , the ratio is

$$\frac{T_2^2}{T_{EW}^2} \sim \frac{\lambda_2 v_2^2 (1 + 2I_{\Phi_2}) + \frac{3m_h^2}{2\pi^2} \left(\ln \frac{T_2^2 \alpha_B}{Q^2} - \frac{3}{2}\right)}{m_h^2}$$

The mass parameter  $m_H$  does not much affect the possibility of the path of PT.



[Qing-Hong Cao, Katsuya Hashino, Xu-Xiang Li<sup>,</sup> Jiang-Hao Yu]



### Gravitational wave







### 重力波の起源

- ・壁の衝突
- ・プラズマの音波 ・プラズマの乱流

▶ 真空泡の核生成率 $\Gamma$ :  $\Gamma(T) \simeq T^4 e^{-\frac{S_3(T)}{T}}$ 

> 3次元ユークリッド作用S<sub>3</sub>: 
$$S_3(T) = \int dr^3 \left\{ \frac{1}{2} \left( \vec{\nabla} \varphi \right)^2 + V_{\text{eff}}(\varphi, T) \right\}$$

▶ 相転移温度T<sub>t</sub>:   

$$\frac{\Gamma}{H^4}\Big|_{T=T_t} \simeq 1 \longrightarrow \frac{S_3(T_t)}{T_t} = 4\ln(T_t/H_t) \simeq 140$$

$$\alpha = \frac{\epsilon(T_t)}{\rho_{rad}(T_t)}, \quad \beta \simeq \frac{1}{\Gamma} \frac{d\Gamma}{dT} \qquad$$
  
潜熱:   

$$\epsilon(T) = -V_{\text{eff}}(\varphi_B(T), T) + T \frac{\partial V_{\text{eff}}(\varphi_B(T), T)}{\partial T}$$
  
輻射エネルギー密度:   

$$\rho_{rad}$$

## 重力波スペクトル [C. Caprini et al., JCAP 1604, no. 04, 001 (2016)]

壁の衝突

$$\widetilde{\Omega}_{\rm sw}h^2 \simeq 2.65 \times 10^{-6} v_b \widetilde{\beta}^{-1} \left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_\star^t}\right)^{1/5}$$

$$\widetilde{\epsilon} = 1.0 \pm 10^{-5} {\rm M}^{-1} \widetilde{\epsilon} \left(-T_t\right) \left(g_\star^t\right)^{1/6}$$

 $f_{\rm sw} \simeq 1.9 \times 10^{-5} \, {\rm Hz} \frac{-}{v_b} \beta \left( \frac{-}{100 \, {\rm GeV}} \right) \left( \frac{\omega}{100} \right)$ 

プラズマの乱流

$$\begin{split} \widetilde{\Omega}_{\rm turb} h^2 &\simeq 3.35 \times 10^{-4} v_b \widetilde{\beta}^{-1} \left(\frac{\epsilon \kappa_v \alpha}{1+\alpha}\right)^{3/2} \left(\frac{100}{g_\star^t}\right)^{1/3} \\ \widetilde{f}_{\rm turb} &\simeq 2.7 \times 10^{-5} \ {\rm Hz} \frac{1}{v_b} \widetilde{\beta} \left(\frac{T_t}{100 \ {\rm GeV}}\right) \left(\frac{g_\star^t}{100}\right)^{1/6} \end{split}$$

 $K_{\phi}, K_{\nu}, \varepsilon$ : efficiency factors  $v_h$ : wall velocity

### **Phase transition**

The bubble nucleation rate per unit volume per unit time:

$$\left(S_3=\int d^3r\left[rac{1}{2}(ec{
abla}arphi_b)^2+V_{ ext{eff}}(arphi_b,T)
ight]
ight)$$

• The bounce solution  $\phi_{b}$  is obtained by equation of motion.

$$\frac{d^2\varphi_b}{dr^2} + \frac{2}{r}\frac{d\varphi_b}{dr} - \frac{\partial V_{\text{eff}}}{\partial\varphi_b} = 0 \qquad \qquad \left( \begin{array}{cc} \text{Boundary} & \left. \frac{d\varphi_b}{dr} \right|_{r=0} = 0, \quad \lim_{r \to \infty} \varphi_b = 0 \end{array} \right)$$

### The spectrum of the stochastic GWs

[C. Caprini, et al., J. Cosmol. Astropart. Phys. 1604 (04) (2016) 001.]

#### Compression wave of plasma

$$\begin{split} \widetilde{\Omega}_{\rm sw}h^2 &\simeq 2.65 \times 10^{-6} v_b \widetilde{\beta}^{-1} \left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_\star^t}\right)^{1/3} & \text{the peak of energy dencity} \\ \widetilde{f}_{\rm sw} &\simeq 1.9 \times 10^{-5} \; {\rm Hz} \frac{1}{v_b} \widetilde{\beta} \left(\frac{T_t}{100 \; {\rm GeV}}\right) \left(\frac{g_\star^t}{100}\right)^{1/6} & \text{the peak frequency} \\ \Omega_{\rm sw}(f)h^2 &= \widetilde{\Omega}_{\rm sw}h^2 \times (f/\widetilde{f}_{\rm sw})^3 \left(\frac{7}{4+3(f/\widetilde{f}_{\rm sw})^2}\right)^{7/2} \end{split}$$

In this talk, we use only the GW spectrum from compression wave pf plasma.

Collision of wall

#### Plasma turbulence

$$\begin{split} \widetilde{\Omega}_{\rm env}h^2 &\simeq 1.67 \times 10^{-5} \times \left(\frac{0.11 v_b^3}{0.42 + v_b^2}\right) \widetilde{\beta}^{-2} \left(\frac{\kappa_{\varphi} \alpha}{1 + \alpha}\right)^2 \left(\frac{100}{g_{\star}^t}\right)^{1/3} \\ \widetilde{f}_{\rm env} &\simeq 1.65 \times 10^{-5} \; {\rm Hz} \times \left(\frac{0.62}{1.8 - 0.1 v_b + v_b^2}\right) \widetilde{\beta} \left(\frac{T_t}{100 \; {\rm GeV}}\right) \left(\frac{g_{\star}^t}{100}\right)^{1/6} \\ \end{array} \\ \widetilde{f}_{\rm turb} &\simeq 2.7 \times 10^{-5} \; {\rm Hz} \frac{1}{v_b} \widetilde{\beta} \left(\frac{T_t}{100 \; {\rm GeV}}\right) \left(\frac{g_{\star}^t}{100}\right)^{1/6} \end{split}$$

[J.R.Espinosa, T.Konstandin, J.M.No and G. Servant, JCAP 1006,028 (2010)]

### **Efficiency factors**

[J. R. Espinosa, T. Konstandin, J. M. No and G. Servant, JCAP 1006, 028 (2010)]



### 電弱一次相転移由来の重力波



次世代重力波干渉計のLISAやDECIGOで一次相転移由来の重力波を将来 的に観測できる可能性がある

## **GW** spectrum from 1stOPT



$$S_3 \equiv \int d^3r \left[ \frac{1}{2} (\vec{\nabla}\varphi)^2 + V_{\text{eff}}(\varphi, T) \right]$$

• The bubble nucleation rate per unit volume per unit time:  $\Gamma(T) \simeq T^4 e^{-\frac{S_3(T)}{T}}$ 

• Transition temperature  $T_t$ :  $\Gamma/H^4|_{T=T_t} = 1$  (S<sub>3</sub>: the three H: the Hubble (The temperature at which phase transition starts)

(S<sub>3</sub> : the three dimensional Euclidean action H : the Hubble parameter)

• The GW spectrum is characterized by α and β :  $\Omega_{\rm GW} \propto \left(\frac{H}{\beta}\right)^n \left(\frac{\kappa \alpha}{1+\alpha}\right)^m$ 

"Bubble collision(Envelope approximation) n=2, m=2" [M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994)]

 $\alpha \approx$  Normalized latent heat released by PT,  $\beta \approx 1/(\text{The duration of PT})$ 

$$\alpha = \frac{\epsilon(T_t)}{\rho_{rad}(T_t)}, \quad \beta \simeq \frac{1}{\Gamma} \frac{d\Gamma}{dT} \qquad \text{Latent heat} : \epsilon(T) = -V_{\text{eff}}(\varphi_B(T), T) + T \frac{\partial V_{\text{eff}}(\varphi_B(T), T)}{\partial T}$$
Radiative energy dencity :  $\rho_{rad}$